# COMMAND GENERATION AND CONTROL OF MOMENTUM EXCHANGE ELECTRODYNAMIC REBOOST TETHERED SATELLITE

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## Introduction

The research completed for this NASA Graduate Student Research Program Fellowship sought to enhance the current state-of-the-art dynamic models and control laws for Momentum Exchange Electrodynamic Reboost satellite systems by utilizing command generation, specifically Input Shaping. The precise control of tethered spacecraft with flexible appendages is extremely difficult. The complexity is magnified many times when the satellite must interact with other satellites as in a momentum exchange via a tether. The Momentum Exchange Electronic Reboost Tether (MXER) concept encapsulates all of these challenging tasks [1]. Input Shaping is a command generation technique that allows flexible spacecraft to move without inducing residual vibration [2], limit transient deflection [3] and utilize fuel-efficient actuation [4].

Input shaping is implemented by convolving a sequence of impulses, known as the input shaper, with a desired system command to produce a shaped input that is then used to drive the system. This process is demonstrated in Figure 1. The shaped command is then use to drive the system without residual vibration while meeting many other performance specifications.

The completed work developed tether control algorithms for retrieval. A simple model of the tether response has been developed and command shaping was implemented to minimize unwanted dynamics. A model of a flexible electrodynamic tether has been developed to investigate the tether's response during reboost. Command shaping techniques have been developed to eliminate the tether oscillations and reduce the tether's deflection to pre-specified levels during reboost. Additionally, a model for the spin-up of a tethered system was developed. This model was used in determining the parameters for optimization the resulting angular velocity.
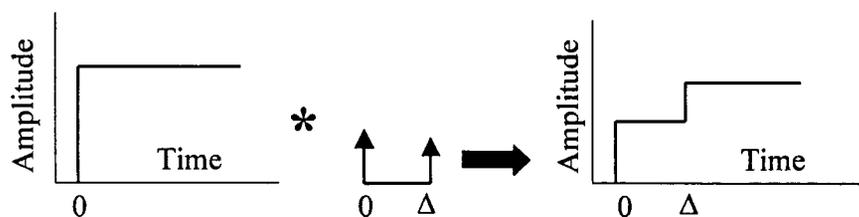


Figure 1: Input Shaping Example.

## Retrieval of Tethered Satellites

The problem of retrieving tethered satellites has seen a tremendous amount of research in recent years. The major conclusions based on the studies suggest that the stationkeeping phase is marginally stable; deployment can become unstable if a critical speed is exceeded; and the retrieval phase is always unstable [5]. One of the biggest issues with tethered satellites is preventing tether slackness. Consequently, there have been many tension control methods proposed for control of tethered satellites. In particular, methods have been developed for deployment [5-7], station-keeping [8] and retrieval [5-7, 9, 10]. Given the inherently unstable nature of the tether retrieval

process, many control strategies have been devised. Fujii and Ishijima [7], and Vadali and Kim [6] used a Lyapunov-based approach to insure stability, while Pines et al [10] used sliding mode and operator-in-the-loop control.

Input shaping has been adapted to work well on other space-based motion control applications. While previous researchers have developed feedback control architectures or pre-computed tether length profiles for the retrieval process, no one to date has explored the benefits of combining command generation and feedback control.

## Model Development

With the ultimate goal of creating a MXER system in mind, a simple 2-D model that assumes a rigid, massless tether system in Keplerian orbit is:

$$\lambda'' - \lambda(1 + \theta')^2 + \lambda + 3\lambda \cos^2 \theta = -\hat{T} \tag{1}$$

$$\theta'' + 2(\lambda' / \lambda)(1 + \theta') + 3\cos\theta \sin\theta = 0 \tag{2}$$

where, $\lambda$ is the nondimensional tether length and $\theta$ is the swing angle. The Lyapunov controller presented in [6] is used in this investigation. For a desired final length of $\lambda_f$, the control law is given by:

$$\hat{T} = 3\lambda + K_1(\lambda - \lambda_f) + (2/3)K_2\theta'(1 + \theta')/\lambda + K_3\lambda' \tag{3}$$

In general, the major trade-off in choosing controller gains $K_1$, $K_2$ and $K_3$ is compromising between the desired fast retrieval time and undesirable levels of swing angle.

A recreation of Vadali's tether length profile, tether swing angle profile and tether tension profile are shown as the solid black lines on Figures 2-4, respectively. Increasing the gains of the feedback controller can lead to improvements in tether retrieval time, but at the expense of increasing the swing angle. These results are shown as the dashed red line on Figures 2-4. Not only is the value of the swing angle increased, but oscillations are also induced.

An input shaper was created to reduce the tether oscillation caused by the faster retrieval (increased gains). The input to the system is a step change in desired length. This step change was convolved with a Unity Magnitude, Zero Vibration (UM-ZV) [11] input shaper to created the new setpoints. The dotted blue line shows the length, angle and tension profiles for the "shaped" tether retrieval in Figures 2-4. As shown in Figure 3, input shaping reduces the magnitude of the oscillations to a mean value approximately equal in value to that of the original gains, but still allows for a faster retrieval.
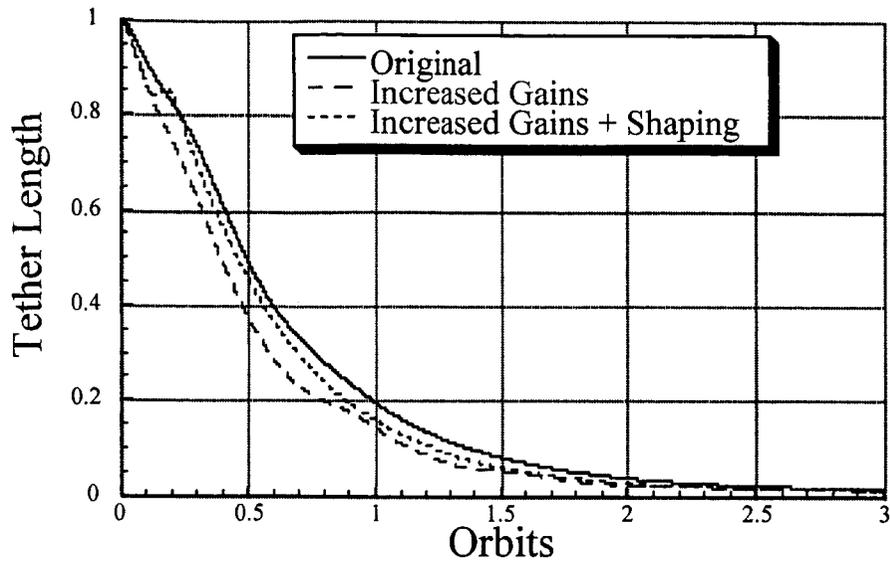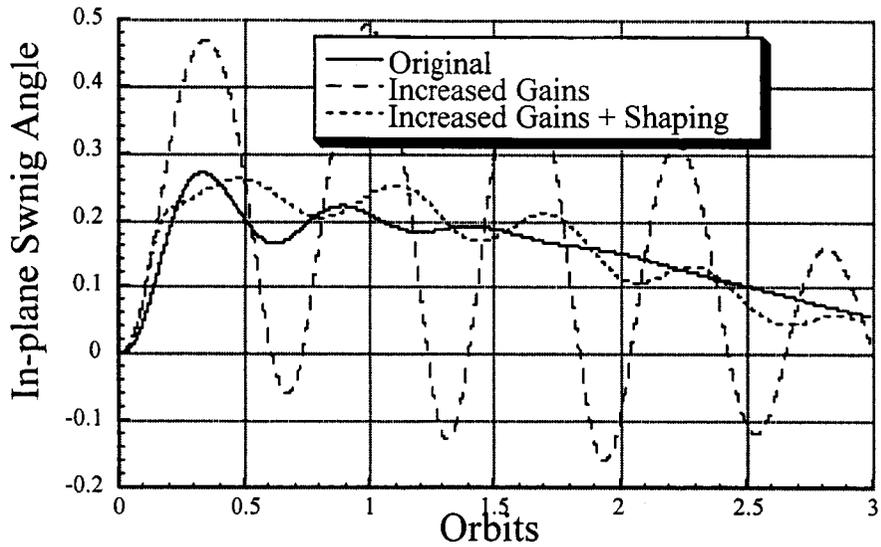
**Figure 2: Length Profile.**



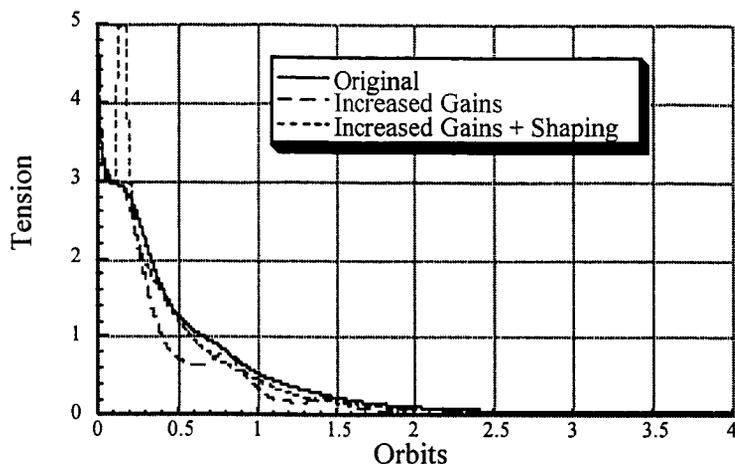**Figure 3: Swing Angle Profile.**

**Figure 4: Tension Profile.**

In the above results, the command shaper was designed for a specific gain set. The shaper switch times were chosen to minimize the tether length at 1 orbit while keeping the maximum swing angle below the toleration limit. Due to the nonlinear nature of the equations of motion, the goal is not zero vibration but reduced swing angle. Figure 3 shows that command shaping reduces the maximum swing angle of the "Increased Gains" case by 50%. A different choice of gains would possibly lead to a vastly different response and require a different command shaper whose retrieval time and swing angle reduction characteristics may be different. In order to fully minimize the retrieval time, both the gain set and the command shaper should be designed together. However, the shaper can be designed independent of the gain set and still lead to significant retrieval time savings. Table 1 shows the quantification of the swing angle reduction and retrieval time reduction for four sample gains sets. (Note: In all cases $K_2$ was set to zero.) In total, 36 different gain sets were chosen. On average, adding command shaping reduces the maximum swing angle by just over 48% with a standard deviation of 0.09 and results in 16% shorter tether lengths after 1 orbit with a standard deviation of 0.08. (Note: The optimal combination of command generation and controller gains resulted in a 23% shorter tether after 1 orbit.)

**Table 1: Tension Controller Time Saving and Swing Angle Reduction Summary.**

| $K_1$ | $K_3$ | % Swing Angle Reduction (Hi vs. Hi+IS) | % Retrieval Time Reduction (Ref. vs. HI+IS) |
|---|---|---|---|
| 3.5 | 9.3 | 72.627 | 44.622 |
| 2.7 | 8.5 | 55.026 | 21.081 |
| 2 | 7.2 | 42.186 | 0.04 |
| 2.2 | 7.5 | 43.93 | 8.973 |
| Averages | | 51.21927 | 20.96926 |

**Compensating for Initial Conditions**

In the previous sections, the command shaper was designed for an initial swing angle of zero radians (or degrees). Figure 5 shows the effect the initial swing angle has on the maximum angle attained during the retrieval process using the Tension controller. Even a 5 degree (0.087 radian) initial condition can have dramatic effects on the performance of the Increased Gains cases. Figure 6 shows that the 5 degree initial condition has little effect on the tether length profile. However, Figure 7 shows that the swing angle for both the increased gains and increased gains with input shaping has increased beyond acceptable levels.

An input shaper was designed for the new set of initial conditions to minimized retrieval time, while keeping the swing angle within acceptable limits. This response is shown as the dotted green lines in Figures 6 and 7. This initial condition (IC) shaper has a swing angle response that is comparable to the original response but has a shorter retrieval time.
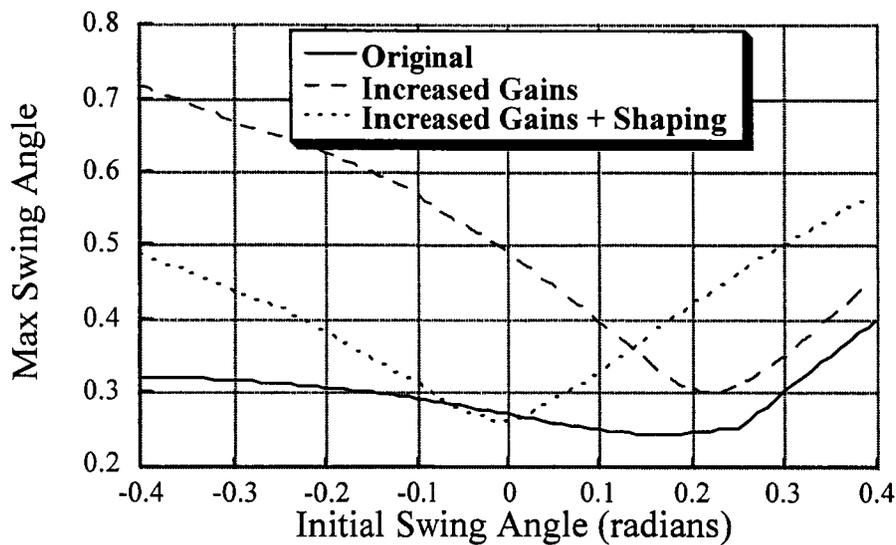
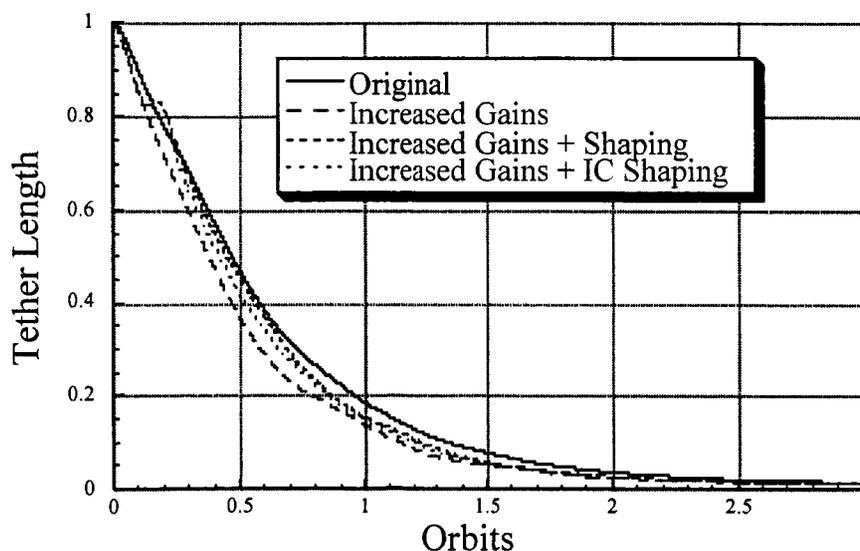**Figure 5: Effect of Initial Swing Angle on Retrieval Process.**

**Figure 6: Tether Retrieval with Initial Conditions.**

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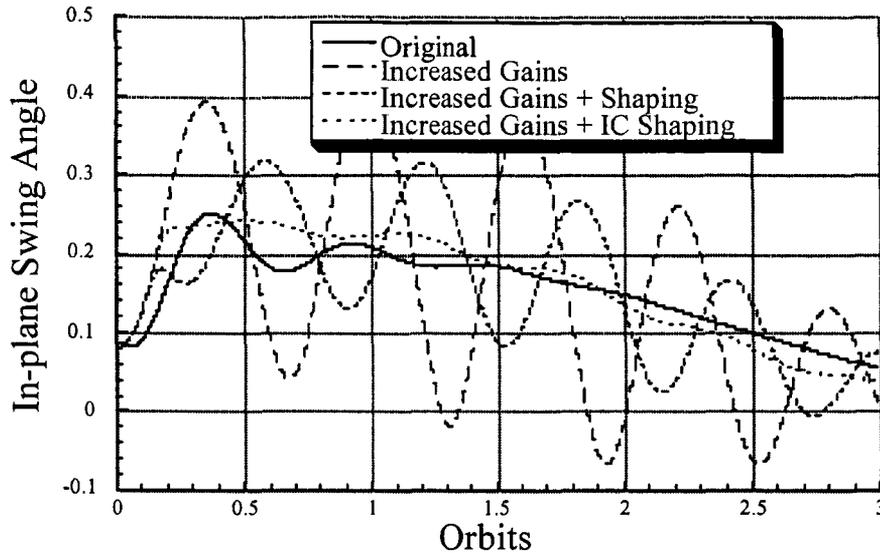Michael J. Robertson – NASA GSRP Final Report

**Figure 7: Swing Angle with Initial Conditions.**

While the IC compensation shaper works well on the specific case for which it was designed, it too suffers from a similar lack of robustness to errors in the estimate of initial conditions. In an effort to produce a command shaper that is more insensitive to the effect of the initial condition, a UM-ZVD shaper was designed for the tether retrieval process. Figure 8 shows that this approach does lead to a significant improvement in robustness. The maximum swing angle is held below the tolerable limit for initial conditions up to 0.15 radians. Unfortunately, this increase in robustness comes with a steep price. Figure 9 shows the length of the tether after 1 orbit. The gain in robustness achieved by using the UM-ZVD shaper is outweighed by the retrieval time penalty. Retrieval using the UM-ZVD shaper is slower than the original case.

In an effort to replicate the robustness of the UM-ZVD, but retain the short retrieval times achieved with the UM-ZV shaper, three UM-ZV shapers designed for different initial conditions were combined into a switching-type shaper. That is, each shaper was assigned to suppress a range of initial conditions. A suppression range of +/- 0.3 radians is shown in Figure 8. Given an initial swing angle, the retrieval process uses the shaper developed for that particular range of swing angles. Figure 9 shows that the retrieval time for the Multi-UM-ZV is shorter than the original case.
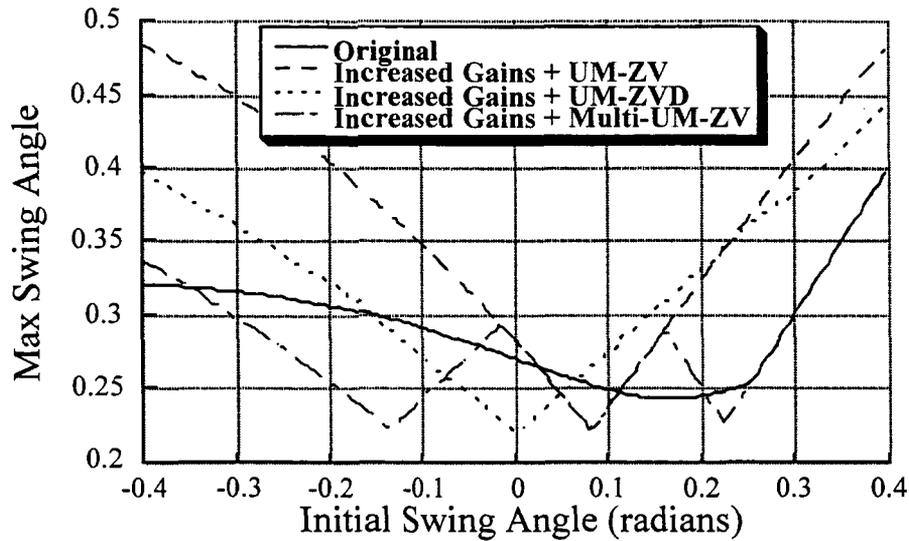
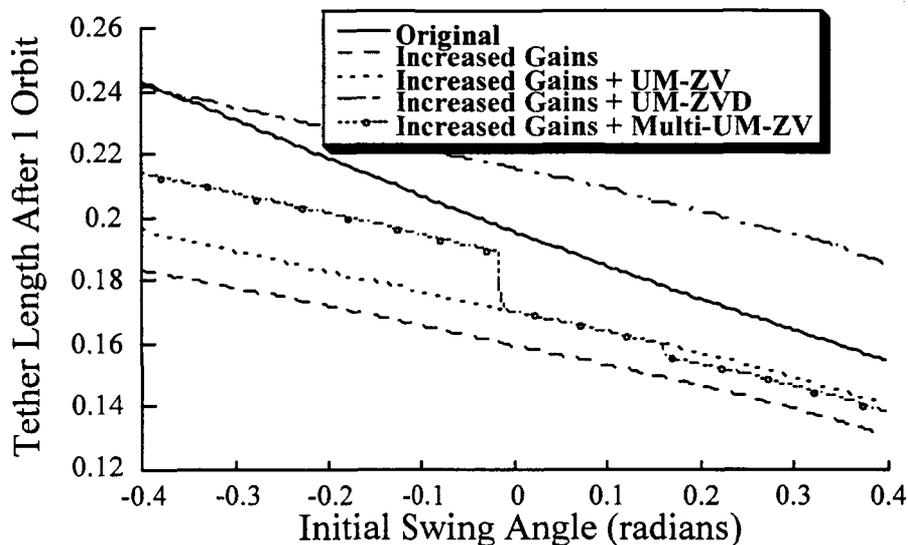**Figure 8: Effect of Initial Swing Angle on Tension Controller.**



**Figure 9: Tether Retrieval Comparison using Tension Controller.**

## Command Shaping on Electrodynamic Tethers

Electrodynamic tethers for de-orbit, power generation and orbit reboost have also seen an increase in research activity. The use of electrodynamic tethers, however, add a new level of complexity and can be a source of instability [12]. A more detailed dynamic model for electrodynamic tethers has been described [13] and suggests methods to control the new dynamics.

To lesson the unwanted dynamic effects in electrodynamic tethers, input shaping can be employed to modify the electric current sent through the tether. If done properly, this will shape the resulting thrust force so that the undesired vibration will be greatly reduced. Tethers have two main modes of oscillation: 1) Libration: low-frequency pendulum motions, and 2) String vibration: numerous high frequency modes.

Given the general qualities of the various input shapers and the needs of the tether system, we utilize a UM-ZV shaper to reduce the libration (pendulum motion) because it is a long duration oscillation that is easy to estimate. It is a function of only the orbital altitude. On the other hand, Zero Vibration and Derivative (ZVD) shapers are employed to deal with the string vibration because it is a more complex function of tether length, line density, tension and motion. Consequently, the input shaper addressing this phenomenon needs more robustness.

Simulations of station-keeping using the dynamic model shown in Figure 10 were conducted to test the hypothesis that applying input shaping to the control of electrical current in a tether system would be beneficial. In simulations, a 2-km long tether was modeled using ten point masses. The tether has a line density of 1/1000 (kg/m) and a stiffness, EA, of 10000 N. The solid line in Figure 11 shows that when the current is simply turned on, the center of the tether deflects approximately 80m forward. However, significant amounts of both libration and string vibrations are induced in the system response. When the current is shaped using a ZVD shaper designed for the first mode of string vibration, the high mode vibration is significantly reduced, as shown by the dotted line in Figure 11. If a negative ZV shaper is convolved with the ZVD shaper to address the libration, then the vibration is nearly eliminated, as shown in Figure 11. Note that the cost of this improvement is a small increase in risetime.
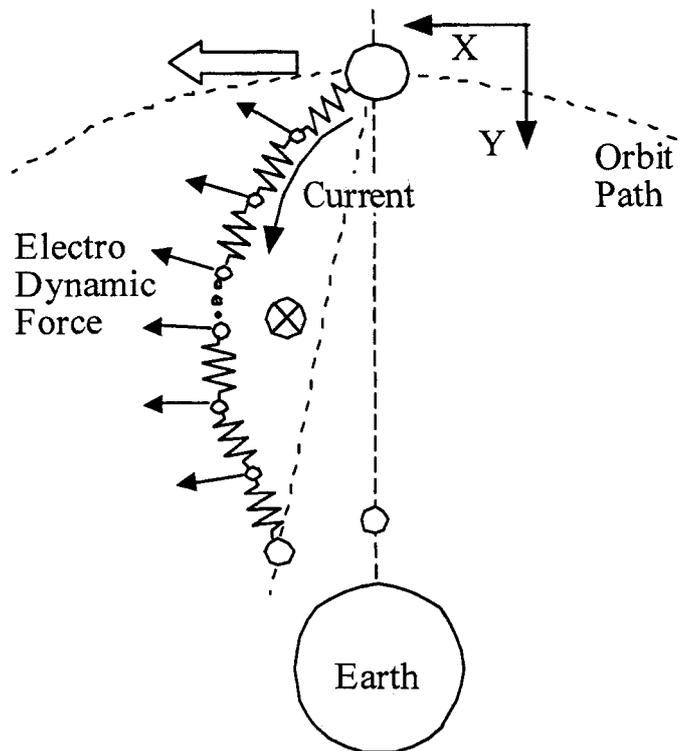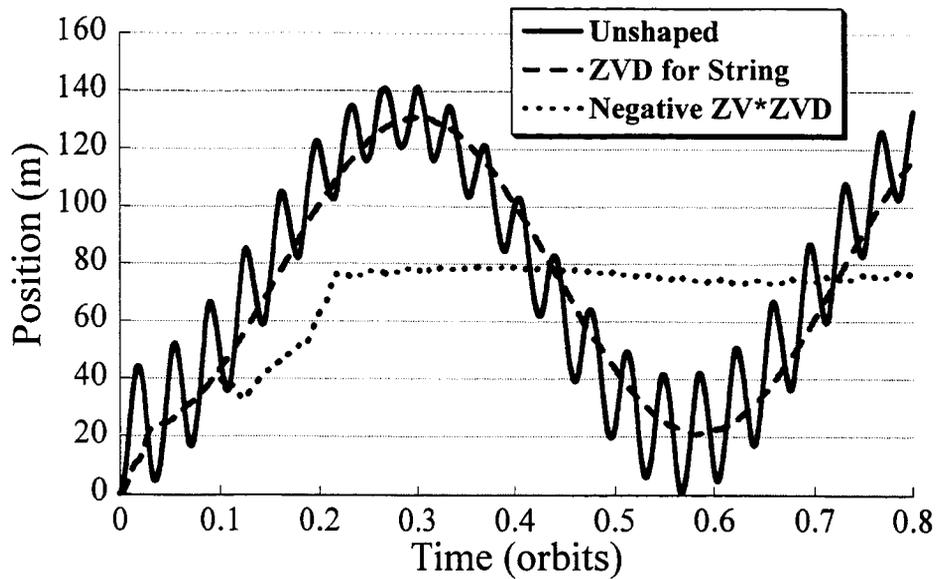
**Figure 10: Electrodynamic Tether Model.**

**Figure 11: Electrodynamic Tether Midpoint Response.**

Using multi-mode shaping allows for a relatively vibration-free electrodynamic reboost. However, the tether mid-point displacement of 78 meters may be unacceptably large. The modified negative ZV shaper developed in the previous section can be used to cancel the libration and limit the mid-point displacement. Figure 12 shows the tether mid-point displacement using a 80% Deflection-Limiting shaper. The overall responses are similar to those shown in Figure 11 except that the amplitude of the mid-point deflection has been reduced by approximately 20% as intended.
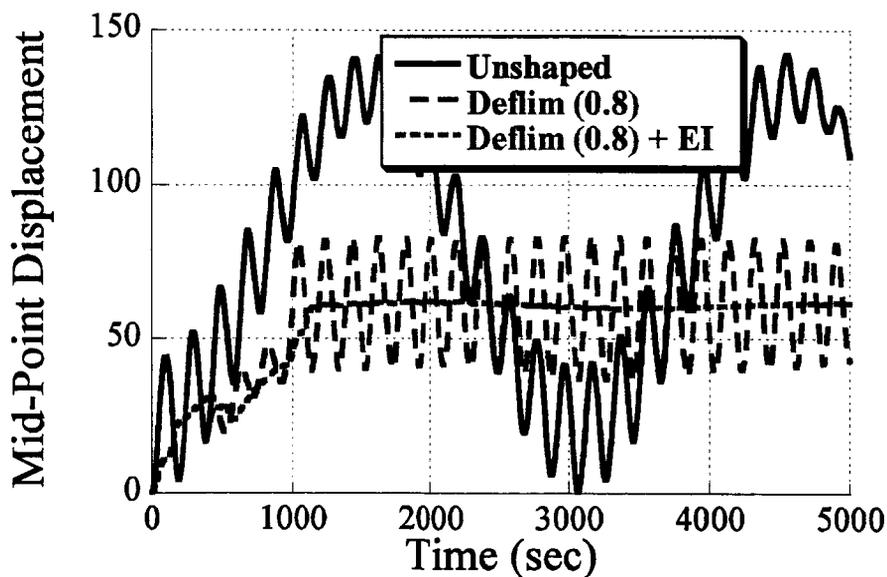


**Figure 12: Deflection-Limited Electrodynamic Tether Midpoint Response.**

The demonstrate more clearly the effectiveness in the Deflection-Limiting shapers to reduce the tether mid-point displacement Figure 13 shows the tether response for three multi-mode shapers.

Using a traditional ZV shaper results in a mid-point deflection of 78 meters. The 80% and the 50% modified negative ZV shapers reduce the mid-point displacement to 61 and 38 meters, respectively.
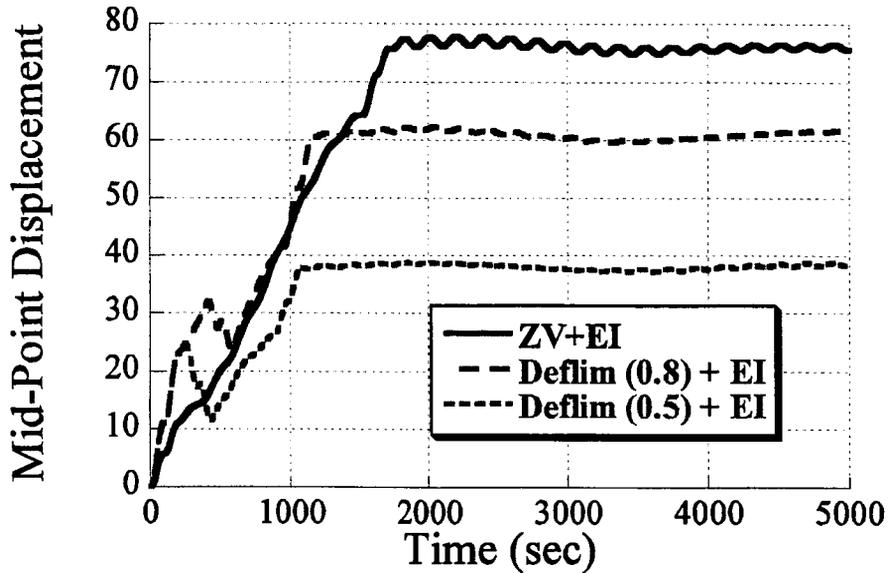


**Figure 13: Comparison of Deflection-Limited Responses.**

## Spinning Tethered Satellite Systems

In addition to reducing retrieval time and swing angle for earth-pointing tethers, command shaping techniques for improved spin-up of spinning tethered satellite systems were developed. The spin-up process is critical to mission success. Variations in angular velocity at the conclusion of the spin-up deteriorates the accuracy of the tether endpoint positioning for momentum exchange and creates fluctuations in the artificial gravity field. Retrieval profiles were developed to reduce these variations and improve performance.

In an effort to get a greater understanding of the spin-up dynamics, the Tethered Artificial Gravity (TAG) satellite project has been proposed [14]. A model of this system is shown in Figure 14. In this project, a satellite will be delivered in to orbit, automatically separated into roughly equal masses and connected by 2000 meters of tether. Retrieval will be used to generate a rapid increase in rotational motion of the system. If the motion of the system is constrained to the orbital plane, the two sections of the satellite are modeled as point masses and the tether is assumed to be massless and remain straight throughout the retrieval process with its elasticity modeled by Hooke's law, then the equations of motion can be given by:

$$\ddot{L} - L\dot{\theta}^2 + 2\Omega\dot{\theta} + 3\Omega\cos^2(\theta) + \frac{EA}{m}\varepsilon U(\varepsilon) = \frac{T}{m}$$

$$\ddot{\theta} + 2\frac{\dot{L}}{L}(\Omega + \dot{\theta}) + \frac{2}{3}\sin(2\theta) = 0$$

where

$$\overline{m} = \frac{m_a m_m}{m_a + m_m}$$

and $m_a$ and $m_b$ are the masses of the two portions of the satellite, L is the length of the tether, $\theta$ is the in-plane swing angle, $\Omega$ is the orbital angular velocity, where E is the modulus of elasticity of the tether, A is the cross sectional area of the tether, $\varepsilon = L/L_0$, $L_0$ is the unstretched length of the tether, U() is the unit step function and T is the tether tension. The TAG will use a simple exponential control for the length of the tether given by

$$L(t) = L_i \exp(-ct)$$

where $L_i$ is the length of the tether at the start of the spin-up process and c is the decay rate. If the desired final length, $L_f$, and the length of time of the spin-up process, $t_r$ are fixed, then c is given by
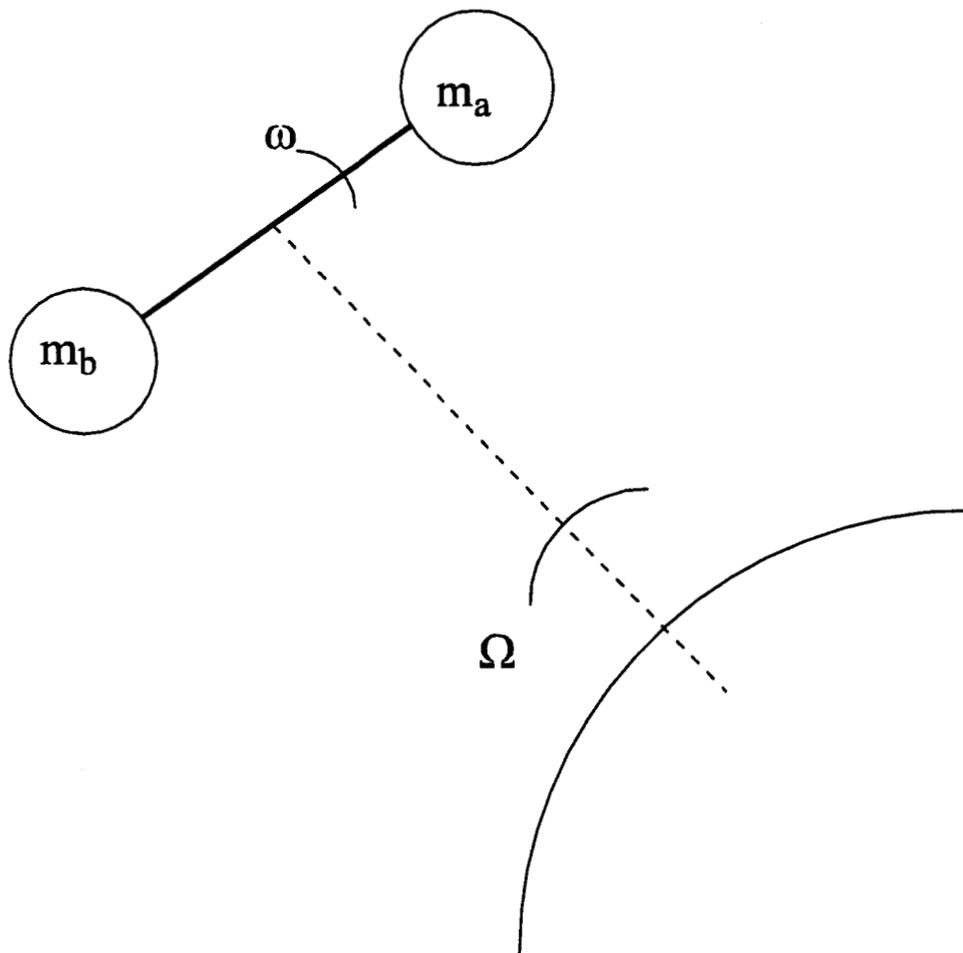
$$c = \frac{L_i}{L_f} t_r$$



**Figure 14: TAG Model.**

The previous results demonstrated that for the simple tether model, the initial swing angle has the greatest influence on the final angular velocity. Negative initial swing angles are ideal, but for positive angles the spin rate can be improved by adjusting the retrieval rate. This section

investigates the effect tether extensibility has on the final angular velocity and presents a command shaping scheme to reduce vibration.

In the TAG satellite project, the portions of the satellite, $m_a$ and $m_b$ will be 40 kg and 25 kg, respectively. The tether stiffness, EA is 10000 N and the orbital rate is 0.0011636 (90-minute orbital period). The tether will be retrieved from an initial length of 2000 meters to a final length of 100 meters in 90 minutes. This results in retrieval rate of c=5.5377*e-4. Figure 15 shows the angular velocity for initial swing angles of 60, 65 and 70 degrees using the extensible tether model. All three cases have oscillation about the final value. Both the frequency and the average angular velocity decrease with initial swing angle. In fact, the angular velocity is negative for the 70 degree initial swing angle. The variations in angular velocity result in variations in the artificial gravity field that can be uncomfortable for humans and may negatively affect experiments and electronic components aboard the system.
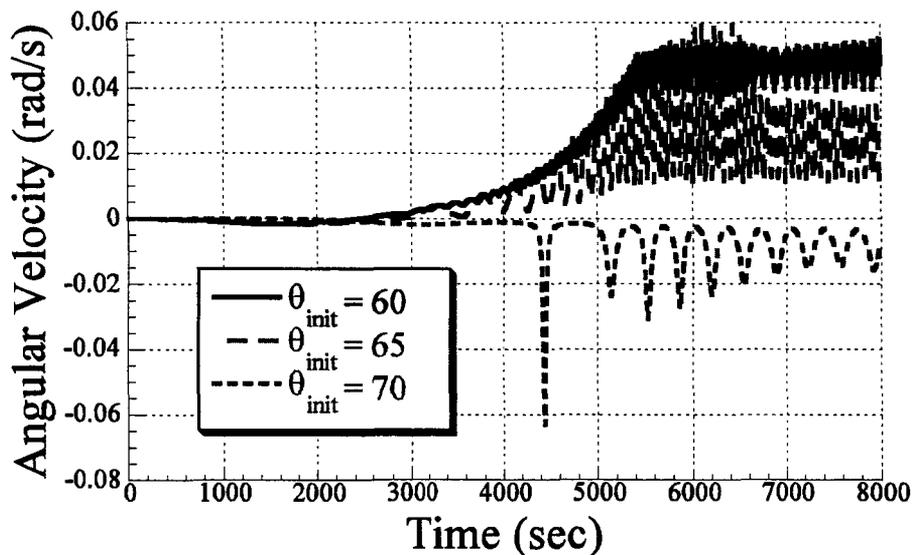


**Figure 15: Angular Velocity using Extensible Tether Model, Positive Initial Swing Angles.**

Figure 16 shows the length profile for these initial swing angles. Here the solid black line label "Rigid" refers to a tether model where elasticity is ignored. There are large oscillations in the tether length and the amplitude of the oscillations increase with initial swing angle. The oscillations occur because the tether can only apply a force when it is in tension (i.e. when the length is greater than 100). The tether force is opposed by the centripetal force $f_c = m\omega^2 L$. Lower angular velocities require longer amounts of time to overcome the tension force applied by the tether. Higher angular velocities mean higher $f_c$ which would keep the tether length closer to the desired final length.
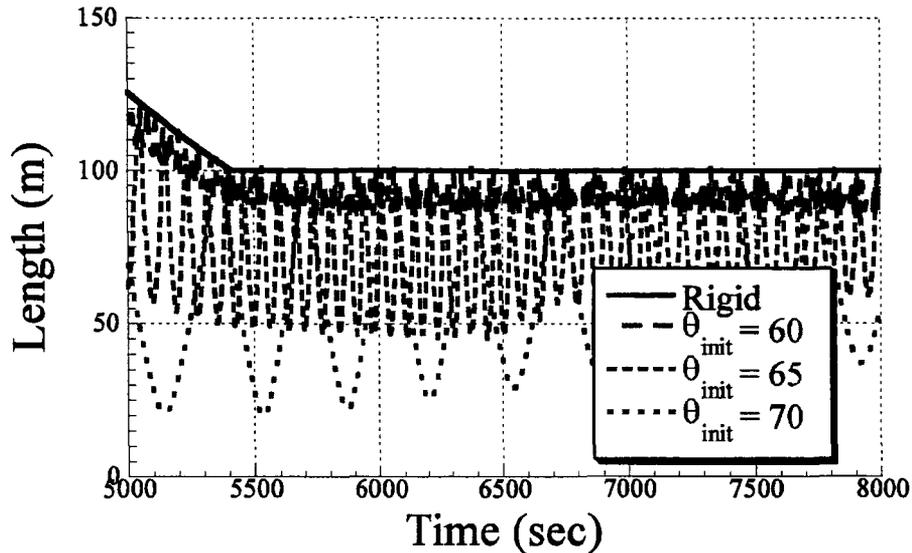
**Figure 16: Length Profile using Extensible Tether Model, Positive Initial Swing Angles.**

Figures 17 shows the effect of the retrieval rate, c, on the angular velocity for a 60 degree initial swing angle. The fastest retrieval, c=7.5*e-4, has the highest angular velocity and the highest variation amplitude. Interestingly, the slowest retrieval has the second highest angular velocity and variation. The length profile for the 60 degree initial conditions is shown in Figure 18. The slowest retrieval rate has the highest variations in length The faster retrieval has the second highest variation. For all retrieval rates except c=4.5*e-4, the tether is in tension. Extremely complicated dynamics exist during satellite spin-up. The results vary greatly for different initial swing angles. Both the initial angle and the retrieval rate greatly affect the final angular velocity and tether length. Arbitrarily setting the retrieval rate will, in general, not result in desired performance.
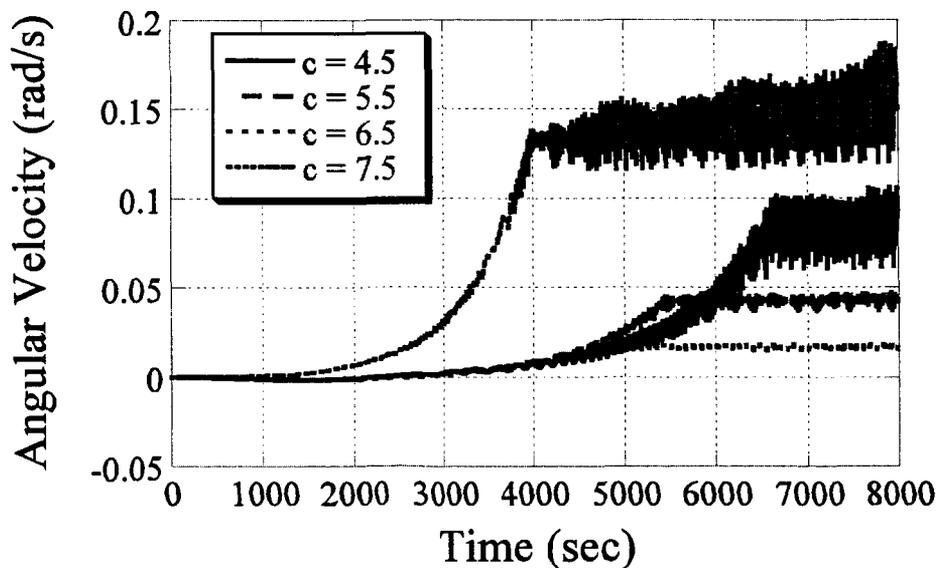


**Figure 17: Angular Velocity using Extensible Tether Model, 60 Degree Initial Swing Angle.**
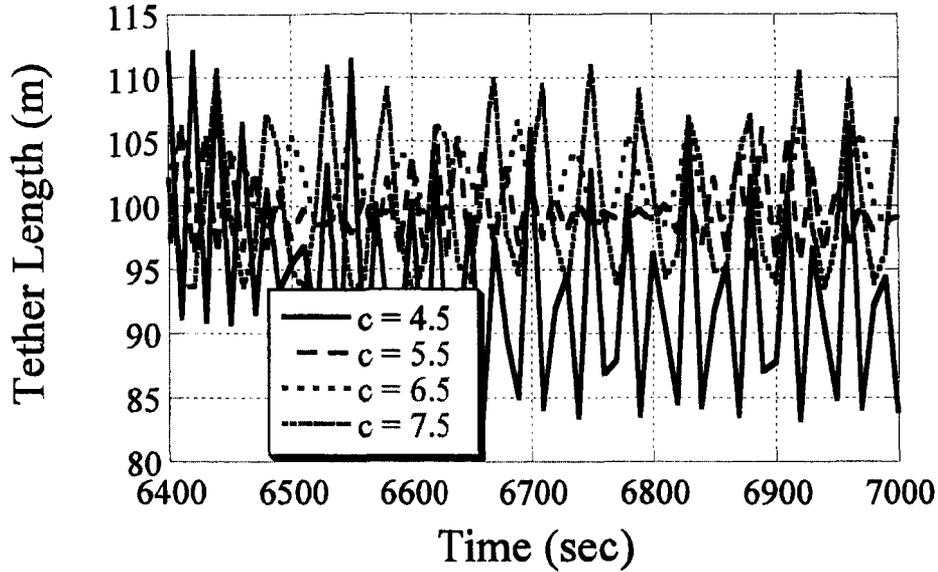
**Figure 18: Tether Length using Extensible Tether Model, 60 Degree Initial Swing Angle.**

Based on simulations of a wide range of initial swing angles and retrieval rates, the following methodology is used to create retrieval profiles: 1) use a retrieval rate, $c_1$, that results in high angular velocity for a brief amount of time to initiate retrieval, 2) use a retrieval rate, $c_2$, that results in low variations to complete the retrieval 3) the time to switch from the first to the second retrieval rate is given by

$$t_{switch} = \frac{L_{init}}{L_{switch}}(c_1 - c_2)$$

where it is assumed that $c_1 > c_2$. The tether length when the retrieval rate changes, $l_{switch}$, is given by

$$l_{switch} = l_{final}\exp(c_2 t_r)$$

The values of $c_1$ and $c_2$ are chosen using an optimization routine with the goal of maximizing angular velocity while keeping variations in angular velocity and tether length low. The retrieval must be completed in 90 minutes, the retrieval time of the unshaped case.

Figure 19 compares the angular velocities resulting from the original and the shaped retrieval profiles. The retrieval rates for the shaped profiles are given in Table 2. For the 60 degree initial swing angle, the shaped profile's angular velocity is slightly less than three times that of the original, with no significant increase in the variation of the angular velocity. For the 65 degree initial swing angle, the shaped profile's angular velocity is approximately nine times that of the original, with a 40% decrease in the variation of the final angular velocity. The 70 degree initial has the largest increase in and overall value of and variation in angular velocity.
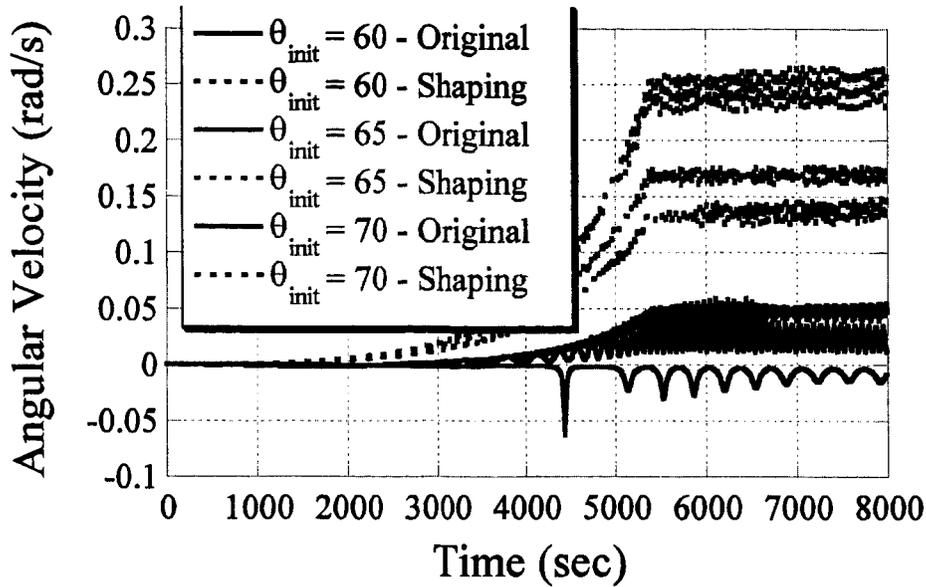
**Figure 19: Angular Velocity.**

**Table 2: Retrieval Rates for Shaped Profiles.**

|    | $c_1$   | $c_2$   |
|----|---------|---------|
| 60 | 7.5 e-4 | 5.0 e-4 |
| 65 | 7.0 e-4 | 5.0 e-4 |
| 70 | 6.5 e-4 | 5.5 e-4 |

Figure 20 shows the tether lengths resulting from the original and the shaped retrieval profiles. The shaped profiles result in tether lengths with variations of approximately six meters, a great improvement of the variations in length exhibited by the original profiles.
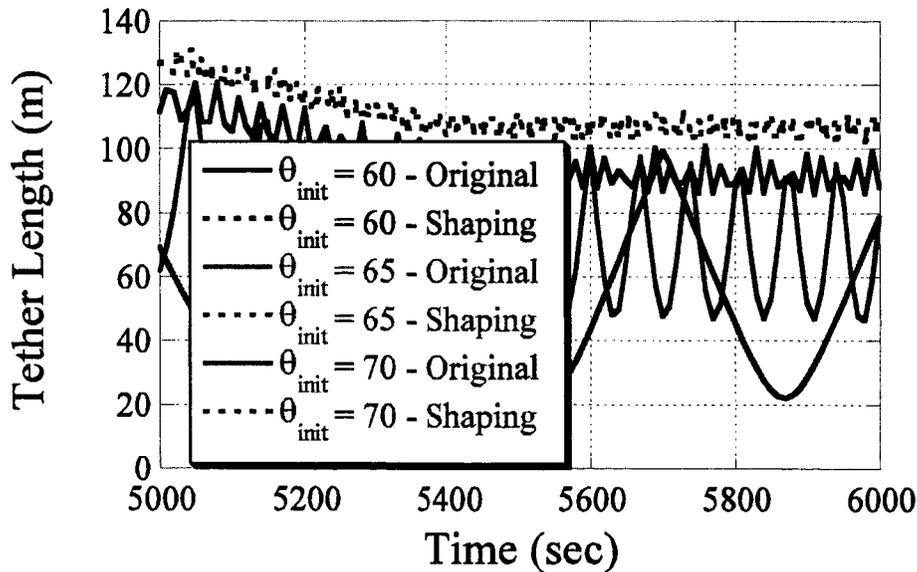


**Figure 20: Tether Length.**

Given the nonlinearity of the system, there are no substantial correlations between initial swing angle and the values $c_1$ and $c_2$ used to create the shaped profile. The effect of the retrieval rate, which has been shown to affect each initial swing angle differently, must first be determined before the retrieval rates can be chosen. Once this effect is known, the shaping procedure can then be applied.

## Publications and Presentations
There have been three conference papers presented and one journal article submitted from the research associated with this project:
1) "Tether-Based Actuators For Space Applications" International Workshop on Advanced Sensors, Structural Health Monitoring and Smart Structures, Keio University, Japan November 2003
2) "Command Generation for Tether Retrieval" AIAA/AAS Space Flight Mechanics Meeting, Maui, Hawaii February 2004
2) "Evaluation of Command Generation Techniques for Tethered Satellite Retrieval" AIAA/AAS Space Flight Mechanics Meeting, Copper Mountain, CO January 2005
1) "Electro-Dynamic Tethers for Orbit Boost" submitted to *Journal of Spacecraft and Rockets*, April 2005

## Future Work
This research demonstrated the how the use of command generation could decrease the retrieval time of tethered satellites. The model used in this stage of the investigation did not account for tether dynamics. Future work should use the commands developed for the simple model on the more complex model. Where necessary, the commands will be modified to optimize the performance of the more complex model. A more complex model for the tether can be modification of the model shown in Figure 10.

Additionally, this more complicated model of the tether can be used in the TAG simulations. Finally, the TAG model should be modified to reflect system parameters more inline with the eventual MXER model and the retrieval profile design procedure adjusted for the new system specifications.

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