NASA/TP—2014–218192

Comparison of the Variations of Sunspot Number, Number of Sunspot Groups, and Sunspot Area, 1875–2013

Robert M. Wilson
Marshall Space Flight Center, Huntsville, Alabama

April 2014

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National Aeronautics and
Space Administration

Marshall Space Flight Center • Huntsville, Alabama 35812

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LIST OF ABBREVIATIONS, ACRONYMS, AND DESIGNATORS

A	area (relation between sunspot area and sunspot number)
ASC	ascent duration in years
G	number of sunspot groups
<G>	mean number of sunspot groups
Gmin	minimum number of sunspot groups
Gmax	maximum number of sunspot groups
M	cycle maximum
m	cycle minimum
N	number of individual spots within spot groups
NOAA	National Oceanic and Atmospheric Administration
NSD	number of spotless days
<NSD>	mean number of spotless days
NSDmax	maximum number of spotless days
PER	period or cycle length (from SSNmin to next cycle SSNmin)
R	relative sunspot number
RGO	Royal Greenwich Observatory
SC	sunspot cycle
SIDC	Solar Influences and Data Analysis Center
SOON	Solar Optical Observing Network
SSA	sunspot area

LIST OF ABBREVIATIONS, ACRONYMS, AND DESIGNATORS (Continued)

<SSA>	mean sunspot area
SSA/G	ratio of SSN to G
<SSA/G>	mean ratio SSN to G
SSAmax	maximum sunspot area
SSAmin	minimum sunspot area
SSA/SSN	ratio of SSA to SSN
<SSA/SSN>	mean ratio of SSA to SSN
SSN	sunspot number
<SSN>	mean sunspot number
SSN/G	ratio of SSN to G
<SSN/G>	mean ratio of SSN to G
SSNmax	maximum sunspot number
SSNmin	minimum sunspot number
USAF	United States Air Force

NOMENCLATURE

cl	confidence level
k	personal reduction coefficient
R_{y12}	coefficient of the sample multiple correlation
r	coefficient of correlation
r^2	coefficient of determination
sd	standard deviation
$S_{\mathrm{y}12}$	coefficient of the sample multiple correlation error of estimate
se	standard error of estimate
t	elapsed time from SSNmin; the t statistic for independent samples
x	independent variable
x_1	first independent variable in the bivariate fit
x_2	second independent variable in the bivariate fit
y	dependent variable
z	normal deviate for the sample

TECHNICAL PUBLICATION

COMPARISON OF THE VARIATIONS OF SUNSPOT NUMBER, NUMBER OF SUNSPOT GROUPS, AND SUNSPOT AREA, 1875–2013

1. INTRODUCTION

Long ago[1–3] Samuel Heinrich Schwabe, a German apothecary and amateur astronomer, demonstrated the existence of the sunspot cycle, a 10- to 12-year variation in the number of sunspots with time (i.e., the elapsed time from cycle minimum to next cycle minimum). He accomplished this by recording the number of 'days when no spots were observed' and the number of 'clusters of spots' that he observed daily over the course of the year from Dessau, Germany, during the interval of 1826–1868, averaging about 290 observing days per year. He reasoned that the peak annual number of spotless days represented a minimum of sunspot activity (occurring concurrently with the minimum annual number of clusters of spots), while the peak annual number of clusters of spots represented a maximum of sunspot activity. In 1857 the Royal Astronomical Society awarded Schwabe its gold medal and in 1868 Schwabe was elected a member of the Royal Society of London.[4]

Unfortunately, the discovery of the sunspot cycle went largely unnoticed for several years. In fact, it was not until about 1851 when Friedrich Wilhelm Heinrich Alexander von Humboldt mentioned it in his *Kosmos* (vol. 3), a treatise on science and nature, that the existence of the sunspot cycle was able to finally draw more widespread attention.[5]

It should be noted, however, that about 1847 Johan Rudolf Wolf, a professional astronomer and Director of the Berne Observatory, had become aware of Schwabe's work on sunspots and in 1849 he began his own systematic recording of sunspot counts based upon the use of his 'relative sunspot number,' an index he had introduced in 1848.[6–8] Wolf determined his relative sunspot number from the observed number of sunspot groups (G, essentially the same as Schwabe's clusters of spots) and the total number of individual spots (N) within all groups, computing it as $R = 10G + N$, where R represents the relative sunspot number. Whereas Schwabe initially published his daily findings as annual values once per year, Wolf instead published his daily findings as monthly values twice per year (between 1849 and 1855). He also recorded the quality of sky conditions (regarding cloudiness) and which of the two telescopes (small or large) he had used in the determination of his relative sunspot number. As to why the number 10 is used instead of some other number,[8] Wolf said, "I chose the number 10, on the one hand, because it seemed to work well for a large number of instances which I studied for this purpose, and, on the other hand, because it was simpler to deal with than any other number close to it." (Wolf apparently had preferred to measure the area of sunspots[7] for describing the sunspot cycle rather than determining his relative sunspot number, but because of his lack of the necessary instrumentation needed to perform such detailed measurements, his relative sunspot number, by default, became the index of choice regarding the description of the sunspot cycle.)

Today, the relative sunspot number remains the main descriptor of the sunspot cycle, primarily because of its continued use for the past 165 years, although one cannot deny the arbitrariness of the method of using G and N (rather than sunspot area). However, as noted by Clette et al.,[9] the historical record of sunspot number actually is a blend of various sunspot indices of different intrinsic nature and accuracy, with systematic observations extending only from 1848 to the present: Wolf sunspot numbers spanning the interval 1848–1882, Zürich sunspot numbers spanning the interval 1882–1980, and Solar Influences Data Analysis Center (SIDC) sunspot numbers (also referred to as International sunspot number) spanning the interval 1981–present. Consequently, relative sunspot number is now calculated as $R = k(10G+N)$, where k is a personal reduction coefficient calculated for each individual observer to bring observations into the semblance of agreement. For the Wolf timespan, $k=1$; for the Zürich timespan, $k=0.6$ (changed to account for the inclusion of smaller spots and weighted according to size and structure of the umbra); and for the SIDC timespan, k is variable, averaging about 0.9. Other sunspot indices[10] include the Boulder sunspot number, having $k=1$, and the American sunspot number, having a variable k that averages about 0.7. The Boulder number typically measures about 25% larger than the International number. More recently, Hoyt and Schatten have introduced the Group sunspot number, which essentially mimics sunspot numbers from 1882 onwards, but which can depart by as much as 25% from Wolf's reconstructed sunspot numbers for the interval prior to 1882 (i.e., the Wolf regime and historical reconstructed interval, back to 1610).[11–15]

The measurement of sunspot areas did not become routinely available until 1874 and later, due to the efforts of the Royal Greenwich Observatory (RGO).[6,16,17] Beginning in May 1874, the RGO began cataloging daily sunspot positions and areas, based on systematic observations at Greenwich, England, Cape Town, South Africa, and Kodaikanal, India. The data were obtained directly from photographs by means of reticules divided into small squares and measured relative to the Sun's center. Unfortunately, the RGO dataset ended in 1976. (From the RGO data, one determines both the number of sunspot groups and the corrected sunspot area, corrected for foreshortening.)

Continuation of the sunspot area dataset beyond 1976 is accomplished today through the use of the worldwide United States Air Force (USAF)/National Oceanic and Atmospheric Administration (NOAA) Solar Optical Observing Network (SOON). Sunspot areas from the USAF/NOAA/SOON system are visual determinations rather than photographic determinations. Studies[16,18] have indicated that the visual determinations of sunspot area *underestimate* the area as determined from photographs by about 40%. Hence, one must multiply the observed visually determined sunspot area by the factor 1.4.

This study examines the variations of sunspot number (SSN), the number of sunspot groups (G), and the corrected sunspot area (SSA) for the interval 1875–2013, with the primary intention being the determination of the formulation for an expression that relates sunspot number to both the combined number of sunspot groups and sunspot area. This study also examines (1) the ratios SSN/G, SSA/G, and SSA/SSN for the interval 1875–2013; (2) sunspot cycle (SC) 24's SSN, G, SSA, number of spotless days (NSD), <SSN/G>, <SSA/G>, and <SSA/SSN> in comparison with the means and standard deviations (*sd*) of these parameters for elapsed times (t) –3 to 10 years from SSN minimum amplitude occurrence; (3) the variation of cyclic averages of <SSN>, <G>, <SSA>, <SSN/G>, <SSA/G>, and <SSA/SSN> for SC12–SC23; and (4) the comparison of parametric means based on the first 6 years of SC14 and SC24.

2. RESULTS

Figure 1 displays the annual values of (a) SSN, (b) G, and (c) SSA (in millionths of a solar hemisphere) for the interval 1875–2013. Figure 1(a) also delineates the SC number, spanning the last 3 years of SC11 through the first 6 years of SC24, the current ongoing SC. Sunspot cycles are reckoned from cycle minimum to next cycle minimum (i.e., from SSN minimum amplitude of a cycle to the next cycle's SSN minimum amplitude, or using NSD, from NSD maximum amplitude of a cycle to the next cycle's NSD maximum amplitude), which corresponds to its period (PER) or cycle length that has always been 10–12 years during the modern era of sunspot observations.[19–25] For the modern era, sunspot cycles are found to have ascent (ASC) lengths (i.e., the elapsed time from a cycle's SSN minimum amplitude to its SSN maximum amplitude) of 3–5 years.[26] On average, during the interval 1875–2013, SSN measures 54.8 (±45), G measures 4.74 (±3.69), and SSA measures 838.8 (±745) millionths of a solar hemisphere, where the values within the parentheses are the parametric standard deviations (*sd*). (SSN is available online at http://sidc.oma.be/index.php3 and SSA and G are available online at http://solarscience.msfc.nasa.gov/greenwch.shtml.)

Table 1 gives the minimum and maximum parametric values and the dates of their occurrence for each of SC12–SC24, as well as each cycle's ASC and PER (based on SSN). Plainly, the weakest cycle to date (i.e., smallest SSNmax) is SC14, having SSNmax = 63.5, and the strongest cycle to date (i.e., largest SSNmax) is SC19, having SSNmax = 190.2. In terms of SSNmax, SC24 appears to be slightly stronger than either SC12 (SSNmax = 63.7) or 14, based on its preliminary SSN value for 2013 (64.9), inferring an ASC = 5 years. SC24 is expected to persist through at least 2017 and possibly through 2019. The reader should notice that, while all Gmin occurrence dates are concurrent with the occurrence dates of SSNmin, only 8 of 12 cycles have had concurrent dates for SSNmax and Gmax, the exceptions include SC12, SC14, SC20, and SC22. Thus, there is the possibility that Gmax for SC24 has not yet been seen and might occur 1–2 years after the occurrence of SSNmax (presumed to be 2013). For SSAmin, only SC21 has an occurrence date different from the occurrence of SSNmin. For SSAmax, all cycles except SC20, SC21, and SC23 have occurrence dates concurrent with the occurrence dates of SSNmax. For NSDmax, all dates of occurrence are concurrent with the occurrence dates of SSNmin. In terms of SSAmax, SC24 is the *smallest* cycle to date, having SSAmax = 860.8 millionths of a solar hemisphere in 2013, smaller than either SC12 (SSAmax = 1,148.9) or SC14 (SSAmax = 1,195.9). Perhaps, this might be an indication that SSAmax (and possibly Gmax) has not yet occurred for SC24 (i.e., it might occur in 2014 or later).

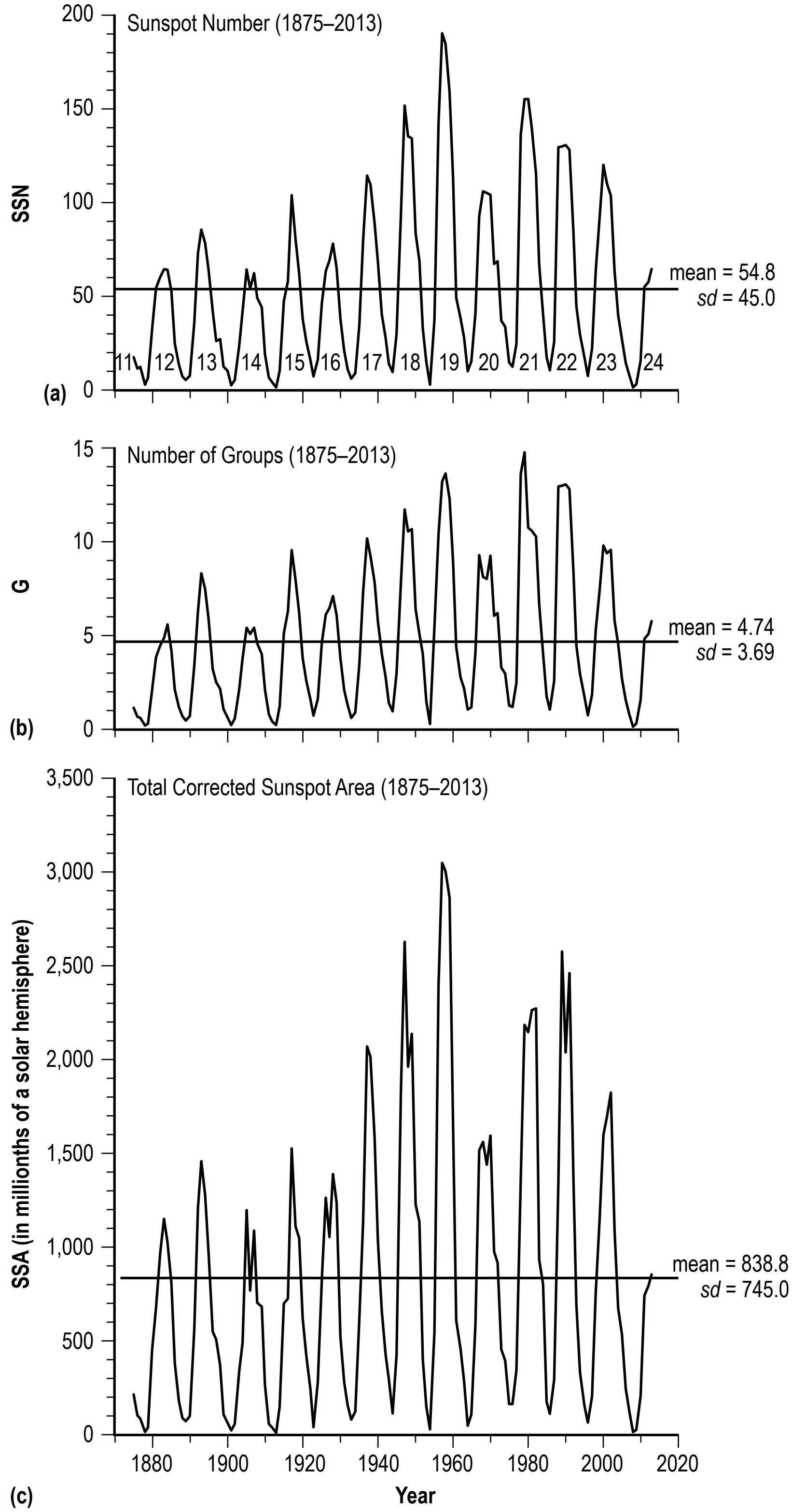

Figure 1. Annual variation of (a) SSN, (b) G, and (c) SSA for the interval 1875–2013.

Table 1. Minimum and maximum parametric values and epochs of occurrence, SC12–SC24.

Cycle	SSNmin (yr)	SSNmax (yr)	ASC	PER	Gmin (yr)	Gmax (yr)	SSAmin (yr)	SSAmax (yr)	NSDmax (yr)
12	3.4 (1878)	63.7 (1883)	5	11	0.22 (1878)	5.57 (1884)	22.2 (1878)	1,148.9 (1883)	280 (1878)
13	6.3 (1889)	85.1 (1893)	4	12	0.52 (1889)	8.42 (1893)	76.7 (1889)	1,460.6 (1893)	212 (1889)
14	2.7 (1901)	63.5 (1905)	4	12	0.22 (1901)	5.35 (1907)	27.9 (1901)	1,195.9 (1905)	287 (1901)
15	1.4 (1913)	103.9 (1917)	4	10	0.17 (1913)	9.62 (1917)	7.5 (1913)	1,533.9 (1917)	311 (1913)
16	5.8 (1923)	77.8 (1928)	5	10	0.67 (1923)	7.14 (1928)	54.7 (1923)	1,388.9 (1928)	200 (1923)
17	5.7 (1933)	114.4 (1937)	4	11	0.62 (1933)	10.15 (1937)	91.3 (1933)	2,072.8 (1937)	240 (1933)
18	9.6 (1944)	151.6 (1947)	3	10	1.01 (1944)	11.78 (1947)	124.7 (1944)	2,634.1 (1947)	159 (1944)
19	4.4 (1954)	190.2 (1957)	3	10	0.45 (1954)	13.74 (1957)	34.6 (1954)	3,048.5 (1957)	241 1954)
20	10.2 (1964)	105. 9 (1968)	4	12	1.07 (1964)	9.29 (1967)	53.9 (1964)	1,601.3 (1970)	112 (1964)
21	12.6 (1976)	155.4 (1979)	3	10	1.16 (1976)	14.90 (1979)	166.4 (1975)	2,283.3 (1982)	105 (1976)
22	13.4 (1986)	157.6 (1989)	3	10	1.10 (1986)	13.06 (1990)	124.7 (1986)	2,579.2 (1989)	129 (1986)
23	8.6 (1996)	119.6 (2000)	4	12	0.83 (1996)	9.81 (2000)	81.9 (1996)	1,828.7 (2002)	165 (1996)
24	2.9 (2008)	64.9 (2013)?	5?	–	0.33 (2008)	5.80 (2013)?	22.8 (2008)	860.8 (2013)?	265 (2008)

Figure 2 compares SC24 against the SC12–SC23 parametric averages of (a) <SSN>, (b) <G>, (c) <SSA>, and (d) <NSD> for $t = -3$ to 10 years relative to SSNmin occurrence. The vertical bars represent the mean ±1 *sd* interval for each t. Relative to the means, SC24 is running slightly more than 1 *sd* lower than the means of <SSN>, <G>, and <SSA>. For NSD (at $t = 0$) SC24 is about 62 days longer than the mean of SC12–SC23; however, as shown in table 1, NSD (at $t = 0$) was longer for SC12, SC14, and SC15 than it was for SC24. From figure 2, one expects SC24 to begin decreasing in amplitudes (i.e., SSN, G, and SSA), probably beginning in 2014, with the next cycle minimum expected probably about 2018–2020. The first spotless day post maximum amplitude (i.e., SSNmax) can occur any time, especially, at $t \geq 6$ years, heralding the demise of the current ongoing cycle and anticipating the onset of the next cycle.

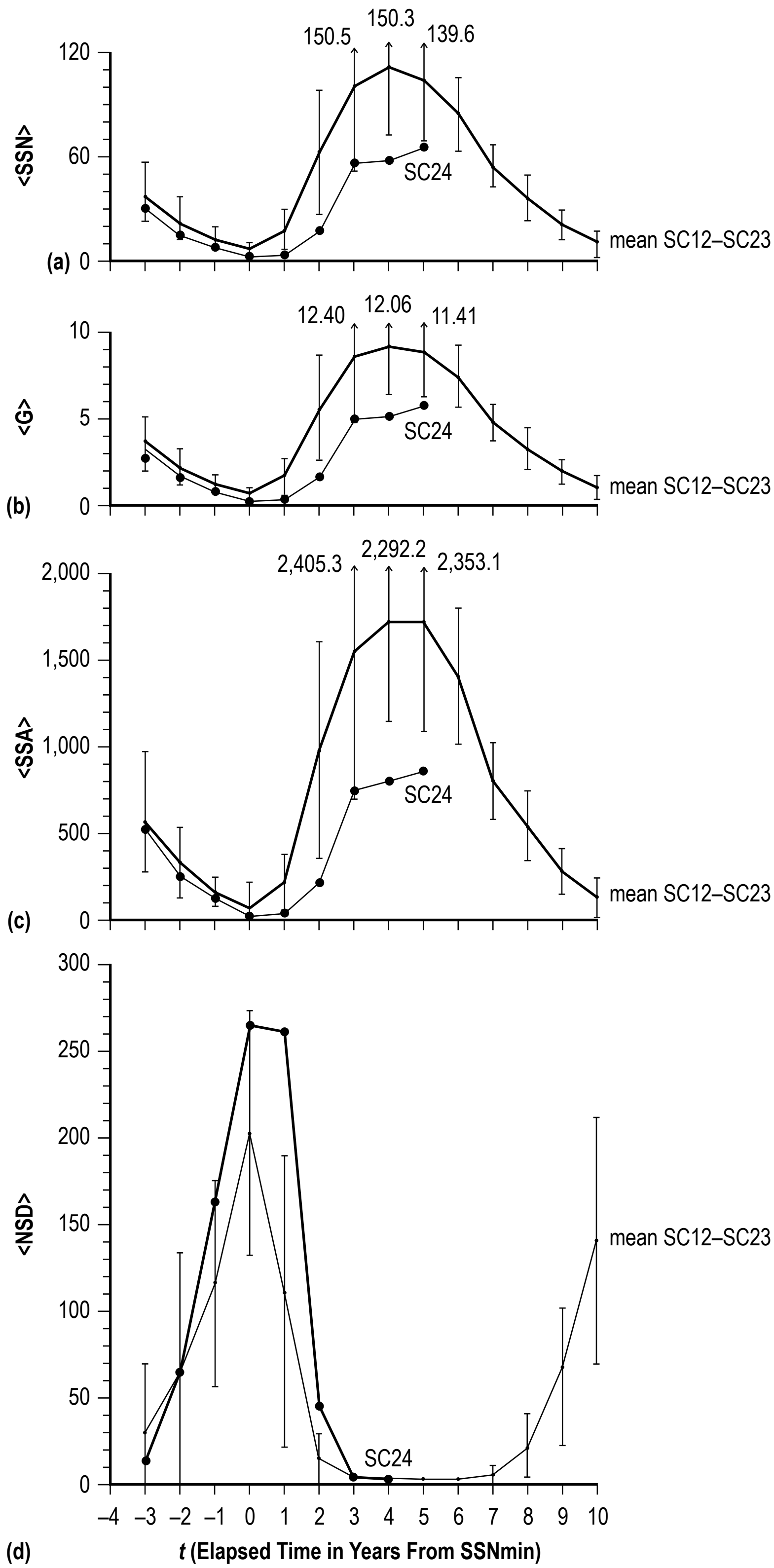

Figure 2. SC24 values of (a) SSN, (b) G, (c) SSA, and (d) NSD in relation to the mean of cycles 12–23 for elapsed time $t = –3$ to 10 years from SSNmin.

Figure 3 displays the ratios (a) SSN/G, (b) SSA/G, and (c) SSA/SSN for the interval 1875–2013. The means of the ratios are 11.38 (±1.83), 158.6 (±40.3), and 14.01 (±3.19), respectively. For SSN/G, the highest ratios occur prior to 1882 (the year when the *k* factor was changed from 1 to 0.6), suggesting perhaps that the SSN values during the early interval 1875–1881 might be slightly too high (≤5 units of SSN). If true, then the early ratios of SSA/SSN could be slightly too low.

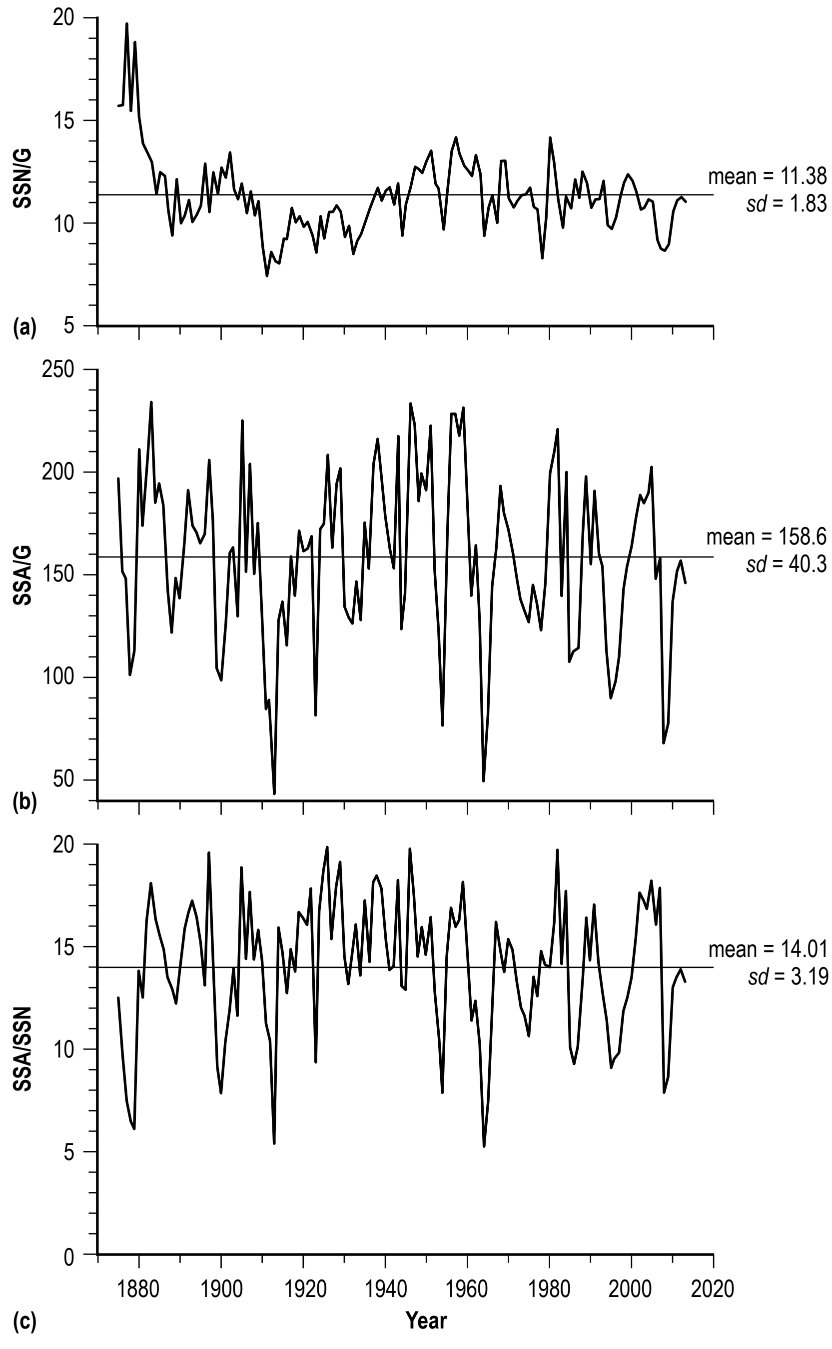

Figure 3. Annual variation of the ratios (a) SSN/G, (b) SSA/G, and (c) SSA/SSWN for the interval 1875–2013.

Close inspection of figure 3 suggests that the ratios might not be randomly distributed. As an example, for the interval 1875–2013, the median value for SSN/G is 11.27, with the number of values greater than or equal to the median being 71 and the number below the median being 68 and with 23 larger-than-median runs. Runs testing[27] yields the normal deviate for the sample results as $z = 4.14$, and by hypothesis testing, one surmises that there is >99.9% probability that SSN/G varies nonrandomly. Hence, simple relationships, such as SSN/G = 11.38 (±1.83), SSA/G = 158.6 (±40.3), or SSA/SSN = 14.01 (±3.19), while approximately true, are inadequate for calculating SSN directly from G or SSA. (In many texts, the oft stated relation between sunspot area and sunspot number is A = 16.7 R, where A refers to the area of sunspots and R refers to sunspot number.)

Figure 4 depicts variations of the ratios (a) <SSN/G>, (b) <SSA/G>, and (c) <SSA/SSN for $t = -3$ to 10 years relative to SSNmin occurrence (a la fig. 2). Clearly, the ratios vary over the SC with a minimum near cycle minimum ($t = 0$) and a maximum near cycle maximum ($t = 3$–6). While SC24 has SSN/G ratio values within the mean ±1*sd* interval, for SSA/G and SSA/SSN SC24 ratios appear to now be running outside-low the mean ±1 *sd* intervals. Figure 4 suggests that the ratios possibly have all peaked at $t = 4$ (the year 2012) and are now destined to decline with the passage of time. (In contrast, individual parameters SSN, G, and SSA possibly have peaked at $t = 5$, the year 2013, as shown in fig. 2.)

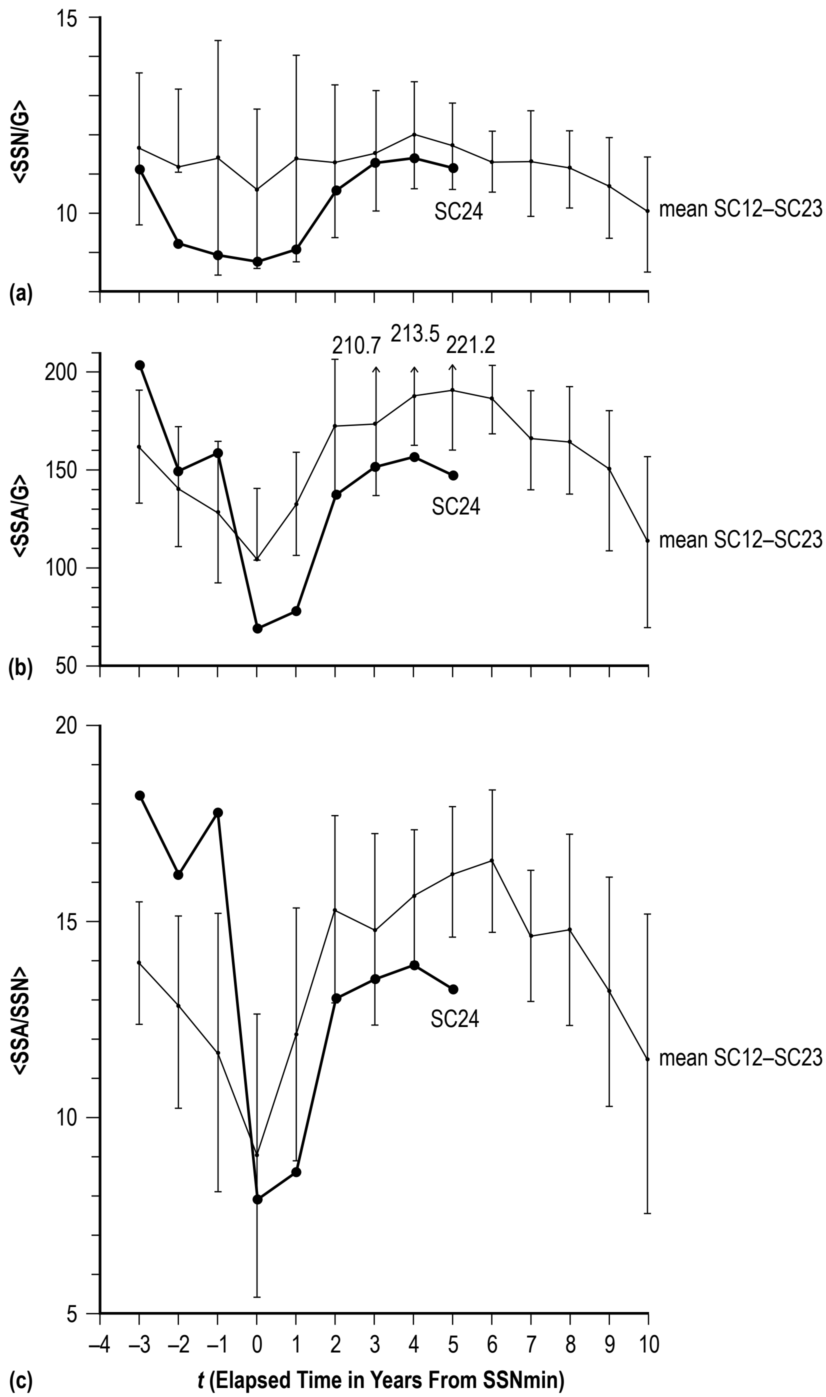

Figure 4. SC24 values of the ratios (a) SSN/G, (b) SSA/G, and (c) SSA/SSN in relation to the mean of cycles 12–23 for elapsed time $t = -3$ to 10 years from SSNmin.

Figure 5 displays cyclic averages of (a) <SSN>, (b) <G>, (c) <SSA>, (d) <SSN/G>, (e) <SSA/G>, and (f) <SSA/SSN> for SC12–SC23. From the preceding, it is apparent that SC24 will have lower values than those of SC23, indicating a continuing downward progression in the parametric values with time.

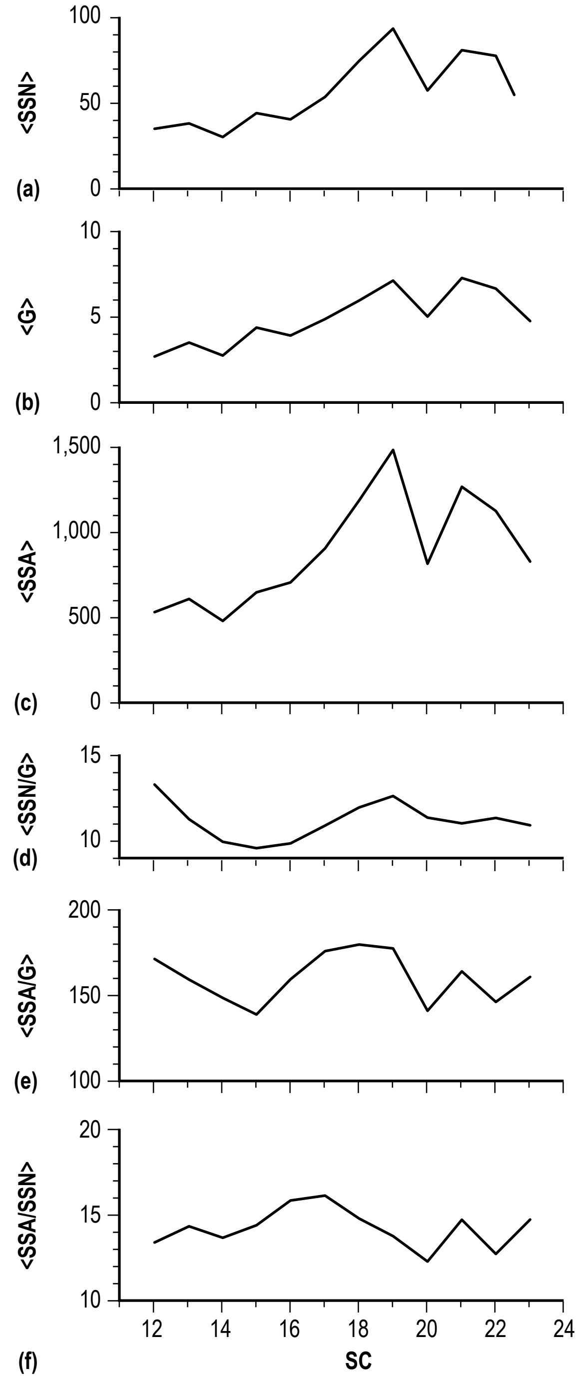

Figure 5. SC-mean values of (a) SSN, (b) G, (c) SSA, (d) SSN/G, (e) SSA/G, and (f) SSA/SSN for SC12–SC23.

Figure 6 compares SC24 and SC14 in terms of (a) SSN, (b) SSA, (c) G, (d) SSN/G, (e) SSA/SSN, and (f) SSA/G. For SSN, SSA, and G the values are very close for both cycles. However, for the ratios, there are obvious differences. For example, while values of SSN/G appear to be trending downwards throughout SC14, for SC24 they appear to be more cyclic in appearance (i.e., rising and flattening). For the other ratios, SC14 tends to be the larger cycle and the ratios are more cyclic in appearance (for both SC14 and SC24).

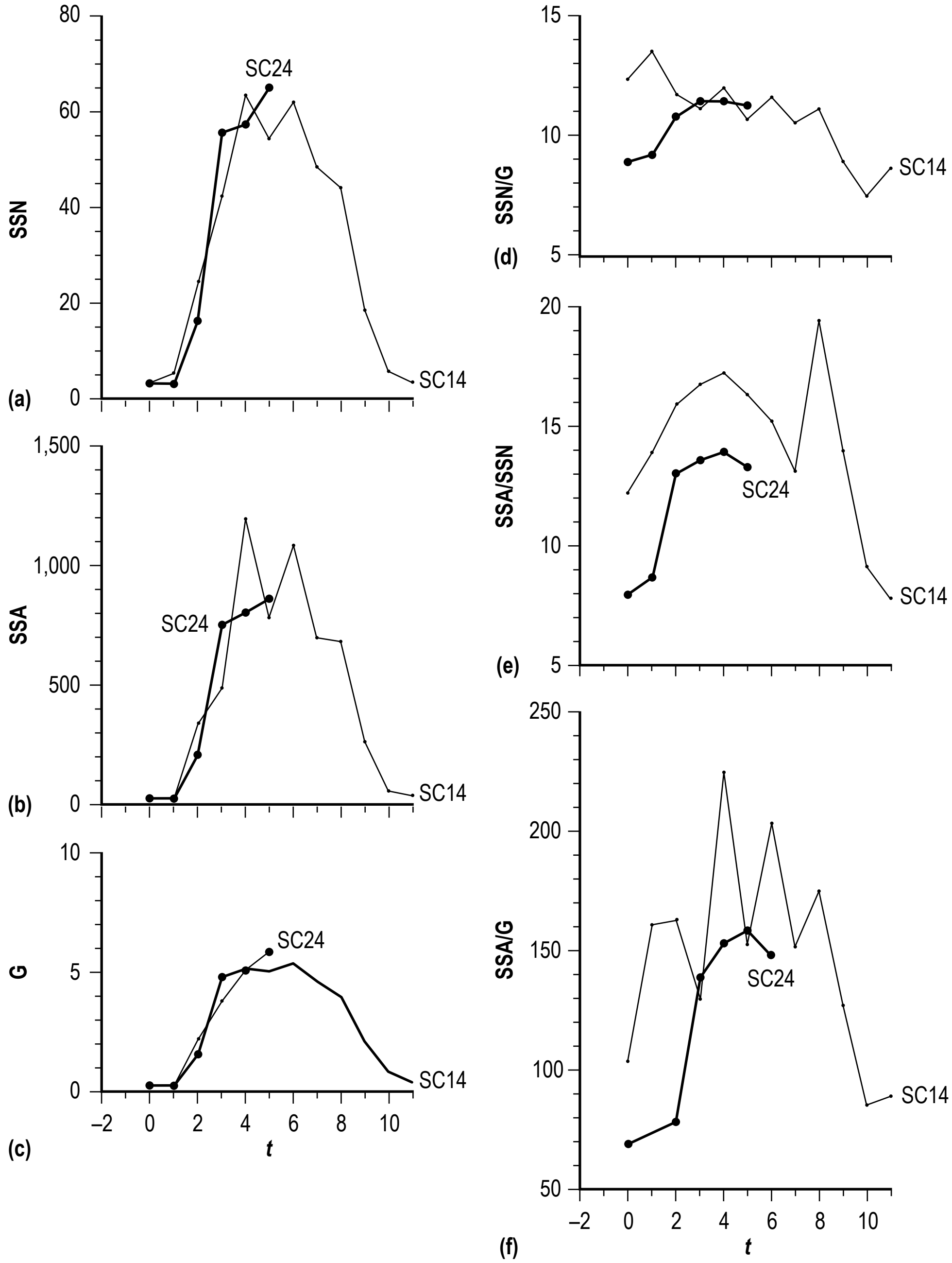

Figure 6. SC24 values of (a) SSN, (b) G, (c) SSA, (d) SSN/G, (e) SSA/G, and (f) SSA/SSN in relation to SC-mean values (SC12–SC23) for elapsed time $t = 0$–10 years.

Table 2 compares the parametric means for SC14 and SC24 during the first 6 years of each cycle. The only parameter that has a possible significant difference, on the basis of the *t* statistic for independent samples,[28] is the one for SSN/G, which has $t = –2.3$, inferring that the difference in the means for the two cycles is statistically important at the 5% level of significance.

Table 2. Results of statistical testing of the parametric means based on the first 6 years of SC14 and SC24 using the *t* statistic for independent samples.

Parameter	SC14	SC24	*t*
SSN	31.9 (25.3)	33.4 (29)	–0.10
SSA	480.9 (447.4)	445.2 (399)	0.15
G	2.81 (2.26)	3.00 (2.53)	–0.14
SSN/G	11.87 (1.01)	10.42 (1.17)	2.30
SSA/G	159.6 (35.5)	124.1 (39.7)	1.63
SSA/SSN	13.50 (3.02)	11.67 (2.7)	–1.11

Figure 7 shows the scatter plots of (a) SSN versus G and (b) SSA versus G. For SSN versus G, the inferred preferential linear regression is $y = –2.019 + 11.993x$, where y is SSN and x is G. The inferred regression has a coefficient of linear regression $r = 0.984$, a coefficient of determination (a measure of the amount of variance explained by the regression) $r^2 = 0.968$, a standard error of estimate $se = 8.049$, and a confidence level cl » 99.9%. For SSA versus G, the inferred preferential linear regression is $y = –90.988 + 196.099x$, having $r = 0.971$, $r^2 = 0.943$, $se = 178.119$, and cl» 99.9%. As an example, given $G = 10$, one expects $SSN = 117.9 \pm 8$ and $SSA = 1{,}870 \pm 178.1$, these being the ± 1 *se* prediction intervals.

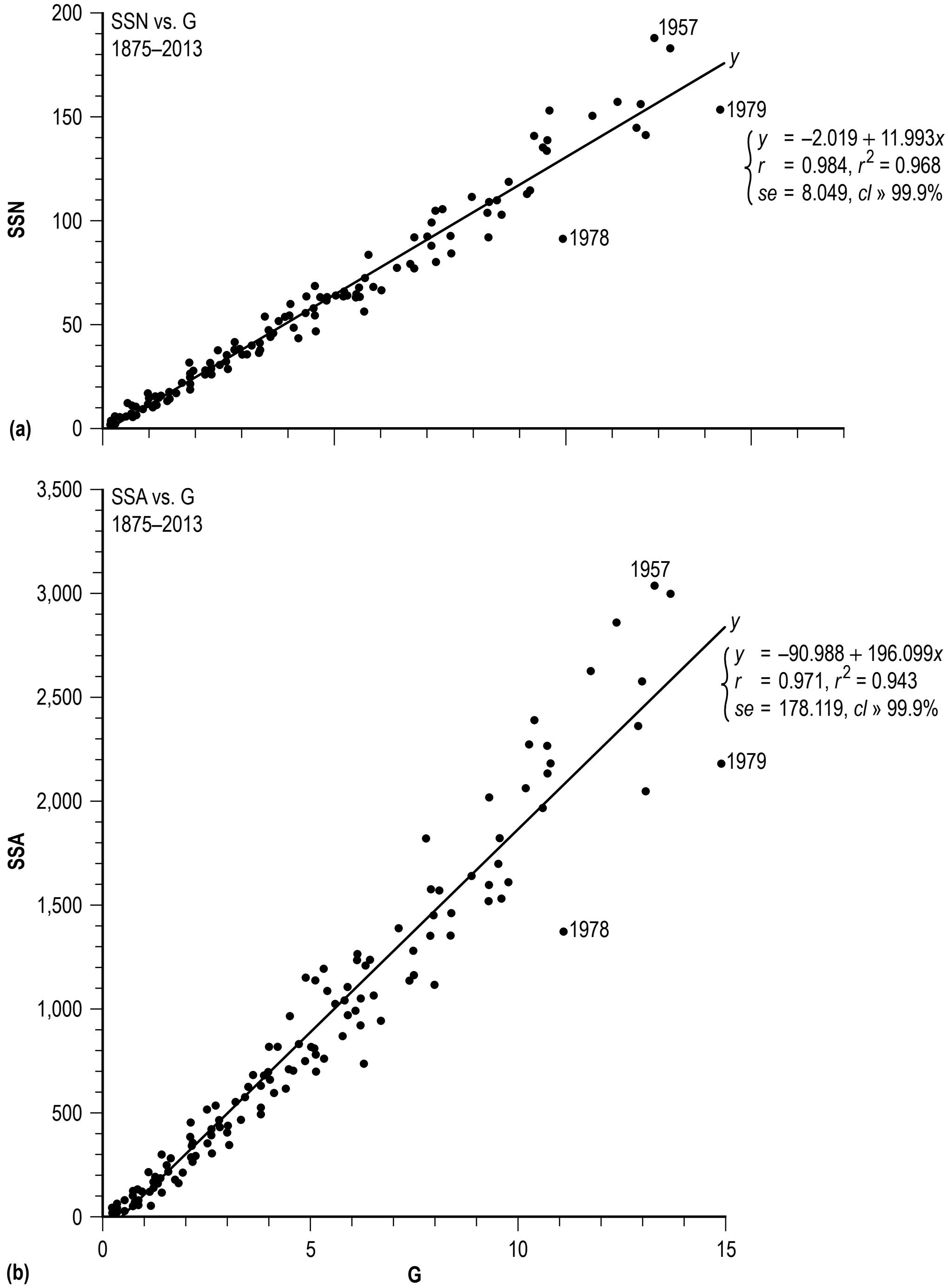

Figure 7. Scatter plot of (a) SSN versus G and (b) SSA versus G for the interval 1875–2013.

In figure 7, the years 1957, 1978, and 1979 are identified because of their large deviations from the regression line. The year 1957 represents the highest observed annual SSN to date (= 190.2, associated with cycle maximum for SC19). Its observed value is 4.06 *se* higher than what one would have predicted for SSN given the observed G (=13.3). Similarly, the years 1978 and 1979 (associated with SC21) have large deviations from the regression line, but now in the opposite sense (i.e., their predicted SSNs are too high). For 1978, given G = 11.05, one would have expected SSN = 130.5; however, the observed SSN measured only 92.5, a difference of –4.7 *se*. Similarly, for 1979, given G = 14.9, one would have expected SSN = 176.7; the observed SSN instead measured only 155.4, a difference of –2.6 *se*.

Figure 8 displays the scatter plot of SSN versus SSA. The inferred preferential linear regression is $y = 4.959 + 0.059x$, having $r = 0.985$, $r^2 = 0.971$, $se = 9.878$, and $cl >> 99.9\%$. Three years are also identified in it as having large deviations: 1946, 1979, and 1982. For 1946 (SC18), given SSA = 1,823.9, one would have expected SSN = 112.6; instead, the observed SSN measured 92.6, a difference of –2 *se*. For 1979, given SSA = 2,194.5, one would have expected SSN = 134.4; the observed SSN measured 155.4, a difference of 2.1 *se*. For 1982, given SSA = 2,283.3, one would have expected SSN = 139.7; the observed SSN measured 115.9, a difference of –2.4 *se*.

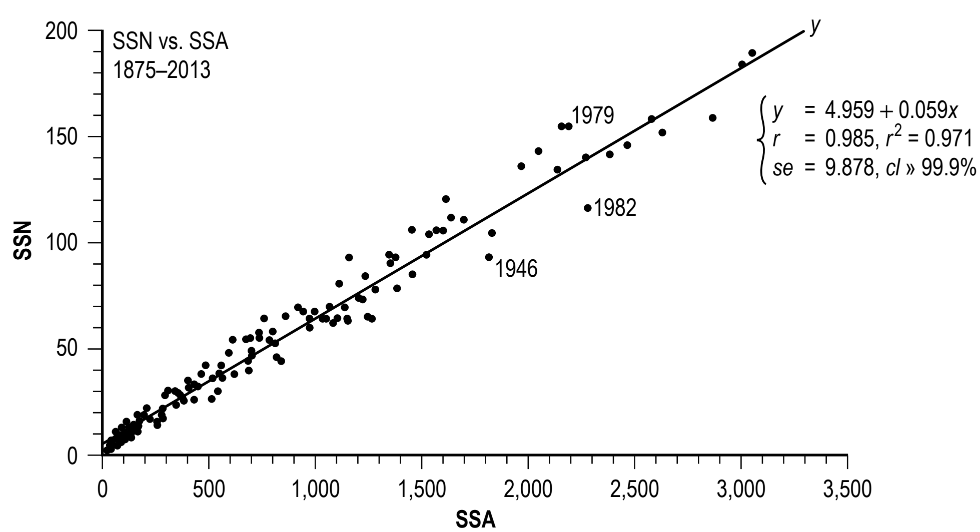

Figure 8. Scatter plot of SSN versus SSA for the interval 1875–2013.

Figure 9 plots the observed SSN versus the bivariate fit of SSN (parameter y) against both G (parameter 1) and SSA (parameter 2). The bivariate equation is SSN (predicted) = 0.973 + 5.877G + 0.031SSA, having $R_{y12} = 0.990$ and $S_{y12} = 6.4$, an improvement over using either G or SSA alone.[29,30]

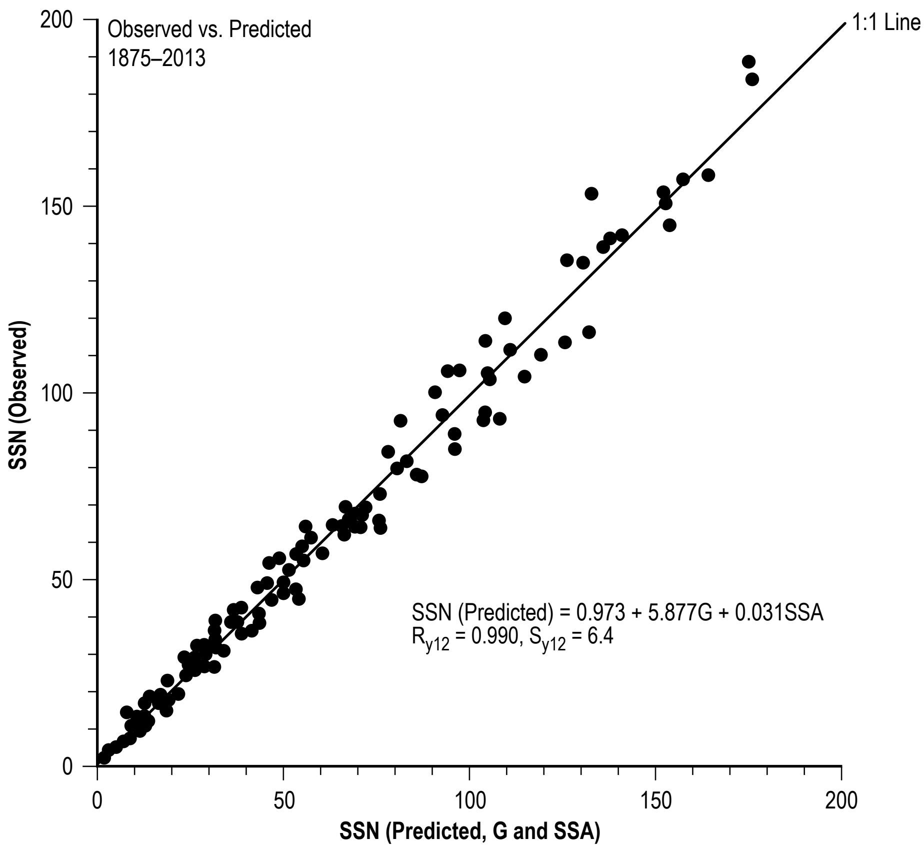

Figure 9. Scatter plot of SSN (observed) versus SSN (predicted, G and SSA) for the interval 1875–2013.

Figure 10 plots the difference (observed – predicted) for SSN based on using the bivariate equation for estimating SSN. Large discrepancies are still apparent, especially for the interval 1978–1982 associated with SC21 (which is also during the changeover interval from Zürich SSN to SIDC SSN). The only years having discrepancies >±2 *se* (i.e., ≥±16.8) include 1957 (too high), 1978 (too low), 1980 (too high), and 1982 (too low), with the worst difference being the one for 1980 (observed – predicted = 23, a difference of 3.6 *se*). For SC24, all differences have measured –0.7 to 3.2 (or ≤±0.5 *se*).

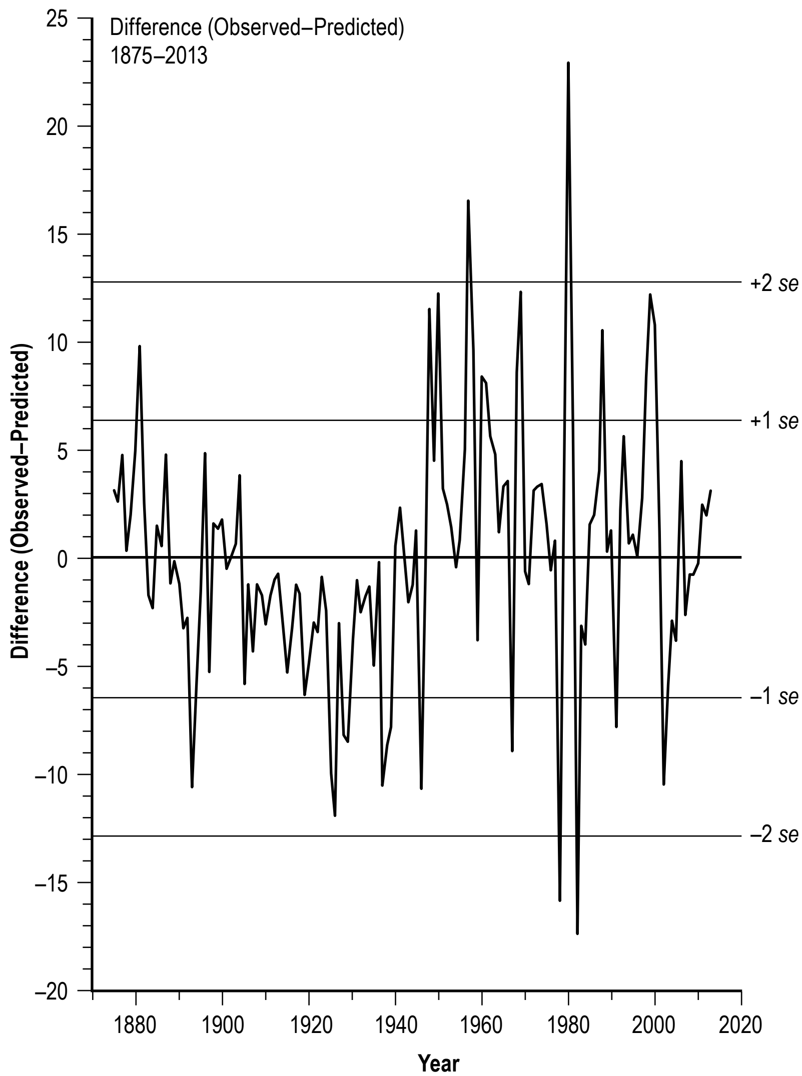

Figure 10. Annual variation of the difference (observed – predicted) for the interval 1875–2013.

Interestingly, it appears that the differences were trending downwards from positive values to negative values prior to about 1930, but were trending upwards towards positive values thereafter until about 1960. While there are about equal numbers of positive and negative differences, runs testing suggests that the overall behavior of the differences is nonrandom in nature. About 80% of the differences lie within the ±1 *se* band (i.e., the difference is with ±1 *se*, or within 6.4 units of sunspot number). Table 3 provides a direct accounting of the observed and predicted (based on G and SSA) values and the difference (observed – predicted).

Table 3. Comparison of observed and predicted SSN, where predicted SSN is the bivariate fit of SSN based on G and SSA.

Year	Observed	Predicted (G,SSA)	Difference	Year	Observed	Predicted (G,SSA)	Difference	Year	Observed	Predicted (G,SSA)	Difference
1875	17.0	13.9	3.1	**Cycle 15**				1950	83.9	76.6	12.3
1876	11.3	8.6	2.7	1913	1.4m	2.2m	–0.7	1951	69.4	66.1	3.3
1877	12.4	7.6	4.8	1914	9.6	12.7	–3.1	1952	31.5	28.9	2.6
Cycle 12				1915	47.4	52.6	–5.2	1953	13.9	12.4	1.5
1878	3.4m	3.0m	0.4	1916	57.1	60.3	–3.2	**Cycle 19**			
1879	6.0	4.0	2.0	1917	103.9M	105.1M	–1.2	1954	4.4m	4.7m	–0.3
1880	32.3	27.3	5.0	1918	80.6	82.2	–1.6	1955	38.0	37.1	0.9
1881	54.3	45.5	9.8	1919	63.6	69.9	–6.3	1956	141.7	136.6	5.1
1882	59.7	57.1	2.6	1920	37.6	42.5	–4.9	1957	190.2M	173.6	16.6
1883	63.7M	65.4	–1.7	1921	26.1	29.1	–3.0	1958	184.8	175.1M	9.7
1884	63.5	65.8M	–2.3	1922	14.2	17.5	–3.3	1959	159.0	162.7	–3.7
1885	52.2	50.7	1.5	**Cycle 16**				1960	112.6	104.2	8.4
1886	25.4	24.8	0.6	1923	5.8m	6.6m	–0.8	1961	53.9	45.6	8.3
1887	13.1	8.3	4.8	1924	16.7	19.1	–2.4	1962	37.6	31.9	5.7
1888	6.8	7.9	–1.1	1925	44.3	54.2	–9.9	1963	27.9	23.1	4.8
Cycle 13				1926	63.9	75.7	–11.8	**Cycle 20**			
1889	6.3m	6.4m	–0.1	1927	69.0	72.0	–3.0	1964	10.2m	8.9m	1.3
1890	7.1	8.2	–1.1	1928	77.8M	86.0M	–8.2	1965	15.1	11.7	3.4
1891	35.6	38.7	–3.1	1929	64.9	75.3	–8.4	1966	47.0	43.4	3.6
1892	73.0	75.8	–2.8	1930	35.7	39.4	–3.7	1967	93.8	102.7	–8.9
1893	85.1M	95.7M	–10.6	1931	21.2	22.2	–1.0	1968	105.9M	97.2	8.7
1894	78.0	84.8	–6.8	1932	11.1	13.6	–2.5	1969	105.5	93.1	12.4
1895	64.0	65.5	–1.5	**Cycle 17**				1970	104.5	105.0M	–0.5
1896	41.8	36.9	4.9	1933	5.7m	7.4m	– 1.7	1971	66.6	67.6	–1.1
1897	26.2	31.4	–5.2	1934	8.7	10.0	–1.3	1972	68.9	65.7	3.2
1898	26.7	25.1	1.6	1935	36.1	41.0	–4.9	1973	38.0	34.6	3.4
1899	12.1	10.6	1.5	1936	79.7	79.8	–0.1	1974	34.5	31.0	3.5
1900	9.5	7.7	1.8	1937	114.4M	124.9M	–10.5	1975	15.5	13.8	1.7
Cycle 14				1938	109.6	118.2	–8.6	**Cycle 21**			
1901	2.7m	3.1m	–0.4	1939	88.8	96.4	–7.8	1976	12.6m	13.1m	–0.5
1902	5.0	5.0	–	1940	67.8	67.2	0.6	1977	27.5	26.7	0.8
1903	24.4	23.7	0.7	1941	47.5	45.1	2.4	1978	92.5	108.3	–15.8
1904	42.0	38.1	3.9	1942	30.6	30.6	–	1979	155.4M	151.3M	4.1
1905	63.5M	69.3M	–5.8	1943	16.3	18.2	–1.9	1980	154.6	131.6	23.0
1906	53.8	55.0	–1.2	**Cycle 18**				1981	140.4	134.5	5.9
1907	62.0	66.3	–4.3	1944	9.6m	10.8m	–1.2	1982	115.9	132.2	–16.3
1908	48.5	49.7	–1.2	1945	33.2	31.9	1.3	1983	66.6	69.7	–3.1
1909	43.9	45.6	–1.7	1946	92.6	103.2	–10.6	1984	45.9	49.8	–3.9
1910	18.6	21.6	–3.0	1947	151.6M	151.9M	–0.3	1985	17.9	16.3	1.6
1911	5.7	7.4	–1.7	1948	136.3	124.7	11.6				
1912	3.6	4.6	–1.0	1949	134.7	130.1	4.6				

Table 3. Comparison of observed and predicted SSN, where predicted SSN is the bivariate fit of SSN based on G and SSA (Continued).

Year	Observed	Predicted (G,SSA)	Difference	Year	Observed	Predicted (G,SSA)	Difference	Year	Observed	Predicted (G,SSA)	Difference
Cycle 22				Cycle 23				Cycle 24			
1986	13.4m	11.3m	2.1	1996	8.6m	8.4m	0.2	2008	2.9m	3.6m	–0.7
1987	29.4	25.3	4.1	1997	21.5	18.6	2.9	2009	3.1	3.8	–0.7
1988	100.2	89.6	10.6	1998	64.3	55.8	8.5	2010	16.5	16.7	–0.2
1989	157.6M	157.2M	0.4	1999	93.3	81.0	12.3	2011	55.6	53.1	2.5
1990	142.6	141.2	1.4	2000	119.6M	108.7	10.9	2012	57.5	55.4	2.1
1991	145.7	153.4	–7.7	2001	111.0	109.7	1.3	2013	64.9M	61.7M	3.2
1992	94.3	91.9	2.4	2002	104.0	114.3M	–10.4				
1993	54.6	48.9	5.7	2003	63.7	69.5	–5.8				
1994	29.9	29.1	0.8	2004	40.4	43.2	–2.8				
1995	17.5	16.3	1.2	2005	29.8	33.5	–3.7				
				2006	15.2	10.6	4.6				
				2007	7.5	10.0	–2.5				

3. DISCUSSION AND SUMMARY

The oldest routinely observed solar index (1826–1868) for measuring activity on the Sun is Schwabe's 'clusters of spots' (essentially, G) and the 'number of days when no spots were observed' (or NSD).[1] Schwabe's simple method for monitoring the spottiness on the Sun was later superseded by the use of Wolf's relative sunspot number,[6] an index that he introduced in 1848 (based on G and N) and used to reconstruct the sunspot record back to 1610, when Galileo first reported seeing spots on the Sun. About 1882, however, a change[7] was made in the methodology for counting sunspots (i.e., the inclusion of smaller spots and the use of a weighting scheme) at the Swiss Federal Observatory such that a factor $k=0.6$ was introduced to bring measurements of sunspot number into agreement with Wolf's earlier values (which neglected smaller spots). Then, in 1981, the Royal Observatory of Belgium[9] (i.e., the SIDC) took over the task of determining sunspot number values from the Swiss Federal Observatory and continues to do so today, employing a variable k (equal to about 0.9, on average, being dependent upon the actual number of observers[10]).

Examined in this study has been the annual variations and ratios of SSN, G, and SSA for the overall interval 1875–2013, correlations of SSN against G and SSA, and the correlation of SSN (observed) against the bivariate fit based on the combined variations of G and SSA. Also, parametric variations during SC24 have been contrasted with the mean values based on SC12–SC23 and SC14, the smallest sunspot cycle of the modern era.

While SSN is found to independently correlate strongly (r = 0.98) against G ($y=-2+11.99x$) and SSA ($y=5+0.059x$), a stronger correlation ($R_{y12}=0.99$) is the one based on the bivariate fit of SSN against the combined variations of G and SSA ($y=1+5.88x_1+0.031x_2$). Also, based on the bivariate fit, while all cycle minima are found to occur concurrently with SSNmin, cycle maxima sometimes are found to occur at different times. For example, based on the inferred bivariate fit, the predicted SSNmax is inferred to have occurred in 1884, 1958, 1970, and 2002, respectively, for SC12, SC19, SC20, and SC23 rather than in 1883, 1957, 1968, and 2000, based on the observed SSNmax. Also, based on the bivariate fit, the observed SSNmax for SC19 seems too high. During the 139-year interval 1875–2013, the difference between the observed and predicted SSN based on the bivariate fit is <1 S_{y12} (<6.4) for 111 years, between 1 and <2 S_{y12} (6.4–<12.8) for 28 years and ≥2 S_{y12} (≥12.8) for only 4 years, these particular years being 1957 (16.6), 1978 (–15.8), 1980 (23), and 1982 (–16.3). For sunspot cycle 24 the difference between observed and predicted SSN has been –0.7 to 3.2 (≤0.5 S_{y12}).

In conclusion, had Wolf used SSA or G and SSA rather than G and N for describing solar activity, the record would look slightly different, especially as related to the amplitudes and occurrences of SSNmax for the years 1957, 1978 1980, and 1982.

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14. ABSTRACT
Examined are the yearly variations and ratios of sunspot number, the number of sunspot groups, and the total corrected sunspot area for the interval 1875–2013. While yearly sunspot number independently correlates strongly (r=0.98) with the yearly number of sunspot groups (y=–2+11.99x) and the total corrected sunspot area (y= 5+0.059x), the strongest correlation (R_{y12}=0.99) is the one based on the bivariate fit of sunspot number against the combined variations of the number of sunspot groups and sunspot area (y=1+5.88x_1+0.031x_2, where y refers to sunspot number, x_1 refers to the number of sunspot groups, and x_2 refers to the sunspot area). While all cycle minima based on the bivariate fit are concurrent with the observed minimum in sunspot number, cycle maxima are sometimes found to differ. For sunspot cycles 12, 19, 20, and 23, cycle maximum is inferred to have occurred in 1884, 1958, 1970, and 2002, respectively, rather than in 1883, 1957, 1968, and 2000, based on the observed sunspot number. Also, cycle 19's maximum amplitude based on observed sunspot number seems too high in comparison to that found using the bivariate fit. During the 139-year interval 1875–2013, the difference between the observed and predicted sunspot number based on the bivariate fit is <1 standard error of estimate (*se*) (<6.4) for 111 years, between 1 and <2 *se* (6.4 to <12.8) for 28 years, and ≥2 *se* (≥12.8) for only 4 years, these years being 1957 (16.6), 1978 (–15.8), 1980 (23), and 1982 (–16.3). For sunspot cycle 24, the difference between observed and predicted values has been only –0.7 and 3.2 (≤0.5 *se*).

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