NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

SPECIAL REPORT # 67

STABILITY OF CASTERING WHEELS FOR AIRCRAFT LANDING GEARS

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September 1937

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SUMMARY

A theoretical study has been made of the shimmy of
castering wheels. The theory is based on the discovery of
a phenomenon called "kinematic shimmy." Experimental
checks, using a model having low-pressure tires, are re-
ported and the applicability of the results to full scale
is discussed. Theoretical methods of estimating the spin-
dle viscous damping and spindle solid friction necessary to
avoid shimmy are given. A method of avoiding shimmy - lat-
eral freedon - is introduced.

INTRODUCTION

In many installations of castering rubber-tired wheels
there is a tendency for the wheel to oscillate violently
about the spindle axis. This phenomenon, popularly called
"shimmy," has occurred in some airplane tail wheels and
has been corrected in two ways: First, by the application
of friction in the spindles of the tail wheels; and, sec-
ond, by locking the wheels while taxying at high speeds.
Shimmy is common with the large wheels used as nose wheels
in tricycle landing gears and, since it is impossible to
lock the wheels, friction in the nose-wheel spindle has
been the sole means of correction. Because the nose wheel
is larger than the conventional tail wheel and usually car-
ries a greater load, the larger amounts of spindle friction
necessary to prevent shimmy are objectionable.

Several experimental investigations undertaken to find
methods of avoiding shimmy showed that shimmy could be pre-
vented in small tail wheels by friction in the spindle.
The most thorough work on the general subject of stability
of castering wheels is that of Becker, Fromm, and Maruhn
(reference 1) on shimmy in automobiles. Because they do
not include the effect of lateral tire deflection, it is be-
lieved that their results would, in general, not be appli-
cable to airplanes.

The present paper presents a theoretical and experi-
mental study of the problem of the stability of castering
wheels for airplane landing gears. On the basis of simpli-
fied assumptions induced from experimental observations, a

theory of the phenomenon is presented. The theory is then
compared quantitatively with the results of model experi-
ments.

THEORY OF THE STABILITY OF CASTERING WHEELS

Kinematic (or Static) Shimmy

Some preliminary experimental results on shimmy were
obtained by the N.A.C.A. with the aid of the belt-machine
apparatus shown in figure 1. This machine consists of a
continuous fabric belt mounted on two rotating drums and
driven by a variable-speed electric motor. Provision is
made for rolling a castering wheel up to about 6 inches
in diameter on the belt in such a way that it is free to
move vertically but not horizontally.

On this belt machine the following phenomenon (see
fig. 1) was discovered while pushing the belt very slowly
by hand. With the wheel set at an angle with the belt as
in (a) and the belt pushed slowly, the bottom of the tire
would deflect laterally as is shown in (b). When the belt
was pushed farther, the wheel straightened out gradually
as is shown in (c). However, the bottom of the tire would
then still be deflected and the wheel would continue to
turn as in (d). The wheel would thus finally overshoot,
as shown in (e) and (f). The process would then be re-
peated in the opposite direction.

Figure 2 is a photostatic record of the track left by
the bottom of the tire on a piece of smoked metal. Two
things will be noticed: First, that the bottom of the
tire did not skid; and second, that the places where the
wheel angle is zero (indicated by zeros on the track) cor-
respond roughly to the places where the lateral deflection
of the tire is a maximum. Thus the wheel angle lags the
tire deflection by 1/4 cycle.

It was noticed that the oscillation could be inter-
rupted at any point in the cycle by interrupting the mo-
tion of the belt without appreciably altering the phenom-
enon. From this observation it was deduced that dynamic
forces play no appreciable part in this oscillation. This
type of oscillation might be called "static" shimmy but
the more rigorous term "kinematic shimmy" is preferred.

It was also found that the distance along the belt

required for one cycle did not vary much with caster angle or caster length. (See fig. 3.) Caster length was therefore considered not to be of fundamental importance in this type of oscillation.

It should be pointed out that, in order to observe the kinematic shimmy, lateral restraint of the spindle is necessary to prevent the spindle from moving laterally when the bottom of the tire is deflected and thus neutralizing the tire deflection. This restraint is supplied by the dynamic reaction of the airplane when the airplane is moving forward rapidly but is not ordinarily present when the airplane is moving forward slowly. It has been observed, however, on airplanes towed slowly with two tow ropes so arranged as to provide some lateral restraint.

It is observed in figure 1(a) that, when the center line of the wheel is at an angle θ (see fig. 3) with the direction of motion, the bottom of the tire deflects. This situation is represented schematically in figure 4. It is seen that a typical point on the peripheral center line must have a component of motion perpendicular to the wheel center line if the tire is not to skid. Thus

$$d\lambda = - \sin \theta \, ds$$

(The minus sign follows from the conventions used as shown in fig. 3.) Since only small oscillations are to be considered, the approximation

$$\frac{d\lambda}{ds} = - \theta \tag{1}$$

may be substituted.

The effect of tire deflection on θ will now be considered. It will be assumed, for the purposes of rough calculation, that as illustrated in figure 5 the projection of the peripheral center line on the ground is a circular arc intersecting the wheel central plane at the extremities of the projection of the tire diameter. (See fig. 1(d).) Thus, in figure 5, r is the tire radius. (It will be assumed for the time being that the caster length and caster angle are zero.) Now if the tire is deflected in the form of a circular arc, then the condition that the torque about the spindle axis be zero is that the strain be symmetrical about the projection of the wheel axle on the ground. Clearly, if the wheel is displaced,

it will be turned about the spindle axis by the asymmetrical elastic forces until, if it is allowed time to reach equilibrium, the symmetrical strain condition is reached. Thus, if the tire is deflected an amount λ as in figure 5(a); and, if the wheel rolls forward a distance ds to the condition shown in figure 5(b), in order for the strain to remain symmetrical the wheel must turn about the spindle axis an amount $d\theta$. From figure 5, $R \, d\theta = ds$. The value of R may be readily obtained from geometry in terms of r and λ. Thus $R^2 = r^2 + (R - \lambda)^2$; from which, if $\lambda^2 \ll r^2$, it is seen that $R = r^2/2\lambda$. Then substituting for R,

$$\frac{d\theta}{ds} = \frac{2}{r^2} \lambda \tag{2}$$

If the caster length is finite, the strain will not be symmetrical about the axle as was assumed here but will be symmetrical with respect to some line parallel to the axle but a certain fixed distance ahead. Hence, the essential elements of the geometry are unchanged and all the reasoning that led to equation (2) is still valid for this case.

Since the phenomena represented by equations (1) and (2) occur simultaneously, they must be combined to get the total effect. Thus

$$\frac{d^2\theta}{ds^2} = - \frac{2}{r^2} \theta \tag{3}$$

This differential equation corresponds to a simple harmonic oscillation occurring every time the wheel moves a distance $2\pi/\sqrt{2/r^2} = \sqrt{2} \, \pi \, r$.

Measurements of the interval of kinematic shimmy S have been made for three similar tires of the type illustrated in figure 1. These tires all had radii of approximately 2 inches so that the theoretical interval was about 0.74 foot. Their experimental intervals were 0.65 foot, 0.74 foot, and 0.79 foot. This agreement is closer than might have been anticipated in view of the roughness of the assumption. It will be seen from equation (1) that λ is 1/4 cycle out of phase with θ, which is in agreement with the information obtained from figure 2.

In order to take account of tires for which the assumption made concerning the projection of the peripheral

center line is not quantitatively valid, an empirical constant K_1 will be used in place of $2/r^2$ in equations (2) and (3), thus obtaining

$$\frac{d\theta}{ds} = K_1 \, \lambda \tag{4}$$

and

$$\frac{d^2\theta}{ds^2} = - \, K_1 \, \theta \tag{5}$$

The constant K_1 can be measured by observing the interval of kinematic shimmy. Where experimental values of K_1 are available, they will be used rather than the rough theoretical value $2/r^2$.

Dynamic Shimmy

In the foregoing derivation for the oscillation called "kinematic" shimmy it was assumed that the strain of the tire was always symmetrical, that is, the wheel was moving so slowly that any torque arising from dynamic effects involved in the oscillation would be negligible. For this case, from equation (4)

$$\left(\lambda - \frac{1}{K_1} \, \frac{d\theta}{ds} \right) = 0$$

If, now, the wheel is assumed to be moving at a velocity such that the effect of the moment of inertia about the spindle axis is significant, then the strain can no longer be symmetrical and for small asymmetries the torque exerted by the tire on the spindle will be proportional to the amount of the asymmetry. Thus, the value in parentheses will no longer be zero but it can be assumed that it will be proportional to the dynamic torque,

$$\frac{d^2\theta}{dt^2} = C_1 \left(\lambda - \frac{1}{K_1} \, \frac{d\theta}{ds} \right)$$

where C_1 is an appropriate constant of proportionality and includes the moment of inertia about the spindle axis I_W. If the forward velocity V of the wheel is constant

$$\frac{d^2\theta}{dt^2} = V^2 \, \frac{d^2\theta}{ds^2}$$

and

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$$V^2 \frac{d^2\theta}{ds^2} = C_1 \left(\lambda - \frac{1}{K_1} \frac{d\theta}{ds} \right) \tag{6}$$

The constant C_1 may be measured by deflecting the bottom of the tire a known amount, moving the wheel forward, and balancing the torque T exerted by the tire on the spindle so that θ stays constant. The method of deflecting the bottom of the tire a known amount will be described later. In this case (6) becomes

$$\frac{T}{I_W} = C_1 \lambda$$

from which C_1 may be found. It has been found that C_1 increases with increasing caster angle. Thus, for a tire like the one in figure 1, C_1 was 71,000 radians per second^2 per foot for a caster length L of 0.17 inch (caster angle, 5°; no fork offset) and was 104,000 radians per second^2 per foot for caster length 0.68 inch (caster angle, 20°).

In the study of kinematic shimmy it was also seen that the only change in λ was due to the fact that a component of the forward motion was perpendicular to the central plane of the wheel. This circumstance is expressed by equation (1). It is, however, found that, if the spindle is clamped at $\theta = 0$ and the bottom of the tire is deflected, the deflection will gradually neutralize itself; that is, the bottom of the tire will roll under the wheel. Thus the asymmetrical ($d\theta/ds = 0$) strain that exists in this case contributes to $d\lambda/ds$. The case of θ and $d\theta/ds$ zero is illustrated in figure 6. If the effect is again supposed to be proportional to the cause, there is obtained

$$\frac{d\lambda}{ds} = - C_2 \lambda \tag{7}$$

In this equation the constant C_2 is a geometrical constant of the tire that can be obtained from static measurements. The order of magnitude of C_2 can be obtained by assuming that the periphery of the tire intersects the extremity of the extended central plane of the wheel. In that case $C_2 = 1/r$.

If θ is not zero, there will be a component of the forward motion contributing to $d\lambda/ds$. As in equation

(1), this component will be $-\theta$. Adding this component to the part of $d\lambda/ds$ due to asymmetry, then ($d\theta/ds$ still assumed zero)

$$\frac{d\lambda}{ds} = - \theta - C_2 \lambda \qquad (8)$$

This equation expresses that for $\frac{d\theta}{ds} = 0$ the contribution of the asymmetrical strain to $d\lambda/ds$ was $- C_2 \lambda$. Also for the symmetrical strain, in which case $\lambda - \frac{d\theta/ds}{K_1} = 0$ (kinematic shimmy), there was, of course, no contribution due to asymmetry. Assume now a linear interpolation between these two limiting cases. Thus, finally,

$$\frac{d\lambda}{ds} = - \theta - C_2 \left(\lambda - \frac{1}{K_1} \frac{d\theta}{ds} \right) \qquad (9)$$

A method of measuring C_2 is provided by equation (7) which, when integrated, gives

$$\log_e \frac{\lambda}{\lambda_0} = C_2 (s - s_0)$$

If, with the spindle clamped at $\theta = 0$, the tire is deflected a known amount λ_0 and rolled ahead a known distance and the new λ measured, C_2 may be computed. It was seen earlier that C_2 was of the order of magnitude of $1/r$, that is, it would be of the order of 6 for a 2-inch radius tire. The constant C_2 was measured on two model tires under different loads and found to be 6.2 and 3.4. Considerable variations of this constant with tire pressure and load have been found.

A method of obtaining the constant known λ necessary for the measurement of C_1 is provided through equation (8). Here it is seen that, if the wheel is pushed along at a constant angle θ, λ will increase (negatively) until $\theta = - C_2 \lambda$, in which case $\frac{d\lambda}{ds} = 0$ and equilibrium is reached.

When the wheel is moving ahead at a finite constant velocity, the phenomena represented by equations (6) and (9) occur simultaneously. Therefore, to get the total ef-

fect, combine the two equations thus obtaining

$$\frac{V^2}{C_1} \frac{d^3\theta}{ds^3} + \left(\frac{1}{K_1} + \frac{C_2}{C_1} V^2\right) \frac{d^2\theta}{ds^2} + \theta = 0 \qquad (10)$$

The solution for the natural modes of motion represented by equation (10) is

$$\theta = A\ e^{\alpha_1 s} + B\ e^{\alpha_2 s} + C\ e^{\alpha_3 s} \qquad (11)$$

where the α's are the three solutions of the so-called "auxiliary" equation

$$\frac{V^2}{C_1} \alpha^3 + \left(\frac{1}{K_1} + \frac{C_2}{C_1} V^2\right) \alpha^2 + 1 = 0 \qquad (12)$$

One of these α's is real and negative and corresponds to a nonoscillatory convergence. The other roots are conjugate complex numbers and correspond to the shimmy under consideration. The roots will be of the form $a \pm \omega i$. If the divergence a, is positive, the oscillation will steadily increase in amplitude (while its amplitude is not large enough for skidding to occur); and, if a is negative, the oscillation will steadily decrease in amplitude and eventually disappear. The meaning of the quantity "divergence" may be illustrated by saying that it is approximately equal to the natural logarithm of the ratio of successive maximum amplitudes to the distance between them. The quantity ω is equal to 2π times the number of oscillations per foot. The frequency therefore is $\omega V/2\pi$. The phase angle is obtained by substituting for θ in equation (6) the value obtained from the foregoing procedure and solving for λ.

The divergence, the frequency, and the phase relations thus derived for typical model tire constants are plotted in figure 7. For small velocities (0 to 6 feet per second) the frequency corresponds to kinematic shimmy; it is proportional to velocity. The divergence increases rapidly, however, because the spindle angle lags on account of the moment of inertia about the spindle axis, thus allowing more lateral deflection than would occur in a kinematic shimmy. On the next half cycle a larger spindle angle is reached and the process repeats. As the velocity is further increased, the lag, and hence the asymmetry of the strain, further increase until the strain becomes almost

entirely asymmetrical. For this condition, $\left(\dfrac{d\theta}{ds}\right)\dfrac{1}{K_1} \ll \lambda.$

Then the restoring torque on the spindle is approximately
proportional to λ. (See equation (6).) However, λ
(measured negatively) will still lag somewhat behind θ
because, after the wheel is turned through a given angle,
a certain forward distance is required for equilibrium
tire deflection to be reached. Thus, the restoring force
will again lag the displacement. As the velocity increases
in the high-velocity range, the frequency stays nearly con-
stant (see fig. 7) and the distance corresponding to a sin-
gle oscillation increases. Hence, this constant lag be-
comes a smaller part of the cycle and the divergence de-
creases at high velocities.

It will be appreciated that the foregoing theory con-
siders only the fundamental phenomena taking place in
shimmy. There are, as would be expected, other phenomena
occurring simultaneously which have been neglected. Some
of the more important of the phenomena neglected are:

1. Miscellaneous strains (other than lateral tire
deflection) occurring in the tire. A rubber tire being an
elastic body will distort in many complicated ways while
shimmying. In particular, there will be a twist in the
tire due to the transmission of torque from the ground to
the wheel.

2. Two effects will cause the stiffness constants of
the tire to change with speed. First, centrifugal force
on the rubber will make the tire effectively stiffer at
high speeds. Second, much of the energy used to deflect
the tire will go into compressing the air. The compressi-
bility of the air will change with the speed of compres-
sion owing to the different amounts of heat being trans-
ferred from it.

3. There will be a gyrostatic torque about the spin-
dle axis caused by the interaction of the rotation of the
wheel on the axle and the effective rotation of part of
the tire about a longitudinal axis on account of the later-
al tire deflection. This torque will later be shown to
have a noticeable effect on the results.

The inclusion of items 1 and 2 in the theory would
obviously be very difficult. It is therefore necessary to
resort to experiment to determine whether the present the-
ory gives an adequate description of the phenomena. If
so, the omission of these and any other items will be jus-
tified.

An experimental check on the theory was obtained by measuring the divergence and frequency of the shimmy on the belt machine at two caster angles and at a series of velocities. These measurements were made by placing a lighted flashlight bulb on a 6-inch sting ahead of a model castering wheel with a ball-bearing spindle and then taking high-speed moving pictures of the flashlight bulb with the wheel free. The photographs were made with time recordings on the film, and the belt carried an object that interrupted light from a fixed flashlight bulb and thus recorded the belt speed on the film.

The divergence and frequency of the shimmy were obtained by measuring the displacements and times corresponding to successive maximum amplitudes (while the amplitude was still small enough to make all the assumptions valid). The results are plotted in figure 8.

In order to compare these results with the theory, the constants C_1, C_2, and K_1 were measured on the same tire at the two caster angles by the previously described methods. It was found that $C_2 = 6.2$ feet^{-1}; $K_1 = 62.5$ feet^{-2}; that for 5° caster angle, $C_1 = 71,100$ feet^{-1}-second^{-2}, and for 20° caster angle, $C_1 = 104,000$ feet^{-1}-second^{-2}. The roots of equation (12) were then found and the divergence and frequency of the shimmy computed for a series of velocities and at caster angles of 5° and 20°. These results are plotted in figure 9 and, for purposes of comparison, the experimental curves are also reproduced.

The agreement between theory and experiment is considered satisfactory as regards qualitative results. It will be noticed, however, that the theoretical values of the divergence are decidedly too large at high velocities, say 25 feet per second. It should be noted that 25 feet per second for the model would correspond to 100 feet per second for even a small airplane, such as the Weick W1-A (reference 2). It was thought that the discrepancy might be attributed to a gyroscopic torque (mentioned earlier) that was due to the interaction of the rotation of the wheel on its axle with lateral motion of the bottom of the tire. This lateral motion may be considered to be equivalent to a rotation of _part_ of the tire about an axis perpendicular to the spindle and wheel axle. This interaction gives rise to a spindle torque, which must be included in equation (6). The numerical value of this torque will be

$I_a \dot\psi\dot\varphi$, where I_a is the moment of inertia about the wheel axle of the part of the tire that engages in the lateral motion (say, possibly 1/4 of the total moment of inertia of the tire); $\dot\psi$ is the angular rotation about the longitudinal axis $(\dot\psi = \dot\lambda/r)$; and $\dot\varphi$ is the angular rotation about the axle $(\dot\varphi = V/r)$. Thus, the value of the gyrostatic torque is $I_a\,\dot\lambda V/r^2$; but $\dot\lambda = V\dfrac{d\lambda}{ds}$ so, finally, the torque is $I_a\dfrac{V^2}{r^2}\dfrac{d\lambda}{ds}$. The angular acceleration due to this torque is $\dfrac{I_a}{I_W}\dfrac{V^2}{r^2}\dfrac{d\lambda}{ds}$. Thus (6) becomes $V^2\dfrac{d^2\theta}{ds^2} - \dfrac{I_a}{I_W}\dfrac{V^2}{r^2}\dfrac{d\lambda}{ds} = C_1\left(\lambda - \dfrac{d\theta/ds}{K_1}\right)$. Combining this equation with (9)

$$\frac{V^2}{C_1}\frac{d^3\theta}{ds^3} + \left(\frac{C_2}{C_1}\,V^2 - \frac{C_2}{C_1 K_1}\frac{V^2}{r^2}\frac{I_a}{I_W} + \frac{1}{K_1}\right)\frac{d^2\theta}{ds^2} + \frac{I_a}{I_W r^2}\frac{V^2}{C_1}\frac{d\theta}{ds} + \theta = 0$$

Here the constants C_1, C_2, and K_1 are the same as before. For the assembly on which the measurements plotted in figure 8 were made, I_W was 1.06×10^{-4} slug-feet^2, I_a was 1.09×10^{-5} slug-feet^2, and r was 0.165 foot. Using these values, new theoretical divergences and frequencies were computed and these are compared with the experiments in figure 10. It will be seen that, when the gyrostatic torques are included, the agreement of the theoretical divergences with experimental values is considerably improved at high velocities.

The gyrostatic torque is proportional to $d\lambda/ds$. Hence, at high velocities it is nearly proportional to $d\theta/ds$ since $-\lambda$ and θ are nearly in phase. Thus, this torque is approximately in the right phase to damp the oscillation.

Effect of Motion of Airplane

When a shimmying wheel is attached to an airplane, the airplane will respond to a certain extent to the forces supplied by the shimmy. This lateral motion of the nose of the airplane will in turn affect the shimmy and does, in fact, reduce the divergence at low velocities,

In order to calculate an approximation to this effect the airplane will be assumed to be a rigid body. By this assumption any deflection occurring in the structure or at the rear tires is neglected. The gyrostatic torque due to lateral tire deflection is also omitted.

The following symbols are used in the calculations:

I_Z, moment of inertia about vertical axis through center of gravity of the airplane.

m, mass of airplane.

E_λ, lateral force exerted by nose-wheel tire per foot deflection.

θ, angle between central plane of wheel and average direction of motion.

ψ, angle between longitudinal axis of airplane and average direction of motion. Measured positive in the same direction as θ.

y, lateral distance between center of gravity of airplane and path of average motion. Positive in same direction as λ.

Y, lateral force exerted by the rear wheels on the airplane.

l_2 and l_1, horizontal distances from nose wheel and rear wheels, respectively, to center of gravity.

$l = l_1 + l_2$.

The side force developed by the shimmy is $E_\lambda \lambda$. The lateral acceleration is

$$\frac{E_\lambda \lambda + Y}{m} = \ddot{y} = V^2 \frac{d^2 y}{ds^2} \tag{13}$$

Similarly, the angular acceleration of the airplane is given by

$$\frac{-l_2 E_\lambda \lambda + l_1 Y}{I_Z} = \ddot{\psi} = V^2 \frac{d^2 \psi}{ds^2} \tag{14}$$

There is, in addition, the geometrical relationship

$$\frac{dy}{ds} = - \psi - \frac{d\lambda}{ds} \frac{l_1}{l} - (\theta - \psi) \frac{l_1}{l} \qquad (15)$$

The lateral motion of the nose per unit forward motion, assuming that the rear wheels do not skid, is

$$\psi + l \frac{d\psi}{ds}$$

This motion will naturally affect the rate of deflection of the nose wheel $d\lambda/ds$; equation (9) becomes

$$\frac{d\lambda}{ds} = - \theta - C_2 \left(\lambda - \frac{1}{K_1} \frac{d\theta}{ds} \right) + \psi + l \frac{d\psi}{ds} \qquad (16)$$

Equation (6) is unaffected. Combining these equations

$$V^4 (I_Z + m \, l_1{}^2) + V^2 \left[l_1 \, m \left(V^2 + l_1 \, \frac{C_1}{K_1} \right) + I_Z \left(\frac{C_1}{K_1} + V^2 \, C_2 \right) \right] \frac{d^3\theta}{ds^3}$$

$$+ V^2 \left[l_1 \, m \left(\frac{C_1}{K_1} + \frac{l_2}{l} V^2 \, C_2 \right) + l^2 \, E_\lambda \right] \frac{d^2\theta}{ds^2}$$

$$+ \left(C_1 \, l_1{}^2 \, m \, V^2 + I_Z \, V^2 \, C_1 + l \, E_\lambda \, V^2 + l^2 \, E_\lambda \frac{C_1}{K_1} \right) \frac{d\theta}{ds}$$

$$+ C_1 \left(l_1 \, m \, V^2 + \frac{l \, E_\lambda}{K_1} \right) \theta = 0 \qquad (17)$$

This differential equation has two real roots, which are of little interest to the problem at hand, and two complex roots, which give the divergence and frequency of the shimmy under consideration.

An experimental check on the theory was obtained by running on a flat surface (see fig. 11) two models similar to the one shown in figure 12. The model was launched by a compressed-air catapult, which gave it a very smooth start. The nose wheel was locked during the acceleration so that the shimmy would not start to build up before constant velocity was reached. The wheel was freed by a lever tripped by a wire. It then came into the field of view of an elevated camera and the flashlight bulb made a track of

light on the camera film. The light entering the camera
was interrupted 60 times a second by a shutter driven by a
synchronous motor. A typical trace is shown in figure 13.
The curve was read only in the region between the two ar-
rows. Several such traces were made at various velocities
and the divergence and frequency of the shimmy were meas-
ured and plotted against velocity in figure 14. Two mod-
els with different mass characteristics and different caster
angles and tires were used to get two checks on the theory.
The theoretical divergences and frequencies in these models
are also shown in figure 14.

It will be noticed that both the theoretical and ex-
perimental divergences are somewhat smaller at low veloc-
ities than in the case of the belt-machine shimmy. In
general, however, the differences between the two cases are
small. The theory therefore seems adequate to account for
the shimmy of freely rolling models when corrections are
made for the response of the model to the shimmy.

Full-scale experiments similar to the ones with the
model would be valuable in further checking the theory.
One very rough check was made on the W1-A airplane (refer-
ence 2). The friction in the spindle of this airplane was
considerable but when the friction was reduced the wheel
shimmied. Motion pictures were taken of the shimmying
wheel and some of the enlarged frames are shown as figure
15. The shimmy diverged and finally reached an equilib-
rium amplitude. It will be seen that deflection of the
bottom of the tire plays a prominent part in the shimmy.
The frequency of the shimmy of this airplane was measured
at a velocity of 47 feet per second and found to be 9 cy-
cles per second. The frequency was then computed for this
velocity (on the basis of constants found by the method to
follow) and 10.5 cycles per second was obtained. These re-
sults are taken to be indications that the theory will be,
at least, approximately valid full scale.

In the application of the theory, it is necessary to
know the constants of the tire to be used. In the absence
of such measurements for full-scale tires, it will be nec-
essary to use the theoretical values of the parameters
$K_1 = 2/r^2$ and $C_2 = 1/r$. For $I_w C_1$ (the spindle torque
per unit lateral tire deflection for a completely asymmet-
rical strain) the formula $n r E_\lambda$ will be used, where E_λ
is the lateral force per unit deflection, r is the radi-
us of the tire, and n is an empirical nondimensional

factor to be obtained from experiments. The factor n will, of course, increase with increasing caster length; n for the model tires was 0.69 at caster angle 5° and 1.00 at caster angle 20°.

These expressions for C_1, C_2, and K_1 will be used for numerical calculations in the rest of the report.

METHODS OF AVOIDING SHIMMY

Spindle Damping

In the past the only method that has been used to avoid shimmy in single swiveling wheels has consisted in the application of friction or hydraulic damping to the spindle. It will therefore be desirable to set up a criterion for the amount of spindle damping necessary to avoid shimmy.

In this calculation and also in later calculations of other methods of avoiding shimmy the damping effect of gyrostatic torque due to lateral tire deflections will be neglected as well as that due to airplane motions; these omissions result in small errors and usually give a conservative result.

First, the effect of idealized viscous damping applied to the spindle will be calculated, that is, the effect of a torque proportional to the angular velocity of the spindle, say $K\dot{\theta} = KV\frac{d\theta}{ds}$. The angular acceleration resulting from this torque $KV \frac{d\theta}{ds}/I_W$ must be added to equation (6).

$$V^2 \frac{d^2\theta}{ds^2} = C_1 \left(\lambda - \frac{1}{K_1} \frac{d\theta}{ds}\right) - \frac{KV}{I_W} \frac{d\theta}{ds} \qquad (18)$$

Combining this equation with equation (9), which is unaffected by the addition of spindle friction,

$$\frac{V^2}{C_1} \frac{d^3\theta}{ds^3} + \left(\frac{V}{C_1} \frac{K}{I_W} + \frac{1}{K_1} + \frac{C_2}{C_1} V^2\right) \frac{d^2\theta}{ds^2} + \frac{C_2}{C_1} V \frac{K}{I_W} \frac{d\theta}{ds} + \theta = 0 \quad (19)$$

In the case of a self-excited oscillation represented by a third-order differential equation, Routh's discrimi-

nant provides a simple method of deciding whether the os-
cillation will or will not diverge. If all the coeffi-
cients are positive (as they are here), then the oscilla-
tion will diverge if and only if the product of the coeffi-
cients of the third- and zero-order terms is greater than
that of the first- and second-order terms. Hence for the
shimmy to have zero divergence

$$\frac{V^2}{C_1} = \left(\frac{V}{C_1}\frac{K}{I_W} + \frac{1}{K_1} + \frac{C_2}{C_1}V^2\right)\frac{C_2}{C_1}V\frac{K}{I_W}$$

or, rearranging and solving for K/I_W,

$$2\frac{K}{I_W} = -\left(\frac{C_1}{VK_1} + C_2 V\right) \pm \sqrt{\left(\frac{C_1^2}{V^2 K_1^2} + 2\frac{C_1 C_2}{K_1} + C_2^2 V^2\right) + 4\frac{C_1}{C_2}} \quad (20)$$

for the condition of zero divergence.

Only the positive value of K is of interest because
the application of negative damping to the spindle would
make the wheel unstable. From equation (20) it is found
that there is a maximum positive value of K at the veloc-
ity $\sqrt{C_1/C_2 K_1}$, which for the model wheels at 5° caster
is 13.6 feet per second. Thus, it is seen (see fig. 9)
that the amount of spindle damping necessary to avoid
shimmy is a maximum near the velocity at which maximum di-
vergence occurs with an undamped wheel. This velocity
will be denoted by V_0. The damping required at this ve-
locity, i.e., the maximum damping required, is

$$K_{max} = I_W \sqrt{\frac{C_1 C_2}{K_1}} \left(\sqrt{1 + \frac{K_1}{C_2^2}} - 1\right) \frac{\text{lb.-ft.}}{\text{radians per sec.}} \quad (21)$$

It will be seen from equation (21) that it is desira-
ble to make the caster length small for in this way C_1,
and hence K_{max}, may be made small. A rough approxima-
tion to K_{max} may be obtained from equation (21) by the
use of the theoretical formulas for the tire parameters
given earlier. Using these formulas and the value of n
obtained for 5° caster angle,

$$K_{max} = 0.43\ r\ \sqrt{I_W\ E_\lambda}\ \frac{\text{lb.-ft.}}{\text{radians per sec.}}$$

Using these formulas, also

$$V_o = 0.59 \ r^2 \sqrt{\frac{E_\lambda}{I_W}} \ \text{ft. per sec.}$$

for the velocity at which this maximum damping is required. For the W1-A airplane, using r = 0.67 ft., I_W = 0.11 slug-ft.^{2} (estimated), and E_λ = 2,400 lb./ft. (from rough measurements)

$$K_{max} = 4.6 \ \text{lb.-ft. per radian per sec., and}$$

$$V_o \quad = 39 \ \text{ft. per sec.}$$

For the Hammond Model Y airplane, which has had considerable difficulty with shimmy, r = 0.83 ft., I_W (estimated) = 0.33 slug-ft.^{2}, E_λ (estimated) = 4,130 lb./ft., which gave

$$K_{max} = 13.2 \ \text{lb.-ft. per radian per sec., and}$$

$$V_o \quad = 46 \ \text{ft. per sec.}$$

The amount of damping required increases with the fourth power of the size of the airplane. The application to an airplane of viscous spindle damping, such as might be applied by a hydraulic damper, might be expensive and rather heavy. It would be necessary, moreover, to insure that adequate damping would be obtained at all amplitudes, that is, it would be necessary to allow for the nonlinearity of the damper.

Most practical shimmy dampers have used solid rather than viscous friction. Solid friction, however, is difficult to deal with analytically in a straightforward manner. Therefore, resort will be had to the following subterfuge. It will be assumed that, if solid friction takes an amount of energy out of an oscillation equal to the energy taken out by a given amount of viscous damping, it is equivalent to viscous damping for purposes of damping the oscillation. The energy taken out by viscous damping in 1/4 cycle is

$$\int_0^{\theta_{max}} K\dot{\theta}\ d\theta = \int_0^{\theta_{max}} KV\frac{d\theta}{ds}\ d\theta = \frac{\pi}{4}\ KV\omega\ \theta_{max}^2$$

where $\theta = \theta_{max} \sin \omega s$. The energy taken out by a solid friction torque T is $T\theta_{max}$. Therefore, to take out the same amount of energy

$$T = \frac{\pi}{4}\ KV\omega\ \theta_{max} \qquad (22)$$

(Velocity enters because the independent variable is distance.)

Equation (22) gives an interesting explanation of the fact that shimmy (and other self-excited vibrations) will usually not start in the presence of solid friction until an initial displacement is supplied. From the theory of small oscillations the viscous friction required to damp a self-excited vibration is independent of the amplitude. (See equation (21).) It will be seen from (22) that the solid friction required to damp a self-excited vibration will be directly proportional to the initial amplitude of the oscillation. If solid friction is used to damp shimmy, it will be desirable to insure that the wheel is centered when it first makes contact with the ground. This precaution has been found useful on airplanes.

The value of ω^2 for the case of equilibrium viscous damping (zero divergence) is equal to the ratio of the coefficient of the $d\theta/ds$ term to the $d^3\theta/ds^3$ term in equation (19). This fact may be readily verified by substituting a simple harmonic solution for θ and equating the sum of the imaginary terms to zero. Thus

$$\omega = \sqrt{\frac{C_2 K}{V I_w}} \qquad (23)$$

It is difficult to obtain a good estimate of the maximum angle that should be used in computing the amount of solid friction necessary to damp shimmy on an actual airplane because the value depends on how much energy will be supplied to the shimmy after the tire starts to skid. If it be supposed that the energy supplied to the shimmy at angles greater than that at which skidding starts is entirely taken out by the solid friction, then the value of

θ for which skidding begins can be put into (22) and a value of the friction will be obtained such that shimmy with all initial angles will be damped out. This supposition is probably true for small caster lengths. The tire will start to skid when the lateral force is large enough to overcome the friction, that is, when $E_\lambda \lambda = \mu W$, where μ is the coefficient of friction and W is the load on the swiveling wheel. Now, if the wheel moves in a direction at an angle θ with the central plane of the wheel the equilibrium deflection of the tire will be given by

$$C_2 \lambda = - \theta$$

from equation (8). In this way a rough estimate can be obtained of the angle θ_0 at which skidding starts. Thus

$$\left| \theta_0 \right| = \frac{C_2 \, \mu \, W}{E_\lambda} \tag{24}$$

Substituting in (22) from (24) for θ_{max}, the value from (23) for ω, V_0 or $\sqrt{C_1/C_2 K_1}$ for V, and the value of K_{max} from (21) for K, then

$$T_{max} = \frac{\pi \, \mu \, W \, I_W \, C_1 \, C_2^2}{4 E_\lambda \, K_1} \left(\sqrt{1 + \frac{K_1}{C_2^2}} - 1 \right)^{3/2} \tag{25}$$

is obtained as the amount of solid spindle friction required to damp shimmy at the velocity requiring most viscous spindle damping (which velocity will probably require the most solid spindle friction). The validity of formula (25) is restricted to small caster lengths; this restriction should not hamper its application because the caster length is preferably small. Using the theoretical formulas for the tire parameters, there is obtained the simple formula (again assuming small caster length),

$$T_{max} = 0.17 \, \mu \, r \, W \text{ lb.-ft.} \tag{26}$$

It should be remembered that the theoretical formulas for the tire parameters used in the derivation of (26) are approximately valid only for tires similar to the model tires used in the experiments. Thus (26) and all other formulas based on the theoretical tire parameters will be approximately valid only for air wheels or low-pressure tires. For high-pressure tires it seems likely that the

effective r would be smaller than the geometrical radius
of the tire. It should be remembered that no conservative
assumptions were made in the derivation of (26).

If now μ = 0.55, a value of 19 foot-pounds spindle
friction is required for the W1-A airplane with W = 310
pounds (including propeller thrust) and 44 foot-pounds
for the Hammond Y airplane with W = 570 pounds. The
amount of spindle friction actually present in the spindle
and steering mechanism of the W1-A was measured and found
to be of the same order of magnitude as the computed value.
This result explains why no shimmy was found with that air-
plane. As mentioned previously, when this solid friction
was reduced, shimmy occurred in the castering front wheel
of this airplane.

It will be seen that the amount of solid spindle fric-
tion necessary to damp shimmy in nose wheels is considera-
ble even at small caster lengths. As is the case for the
hydraulic damper, it increases with the fourth power of the
size of the airplane.

Lateral Freedom

It has been seen that the divergence of the shimmy
depends on the interaction of the lateral tire deflection
with the angular rotation of the spindle. If the coupling
between these two degrees of freedom could be sufficiently
reduced, the shimmy would not diverge.

One method of reducing the coupling is to introduce
a new degree of freedom into the oscillation. Thus, if
the wheel is free to move in the direction of the axle,
the tire deflections will depend on the inertia reactions
of the wheel (and of any other members moving laterally).
The coupling present between λ and θ will depend on
the mass of the wheel. The maximum mass that the wheel
could have and still reduce the coupling sufficiently to
avoid shimmy will now be calculated.

If the lateral acceleration of the wheel is $\ddot{x}$ and
the mass of the wheel and tire (and other members moving
laterally) m', then

$$E_\lambda \lambda = m' \ddot{x} = m' V^2 \frac{d^2 x}{ds^2}$$

or

$$\int \frac{E_\lambda \lambda}{m' V^2} ds = \frac{dx}{ds} \tag{27}$$

This lateral motion of the wheel must be added to equation (9),

$$\frac{d\lambda}{ds} = -\theta - C_2\left(\lambda - \frac{d\theta/ds}{K_1}\right) - \frac{dx}{ds}$$

or

$$\frac{d\lambda}{ds} = -\theta - C_2\left(\lambda - \frac{d\theta/ds}{K_1}\right) - \int \frac{E\lambda\lambda}{m'V^2}\,ds \quad (28)$$

The effect on equation (6) will be small since only small lateral motions will be involved. Combining equation (6) with equation (28),

$$\frac{V^2}{C_1}\frac{d^3\theta}{ds^3} + \left(V^2\frac{C_2}{C_1} + \frac{1}{K_1}\right)\frac{d^2\theta}{ds^2} + \frac{E\lambda}{C_1 m'}\frac{d\theta}{ds} + \left(1 + \frac{E\lambda}{K_1 m'V^2}\right)\theta = 0 \quad (29)$$

It is seen that a damping term has been added to the final equation of motion. (Cf. equation (10).) If this term is sufficiently large, it will counteract the divergence of the oscillation. Applying Routh's discriminant to obtain the condition of zero divergence and simplifying,

$$m_0' = \frac{C_2 E\lambda}{C_1} \text{ slugs}$$

It is noted that m_0' is independent of velocity. If m' is smaller than m_0' the shimmy will converge; if greater, it will diverge. Putting in the theoretical formulas for the constants

$$m_0' = 1.45\frac{I_W}{r^2} \text{ slugs} = 47\frac{I_W}{r^2} \text{ pounds}$$

For the Wl-A airplane this value would be 11 pounds and for the Hammond Y, 22 pounds. Fortunately, these values are of the same order of magnitude as the weights of the wheels used on these airplanes.

The necessary amount of lateral freedom will now be estimated. This distance is determined by the largest initial deflections that might be encountered. If the castor length is small, very little energy will be added to the shimmy after skidding starts, when $E\lambda\lambda = \mu W$. It

is necessary to find the amplitude of the oscillation in x for an oscillation in λ with the amplitude $\mu W/E_\lambda$. Since $m' \ddot{x} = E_\lambda \lambda$ at maximum amplitude $m' \ddot{x} = \mu W$.

Suppose the value of m' used is that for equilibrium damping $(m_0' = C_2 E_\lambda / C_1)$, then the maximum possible $\ddot{x}$ is

$$\ddot{x}_{max} = \frac{\mu W C_1}{E_\lambda C_2}$$

but

$$\ddot{x}_{max} = V^2 \omega^2 x_{max}$$

hence

$$x_{max} = \frac{\mu W C_1}{V^2 \omega^2 E_\lambda C_2}$$

As before, for equilibrium damping, ω^2 equals the ratio of the coefficient of the $d\theta/ds$ term to that of the $d^3\theta/ds^3$ term in equation (29), for $m' = C_2 E_\lambda / C_1$,

$$\omega^2 = \frac{C_1}{C_2 V^2}$$

and

$$x_{max} = \frac{\mu W}{E_\lambda}$$

Therefore, it will be seen that the minimum amount of lateral freedom necessary on either side of the wheel is equal to the amount that the tire will deflect laterally before skidding when the weight of the wheel is the required value. This approximation is rough and no conservative assumptions were made other than the one implicit in the neglect of whatever spindle friction may be present. For the Wl-A airplane the calculated amount of lateral freedom required is 0.85 inch on either side of the wheel and for the Hammond Y airplane, 0.91 inch.

In the application of lateral freedom to an airplane wheel, means for keeping the wheel centered in the lateral travel must be provided. It can be shown that the application of spring-restoring forces sufficiently large to accomplish this purpose would tend to make the wheel shimmy. It was found, however, in a lateral-freedom arrangement that, when the wheel was tilted, it would tend to

ride on the lowest portion of the axle. From figure 16
the point A has a smaller rolling radius than the point
B but the point A must make as many revolutions as
point B. Hence, if no skidding occurs, point A will
tend to lag behind point B. Then the resultant couple
will turn the wheel about the spindle axis in such a direc-
tion that the forward motion will supply a side force caus-
ing the wheel to slide "downhill" on the axle. This effect
is found to be more than large enough to overcome the com-
ponent of the weight of the airplane that would tend to
make the wheel slide "uphill." Therefore, if an axle that
is convex downward is used, the wheel will always seek the
lowest point, which in this case will be the center of the
lateral travel. Both theoretically and experimentally,
this has been found to be a point of stable equilibrium.

Two methods of applying lateral freedom to a caster-
ing wheel were tested on the belt machine. The first
method was simply to allow the wheel to slide along the
axle. The second method was to allow the wheel to rotate
about two axes in a vertical plane perpendicular to the
axle. Shimmying in models could be avoided by either
method. The second method, however, did not involve mak-
ing the fork any wider.

Additional full-scale tests of the first system were
made of the Wl-A airplane. An axle curved downward was
used to keep the wheel centered. A radius of curvature
equal to 6 times the tire radius was arbitrarily chosen.
It was found that the wheel centered satisfactorily and
that on a rough turf-covered flying field shimmy was pre-
vented by the use of 1/2 inch of latteral freedom on ei-
ther side of the wheel. On concrete with a lateral free-
dom of 3/4 inch the wheel could be made to shimmy by taxy-
ing over a severe bump at a moderate speed and an angle
that would introduce a large initial angular deflection of
the wheel. In order to prevent shimmy under these condi-
tions, the rotation of the spindle was limited by stops to
$\pm 13^{\circ}$. The pilot did not regard this restriction of spin-
dle freedom as objectionable for the ground handling of
the airplane.

The numerical values given for the trials of the Wl-A
airplane are not to be taken as representative of the op-
timum arrangement. The presentation of more conclusive
design data must await the completion of further tests.
One point worth noting is that recent model tests have in-
dicated the possibility of dispensing with the use of

stops to limit the angular spindle notion, such as were found to be necessary in the trials with the W1-A airplane.

CONCLUDING REMARKS

The phenomenon called "kinematic shimmy" seems to be the fundamental phenomenon in the oscillation of model wheels. A dynamical theory based on this discovery gives an adequate description of the shimmy of small model wheels. It seems unlikely that shimmy on full-scale airplanes will be fundamentally different.

On the basis of this theory a method has been provided of numerically estimating the spindle friction necessary to damp shimmy. An objection to this method of avoiding shimmy lies in its indicated interference with the steering of nose wheels.

A new method of avoiding shimmy by the use of lateral freedom for the wheel is introduced but requires further investigation before complete design data can be given.

Langley Memorial Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
      Langley Field, Va., August 11, 1937.

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1. Becker, Gabriel; Fromm, Hans; and Maruhn, Herbert: Shwingungen in Autonobillenkungen ("Shimmy"). M. Krayn (Berlin), 1931.

2. Weick, Fred E.: Everyman's Airplane - A Development Toward Simpler Flying. S.A.E. Jour., vol. 38, no. 5, May 1936, pp. 176-187.
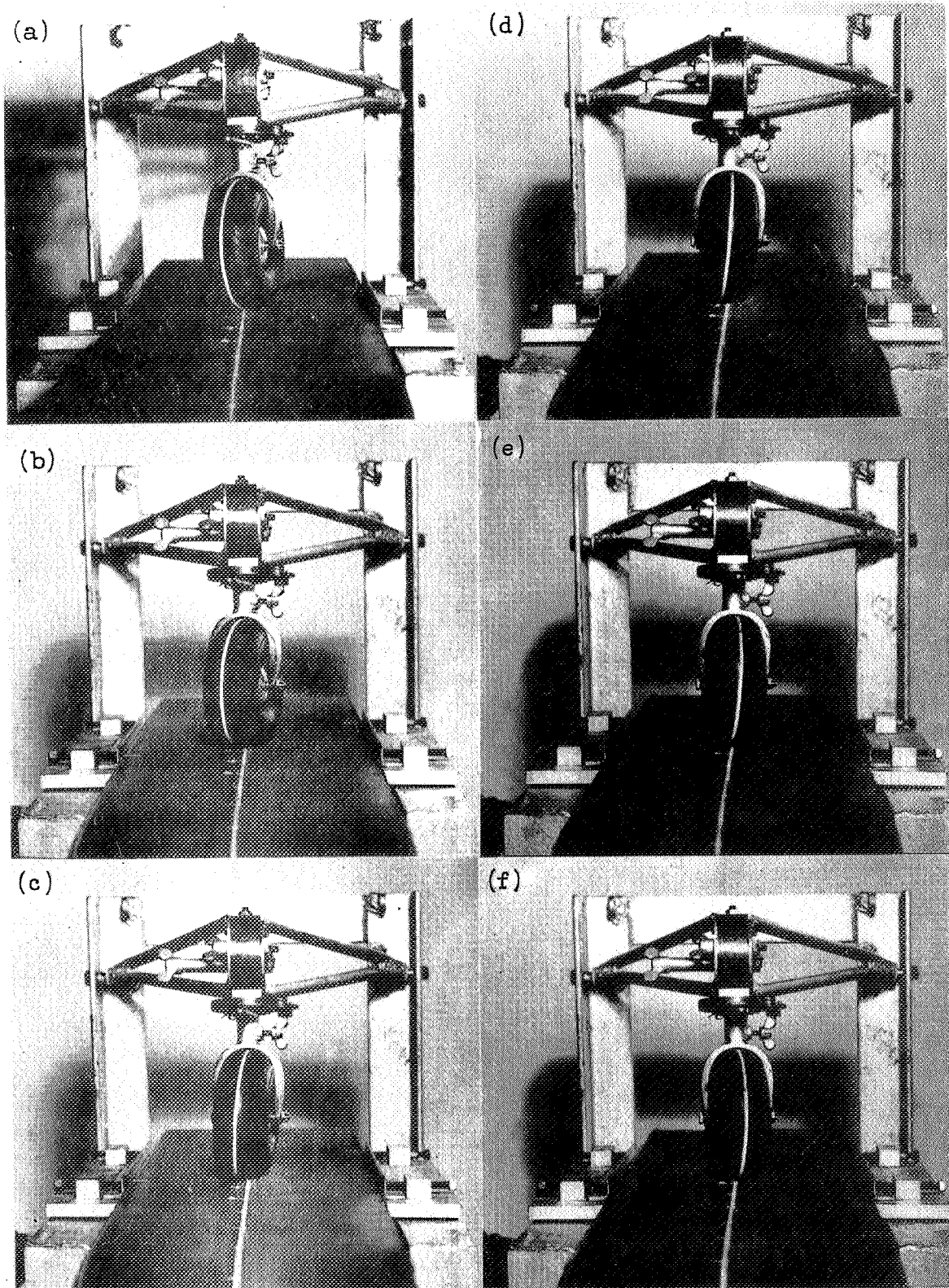
Figure 1.- Kinematic shimmy.

0	0	0	0
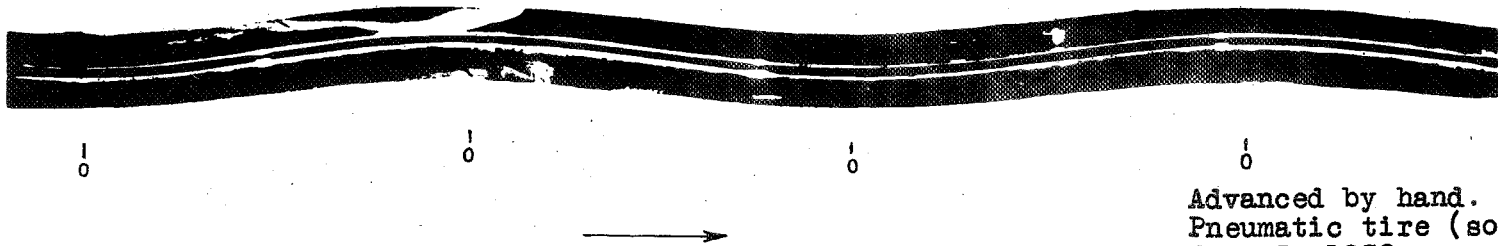
Advanced by hand.
Pneumatic tire (soft).
June 3, 1936.

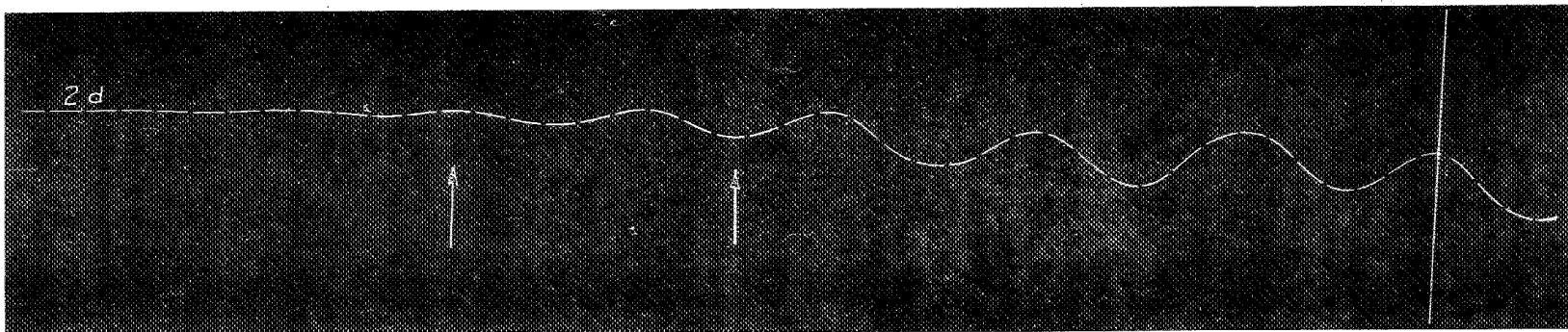Figure 2.- Record of the track of a wheel executing a kinematic shimmy.

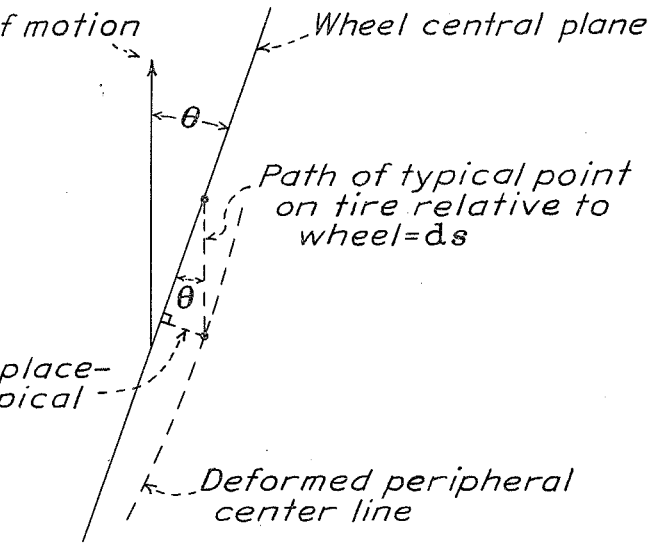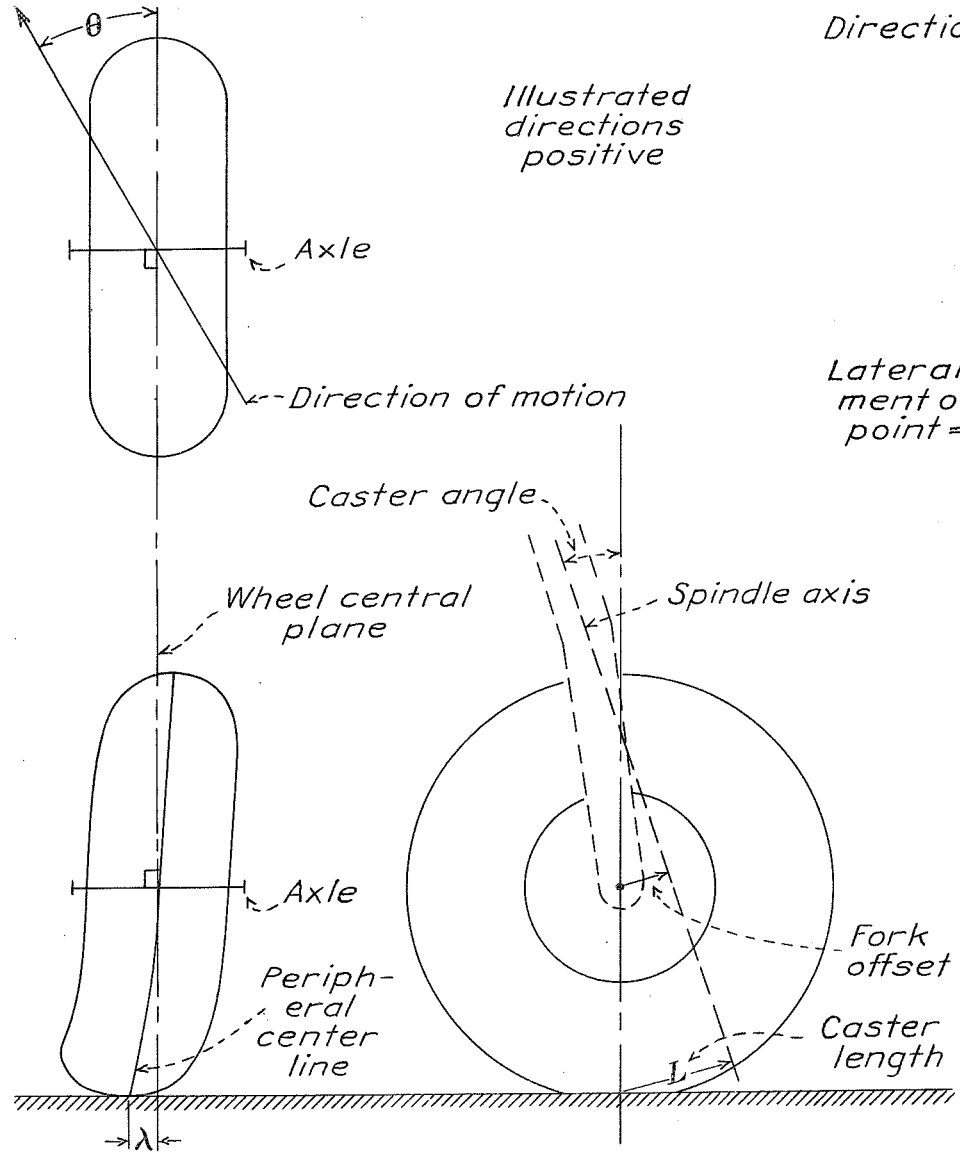Figure 13.- Record from free-model apparatus.

θ

Illustrated
directions
positive

Direction of motion

Wheel central plane

Axle

θ

Path of typical point
on tire relative to
wheel = ds

θ

Direction of motion

Lateral displace-
ment of typical
point = dλ

Deformed peripheral
center line

Caster angle

Wheel central
plane

Spindle axis

Figure 4.- Geometrical relationships involved
in kinematic shimmy, illustrating
that dλ = -θds

Axle

Periph-
eral
center
line

Fork
offset

Figure 3.- Definition of terms.

Caster
length

L

λ

Wheel central plane

(a)

r

R

Center line of dis-
torted periphery

0

λ

Axle and axis of symmetry

Projection of peripheral center
line not in contact with ground

Wheel central plane

dθ

(b)

Center line of dis-
torted periphery

λ

0'

Axle and axis of symmetry

ds

R

dθ

Former axis of symmetry
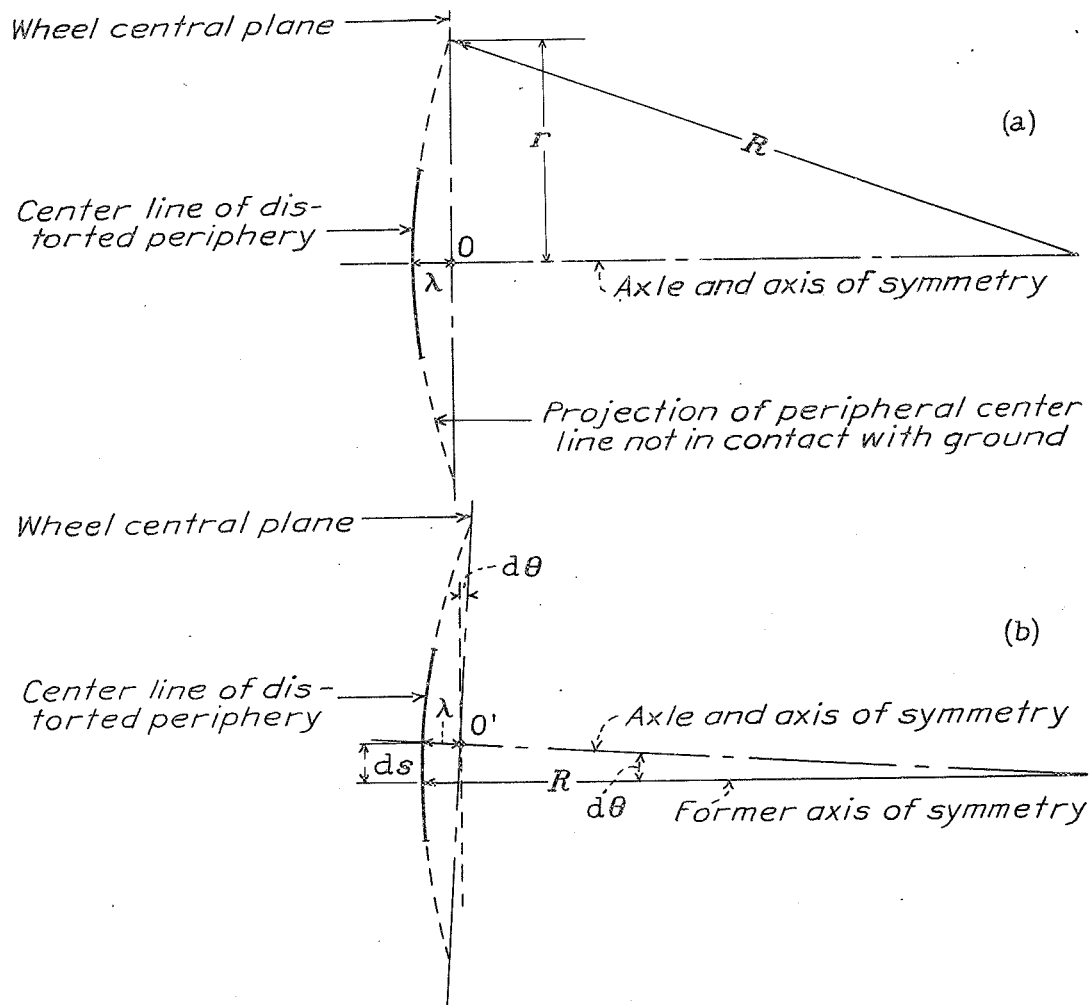
Figure 5.- Geometrical relationships involved in kinematic
shimmy; illustrating that $d\theta = 2\lambda/r^2 \propto K_1\lambda$

Peripheral
center line

$\frac{1}{C_2}$

dλ

0'

ds

0

Wheel
axle

λ

Contacted
region

Wheel
central
plane

(0-0' represents the
path of the point
under the axle along
the ground)
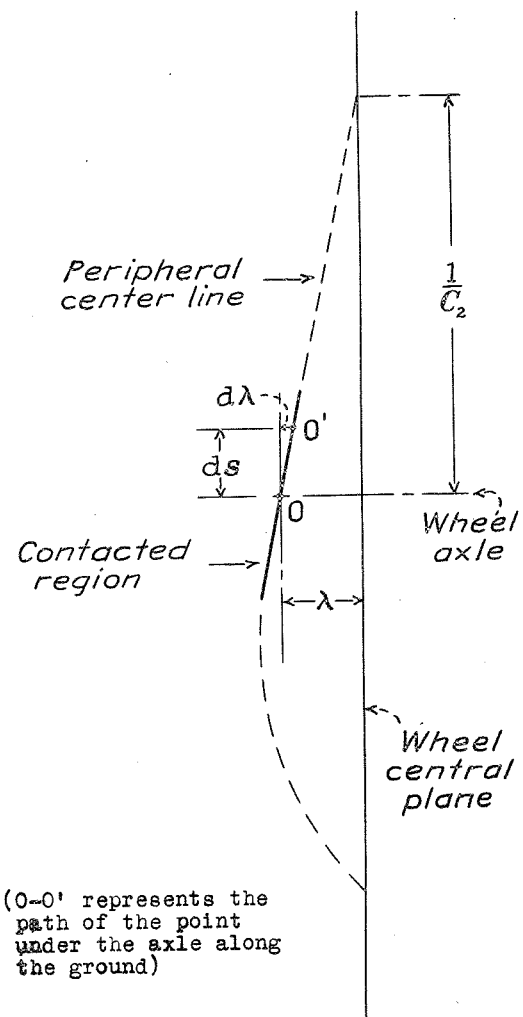
Figure 6.- Illustrating the contribution
of asymmetrical strain to dλ/ds
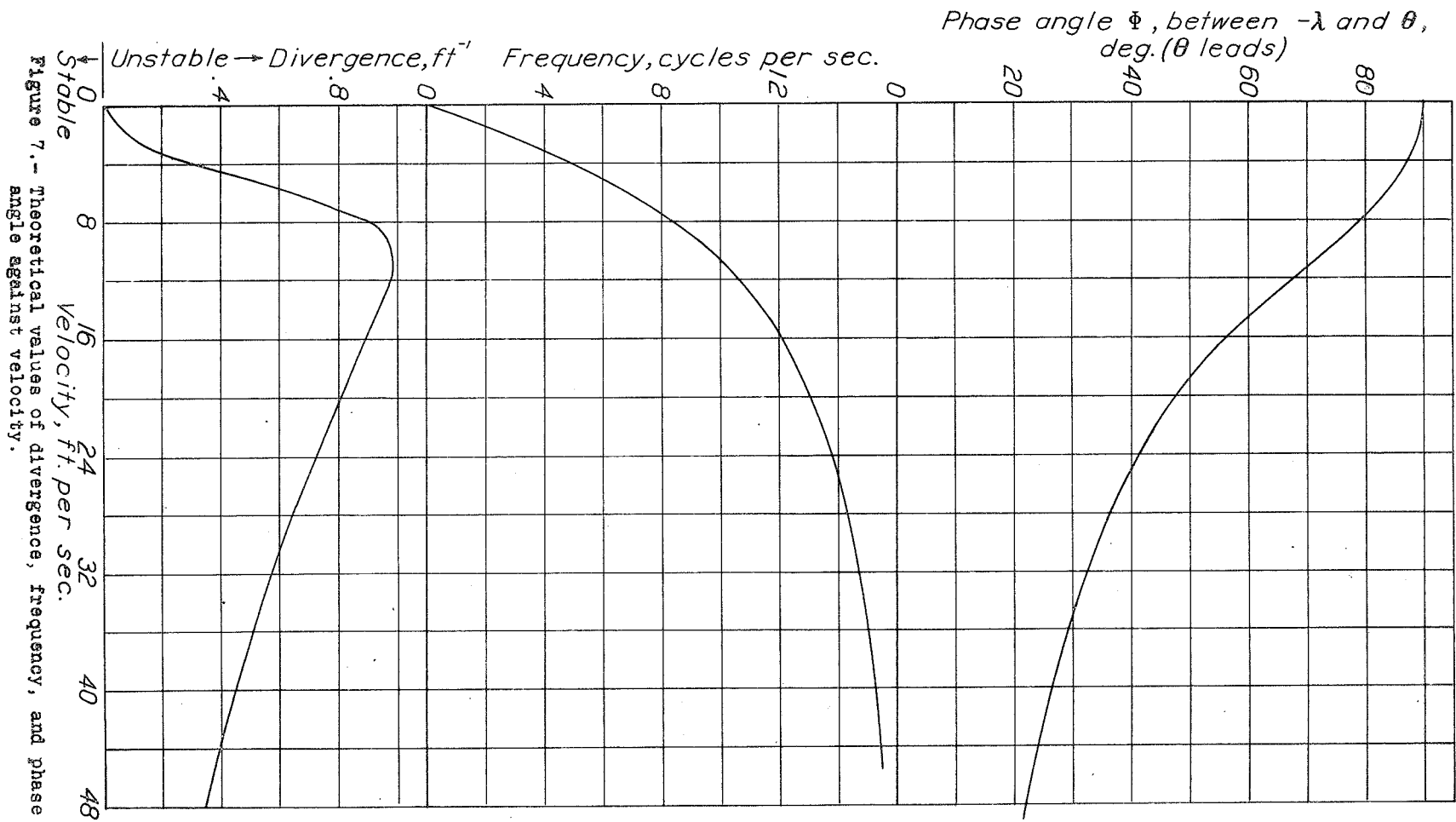
Figure 7.— Theoretical values of divergence, frequency, and phase angle against velocity.

Fig. 7

(a) Caster angle, 5°

(b) Caster angle, 20°
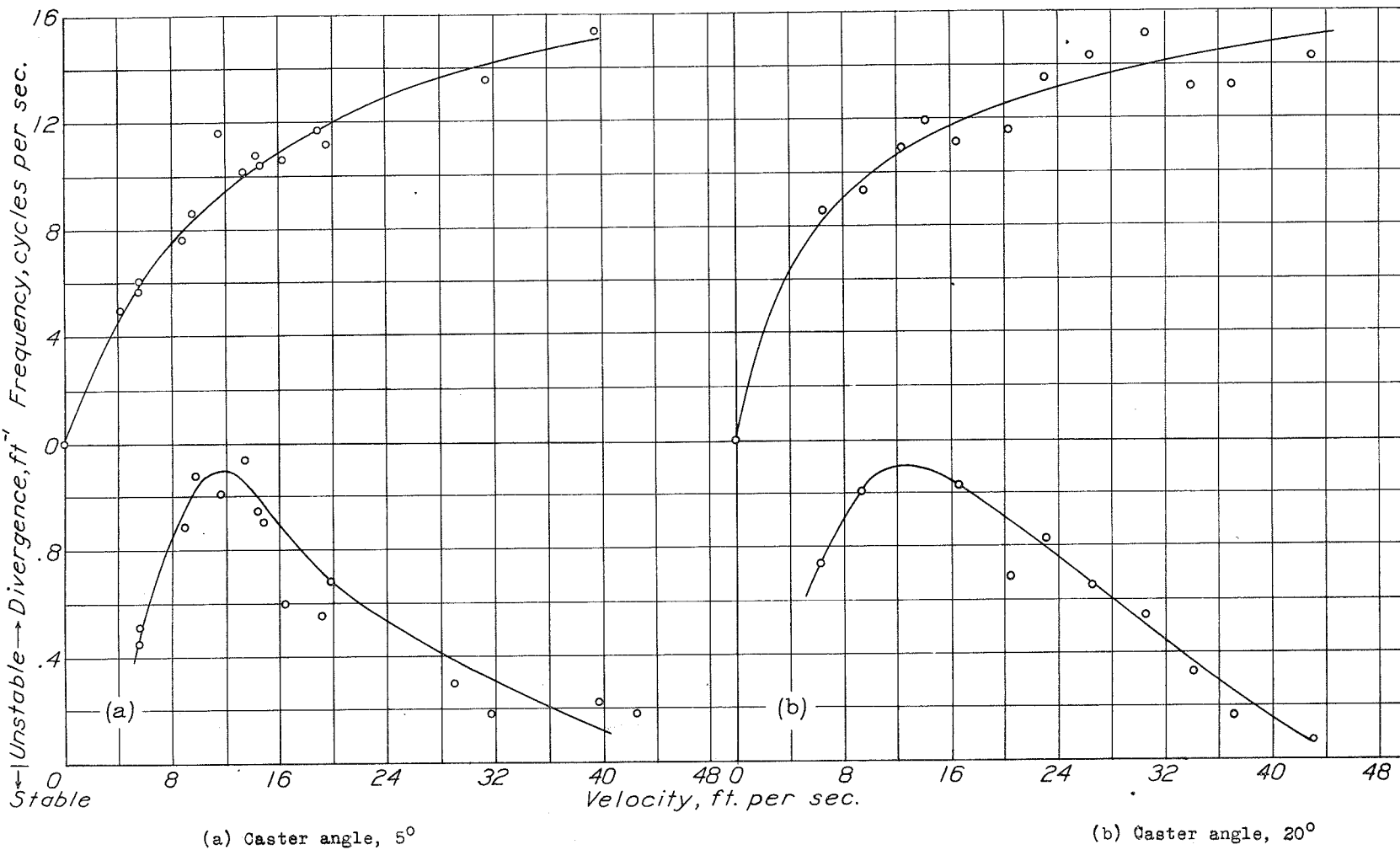
Figure 8.- Experimental values of divergence and frequency of shimmy against velocity as determined on belt machine.

Fig. 8
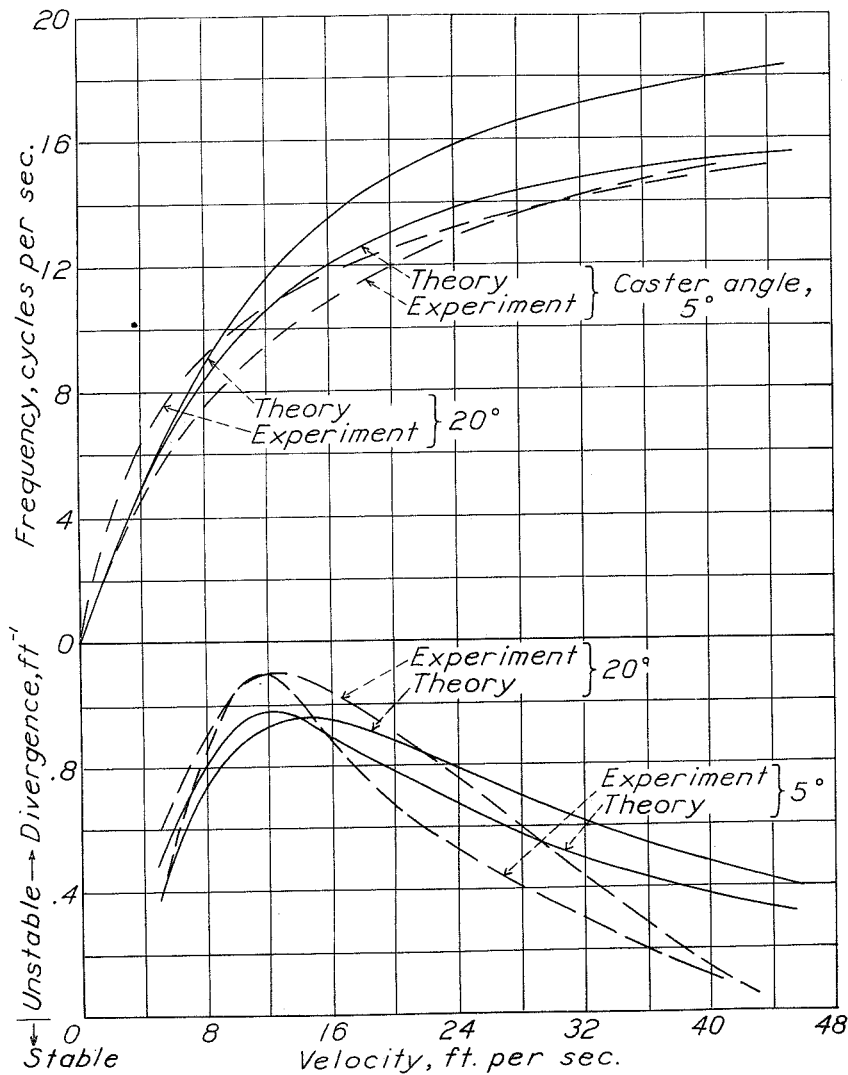
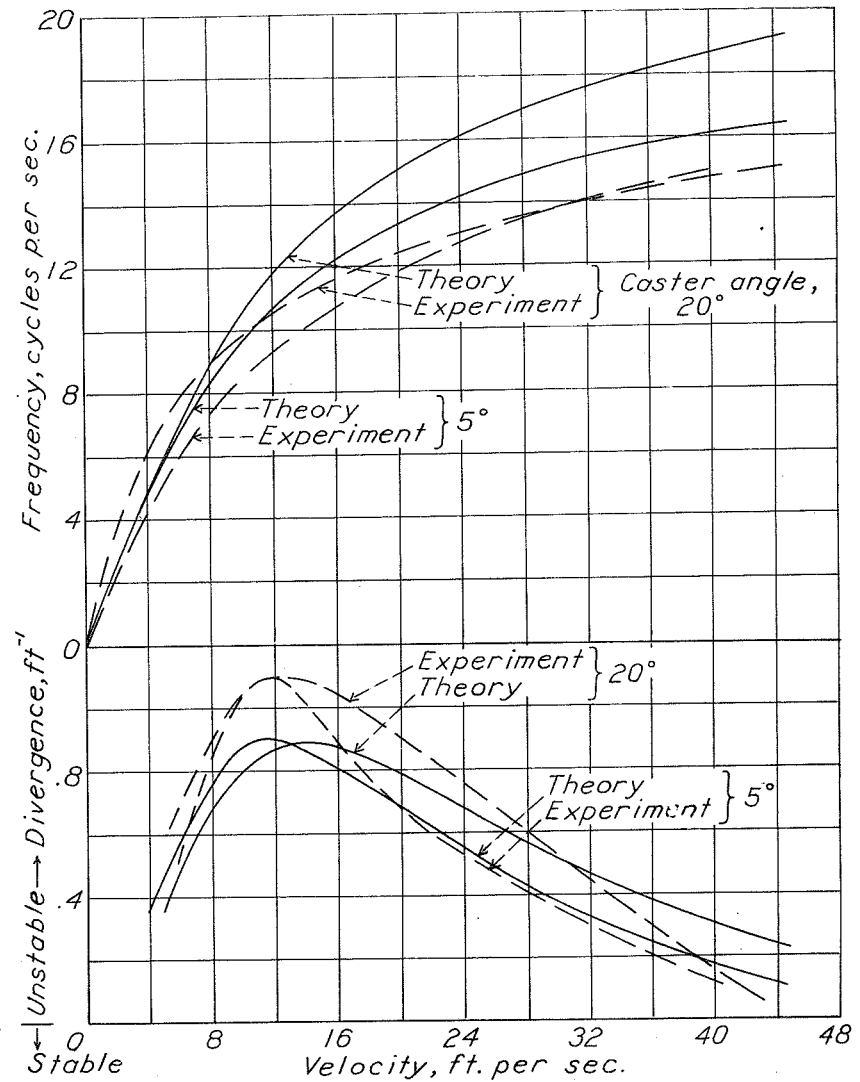Figure 9.- Divergence and frequency of shimmy on belt machine compared with theory.

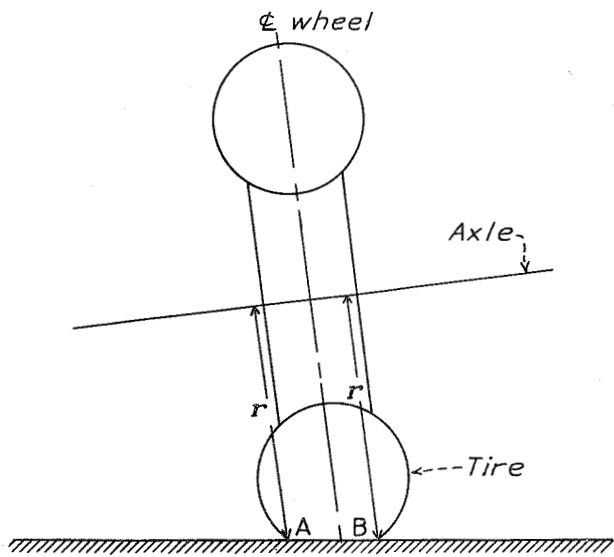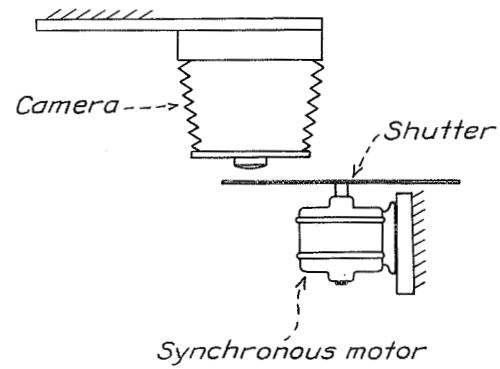Figure 10.- Divergence and frequency of shimmy on belt machine compared with theory including gyrostatic torque.
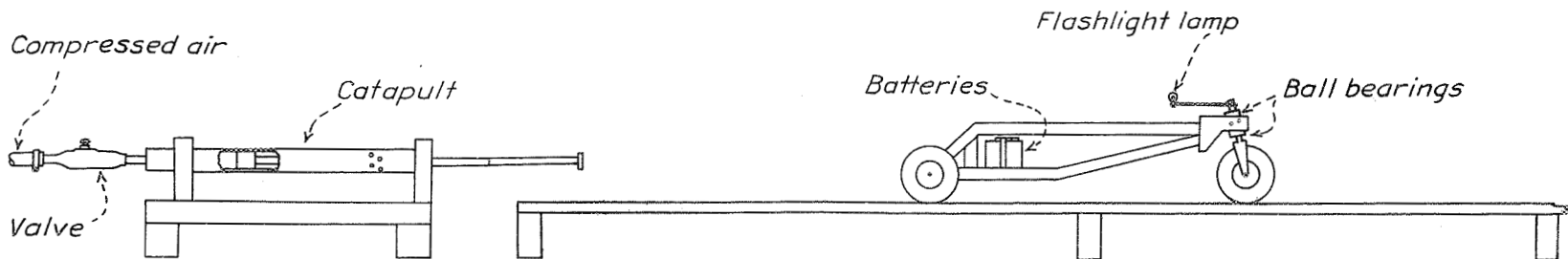
Figure 16.- Tilted axle.

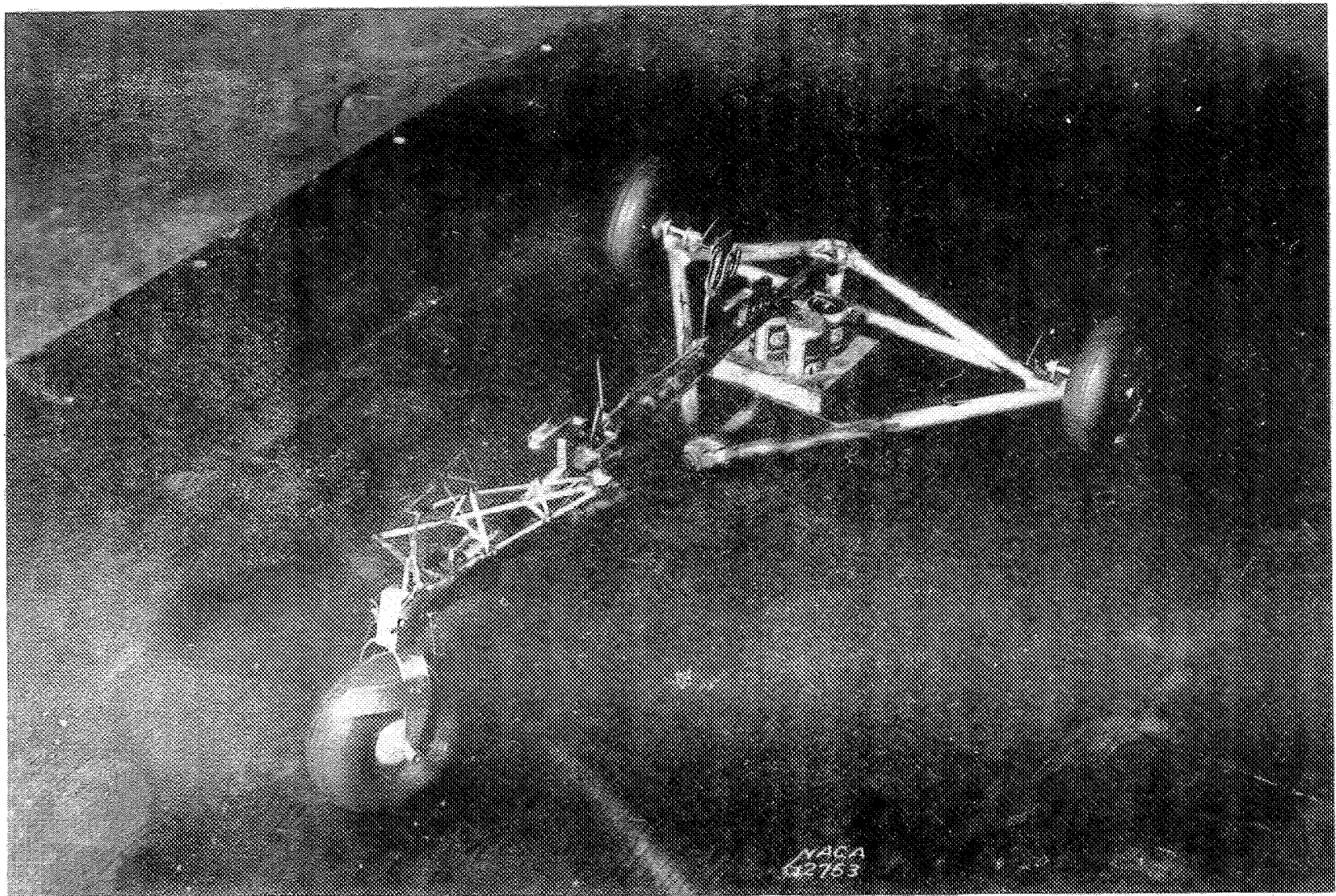Figure 11.- Apparatus for conducting free-model experiments. (Elevation view).

Fig. 12

Figure 12.- Model used for free-model test.

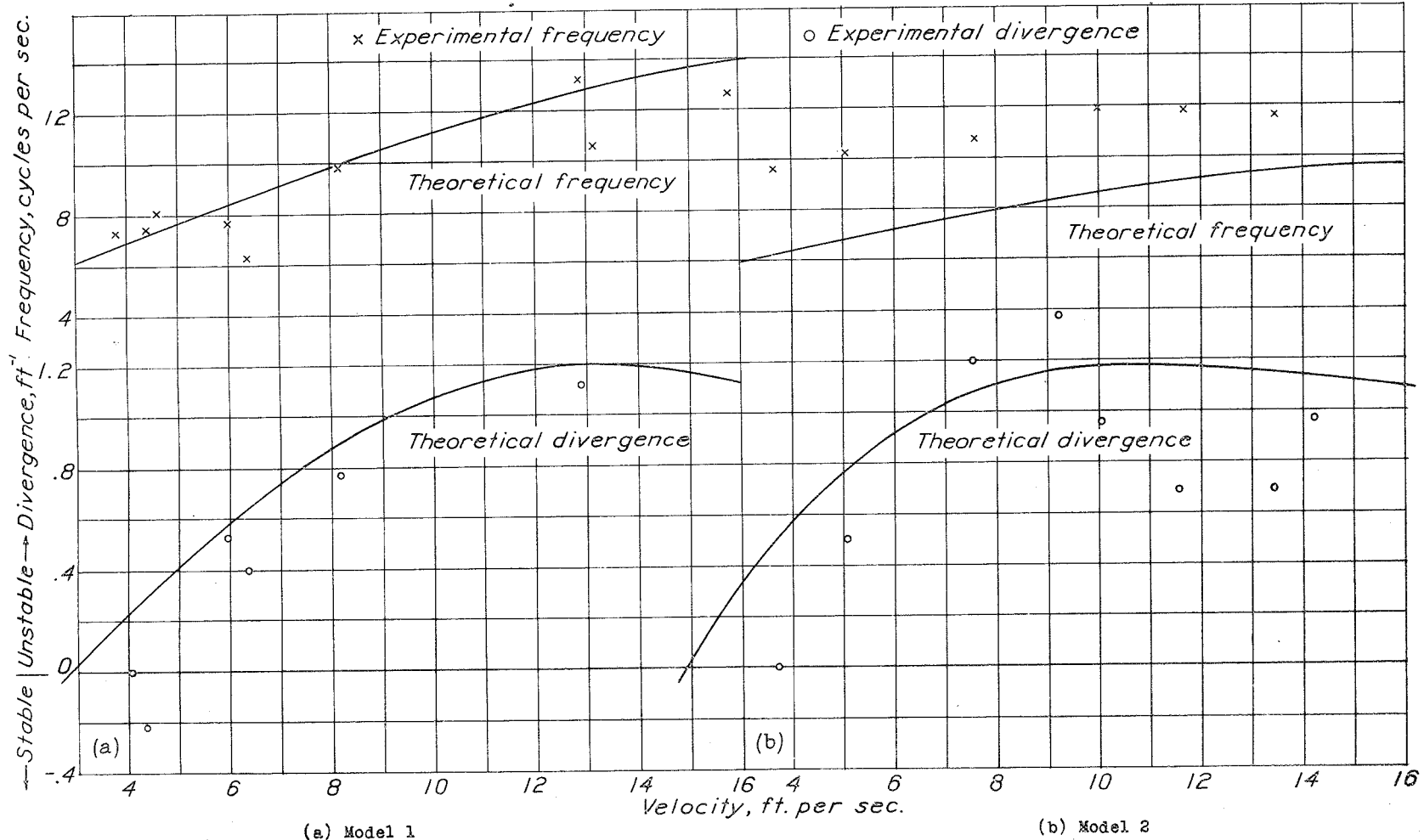(a) Model 1

(b) Model 2

Figure 14.— Divergence and frequency of shimmy of nose-wheel models compared with theory.

Fig. 14

Figure 15.-

Shimmy
in Wl-A
airplane.
64 frames
per second.

Fig. 15