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**On the Minimum Induced Drag of Wings  
-or-  
Thinking Outside the Box**

Albion H. Bowers  
NASA Dryden Flight Research Center

04 Sep11

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# Introduction

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- The History of Spanload  
Development of the optimum spanload  
Winglets and their implications
  - Horten Sailplanes
  - Flight Mechanics & Adverse yaw
  - Concluding Remarks
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# Birds

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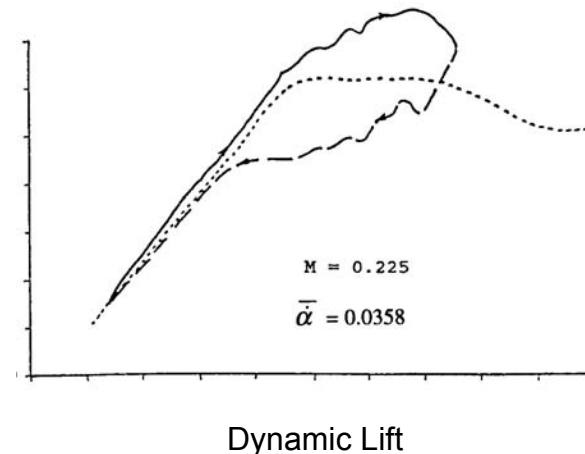


# Bird Flight as a Model

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## or “Why don’t birds have vertical tails?”

- Propulsion
  - Flapping motion to produce thrust
  - Wings also provide lift
  - Dynamic lift - birds use this all the time (easy for them, hard for us)
- Stability and Control
  - Still not understood in literature
  - Lack of vertical surfaces
- Birds as an Integrated System
  - Structure
  - Propulsion
  - Lift (performance)
  - Stability and control
- Wright dis-integrated the bird
- Its time to reintegrate bird-flight



# Spanload Development

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- Ludwig Prandtl
    - Development of the boundary layer concept (1903)
    - Developed the “lifting line” theory
    - Developed the concept of induced drag
    - Calculated the spanload for minimum induced drag (1908?)
    - Published in open literature (1920)
  - Albert Betz
    - Published calculation of induced drag
    - Published optimum spanload for minimum induced drag (1914)
    - Credited all to Prandtl (circa 1908)
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# Spanload Development (continued)

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- Max Munk  
General solution to multiple airfoils  
Referred to as the “stagger biplane theorem” (1920)  
Munk worked for NACA Langley from 1920 through 1926
  - Prandtl (again!)  
“The Minimum Induced Drag of Wings” (1932)  
Introduction of new constraint to spanload  
Considers the bending moment as well as the lift and induced drag
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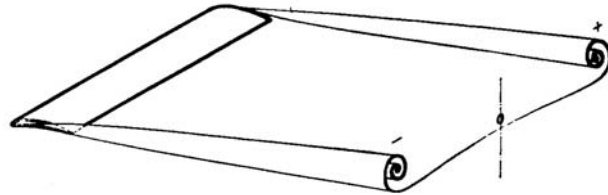
# Practical Spanload Developments

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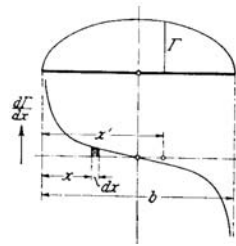
- Reimar Horten (1945)
    - Use of Prandtl's latest spanload work in sailplanes & aircraft
    - Discovery of induced thrust at wingtips
    - Discovery of flight mechanics implications
    - Use of the term "bell shaped" spanload
  - Robert T Jones
    - Spanload for minimum induced drag and wing root bending moment
    - Application of wing root bending moment is less general than Prandtl's
    - No prior knowledge of Prandtl's work, entirely independent (1950)
  - Armin Klein & Sathy Viswanathan
    - Minimum induced drag for given structural weight (1975)
    - Includes bending moment
    - Includes shear
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# Prandtl Lifting Line Theory

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- Prandtl's "vortex ribbons"

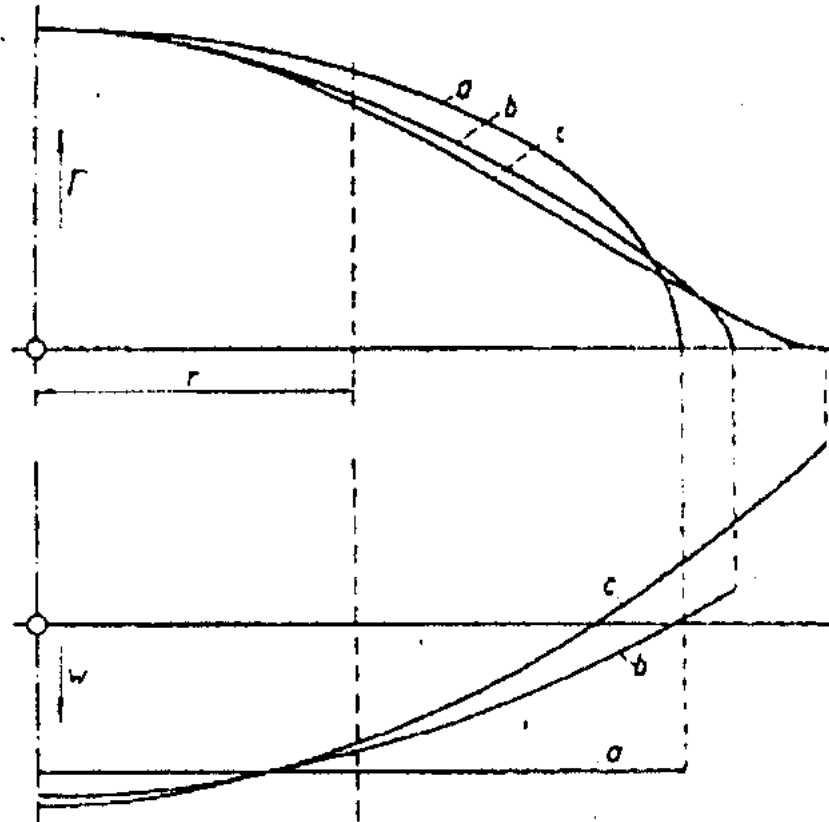


- Elliptical spanload (1914)
  - "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift."  $y = c$
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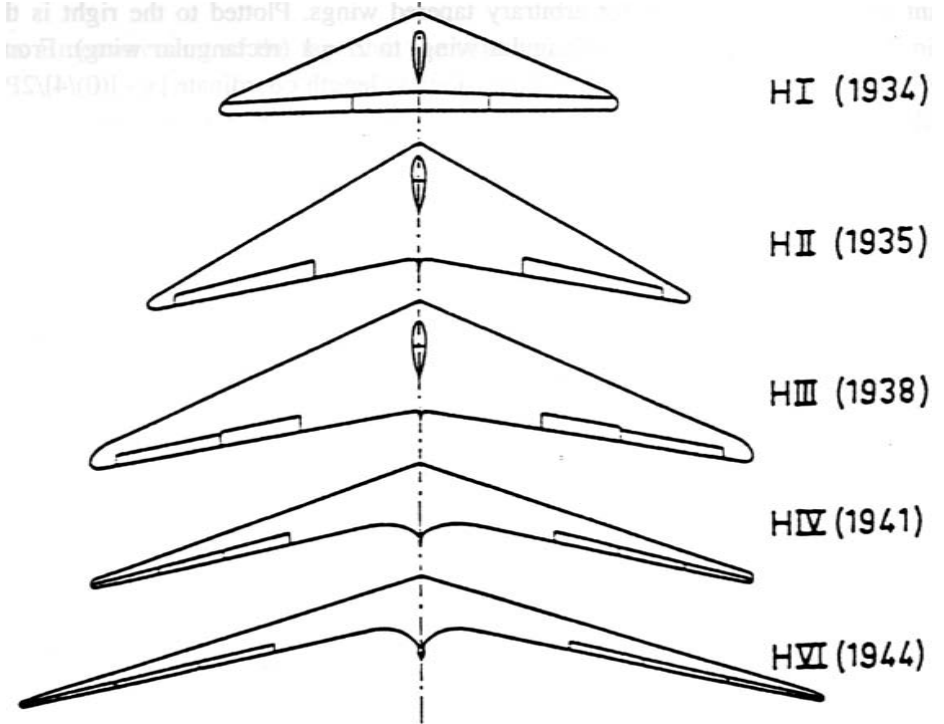
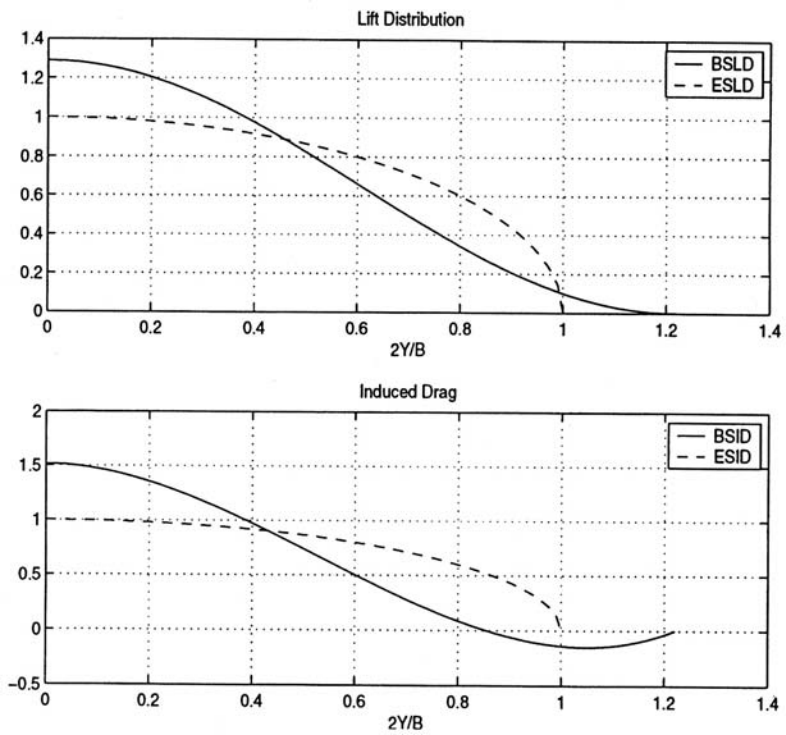
# Minimum Induced Drag & Bending Moment

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- Prandtl (1932)  
Constrain minimum induced drag  
Constrain bending moment  
22% increase in span with 11% decrease in induced drag
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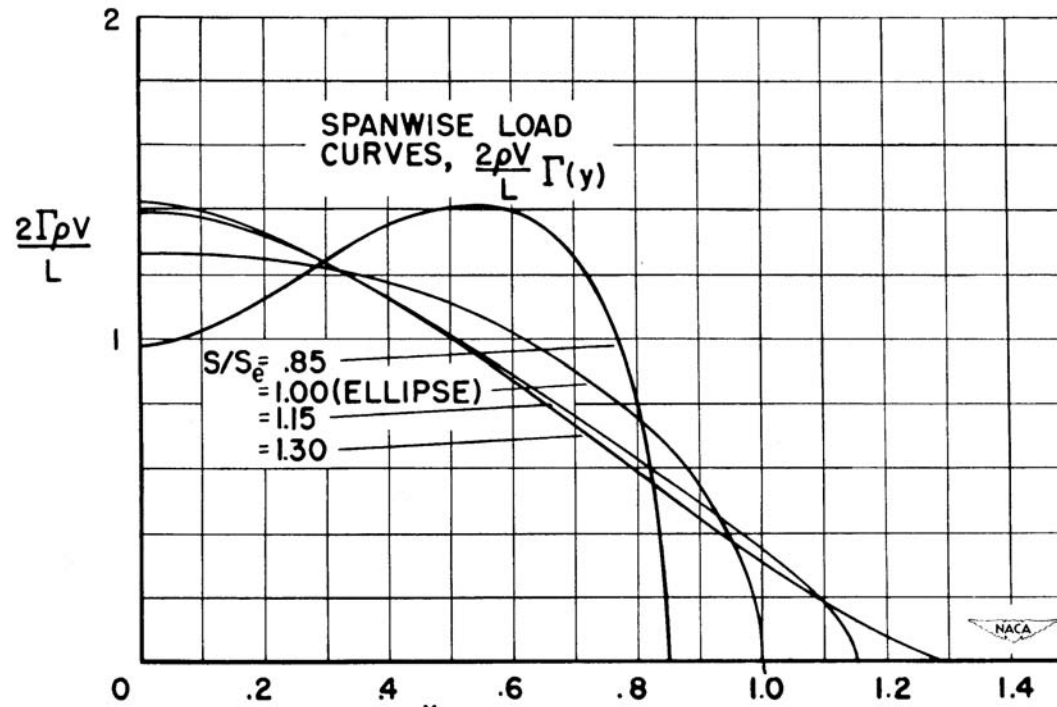
## Horten Applies Prandtl's Theory



Horten Sailplanes

- Horten Spanload (1940-1955)  
induced thrust at tips  
wing root bending moment

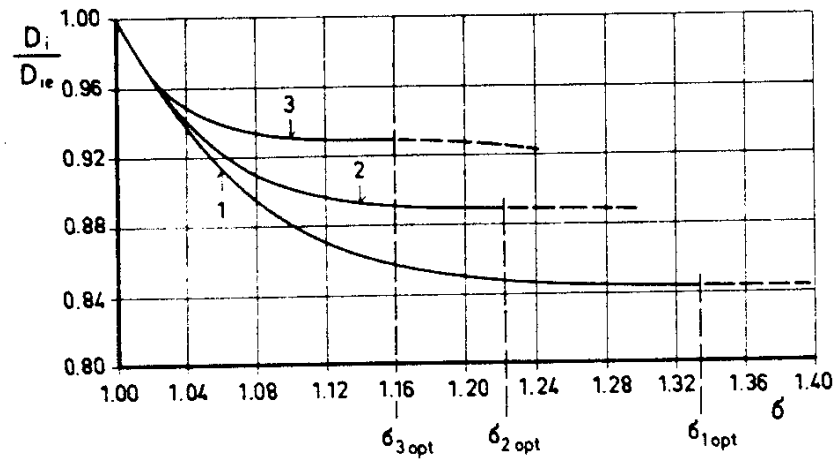
# Jones Spanload



- Minimize induced drag (1950)  
Constrain wing root bending moment  
30% increase in span with 17% decrease in induced drag
- “Hence, for a minimum induced drag with a given total lift and a given bending moment the downwash must show a linear variation along the span.”  $y = bx + c$

# Klein and Viswanathan

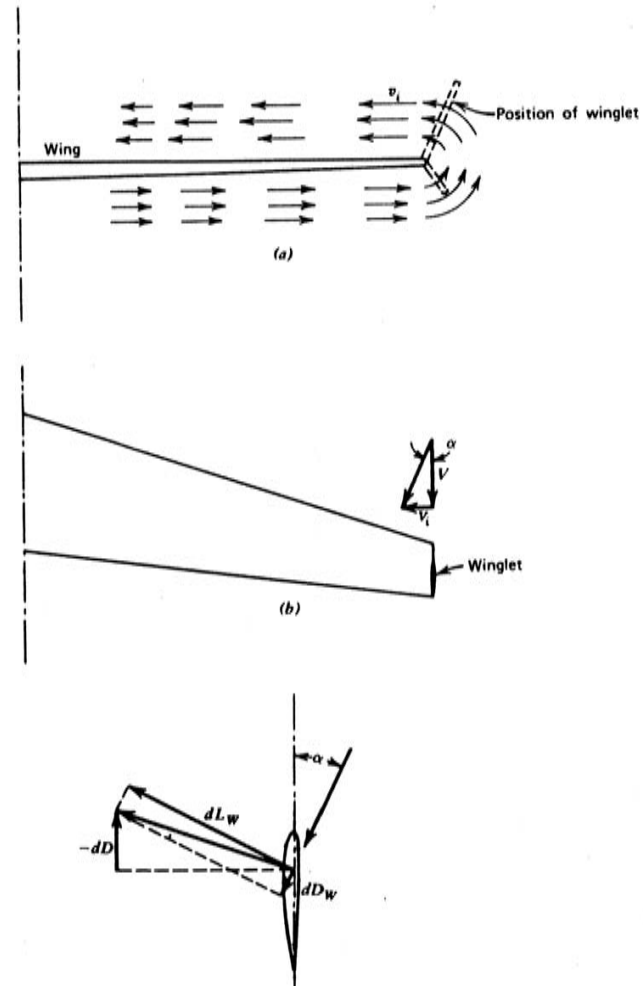
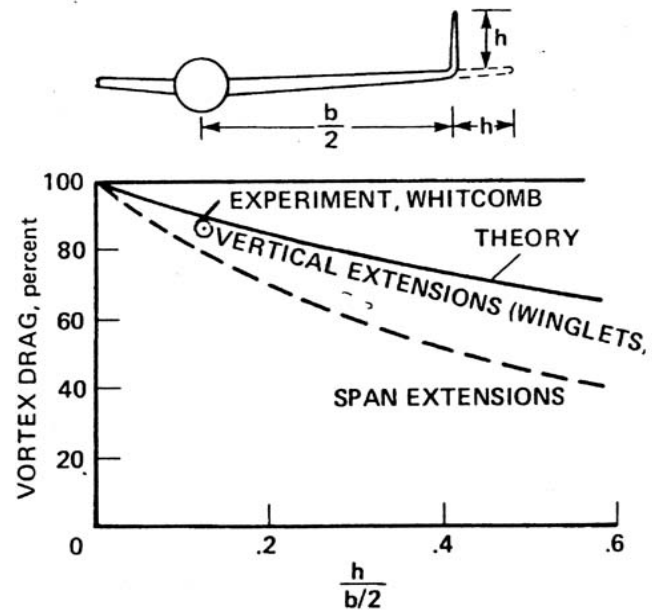
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- Minimize induced drag (1975)  
Constrain bending moment  
Constrain shear stress  
16% increase in span with 7% decrease in induced drag
  - “Hence the required downwash-distribution is parabolic.”  
 $y = ax^2 + bx + c$
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# Winglets

- Richard Whitcomb's Winglets
  - induced thrust on wingtips
  - induced drag decrease is about half of the span "extension"
  - reduced wing root bending stress



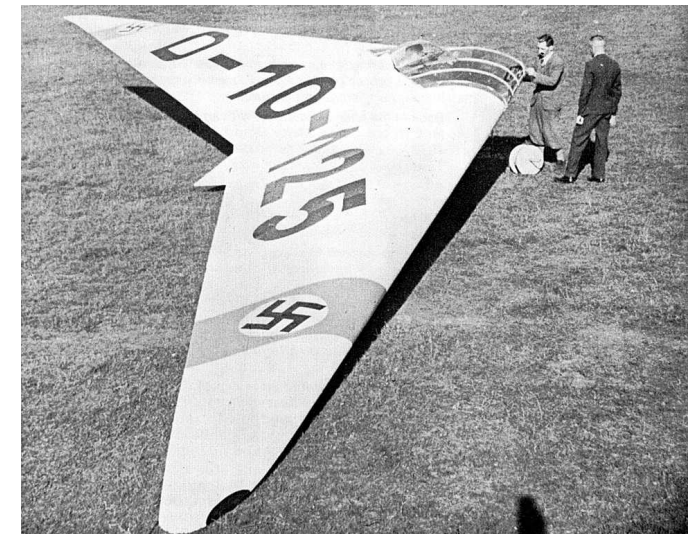
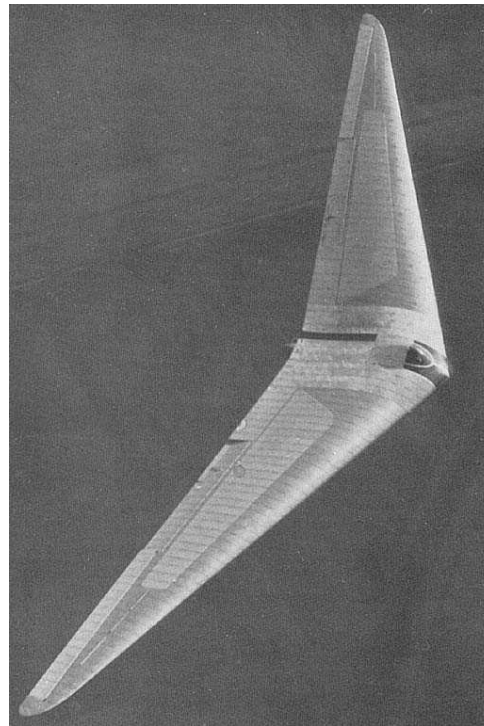
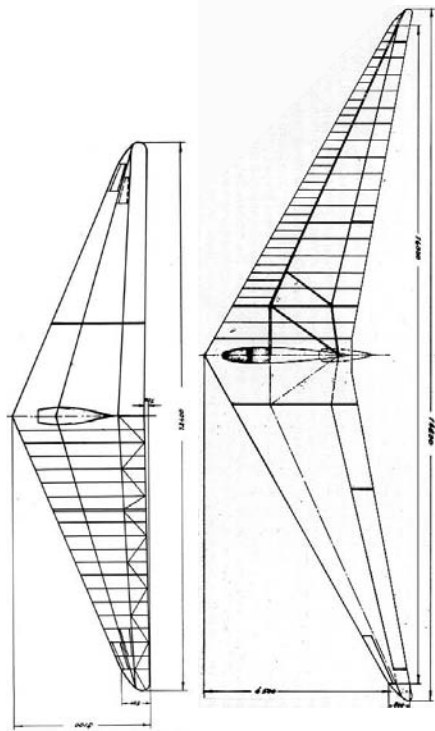
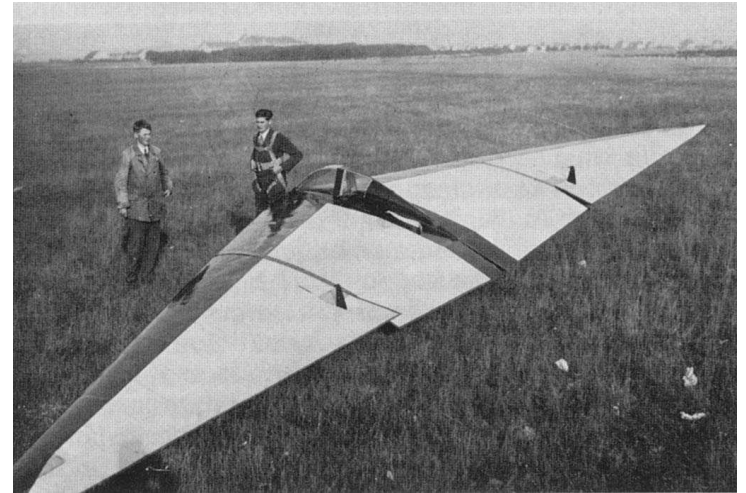
# Spanload Summary

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- Prandtl/Munk (1914)  
Elliptical  
Constrained only by span and lift  
Downwash:  $y = c$
  - Prandtl/Horten/Jones (1932)  
Bell shaped  
Constrained by lift and bending moment  
Downwash:  $y = bx + c$
  - Klein/Viswanathan (1975)  
Modified bell shape  
Constrained by lift, moment and shear (minimum structure)  
Downwash:  $y = ax^2 + bx + c$
  - Whitcomb (1975)  
Winglets
  - Summarized by Jones (1979)
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# Early Horten Sailplanes (Germany)

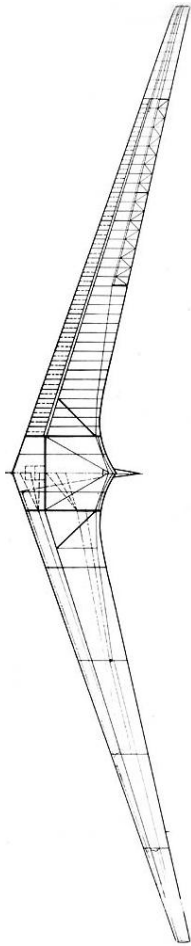
- Horten I - 12m span
- Horten II - 16m span
- Horten III - 20m span



# Horten Sailplanes (Germany)

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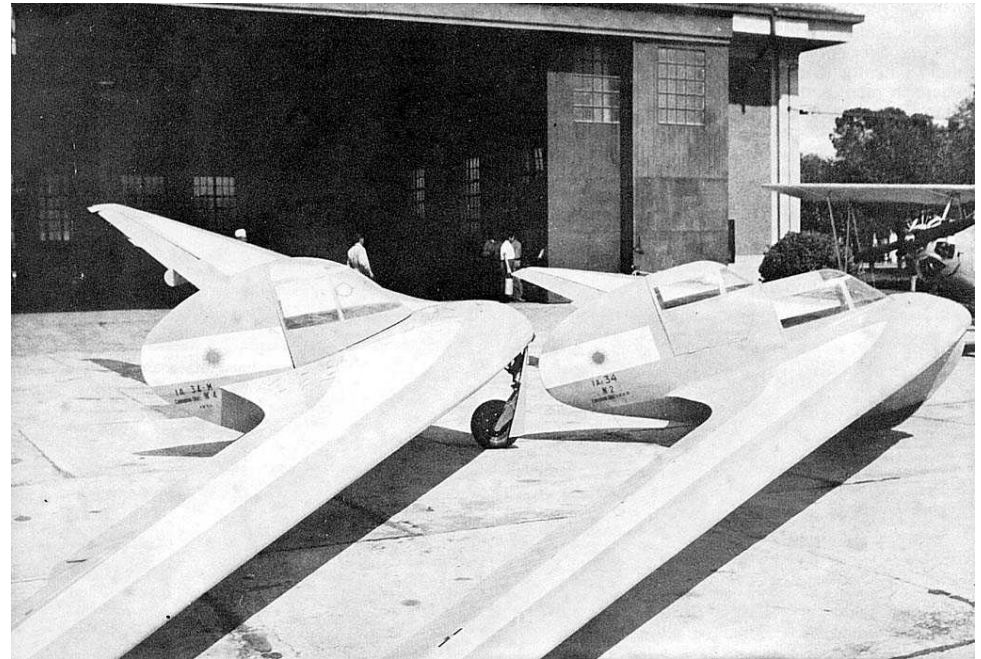
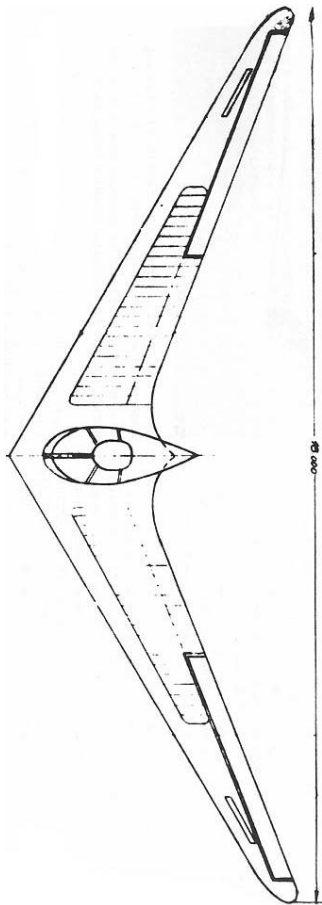
- H IV - 20m span
- H VI - 24m span





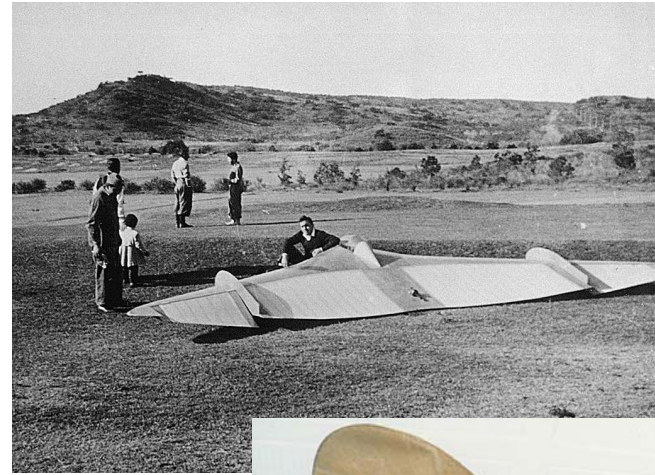
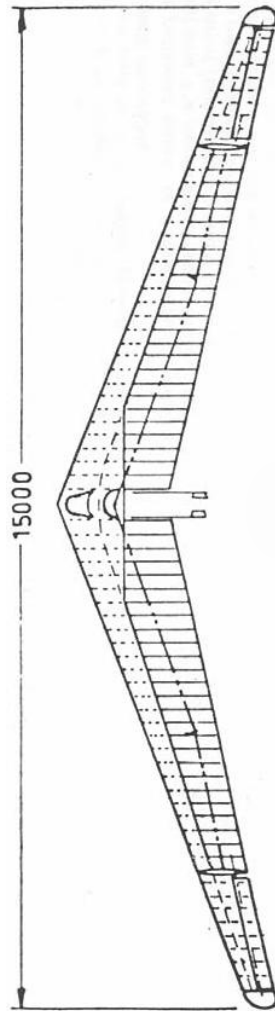
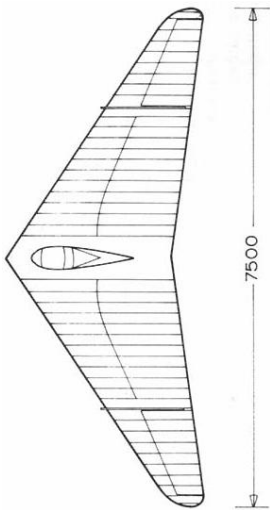
# Horten Sailplanes (Argentina)

- H I b/c - 12m span
- H XV a/b/c - 18m span



# Later Horten Sailplanes (Argentina)

- H Xa/b/c  
7.5m,  
10m, &  
15m



# Bird Flight Model

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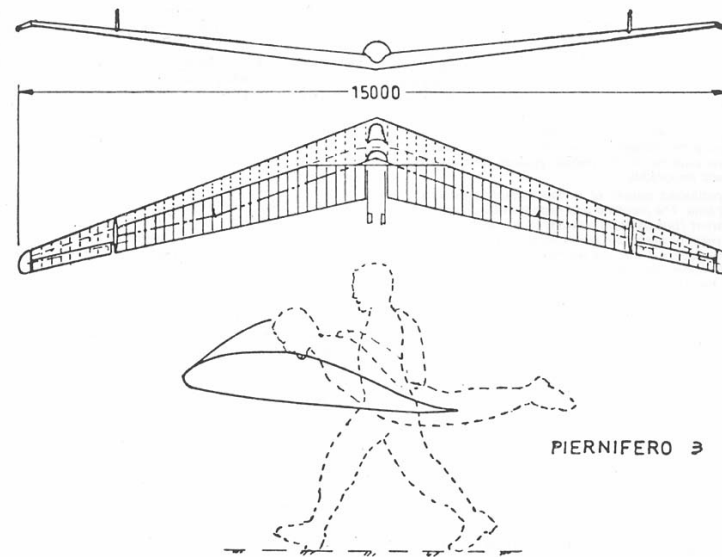
- Minimum Structure
- Flight Mechanics Implications
- Empirical evidence
- How do birds fly?



# Horten H Xc Example

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- Horten H Xc  
footlaunched  
ultralight sailplane  
1950

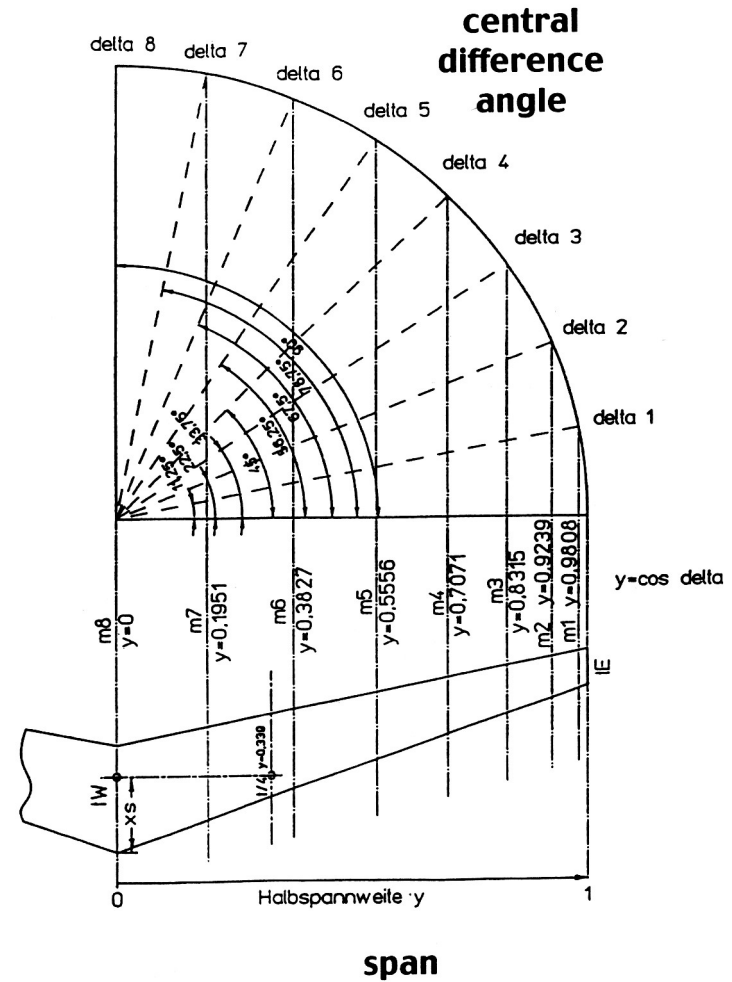
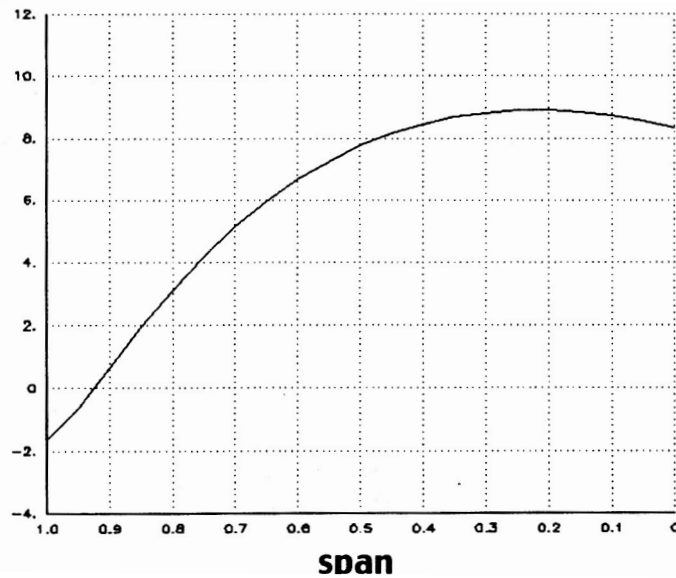


Skizze der H X c mit 15 Meter Spannweite. (Zeichnung Jan Scott)

# Calculation Method

- Taper
- Twist
- Control Surface Deflections
- Central Difference Angle

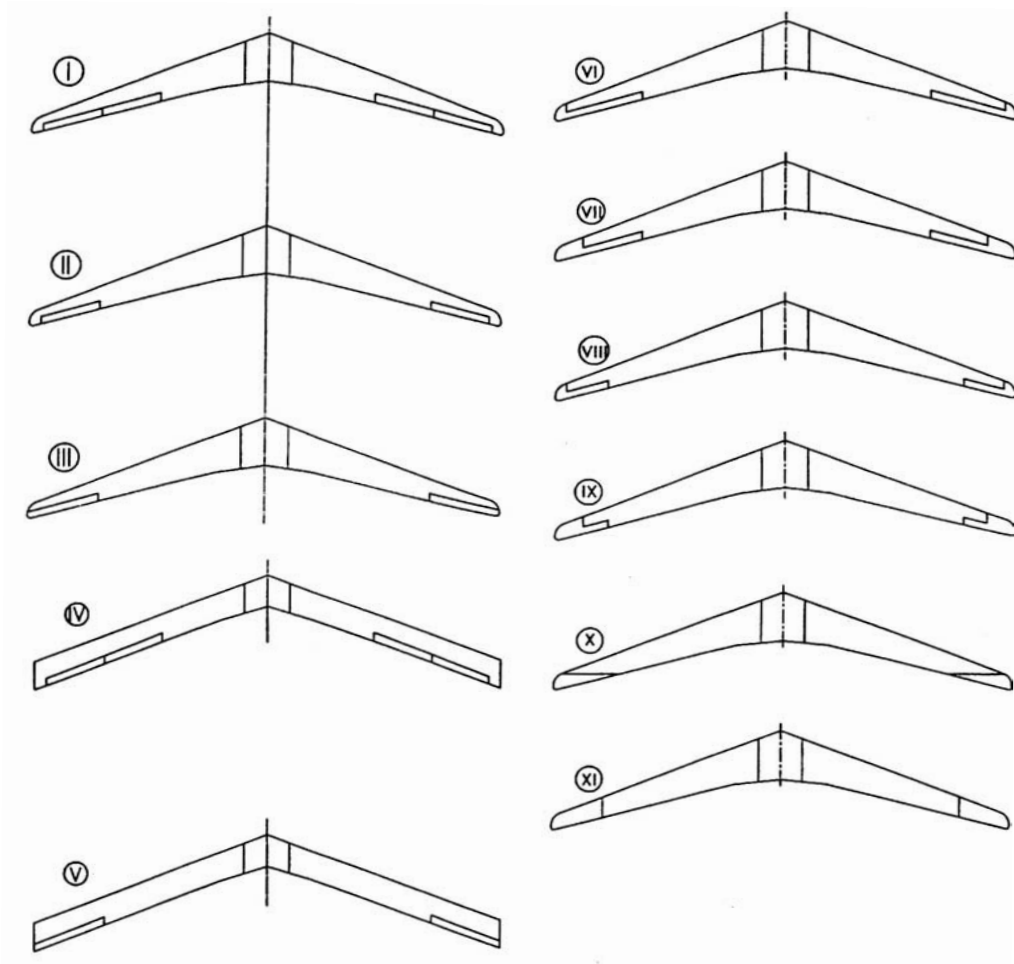
twist  
for  
HXc



# Dr Edward Udens' Results

- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

| Elevon Config | $C_{n\delta a}$ | Spanload   |
|---------------|-----------------|------------|
| I             | -.002070        | bell       |
| II            | .001556         | bell       |
| III           | .002788         | bell       |
| IV            | -.019060        | elliptical |
| V             | -.015730        | elliptical |
| VI            | .001942         | bell       |
| VII           | .002823         | bell       |
| VIII          | .004529         | bell       |
| IX            | .005408         | bell       |
| X             | .004132         | bell       |
| XI            | .005455         | bell       |

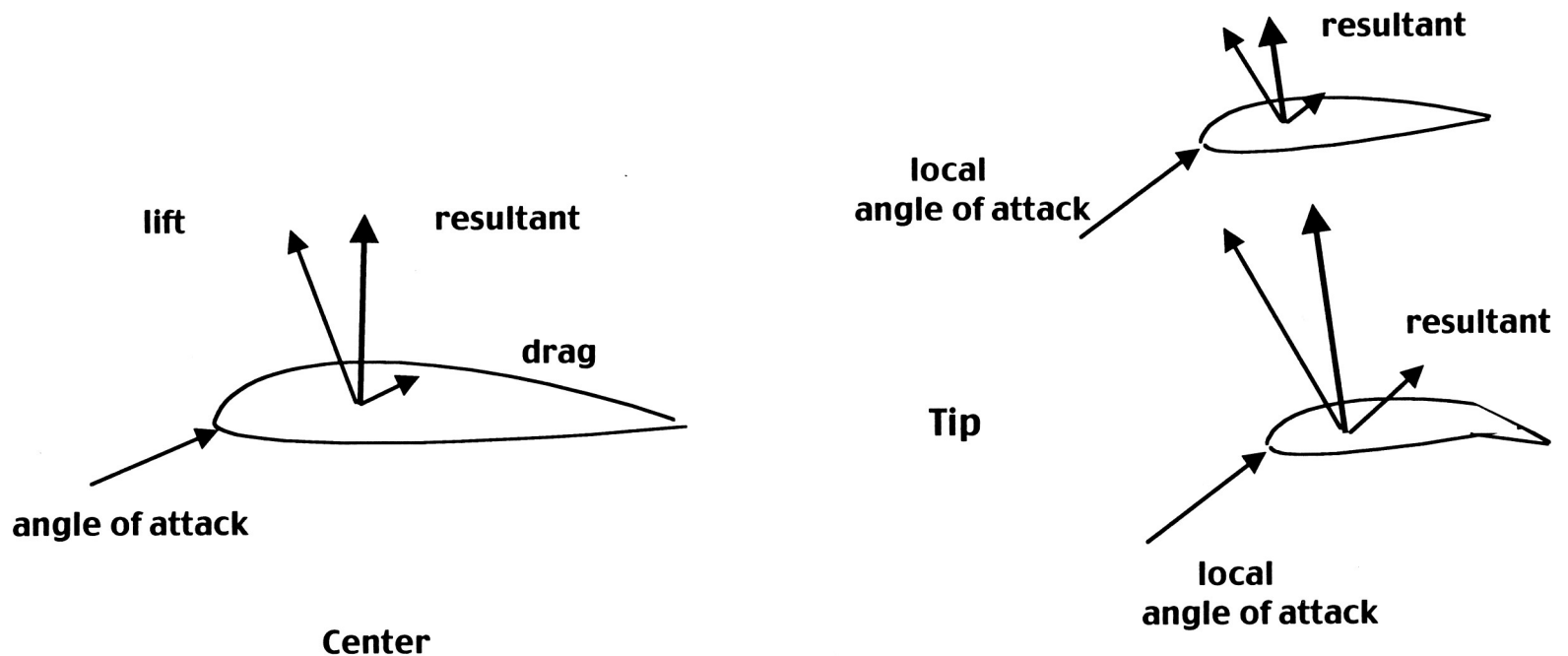


# Horten H Xc Wing Analysis

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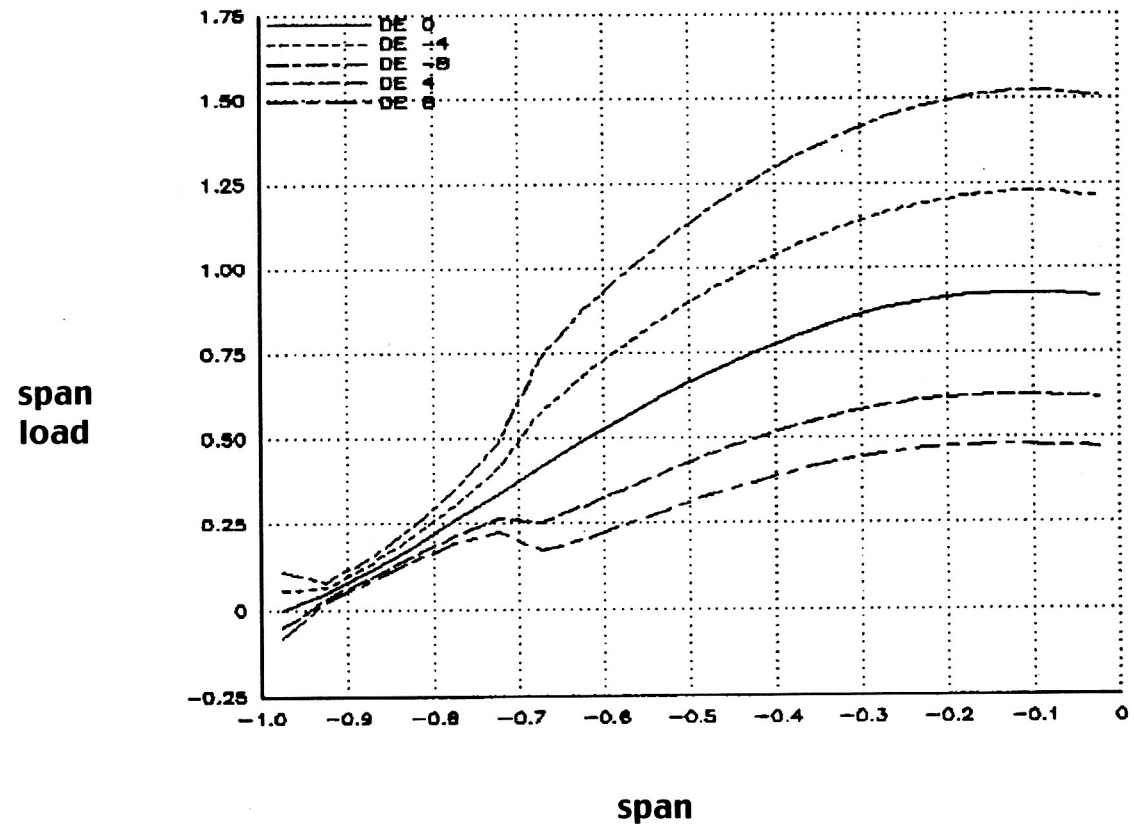
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- Vortex Lattice Analysis
- Spanloads (longitudinal & lateral-directional) - trim & asymmetrical roll
- Proverse/Adverse Induced Yawing Moments handling qualities
- Force Vectors on Tips - twist, elevon deflections, & upwash
- 320 Panels: 40 spanwise & 8 chordwise



# Symmetrical Spanloads

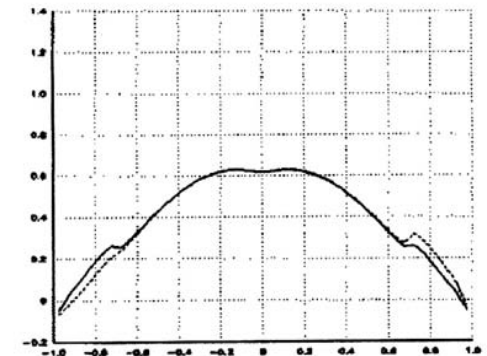
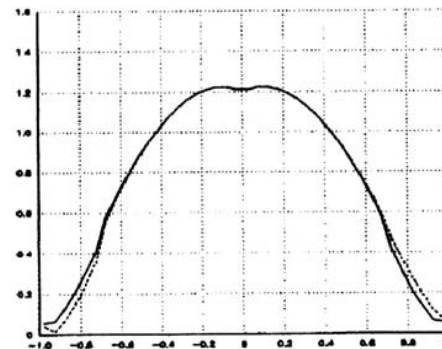
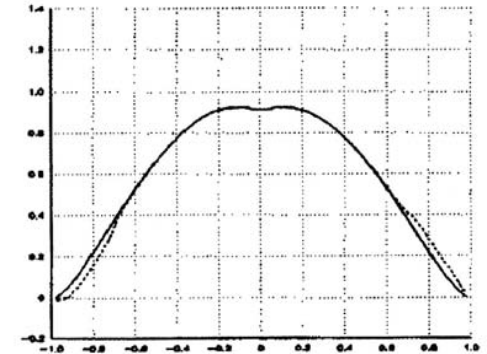
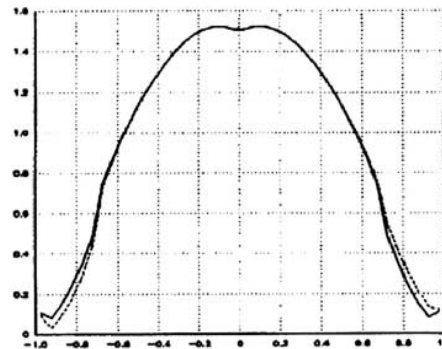
- Elevon Trim
- CG Location



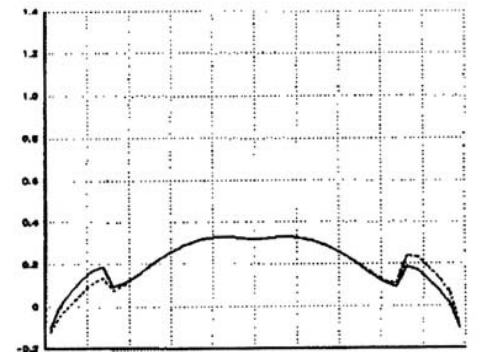


# Asymmetrical Spanloads

- $Cl_{\partial a}$  (roll due to aileron)
- $Cn_{\partial a}$  (yaw due to aileron)  
induced component  
profile component  
change with lift
- $Cn_{\partial a}/Cl_{\partial a}$
- CL (Lift Coefficient)  
Increased lift:  
increased  $Cl_{\partial a}$   
increased  $Cn_{\partial a}^*$   
Decreased lift:  
decreased  $Cl_{\partial a}$   
decreased  $Cn_{\partial a}^*$



| CL   | Cl     | Cn      |
|------|--------|---------|
| .966 | .01384 | .00055  |
| .774 | .01384 | .00037  |
| .582 | .01345 | .00021  |
| .390 | .01384 | .00003  |
| .198 | .01345 | -.00015 |



# Airfoil and Wing Analysis

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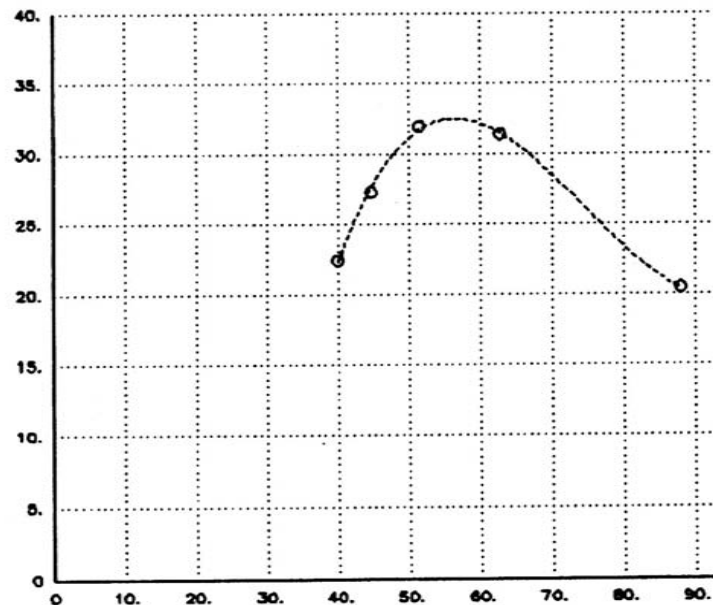
- Profile code (Dr Richard Eppler)
  - Flap Option (elevation deflections)
  - Matched Local Lift Coefficients
  - Profile Drag
  - Integrated Lift Coefficients  
match Profile results to Vortex Lattice  
separation differences in lift
  - Combined in MatLab
-

# Performance Comparison

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- Max L/D: 31.9
- Min sink: 89.1 fpm
- Does not include pilot drag
  
- Predicted L/D: 30
- Predicted sink: 90 fpm

L/D



velocity

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# Horten Spanload Equivalent to Birds

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- Bell Shaped Span Load is equivalent to bird span load (shear not considered in Horten designs)
  - Flight mechanics are the same - turn components are the same
  - Both attempt to use minimum structure
  - Both solve minimum drag, turn performance, and optimal structure with one solution
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# Concluding Remarks

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- Birds as as the first model for flight
  - Theoretical developments independent of applications
  - Applied approach gave immediate solutions, departure from bird flight
  - Eventual meeting of theory and applications (applied theory)
  - Spanload evolution (Prandtl/Munk, Prandtl/Horten/Jones, Klein & Viswanathan)
  - Flight mechanics implications
  - Hortens are equivalent to birds
  - Thanks: Juliana Plumb, Dr Bob Liebeck, Nalin Ratnayake, Kia Davidson, Mike Allen, Walter Horten, Georgy Dez-Falvy, Rudi Opitz, Bruce Carmichael, R.T. Jones, Russ Lee, Phil Barnes, Dan & Jan Armstrong, Dr Phil Burgers, Ed Lockhart, Andy Kesckes, Dr Paul MacCready, Reinhold Stadler, Edward Udens, Dr Karl Nickel & Jack Lambie
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