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Approximate Pressure Distribution in an Accelerating Launch-Vehicle Fuel Tank

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Summary

A detailed derivation of the equations governing the pressure in a generic liquid-fuel launch vehicle tank subjected to uniformly accelerated motion is presented. The equations obtained are then for the Space Shuttle Superlightweight Liquid-Oxygen Tank at approximately 70 seconds into flight. This generic derivation is applicable to any fuel tank in the form of a surface of revolution and should be useful in the design of future launch vehicles.

Introduction

An important consideration in the design of launch vehicles is the pressure exerted on the wall of a fuel tank during flight. For example, studies of the Space Shuttle Superlightweight Liquid-Oxygen (LO_2) Tank shown in figure 1 have been presented that include the effects of the LO_2 pressure on its nonlinear and buckling behavior under flight loads.[1, 2] In these very complex studies, the LO_2 pressure acting on the tank wall was approximated by using several simplifying presumptions that are based on the specific characteristics of the vehicle dynamics, at specific times in the flight trajectory. For example, stratification of the cryogenic fluid was neglected and the fluid was presumed to be homogeneous and, as a result, to have a uniform density. Additionally, fluid sloshing effects were neglected. The fluid was taken to be incompressible and inviscid, and flow effects were neglected. Moreover, every particle of fluid in the tank was presumed to be subjected to uniformly accelerated motion.

The objective of this memorandum is to present a detailed derivation of the fomulas used in references 2 and 3 to simulate the the LO_2 pressure field in the Space Shuttle Superlightweight Liquid-Oxygen Tank. Toward this objective, a generic presentation of the fundamental equations is given first, followed by an example problem that corresponds to approximately 70 seconds into flight, given in reference 1. This generic derivation is applicable to any fuel tank in the form of a surface of revolution and should be useful in the design of future launch vehicles.

Derivation

In the analysis presented subsequently, the fluid is presumed to be incompressible and inviscid, and to have a uniform density. The tank and the fluid are presumed to be moving together as a uniformly accelerated rigid body without sloshing, and the tank has an internal ullage pressure p_u. For these presumptions, the pressure field, $p(x, y, z)$, is given by (see reference 3)

$$-\overrightarrow{\nabla p} + \rho\vec{g} = \rho\vec{a} \tag{1}$$

In this equation, ρ is the density of the fluid, $\vec{a}$ is the acceleration vector, $\rho\vec{g}$ is the body force associated with gravity, and $\vec{\nabla}$ is the gradient operator for the (x, y, z) coordinate system fixed to the generic tank shown in figure 2. Associated with the (x, y, z) coordinates are the corresponding unit vectors $\{\hat{i}, \hat{j}, \hat{k}\}$. It is convenient to express equation (1) as

$$-\vec{\nabla} p = \rho\vec{\alpha} \tag{2a}$$

where

$$\vec{\alpha} = \vec{a} - \vec{g} \tag{2b}$$

is the net acceleration vector.

To obtain the desired expression for the pressure field acting on the tank wall, it is convenient to first determine the magnitude and direction of net lateral acceleration, denoted by η, as shown in figure 3. The magnitude of the net lateral acceleration is given by

$$\alpha_\eta = \sqrt{\alpha_y^2 + \alpha_z^2} \tag{3}$$

and the corresponding direction is given by the unit vector

$$\hat{\eta} = \cos\psi\ \hat{j} + \sin\psi\ \hat{k} \tag{4a}$$

where

$$\cos\psi = \frac{\alpha_y}{\alpha_\eta} \tag{4b}$$

and

$$\sin\psi = \frac{\alpha_z}{\alpha_\eta} \tag{4c}$$

Next, the coordinate system (x, η, ζ) is introduced, as shown in figure 3. The $(0, \eta, \zeta)$ coordinates shown in this figure span the same plane as the $(0, y, z)$ coordinates. With the (x, η, ζ) coordinates, equation (1) is expressed in a simpler component form as

$$-\frac{\partial p}{\partial x} = \rho\alpha_x \tag{5a}$$

$$-\frac{\partial p}{\partial \eta} = \rho\alpha_\eta \tag{5b}$$

$$-\frac{\partial p}{\partial \zeta} = 0 \tag{5c}$$

From equation (5c), it follows that the functional dependence of the pressure field is given by

$$p = p(x, \eta) \tag{6}$$

On a surface of constant pressure, the pressure differential dp must obey

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial \eta}d\eta + \frac{\partial p}{\partial \zeta}d\zeta = 0 \tag{7}$$

Using equation (5c) with equation (7) yields

$$\frac{dx}{d\eta} = -\frac{\frac{\partial p}{\partial \eta}}{\frac{\partial p}{\partial x}} \tag{8}$$

Substituting equations (5a) and (5b) into equation (8) yields

$$\frac{dx}{d\eta} = -\frac{\alpha_\eta}{\alpha_x} \tag{9}$$

which is easily integrated to yield the equation of plane

$$x + \frac{\alpha_\eta}{\alpha_x}\eta = C \tag{10}$$

where C is a constant. It is useful to point out that the net acceleration vector in the x-η plane is given by

$$\vec{\alpha} = \alpha_x \hat{i} + \alpha_\eta \hat{\eta} \tag{11}$$

and its slope in this plane is α_x/α_η, which is the negative reciprocal of the slope of the plane of constant pressure given by equation (10). Thus, the planes of constant pressure are perpendicular to the net acceleration vector in the x-η plane. The angle of inclination, β, of the lines of constant pressure is given by

$$\tan\beta = -\frac{dx}{d\eta} \tag{12}$$

as seen in figure 4. By using equations (3) and (9), equation (12) becomes

$$\tan\beta = \frac{\alpha_\eta}{\alpha_x} = \frac{\sqrt{\alpha_y^2 + \alpha_z^2}}{\alpha_x} \tag{13}$$

In addition, figure 4 indicates that

$$\cos\beta = \frac{\alpha_x}{|\vec{\alpha}|} \tag{14a}$$

and

$$\sin\beta = \frac{\alpha_\eta}{|\vec{\alpha}|} \tag{14b}$$

To obtain an expression for the pressure gradient that can be easily integrated, it is convenient to let $(\bar{x}, \bar{\eta})$ denote the coordinates of points in the plane perpendicular to the planes of constant pressure, as shown in the figure 4. For this coordinate system, equation (2a) is expressed simply as

$$-\frac{dp}{d\bar{x}} = \rho|\vec{\alpha}| \tag{15}$$

Integrating this equation gives

$$p = -\rho|\vec{\alpha}|\bar{x} + \bar{C} \tag{16}$$

where $\bar{C}$ is a constant. At the top of the fluid surface, $\bar{x} = \bar{x}_f$, where

$$\bar{x}_f = x_f \cos\beta = x_f \frac{\alpha_x}{|\vec{\alpha}|} \tag{17}$$

where equation (14a) has been used. At $\bar{x} = \bar{x}_f$, the pressure field is equal to the ullage pressure p_u, as depicted in figure 4. Therefore, equations (16) and (17) yield

$$p = p_u + \rho|\vec{\alpha}|\left(x_f \frac{\alpha_x}{|\vec{\alpha}|} - \bar{x}\right) \tag{18a}$$

for

$$\bar{x} \leq \bar{x}_f = x_f \frac{\alpha_x}{|\vec{\alpha}|} \tag{18b}$$

The coordinate $\bar{x}$ is expressed in terms of the (x, η, ζ) coordinates by

$$\bar{x} = \cos\beta\, x + \sin\beta\, \eta = \frac{\alpha_x}{|\vec{\alpha}|} x + \frac{\alpha_\eta}{|\vec{\alpha}|} \eta \tag{19}$$

Substituting equation (19) into equations (18) gives

$$p = p_u + \rho\alpha_x\left(x_f - x - \frac{\alpha_\eta}{\alpha_x}\eta\right) \tag{20a}$$

for

$$x + \frac{\alpha_\eta}{\alpha_x}\eta \leq x_f \tag{20b}$$

Next, the η coordinate is expressed as (see figure 3)

$$\eta = \cos\psi\, y + \sin\psi\, z = \frac{\alpha_y}{\alpha_\eta}y + \frac{\alpha_z}{\alpha_\eta}z \tag{21}$$

where equations (4) have been used. Substituting equation (21) into equations (20) gives

$$p = p_u + \rho\alpha_x\left(x_f - x - \frac{\alpha_y}{\alpha_x}y - \frac{\alpha_z}{\alpha_x}z\right) \tag{22a}$$

for

$$x + \frac{\alpha_y}{\alpha_x}y + \frac{\alpha_z}{\alpha_x}z \leq x_f \tag{22b}$$

For a tank in the form of a surface of revolution, the y and z coordinates of a point (x, y, z) on the surface are expressed as

$$y = r(x)\cos\theta \tag{23a}$$

$$z = r(x)\sin\theta \tag{23b}$$

where $r(x)$ is the horizontal radius of the tank wall and θ is the polar angle, as shown in figure 5. By using equations (23) with equations (22), the pressure field acting on the tank wall is given by

$$p(x, \theta) = p_u \quad \text{for} \quad x > x_s(x, \theta) \tag{24a}$$

and

$$p(x, \theta) = p_u + \rho\alpha_x\left[x_s(x, \theta) - x\right] \quad \text{for} \quad x \leq x_s(x, \theta) \tag{24b}$$

where

$$x_s(x, \theta) = x_f - r(x)\left[\frac{\alpha_y}{\alpha_x}\cos\theta + \frac{\alpha_z}{\alpha_x}\sin\theta\right] \tag{24c}$$

Example Problem

Formulas similar to equations (24) were used in the structural analysis of the Space Shuttle Superlightweight External Tank (SLWT). In particular, an early booster ascent loading condition was examined in reference 1 that corresponds to approximately 70 seconds into flight and a LO_2 tank that is approximately seven-eighths full. At this instant of the trajectory, the acceleration field was approximated by a uniform field that is given in terms of the coordinates shown in figure 5 by $\alpha_x = 2.011g$, $\alpha_y = 0.049g$, and $\alpha_z = 0.440g$, where g is the magnitude of uniform gravitational acceleration. For this acceleration field, equations (3) and (4) give $\psi = 83.65$ degrees for the plane of the resultant lateral acceleration vector. Likewise, equation (13) gives $\beta = 12.42$ degrees for the inclination of the fluid surface, in the x-η plane, shown in figure 4. These two angles are independent of the shape of the tank.

The SLWT LO_2 tank, without its foam insulation, is shown in figure 6 and the corresponding geometry and dimensions of a corresponding idealized wall reference surface are shown in figure 7. The major tank-wall components consists of a forward ogive, an aft ogive, a cylindrical barrel section, and an aft dome. The aft dome consists of a truncated elliptical dome section with a spherical cap. Each of these tank elements has a circular cross section.

Determination of the pressure acting on tank wall requires an expression for the horizontal radius r(x), appearing in equation (24b), for each tank component. The geometry of the ogive section, consisting of the forward and aft ogives, is shown in figure 8. The ogive is formed by rotating a segment of an eccentric circle about the x-axis. This tangent line of the segment at $x = h_2$ is parallel to the x-axis. Inspection of this figure reveals

$$x = h_2 + R \sin\phi \tag{25a}$$

and

$$r(x) = R(\cos\phi - 1) + r_2 \tag{25b}$$

From equation (25a) it follows that

$$\cos\phi = \frac{1}{R} \sqrt{R^2 - (x - h_2)^2} \tag{26}$$

Substituting equation (26) into equation (25b) gives the desired expression

$$r(x) = \sqrt{R^2 - (x - h_2)^2} + r_2 - R \quad \text{for} \quad h_2 \le x \le h_1 + h_2 \tag{27}$$

For a given value of r_1 shown in figure 7, the height h_1 is found from equation (27) as

$$h_1 = \sqrt{(r_2 - r_1)[2R - r_2 + r_1]} \tag{28}$$

For the barrel section, inspection of figure 7 gives

$$r(x) = r_2 \quad \text{for} \quad 0 \le x \le h_2 \tag{29}$$

The corresponding expression for the truncated elliptical section of the aft dome is obtained directly by using the equation for an ellipse in x-r coordinates along with the geometry given in figure 7. The result is

$$r(x) = r_2 \sqrt{1 - \left(\frac{x}{h_3}\right)^2 \left[1 - \left(\frac{r_3}{r_2}\right)^2\right]} \quad \text{for} \quad -h_3 \le x \le 0 \tag{30}$$

Likewise, the expression for the spherical part of the aft dome is obtained by using the equation for a circle whose origin is at $r = 0$ and $x = R_s - h_3 - h_4$, where R_s is the radius of the spherical cap shown in figure 7. From this equation, it follows that

$$r(x) = \sqrt{R_s^2 - \left(x - R_s + h_3 + h_4\right)^2} \quad \text{for} \quad -\left(h_3 + h_4\right) \le x \le -h_3 \tag{31a}$$

where

$$R_s = \frac{r_3^2 + h_4^2}{2h_4} \tag{31b}$$

A "to scale" graphical image based on equations (27)-(31) is shown in figure 9. For this image, a value of $r_1 = 0$ was used to obtain the sharp point at the nose of the tank. The corresponding value of h_1 given by equation (28) is 418.5 in.

The approximate pressure distribution on the interior surface of the LO_2 tank is given by equations (24), where x is the axial coordinate shown in figure 7 and r(x) is given by equations (27)-(31). The fill level for this example is given by $x = x_f = 305.9$ in., and the density ρ is given by $\rho g = \gamma$, where $\gamma = 0.04123$ lb/in^3 is the specific weight of the LO_2. The ullage pressure for this example is $p_u = 19.6$ psi and the corresponding tank dimensions are given in figure 7. Pressure calculations were made for these specific values and for a closed, pointed ogive with $r_1 = 0$, and are shown in figures 10 and 11. The results in figure 10 show the variation of the pressure acting on the shell wall as a function of the polar angle θ. Five curves are shown in the figure that correspond to different axial positions along the tank. The straight line shown for x = 500 in. is in the unfilled region of the ogive where the pressure is equal to the 19.6 psi ullage pressure, and is independent of θ. The curves shown for x = 250, 100, and 0 in. correspond to a location in the ogive below the fluid surface, a location within the barrel section, and the location where the barrel and aft dome meet, respectively. The sinusoidal shape of these curves corresponds to the inclination of the fluid surface caused by the uniformly accelerated motion. The straight line shown for x = -114 in. is almost at bottom of the aft dome, where the variations of the pressure with θ are negligible.

The results in figure 11 show the variation of the pressure acting on the shell wall as a function of the axial coordinate x. Two curves are shown in this figure that correspond to θ = 83.65 and 263.65 degrees. These two tank meridians span the plane of maximum lateral acceleration. The differences in the two curves are also associated with the inclination of the fluid surface. The two curve intersect at x = -114.9 in., the center of the aft dome, where a maximum pressure equal to 54.5 psi occurs.

Concluding Remarks

A detailed derivation of the equations governing the pressure in a generic liquid-fuel launch vehicle tank subjected to uniformly accelerated motion has been presented. The equations obtained have been applied to detemine the pressure acting on the shell wall of the Space Shuttle Superlightweight Liquid-Oxygen Tank at approximately 70 seconds into flight. This generic derivation is applicable to any fuel tank in the form of a surface of revolution and should be useful in the design of future launch vehicles.

References

1. Young, R. D.; Nemeth, M. P.; Collins, T. J.; and Starnes, J. H., Jr.: Nonlinear Behavior of Space Shuttle Superlightweight Tank under Booster Ascent Loads. J. Spacecraft and Rockets, vol. 36, 1999, pp. 820-827.

2. Nemeth, M. P.; Young, R. D.; Collins, T. J.; and Starnes, J. H., Jr.: Nonlinear Behavior of Space Shuttle Superlightweight Tank under End-of-Flight Loads. J. Spacecraft and Rockets, vol. 36, 1999, pp. 828-835.

3. Fox, R. W. and McDonald, A. T.: Introduction to Fluid Mechanics. Third ed., John Wiley & Sons, 1985, pp. 74-77.

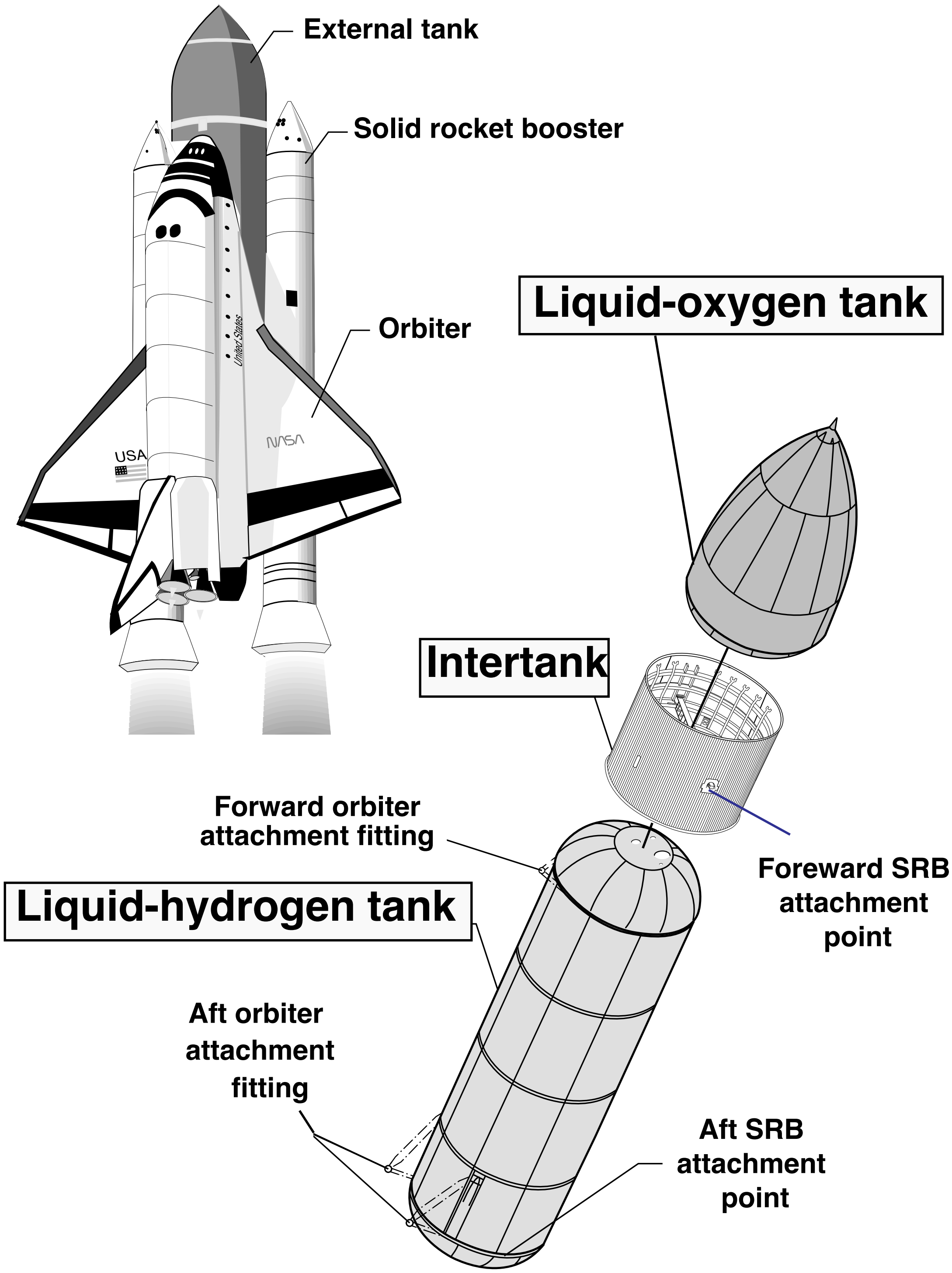

Figure 1. Space Shuttle external tank components.

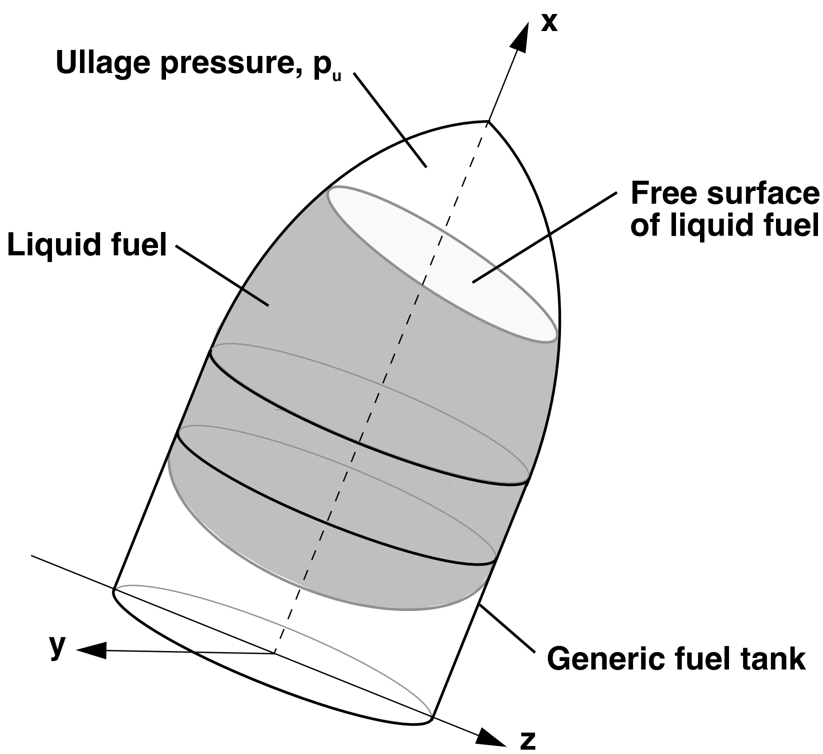

Figure 2. Baseline coordinate frame of generic fuel tank.

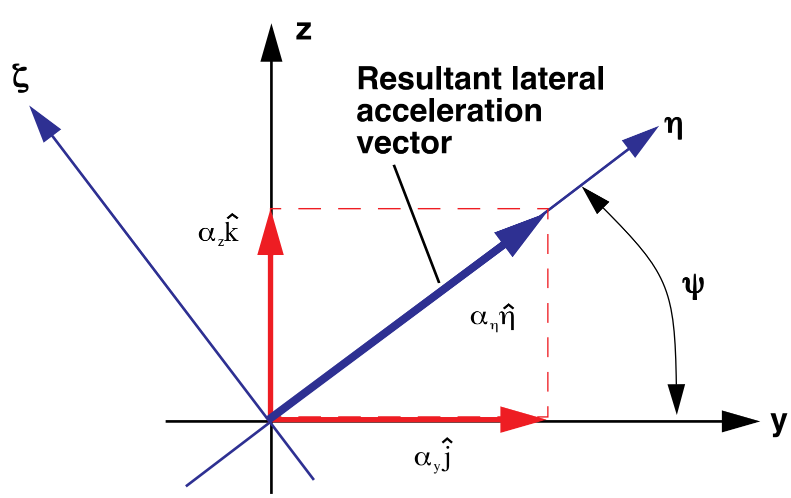

Figure 3. Coordinate frame of generic fuel tank associated with the resultant lateral acceleration.

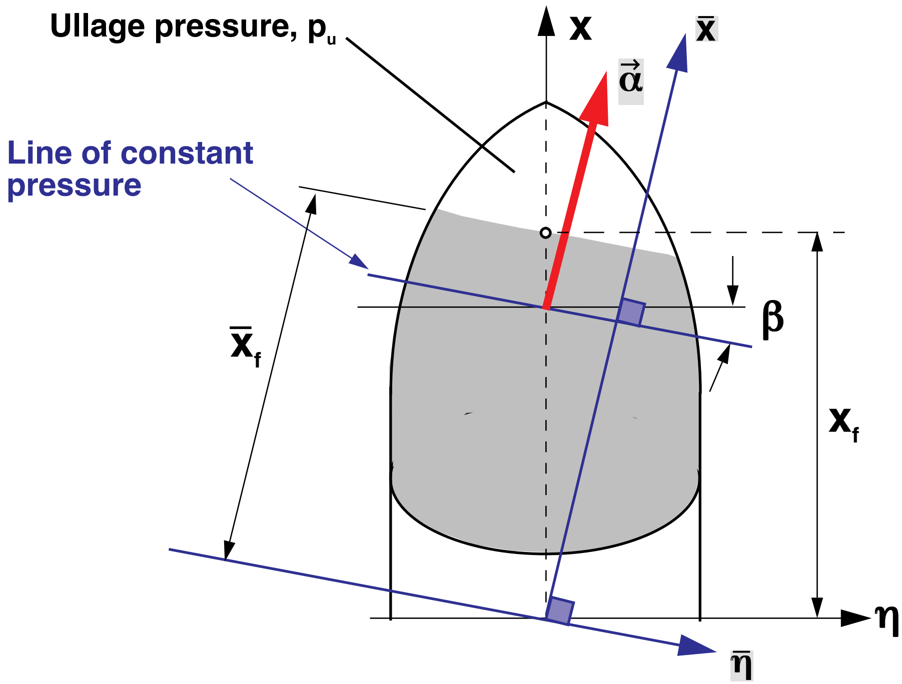

Figure 4. Coordinate frame of generic fuel tank associated with lines of constant pressure in the plane of resultant lateral acceleration.

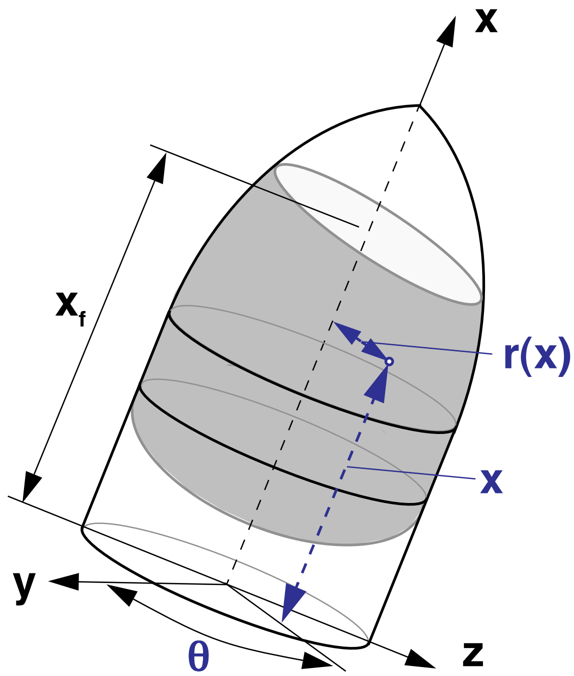

Figure 5. Cylindrical coordinate frame for a generic fuel tank in the form of a surface of revolution.

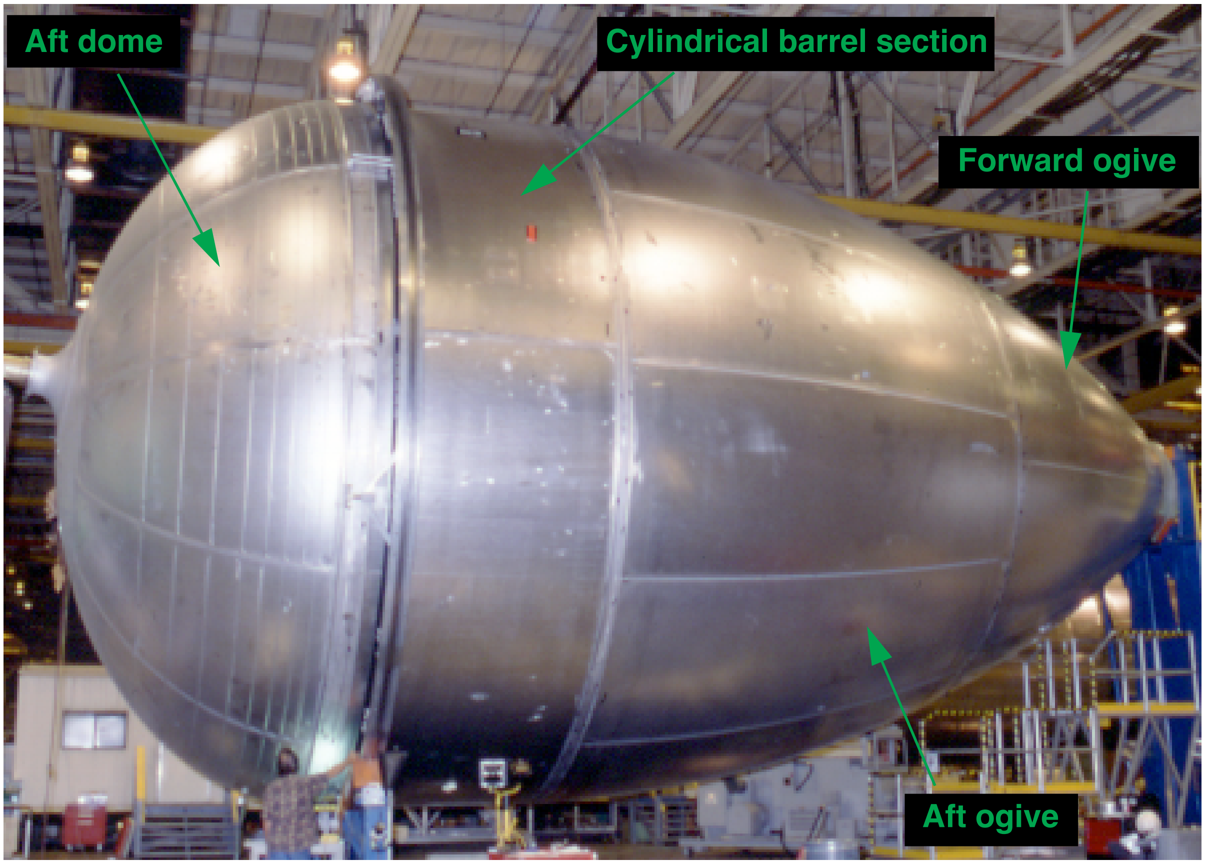

Figure 6. Space Shuttle Superlightweight Liquid-Oxygen Tank.

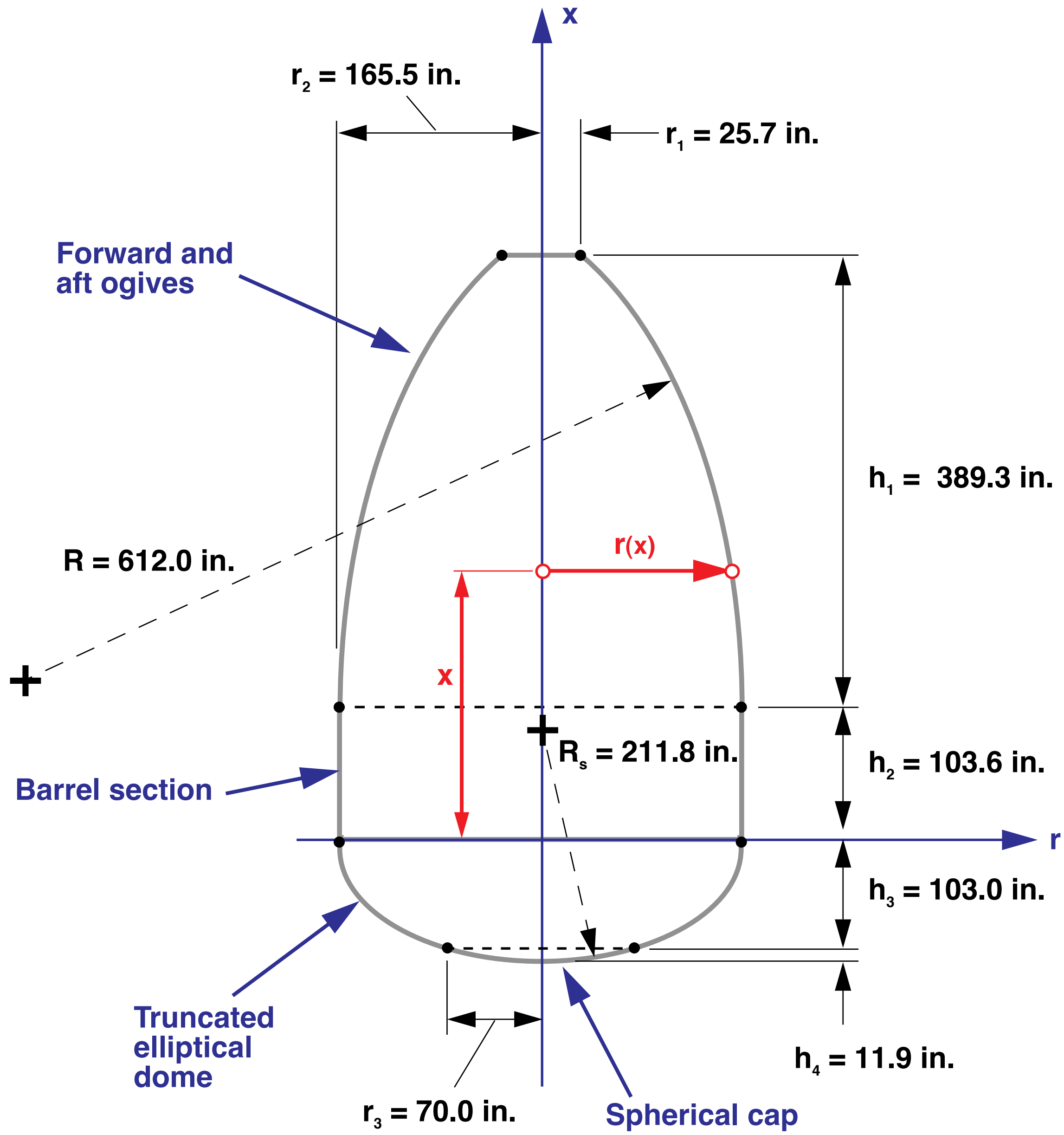

Figure 7. Geometry and dimensions of Space Shuttle Superlightweight Liquid-Oxygen Tank.

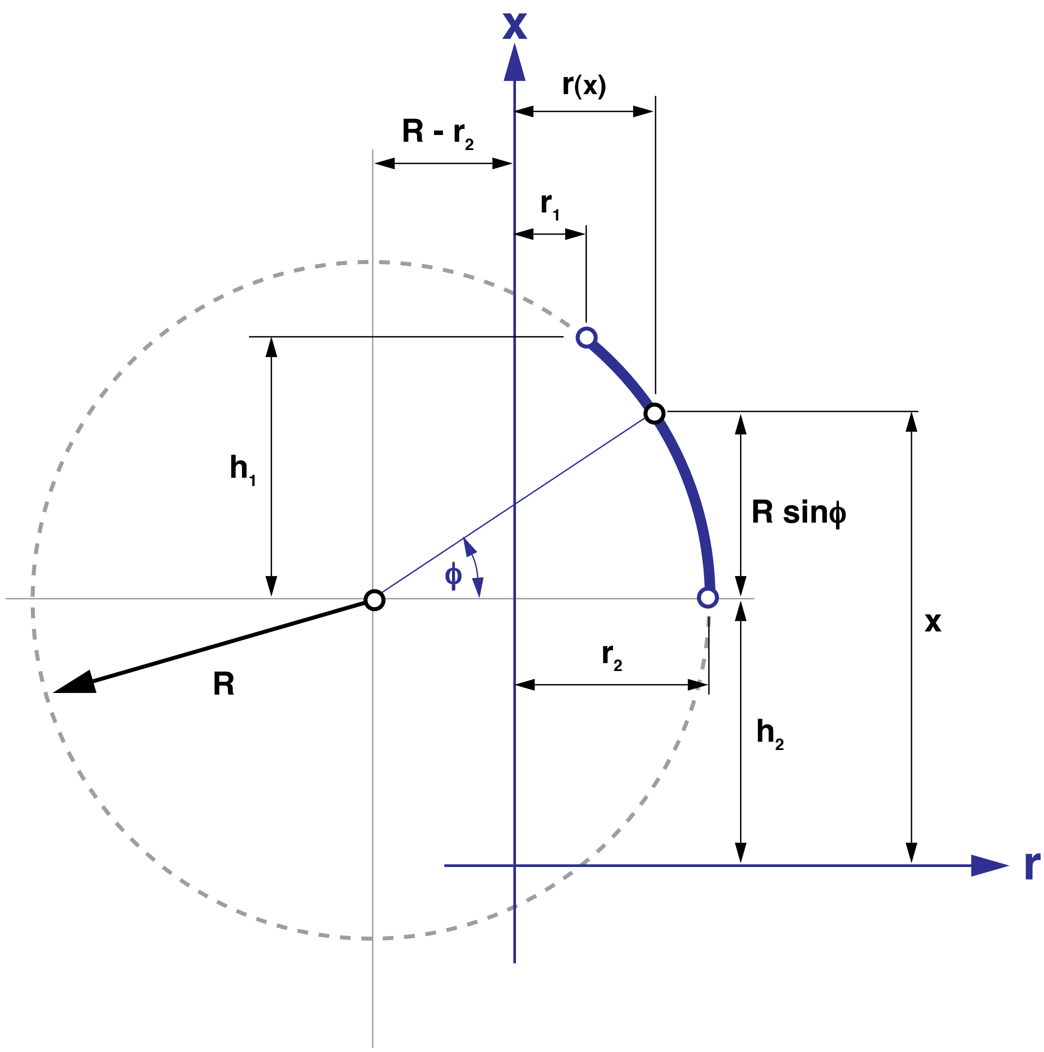

Figure 8. Geometry of the ogive.

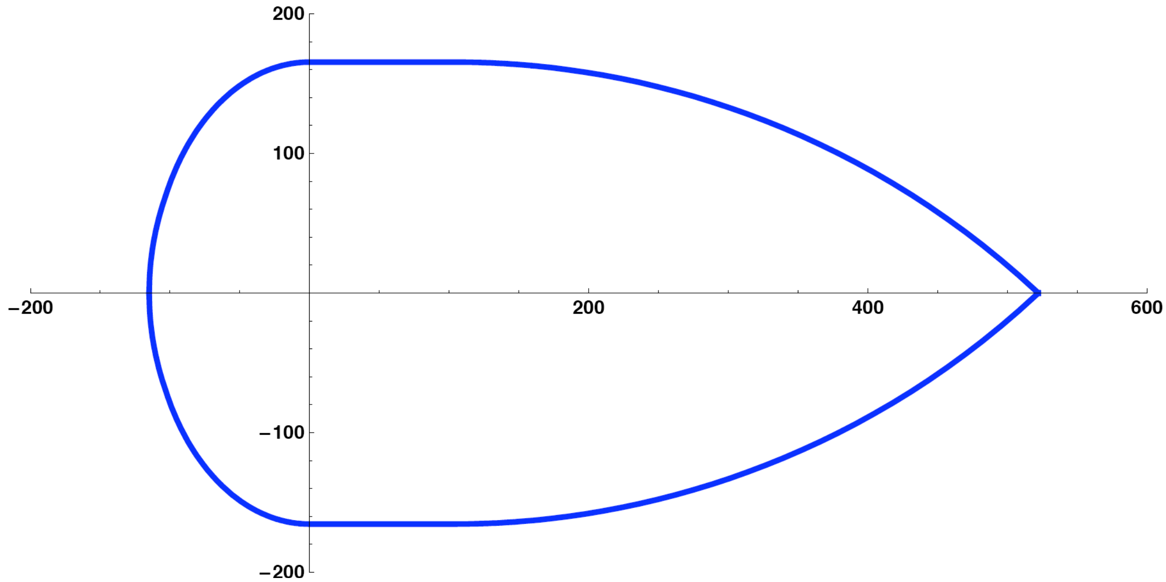

Figure 9. Graphical output of equations (27)-(31) for the Space Shuttle Superlightweight Liquid-Oxygen Tank with the dimensions given in figure 7, except $r_1 = 0$.

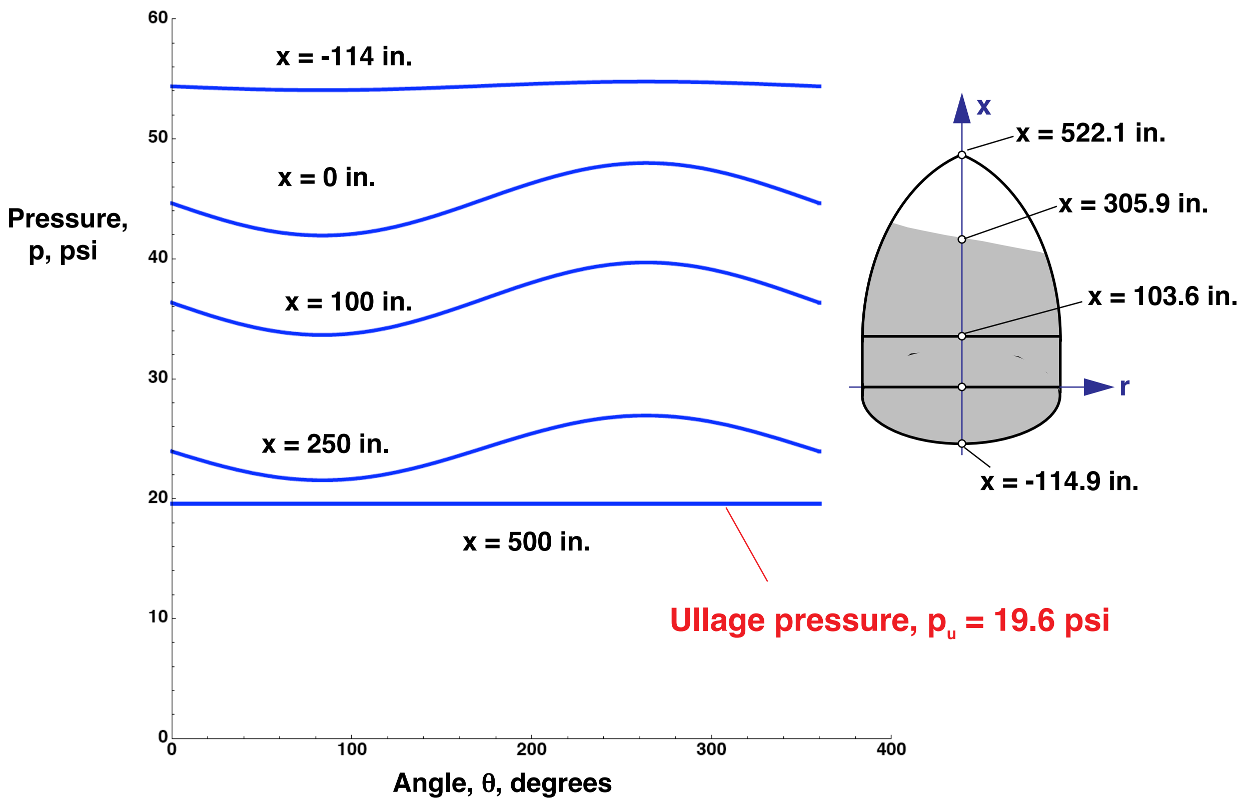

Figure 10. Pressure acting on the shell wall of the Space Shuttle Superlightweight Liquid-Oxygen Tank as a function of the polar angle θ (see figure 5).

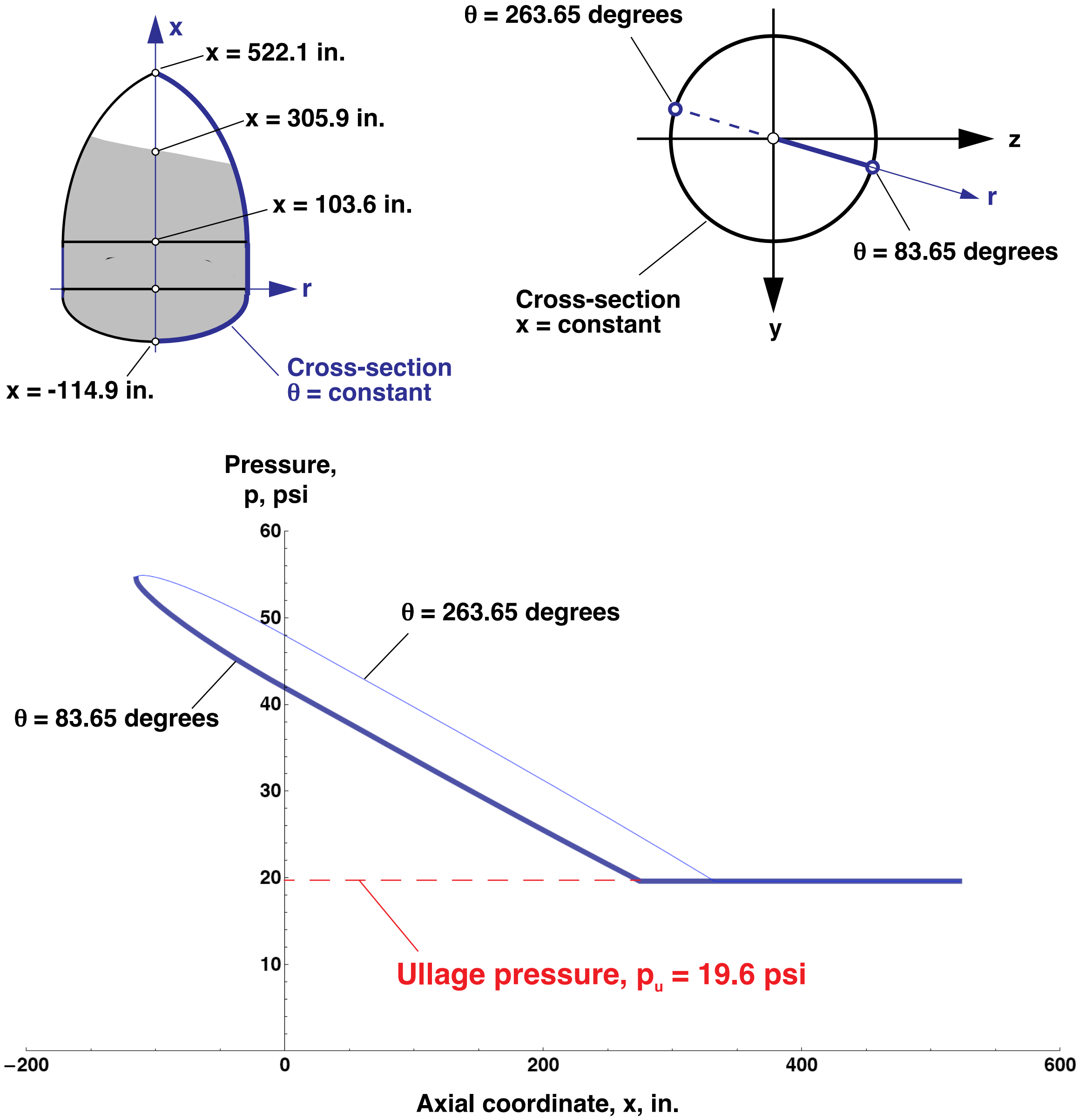

Figure 11. Pressure acting on the shell wall of the Space Shuttle Superlightweight Liquid-Oxygen Tank as a function of the axial coordinate x (see figure 5).

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