ENHANCED DAMPING FOR CAPILLARY BRIDGE OSCILLATION USING VELOCITY FEEDBACK

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The stability of cylindrical liquid bridges in reduced gravity is affected by ambient vibrations of the spacecraft. Such vibrations are expected to excite capillary modes of the bridge. The lowest-order unstable mode is particularly susceptible to vibration as the length of the bridge approaches the stability limit. This low-order mode is known as the (2,0) mode and is an axisymmetric varicose mode of one wavelength in the axial direction. In this work, an optical system is used to detect the (2,0)-mode amplitude. The derivative of the error signal produced by this detector is used to produce the appropriate voltages on a pair of ring electrodes which are concentric with the bridge. A mode-coupled Maxwell stress profile is thus generated in proportional to the modal velocity. Depending on the sign of the gain, the damping of the capillary oscillation can be either increased or decreased. This effect has been demonstrated in Plateau-tank experiments. Increasing the damping of the capillary modes on free liquid surfaces in space could be beneficial for containerless processing and other novel technologies. [work supported by NASA]

Enhanced damping for capillary bridge oscillation using velocity feedback

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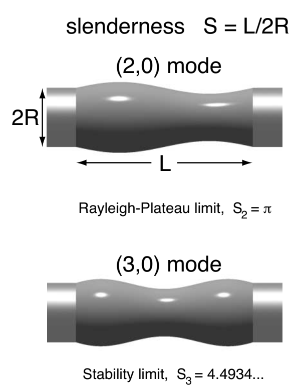
Abstract

The stability of cylindrical liquid bridges in reduced gravity is affected by ambient vibrations of the spacecraft. Such vibrations are expected to excite capillary modes of the bridge. The lowest-order unstable mode is particularly susceptible to vibration as the length of the bridge approaches the stability limit. This low-order mode is known as the (2,0) mode and is an axisymmetric varicose mode of one wavelength in the axial direction. In this work, an optical system is used to detect the (2,0)-mode amplitude. The derivative of the error signal produced by this detector is used to produce the appropriate voltages on a pair of ring electrodes which are concentric with the bridge. A mode-coupled Maxwell stress profile is thus generated in proportion to the modal velocity. Depending on the sign of the gain, the damping of the capillary oscillation can be either increased or decreased. This effect has been demonstrated in Plateau-tank experiments. Increasing the damping of the capillary modes on free liquid surfaces in space could be beneficial for containerless processing and other novel technologies.

1 Introduction

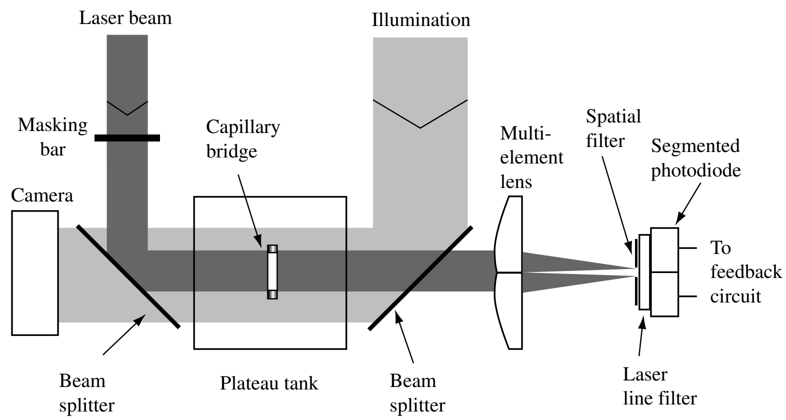
Various capillary modes of liquid bridges are susceptible to vibration in a zero gravity environment. Under zero gravity, in the absence of any stabilization method, the (2,0) mode shown in the graph below is the first mode to become unstable and breaks if the slenderness $S = L/2R$ exceeds π. This is known as Rayleigh-Plateau limit.



When the vibration frequency of the environment is near the natural frequency of a capillary bridge mode, mode-coupled electrostatic stress in proportion to the modal amplitude can be used to shift the frequency of the mode higher, and at same time the electrostatic stress in proportion to the modal velocity can be used to further enhance the stability of the bridge. This is useful for reducing the effect of g-jitter.

2 Setup

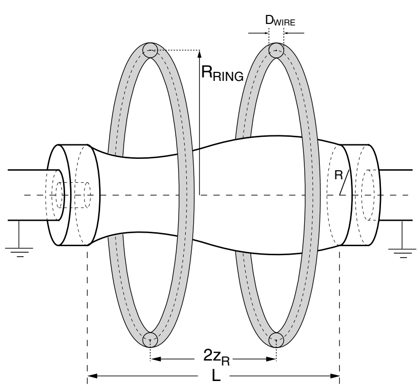
2.1 System configuration

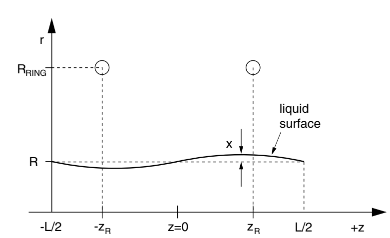


As shown in the above illustration, the laser beam illuminates the bridge which has a composition of 52 wt% CsCl and 48 wt % H_2O. The tank liquid is a 3M product, HFE 7500 which has the same density as the bridge liquid so that a weightless condition is simulated. A multi-element lens is used to focus the laser beam onto the segmented photo diode, which detects the deformation of the bridge and generates a signal to the feedback circuit which will generate the proper voltage (in proportion to the square root of the modal velocity) on the ring.

The bridge is horizontal in the Plateau tank, and this is the top view.

2.2 Bridge profile





The left figure above shows a grounded bridge with two concentric ring electrodes. In this figure the bridge has a (2,0) mode shape. Since the voltage on the ring is proportional to the square root of the modal velocity, $(dx/dt)^{1/2}$, so is the electric field. Note that the stress is proportional to the square of the electric field, therefore, the generated stress on the surface of the bridge is proportional to the modal velocity dx/dt, which is the derivative of the modal amplitude, shown as x in the above graph.

3 Simple model and model prediction

The bridge is similar to a driven, damped harmonic oscillator. The feedback force can be expressed as: $F_{fb} = -R^2 \left[Gx(t-\tau) + G_v \frac{dx}{dt}|_{t-\tau_v} \right]$. where G, G_v are the modal amplitude gain and velocity gain. Therefore the equation of motion is

$$\begin{aligned} m_0 \frac{d^2x}{dt^2} = & - \quad k_0 x(t) - \gamma_0 \frac{dx}{dt} - R^2 \left[Gx(t-\tau) + G_v \frac{dx}{dt}|_{t-\tau_v} \right] - \\ & - \quad \alpha\sqrt{2} \int_{-\infty}^{t} \frac{1}{[\pi(t-t')]^{1/2}} \frac{d^2x}{dt'^2} dt', \end{aligned}$$

where $x(t)$ is instantaneous modal amplitude. For the eigenmode solution $x(t) = x_0 e^{i\Omega t}$, the characteristic equation is

$$\begin{aligned} (k_0 + R^2 G) \quad & - \quad \Omega^2 \left(m_0 + \frac{1}{2} R^2 G \tau^2 - R^2 G_v \tau_v \right) + \alpha i \left(1+i\right) \Omega^{3/2} \\ & + \quad i\Omega \left(\gamma_0 - R^2 G\tau + R^2 G_v \right) = 0. \end{aligned}$$

This equation can be rewritten as

$$k_e - \Omega^2 m_e + \alpha i \left(1+i\right) \Omega^{3/2} + i\Omega\gamma_e = 0$$

where k_e, γ_e are the effective spring constant and damping rate respectively,

$$\begin{aligned} k_e &= k_0 + R^2 G, \\ k_0 &\propto \left[(\frac{\pi}{S})^2 - 1 \right], \\ \gamma_e &= \gamma_0 - R^2 G\tau + R^2 G_v, \\ m_e &= m_0 + \frac{1}{2} R^2 G \tau^2 - R^2 G_v \tau_v. \end{aligned}$$

Inspection of the above equations suggests that:

- When the slenderness of the bridge is larger than π, k_0 becomes negative, thus the bridge is unstable, but by introducing amplitude gain G, k_e can be made positive, the bridge can therefore be stabilized.

- Increasing the velocity gain can increase the damping rate γ_e linearly.

4 Experimental procedure

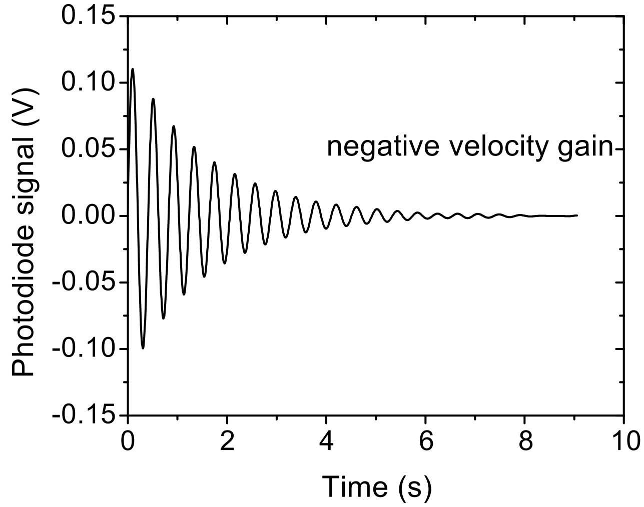
- Check the density match between bridge liquid and tank liquid,deploy the bridge

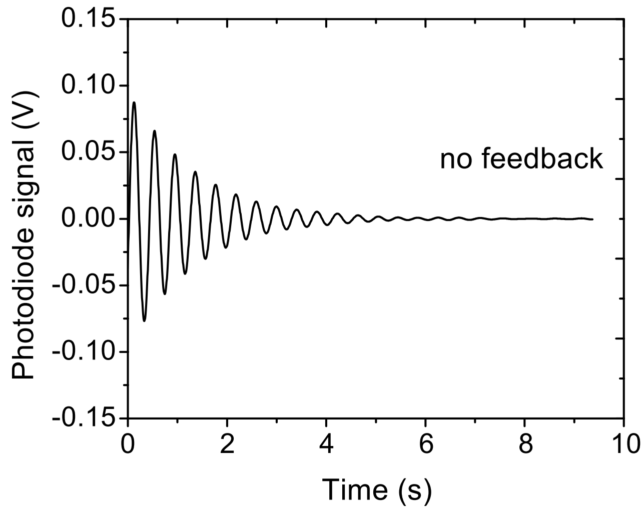
- Excite the bridge with a 20 cycle sine wave burst

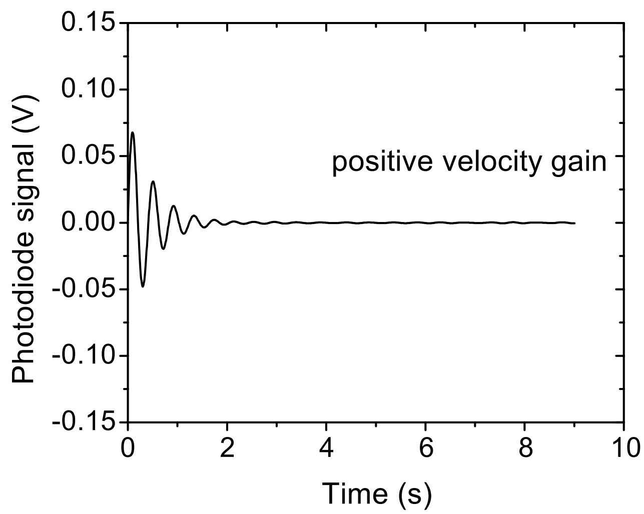
- Record the P.D signal, fit decay to Duffing equation to get $\Omega = \omega + i\gamma$, where ω is the frequency and γ the damping rate.

$$\frac{d^2x}{dt^2} + (\omega^2 - ax^2)x \quad = \quad -2\gamma \frac{dx}{dt},$$

5 Experiment results







This figure shows the qualitative effect of velocity feedback. One can see that positive velocity gain can enhance the damping while negative velocity gain decreases the damping.

The experimental results also show that

- The velocity gain doesn't affect the frequency a lot.
- Increasing the velocity gain can increase the damping rate.
- Damping increases linearly with the velocity gain, after the gain correction factor (due to non-ideal feedback electronics) is taken into account.
- when the velocity gain becomes sufficiently negative, the quality factor or the damping rate can become negative, this will cause the naturally stable bridge to break up.

6 Conclusions

- Enhanced damping of the axisymmetric (2,0) capillary mode is demonstrated by applying mode-coupled electrostatic Maxwell stress that is proportional to the modal velocity.
- Damping increases linearly with velocity gain as predicted from the model.
- Amplitude feedback shifts the natural frequency higher and has been used for bridge stabilization. Amplitude and velocity feedback can be used together to further enhance the stability.

Acknowledgment

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