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## ➤ Rapid Calculation of Spacecraft Trajectories Using Efficient Taylor Series Integration

Software greatly accelerates the calculation of spacecraft trajectories.

*John H. Glenn Research Center, Cleveland, Ohio*

A variable-order, variable-step Taylor series integration algorithm was implemented in NASA Glenn's SNAP (Spacecraft N-body Analysis Program) code. SNAP is a high-fidelity trajectory propagation program that can propagate the trajectory of a spacecraft about virtually any body in the solar system. The Taylor series algorithm's very high order accuracy and excellent stability properties lead to large reductions in computer time relative to the code's existing 8th order Runge-Kutta scheme. Head-to-head comparison on near-Earth, lunar, Mars, and Europa missions showed that Taylor series integration is 15.8 times faster than Runge-Kutta on average, and is more accurate. These speedups were obtained for calculations involving central body, other body, thrust, and drag forces. Similar speedups have been obtained for calculations that include J2 spherical harmonic for central body gravitation. The algorithm includes a step size selection method that directly calculates

the step size and never requires a repeat step.

High-order Taylor series integration algorithms have been shown to provide major reductions in computer time over conventional integration methods in numerous scientific applications. The objective here was to directly implement Taylor series integration in an existing trajectory analysis code and demonstrate that large reductions in computer time (order of magnitude) could be achieved while simultaneously maintaining high accuracy.

This software greatly accelerates the calculation of spacecraft trajectories. At each time level, the spacecraft position, velocity, and mass are expanded in a high-order Taylor series whose coefficients are obtained through efficient differentiation arithmetic. This makes it possible to take very large time steps at minimal cost, resulting in large savings in computer time. The Taylor series algorithm is implemented primarily through three subroutines: (1) a driver routine that automatically intro-

duces auxiliary variables and sets up initial conditions and integrates; (2) a routine that calculates system reduced derivatives using recurrence relations for quotients and products; and (3) a routine that determines the step size and sums the series. The order of accuracy used in a trajectory calculation is arbitrary and can be set by the user. The algorithm directly calculates the motion of other planetary bodies and does not require ephemeris files (except to start the calculation). The code also runs with Taylor series and Runge-Kutta used interchangeably for different phases of a mission.

*This work was done by James R. Scott and Michael C. Martini of Glenn Research Center. Further information is contained in a TSP (see page 1).*

*Inquiries concerning rights for the commercial use of this invention should be addressed to NASA Glenn Research Center, Innovative Partnerships Office, Attn: Steve Fedor, Mail Stop 4-8, 21000 Brookpark Road, Cleveland, Ohio 44135. Refer to LEW-18445-1.*

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## ➤ Efficient Kriging Algorithms

*Goddard Space Flight Center, Greenbelt, Maryland*

More efficient versions of an interpolation method, called kriging, have been introduced in order to reduce its traditionally high computational cost. Written in C++, these approaches were tested on both synthetic and real data.

Kriging is a best unbiased linear estimator and suitable for interpolation of scattered data points. Kriging has long been used in the geostatistic and mining

communities, but is now being researched for use in the image fusion of remotely sensed data. This allows a combination of data from various locations to be used to fill in any missing data from any single location.

To arrive at the faster algorithms, sparse SYMMLQ iterative solver, covariance tapering, Fast Multipole Methods (FMM), and nearest neigh-

bor searching techniques were used. These implementations were used when the coefficient matrix in the linear system is symmetric, but not necessarily positive-definite.

*This work was done by Nargess Memarsadeghi of Goddard Space Flight Center. For further information, contact the Goddard Innovative Partnerships Office at (301) 286-5810. GSC-15555-1*

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## ➤ Predicting Spacecraft Trajectories by the WeavEncke Method

*Lyndon B. Johnson Space Center, Houston, Texas*

A combination of methods is proposed of predicting spacecraft trajectories that possibly include multiple maneuvers and/or perturbing accelerations, with greater speed, accuracy, and repeatability than were heretofore achievable. The

combination is denoted the WeavEncke method because it is based on unpublished studies by Jonathan Weaver of the orbit-prediction formulation of the noted astronomer Johann Franz Encke. Weaver evaluated a number of alternatives that

arise within that formulation, arriving at an orbit-predicting algorithm optimized for complex trajectory operations.

In the WeavEncke method, Encke's method of prediction of perturbed orbits is enhanced by application of mod-

ern numerical methods. Among these methods are efficient Kepler's-equation time-of-flight solutions and self-starting numerical integration with time as the independent variable. Self-starting numerical integration satisfies the require-

ments for accuracy, reproducibility, and efficiency (and, hence, speed). Self-starting numerical integration also supports fully analytic regulation of integration step sizes, thereby further increasing speed while maintaining accuracy.

*This work was done by Jonathan K. Weaver of Johnson Space Center and Daniel R. Adamo of United Space Alliance. For further information, contact the JSC Innovation Partnerships Office at (281) 483-3809. MSC-23802-1*

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## ➤ An Augmentation of G-Guidance Algorithms

**This augmented algorithm can be used in small-body proximity operations utilizing model predictive control with a need for safety from surface-constraint uncertainty.**

*NASA's Jet Propulsion Laboratory, Pasadena, California*

The original G-Guidance algorithm provided an autonomous guidance and control policy for small-body proximity operations that took into account uncertainty and dynamics disturbances. However, there was a lack of robustness in regards to object proximity while in autonomous mode. The modified G-Guidance algorithm was augmented with a second operational mode that allows switching into a safety hover mode. This will cause a spacecraft to hover in place until a mission-planning algorithm can compute a safe new trajectory. No state or control constraints are violated. When a new, feasible state trajectory is calculated, the spacecraft will return to standard mode and maneuver toward the target. The main goal of this augmentation is to protect the spacecraft in the event that a landing surface or obstacle is closer or further than anticipated. The algorithm can be used for the miti-

gation of any unexpected trajectory or state changes that occur during standard mode operations.

In order to have the G-Guidance algorithm detect an unsafe condition, it required some modification. This modification provides a policy to safely maneuver the spacecraft between its current state and a desired target state while ensuring satisfaction of thruster and trajectory constraints, along with safety constraints. In standard mode, this modification brings the spacecraft from its current position closer to its target state. In safety mode, the algorithm maintains the spacecraft's current state at zero velocity. Since the safety mode is designed to be temporary, the destination location in this mode is also temporary, and once a new destination location is provided, the spacecraft returns to standard mode.

The G-Guidance algorithm uses both a planned trajectory (feedforward) and

a control policy (feedback), along with sensors to monitor actual spacecraft state. The feedback is designed to ensure that the spacecraft stays within a specified proximity to the feedforward. The feedforward is designed to achieve the goals of each mode: hover for safety mode and maneuver toward target for standard mode. By giving the spacecraft the ability to re-compute its trajectory on-the-fly in response to local conditions, minimization of fuel usage is provided. The original G-Guidance algorithm provides robustness to uncertainty affecting the dynamics. The safety augmentation provides a form of state-constraint robustness, which further mitigates risk.

*This work was done by John M. Carson III and Behcet Acikmese of Caltech for NASA's Jet Propulsion Laboratory. For more information, contact [iaoffice@jpl.nasa.gov](mailto:iaoffice@jpl.nasa.gov). NPO-46452*

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## ➤ Comparison of Aircraft Icing Growth Assessment Software

**The goal is to provide software that can predict ice growth under any condition for any aircraft surface.**

*John H. Glenn Research Center, Cleveland, Ohio*

A research project is underway to produce computer software that can accurately predict ice growth under any meteorological conditions for any aircraft surface. An extensive comparison of the results in a quantifiable manner against the database of ice shapes that have been generated in the NASA Glenn Icing Research Tunnel (IRT) has been performed, including additional data taken to extend the database in the Super-cooled Large Drop (SLD) regime. The project shows the differences in ice shape between LEWICE 3.2.2, GlenNICE, and experimental data.

The Icing Branch at NASA Glenn has produced several computer codes over the last 20 years for performing icing simulation. While some of these tools have been collaborative projects, most have been developed primarily by one person, with some assistance by others. The state of computing has also changed dramatically in that time period. As these codes have grown in complexity and have been accepted by users as production icing tools, there has arisen a need for the developers to adhere to standard software practices used to develop commercial software.

The project addresses the validation of the software against a recent set of ice-shape data in the SLD regime. This validation effort mirrors a similar effort undertaken for previous validations of LEWICE. Those reports quantified the ice accretion prediction capabilities of the LEWICE software. Several ice geometry features were proposed for comparing ice shapes in a quantitative manner. The resulting analysis showed that LEWICE compared well to the available experimental data.

The effects of super-cooled large droplets in icing have been researched