Numerical simulations studies of the convective instability onset in a supercritical fluid

A. Furukawa[1], H. Meyer[2] and A. Onuki[1]
1. Department of Physics, Kyoto University, Kyoto 606-8502, Japan
2. Department of Physics, Duke University, Durham, NC 27708-0305, USA.
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Numerical simulation studies are reported for the convection of a supercritical fluid, ^{3}He, in a Rayleigh-Bénard cell. The calculations provide the temporal profile $\Delta T(t)$ of the temperature drop across the fluid layer. In a previous article, systematic delays in the onset of the convective instability in simulations relative to experiments were reported, as seen from the $\Delta T(t)$ profiles. They were attributed to the smallness of the noise which is needed to start the instability. Therefore i) homogeneous temperature noise and ii) spatial lateral periodic temperature variations in the top plate were programmed into the simulations, and $\Delta T(t)$ compared with that of an experiment with the same fluid parameters. An effective speed-up in the instability onset was obtained, with the best results obtained through the spatial temperature variations with a period of $2L$, close to the wavelength of a pair of convections rolls. For a small amplitude of 0.5 μK, this perturbation gave a semiquantitative agreement with experimental observations. Results for various noise amplitudes are presented and discussed in relation to predictions by El Khouri and Carlès.

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I. INTRODUCTION

In recent papers, convection experiments of supercritical ^{3}He at the critical density in a Rayleigh-Benard cell with a constant heat current q were reported[1, 2]. After q is started, the temperature drop $\Delta T(t)$ across this highly compressible fluid layer increases from zero, an evolution accelerated by the "Piston Effect" [3–5]. Assuming that q is larger than a critical heat flux necessary to produce fluid instability, $\Delta T(t)$ passes over a maximum at the time $t = t_p$, which indicates that the fluid is convecting and that plumes have reached the top plate. Then truncated or damped oscillations, the latter with a period $t_{\rm osc}$, are observed under certain conditions before steady-state conditions for convection are reached, as described in refs.[1, 2]. The scenario of the damped oscillations, and the role of the "piston effect" has been described in detail in refs.[6] and [7] and will not be repeated here. The height of the layer in the RB cell was $L = 0.106$ cm and the aspect ratio Γ=57. The ^{3}He convection experiments extended over a range of reduced temperatures between $5\times10^{-4} \le \epsilon \le 0.2$, where $\epsilon = (T-T_c)/T_c$ with $T_c = 3.318$ K, the critical temperature. The truncated - or damped oscillations were observed for $\epsilon \ge 0.009$ and over this range the fluid compressibility varies by a factor of about 30.

The scaled representation of the characteristic times $t_{\rm osc}$ and $t_{\rm p}$ versus the Rayleigh number, and the comparison with the results from simulations has been described in ref.[8]. Good agreement for the period $t_{\rm osc}$was reported. However a systematic discrepancy for the times $t_{\rm p}$ shows that in the simulations the development of convection is retarded compared to the experiments. This effect increases with decreasing values of $[Ra^{\rm corr} - Ra_c]$, where $Ra^{\rm corr}$ is the Rayleigh number corrected for the adiabatic temperature gradient as defined in refs.[1, 2] and Ra_c is the critical Rayleigh number, 1708. This is shown in Fig.1 of ref.[8], in particular in Fig.1b) for $\epsilon = 0.2$ and $q = 2.16\times10^{-7}$ W/cm^2 ($[Ra^{\rm corr} - Ra_c] = 635$), where an experimental run is compared with simulations for the same parameters. Here clearly the profile $\Delta T(t)$ from the simulations shows the smooth rise until the steady-state value, $\Delta T = qL/\lambda$

= 95 μK has been reached, where λ is the thermal conductivity. Only at t $\approx$ 90 s. does convection develop, as shown by a sudden decrease of $\Delta T(t)$. By contrast, the experimental profile shows a much earlier development of convection. Fig.1 of ref.[8] is representative for the observations at low values of $[Ra^{\rm corr} - Ra_c]$. At high values, both experiment and simulations show the convection development to take place at comparable times, as indicated in Fig.5b) of ref.[8], and specifically in Fig.2 a) of ref.[7], where $[Ra^{\rm corr} - Ra_c]$ =4.1$\times 10^5$. It is the purpose of this report to investigate the origin of this discrepancy by further simulation studies.

II. CONVECTION ONSET CALCULATIONS, SIMULATIONS AND COMPARISON WITH EXPERIMENTS

El Khouri and Carlès[9] studied theoretically the stability limit of a supercritical fluid in a RB cell, when subjected to a heat current q started at the time $t = 0$. Their fluid was also ^{3}He at the critical density, and the same parameters as in ref.[1] were used. They calculated the time $t_{\rm instab}$ and also the corresponding $\Delta T(t_{\rm instab})$ for the onset of fluid instability and they determined the modes and the wave vectors of the perturbations for different scenarios of q and ϵ. For $t > t_{\rm instab}$ inhomogeneities in the RB cell and noise within the fluid will produce perturbations which will grow, from which the convection will develop. An indication of the growth of convection is a deviation of the $\Delta T(t)$ profile in the experiments or in the simulations from the calculated curve for the stable fluid (see for instance Eq.3.3 of ref[6]). It is readily seen from simulation profiles such as Fig.1a) and b) in ref.[6] that the deviation becomes significant for t only slightly below $t_{\rm p}$ - the maximum of $\Delta T(t)$. In simulations, the effective start of convection can also be seen from snapshots in 2D of the fluid temperature contour lines at various times, as shown in Fig. 5 of ref.[10].

P.Carlès [11] has argued that the reason for the discrepancy for the time $t_{\rm p}$ between experiment and simulation is that in the former, the physical system has noise and inhomogeneities which cause the perturbations beyond $t_{\rm instab}$ to grow into the developed convection. By contrast simulations have a much smaller noise. Therefore in the simulations the perturbations take a longer time to grow than in the physical system, leading to a larger $t_{\rm p}$ than observed. Carlès' comment led us to try as a first step imposing a thermal random noise on the top plate of the RB cell, which was to simulate fluctuations in the upper plate temperature control of the laboratory experiment. The temperature of the plate was assumed to be uniform, because of the large thermal diffusivity $D_T \approx 2 \times 10^4$ cm^2/s. of the copper plate in the experiments. Accordingly simulations were carried out with a homogeneous time-dependent temperature random fluctuation of given rms amplitude imposed on the upper plate. This implementation consisted in adding or subtracting randomly temperature spikes T_t at the time t with a programmed rms amplitude at steps separated by 0.02 s. This interval is much larger than the estimated relaxation time of the top plate over a distance $2L$, approximately the wavelength of convection roll pair. Values of the variance A = $\sqrt{< (T_t - < T_t >)^2 >}$ were chosen between 0 and 40 μK. The range of the A values was taken well beyond the estimated fluctuation rms amplitude during the experiments[1] of $\approx 1 \mu K/\sqrt{Hz}$. Three representative curves with 0, 3 and 40 μK are shown in Fig.1a) by dashed lines for $\epsilon = 0.2$ for q= 2.16$\times 10^{-7}$ W/cm^2, L = 0.106cm and Γ = 5.1. For this value of q, the calculation by El Khouri and Carlès [12] give $t_{\rm instab}$ = 6.3 s and $\Delta T(t_{\rm instab})$ = 75 μK. In the simulation without imposed noise, the start of convection has therefore been considerably delayed relative to $t_{\rm instab}$. The injection of random noise has a significant effect in developing convection at an earlier time. In Fig.1a) the three curves are also compared with the experimental one, shown by a solid line. Here we have not incorporated into the simulations the delay affecting the experimental temperature recording, so that they could be intercompared more readily, and also with predictions[12]

However this operation will be presented in Fig.4. Further simulations with added random noise were carried out for $\epsilon = 0.2$ and 0.05 where the $\Delta T(t)$ time profiles are not shown here.

Fig.2a) shows a plot of the time of the developed convection, represented by t_p, versus the random rms amplitude A for three series of simulations, all taken for a cell with $\Gamma = 5.1$. They are a) and b) $\epsilon = 0.2$, $q = 2.16$ and 3.89×10^{-7}W/cm^2, and c) $\epsilon = 0.05$, $q = 60$ nW/cm^2, ($[Ra^{\mathrm{corr}} - Ra_c] = 635$, 1740 and 4200). The simulation results, shown by solid circles, are compared with the experimentally observed t_p shown by horizontally dot-dashed lines. It can be clearly seen that noise imposition, which creates a vertical disturbance across the fluid layer, reduces the time of convection development. While the decrease of t_p is strong for small values of A, it saturates at a certain level of noise amplitude. The gap between simulations and experiment increases with a decrease of $[Ra^{\mathrm{corr}} - Ra_c]$, namely as the fluid stability point is approached. A "critical slowing down" is seen in the effectiveness of the perturbations in triggering the instability. Hence this mode of noise introduction fails, because its amplitude is limited to the vertical z direction and it evidently couples only weakly into the convective motion.

In parallel with the present experiments, S. Amiroudine[13] also carried out a systematic study of simulations on supercritical ^{3}He in a RB cell for several values of ϵ and q. The resulting profiles $\Delta T(t)$ could be compared with those from experiments done under nearly the same conditions. In his simulations, homogeneous temperature random noise was again imposed on the top plate. The shift in t_p showed less systematic trends than in the results described in this report. However for the same values of ϵ and q as those reported above, and at zero noise, the t_p values tended to be somewhat smaller than in the results of Fig 2a).

Here we mention that the onset of convection in the simulations is further influenced by the aspect ratio Γ. The simulations described above, but without noise, were carried out in a cell $\Gamma = 5.1$ having periodic lateral boundaries. Further simulations with zero noise for $\epsilon = 0.2$ with $\Gamma = 8.0$, 10.2, 20.5 and 41.0 were carried out, and showed a decrease of the convection development time from ≈ 90 s, tending to a constant value of ≈ 60 s. above $\Gamma = 20$. This shift in the onset of instability is probably due to the decreased finite size effect which the rising plumes experience with increasing Γ, in spite of the periodic boundary conditions.

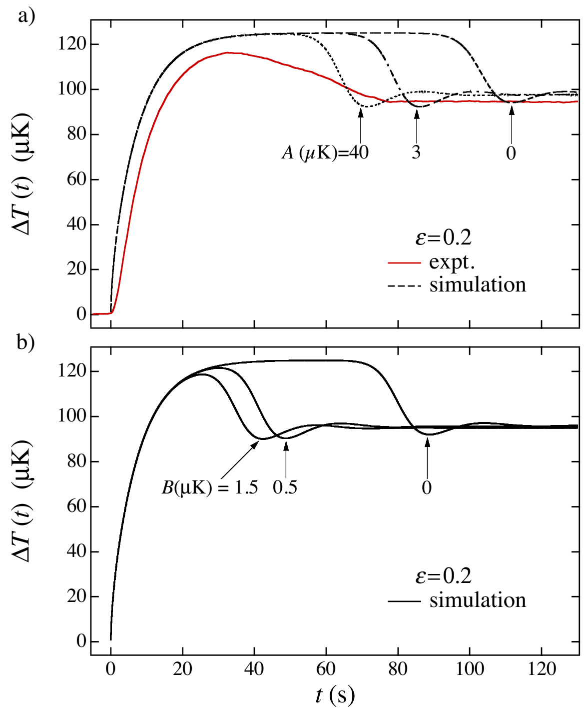

FIG. 1: a) the temperature profile $\Delta T(t)$ from experiments (solid line with noise) and from several simulations (dashed lines) at $\epsilon = 0.2$, q= 2.16×10^{-7} W/cm^2. In the simulations, Γ=5 and uniform temperature noise has been imposed on the top plate with variance $A(\mu K)$ = 0, 3 and 40, as described in the text. b) Temperature profile $\Delta T(t)$ from several simulations at $\epsilon = 0.2$, q= 2.16×10^{-7} W/cm^2, $\Gamma = 8$ and imposed lateral periodic, time independent temperature variations on the top plate with period $2L$ and amplitude $B(\mu K) = 0$, 0.5 and 1.5.

The next step in our attempts, stimulated by communications with P. Carlès, was introducing perturbations into the simulations via some time-independent lateral variation proportional to sin

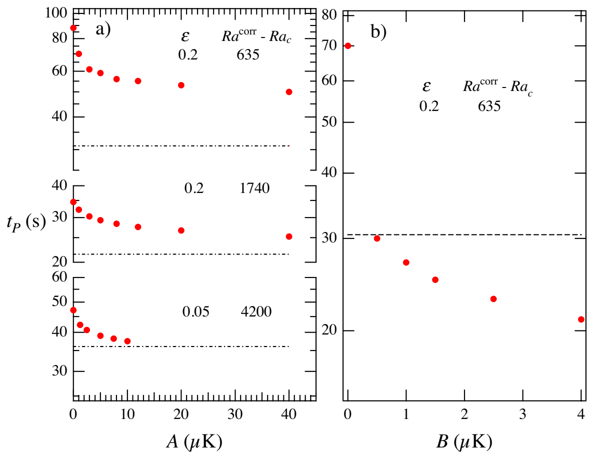

FIG. 2: a) The time for effective development of convection, characterized by t_p, versus A (homogeneous temperature noise imposed on the top plate). The horizontal dot-dashed lines indicate the observed t_p, corrected for instrumental recording delay. b) The time for effective development of the convection, labeled by t_p versus B (lateral time-independent periodic temperature variations). The horizontal dashed line indicates t_p as obtained by experiment, corrected as before.

$(2\pi x/P)$ where P is the period. A periodic variation in the spacing of the plates proved not to be feasible for our simulation code, and instead we opted to introduce again a temperature variation in the top plate with an amplitude B (in μK) and period P=2L, nearly the same as the wavelength of a pair of convection rolls. The temperature of the bottom plate was kept homogeneous. This "Gedanken Experiment" implies that the material of the top plate permitted a temperature inhomogeneity, which of course is not realized in the experiment. As a control experiment, we also made a simulation with $P = L$.

Fig 1b) shows representative profiles $\Delta T(t)$ for the parameters $\epsilon = 0.2$ and q= 2.16×10^{-7} W/cm^2 and with B = 0, 0.5 and 1.5 μK, and for Γ=8. For B = 0, the effect of having a larger Γ, namely 8 instead of 5.1, can be seen by comparing with the curve A=0 in Fig.1a). As B is increased from zero, there is a large decrease in the time for convection development, represented by t_p, which is plotted versus B in Fig 2b). The horizontal dashed line shows the t_p from the experiment, and this plot is to be compared with Fig. 2a). For an inhomogeneity amplitude of only B= 0.5μK, t_p is nearly the same for simulations and experiment. By contrast, simulations with B=2μK and P=L (not presented here) show no difference from those with B=0. Hence the nucleation of the convection is accelerated if the period is in approximate resonance with the wavelength of a convection roll pair. The values of steady-state ΔT and t_osc are only marginally affected by the noise.

We note from Fig.1b) that the simulation curve calculated for B = 0 shows the fluid not convecting until $\approx$ 70 s. For the curves with B= 0.5 μK., the start of deviations from the stable fluid curve cannot be estimated well from Fig.1b) but is readily obtained from the data files, which tabulate $\Delta T(t)$ to within 1 nK. For B = 0.5 μK, systematic deviations $\delta\Delta T(t, B) \equiv [\Delta T(t$,B=0) - $\Delta T(t$,B)] increase rapidly from 1 nK for t >8 s (where $\Delta T \approx$85 μK), a value comparable with the predicted t_instab= 6.3 s., $\Delta T(t_\mathrm{instab})$ = 75μK[12]. However a comparison with predictions becomes more uncertain as B is increased. This is because the changes in the base heat flow by the periodic perturbations are not considered in the theory[12]. We also note that the time interval $\delta t \equiv [t_\mathrm{p} - t_\mathrm{instab}]$ between the first sign of instability ($\delta\Delta T > 0$) and t_p is $\approx$ 20 s, and roughly independent of B. This represents approximately the period taken by the convection to develop and for the plumes to reach the top plate boundary.

In Fig.3, we present a series of 2D "snapshots" at various times for the simulation with B = 0.5μK, showing the temperature contour lines (in color) for the RB cell. The "warm" side is shown by red, $T(t, z = 0)$ and the "cold" side by mauve, $T(z = L)$ = const. At t= 8 s. the fluid instability has just started near the top of the layer, while near the bottom the temperature contour lines are still horizontal. At t = 27 s., where the peak of $\Delta T(t)$ at $z = L$ has been reached, the warm plumes have reached the top plate, and the "cold" piston effect is about to start, causing the bulk fluid temperature to drop and $\Delta T(t)$ to decrease. The transient process con-

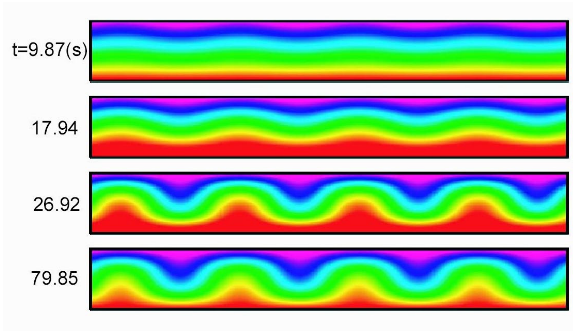

FIG. 3: Snapshots in 2D for the RB cell with an aspect ratio of 8, of simulations with B = 0.5 μK at various times t after starting the heat current q. The temperature contour lines and their evolution are described in the text. At the time t = 9.9s, the fluid instability has just started near the top.

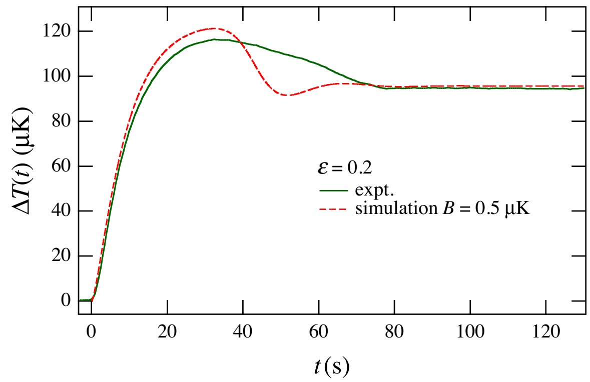

FIG. 4: Comparison of the profile $\Delta T(t)$ from experiment and from simulations with B = 0.5 μK . To make the comparison realistic, the simulations have been convoluted with the same "instrumental" delay time τ =1.3 s. which has influenced the shape of the experimental curve.

tinues with damped oscillations of $\Delta T(t)$. Steady state convection is reached at t= 80s, with a pair of convection rolls having a wavelength of $\approx 2L$, as expected.

In Fig.4 we show the profiles $\Delta T(t)$ from the experiment and from the simulations with a periodic perturbation amplitude B = 0.5 μK. For an optimal comparison, the delay affecting the experimental temperature recording was incorporated into the simulation curve. For this, the delay function with the instrumental time constant τ = 1.3 s. [1] was folded into the simulation curve by a convolution method. This operation retards the initial rise of the temperature drop by the order of 2-3 seconds, and brings both experiment and simulations into fair agreement in the regime where the fluid is stable. The time $t_{\rm p}$ for the maximum is now closely the same for both experiments and simulations. However beyond the predicted instability time $t_{\rm instab}$= 6.3 s., the experimental curve starts to deviate more rapidly with time than do the numerical simulations from the calculated curve for the fluid in the stable regime. As discussed previously[2], for these parameters of ϵ and q the experiment does not show damped oscillations, which are observed for higher values of q. In the steady-state, the agreement is very good.

Our goal has been to show that injecting a small temperature perturbation into the top plate, produces for the simulations an earlier start in the convective instability, which becomes consistent with experimental observations. For this, we have limited ourselves to an example at a low value of $[Ra^{\rm corr} - Ra_c]$, where the delay has been particularly large with respect to the experiment.

III. SUMMARY AND CONCLUSION

We have presented a comparison of numerical simulations with experimental data investigating the transient to steady convection after the start of a heat current through a supercritical ^{3}He layer in a RB cell. Here the temperature drop $\Delta T(t)$ across the fluid layer versus time t was studied. The aim was to understand and to reduce the discrepancy between experiment and simulations in the time of the convection development, as detected by $\Delta T(t)$. Simulations for one set of fluid parameters (where the largest discrepancy had been observed) are reported with imposed temperature variations on the top plate. Satisfactory results were obtained for spatial lateral temperature variations with an amplitude of 0.5 μK and a period approximately equal to that of the wavelength of a convection roll pair. As the perturbation amplitude is further increased, the development of convection occurs earlier than the observed one.

IV. ACKNOWLEDGMENT

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