# A Study of Visualization for Mathematics Education

by

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**Abstract**

Graphical representations such as figures, illustrations, and diagrams play a critical role in mathematics and they are equally important in mathematics education. However, graphical representations in mathematics textbooks are static, i.e. they are used to illustrate only a specific example or a limited set of examples. By using computer software to visualize mathematical principles, virtually there is no limit to the number of specific cases and examples that can be demonstrated. However, we have not seen widespread adoption of visualization software in mathematics education. There are currently a number of software packages that provide visualization of mathematics for research and also software packages specifically developed for mathematics education. We conducted a survey of mathematics visualization software packages, summarized their features and user bases, and analyzed their limitations. In this survey, we focused on evaluating the software packages for their use with mathematical subjects adopted by institutions of secondary education in the United States (middle schools and high schools), including algebra, geometry, trigonometry, and calculus. We found that cost, complexity, and lack of flexibility are the major factors that hinder the widespread use of mathematics visualization software in education.

## 1. Introduction

Mathematics is a difficult and frustrating subject to learn without proper teaching and guidance. Computer visualization offers an extraordinary potential for communication of complex ideas. Utilizing computer visualization in mathematics education will aid in the comprehension of a variety of topics, from algebra to geometry, which otherwise might only be partially understood. By using computer software to visualize mathematical principles, virtually there is no limit to the number of specific cases and examples that can be demonstrated. Computer visualization of mathematics topics will also facilitate increasing the student's interest and involvement in engineering, computer science, and other related fields in a rapidly growing, worldwide technology marketplace.

Considering the impact computer visualization will have on mathematics education, specifically in secondary education in the United States, this paper will first describe current software packages for mathematics visualization including their features and user bases. Then we will analyze the limitations that prevent their widespread use for mathematics education and discuss the applicable topics in mathematics education which visualization will prove the most effective. Finally, we will discuss our future plans for research in this area and overcoming the current software packages limitations with an open source software architecture.

## 2. Mathematics Software for Research

In this section, we will discuss some mathematics visualization software packages intended for research and development. These packages could be used for education but we will describe reasons why it is not practical and would not be effective in education.

### 2.1 Software Packages

The following lists some of the mathematics visualization software packages for research and development:

- MATLAB is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numeric computation [1]. It contains built-in mathematical functions for a wide range of topics including linear algebra, statistics, Fourier analysis, filtering, optimization, and numerical integration. MATLAB also has the ability to display 2D and 3D graphic functions for visualizing data. Its algorithms can be integrated with external applications and languages such as C, C++, Java, and Microsoft Excel [1]. MATLAB has a large user base, focused primarily in industry and universities.

- Mathematica is another software package for research and development. It contains a numeric and symbolic computational engine, solves equations, acts as a graphing calculator, generates code, and can easily integrate code and libraries from other programming languages such as C++ and Java [2].Like MATLAB, it has a large user base, focused primarily in industry and universities.

- Maple is a third software package which has a large user base intended for use in industry and academia in the practical application of mathematics. Maple is a high performance numeric and symbolic computation engine and interactive environment for designing, creating, and analyzing [3]. It also includes a handwriting recognition tool, plots and animations, graphing calculator, code generation, and equation editor with symbol template. Students can run Maple-created learning materials through a web browser and Maple eBooks and study guides provide examples and step-by-step solutions for topics from algebra to calculus [3]. Even though Maple can be a useful teaching tool, it's primarily used in industry for scientific and operations research, engineering design, and financial analysis due to its advanced mathematical capabilities.

- MuPAD Pro is a computer algebra system and problem solving environment for exact symbolic and numeric computations [4]. MuPAD Pro has its own programming language and allows imperative and functional, as well as object-oriented, programming [4]. It also has interactive 2D and 3D graphics for visualization and animation. The user base of MuPAD Pro is smaller compared to that of MATLAB, Mathematica, and Maple, but still focused on industry and research.

### 2.2 Limitations

Software packages designed specifically for research and development can be used as teaching tools; however, it is not practical because the topics for which they are designed are too complex for those of secondary education. These software packages are excellent tools for applications of mathematics but not necessarily for the foundation and learning of mathematical subjects like algebra, geometry, trigonometry, and calculus. Another factor which deters these software packages from being used in education is their cost. From over a hundred dollars for a single user license to over a thousand dollars for a school site license, the packages are not affordable on an education budget.

## 3. Mathematics Software for Education

In this section, we will discuss some mathematics visualization software packages intended specifically for mathematics education. Although these packages are effective teaching tools, we will describe some of their limitations.

### 3.1 Software Packages

The following lists some of the mathematics visualization software for mathematics education:

- MathAid is interactive educational software for learning mathematics. Its features include tutorials and self-tests, problems containing theoretical background as well as practical examples, and interactive examples to illustrate dependence of mathematical objects on parameter values [5]. Topics addressed by MathAid software are algebra, trigonometry, geometry, and pre-calculus. The user base of MathAid is small and is directed towards high school students.

- The Geometer's Sketchpad is a construction and exploration tool designed specifically for students in middle school or high school math. It is an environment for creating, exploring, and presenting mathematics interactively and dynamically. Students can construct objects, figures, and diagrams and explore their mathematical properties by dragging objects with the mouse [6]. The Sketchpad contains separate modules for algebra, trigonometry, geometry, and calculus.

- Crocodile Mathematics is mathematical modeling software for algebra and geometry topics in secondary education. Its interactive environment allows users to link shapes, numbers, equations and graphs to understand mathematical relationships and theoretical concepts [7].

- Autograph is interactive software for teaching calculus, algebra, and coordinate geometry. Its environment has 2D and 3D graphing capabilities for topics such as transformations, conic sections, vectors, slope, and derivatives [8]. In real-time, users can observe how functions, graphs, equations, and calculations change due to modifications in parameter values.

### 3.2 Limitations

Software packages designed specifically for mathematics education prove to be effective teaching tools. Upon deeper investigation by using some of the software packages, we found they also have limitations which prevent them from being widely used. One such limitation is that some of these packages do not allow users to define their own function, equation, or graph and investigate it; they can only explore mathematics using what is pre-defined by the software. Another limitation is cost. These packages are between $100 and $150 dollars for a single user license, and more for multiple user or site licenses. There is also a lack of packages which include topics from trigonometry. It seems all that is covered by most of the packages are basic sine, cosine, and tangent functions and graphs; however, trigonometry topics in secondary education encompass much more, such as inverse trigonometric functions, standard equations of the trigonometric functions, and special right triangles.

## 4. Applicable Mathematics Topics for Visualization

Despite the obvious ability for computer visualization to aid in the comprehension of mathematics, not every topic in mathematics can be visualized effectively or provide the most useful learning experience. In mathematics, there exist many properties and identities of numbers, functions, and expressions that must simply be known. In these cases, it is not straightforward to use visualization, nor is it worthwhile since memorization is all that is required. Using concurrent information from the Virginia Department of Education (VDOE) and National Council of Teachers of Mathematics (NCTM), we have compiled all the principles and standards of secondary education mathematics and determined the specific topics for visualization which will provide the most effective educational experience for users.

### 4.1 Middle School Mathematics Topics

Mathematics topics for middle school are divided into 6 major categories. This section will describe a couple examples of the specific topics under each category that will serve as an effective visualization for students. [9], [10]

- Number and Number Sense – comparing and ordering whole numbers, fractions, and decimals using a number line where users can drag and drop numbers to reorder them and choose the correct symbol (less than, greater than, equal to) to describe the location of two numbers on the number line; relationships among the subsets of the real number system can be illustrated by using Venn diagrams.

- Computation and Estimation – estimating and determining length, weight/mass, area, and liquid volume/capacity by watching volume change as a glass fills up with water or as weights are added to a balance; applying area/circumference and perimeter formulas of triangles, rectangles, and circles by users observing calculation changes in real-time as they change the size of the geometric figures using the mouse.

- Measurement – estimating the area of polygons by using the mouse to draw lines subdividing them into rectangles and right triangles; measuring and describing the relationships among intersecting lines and vertical, supplementary, and complementary angles by two lines intersecting in a variety of orientations where the intersection points can be dragged to different positions or line orientations and the angle measures will change simultaneously.

- Geometry – classifying triangles, quadrilaterals, pentagons, and hexagons can be visualized by viewing them in different orientations and categorizing; applying transformations by starting with the original object and using the mouse to draw a new object under a specific transformation or to drag and drop each point from original object to its new location; explore the Pythagorean Theorem by creating right triangles with different lengths and missing sides.

- Probability and Statistics – users will understand probability and statistics concepts by analyzing, displaying, and interpreting data using graphical methods such as line, bar, and circle graphs, stem-and-leaf plots, and box-and-whisker plots. Users will be able to draw their own graphs and plots on the screen as well as observe how they change when the data is changed to represent a new mean or median.

- Patterns, Functions, and Algebra – users can represent a one-step equation using materials such as colored chips on an equation mat, algebra tiles, or weights on a balance scale; graphing a linear equation in two variables using a table of ordered pairs next to the coordinate plane where users will see corresponding points in the graph and table highlighted.

### 4.2 High School Mathematics Topics

Mathematics topics for high school are divided into 5 major subjects. This section will describe a couple examples of the specific topics under each category that will serve as an effective visualization for students. [9], [10]

- Algebra I and Algebra II – factor polynomials visualized using Pascal's Triangle with corresponding triangle levels and coefficients highlighted; graph user-defined inequalities, including shading the appropriate region of the solution; view different graphs/equations of conic sections and using scrolling or sliding bars to manipulate their parameters and observe real-time changes.

- Geometry – parallel lines and traversals could be visualized by two parallel lines with a perpendicular bisector, where the intersection points can be dragged to different positions along the parallel lines and the angle measures change simultaneously; arcs, chords, and tangents of circles can be visualized by moving/dragging points on the circle and increasing/decreasing the size of the circle.

- Trigonometry – users will understand trigonometric functions and their inverses from their graphs, where parameters (amplitude, period, etc.) can be changed using scroll/slide bars and the corresponding standard equations will change as well. The unit circle can be visualized by dragging a point around the circle and observing changes in the interior right triangle angle measures and lengths of sides.

- Math Analysis – graphs of polynomial and rational functions can be explored by dragging points with the mouse to change parameters; limit of an algebraic function by observing it graphed in time steps towards either negative or positive infinity; understand derivatives by viewing a graph of the original function and the slope of the tangent line to the graph, where the tangent line can be moved along the function and the derivative will change.

## 5. Conclusion

From our survey of current software packages available for mathematics visualization, we found some that are designed specifically for industry and research and others that were developed specifically for mathematics education. Upon analyzing the limitations of both types of mathematics visualization software, we found that cost, complexity, and lack of flexibility are the factors which prevent their widespread use in mathematics education. Complexity and flexibility are directly related. The packages which are very flexible are highly complex, like MATLAB, Mathematica, and MuPAD Pro. Packages that have limited flexibility are much simpler, like MathAid, Autograph, and Crocodile Mathematics. We also determined the applicable topics in mathematics of secondary educations which will provide the most enhanced educational outcome for the user.

## 6. Future Work

We plan to further investigate the factors that hinder the widespread use of computer visualization in mathematics education. To exceed these limitations, we will propose an open source software architecture for mathematics education and address the key issues and challenges in the development of such an open software architecture, such as architecture design, user interface design, and content selection. Particularly, we will implement an Internet-based software architecture that will make it readily accessible to the largest user base possible, similar to the successes that popular Internet software has achieved, such as various search engines and Wikipedia.

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