Geopositional Statistical Methods

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Outline

- Background
- Sources of error in geopositional assessment
- Error model
- Discussion of geopositional error computation methods
- Modeled performance of geopositional error computation methods
- Conclusions and recommendations

Background

- 1947 – U.S. Bureau of the Budget. National Map Accuracy Standards.
  - Establishes equivalent of circular error criteria as error standard of maps of various scales.
- 1962 – Clyde Greenwalt and Melvin Shultz. Principles of Error Theory and Cartographic Applications.
  - Provides rigorous treatment of circular error assuming that error is
    - Zero mean (no horizontal bias)
    - Normally distributed
    - Near-circular
- 1963 – Melvin Shultz. Circular Error Probability of a Quantity Affected by a Bias.
  - Provides limited treatment of error with horizontal bias.
- 1990 – MIL-STD-600001. Mapping, Charting and Geodesy Accuracy.
  - Adopts the 1963 Shultz approach to horizontal bias. Discusses empirical approach as an alternative estimate.
- 1998 – Federal Geographic Data Committee. National Standard for Spatial Data Accuracy (NSSDA).
  - Adopts Greenwalt and Shultz approach, but swaps RMSE for standard deviation. No provision for horizontal bias.
- 2003 – Joseph McCollum (USFS). Map Error and Root Mean Square.
  - Paper calls Greenwalt and Shultz method into question.
- 2003 – USGS Proposal for Revision of NSSDA.
  - Out of Geography Discipline. POC: John Conroy, jconroy@usgs.gov.
- 2004 (first version 2002?) – Tom Ager (NIMA InnoVision). An Analysis of Metric Accuracy Definitions and Methods of Computation.
  - White paper supports empirical approach. Also modifies Shultz approach to provide for large horizontal bias.

Revision Status

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- The revision of the NSSDA standard is currently in step 4, or the draft stage, of the 12-step FGDC standards approval process (http://www.fgdc.gov/standards/directives/dir1.html).
- Progress on the standard development will continue based on funding priorities.

Proposal Stage
Step 1, Develop Proposal
Step 2, Review Proposal
Project Stage
Step 3, Set Up Project
Draft Stage
Step 4, Produce Working Draft
Step 5, Review Working Draft
Review Stage
Step 6, Review and Evaluate Committee Draft
Step 7, Approve Standard for Public Review
Step 8, Coordinate Public Review
Step 9, Respond to Public Comments
Step 10, Evaluate Responsiveness to Public Comments
Step 11, Approve Standard for Endorsement
Final Stage
Step 12, Endorsement

Sources of Error in Geopositional Assessment

- Assessment Error
  - Ground Control Error
    - Pointing
    - Measurement
  - Analyst Error
    - Pointing
- Product Error (potential)
  - Spatial Resolution
  - Pointing (Displacement)
  - Azimuth
  - Scale
  - Orthogonality
  - Other product distortion
  - Terrain effects

- "Pointing error" for surveyors & analysts is here intended to mean the errors these individuals have in picking their target.

- **random error**

- "Measurement error" for ground control is here intended to mean the error inherent in the measuring instrument or system (GPS in this case).

- **constant systematic error**

- "Pointing error" for a geo-imaging system is here intended to mean the constant separation between estimated target coordinates and actual target coordinates.

- **functional systematic error**

Check Point Error

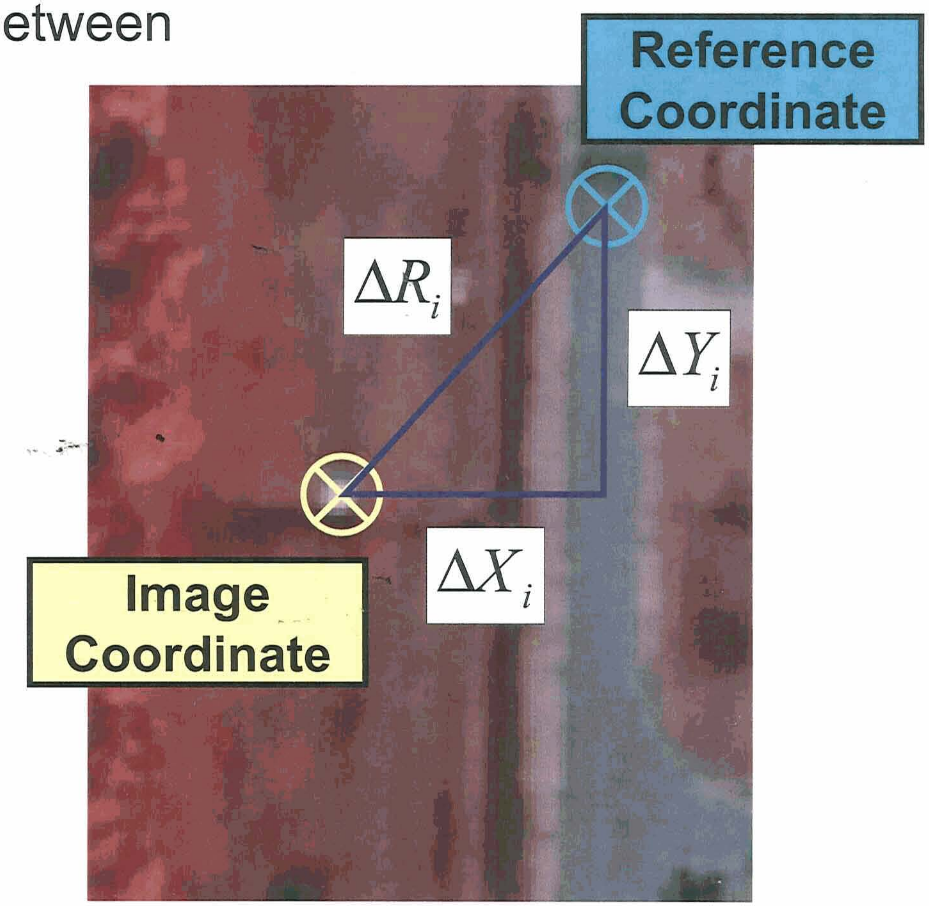
- Check Point Error – differences between image and reference coordinates

$$\Delta X_i = X_{image,i} - X_{reference,i}$$

$$\Delta Y_i = Y_{image,i} - Y_{reference,i}$$

- Check point error radial magnitude calculated by

$$\Delta R_i = \sqrt{\Delta X_i^{\,2} + \Delta Y_i^{\,2}}$$



Error Component Estimates

- The error model chosen for generalized assessment

$$X_{image} = X + \varepsilon \quad \text{where} \quad \varepsilon = \varepsilon_{constant} + \varepsilon_{zero-mean}$$

- Horizontal Bias – an estimate of the constant error, designated here as μ_H, is the magnitude of the vector sum of the average error in the *X* and the *Y*

$$\mu_H = \sqrt{\left(\overline{\Delta X}\right)^2 + \left(\overline{\Delta Y}\right)^2}$$

- Circular Standard Error – an estimate of the zero-mean circular equivalent error valid even for elliptical error distributions with minimum to maximum error ratios as low as 0.6

$$\sigma_C \cong \frac{\sigma_{\Delta X} + \sigma_{\Delta Y}}{2} \quad \text{where} \quad \sigma_{\Delta X} = \sqrt{\frac{\sum\left(\Delta X_i - \overline{\Delta X}\right)^2}{n-1}} \quad \& \quad \sigma_{\Delta Y} = \sqrt{\frac{\sum\left(\Delta Y_i - \overline{\Delta Y}\right)^2}{n-1}}$$

Tom Ager used the horizontal error defined on the right, but Greenwalt and Shultz found this to be invalid for minimum to maximum error ratios less than 0.8.

$$\sigma_H = \sqrt{\frac{\left(\sigma_{\Delta X}^{\;2} + \sigma_{\Delta Y}^{\;2}\right)}{2}}$$

RMSE Definitions

- RMSE – Root mean square error (horizontal bias & zero-mean error not decoupled)
  - 1D

$$RMSE_x = \sqrt{\sum \frac{\Delta X_i^2}{n}} \qquad RMSE_y = \sqrt{\sum \frac{\Delta Y_i^2}{n}}$$

  - 2D (NSSDA General)

$$RMSE_r = \sqrt{RMSE_x^2 + RMSE_y^2}$$

  - 2D (NSSDA Case 2*)

$$RMSE_c = 0.5 * (RMSE_x + RMSE_y)$$

* $RMSE_c$ is a recasting of terms in formula from NSSDA Appendix A Case 2. It is not found explicitly in the NSSDA.

Circular Error Definitions

- CE_{90} – The radial error which 90% of all errors in a circular distribution will not exceed (adapted from Greenwalt and Shultz, 1962)
  - Equivalent to the Circular Map Accuracy Standard (CMAS)
- CE_{95} – The radial error which 95% of all errors in a circular distribution will not exceed (adapted from Greenwalt and Shultz, 1962)
  - Equivalent to $Accuracy_r$ (from NSSDA)
- In the normal case, circular error may be generally defined as the circle radius, R, that satisfies the conditions of the equation below (where *C.L.* is the desired confidence level); however, *there is no analytical solution to this equation*.

$$C.L. = \int_{-R}^{R} \int_{-\sqrt{R^2 - x^2}}^{\sqrt{R^2 - x^2}} \frac{1}{2\pi\sigma_x\sigma_y\left(1-\rho^2\right)} \exp\left[\frac{-1}{2\left(1-\rho^2\right)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right] dydx$$

Common CE_{90} Estimates

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- RMSE based (NSSDA)
  - Appendix A: General → $CE_{90} = 1.5175 \cdot \mathrm{RMSE}_r$
  - Appendix A: Case 2 → $CE_{90} = 2.1460 \cdot \mathrm{RMSE}_c$

- Bias and Standard Circular Error based
  - Sum of squares → $CE_{90} = \sqrt{(2.1460 \cdot \sigma_C)^2 + {\mu_H}^2}$
  - Shultz approach accounting for bias → $CE_{90} = 2.1272\sigma_C + 0.1674\mu_H + 0.3623\frac{{\mu_H}^2}{\sigma_C} - 0.055\frac{{\mu_H}^3}{{\sigma_C}^2}$
  - Ager approach accounting for bias (modified Shultz) →
    - When $\mu_H/\sigma_C \leq 0.1$ $CE_{90} = 2.1460\sigma_C$
    - When $0.1 < \mu_H/\sigma_C \leq 3$ apply equation from Shultz
    - When $\mu_H/\sigma_C > 3$ $CE_{90} = 0.986\mu_H + 1.4548\sigma_C$

- Empirically estimated
  - 90th percentile → $CE_{90} = 90^{th}$ percentile of ΔR
  - Radial error for 1st point of percentile rank > 90

Circular Error Modeling Study

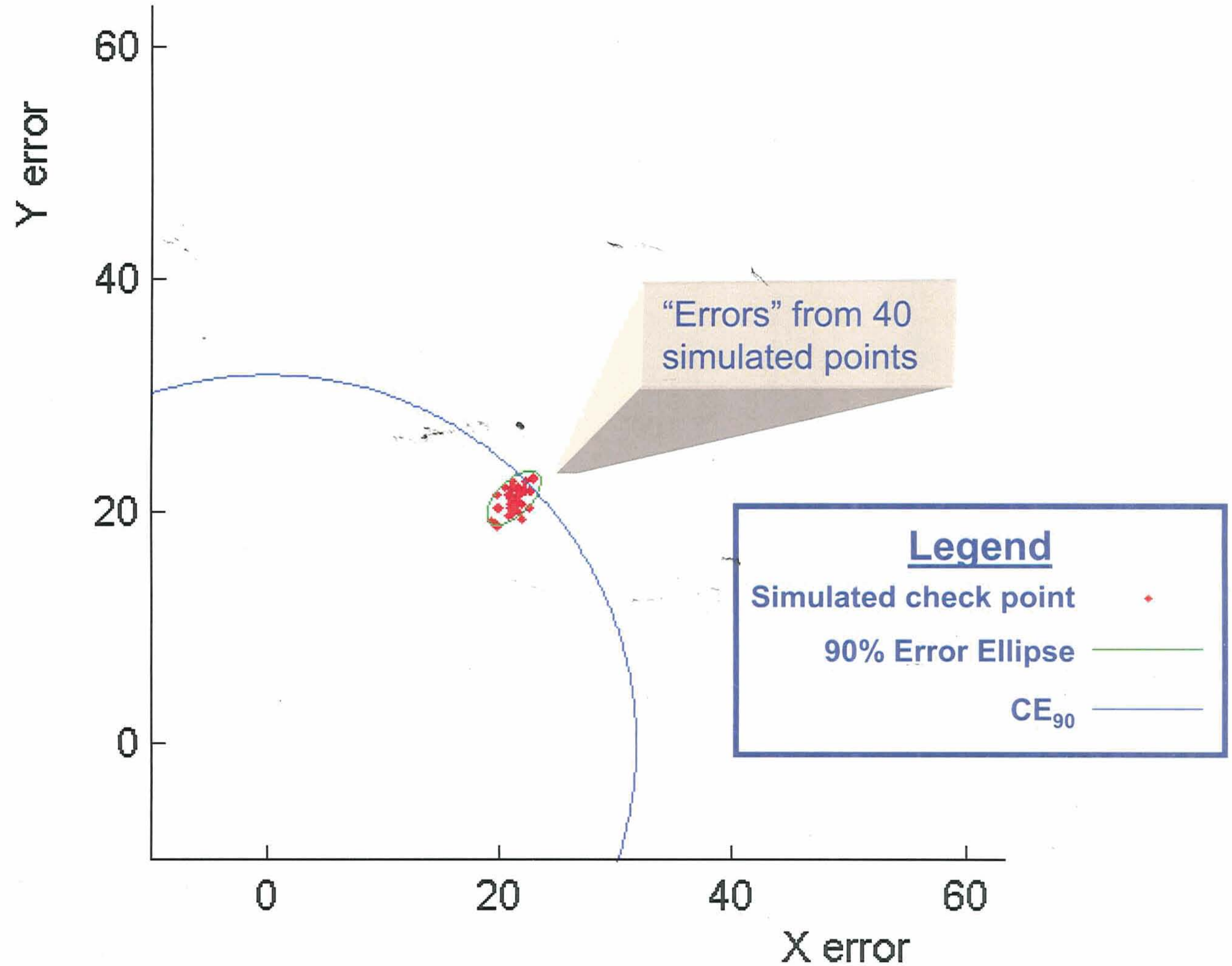
- Assumed bivariate normal distribution of errors
- Modeled population (*all possible check points*) as 1M points
- Modeled sample (*simulated target range*) as 40 points (generated 10,000 trials of 40)
- Constrained σ_C to 1 (*unitless for modeling purposes, but for spaceborne commercial imaging σ_C ~ 1 meter*)
- Varied $\sigma_{min}/\sigma_{max}$ from 0 to 1 (*distributions from univariate through elliptical to perfectly circular*)
- Varied μ_H from 0 to 10,000

Example Trial

$$\text{Bias Direction} = 45^\circ$$

$$\frac{\sigma_{\min}}{\sigma_{\max}} = 0.5$$

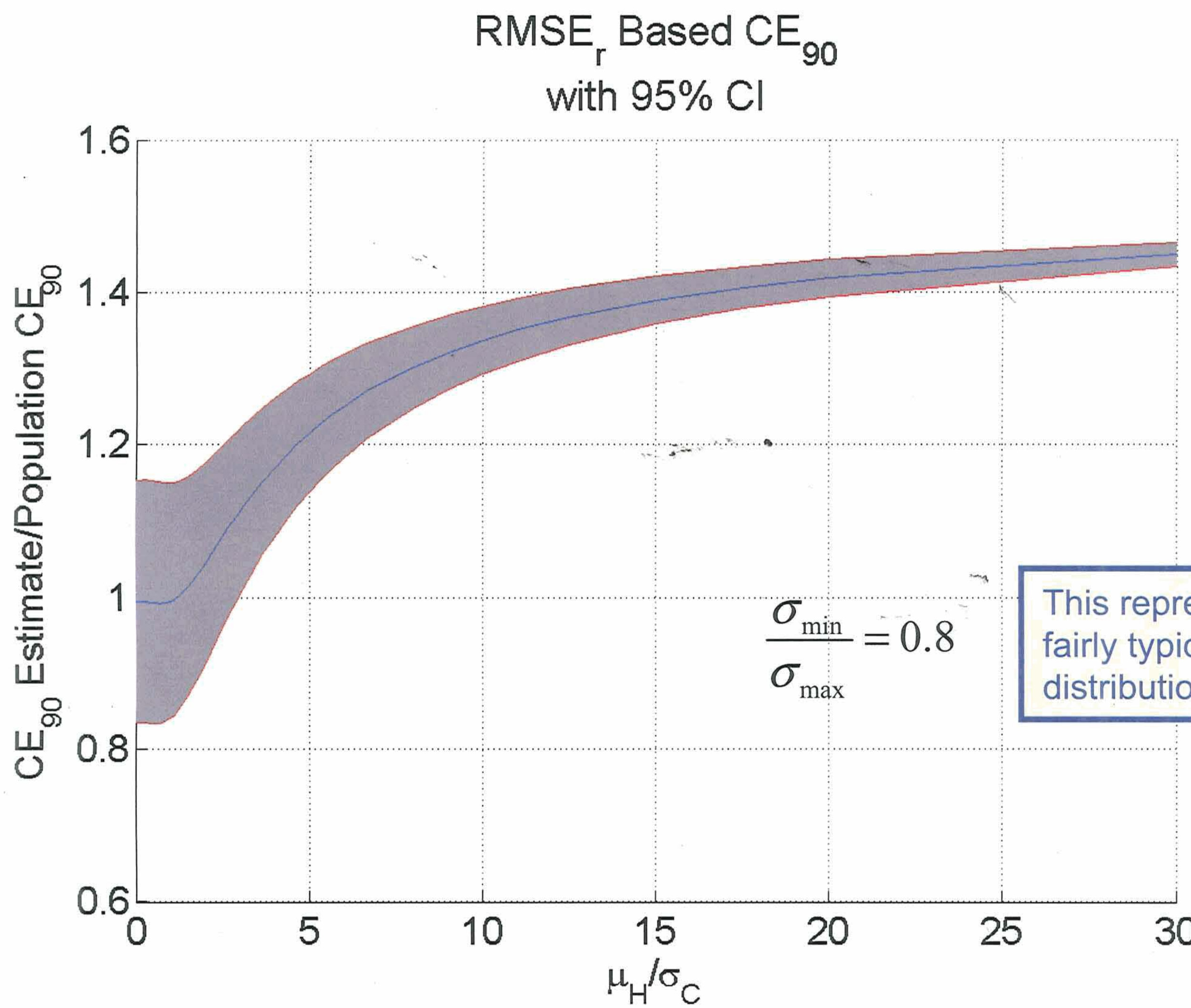
$$\frac{\mu_H}{\sigma_C} = 30$$



NSSDA $RMSE_r$ Based

$$CE_{90} = 1.5175 \cdot \mathrm{RMSE}_r$$

NSSDA $RMSE_r$ Based Confidence Interval



This represents a fairly typical error distribution shape

NSSDA Case 2

$$CE_{90} = 2.1460 \cdot \mathrm{RMSE}_c$$

NSSDA Case 2 Confidence Interval (cardinal direction)

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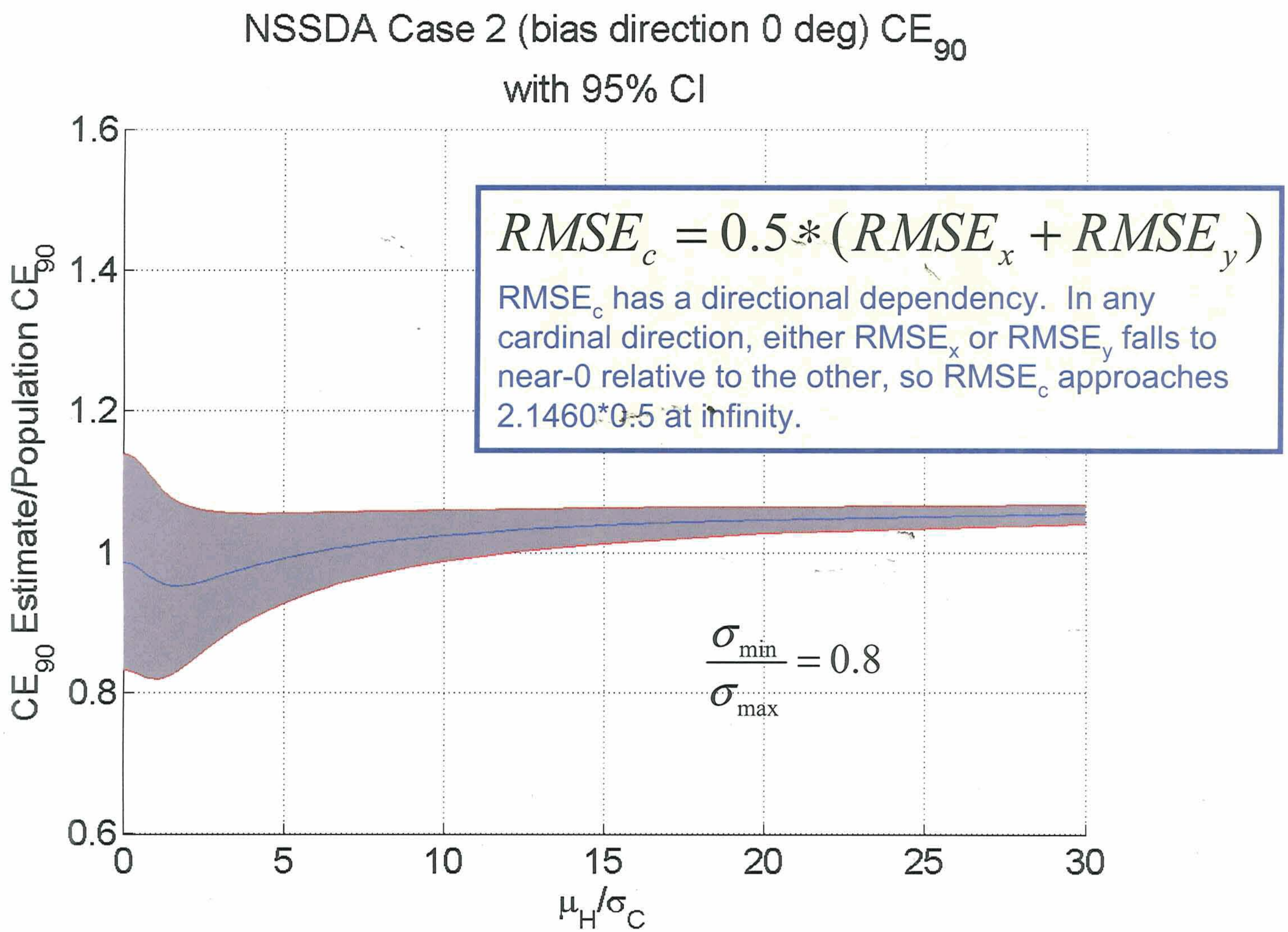
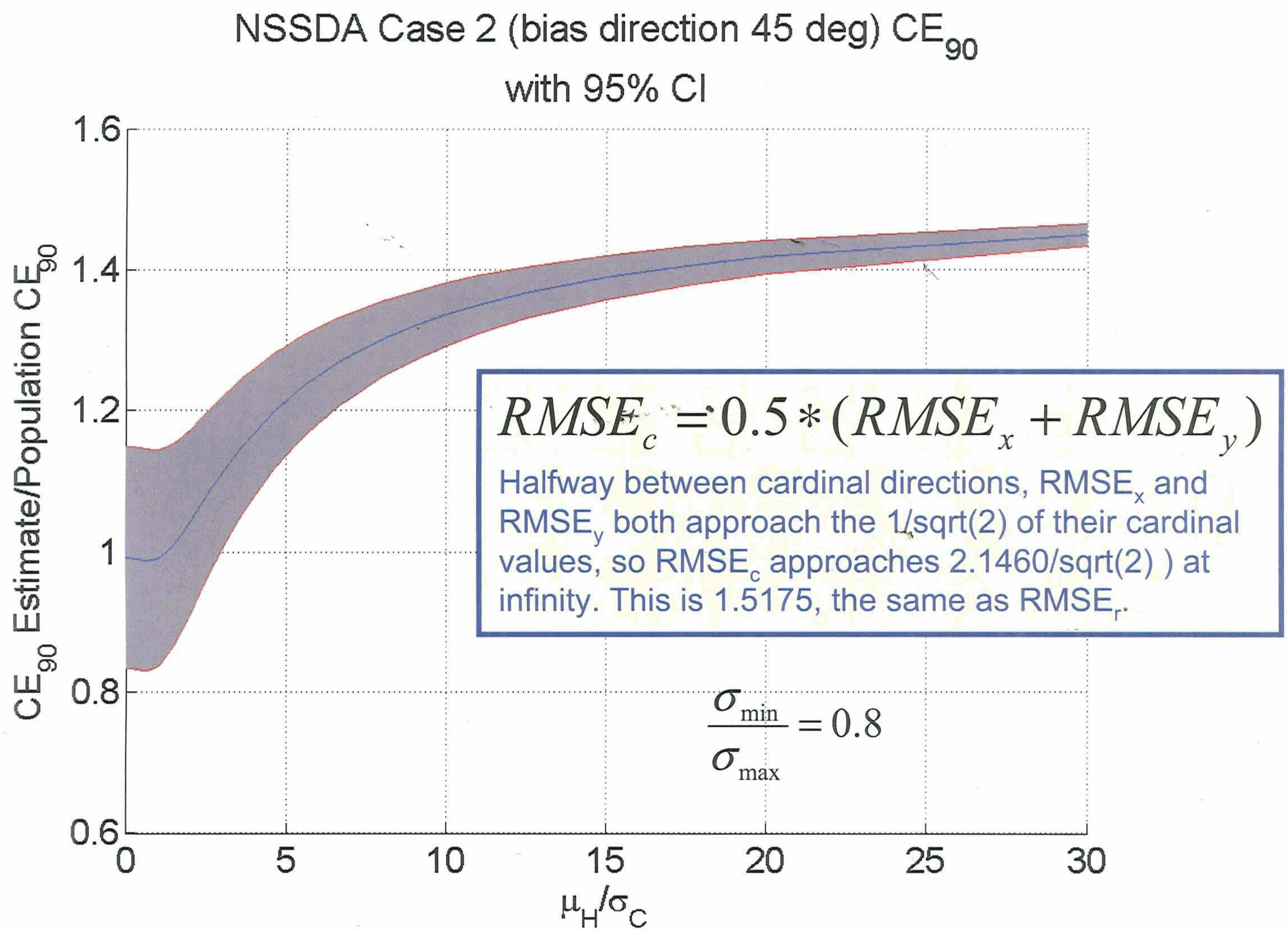


$$RMSE_c = 0.5*(RMSE_x + RMSE_y)$$

$RMSE_c$ has a directional dependency. In any cardinal direction, either $RMSE_x$ or $RMSE_y$ falls to near-0 relative to the other, so $RMSE_c$ approaches 2.1460*0.5 at infinity.

NSSDA Case 2 Confidence Interval (45° off axis)



Sum of Squares

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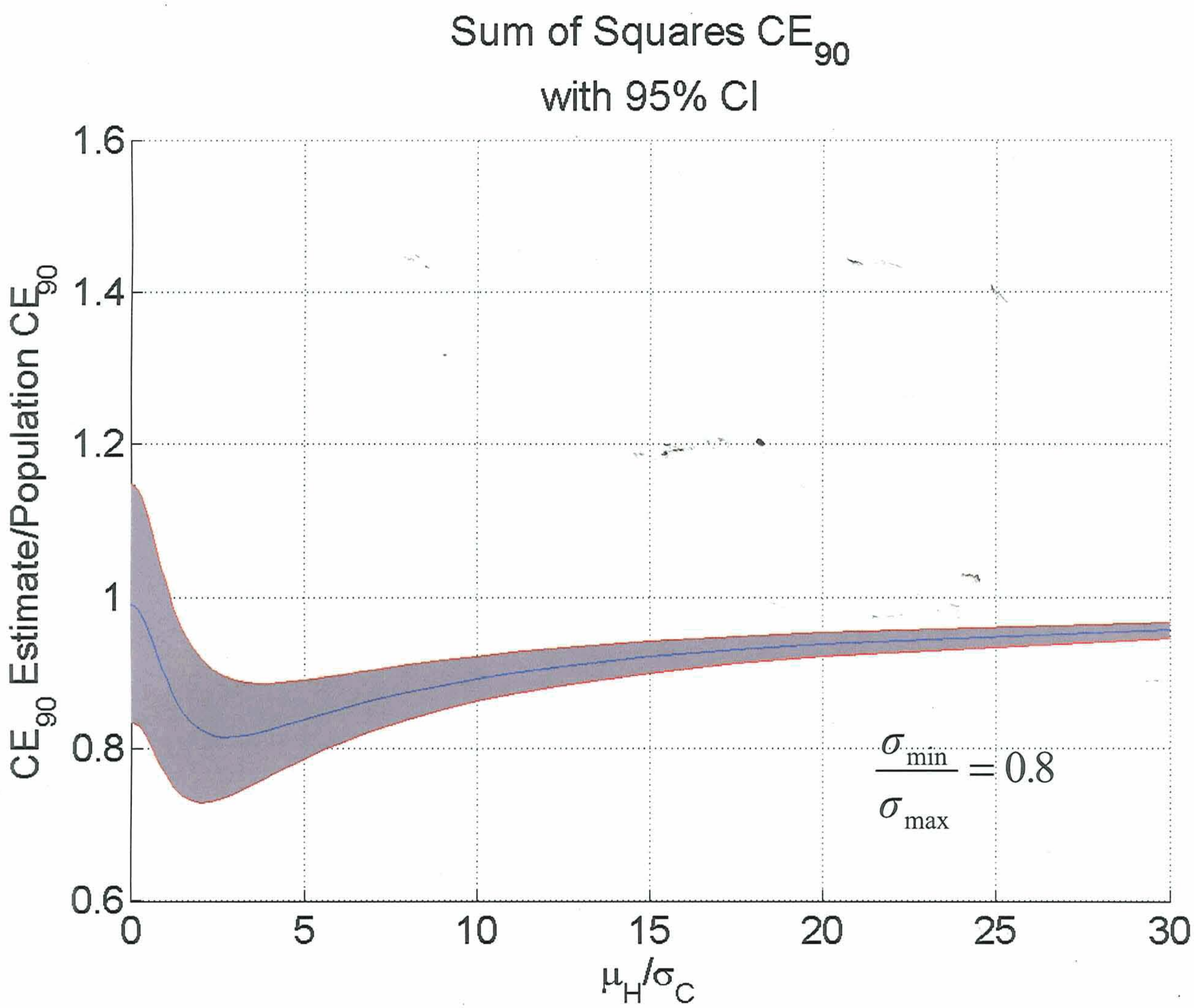
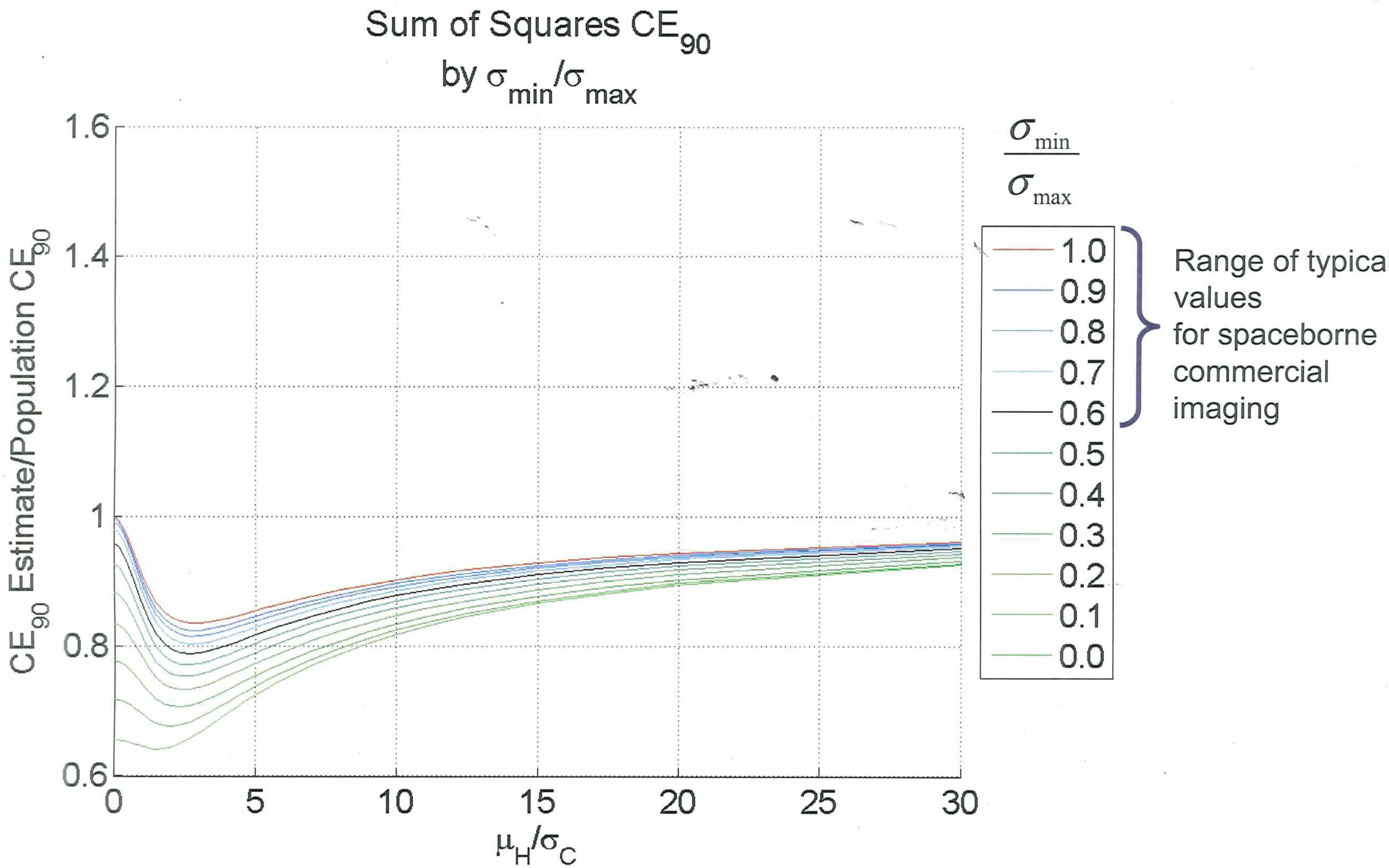
$$CE_{90} = \sqrt{(2.1460 \cdot \sigma_C)^2 + {\mu_H}^2}$$

Sum of Squares Confidence Interval



Sum of Squares Results by Error Distribution Shape



Ager Approach

- When $\mu_H/\sigma_C \leq 0.1$

$$CE_{90} = 2.1460\sigma_C$$

- When $0.1 < \mu_H/\sigma_C \leq 3$

$$CE_{90} = 2.1272\sigma_C + 0.1674\mu_H + 0.3623\frac{\mu_H^{\ 2}}{\sigma_C} - 0.055\frac{\mu_H^{\ 3}}{\sigma_C^{\ 2}}$$

- When $\mu_H/\sigma_C > 3$

$$CE_{90} = 0.986\mu_H + 1.4548\sigma_C$$

Ager Approach Confidence Interval

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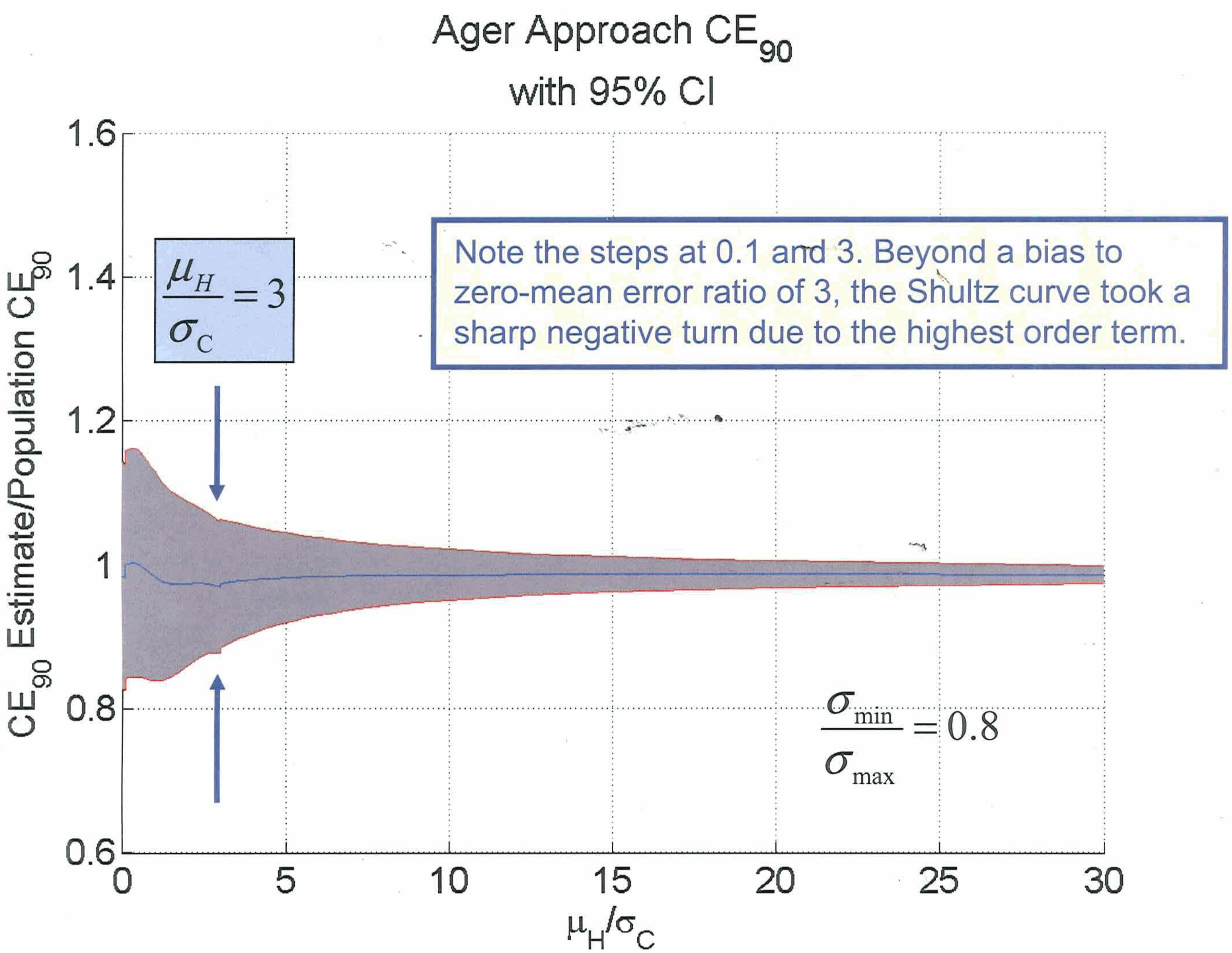
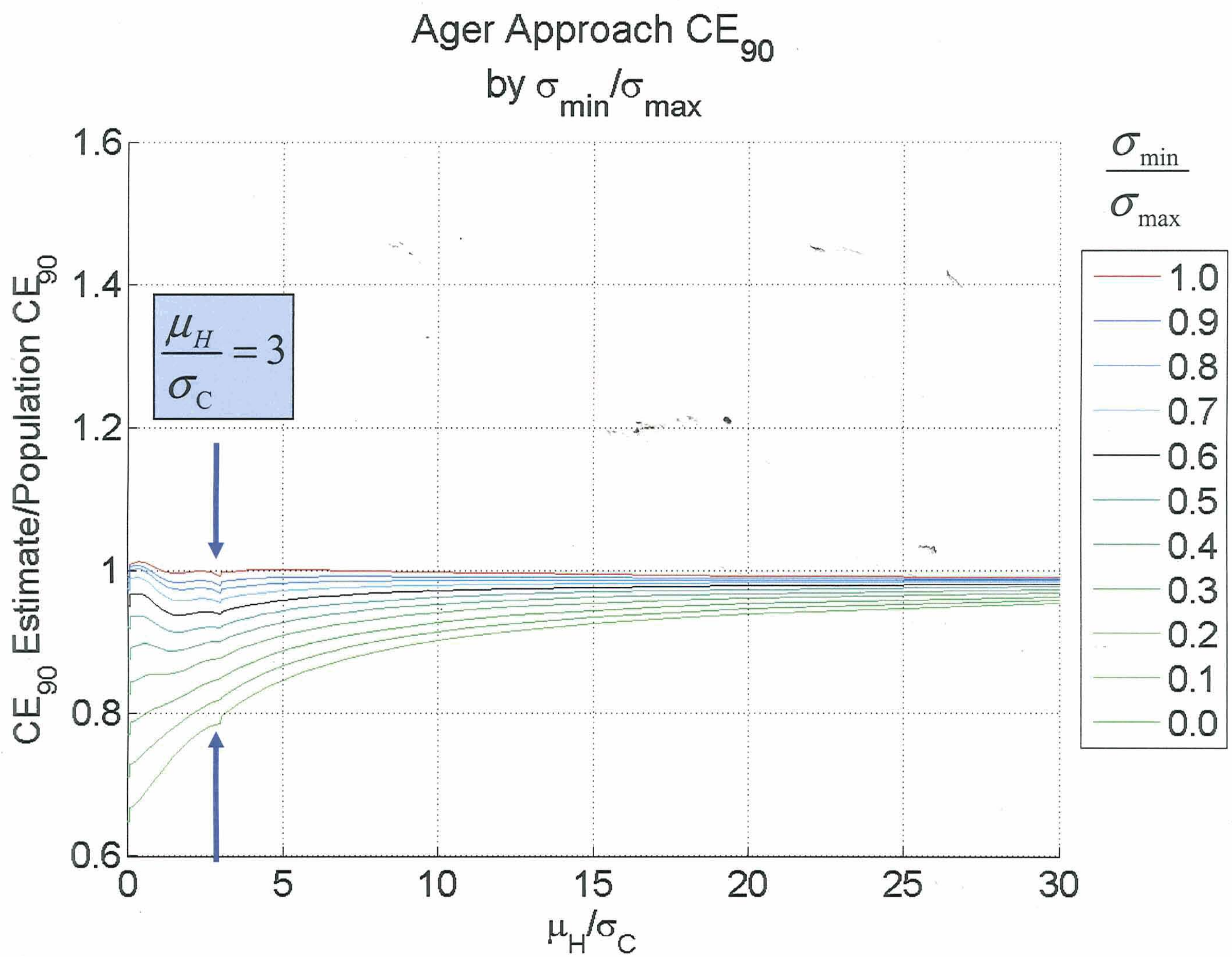


Ager Approach Results by Error Distribution Shape



"Empirical" Approach

- Given radial error magnitude calculated by

$$\Delta R_i = \sqrt{\Delta X_i^{\,2} + \Delta Y_i^2}$$

$$CE_{90} = 90^{\text{th}} \text{ percentile of } \Delta R$$

"Empirical" Approach Confidence Interval

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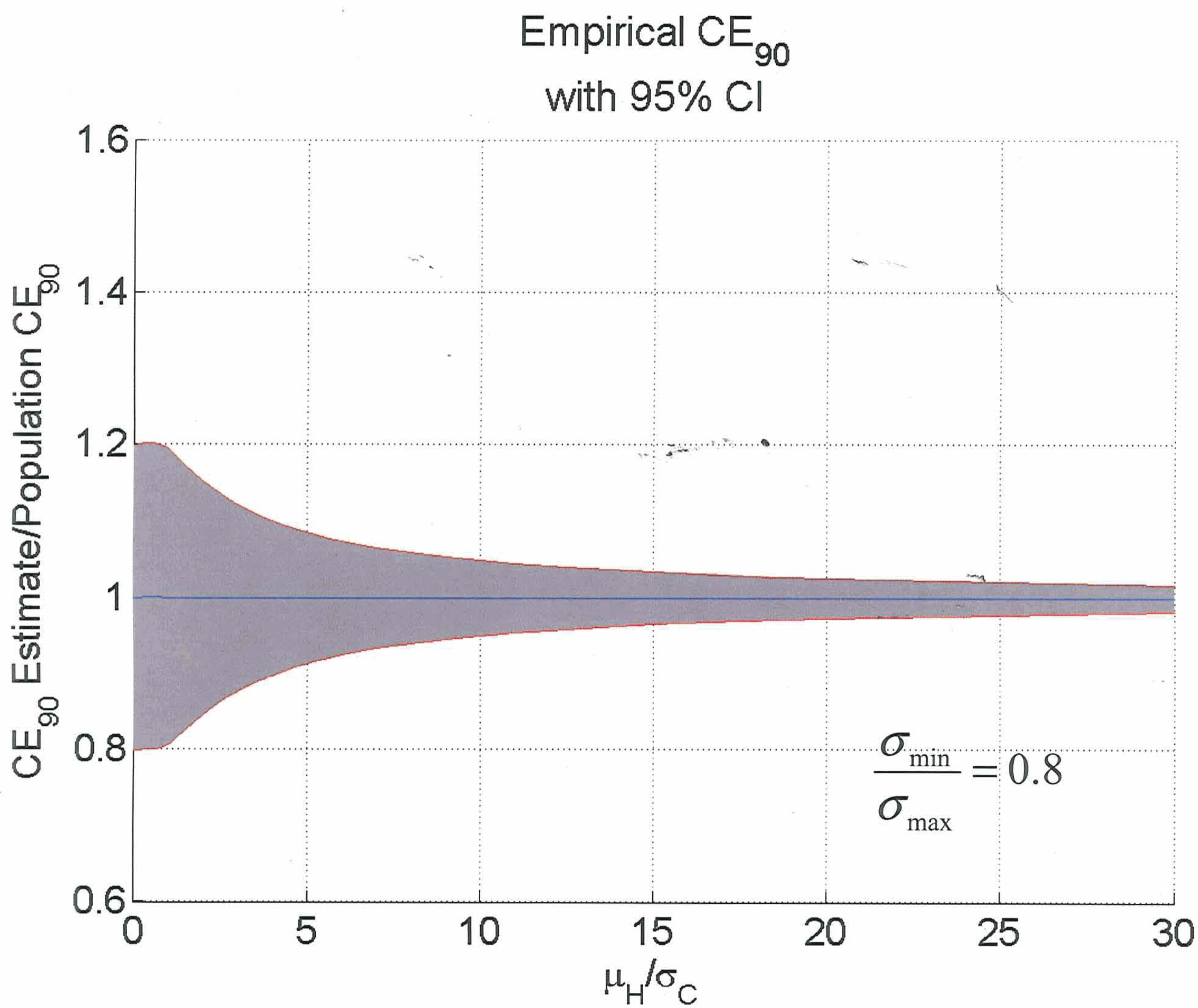
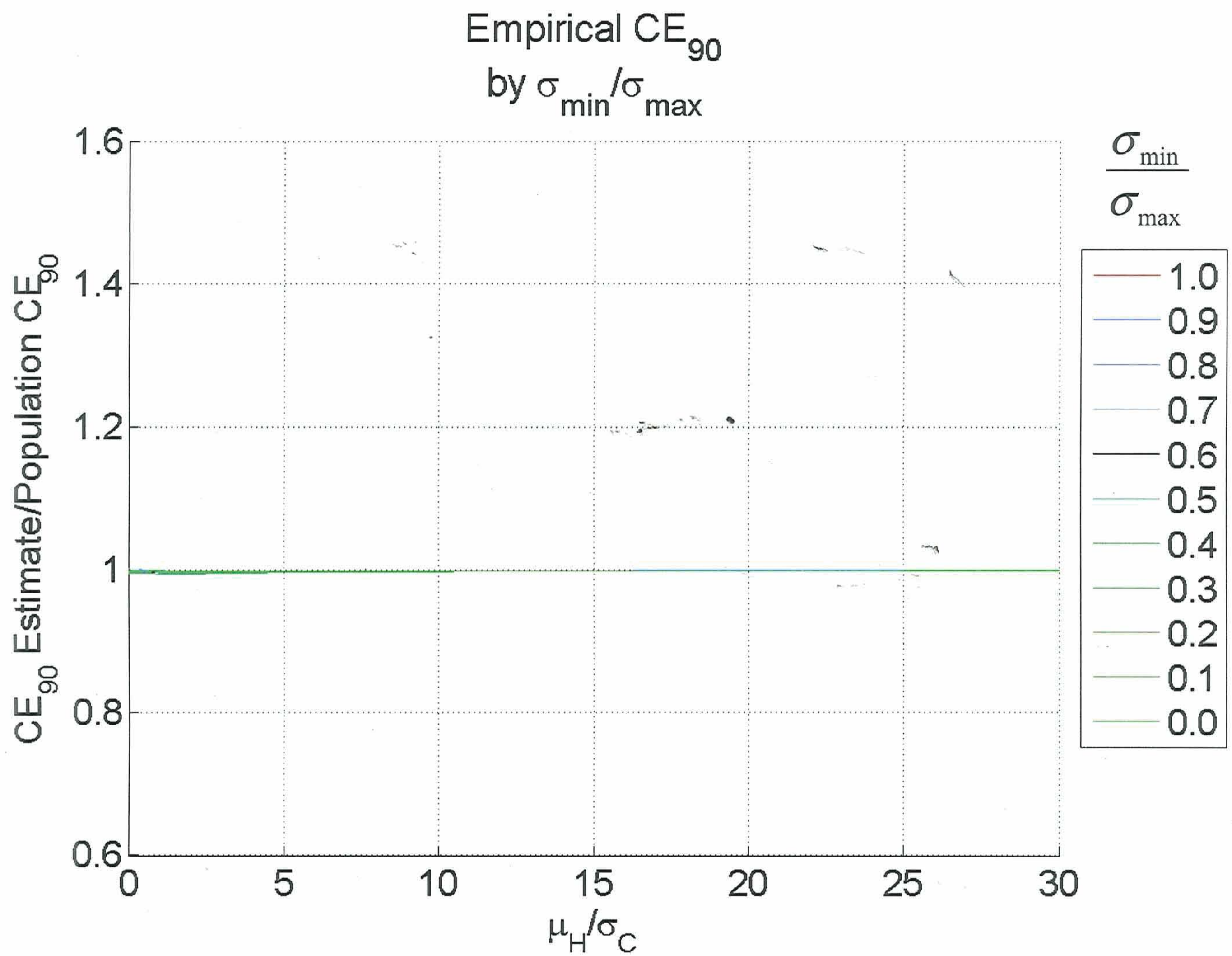
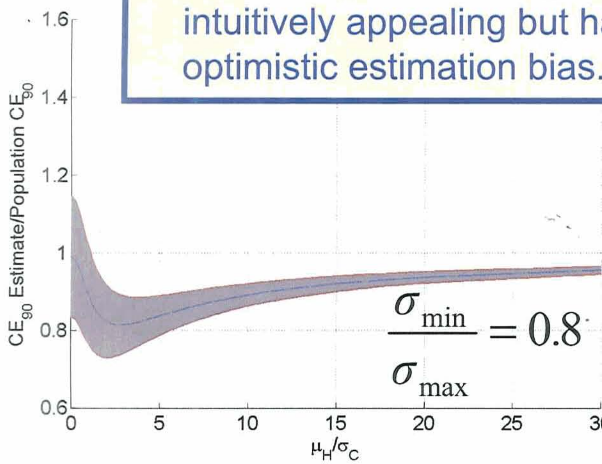


"Empirical" Approach Results by Error Distribution Shape

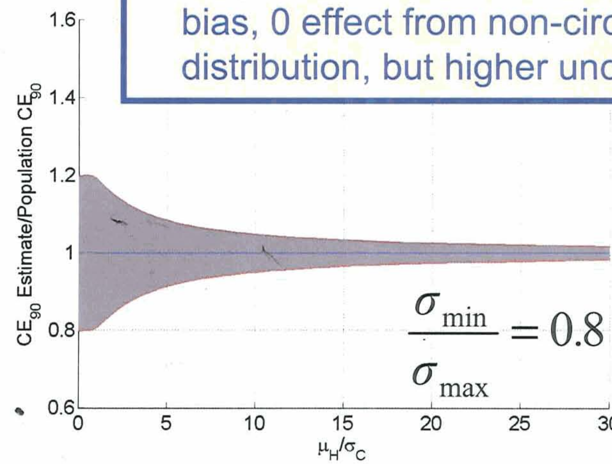


Side-by-Side Summary

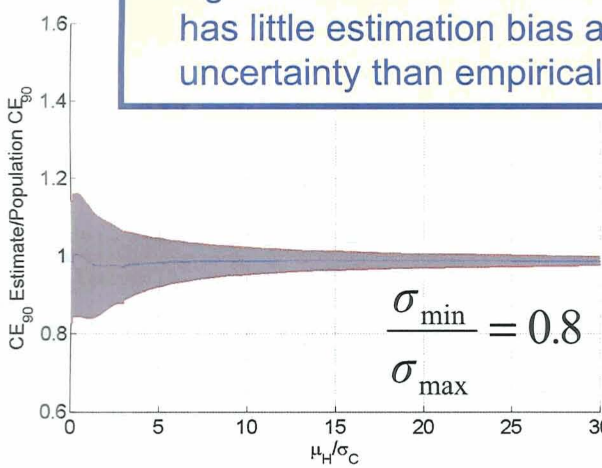
- Sum of squares is simple and intuitively appealing but has optimistic estimation bias.

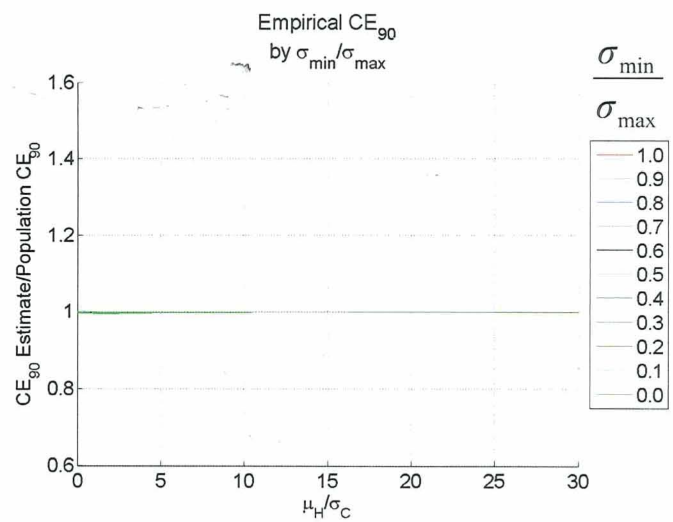


- Empirical method has 0 estimation bias, 0 effect from non-circular distribution, but higher uncertainty.



- Ager modification to Shultz method has little estimation bias and less uncertainty than empirical.





Conclusions and Recommendations

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- RMSE based methods distort circular error estimates (up to 50% overestimation).
- The empirical approach is the only statistically unbiased estimator offered.
- Ager modification to Shultz approach is nearly unbiased, but cumbersome.
- All methods hover around 20% uncertainty (@ 95% confidence) for low geopositional bias error estimates. *This requires careful consideration in assessment of higher accuracy products.*