Physical Sciences

## Nonlinear Thermal Compensators for WGM Resonators

At one target temperature, thermal frequency fluctuations would vanish to first order.

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In an alternative version of a proposed bimaterial thermal compensator for a whispering-gallery-mode (WGM) optical resonator, a mechanical element having nonlinear stiffness would be added to enable stabilization of a desired resonance frequency at a suitable fixed working temperature. The previous version was described in "Bimaterial Thermal Compensators for WGM Resonators" (NPO-44441), NASA Tech Briefs, Vol. 32, No. 10 (October 2008), page 96. Both versions are intended to serve as inexpensive means of preventing (to first order) or reducing temperature-related changes in resonance frequencies.

A bimaterial compensator would apply, to a WGM resonator, a force that would slightly change the shape of the resonator and thereby change its resonance frequencies. Through suitable choice of the design of the compensator, it should be possible to make the temperature dependence of the force-induced frequency shift equal in magnitude and opposite in sign to the



A **Component Having Nonlinear Stiffness** and a means of temperature control would be added to a previous, basic version of a bimaterial compensator for a WGM resonator. In both versions, a temperature-dependent stress would be applied to counteract the temperature dependence of the spectrum of the uncompensated resonator.

temperature dependence of the frequency shift of the uncompensated resonator so that, to first order, a change in temperature would cause zero net change in frequency.

Because the version now proposed is similar to the previous version in most respects, it is necessary to recapitulate most of the description from the cited prior article, with appropriate modifications. In both the previous and present versions (see figure), a compensator as proposed would include (1) a frame made of one material having a thermalexpansion coefficient  $\alpha_1$  and (2) a spacer made of another material having a thermal-expansion coefficient  $\alpha_2$ . The WGM resonator would be sandwiched between disks, and the resulting sandwich would be squeezed between the frame and the spacer. Assuming that the cross-sectional area of the frame greatly exceeded the cross-sectional area of the spacer and that the thickness of the sandwich was small relative to the length of the spacer, the net rate of change of a resonance frequency with changing temperature would be given by

## $df/dT \approx \partial f/\partial T + (\partial f/\partial F)S_2E_2(\alpha_2 - \alpha_1)$

where *f* is the resonance frequency, *T* is temperature,  $\partial f/\partial T$  is the rate of change of resonance frequency as a function of temperature of the uncompensated resonator,  $\partial f/\partial F$  is the rate of change of frequency as a function of applied force *F* at constant temperature,  $S_2$  is the effective cross-sectional area of the spacer, and  $E_2$  is the modulus of elasticity of the spacer.

In principle, through appropriate choice of materials and geometry, one could obtain temperature compensation — that is, one could make  $df/dT \approx 0$ . For example, the effective spacer cross-sectional area for temperature compensation is given by

 $S_2 \approx (\partial f/\partial T) / [(\partial f/\partial F) E_2(\alpha_1 - \alpha_2)].$ 

In practice, because of inevitable manufacturing errors and imprecise knowledge of thermomechanical responses of structural components, it is difficult or impossible to obtain exact temperature compensation of frequency through selection of  $S_2$ .

According to the present proposal, to make it possible to obtain exact temperature compensation, one would add a component having a nonlinear stiffness to the mechanical load path and would place the entire resonator-and-compensator assembly on a thermoelectric controller, in an oven, or both. Then the temperature dependence of frequency would be approximately quadratic and the net derivative of frequency with respect to temperature would be given by

 $df/dT \approx \partial f/\partial T + (\partial f/\partial F)S_2E_2(\alpha_2 - \alpha_1) + A\Delta T$ 

where A is a parameter that characterizes the nonlinearity to lowest order in temperature and  $\Delta T$  is the difference between the present temperature and some other temperature, which could be a target temperature. To find the target temperature that gives exact temperature compensation, one sets the derivative equal to zero and solves for  $\Delta T$ :

$$\Delta T \approx -A^{-1} \left[ \frac{\partial f}{\partial T} + (\frac{\partial f}{\partial F}) S_2 E_2(\alpha_2 - \alpha_1) \right]$$

The oven and/or the thermoelectric controller could be used to set the temperature to the exact compensation temperature. Even if the exact values of *A*,  $\partial f/\partial T$ ,  $\partial f/\partial F$ ,  $S_2$ ,  $E_2$ ,  $\alpha_1$ , and  $\alpha_2$  were not known in advance, one could still determine the exact compensation temperature by measuring frequency as a function of temperature and finding the lowest point on the approxi-

mately quadratic frequency-versus-temperature curve.

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page number.

## Operation Self-Locking of an OEO Containing a VCSEL

This is an alternative scheme for developing small, low-power atomic clocks.

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A method of dynamic self-locking has been demonstrated to be effective as a means of stabilizing the wavelength of light emitted by a vertical-cavity surface-emitting laser (VCSEL) that is an active element in the frequencycontrol loop of an optoelectronic oscillator (OEO) designed to implement an atomic clock based on an electromagnetically-induced-transparency (EIT) resonance. This scheme can be considered an alternative to the one described in "Optical Injection Locking of a VCSEL in an OEO" (NPO-43454), NASA Tech Briefs, Vol. 33, No. 7 (July 2009), page 33. Both schemes are expected to enable the development of small, low-power, high-stability atomic clocks that would be suitable for use in

applications involving precise navigation and/or communication.

To recapitulate from the cited prior article: In one essential aspect of operation of an OEO of the type described above, a microwave modulation signal is coupled into the VCSEL. Heretofore, it has been well known that the wavelength of light emitted by a VCSEL depends on its temperature and drive current, necessitating thorough stabilization of these operational parameters. Recently, it was discovered that the wavelength also depends on the microwave power coupled into the VCSEL. This concludes the background information.

From the perspective that led to the conception of the optical injection-



This **Optoelectronic Oscillator** is a compact, relatively simple implementation of an atomic clock. The cell contains the optically absorbing atoms upon which the clock is based.

locking scheme described in the cited prior article, the variation of the VCSEL wavelength with the microwave power circulating in the frequency-control loop is regarded as a disadvantage and optical injection locking is a solution of the problem of stabilizing the wavelength in the presence of uncontrolled fluctuations in the microwave power. The present scheme for dynamic self-locking emerges from a different perspective, in which the dependence of VCSEL wavelength on microwave power is regarded as an advantageous phenomenon that can be exploited as a means of controlling the wavelength.

The figure schematically depicts an atomic-clock OEO of the type in question, wherein (1) the light from the VCSEL is used to excite an EIT resonance in selected atoms in a gas cell (e.g., <sup>87</sup>Rb atoms in a low-pressure mixture of Ar and Ne) and (2) the power supplied to the VCSEL is modulated by a microwave signal that includes components at beat frequencies among the VCSEL wavelength and modulation sidebands. As the VCSEL wavelength changes, it moves closer to or farther from a nearby absorption spectral line, and the optical power transmitted through the cell (and thus the loop gain) changes accordingly. A change in the loop gain causes a change in the