

CONVECTION IN ICE I WITH COMPOSITE NEWTONIAN/NON-NEWTONIAN RHEOLOGY: APPLICATION TO THE ICY GALILEAN SATELLITES. Amy C. Barr^{1,2} and Robert T. Pappalardo, Laboratory for Atmospheric and Space Physics, University of Colorado Boulder, CO 80309 (robert.pappalardo@colorado.edu); ² now at Department of Earth and Planetary Sciences, Washington University, Saint Louis, MO 63130 (amyb@levee.wustl.edu)

Introduction: Ice I exhibits a complex rheology at temperature and pressure conditions appropriate for the interiors of the outer ice I shells of Europa, Ganymede, and Callisto. We use numerical methods to determine the conditions required to trigger convection in an ice I shell with a stress-, temperature-, and grain-size-dependent rheology measured in laboratory experiments by *Goldsby and Kohlstedt* [2001] (henceforth GK2001). Triggering convection from an initially conductive ice shell with a non-Newtonian rheology for ice I requires that a finite-amplitude temperature perturbation be issued to the ice shell [2]. Here, we characterize the amplitude and wavelength of temperature perturbation required to initiate convection in the outer ice I shells of Europa, Ganymede, and Callisto using the GK2001 rheology for a range of ice grain sizes.

Numerical Implementation of Ice Rheology:

The strain rate for each mechanism in the composite rheology is described by

$$\dot{\epsilon} = A \frac{\sigma^n}{d^p} \exp\left(\frac{-Q^*}{RT}\right), \quad (1)$$

where $\dot{\epsilon}$ is the strain rate; A , n , p , and Q^* are experimentally determined rheological parameters; d is the ice grain size, R is the gas constant, and T is temperature (see Table 1).

GK2001 provide an alternate set of creep parameters for grain boundary sliding and dislocation creep in ice near its melting point, but we do not include this effect in our present models. If the viscosities due to GBS and dislocation creep in the warm ice near the base of the shell are much smaller than described here, convection might be possible in ice shells thinner than described by our models.

To implement a viscosity due to all four deformation mechanisms simultaneously, we rephrase the composite flow law of GK2001 in terms of viscosities using $\eta = \sigma/\dot{\epsilon}$, which allows an approximate solution for the total viscosity [3],

$$\eta_{tot} = \left[\frac{1}{\eta_{diff}} + \frac{1}{\eta_{disl}} + (\eta_{GBS} + \eta_{bs})^{-1} \right]^{-1}, \quad (2)$$

due to volume diffusion (*diff*), dislocation creep (*disl*), grain boundary sliding (*GBS*), and basal slip (*bs*). An explicit stress-dependent rheology of form:

$$\eta = \frac{d^p}{A} \sigma^{(1-n)} \exp\left(\frac{Q^*}{RT}\right) \quad (3)$$

is used for each term in equation (2). To non-dimensionalize the rheology, we divide each term in equation (2) by the viscosity due to volume diffusion at the melting temperature of ice,

$$\eta_o = \frac{d^2}{A} \exp\left(\frac{Q_v^*}{RT_m}\right). \quad (4)$$

The transition stresses between the deformation mechanisms are mathematically represented by a series of weighting factors (β) between the four component rheologies, which govern the relative importance of each mechanism as a function of temperature and grain size [3]. Each viscosity function is expressed in non-dimensional coordinates (primed quantities) as

$$\eta' = \frac{1}{\beta} \sigma'^{(1-n)} \exp\left(\frac{E}{T'+T'_o} - \frac{E_v}{1+T'_o}\right), \quad (5)$$

where $\sigma' = \sigma/(\eta_o \dot{\epsilon}_o)$ is the non-dimensional stress, $E=Q^*/nRAT$ is the non-dimensional activation energy, $E_v=Q_v^*/nRAT$, and T'_o is the reference temperature.

We use a reference Rayleigh number defined by

$$Ra = \frac{\rho g \alpha \Delta T D^3}{\kappa \eta_o} \quad (6)$$

where $\rho=930 \text{ kg m}^{-3}$ is the density of ice, g is the acceleration of gravity, $\alpha=10^{-4} \text{ K}^{-1}$ is the coefficient of thermal expansion, ΔT is the temperature difference between the surface and the base of the shell, D is the thickness of the ice shell, and $\kappa=10^{-6} \text{ m}^2 \text{ s}^{-1}$ is the thermal diffusivity. In a non-Newtonian fluid, viscosities in the layer may evolve to values larger or smaller than η_o depending on the vigor of convection. The viscosity at the melting point near the base of the ice shell is 10^{13} Pa s when volume diffusion and dislocation creep are the dominant rheologies. When GSS creep accommodates convective strain, the viscosity in the convecting interior of the ice shell is 10^{14} to 10^{15} Pa s .

Initial Conditions: The approach we use to numerically determine the critical Rayleigh number is similar to linear stability analysis [4,5]. The convection simulations are started from an initial condition of a conductive ice shell plus a temperature perturbation expressed as a single Fourier mode,

$$T(x, z) = T_s - \frac{z \Delta T}{D} + \delta T \cos\left(\frac{2\pi D}{\lambda} x\right) \sin\left(\frac{-z\pi}{D}\right), \quad (7)$$

where δT and λ are the amplitude and wavelength of the temperature perturbation, and $z=-D$ at the warm base of the ice shell.

The simulation is run for a short time to determine whether the initial perturbation grows and convection begins, or decays with time due to thermal diffusion and viscous relaxation, causing the ice layer to return to a conductive equilibrium [Barr *et al.*, 2004]. For a given pair of δT , λ values, we run a series of convection simulations with decreasing values of Ra_o . The critical Rayleigh number is defined as the minimum value of Ra_o where the system convects for a given initial condition, and here is determined to three significant figures.

Following the procedure described in [2], we determine the wavelength of perturbation for which Ra_{cr} is minimized, and investigate how Ra_{cr} varies with the amplitude of temperature perturbation. Over the range of grain sizes used in this study ($d=0.1$ mm – 10 cm), the critical wavelength is constant at $1.75D$.

Results: We find that the critical Rayleigh number for convection varies as a power (-0.24) of the amplitude of initial temperature perturbation,

$$Ra_{cr} = Ra_{cr,0} \left(\frac{\delta T}{\Delta T} \right)^{-0.24}, \quad (8)$$

for perturbation amplitudes between 3 K and 30 K. Based on the results of [2], we expect the critical Rayleigh number to reach a constant asymptotic value for $\delta T > 0.25 \Delta T$.

Values of the fitting coefficient $Ra_{cr,0}$ depend on the grain size of ice, and can be fit to a polynomial in log-log space,

$$\ln(Ra_{cr,0}) = -0.129(\ln d)^3 - 2.98(\ln d)^2 - 22.6(\ln d) - 42.7, \quad (9)$$

which can be combined with the definition of the Rayleigh number to obtain a scaling between the critical ice shell thickness for convection, the grain size of ice, and the amplitude of temperature perturbation.

Implications for the Icy Galilean Satellites:

Figure 1 illustrates the variation in critical shell thickness (D_{cr}) for convection in Europa as a function of grain size. If the ice grain size is < 1 mm, $D_{cr} < 30$ km because relatively low thermal stresses are not sufficient to activate weakly non-Newtonian GSS creep, so plume growth is controlled by Newtonian volume diffusion. For grain size > 1 cm, $D_{cr} < 30$ km because thermal stresses can activate strongly non-Newtonian dislocation creep, and the ice softens as it flows. For intermediate grain sizes (1-10 mm), weakly non-Newtonian GSS creep controls plume growth, yielding critical shell thicknesses close to the

maximum permitted shell thickness for each of the Galilean satellites.

Future Work: GK2001 provide an alternate set of creep parameters for GBS and dislocation creep in ice near its melting point, but we have not yet included this effect in our models. If the viscosities due to GBS and dislocation creep in the warm ice near the base of the shell are much smaller than described here, convection might be possible in ice shells thinner than found here.

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References: [1] Goldsby, D. L., and D. L. Kohlstedt, *JGR* v. 106, p. 11,017-11,030, 2001; [2] Barr, A. C., R. T. Pappalardo, and S. Zhong, *JGR*, 109, E12008, doi:10.1029/2004JE0022962004; [3] Barr, A. C. and R. T. Pappalardo, *JGR* submitted, 2004; [4] Turcotte, D. L. and G. Schubert, *Geodynamics: Applications of Continuum Physics to Geological Problems*, John Wiley & Sons, 1982; [5] Chandrasekhar, S., *Hydrodynamic and Hydromagnetic Stability*, Oxford, Clarendon, 1961.

Table 1. Rheological Parameters^a

Rheology	A ($m^p Pa^{-n} s^{-1}$)	n	p	Q^* ($kJ mol^{-1}$)
Volume Diffusion	3.5×10^{-10} ^b	1	2	59.4
Basal Slip	2.2×10^{-7}	2.4	0	60
GBS	6.2×10^{-14}	1.8	1.4	49
Dislocation Creep	4.0×10^{-19}	4.0	0	60

^aAfter Goldsby and Kohlstedt [2001]. ^bFor $T_m=260$ K.

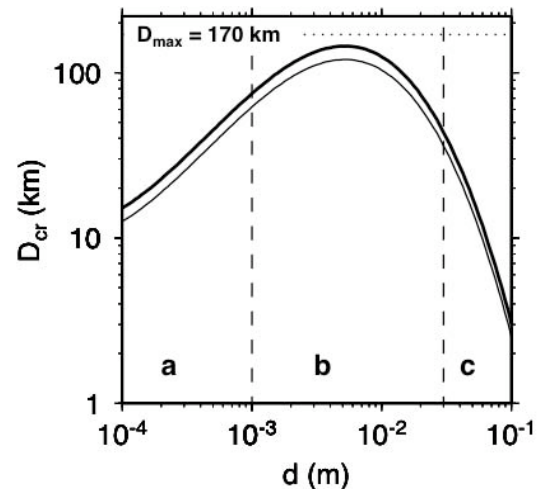


Figure 1. Critical shell thickness for convection in Europa as a function of grain size, from equation (9). (a) Volume diffusion accommodates creep during initial plume growth, and the critical shell thickness for convection increases as a strong function of grain size. (b) GSS creep controls plume growth, and the critical shell thickness achieves a maximum value close to the maximum permitted shell thickness. (c) Dislocation creep accommodates flow, yielding smaller values of critical shell thickness.