MORE ON MAGNETIC SPECTRA FROM CORRELATED CRUSTAL SOURCES ON MARS. C. V. Voorhies, Planetary Geodynamics, Code 921/698, NASA, Goddard Space Flight Center, Greenbelt MD 20771, USA Coerte.V.Voorhies@nasa.gov.

Introduction: The spectral method for distinguishing crustal from core-source magnetic fields has been re-examined, modified and applied to both a comprehensive geomagnetic field model and an altitude normalized magnetic map of Mars [1]. These observational spectra are fairly fitted by theoretical forms expected from certain elementary classes of magnetic sources [2]. For Earth we found fields from a core of radius 3512 +/- 64 km, in accord with the 3480 km seismologic radius, and a crust represented by a shell of random dipolar sources at radius 6367 +/- 14 km, just beneath the 6371.0 km mean radius. For Mars we found only a field from a crust represented in same way, but 46 +/- 10 km below the planetary mean radius of 3389.5 km, and with sources about 9.6 +/- 3.2 times stronger than Earth's [1].

It is remarkable that the same simple theoretical form should fairly fit crustal magnetic spectra for both worlds and return crustal-source depth estimates of plausible magnitude. Evidently, the idea of an ensemble of compact, quasi-independent, magnetized regions within these planetary crusts has some merit. Yet such estimates, at best a kind of average, depend upon both the observational spectrum fitted and the physical basis of the theoretical spectrum.

Observational Spectra: The magnetic spectrum of a planet is the mean square magnetic induction configured in spherical harmonics of degree n, averaged over a centered sphere of radius r that contains the sources [3],

$$R_n(r) = (n + 1)(a/r)^{2n+4} \sum_{m=0}^{n} [(g_n^m)^2 + (h_n^m)^2]. \quad (1)$$

Here a is the radius of a reference sphere and (g_n^m, h_n^m) are the Gauss coefficients of degree n and order m in a Schmidt-normal spherical harmonic expansion of the scalar potential V: $\boldsymbol{B} = -\nabla V$. Observational spectra can be calculated from coefficients obtained via harmonic analyses of either measured data, or other models fitted to such data. These spectra depend on the data fitted and modeling technique.

For Earth, the magnetic field model we used has since been revised [4], updated with high precision Ørsted data, and upgraded with high resolution Champ data [5]; at high degrees, the refined spectra appear softer than before. For Mars, there are now many fine models of the outstanding global MGS MAG/ER data [6 - 13]; these agree well through intermediate degrees, but differ at hard to resolve high degrees.

Theoretical Spectra: Consider a thin crust with compact, effectively dipolar, sources. If we expect dipole positions to be uncorrelated, random samples of a uniform distribution on a spherical shell, and vector dipole moments to be uncorrelated, random samples of a zero mean distribution, then our expectation spectrum from an ensemble of such random dipoles on a shell of radius r_x is [1]

$$\{R_n(a)\}^{ss} = A_x\, n\, (n + ½)\, (n + 1)\, (r_x/a)^{2n-2}. \quad (2)$$

In spectrum (2), amplitude A_x is proportional to the mean square moment of the sources and a cubic polynomial of n modulates the exponential attenuation function of $r_x/a < 1$. This spectrum increases with n at low degrees, peaks near degree $3/2ln(a/r_x)$, and falls of exponentially at high degrees. Curiously, the mathematical derivation of (2) [2] is analogous to that for magnetic change induced by compact hydromagnetic eddies near the top of a fluid conductor [14].

More realistic theoretical spectra, which allow for crustal thickness, oblateness and magnetization by a planet centered dipole, have already been derived and discussed, as have important spectral effects of laterally correlated sources [1]. The latter were described via an ensemble of vertically and uniformly magnetized spherical caps; yet interesting complications arise from trial distribution functions for the sizes and magnetizations of extended sources. These are addressed in hopes of improving source-depth estimates, resolving joint distribution parameters, and determining whether or not the different crustal magnetic dichotomies of the two planets, evident in the phase information, have distinct spectral signatures.

Correlated Sources: The main effect of laterally correlated sources is to soften the spectrum at high degrees [1]. We tend to over-estimate source shell depth when this is omitted. To include this effect simply, size and magnetization distribution functions for extended sources are recast as the characteristic half-angle ψ_0 (hence diameter) and mean square total moment $\{T^2\}$ for an ensemble of vertically and uniformly magnetized spherical caps on the shell of radius r_x. The resulting theoretical spectrum is

$$\{R_n(a)\} = A\, (n/2)\, [Z_n(\psi_0)]^2\, (r_x/a)^{2n-2}. \quad (3)$$

Amplitude A is proportional to $\{T^2\}$ and, in terms of the Schmidt-normal associated Legendre polynomials $P_n^m(\cos\psi)$, $Z_n(\psi) = \sin\psi\, P_n^1(\cos\psi)/[1-\cos\psi]$.

The physics of spectrum (3) is perhaps most easily seen via its finite Taylor expansion in $\varepsilon = 1 - \cos\psi_0$, a quantity proportional to the characteristic area of small source regions. To first order in small ε,

$$\{R_n(a)\} \approx A\, n^2 (n+1) (r_x/a)^{2n-2} \times [1 - (\psi_0^2/4)n(n+1)]. \qquad (4)$$

For small caps, and at moderate degrees $n\psi_0 << 1$, the partial derivatives of the logarithm of theoretical spectrum (4) with respect to amplitude A, shell radius r_x, and cap half-angle ψ_0 are approximately proportional to 1, n, and $-n^2$, respectively. Separation of characteristic source diameter from amplitude and depth should thus be straightforward, unlike separation of amplitude from layer thickness. The negative sign of the partial w.r.t. ψ_0 describes the softening of the spectrum due to the small, but non-zero, area of the still unresolved sources.

Indeed, with $n\psi_0 << 1$, $ln[1 - (\psi_0^2/4)n(n+1)]$ is about $-(\psi_0^2/4)n(n+1)$ and the logarithm of demodulated spectrum (4) is about

$$ln[\{R_n(a)\}/n^2(n+1)] \approx lnA + 2(n-1)ln(a/r_x) - (\psi_0^2/4)n(n+1). \qquad (5)$$

This is a parabola in n. Of course, spectrum (3) is positive, so approximate spectrum (4) and relation (5) require $\psi_0^2 << 4/n(n+1)]$. For high degrees and/or broader sources, higher order terms must be retained in the expansion of spectrum (3). For example, the second order correction to spectrum (4) is less than the first order term only for $\psi_0^2 < (9.6)^{1/2}/n(n+1)$. For degrees $n \leq 90$, this is so for source regions up to a 1.1 degree half angle, or 130 km diameter, on Mars.

Summary: Perhaps the simplest statistical model for compact sources in the magnetic crust of a planet is that of random dipoles on a shell. By carefully fitting a theoretical spectrum like (2) to an observational magnetic spectrum, we can estimate the characteristic amplitude and depth of such sources [1].

A more realistic model is needed to improve such estimates.. Perhaps the most important improvement is to include the small but non-zero areas of the source geologic structures. To this end, theoretical spectra have been developed for a simple class of laterally correlated sources.

The results, here summarized simply via approximate spectrum (4) and relation (5), yield spectra softer than (2). Moreover, comparison with previous spectral forms indicates that, in addition to source amplitude and depth, observational magnetic spectra from a planetary crust are likely telling us far more about the typical breadth of source regions than about typical source layer thickness.

Estimation of three fundamental properties of a thin magnetized crust, the characteristic magnetization, depth, and breadth of its presumably small constituent source regions, boils down to a parabolic fit to the log of its cubically demodulated magnetic spectrum. Moreover, the restrictive suppositions of small source regions and moderate harmonic degrees can be eliminated, though the computations, notably the formulas for the partials, become more complicated. Preliminary results from applications to observational spectra are to be presented and discussed.

References: [1] Voorhies, C. V, T. J. Sabaka & M. Purucker (2002) *JGR, 107,* E6, doi:10.129/2001JE001534, June. [2] Voorhies, C. V. (1998) *NASA Technical Paper* 1998-208608, 38pp, Dec.. [3] Lowes, F. J. (1966) *JGR, 71,* 2179. [4] Sabaka, T. J., N. Olsen & R. A. Langel (2002) *GJI, 151,* 32-68. [5] Sabaka, T. J., N. Olsen & M. Purucker (2004), *GJI, 159,* 521-547. [6] Cain J., B. Ferguson & D. Mozzoni (2000) *Trans AGU, 81, Spring Meeting Supp.*, S171. [7] Connerney, J.E.P., et al. (2001) *GRL, 28,* 4015-4018. [8] Arkani-Hamed, J., (2001) *JGR, 106,* 23,197-23,208. [9] Arkani-Hamed, J., (2002) *JGR, 107,* E5, doi: 10.1029/JE2001496, May. [10] Hutchinson, W. E., & M. T. Zuber (2002) *LPS XXXIII*, 75. [11] Langlais, B., M. E. Purucker & M. Mandea (2004) *JGR, 109,* E02008, doi:10.1029/2003JE002048, Feb.. [12] Cain, J., B. Ferguson & D. Mozzoni (2003) *JGR., 108,* E2, doi: 10.1029/2000JE001487, Feb.. [13] Whaler, K. A., & M. E. Purucker (2005) *JGR,* 2004JE002393, submitted. [14] Voorhies, C. V., *JGR, 109,* B03106, doi: 10.1029/2003JB002833, March (see Appendix C).