

# MAGNETOSPHERIC MULTISCALE (MMS) MISSION COMMISSIONING PHASE ORBIT DETERMINATION ERROR ANALYSIS

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The Magnetospheric MultiScale (MMS) mission commissioning phase starts in a 185 km altitude x 12 Earth radii ( $R_E$ ) injection orbit and lasts until the Phase 1 mission orbits and orientation to the Earth-Sun line are achieved. During a limited time period in the early part of commissioning, five maneuvers are performed to raise the perigee radius to 1.2  $R_E$ , with a maneuver every other apogee. The current baseline is for the Goddard Space Flight Center Flight Dynamics Facility to provide MMS orbit determination support during the early commissioning phase using all available two-way range and Doppler tracking from both the Deep Space Network and Space Network. This paper summarizes the results from a linear covariance analysis to determine the type and amount of tracking data required to accurately estimate the spacecraft state, plan each perigee raising maneuver, and support thruster calibration during this phase. The primary focus of this study is the navigation accuracy required to plan the first and the final perigee raising maneuvers. Absolute and relative position and velocity error histories are generated for all cases and summarized in terms of the maximum root-sum-square consider and measurement noise error contributions over the definitive and predictive arcs and at discrete times including the maneuver planning and execution times. Details of the methodology, orbital characteristics, maneuver timeline, error models, and error sensitivities are provided.

## INTRODUCTION

The Magnetospheric MultiScale (MMS) mission, currently planned for launch in 2014, consists of four identically instrumented spacecraft that will be used to investigate magnetic reconnection, a process that converts magnetic field energy to heat and kinetic energy. A single launch vehicle will sequentially inject the MMS spacecraft into 28.5 degree inclination orbits, and the spacecraft will be maneuvered into a tetrahedron formation for the mission. The highly eccentric mission orbits will range from 1.2 Earth radii ( $R_E$ ) x 12  $R_E$  during Phase 1 to 1.2  $R_E$  x 25  $R_E$  during Phase 2 to allow MMS to study two regions of interest in the magnetic reconnection: the daytime magnetopause and the nighttime magnetotail, respectively..

The MMS mission commissioning phase starts in a highly elliptical (eccentricity approximately 0.84) injection orbit characterized by a 185 km altitude x 12  $R_E$  with a 1 day

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period. During a limited time period in the early part of commissioning, a nominal perigee raising maneuver sequence of five maneuvers every other apogee will be performed to raise the perigee radius to  $1.2 R_E$ . The commissioning phase ends when the spacecraft reach the Phase 1 mission orbit and the orientation to the Earth-Sun line are achieved.

The current navigation concept is for the Goddard Space Flight Center (GSFC) Flight Dynamics Facility (FDF) to provide MMS orbit determination support during the early commissioning phase using two-way range and Doppler tracking from both the Deep Space Network (DSN) and Space Network (SN). The overall goal of this study is to determine the expected orbit determination (OD) accuracy during the early commissioning phase. This paper examines the sensitivity of OD accuracy prior to the initial and final perigee raising maneuvers to the tracking schedule, tracker type, measurement type, OD tracking arcs, dynamic and measurement error sources, and additional estimated OD parameters.

The navigation performance is assessed versus the following MMS commissioning phase ground navigation requirements:

- Maneuver planning--The error in the velocity vector solution predicted ahead to the next apogee shall not exceed 5 mm/sec with 99% probability
- Open-loop thruster calibration--The velocity error in each component of the definitive OD ground solutions within one revolution after an open-loop thruster calibration maneuver shall not exceed either 1% of the associated components of the equivalent delta-V vector or 5 mm/s at an apogee time, whichever is greater, with 99% probability.
- Acquisition data prediction--The absolute position error in a 24-hour prediction transmitted to DSN 24 hours before the first contact shall not exceed 0.125 degrees in equivalent orbital error in the RSS of the along-track and cross-track orbital components as seen from the ground station. For the first perigee raising maneuver, the acquisition prediction requirement translates to about 400 m at perigee and 153 km at apogee. For the final perigee raising maneuver, the acquisition prediction requirement translates to about 2.8 km at perigee and 153 km at apogee.
- Relative position prediction--Position errors for 5 days ahead should be less than 1 km to satisfy collision assessment requirements

This paper discusses the navigation concepts for the early commissioning phase including the first and final perigee raising timelines and the definitive OD arcs and predictive arcs studied in each. The analysis methodology section provides a brief overview of the ODEAS tool used for the analysis and lists the major assumptions. The results sections follow with the first perigee raising maneuver results and then the final perigee raising maneuver. These sections discuss the tracking schedules and simulation scenarios and present the absolute and relative error results for the definitive and predictive position and velocity. The final section summarizes the conclusions and identifies future work. Additional information is provided in the appendix.

## NAVIGATION CONCEPTS FOR COMMISSIONING PHASE

The initial and final perigee raising maneuvers are the smallest maneuvers (about 2-3 m/sec delta-V) and therefore pose the biggest challenge to meeting the maneuver planning navigation accuracy requirement. The initial perigee raising maneuver scenario starts with the injection state vector. The final maneuver scenario uses the Phase 1a initial state. There is an assumption in this analysis that the four MMS are kept relatively close together for each OD arc. The same epoch is applied for all MMS spacecraft in this simulation; the chosen tracking schedule differentiates the cases.



Figure 1 shows the initial perigee raising maneuver simulation timeline. Spacecraft separation from the launch vehicle occurs at first perigee (P1) on October 15, 2014 12:00:00.000 UTC. The 1st perigee raising maneuver planning time occurs 79 hours after separation. OD arc lengths of 24 hours (OD 1) and 37 hours (OD 2) are studied. The tracking schedule during these OD arcs is described in the first perigee raising maneuver analysis section. A 5-hour predictive time interval follows up to the time of the first perigee maneuver execution.

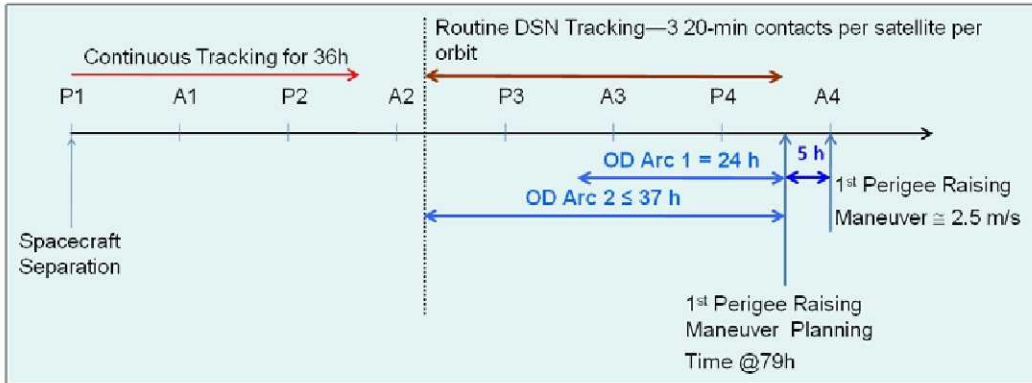


Figure 1. Timeline for Initial Perigee Raising Maneuver Simulation.

Figure 2 shows the final perigee raising maneuver simulation timeline. Three OD tracking arcs are studied: 42-, 37-, and 24-hour, which are measured back from the 5th perigee raising maneuver planning time. In addition to the 5-hour prediction to the final perigee raising maneuver, 24-hour and 48-hour prediction accuracies are also studied for acquisition data generation.

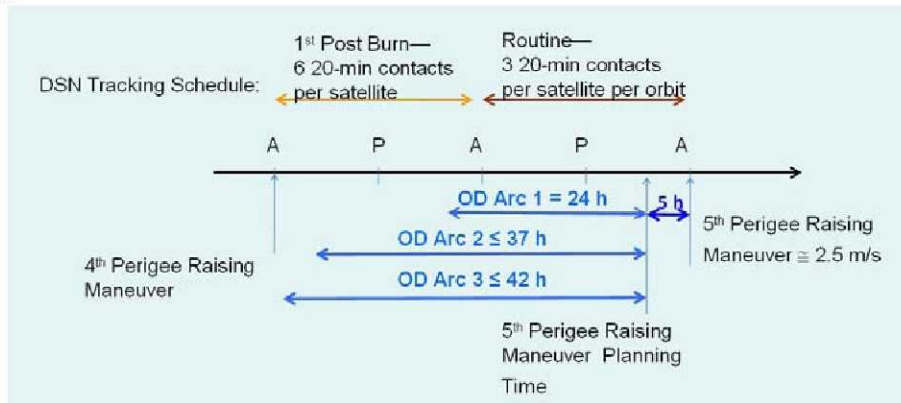


Figure 2. Timeline for Final Perigee Raising Maneuver Simulation.

## ANALYSIS METHODOLOGY

### Orbit Determination Error Analysis System (ODEAS)

This study was performed using GSFC’s Orbit Determination Error Analysis System (ODEAS). This is an institutional tool used extensively for pre-mission OD accuracy analysis as a function of tracking data types and tracking schedules for a wide variety missions including early orbit trajectories for GOES and STEREO. ODEAS performs linear covariance analysis for a batch-least-squares orbit determination of each spacecraft independently. The ODEAS analysis assumes independent estimation of each MMS and TDRS; however, operationally MMS and

TDRS will be simultaneously estimated using a state-of-the-art sequential estimator, which can process through maneuvers and support accurate bias estimation to reduce errors.

ODEAS solves for (i.e., estimates) errors on a state that include a single satellite position and velocity and additional optional parameters such as atmospheric drag coefficient, solar radiation pressure coefficient, clock drift and acceleration, and measurement biases. The state estimate covariance is computed at an epoch time and the errors are propagated over the definitive (i.e., period over which the measurements are taken) and predictive (i.e., the propagated portion) arcs. Table A-1 in Appendix A lists the spacecraft propagation models. ODEAS error estimates are conservative with 3-sigma uncertainties used for all error sources as listed in Table A-2 in Appendix A for this problem.

The total root-sum-square (RSS) and individual contributions to the estimated parameter covariance due to measurement noise and measurement and dynamic modeling error parameters that are “considered” (i.e., applied) but not estimated (i.e., solved-for) are computed. Equation 1 shows a simplification of the error calculation (see Reference 1 for more detail about ODEAS mathematics).

$$P_{\text{Total}}(t) = P_{\text{Noise}}(t) + P_{\text{Consider}}(t) \quad (1)$$

where

$P_{\text{Noise}}(t)$  = State estimate covariance due to the initial state uncertainty and the measurement noise propagated to time  $t$  using the state transition matrix

$P_{\text{Consider}}(t)$  = State estimate covariance due to dynamic and local consider parameter uncertainties propagated to time  $t$  using the state transition matrix

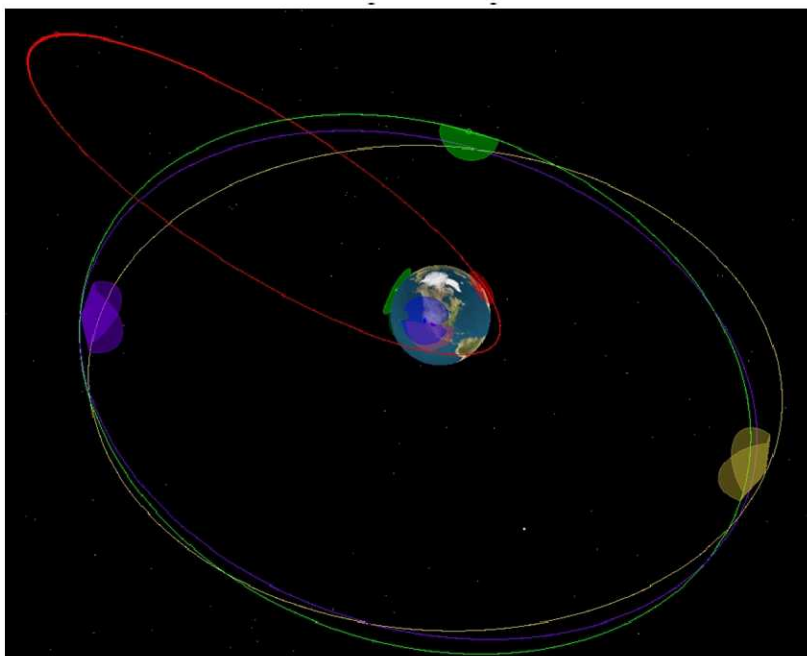
### Simulation Parameters

The three DSN ground tracking stations selected to generate tracking data are: Goldstone (GDSA), Madrid (MAD8) and Canberra (DS46). The TDRS satellites used are TDRS-3 (Zone of Exclusion, ZOE), TDRS-6 (West) and TDRS-10 (East). TDRS-3 communicates with the Guam (GWMK) ground station antenna and TDRS-6 and TDRS-10 communicate with White Sands, New Mexico station antennas (WHSK and WH2K respectively). Figure 3 depicts the highly elliptical shape (eccentricity approximately 0.84) of the MMS orbit (dark pink) during the 48-hours prior to the first perigee raising maneuver, TDRS (TDRS-3: green, TDRS-6: purple, TDRS-10: yellow) spacecraft and the ground station locations are also indicated.

A slant range distance of  $17.3 R_E$  (110,341.7 km) is the maximum limit for TDRS acquisition of a low data rate signal. When evaluating the visibility of MMS, this restriction on the slant range must be considered due to the MMS orbit extending beyond the altitude of the geosynchronous TDRSS constellation. A nominal elevation angle mask of 5 degrees was used for all the TDRS and DSN measurements. For TDRS tracking, a 1,000 km minimum height of ray path (HORP) restriction was used to eliminate geometries with long paths through the lower atmosphere. Also assumed was a rectangular transmitting antenna pattern on each TDRS with a half-angle width<sup>5</sup> (N-S) of 30.5 degrees and a half-angle height (E-W) of 76.8 degrees in order to simulate a 2nd and 3rd generation TDRS. Table A-3 in Appendix A lists additional nominal DSN/SN tracking parameters. The slant range restriction eliminated some TDRS visibility and modeling the TDRS transmitting antenna pattern further reduced the periods of visibility. Visibility of the TDRS and ground stations will also vary with the geometry imposed by each



MMS launch opportunity. Furthermore, there is a spacecraft thermal and power constraint that limits the tracking to 2.5 hours every 6 hours per spacecraft.



**Figure 3. MMS and TDRS Orbits with SN and DSN Ground Station Visibility Prior to Final Perigee Raising Maneuver.**

## FIRST PERIGEE RAISING MANEUVER ANALYSIS

### First Perigee Raising Tracking Schedule

Routine DSN tracking for the first perigee raising maneuver consists of three 20-minute DSN contacts per spacecraft each orbit with 10-minute handover intervals to switch between spacecraft. When SN tracking is added to the schedule, 20-minute SN contacts per spacecraft are scheduled when not being tracked by DSN, with 10-minute handover intervals. Figure 4 shows the total visibility (solid rectangles) and the selected tracking passes for each MMS chosen in round robin fashion (MMS1 followed by 2, 3, 4) for the 24-hour definitive case. The three DSN ground stations are at the bottom of the graph and the three TDRS are above. The 24-hour DSN-only and DSN + SN tracking schedule used for each MMS can be extracted from this graph. Figure 5 shows the total visibility (solid rectangles) and tracking passes selected for each MMS for the 37-hour definitive case.

Table 1 lists the tracking scenarios studied. The atmospheric drag coefficient, a major error source for this orbit, is estimated in all cases in addition to position and velocity. Clock drift and acceleration were only estimated in the one-way Doppler cases, which were initially analyzed before moving to the current two-way range and Doppler baseline. Processing only DSN one-way Doppler did not provide adequate OD accuracy for planning the maneuver (see Reference 2), with noise being a major error contributor; and the atmospheric drag coefficient being poorly estimated for the 24-hour and 37-hour cases, which implies an unstable solution in the DSN-only Scenarios 1 and 2 (which are differentiated by a minor adjustment in tracking schedule). Adding one-way SN Doppler provided an improvement when solving for the atmospheric drag

coefficient; however, when solving for the clock drift and acceleration in addition, noise increases and these additional parameters are not well estimated.

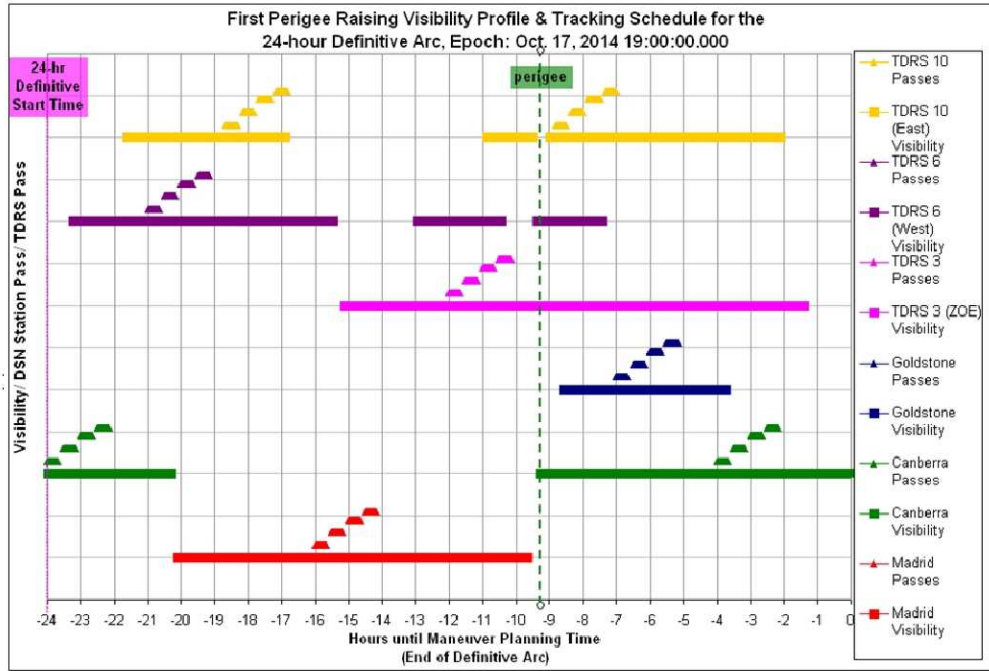


Figure 4. Initial Perigee Raising Maneuver 24-hour Visibility and Tracking Schedule.

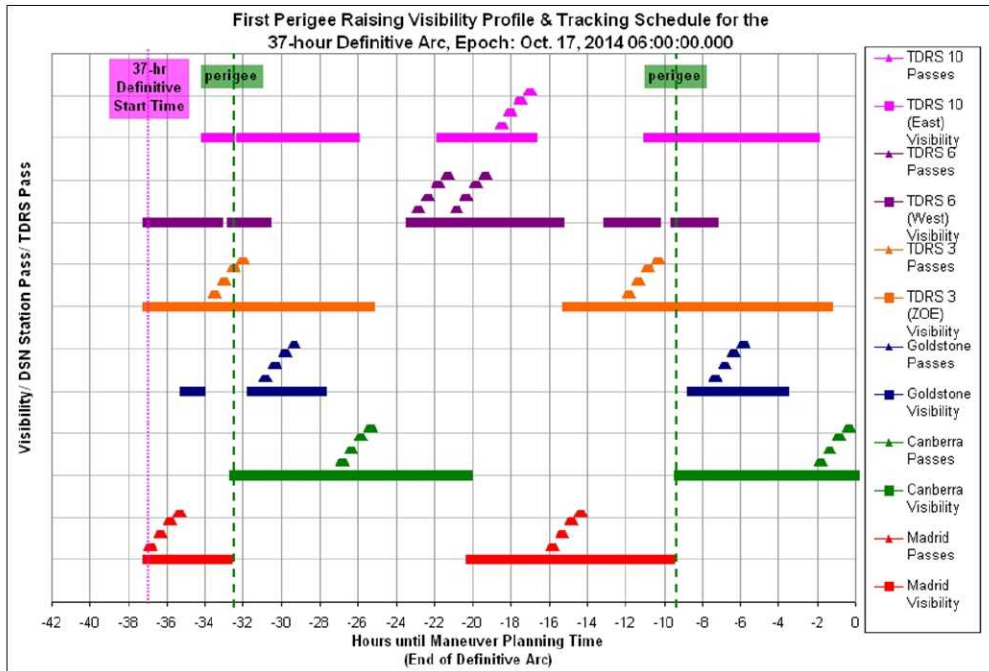


Figure 5. Initial Perigee Raising Maneuver 37-hour Visibility and Tracking Schedule.

Table 1. Initial Perigee Raising Maneuver Tracking Scenarios.

Scenario	Tracking Systems	Measurement Types	Additional Estimated Parameters
1,2	DSN	1-Way Doppler	Drag
3	DSN & SN	1-Way Doppler	Drag
4	DSN & SN	1-Way Doppler	Drag, Clock Drift
5	DSN & SN	1-Way Doppler	Drag, Clock Drift, Clock Acceleration
7	DSN	2-Way Range and Doppler	Drag
8	DSN & SN	2-Way Range and Doppler	Drag
9	DSN & SN	2-Way DSN Doppler 2-Way SN Range and Doppler	Drag

The best case is Scenario 8, which uses DSN and SN two-way range and Doppler with drag estimated. The addition of SN measurements improves the solution from just having DSN-only. Scenario 9 is similar to 8, except with DSN measurements, where only two-way Doppler is used. The results of cases Scenarios 8 and 9 will be the focus in the following sections.

**Initial Perigee Raising: Absolute Tracking Sensitivity**

Figure 6 compares the maximum absolute total root-sum-square and Noise (RSS+Noise) position error for each MMS spacecraft during the 24-hour and 37-hour definitive arcs for Scenarios 8 and 9. Figure 7 compares the maximum absolute RSS+Noise position error during the 5-hour predictive arc for both cases for Scenarios 8 and 9.

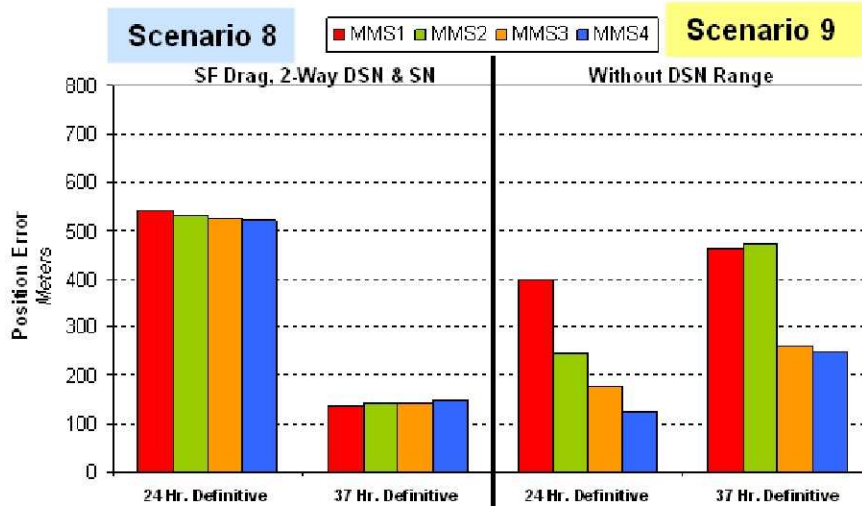


Figure 6. Initial Perigee Raising Maneuver Maximum Absolute RSS+Noise Position Error During the Definitive Arc.



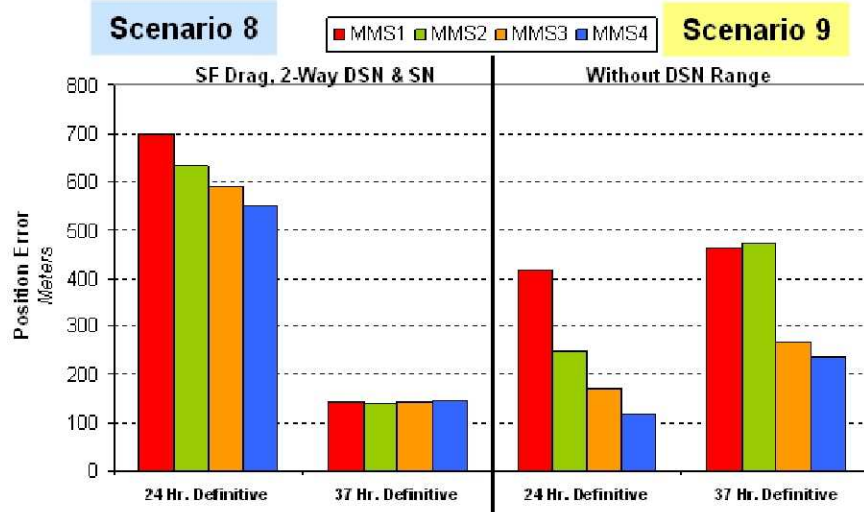


Figure 7. Initial Perigee Raising Maneuver Maximum Absolute RSS+Noise Position Error During 5-Hour Predictive Arc.

Removing DSN range (i.e. Scenario 9) produced larger position error variations between the four spacecraft; the errors decreased for the 24-hr OD arc and increased for 37-hr OD arc. Having both DSN range and Doppler tracking (i.e., Scenario 8) and extending the definitive arc length to 37 hours significantly reduces absolute maximum RSS+Noise position to roughly 150 m for both definitive and predictive arcs. The shorter 24-hour tracking arc is better for the cases where DSN range is removed. The errors are reduced from over 500 m (Scenario 8) to less than 400 m (Scenario 9) for the definitive arc and from 550 m to 700 m (Scenario 8) to 250 m to 480 m (Scenario 9) for the predictive arc.

Figure 8 compares the maximum absolute RSS+Noise velocity error at the first apogee (approximately 21 hours prior to the first perigee maneuver planning time) in both definitive arc cases for Scenarios 8 and 9 versus the 5 mm/s thruster calibration requirement (note that 1% of the delta-V is probably on the order of 5 mm/s). Figure 9 shows the maximum absolute RSS+Noise velocity error at the first perigee maneuver execution time (5 hours after the maneuver planning time) versus the 5 mm/s maneuver planning prediction requirement. The impact of removing DSN range allowed both 24- and 37-hour definitive arcs to satisfy the thruster calibration requirement and the 24-hour definitive arc to satisfy the maneuver planning requirement. The velocity error between Scenarios 8 and 9 decreased for the 24-hr OD arc, but slightly increased for 37-hr OD arc.



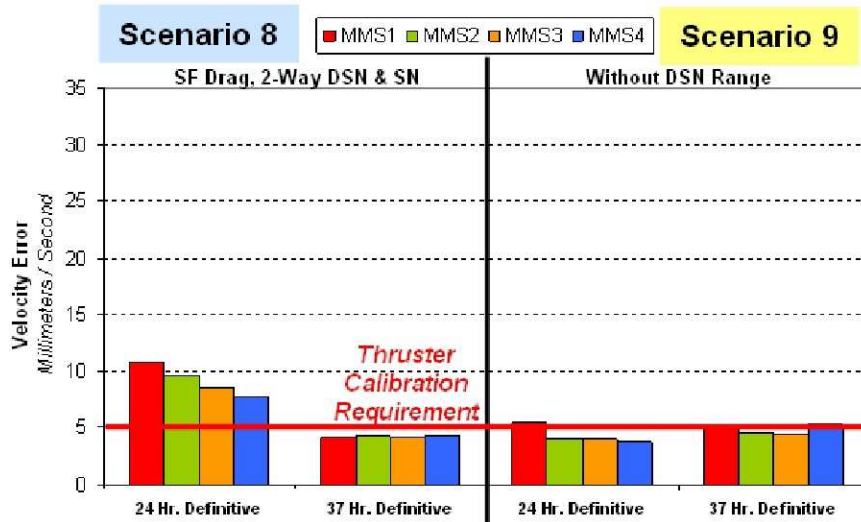


Figure 8. Initial Perigee Raising Maneuver Maximum Absolute RSS+Noise Velocity Error at First Apogee during the Definitive Arc.

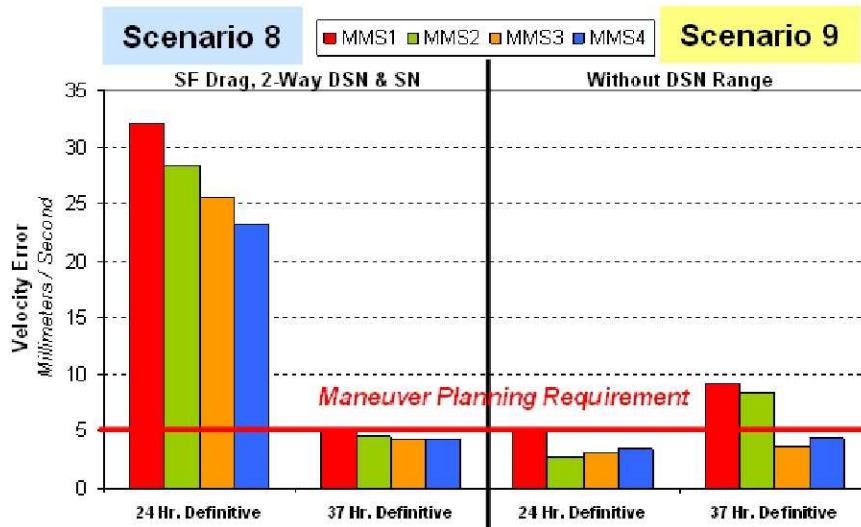


Figure 9. Initial Perigee Raising Maneuver Maximum Absolute RSS+Noise Velocity Error at Maneuver Execution time in the 5-Hour Predictive Arc.

**Initial Perigee Raising: Error History Profiles**

Figures 10 through 13 are a series of position and velocity error history profiles for 37-hour definitive OD arc comparing Scenarios 8 and 9 (Reference 2 provides additional history graphs). For position errors, the maximum errors are at apogee, and the minimum are at perigee. The opposite is true for velocity. Major position consider error sources include station ionosphere delays (i.e., ION-GWMK), TDRS ephemeris errors (i.e., EPHERTDR) and DSN range biases when processed (i.e., MEASBI 1).

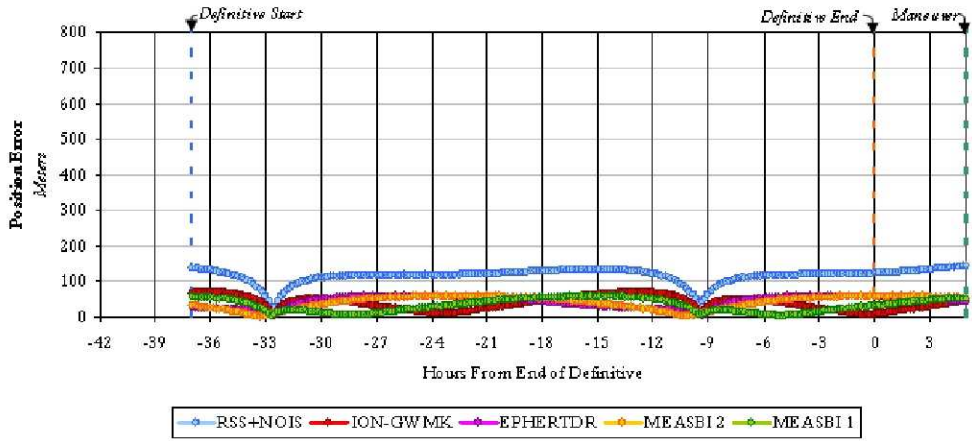


Figure 10. Initial Perigee Raising Maneuver Scenario 8: MMS1 37-hour OD Arc, Position Major Error Source History.

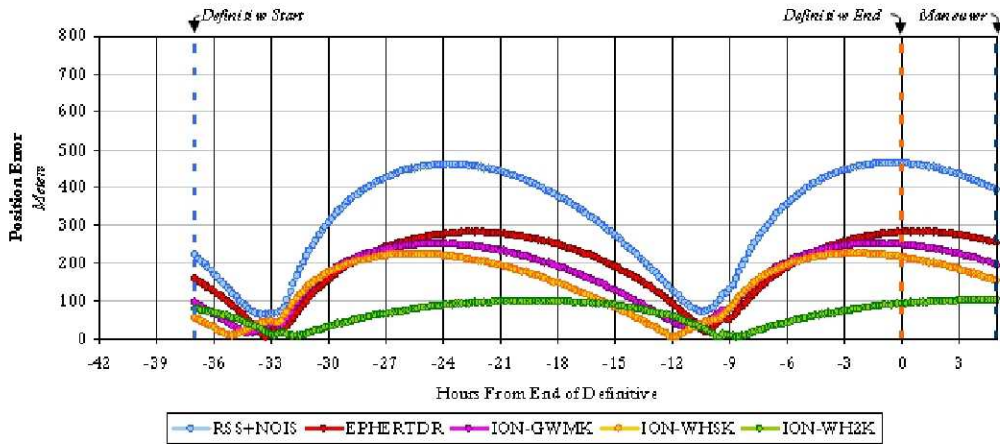


Figure 11. Initial Perigee Raising Maneuver Scenario 9: MMS1 37-hour OD Arc, Position Major Error Source History.

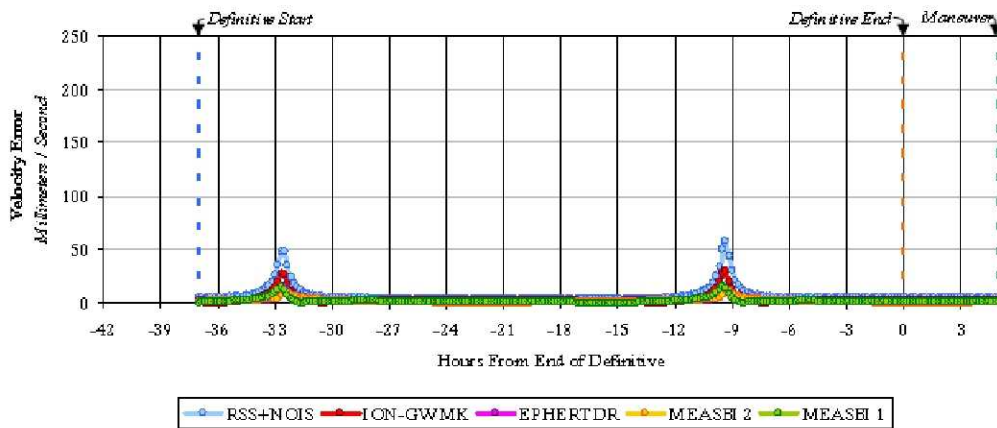


Figure 12. Initial Perigee Raising Maneuver Scenario 8: MMS1 37-hour OD Arc, Velocity Major Error Source History over the Definitive and Predictive Arcs.

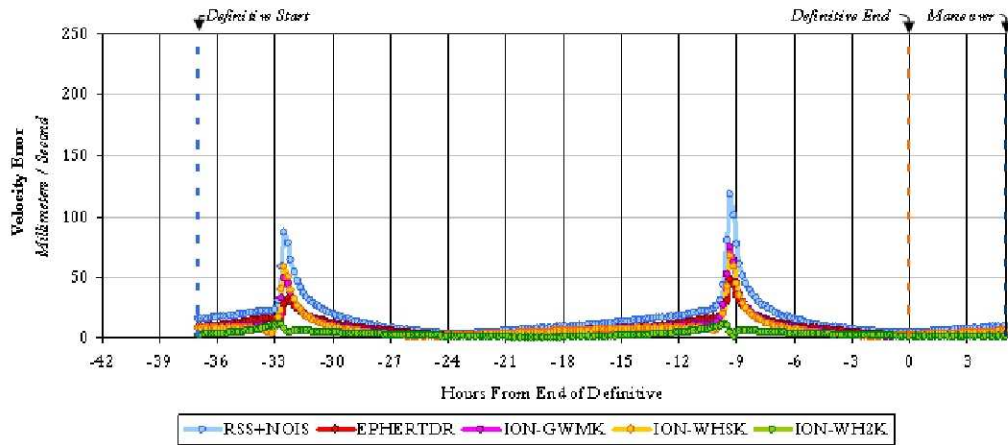


Figure 13. Initial Perigee Raising Maneuver Scenario 9: MMS1 37-hour OD Arc, Velocity Major Error Source History over the Definitive and Predictive Arcs.

**Initial Perigee Raising: Relative Tracking Sensitivity**

The average relative position and velocity errors of MMS2, 3 and 4 with respect to (w.r.t.) MMS 1 at the maneuver planning time are shown in Table 2. Appendix B discusses how the relative errors are calculated.

Table 2. Initial Perigee Raising Maneuver Relative Errors at the Maneuver Planning Time.

Scenario	Definitive Tracking Arc	Average w.r.t. MMS1 Relative Position Error (m)	Average w.r.t. MMS1 Relative Velocity Error (mm/s)
8	24-hour	646.16	23.867
	37-hour	158.59	5.867
9	24-hour	348.527	9.033
	37-hour	537.977	6.400

**FINAL PERIGEE RAISING MANEUVER ANALYSIS**

**Final Perigee Raising Tracking Schedule**

For the final perigee maneuver, the nominal DSN tracking schedule is six 20-minute contacts per spacecraft for the first post-burn orbit and three 20-minute contacts per spacecraft for the following orbit, with 10-minute downtime to switch between spacecraft. The SN tracking is added when not being tracked by DSN as 20-minute contacts per spacecraft, with 10-minute handover interval between spacecraft. Three OD tracking arcs were investigated and Figure 14 shows the complete 42-hour schedule, the 37-hour, 24-hour and DSN-only and DSN & SN cases can be extracted from this plot. Figure 15 shows an additional tracking schedule for the 24-hour OD arc with 2 additional DSN passes from Madrid and Canberra.



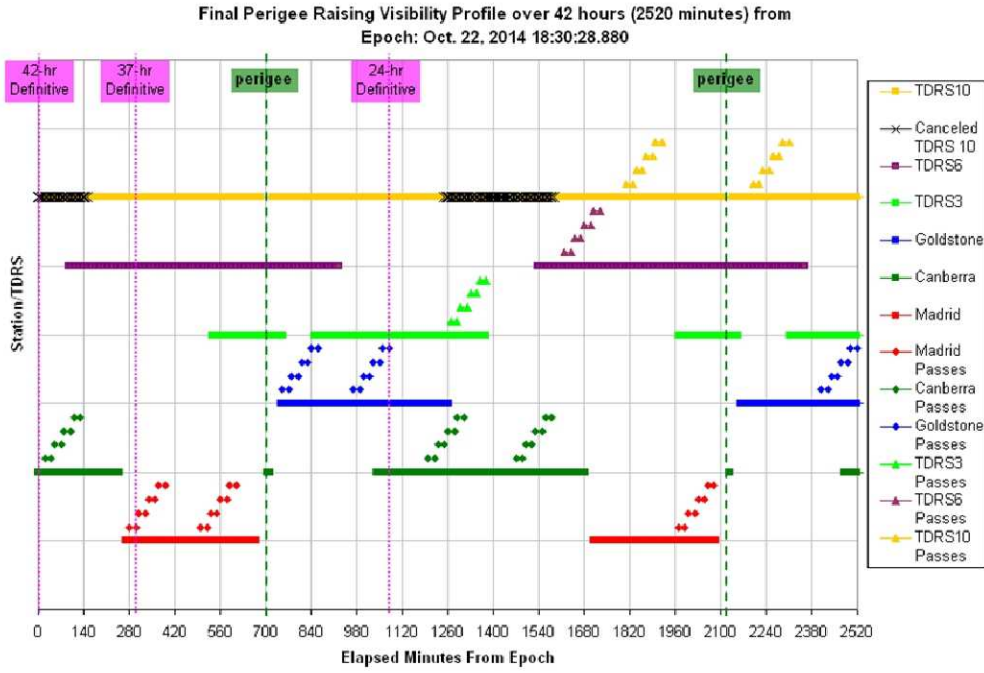


Figure 14. Final Perigee Raising Maneuver 42-hour Total Visibility and Tracking Schedule.

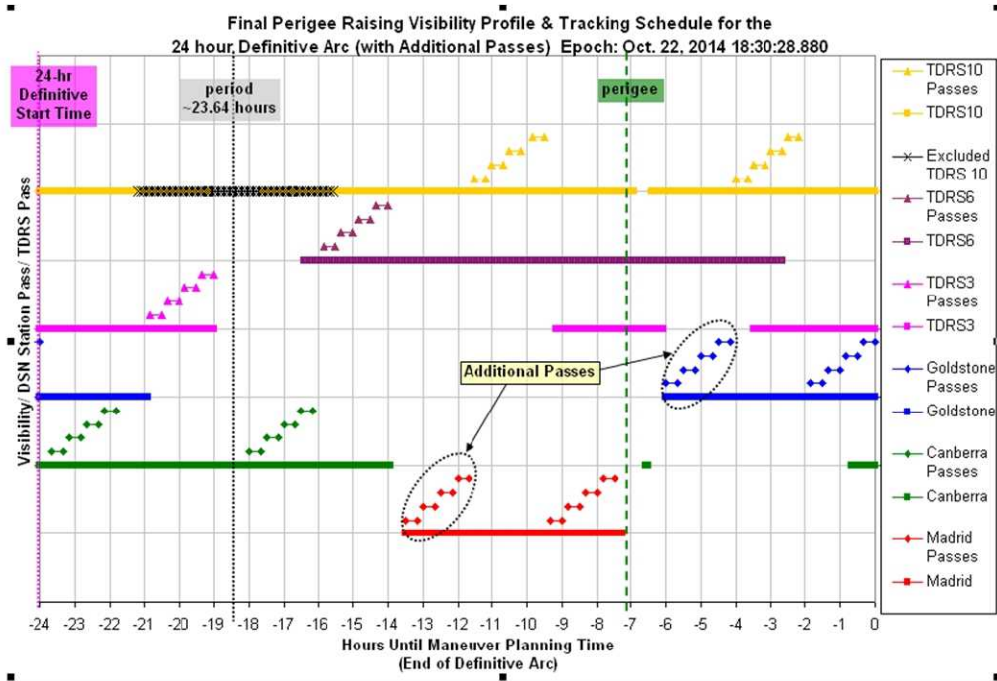


Figure 15. Final Perigee Raising Maneuver 24-hour Additional Visibility and Tracking Schedule.

Table 3 lists all the scenarios analyzed for the final perigee (see Reference 3 for more details). Scenarios 4 and 7 use tracking schedule derived from Figure 14; and Scenarios 5 and 8 use the additional 24 hour tracking schedule of Figure 15; the scenario pairs are differentiated by whether

each uses the two-way DSN range measurement type. These four scenarios are discussed in more detail in the remainder of this section

**Table 3. Final Perigee Raising Maneuver Tracking Scenarios.**

Scenario	Tracking Systems	Measurement Types	Additional Estimated Parameters*
1	DSN	2-Way Doppler and Range	Drag
2	DSN	2-Way Doppler and Range	
3	DSN	2-Way Doppler and Range	Solar Radiation Pressure (SRP)
4	DSN & SN	2-Way Doppler and Range	
5**	DSN & SN	2-Way Doppler and Range	
6**	DSN-only	2-Way Doppler and Range	
7	DSN & SN	2-Way DSN Doppler 2-Way SN Range and Doppler	
8**	DSN & SN	2-Way DSN Doppler 2-Way SN Range and Doppler	

### Final Perigee Raising: Absolute Tracking Sensitivity

Figure 16 compares the average (among all 4 MMS spacecraft) absolute position errors during the definitive and the entire 48-hour predictive arcs. For DSN+SN, two-way range and Doppler (Scenarios 4 and 5), the best solutions are with 37- and 42-hour OD arcs, which are very similar. There is little prediction error growth for longer OD arcs. Additional DSN contacts for 24-hr arc slightly beneficial when comparing the 24-hour arcs of Scenario 4 to 5 and 7 to 8. Without DSN two-way range (Scenarios 7 and 8), the 24-hr definitive solution improves, yet it degrades 37-hr solution. Having additional DSN contacts for 24-hr arc is not beneficial when DSN range is removed. However, this result may be different if the two extra passes were TDRS instead of DSN. All predictive position solutions satisfy 1-day acquisition data requirement of 2.8 km at perigee and 153 km at apogee.

Figure 17 compares the average absolute velocity error at first apogee (18.4 hours prior to the end of the definitive arc) and the absolute velocity error at maneuver execution time (5 hours into prediction). There is little difference in position and velocity errors between the 42-hour and 37-hour definitive results across all scenarios, implying there is no real benefit of extending the orbit determination arc. However, there is a sizeable difference between the 24-hour and 37-hour solutions with DSN range and Doppler (Scenario 4) indicating that there may be a definitive time length in between the arcs that provides a solution with equivalent results to the 37-hour arc, but in a shorter time. Additional 24-hour contacts, or removing DSN range for this OD arc length improves the solution.

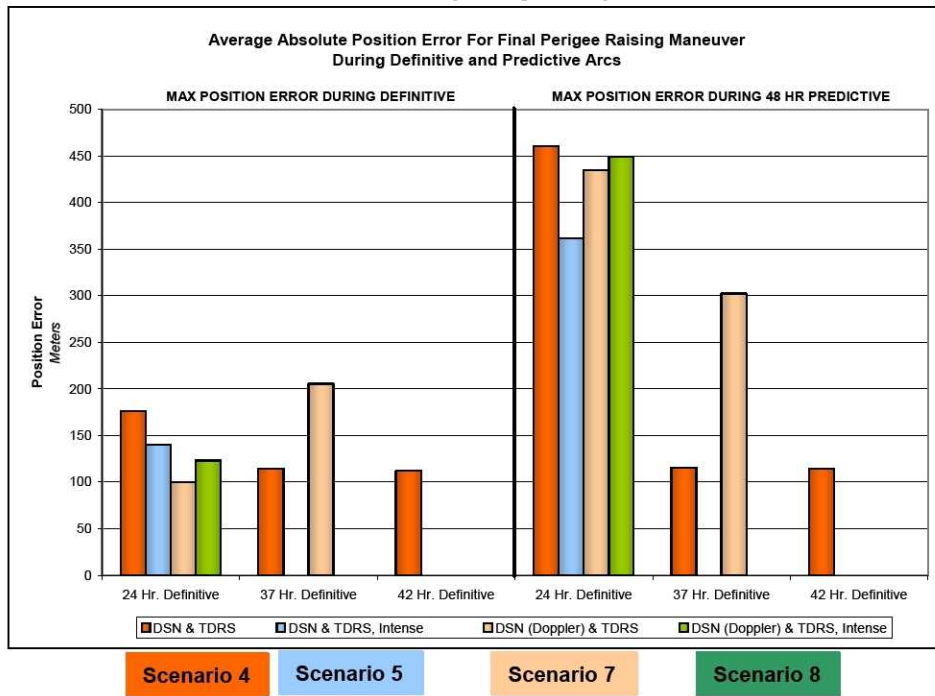
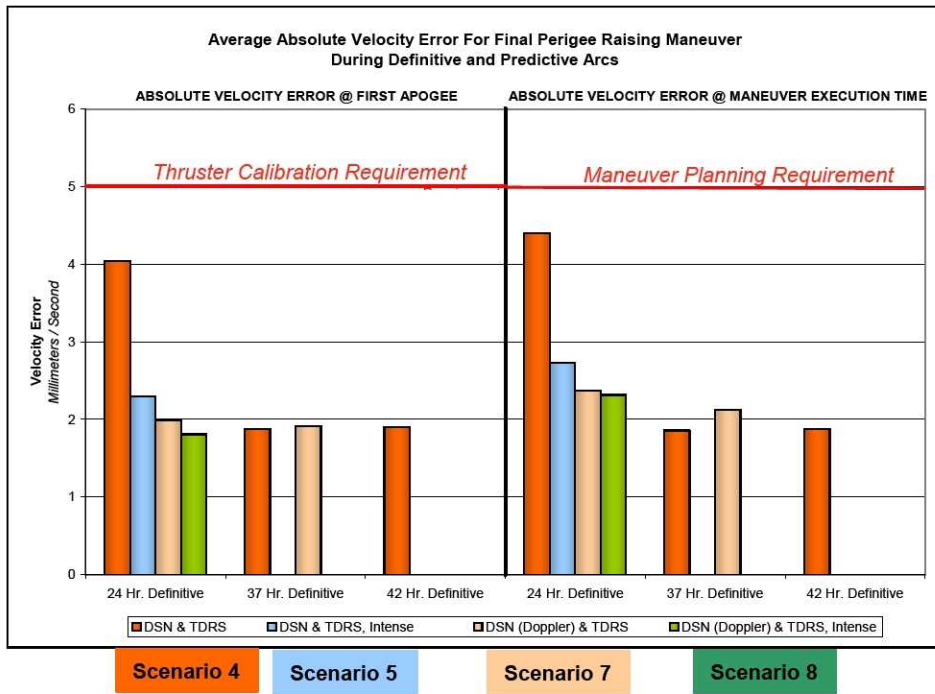


Figure 16. Final Perigee Raising Maneuver Maximum Average Absolute RSS+Noise Position Error During the Definitive and Predictive Arc.



7. Final Perigee Raising Maneuver Average Absolute RSS+Noise Velocity at First Apogee and at the Maneuver Execution Time.



Final Perigee Raising: Error History Profiles

Figures 18 through 21 are a series of position and velocity error history profiles comparing the 37-hour OD arc of Scenarios 4 (with DSN range and Doppler) and 24-hour additional tracking OD arc Scenario 8 (with DSN Doppler) (Reference 4 provides additional history graphs). For position errors, the maximum errors are at apogee, and the minimum are at perigee. The opposite is true for velocity. Major position consider error sources include station ionosphere delays (i.e., ION-GWMK), TDRS ephemeris errors (i.e., EPHERTDR), DSN range biases when processed (i.e., MEASBI 1), and troposphere (i.e., TROPO 1).

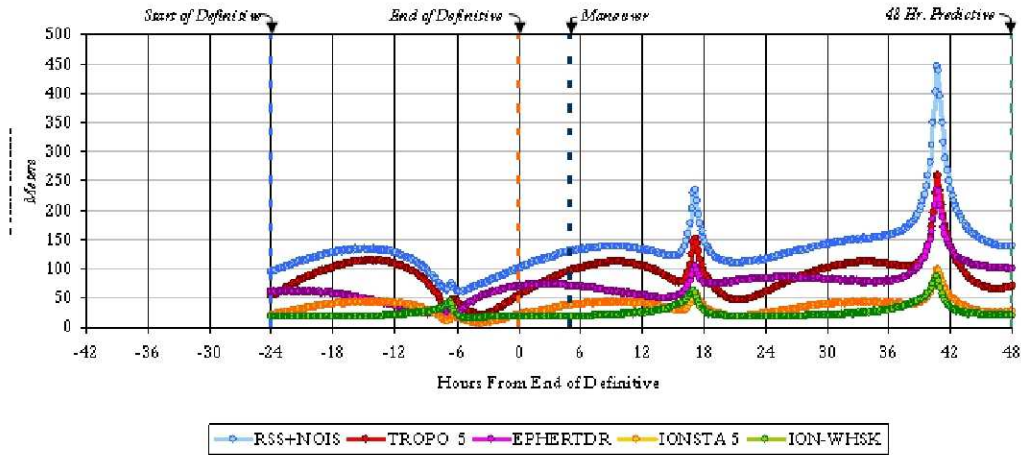


Figure 18. Final Perigee Raising Maneuver Scenario 8: MMS1 24-hour Additional Tracking OD Arc, Position Major Error Source History over the Definitive and Predictive Arcs.

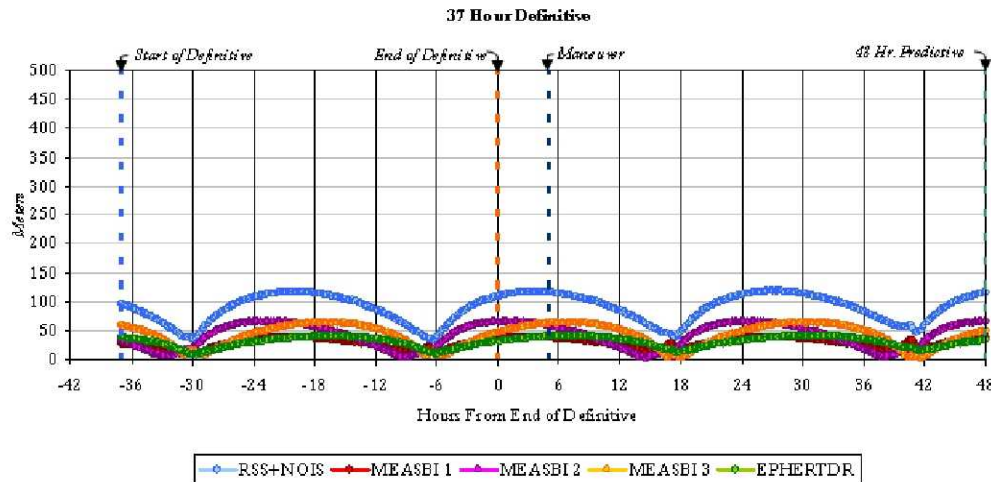


Figure 19. Final Perigee Raising Maneuver Scenario 4: MMS1 37-hour OD Arc, Position Major Error Source History over the Definitive and Predictive Arcs.

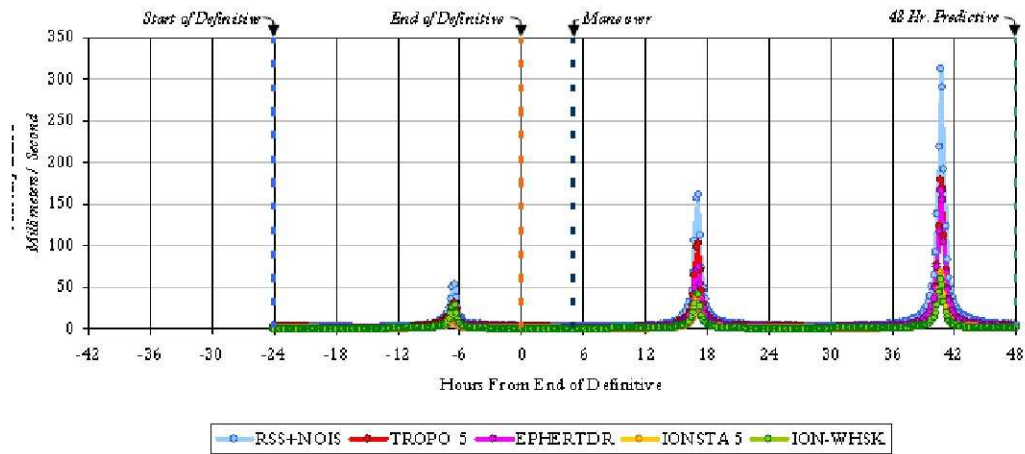


Figure 20. Final Perigee Raising Maneuver Scenario 8: MMS1 24-hour Additional Tracking OD Arc, Velocity Major Error Source History over the Definitive and Predictive Arcs.

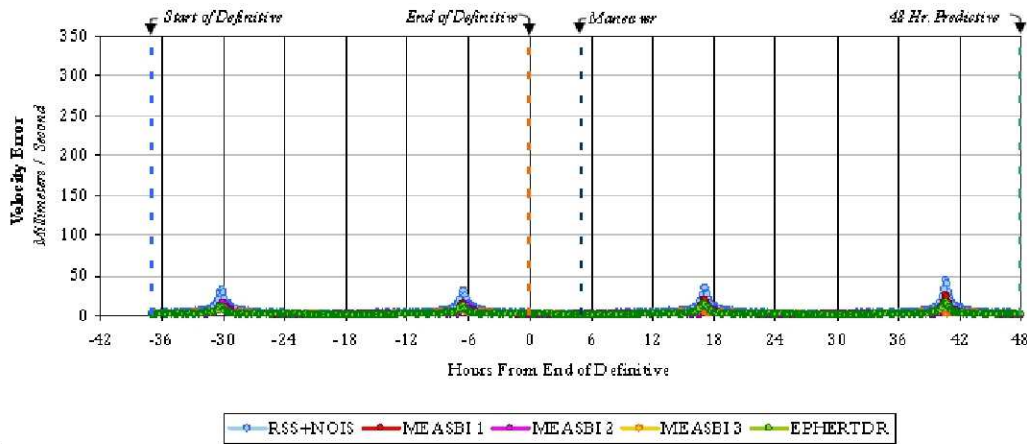


Figure 21. Final Perigee Raising Maneuver Scenario 4: MMS1 37-hour OD Arc, Velocity Major Error Source History over the Definitive and Predictive Arcs.

### Final Perigee Raising: Relative Tracking Sensitivity

Figures 21 and 22 show the relative position error history for Scenario 8 (24-hour additional OD arc) and Scenario 4 (37-hour OD arc) for each MMS spacecraft w.r.t. MMS 1. Maximum relative position errors are near apogee, minimum near perigee. The relative errors were comparable for MMS1 w.r.t. MMS2, 3 and 4 within each scenario, except for the DSN-only MMS3 outlier case which produced relative position errors as large as 25 km at the maneuver planning time.

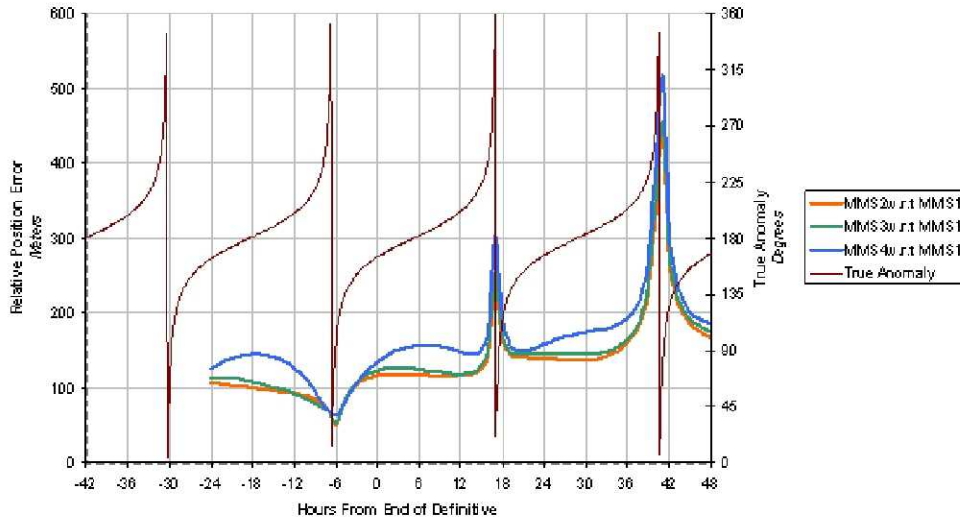


Figure 21. Final Perigee Raising Maneuver Scenario 8: MMS1 24-hour Additional Tracking OD Arc, Relative Position Error History over the Definitive and Predictive Arcs.

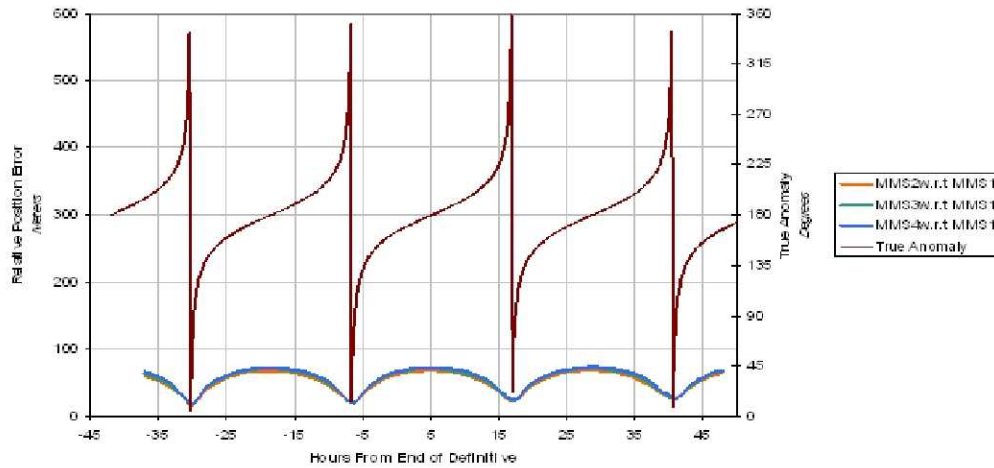


Figure 22. Final Perigee Raising Maneuver Scenario 4: MMS1 37-hour OD Arc, Relative Position Error History over the Definitive and Predictive Arcs.

## CONCLUSIONS AND FUTURE WORK

Tables 4 and 5 are a summary of the initial and final perigee maneuver absolute results and indicate if they satisfy the associated requirements. The values were averaged among all 4 spacecraft. Note the maneuver planning value is taken at the first apogee time (5 hours) into the predictive arc. The thruster calibration value is at the apogee time during the definitive arc, which is about 21 hours prior to the end of definitive OD arc for the initial perigee and about 18.4 hours prior for the final perigee. The initial perigee results were not propagated out to 24 hours, therefore acquisition data prediction requirement following maneuvers still needs to be verified. The collision probability 2 day + 1 revolution prediction accuracy verification is also future work for both maneuvers. Future ground navigation requirement verification analyses will be performed using a simulation approach rather than the linear covariance approach that was used in the current study. In these studies, a high-accuracy OD software system similar to that expected to be used for actual mission support will be used.



Table 4. Initial Perigee Raising Maneuver: Performance vs. Requirements.

Scenario	Early Orbit Navigation Requirements		
	Maneuver Planning: Prediction Error $\leq 5$ mm/s at apogee	Thruster Calibration: Definitive Velocity Error $\leq 1\%$ of Delta-V or 5 mm/sec at apogee	Acquisition Prediction: 24-hr Prediction Error < 0.125 deg
<b>First Perigee Raise:</b>			400 m @ perigee 153 km @ apogee
DSN+SN 24-hr arc	27.3 mm/s	9.2 mm/s	TBD
DSN+SN 37-hr arc	4.5 mm/s	4.2 mm/s	TBD
DSN (no range)+SN 24-hr arc	3.6 mm/s	4.3 mm/s	TBD
DSN (no range)+SN 37-hr arc	6.4 mm/s	4.9 mm/s	TBD

Table 5. Final Perigee Raising Maneuver: Performance vs. Requirements.

Scenario	Early Orbit Navigation Requirements		
	Maneuver Planning: Prediction Error $\leq 5$ mm/s at apogee	Thruster Calibration: Definitive Velocity Error $\leq 1\%$ of Delta-V or 5 mm/sec at apogee	Acquisition Prediction: 24-hr Prediction Error < 0.125 deg
<b>Final Perigee Raise:</b>			2.8 km @ perigee 153 km @ apogee
DSN+SN 24-hr arc	4.0 mm/s	3.69 mm/s	<250 m
DSN+SN 37-hr arc	1.9 mm/s	2.59 mm/s	<150 m
DSN+SN 24-hr arc-Intense	2.3 mm/s	3.20 mm/s	<250 m
DSN (no range) +SN 24-hr arc	2.0 mm/s	2.93 mm/s	<250 m
DSN (no range) +SN 37-hr arc	1.9 mm/s	2.57 mm/s	<250 m
DSN (no range) +SN 24-hr arc- Intense	1.8 mm/s	3.41 mm/s	<250 m

## REFERENCES

- [1] NASA GSFC and Computer Sciences Corporation(CSC), ODEAS Mathematical Specifications Revision 2, September 1993.
- [2] Chung, L., Novak, S., *MMS Mission First Perigee Raising Maneuver Analysis Memorandum*, MMS-FD-2008-140 Revision 1, April 22, 2009.
- [3] Chung, L., Novak, S., *MMS Mission Final Perigee Raising Maneuver Analysis Memorandum*, MMS-FD-2009-104 Revision 1, April 22, 2009.
- [4] Schanzle, A., Kelbel, D., and Oza, D., "Error Sources and Nominal 3-sigma Uncertainties for Covariance Analysis Studies using ODEAS (Update No. 2)," Task Assignment 55 737, May 31, 1995.

APPENDIX A: ERROR MODELS

Table A-1. ODEAS Trajectory Propagation Models.

Simulation Parameter	Nominal Values
Non-spherical Earth Gravity Model	30x30 JGM2
Point Mass Gravity	Sun, Moon
Atmospheric Drag	Harris-Priester model with solar flux=125 (or Jacchia Roberts) and $C_D=2.2$
Solar Radiation Pressure	Spherical model $C_R=1.4$
Spacecraft Area	3.3 meters <sup>2</sup>
Spacecraft Mass	1140 kg

Table A-2. ODEAS Error Sources and Associated 3-Sigma Uncertainties

Parameter	ODEAS Acronym	3-Sigma Uncertainty
GM	EARTH-GM	$GM_{EARTH}$ ( $0.03 \times 10^{-6}$ )
	LUNAR-GM	$GM_{LUNAR}$ ( $10 \times 10^{-6}$ )
	SOLAR-GM	$GM_{SOLAR}$ ( $10 \times 10^{-6}$ )
Planetary positions	LUNAR-X, Y, Z	30 meters for X and Y and 60 meters for Z
	SOLAR-X, Y, Z	5000 meters each for X, Y, and Z
Gravity Error Model	POTERROR	300% JGM2F70-JGM2CL7 Error Model, degree and order up to 70 (30x30)
$C_R$	<b>Satellite number = 8</b> (MMS)	
	SOLRAD-Satellite Number	30% when considered 500% when solved for
$C_D$	USERDRAG	30% when considered
		500% when solved for
Station positions	<b>Station name = DS46</b> (Canberra) <b>Station number = 1</b> (Madrid, MAD8); <b>5</b> (Goldstone, GDSA); <b>3</b> (Guam, GWMK); <b>6</b> (White Sands, WHSK); <b>10</b> (White Sands, WH2K)	
Local X	XLT- station name, or XLTSTA station number	3 m
Local Y	YLT- station name, or YLTSTA station number	3 m
Local Z	ZLT- station name, or ZLTSTA station number	3 m

Parameter	ODEAS Acronym	3-Sigma Uncertainty		
TDRS Ephemeris errors	EPHERTDR	5 m = Height error 10 m = Cross-track error 20 m = Along-track error		
Troposphere	TRP- station name TROPO-station number	45% 45%		
Ionosphere	<b>TDRS satellite number = 3 (TDRS3), 6 (TDRS6), 10 (TDRS10),</b> ION- station name IONSTAT-station number IONSAT-TDRS satellite name	100% 100% 100%		
Measurements*		<b>Noise Sigma</b>	<b>Weight Sigma</b>	<b>Bias</b>
<u>Two-way Return</u>				
DSN Range (m)	NOIS n/a MEASBI-Station Number	3.00	20.00	15.0 (consider)
DSN Range-Rate (m/s)		0.001	0.1	0 (consider)
SN Range (m)		1.50	30.00	7.0 (consider)
SN Doppler (m/s)		0.00282	0.131	0 (consider)
* A 10-second data rate was used in the tracking file for all measurement types.				

**Table A-3. Nominal DSN/SN Tracking Parameters**

Simulation Parameter	Nominal Value
<b>DSN</b>	
DSN Ground Stations:	Goldstone, Canberra, and Madrid
DSN Tracking Passes:	Consistent with schedule provided in "Commissioning Phase Details-07/27/07"
Data Rate:	One measurement every 10 seconds
Elevation angle mask:	5 degrees
<b>SN</b>	
TDRSs:	TDRS-E, TDRS-W, TDRS-Z
TDRSS Tracking Passes:	No nominal tracking from TDRSS
TDRSS Data Rate:	One measurement every 10 seconds
WSC Elevation angle mask:	5 degrees



Simulation Parameter	Nominal Value
TDRS Sensor Specifications:	Rectangular Mask shape for sensor boresight Width = 30.5 degrees (NOTE: N-S) (i.e. half angle projection in sensor Y-frame) Height = 76.8 degrees (NOTE: E-W) (i.e. half angle projection in sensor X-frame)
Slant Range Restrictions	Maximum of 17.3 RE for low data rate signal
Height of Ray Path (HORP)	1000 km

## APPENDIX B: RELATIVE ERROR CALCULATION

The calculation of the total relative error between the MMS spacecraft takes into account that there are correlated and uncorrelated absolute errors. Correlated errors are the dynamic errors among the satellites including drag, solar flux, solar radiation pressure and station location and range bias errors. For example, the correlated relative root-variance for satellites 1 and 2 is calculated using the following equation:

$$\left(\sigma_{\text{Correlated}}^{\text{Relative}}\right)_2 = \sqrt{\sum_{\text{Correlated}} \left[ \left(\Delta X_1^j\right) - \left(\Delta X_2^j\right) \right]^2}; \text{ where, } j \text{ represents a correlated error source}$$

Uncorrelated absolute errors are assumed to be the measurement-related errors and clock errors such as measurement noise, clock drift, clock acceleration, troposphere, and ionosphere. The TDRS ephemeris errors would be expected to be partially correlated for tracking from common TDRSs and partially uncorrelated because the TDRS track the MMS spacecraft at different times. For this analysis, the TDRS ephemeris errors are assumed to be uncorrelated, which is the less optimistic choice. The uncorrelated relative root-variance for satellites 1 and 2 is calculated using the following equation:

$$\left(\sigma_{\text{Uncorrelated}}^{\text{Relative}}\right)_2 = \sqrt{\sum_{\text{Uncorrelated}} \left[ \left(\Delta X_1^i\right)^2 + \left(\Delta X_2^i\right)^2 \right]}; \text{ where, } i \text{ represents an uncorrelated error source}$$

The full relative root-variance between MMS 1 and MMS 2 is the square root of the sum of the squares of the correlated and uncorrelated root-variances:

$$\left(\sigma_{\text{Total}}^{\text{Relative}}\right)_2 = \sqrt{\left(\sigma_{\text{Correlated}}^{\text{Relative}}\right)_2^2 + \left(\sigma_{\text{Uncorrelated}}^{\text{Relative}}\right)_2^2}$$