

# STATISTICAL ISSUES FOR UNCONTROLLED REENTRY HAZARDS

## EMPIRICAL TESTS OF THE PREDICTED FOOTPRINT FOR UNCONTROLLED SATELLITE REENTRY HAZARDS

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### ABSTRACT

A number of statistical tools have been developed over the years for assessing the risk of reentering objects to human populations. These tools make use of the characteristics (e.g., mass, material, shape, size) of debris that are predicted by aerothermal models to survive reentry. The statistical tools use this information to compute the probability that one or more of the surviving debris might hit a person on the ground and cause one or more casualties.

The statistical portion of the analysis relies on a number of assumptions about how the debris footprint and the human population are distributed in latitude and longitude, and how to use that information to arrive at realistic risk numbers. Because this information is used in making policy and engineering decisions, it is important that these assumptions be tested using empirical data.

This study uses the latest database of known uncontrolled reentry locations measured by the United States Department of Defense.

The predicted ground footprint distributions of these objects are based on the theory that their orbits behave basically like simple Kepler orbits. However, there are a number of factors in the final stages of reentry - including the effects of gravitational harmonics, the effects of the Earth's equatorial bulge on the atmosphere, and the rotation of the Earth and atmosphere - that could cause them to diverge from simple Kepler orbit behavior and possibly change the probability of reentering over a given location. In this paper, the measured latitude and longitude distributions of these objects are directly compared with the predicted distributions, providing a fundamental empirical test of the model assumptions.

### 1. INTRODUCTION

One of the hazards of the space age is that many objects in orbit eventually reenter the Earth's atmosphere. Often for large objects, some components tend to survive reentry and pose a hazard to persons on the ground. A whole science has developed around

predicting what portions of a satellite survive the violent forces and heating of reentry. Such aerothermal models as ORSAT and SCARAB [1] use detailed information on shape and material type of various components to predict what will and will not survive to reach the ground. From this information, a reentry "footprint" can be computed that can be used to determine risk on the ground.

For controlled reentries, the reentry target zone can be chosen to avoid populated areas on the Earth. For uncontrolled reentries, however, statistical tools must be used to map human distributions on the Earth under the spacecraft orbit. The exact time and location for uncontrolled reentries are notoriously difficult to predict with any accuracy. In the hours immediately preceding a reentry, it may be possible to narrow the possible ground tracks of the reentering object. But when the risk calculation is needed weeks, months, or years before reentry, essentially any location on Earth over which the spacecraft flies is a potential debris landing site. Nevertheless, it is possible to use even this vague information to compute meaningful risk statistics.

### 2. BASIC STATISTICAL TOOLS

Consider a piece of a reentering satellite that falls into a small geographical region representing a tiny fraction of the Earth's surface with area  $A$ . The region contains a number of individual human beings  $N$ . If the area of the surviving piece  $a$  is smaller than the person-area of a typical human being, then the risk is driven by the number of people in the region, not the size of the surviving debris piece. If, however, the surviving piece is large, then the risk increases with increasing size. This "enhanced" debris-person area  $a'$  can be computed based on assumptions about the size of the debris and a "typical" human.

Assuming that the people in the small geographic region are distributed randomly and that the piece falls at a random location within the region, then the probability becomes a simple binomial distribution - what is the probability that a given number of people will be under the piece when it falls versus those who will be elsewhere in the geographic region.

A simple approximation to the probability of one or more casualties within a small geographic area can be computed using the Poisson relation

$$p_c = 1 - \text{Exp}\left(-N \frac{\alpha'}{A}\right) \approx -N \frac{\alpha'}{A} \quad (1)$$

Note that the last approximation is only true when  $N \alpha' \ll A$ , which is typically the case for standard types of re-entries where the risk is primarily from being struck by pieces as they fall.

Computing the total ground risk simply consists of summing over all possible small geographic regions under the ground-track of the reentering satellite and weighting them for the fraction of time the satellite spends over that region.

### 3. RISK CALCULATIONS FOR ORBITS

Of course, orbiting objects do not orbit an Earth with homogeneously distributed populations. Approximately three quarters of the Earth's surface is covered by oceans, and the population on land is unevenly distributed.

For computing ground risks, NASA's Orbital Debris Program Office uses the databases from the Socioeconomic Data and Applications Center (SEDAC) at Columbia University [2]. Currently, NASA uses the Gridded Population of the World, version 3 (GPWv3). This data set estimates the population in latitude/longitude grid positions on the Earth divided into  $2.5 \times 2.5$  arc minute cells for reference years 1990-2015 in 5-year intervals. These cells are approximately 4.6 km long in the north-south direction. At the equator, the cells are also approximately 4.6 km wide in the east-west direction, but are narrower at more northerly and southerly latitudes.

As mentioned above, predicting accurate reentry footprints weeks, months, or years in advance is not possible with current computer models. Instead, planners are left predicting the generalized likelihood of a reentering object ending up in each latitude/longitude bin.

Note that the calculations described in this paper assume the Earth is a perfect sphere, ignoring the minor difference between the *geocentric* and *geographic* latitudes [3]. For these calculations, the differences are very minor.

The distribution in longitude is relatively easy to estimate. The rotation of the Earth and the precession

of the ascending node of the orbit should result in a random distribution in longitude. Note that this should be true of orbits designed with inclinations to maintain a particular plane orientation (e.g., with respect to the Sun). By the time the orbits decay to low altitudes, the orbit plane "lock" – which is dependent upon semi-major axis as well as inclination – will no longer apply.

The other important distribution should be that in latitude. For a satellite with a given inclination  $i$ , there exists a maximum and minimum latitude  $\lambda_{max}$  and  $\lambda_{min}$

$$\begin{aligned} \text{Sin}(\lambda_{max}) &= \text{Sin}(i) \\ \text{Sin}(\lambda_{min}) &= -\text{Sin}(i) \end{aligned} \quad (2)$$

The ground-path of the satellite will not go north of the maximum latitude, nor south of the minimum latitude.

For a perfectly circular Kepler orbit, the object's location should be uniformly distributed not in latitude, but in argument of latitude. Argument of latitude is the angle in the orbit plane of the object measured from its last north-bound equator crossing. This corresponds to the sum of the argument of perigee and the true anomaly angles. Of course, in reality, perfect Kepler orbits do not exist. But for any generalized elliptical orbit with randomized (uniformly distributed) argument of perigee, a reasonable assumption would be that the argument of latitude would be uniformly distributed for such an orbit as well.

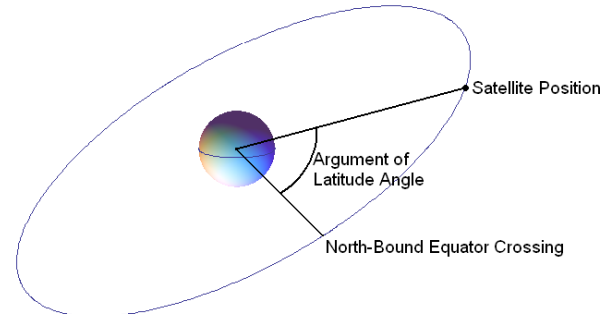


Figure 1. The argument of latitude is the angle in the orbit plane of a satellite measured from the last north-bound/ascending equator crossing.

The relationship between the inclination  $i$ , the latitude  $\lambda$ , and the argument of latitude  $u$  is given by

$$\text{Sin}(u) = \frac{\text{Sin}(\lambda)}{\text{Sin}(i)} \quad (3)$$

Because  $u$  can be in any quadrant, further information must be sought on whether the object is moving north/ascending or south/descending at the particular latitude. Using inverse trigonometric equations,

$$u = \text{ArcSin}\left(\frac{\text{Sin}(\lambda)}{\text{Sin}(i)}\right) \quad (4a)$$

if  $\dot{\lambda} \geq 0$

$$u = \pi - \text{ArcSin}\left(\frac{\text{Sin}(\lambda)}{\text{Sin}(i)}\right) \quad (4b)$$

if  $\dot{\lambda} < 0$

Using this distribution and the gridded population of the world, the NASA Orbital Debris Program Office has computed average population density under orbits as a function of year and inclination [4]. An example chart with two reference dates (showing the projected future growth of the Earth's population) is shown in Fig. 2. Note that sheltering by buildings or other structures is ignored for these calculations.

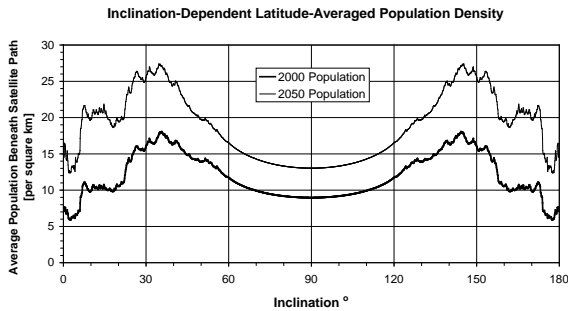


Figure 2. This chart shows the average population beneath an orbit as a function of orbit inclination for two reference years (based on global population databases and future population predictions). The population is weighted by the time each object spends over various latitudes and longitudes based on the distributions outlined in the text.

#### 4. POSSIBLE UNMODELED EFFECTS

These theoretical calculations assume the orbiting objects are behaving similar to perfect Kepler orbits. There are several reasons to believe these assumptions might not be valid (for a summary of the orbital effects described here, refer to [5]).

First, satellites do not orbit a perfectly spherical Earth. There is a latitudinal variation in mass that is enhanced near the equator and is reflected in the  $J_2$  spherical harmonic term of the Earth's gravitational field. The effect of this term is to change the speed and azimuth of the orbit slightly as it passes over the equator relative to an ideal Kepler orbit.

Second, in addition to equatorial enhancement of the gravitational field, the radius of the Earth is somewhat greater at the equator than at the poles. This results in the atmosphere extending farther out into space near the equator relative to the center of the Earth. In addition, there is an enhancement of the atmospheric density in the subsolar regions due to solar heating. This "diurnal bulge" always stays in the tropics, so it further enhances the equatorial atmospheric density at a particular radius from the center of the Earth relative to that near the poles. There can also be density enhancements near the poles due to magnetospheric heating, so the exact latitudinal distribution of atmospheric density can be quite complex.

Third, the atmosphere rotates with the Earth, and the drag is a function of the relative velocity between the orbiting satellite and the local velocity of the atmosphere. Consequently, as the satellite crosses the equator, the relative velocity of the satellite with respect to the atmosphere will be different than when the satellite is at the northernmost or southernmost regions of its orbit.

All of these effects have the potential of making the drag on an orbiting satellite non-uniform in argument of latitude, possibly resulting in unmodeled latitude biases in the reentry location and perhaps altering the resulting ground risk.

#### 5. EMPIRICAL VALIDATION

To answer whether the idealized orbit distribution assumptions described above are applicable, we can use a statistical database of reentry locations and compare the distributions in latitude and longitude to those predicted by theory.

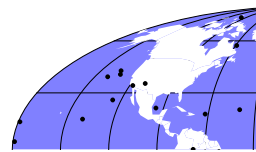


Figure 3. The geographic coordinates of the 81 random object reentries described in the text are plotted here on a map of the Earth. Many of the satellites were in orbits with low inclinations, so there tend to be more reentries nearer the equator. Note that most of the reentries occurred over water.

In 2008, a similar analysis was done with a list of 47 satellites (rocket bodies and spacecraft) that reentered

between 2003 and 2007 [6]. The database at that time had insufficient sample to make positive conclusions about the geographic distributions of reentries. Since that time, more re-entries have occurred – enough to revisit the calculations to see if any new conclusions can be reached.

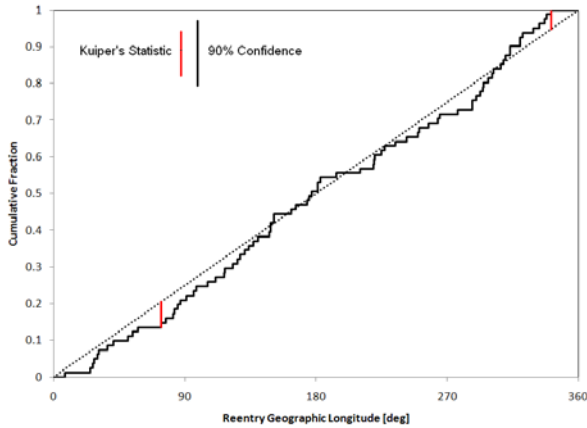


Figure 4. The cumulative distribution in longitude of 81 random object reentries (solid curve) is compared to the theoretical uniform distribution (dotted line). Kuiper's statistic is the sum of the maximum differences between the two distributions both above and below the theoretical curve. The value is shown compared to the 90% confidence value for 81 random points drawn from a uniform distribution. As can be seen, this data set has a Kuiper's statistic smaller than the 90% confidence value, indicating this data is consistent with a random draw from the uniform distribution.

For this study, only reentries were used that had been in orbit at least 15 days (to make sure the orbit nodes had time to be thoroughly “randomized”), and had an accurate latitude/longitude determination by the U.S. Department of Defense. In addition, only near-circular orbits were chosen. These were defined as objects with final orbit eccentricity values (based on two-line element sets) less than 0.0075 – corresponding to orbits with differences between apogee and perigee of less than 100 km. Figure 3 shows the geographic distribution of the 81 orbits used for this study.

The easiest parameter to test is the distribution in longitude, because it should exhibit a simple uniform random distribution around the Earth. Because longitude “wraps around” the Earth, Kuiper's statistic is appropriate to use [7]. This statistic, similar to the more familiar Kolmogorov-Smirnov test, measures the difference between a cumulative data distribution and the corresponding cumulative theoretical distribution. Kuiper's statistic is the sum the maximum difference both above and below the theoretical distribution. Kuiper's statistic works so well with periodic distributions because it gives the same result, no matter

where the “zero” value is defined. Note that Kuiper's statistic is useful for other distributions, not just those that are periodic.

Fig. 4 shows the longitude distribution analysis. The data are well within the 90% confidence range for a random sample of 81 points from a uniform distribution. Therefore, to the limits of finite sampling, these data are completely consistent with random longitude of reentry.

Figure 5 shows the distribution of reentry latitude as a function of inclination.

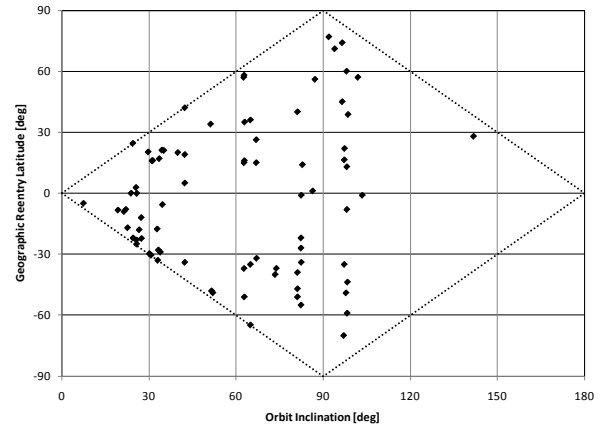


Figure 5. The reentry latitude of the 81 objects in the reentry database is here shown as a function of inclination. The diamond represents the envelope of maximum and minimum latitudes possible for a given inclination.

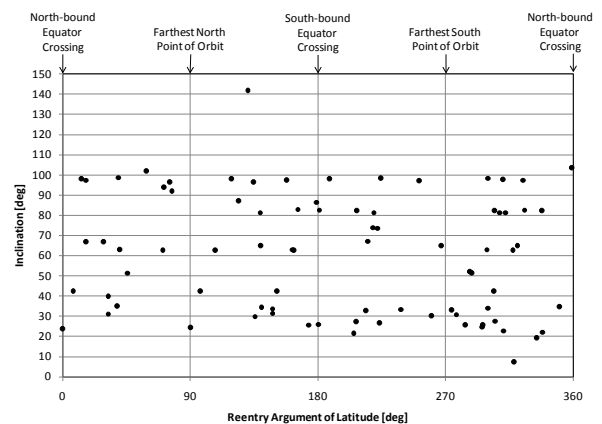


Figure 6. In this plot, the inclination of the 81 objects in the reentry database is here shown as a function of argument of latitude. Markers are added to show the geometric significance of the argument of latitude angle. For example, an argument of perigee value of 0° corresponds to the north-bound equatorial crossing, and 90° corresponds to the farthest north point in the orbit.

Figure 6 shows the same data, but this time showing the distribution in argument of latitude.

Figure 7 shows the argument of perigee cumulative distribution and the Kuiper statistic for that data. As with the longitude distribution, the Kuiper's statistic is well within the 90% confidence limit of a random selection from a uniform distribution.

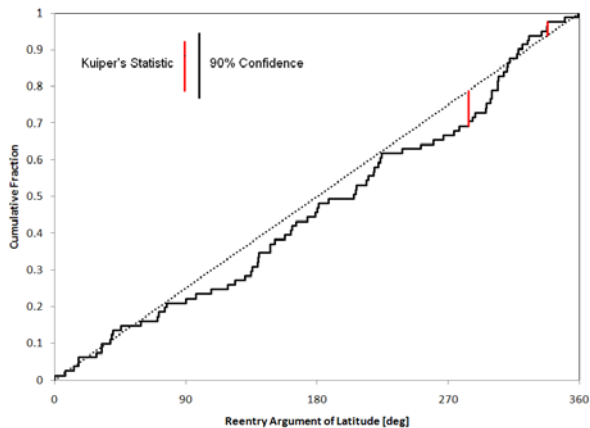


Figure 7. The cumulative distribution in argument of latitude of 81 random object reentries (solid curve) is compared to the theoretical uniform distribution (dotted line). This data set has a Kuiper's statistic smaller than the 90% confidence value, indicating this data is consistent with a random draw from the uniform distribution.

Figure 7 also demonstrates one of the limitations of Kuiper's statistic. Visual examination of the curve seems to indicate that the cumulative distribution dips for both the northern hemisphere segment ( $0^\circ$  to  $180^\circ$ ) and the southern hemisphere segment ( $180^\circ$  to  $360^\circ$ ). Kuiper's statistic may not be the best tool to identify periodicity with twice the frequency of the argument of latitude. However, it is possible to separate the data for these two hemispheres and examine them separately. Figure 8 shows the 37 northern hemisphere reentries, and figure 9 shows the 44 southern hemisphere reentries. Note that this apparent imbalance between the total number of northern and southern hemisphere re-entries (the probability of which should be approximately equal) is well within the 90% confidence limit for 81 random selections from a binomial distribution (analogous to the number of "heads" and "tails" from 81 random coin tosses).

Figures 7 and 8 show that even though there appears to be significant variation from the uniform distribution, the finite number of samples is insufficient to show any deviation from the uniform distribution.

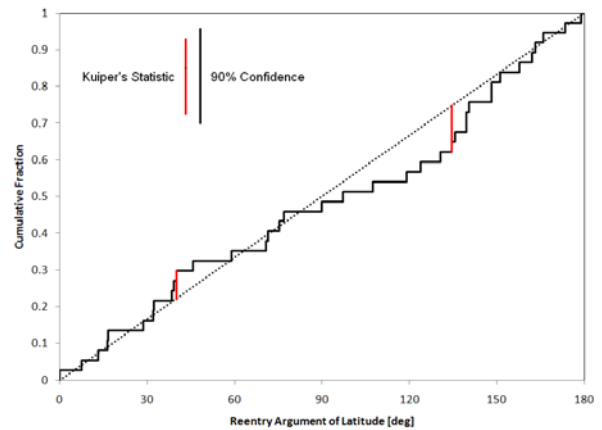


Figure 8. The cumulative distribution in argument of latitude of 37 random object northern hemisphere reentries (solid curve) is compared to the theoretical uniform distribution (dotted line). This data set has a Kuiper's statistic smaller than the 90% confidence value, indicating this data is consistent with a random draw from the uniform distribution.

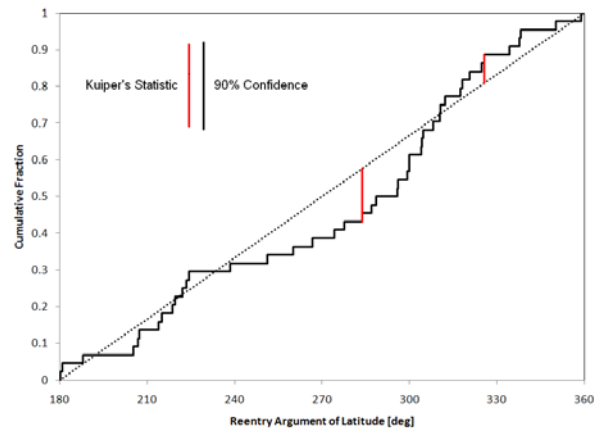


Figure 8. The cumulative distribution in argument of latitude of 44 random object southern hemisphere reentries (solid curve) is compared to the theoretical uniform distribution (dotted line). This data set has a Kuiper's statistic smaller than the 90% confidence value, indicating this data is consistent with a random draw from the uniform distribution.

One more possible test is to combine the northern and southern hemisphere reentries using a parameter that represents the argument of latitude angle measured from the last equatorial crossing (either north-bound or south-bound). Such an angle would take values from  $0^\circ$  to  $180^\circ$ , with  $90^\circ$  representing the farthest north or south point in the orbit. Figure 9 shows the distribution of these "combined" data.

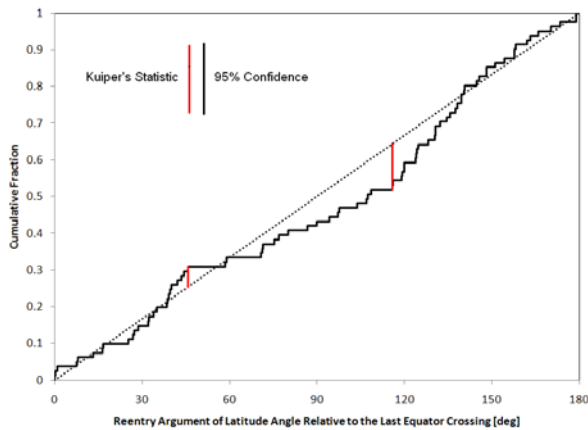


Figure 9. The cumulative distribution of 81 random object reentries (solid curve) is compared to the theoretical uniform distribution (dotted line). The parameter is the combination of the data in Figures 7 and 8, and shows the argument of latitude measured from the last equator crossing (either north-bound or south-bound). This data set has a Kuiper's statistic larger than the 90% confidence value, but is still within the 95% confidence values, indicating this data is consistent with a random draw from the uniform distribution – but may hint at some sort of bias in the reentry location. A larger data set is needed to resolve the question.

## 6. CONCLUSIONS

In this paper I used empirical data to examine several of the assumptions used in computing ground risk from reentering objects. It was shown that for the 81 near-circular orbits examined, the geographic distribution was statistically consistent with the predicted ideal orbit distributions. This indicates that the current methods of computing ground risk are valid.

However, there appeared to be possible biases in the argument of latitude distributions that can only be resolved with better statistics. Further study is encouraged. In addition, there are probably more reentry data available than I have accumulated here. I hope to be able to expand the database for future studies.

I note that I did not examine the behaviour of more elliptical orbits. Examination of the database indicated an unexpected number of moderately elliptical reentries ( $e > 0.0075$ ). Future studies will include the statistics of these elliptical orbits.

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