



Implementation of Speed Variation in the Structural Dynamic Assessment of Turbomachinery Flow-Path Components

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Introduction and Motivation



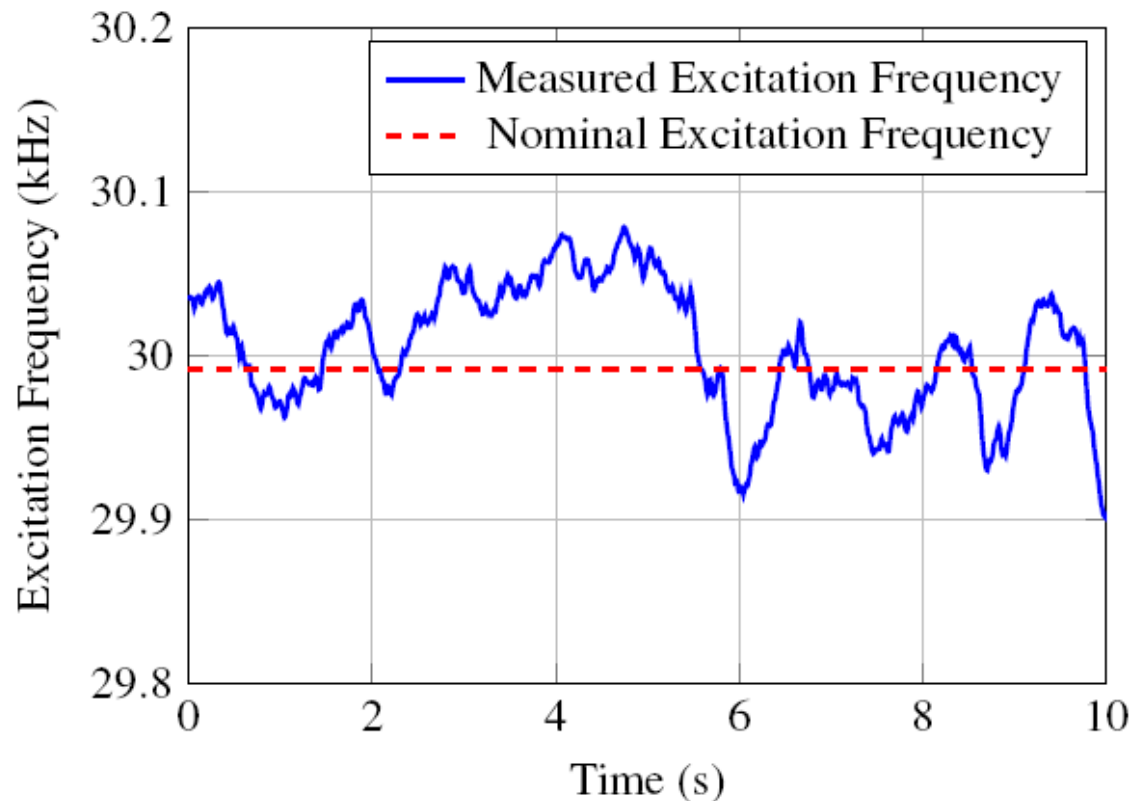
- Structural (S_{ult} & HCF) assessment critical for turbomachinery flow path components undergoing possible resonance.
- Resonance generally avoided, but impossible for higher modes found with modern analysis, especially with wide speed ranges.
 - Space Launch System upper stage J2-X Lox-H2 Engine Fuel Pump turbine stator operates from 26Krpm-34Krpm; 69N forcing excites modes 10-18 between 30KHz-40Khz.
- Criteria triggers forced response analysis at worst case resonant condition.
- Finite life analysis, where actual fatigue damage during operational time is calculated, frequently used if endurance limit criteria violated.



Many Turbopumps “Dither”



- May be beneficial to incorporate fact that real turbopumps dither about a nominal mean speed.



J2-X Powerpack
Adjusted Speed
Trace

- During time speed is not exactly at natural frequency, damage accumulation is significantly reduced.



Agenda



1. Introduction
 - a) Motivation
 - b) Literature Survey, Purpose Statement
 - c) Characteristics of Excitation Frequency
2. Theory and Development of Analysis Methods for Measured Speed Time Histories.
 - a) Numerical Method, Time Step Convergence Study
 - b) Analytical Method
 - c) Calculation of Dither Life Ratio
3. Monte Carlo Method for Unknown Speed History (Design).
 - a) Sensitivity to Damping, Speed standard deviation
4. Conclusion



Literature, Purpose



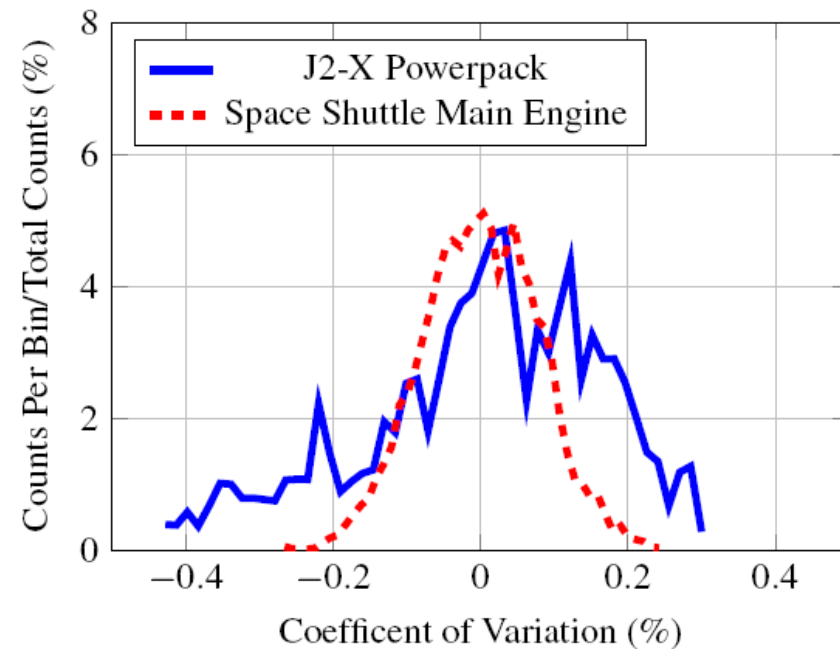
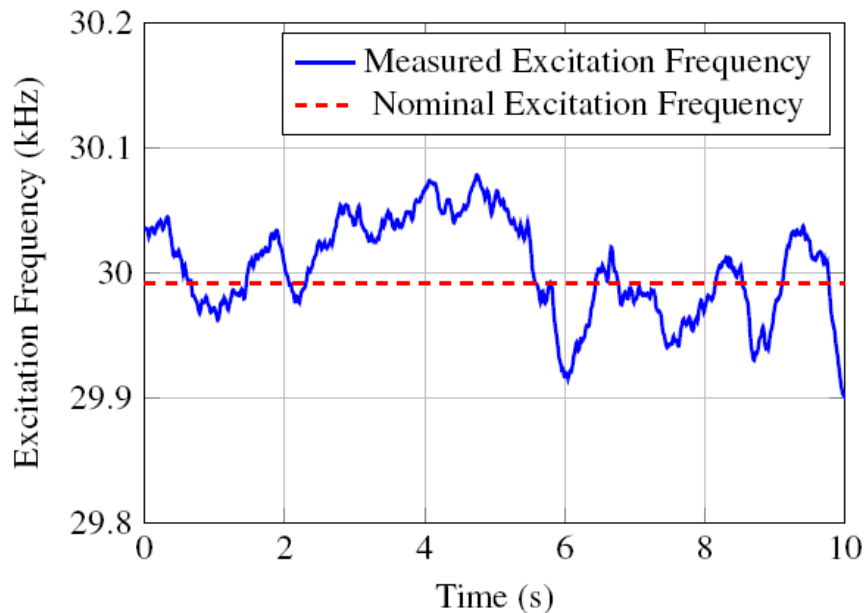
- Initial studies of response of systems with time varying excitation frequency Ω by Lewis- 1932, Cronin- 1965.
- Lollack, 2002, defined reduction in peak response for monotonically varying Ω , useful for defining rate of sine-sweep tests.
- Henson, 2008, studied harmonically varying Ω .
- For rocket engines, Ω varies non-deterministically. Motivated previous work by authors (2010) that developed numerical approach for calculating response and general sensitivities.
- Unacceptable HCF factor for J2-X stator resonant 30Khz mode prompted need for practical technique.
- Purpose of this research
 - *to develop practical design techniques that account for excitation frequency stochasticity in the fatigue life of turbomachinery components.*



Excitation Data



- Taken from hot-fire testing of J2-X and SSME.
- Ω = engine speed (hz)*[forcing pressure distortions/Rev] (FPR).
- Since purpose is to examine fatigue life at resonance, actual mean speed adjusted to natural frequency for analysis.
- Histograms for two different engines show \sim Gaussian distribution of speed.





Theoretical Basis, Numerical Transient Solution



- SDOF EoM

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = \frac{f(t)}{m}$$

where

$$f(t) = A \sin(\phi(t))$$

- Ω is derivative of $\phi(t)$, constant in classical vibration analysis.
For specified time-varying Ω ,

$$\phi(t) = \int_0^t \Omega(\tau) d\tau$$

- Calculate A necessary to generate peak resonant value of σ_{alt} previously obtained by FEA,

$$\sigma_{alt} \equiv x = \frac{A}{\omega^2 2\zeta}$$

- Now can solve for σ_{alt} in EOM with using numerical Runge-Kutte procedure implemented in Matlab; agrees with Lollack's results for linearly varying Ω .

- Finally, Calculate damage fraction Φ using Miner's rule, $\Phi = \sum_{i=1}^K \frac{n}{N}$, which becomes

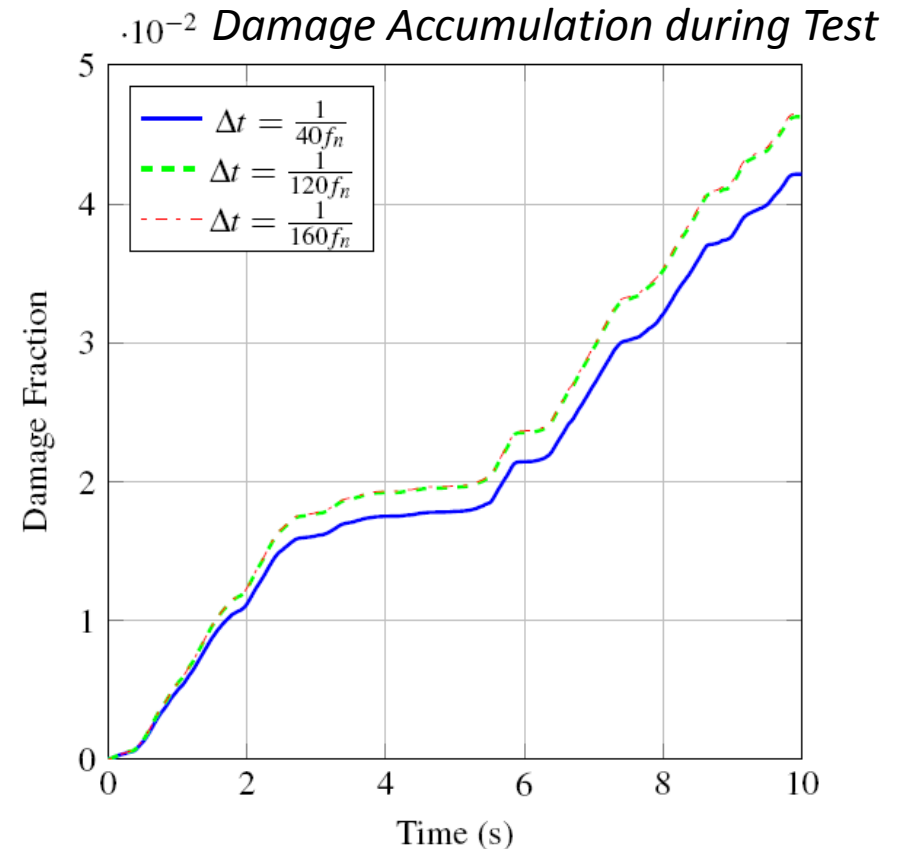
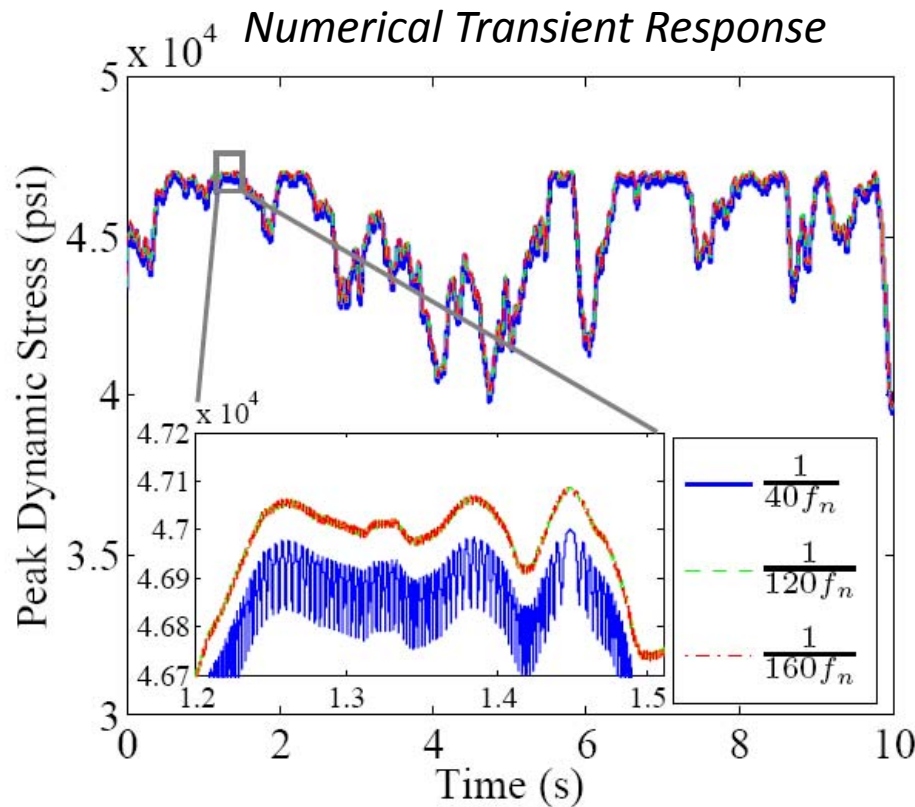
$$\Phi(t) = \int_0^t \frac{\Omega(\tau)}{N(\tau)} d\tau$$



Convergence of Time Step in Transient Solution



- Previous work indicated convergence at $\Delta t = 1/40f_n$.
- Initial studies here showed high frequency oscillation, so response and damage convergence studies performed $\rightarrow \Delta t = 1/120f_n$.





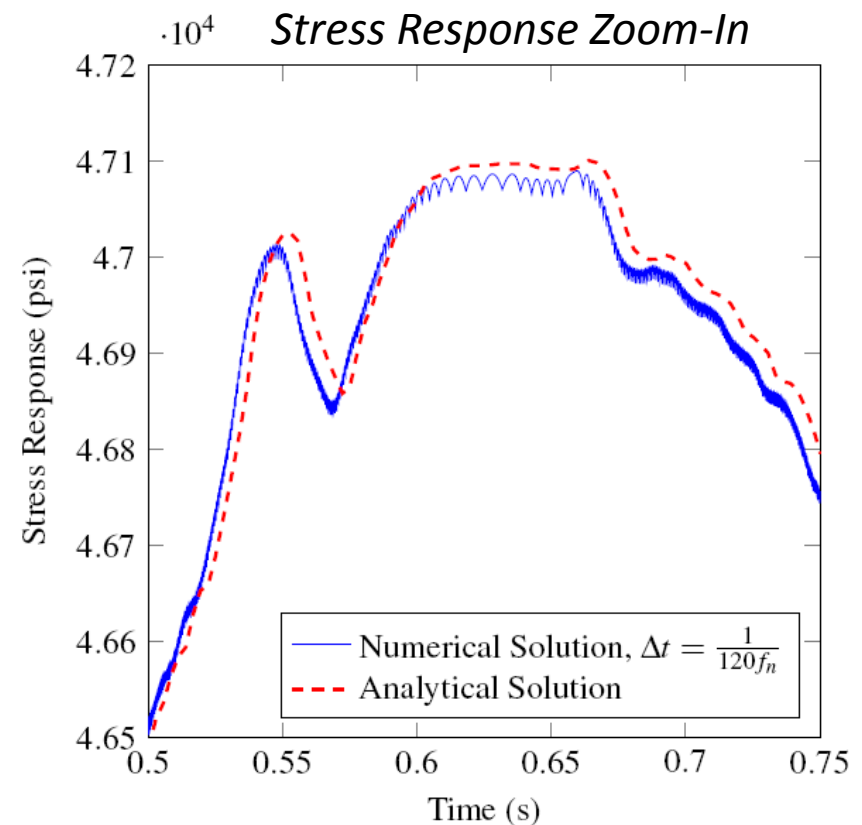
Analytical Solution



- Hypothesis from previous work that if $f_n \ll \frac{d(\text{speed})}{dt}$, then closed-form (computationally fast) standard analytical equation for SDOF steady-state response would be accurate.

$$x_{\text{steady-state}} = \frac{A/\omega^2}{\sqrt{\left(1 - \left(\frac{\Omega}{\omega}\right)^2\right)^2 + (2\zeta \frac{\Omega}{\omega})^2}}$$

- Validation by comparing response with numerical solution.

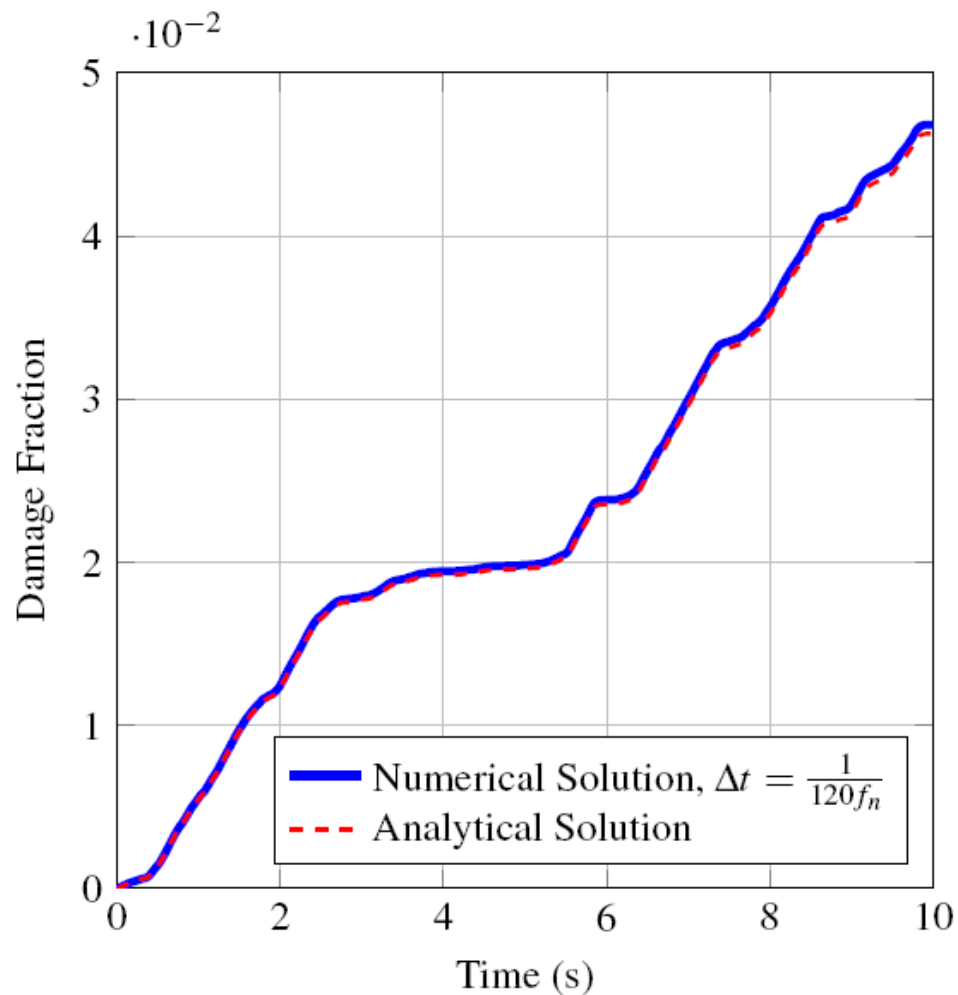




Analytical Solution Validation



- Validation also shown in damage accumulation plot; error in analytical steady-state method is $<1\%$ ($\Delta t \leq 1/120f_n$ required).

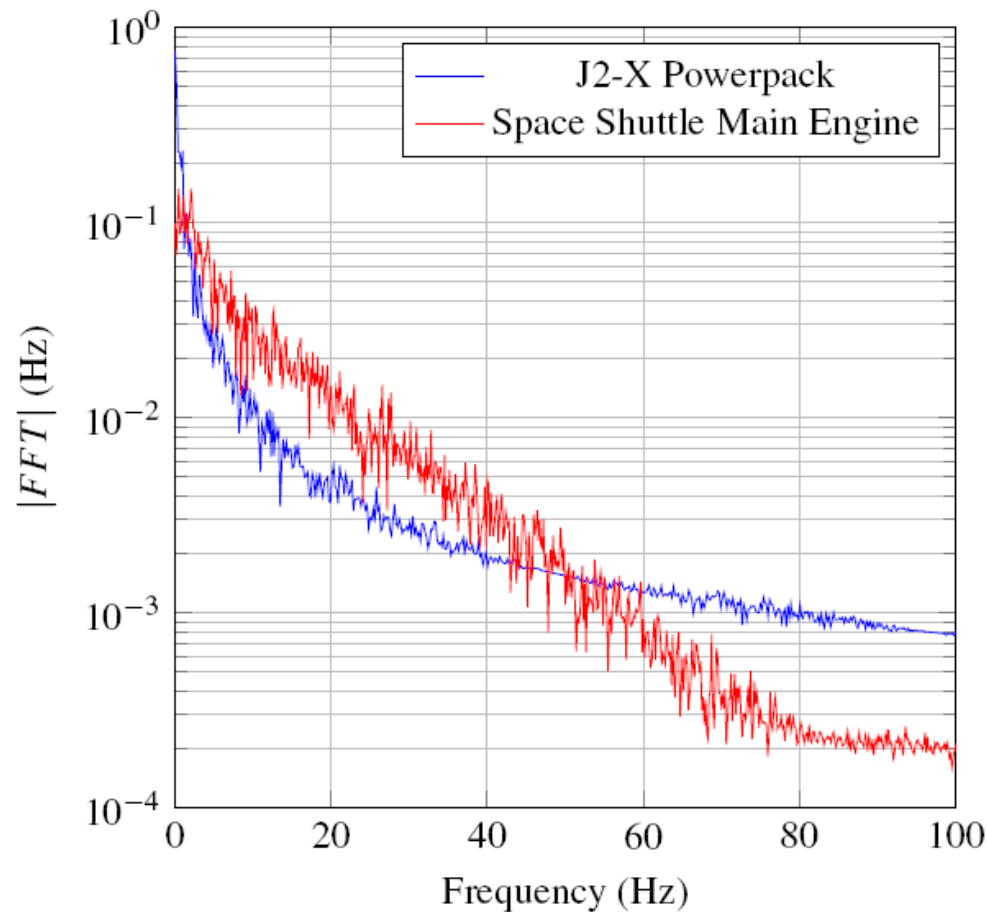




FFT of Speed also shows Analytical Sol'n Validity



- This assumption good with high FPR, driving $f_n / \frac{d(speed)}{dt}$ ratio up.
- FFT of speed shows mostly below 100 hz, very low compared with natural frequency.

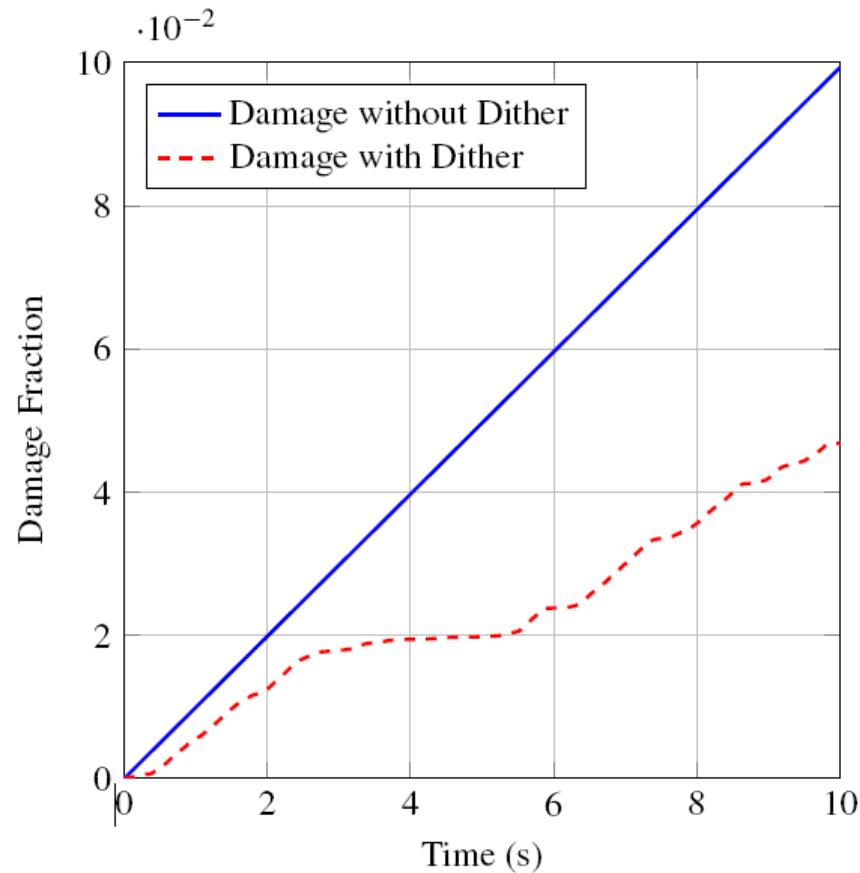




“Dither Life Ratio” for Specified Excitation History



- Calculation of damage performed considering dither for specific 10 sec. window.
- Damage calculation assuming constant resonant excitation → 2.135 times more damage, call it “Dither Life Ratio”.

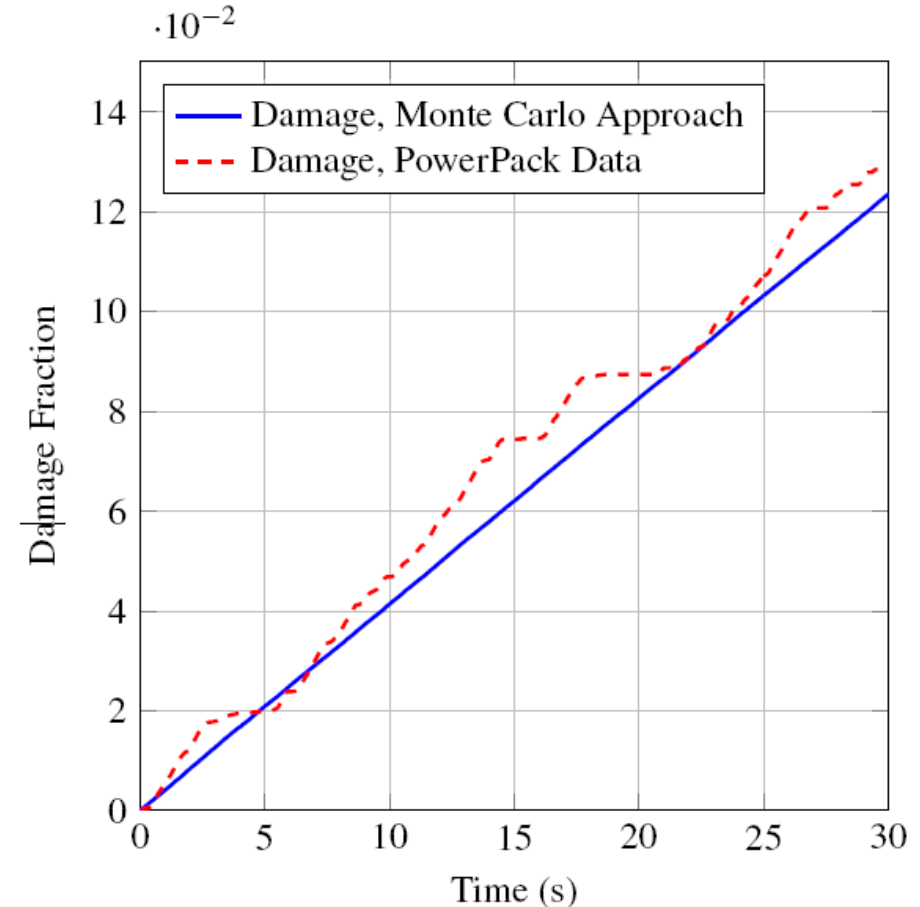




Monte Carlo for Unknown Frequency History

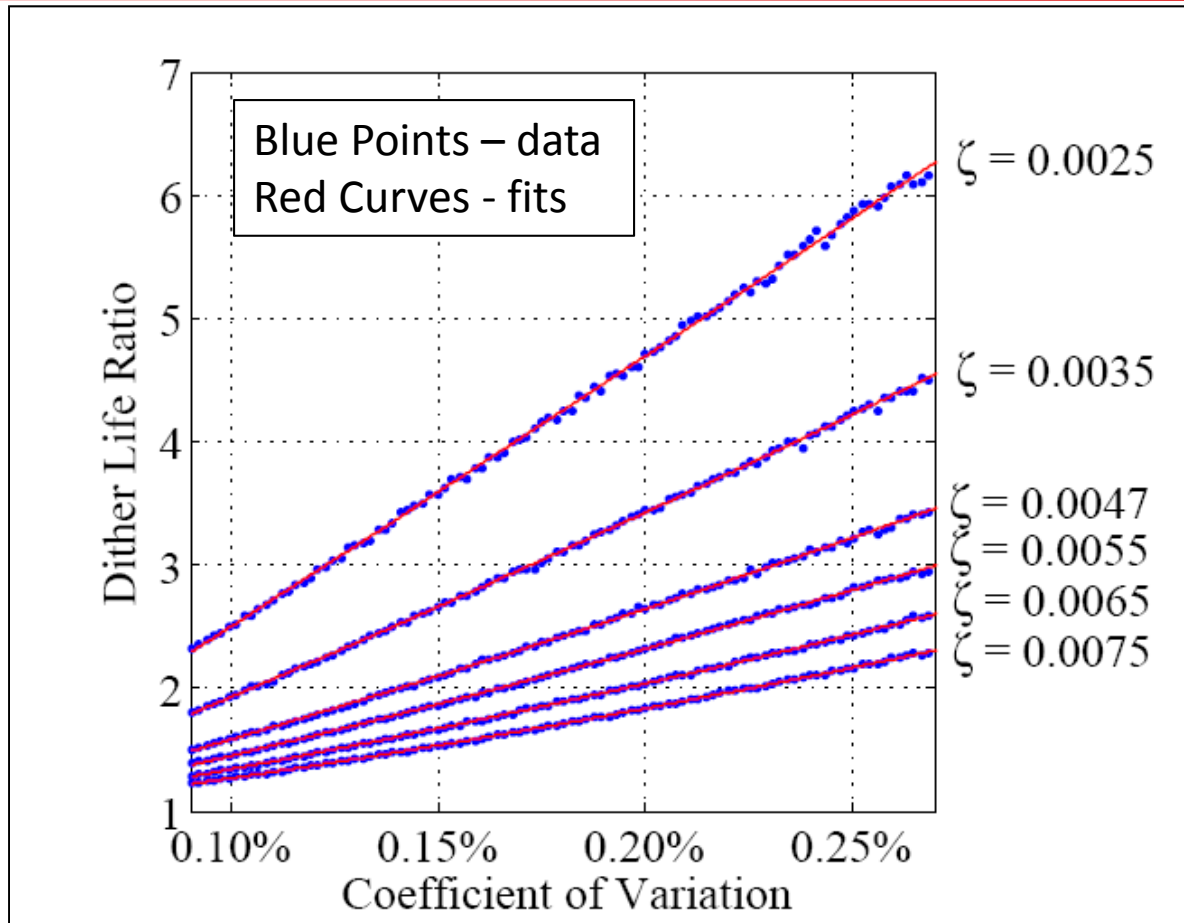


- During design phase, actual speed time histories unknown, but statistics from similar engines known.
- Prompted development of Monte Carlo method using rapid analytical solution.
- Speed vector created using Normal statistical distribution.
- Powerpack data \rightarrow std dev = 38.6 hz (cov=0.129%).
- MC results linear because rate of change of frequency variation not correct (and very high), but damage accumulation is accurate on the average.





Sensitivity of DLR to speed COV and ζ



- Larger for high COV for speed, since more time spent off-resonance.
- Larger for small ζ , since peaks are sharper and time spent off-resonance will have less response.



Conclusions



- Numerical and Analytical methods developed to determine damage accumulation in specific engine components when speed variation included.
- Dither Life Ratio shown to be well over factor of 2 for specific case.
- Steady-State assumption shown to be accurate for most turbopump cases, allowing rapid calculation of DLR.
- If hot-fire speed data unknown, Monte Carlo method developed that uses speed statistics for similar engines.
- Application of techniques allow analyst to reduce both uncertainty and excess conservatism.
- High values of DLR could allow previously unacceptable part to pass HCF criteria without redesign.
- Given benefit and ease of implementation, recommend that any finite life turbomachine component analysis adopt these techniques.