Shape of Strained Solid ^{4}He at Low Temperatures

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outline

1. introduction to stress-driven instability
2. motivation: related phenomena on geologic, kitchen and nano-scales
3. intuitive picture of stress-driven instability

   (without and with gravity, critical stress, why use helium?)
4. apparatus: interferometry set up
5. results
6. summary

1. Introduction to stress-driven instability

Start with a solid in equilibrium with its melt.
Apply external stress.

Q: what happens to its shape?

Theoretical study by: Asaro-Tiller-Grinfeld*

(Asaro-Tiller started their theory work on cracks and corrosions induced by stress. Grinfeld independently generalized the theory to apply to other situations.)

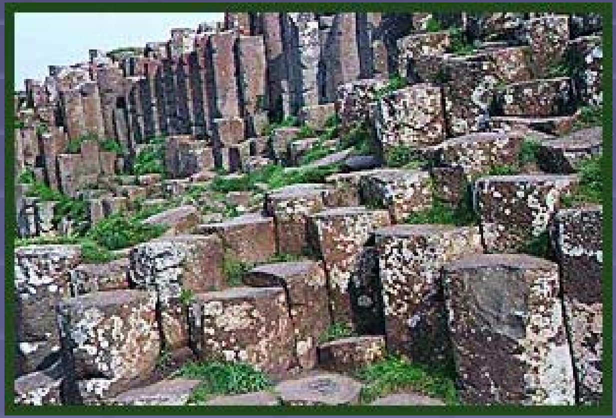
A: beyond a threshold stress, the surface is predicted to become unstable and to develop corrugations.

Present research: use solid ^{4}He in contact with superfluid melt to experimentally study the predicted instability.

*Asaro, R.J.Tiller, W.A, Mettal. Trans. **3**, 1789(1972).
Grinfeld, M., Soviet Phys. Dokl. **31**, 31 (1986).

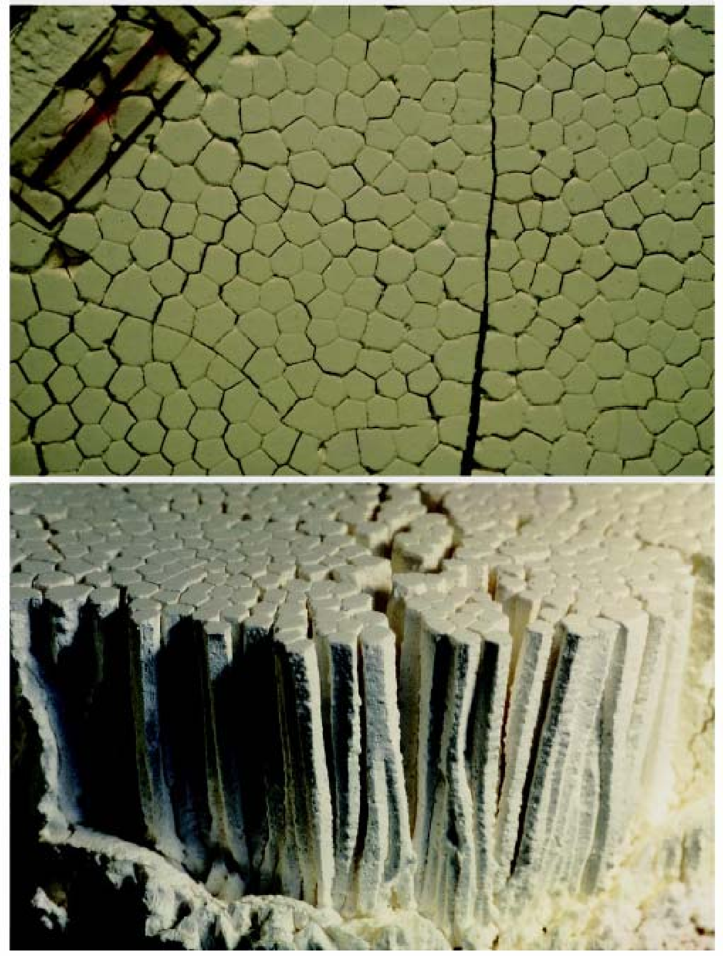
2. Phenomena related to stress-driven instability

(1)Giant's Causeway Northern Ireland



These are photographs for Giant's Causeway in Northern Ireland. These geologic formations are thought be formed by the rapid cooling of molten lava. The rapid cooling can produce large stresses within the lava. These stresses are thought to have produced these patterned surface formation. So this is (possibly) an example of stress-driven instability on a grand geologic scale. Of course, the natives have another theory based on a fight between Giants. But, that's another story.

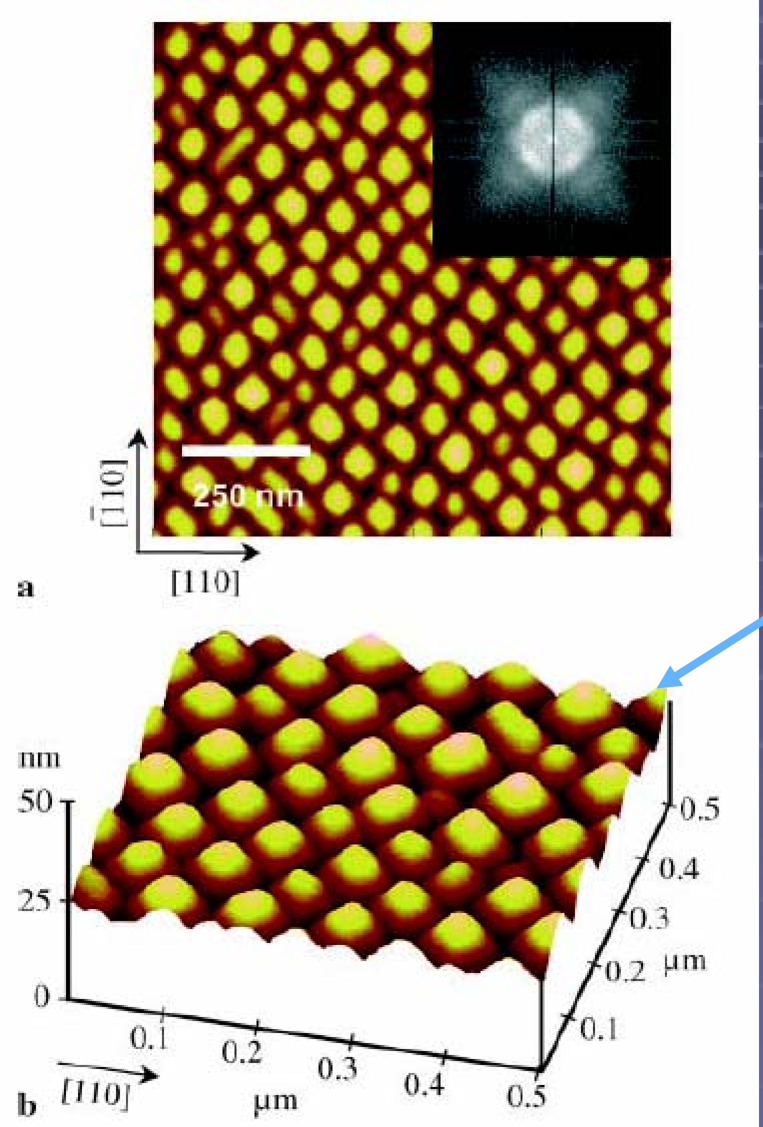
(2)Drying starch



Structures somewhat similar to the Giant's causeway can be produced in a kitchen. Muller* made the observation by drying a beaker of wet starch. The photos at right show the regular pattern seen after drying. This may be another example of stress-driven instability effect, now on the mm scale rather than meter scale.

*G. Muller, J. Volcanology and Geothermal Research 86, 93(1998)

(3) nano-structures by self-organization



Strain mediated self-organization of {105} facetted SiGe crystallites in a SiGe/Si superlattice grown on Si(001). **a** 1 µm×1 µm AFM image of the 20th alloy layer of a 20×(2.5 nmSi0.25Ge0.75/10 nm Si) multilayer film, color scale range: 20 nm. (The *inset:* the 2D power spectrum
calculated from a 5 µm×5 µm image).

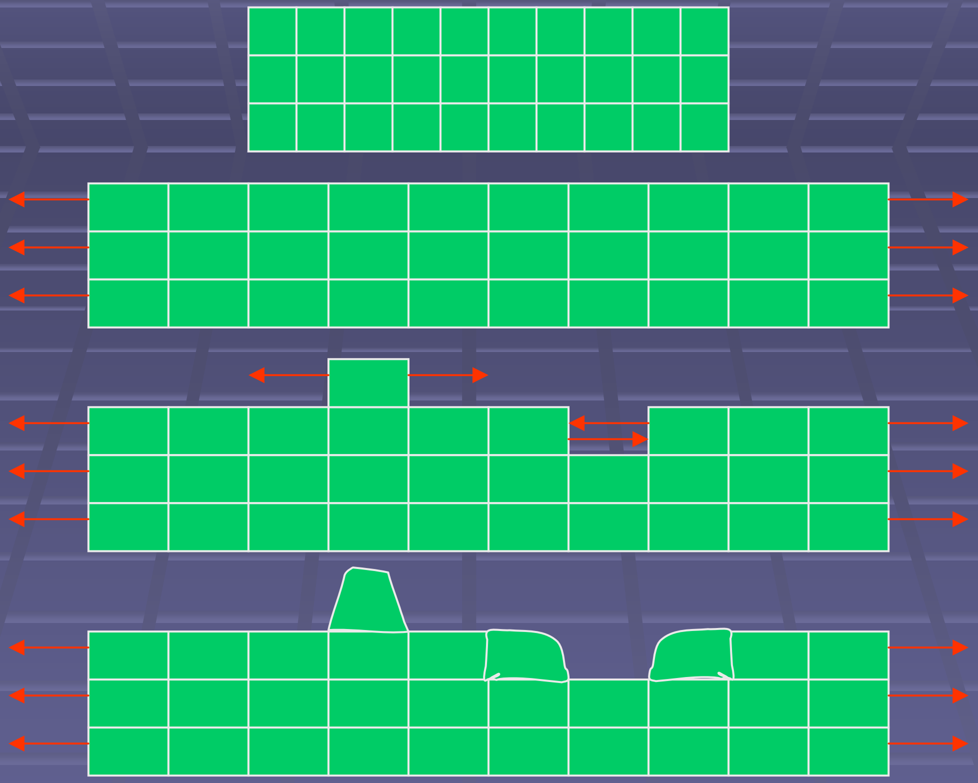
3D AFM image of multiple adlayers of Si-Ge grown on Si substrate.

C. Teichert, Applied Phys. **A76**, 653 (2003)

The lower figure is an AFM image of the multiple layers of Silicon-Germanium grown on top of Silicon substrate. As the number of adlayers is increased, there is an island like pattern that develops on a nano-meter scale. The driving mechanism for this is thought to be the shear stress between the substrate and the adlayer. The stress comes from the lattice mismatch. The mass motion is the atomic diffusion on the surface.

3. Intuitive picture of stress-driven instability

(no gravity, no surface tension)



begin with solid
under zero stress

apply uniaxial stress

$E_{regular}$

fluctuation moves
one element, with
constant stress

$E_{corrugated} = E_{regular}$

without stress,
solid relaxes

$E_{corrugated} < E_{regular}$
→ instability!

The idea of stress-driven instability is illustrated with the help of these cartoons. Start with a piece of solid with no stress. Apply uniaxial stress from the side. The solid accumulates an elastic energy with regular strain. Suppose, by fluctuation, a cell melts and forms on top surface. If the all cells retain the regular shape, there is no change in elastic energy. If the cells at the bump and the dip are allowed to relax, the elastic energy will decrease. The process is then unstable and leads to instability. The effect is different from Euler buckling instability which occurs under compression only. And it occurs without mass transfer. I emphasize that the mass transport is important in this rearrangement instability.

quantitative basis

(<u>including</u> gravity and surface tension -- these oppose surface deformation)

Assume: surface height profile: $h(x) = \eta \, \mathrm{Cos}(kx)$

net energy cost: U = (gravity) + (surface tension)+ (elastic energy)

$$U = \frac{\eta^2}{4}\left[(\rho_{solid} - \rho_{liq})g + \gamma k^2 - 2\frac{(1-\sigma^2)}{E}\sigma_0^2 k\right]$$

g = gravitational constant

γ = surface tension ~ 0.3 dyn/cm

σ = Poisson's ratio

E = Young's modulus

σ_o = Applied stress

critical or threshold effects:

$$\text{critical stress}: \sigma_C = \sqrt{\frac{2\pi\gamma E}{\lambda_C(1-\sigma^2)}}$$

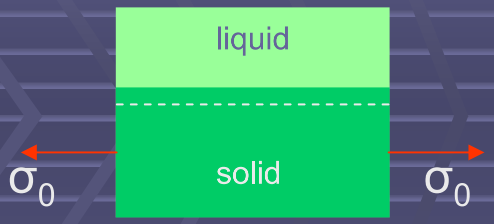
$$\text{critical wavelength}: \lambda_c = \sqrt{\frac{\gamma}{g(\rho_{solid} - \rho_{liq})}}$$

When the critical stress (or strain) is exceeded, corrugation with wavelength (λ_C) of ~ 6 mm should appear.

$\sigma_c = 2.4\mathrm{x}10^4$ dyn/cm^2 or $u_c = 8\mathrm{x}10^{-5}$ (critical strain)

The instability on the preceding page due to the decrease in elastic energy is opposed by gravity and surface tension. So at low stress levels, the surface remains flat. Beyond some critical stress, the instability sets. The instability can be made quantitative by considering sinusoidal surface height profile. Analysis shows that when a critical stress is exceeded, corrugation of wavelength of about 6 mm should appear. We wish to study this stress-induced instability on solid He-4. The critical stress or critical strain is easily achievable on solid He-4. Note the importance of gravitational acceleration constant.

small stress below critical stress



applied stress increases the elastic energy of solid … chemical potential increases … equilibrium is broken

→

chemical potential of liquid must increase to come to equilibrium … solid melts!

Assume isotropic solid for simplicity

equilibrium is maintained if →

$$\delta\mu_{solid} = \delta\mu_{liquid}$$

$$\delta\left(\frac{f_{solid} - \sigma_{zz}}{\rho_{solid}}\right) = \frac{\delta P}{\rho_{liquid}}$$

$$\delta h = -\frac{(1-\sigma)\sigma_0^2}{2Eg(\rho_{solid} - \rho_{liquid})}$$

f_{solid} = free energy/vol

σ = Poisson's ratio

E = Young's modulus

= $3\text{x}10^8$ dyn/cm^2 solidHe

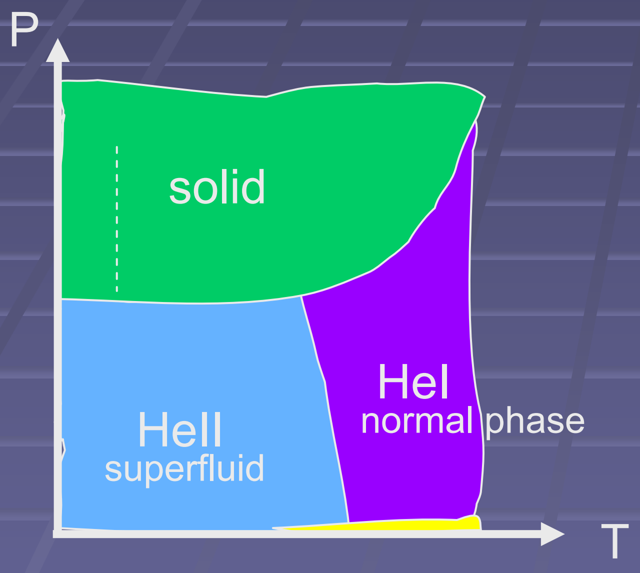
= ~10^{11} dyn/cm^2 Lead

The height of solid decreases under stress.

$\delta h = -\,8.2\text{x}10^{-11}\,\sigma_0^2$ (cgs)

What happens if the applied stress is less than the critical stress? Under stress, the chemical potential of the solid increases. In equilibrium, the chemical potentials of the two phases remain equal. To maintain equilibrium, the chemical potential of the liquid must then increase. That means, the solid should melt and the liquid depth should increase. We wish to observe this as a check on the applied stress in the linear response regime.

Why use solid He-4?



Advantages:

- **high purity**
- **superfluid "melt"**

  rapid heat transfer
- **small latent heat**
- **rapid melting and freezing**
- **relative ease in growing crystals**
- **mm scale corrugations**

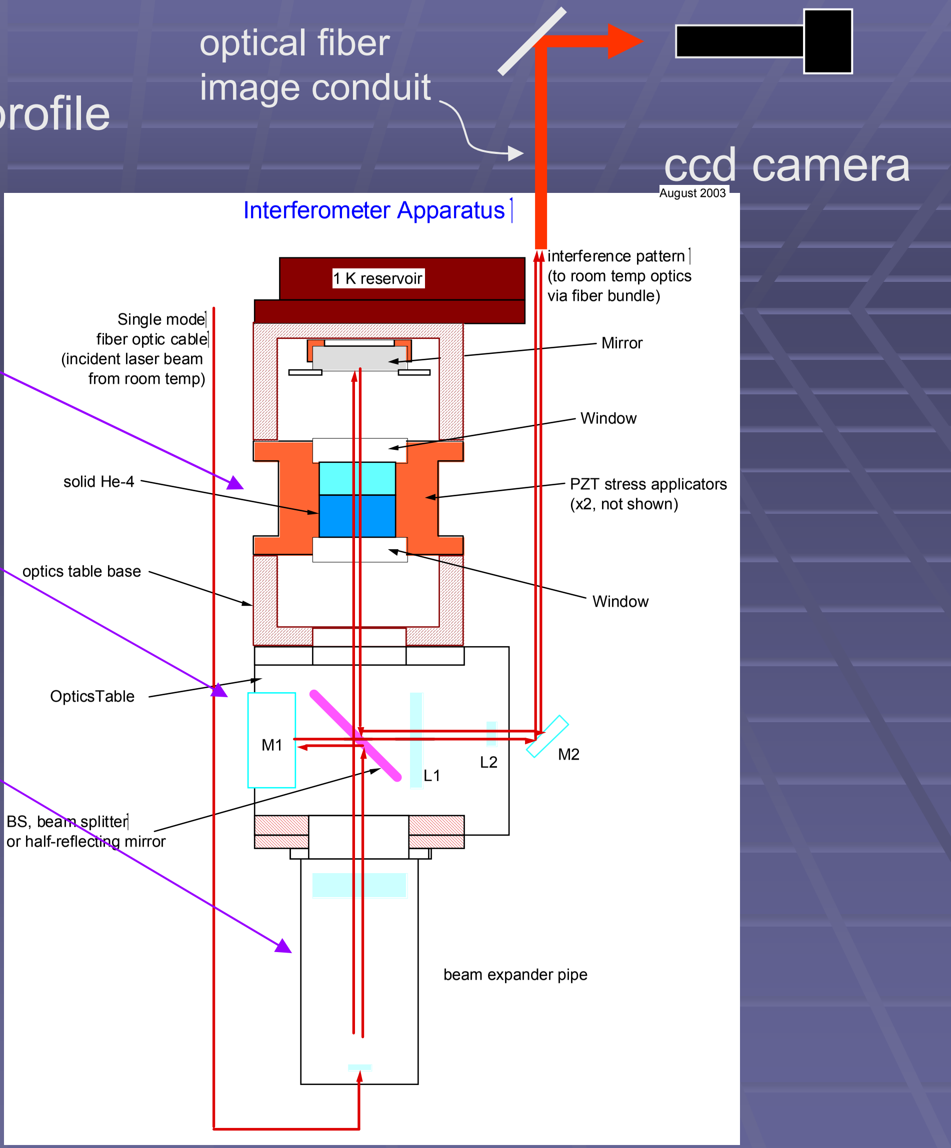
4. Apparatus

interferometer method to detect surface profile

solid He chamber

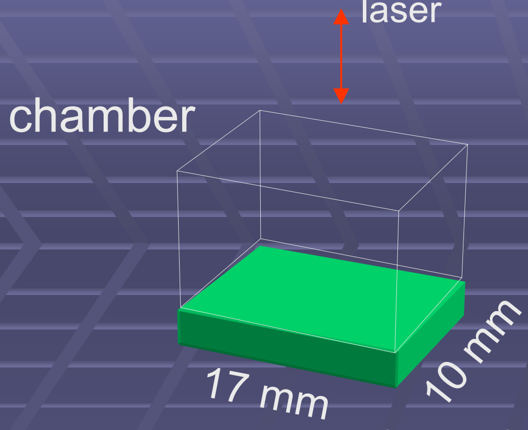
optics table

beam expander



We have constructed an interferometer apparatus to observe the solid height profile. Illumination He-Ne laser beam is fed into vacuum can at low temperature with an optical fiber. The beam goes through an expander and split into a reference beam and a probe beam. The probe beam goes through the chamber containing solid and liquid. The two beams combine to form an interference image. The difference in index of refraction and the spatial dependence of the solid height leads to changes in interference pattern. The interference image is captured with a CCD camera.

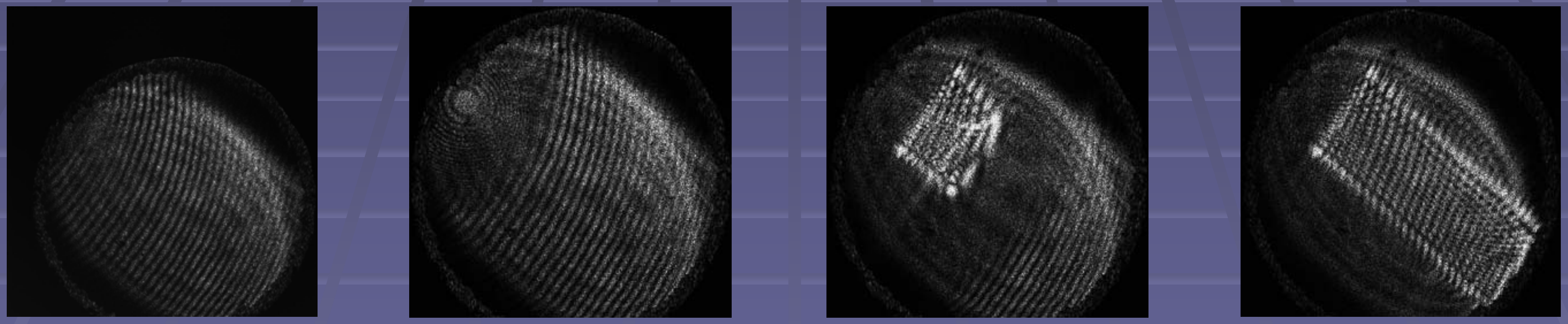
5. Results



Process of formation, growth and melting of solid ^{4}He

$T = 1.2$ K $P_{\text{melting}} \sim 25$ bars

time sequence of interference pattern as solid grows in the chamber
(total elapsed time = 70 min.)

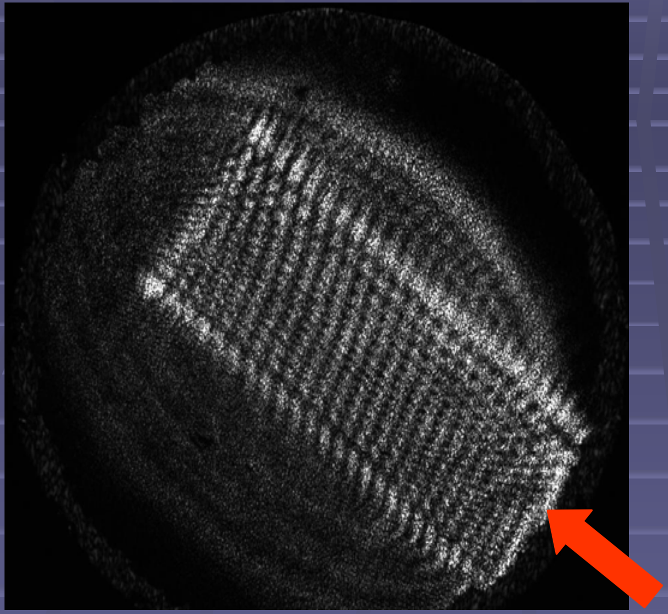


all liquid → solid seed appears → solid spreading → flat solid surface

A schematic of the chamber is shown. The movie shows the development of interference pattern from liquid. The temperature is near 1.2 K. Initially, the chamber contains all liquid near melting pressure. The pattern is caused by the misalignment of the two mirrors. The pressure is increased by forcing liquid into the chamber. A solid seed can be seen in the second video frame. Solid begins to grow and spread across the chamber bottom as in the third frame. As more helium is introduced, solid spreads all over the bottom surface of the chamber.

applying stress to solid

Piston attached to PZT pushes or pulls on solid.



stress by piston
attached to PZT

strain = 5.8×10^{-8} x (applied voltage), $0 < \text{strain} < 2\times10^{-4}$

Convert fringe (or phase) shift → height change (1 fringe shift = 93 μm)

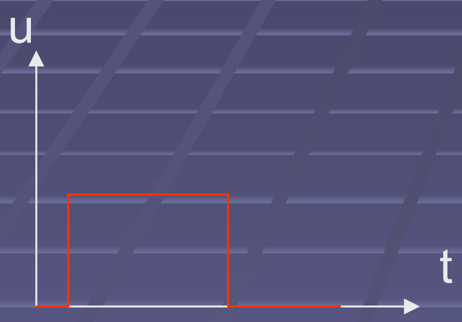
Stress is applied from one end of the chamber as shown. We use a tubular PZT to push a piston which in turn pushes or pulls on the edge of solid. We can apply more than 4500 V to PZT. The fringe pattern moves as the stress is applied. The changes of fringe pattern is converted to phase shift and finally to height changes.

apply strain, $u < u_c$

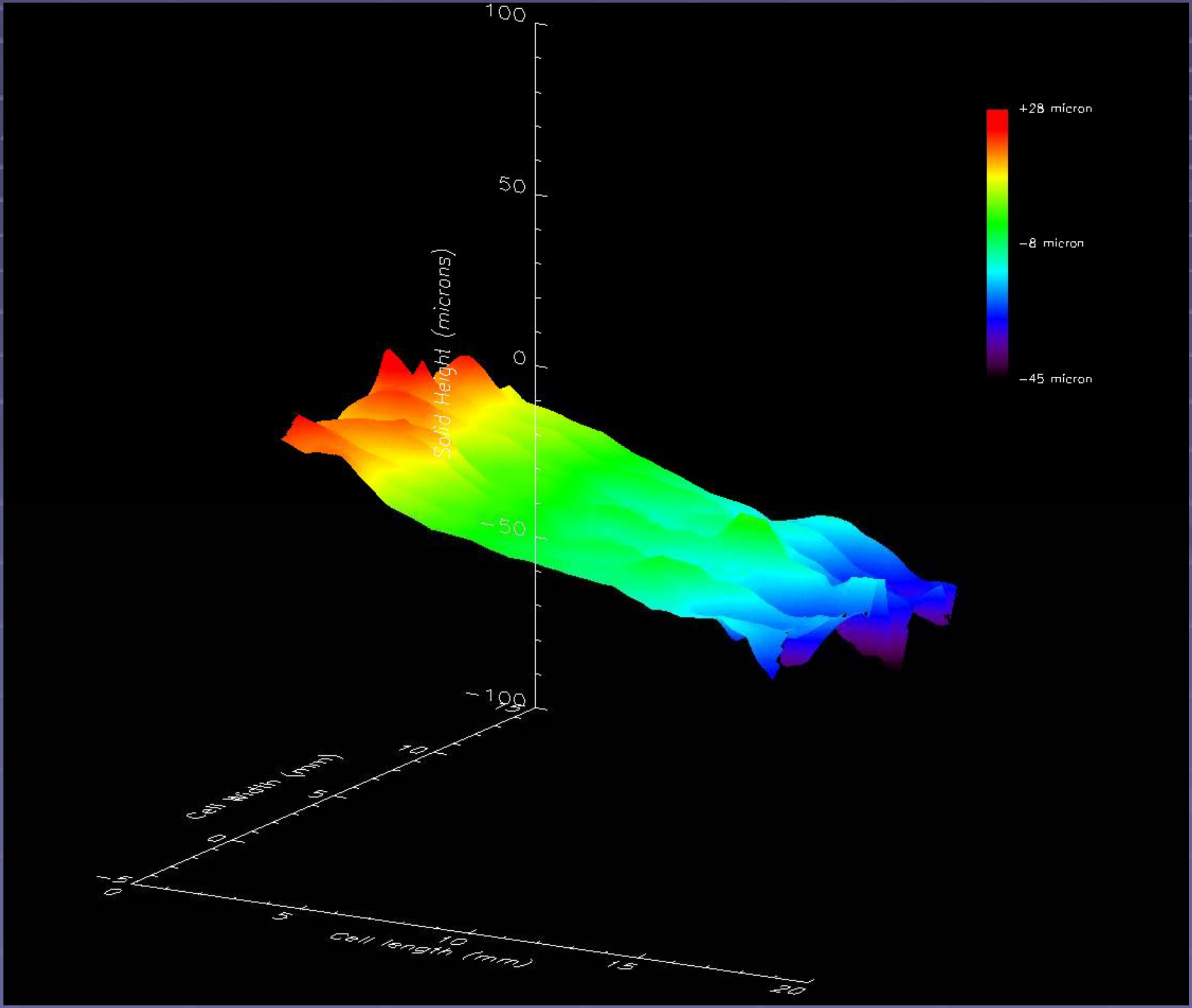
Uniform decrease in solid surface height is expected.

$u = -5 \times 10^{-5}$

$|u_c| \sim 8 \times 10^{-5}$



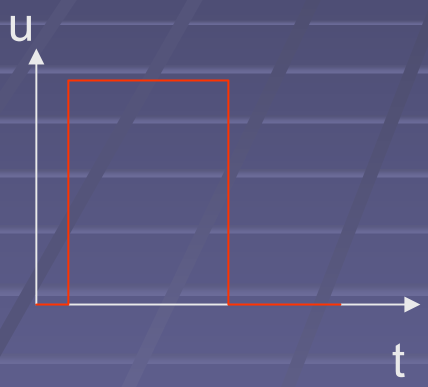
A strain smaller than the critical strain is applied at the beginning and is removed a little later. We expect to see a uniform decrease in solid height. We see a linear slope appearing as shown. When the strain is removed, the surface profile returns to the original flat shape. The profile motion is reversible but different from the expected behavior.

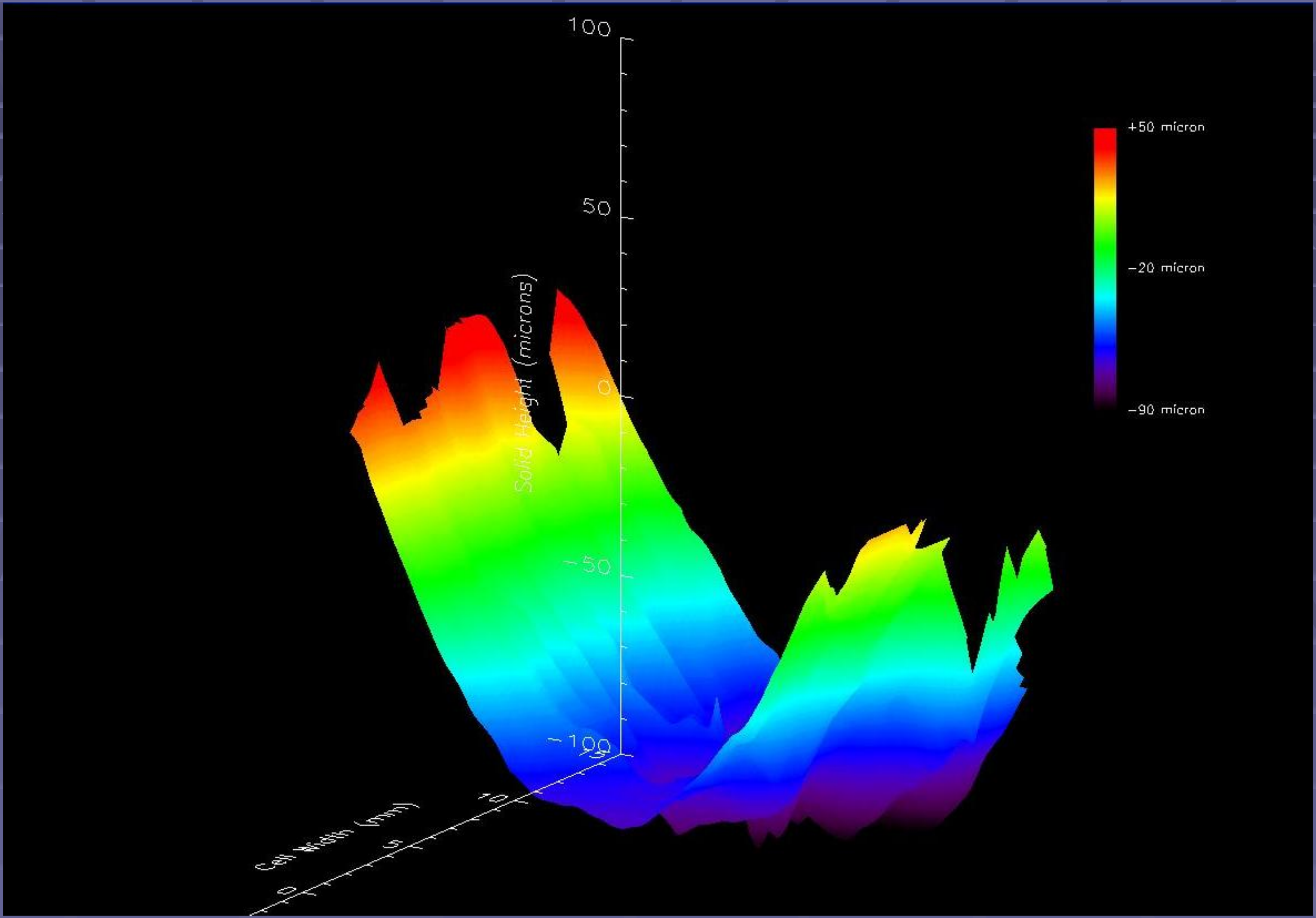


apply strain, $u > u_c$

Corrugation on solid surface height is expected.

$u = -11\times10^{-5}$
$|uc| \sim 8\times10^{-5}$





Let's see what happens when the applied strain is greater than the expected critical threshold. We expect to see corrugations according to theory. After the strain is applied, the surface profile develops an undulation as shown. When the strain is removed, the surface profile does not return to the original flat profile. The process is irreversible. The wavelength of the undulation is 15 mm, not 6 mm as expected. The onset of the undulation is not sudden as a function of strain. The undulation sets in more or less gradually

Summary

(1) Interferometer apparatus for measuring surface profile

(2) For small strains, the expected linear decrease in height is <u>not</u> seen.

(3) For large strains, undulation and irreversible deformations begin to set in, but we cannot yet make clear connection with stress-driven instability theory.

- Torii and Balibar have observed appearances of deformations beyond threshold stress on ^{4}He solid surface.
- The difference of our results from theory real? We are not ready to claim in affirmative.
- To be able to answer:
  - improve crystal growth techniques … orientation, annealing, better pressure control
  - improve homogeneity of stress … better alignment with vertical, better understanding of interaction between solid He-4 and walls
  - improve optics

acknowledgements

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