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# Consensus-Based Formation Control of a Class of Multi-Agent Systems

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## Abstract

This paper presents a consensus-based formation control scheme for autonomous multi-agent systems represented by double integrator dynamics. Assuming that the information graph topology consists of an undirected connected graph, a leader-based consensus-type control law is presented and shown to provide asymptotic formation stability when subjected to piecewise constant formation velocity commands. It is also shown that global asymptotic stability is preserved in the presence of  $(0, \infty)$ -sector monotonic non-decreasing actuator nonlinearities.

## 1 Introduction

Cooperative control of autonomous multi-agent systems has been a subject of considerable research in recent years. Although a large volume of literature exists on this subject, only a few representative references are included in the list of references in this paper in the interest of brevity. In particular, a comprehensive literature review may be found in [1], while [2] and [3] contain an excellent introduction. In the context of aviation, large numbers of autonomous vehicles are expected to operate in the national air space in the future. Autonomous formation flight of multiple air vehicles has the potential for significantly increasing airspace utilization as well as fuel efficiency. In other application areas, autonomous multi-agent control strategies are being investigated for optimally performing common tasks. In an abstract sense, a “formation” may be generically considered to be an arrangement of networked agents. An optimal formation can be designed to maximize some measure of performance (such as system throughput in airspace or communication networks).

This paper investigates a consensus-based formation control scheme for multi-agent systems represented by double integrator agent dynamics, wherein each agent has the ability of bidirectional information exchange with a group of other agents (referred to as its ‘neighbors’), and at least one information path exists between every pair of agents, i.e., the information topology is represented by a connected undirected graph. (The reader is referred to [2] for detailed definitions) In Section 2, a double integrator multi-agent system is defined and a consensus-type formation control law is presented for a leader-follower architecture wherein the leader is required to follow a desired piecewise constant velocity command, and the followers are required to match the commanded velocity while maintaining the formation. To facilitate the analysis, 1-dimensional systems are considered initially. The results are readily extendable to higher dimensional motion (e.g., planar or 3-dimensional) using tools such as Kronecker product algebra, as presented in a latter section. Assuming that each follower agent has access only to the relative positions and velocities of its neighbors and the knowledge of the desired formation geometry, it is shown via Lyapunov approach that the formation is asymptotically stable, i.e., starting from any initial positions and velocities, the agents will asymptotically attain the desired formation velocity and the formation shape. In Section 3, actuator nonlinearities are considered and it is proved that the global asymptotic stability of the formation is preserved in the presence of  $(0, \infty)$ -sector monotonic non-decreasing actuator non-

linearities. Extension to the multi-dimensional motion case is addressed in Section 4. An example consisting of a 5-agent system is presented in Section 5, and Section 6 contains concluding remarks.

## 2 Consensus-Based Leader-Follower Type Control

In a system consisting of  $N$  agents, suppose the dynamics of the agents are described by

$$\ddot{\xi}_i = u_i, \quad i = 1, 2, \dots, N, \quad \xi_i, u_i \in \mathbb{R}^n \quad (1)$$

where  $\xi_i$  denotes the position of the  $i$ th agent and  $u_i$  is the input. (In multi-agent systems of this form, in general, the “position” is defined in an abstract sense (e.g., information state). For vehicles represented by point masses, the position is usually planar ( $n = 2$ ) or 3-dimensional ( $n = 3$ )). Suppose Agent 1 is designated as the leader, and agents  $2, \dots, N$  are the followers. In undirected graph topology, agents  $i$  and  $j$  are defined to be neighbors of each other if there exists an edge (which represents information exchange) between them. The formation geometry is described by the relative position set  $\mathcal{D}$

$$\mathcal{D} = \{d_{ij} := \xi_{iD} - \xi_{jD}\} \quad (2)$$

where  $\xi_{iD}, \xi_{jD}$  are the desired position states of agents  $i$  and  $j$  who are neighbors of each other.  $\mathcal{D}$  is assumed to be fixed (time-invariant). The objective is to get all the agents to converge to the pre-defined formation shape and the commanded velocity starting from any initial condition, and maintain the formation shape in the presence of any piecewise constant velocity commands.

The basic consensus protocol can be modeled using the degree matrix  $D$  and the adjacency matrix  $A$ . The degree matrix is an  $N \times N$  diagonal matrix with  $D_{ii} = \text{degree of node } i$ , i.e, the number of neighbors of the  $i$ th agent. The  $N \times N$  adjacency matrix denotes the connectivity between nodes (in the sense of information exchange), with  $A_{ij} = 1$  if  $j$  is a neighbor of  $i$ ; otherwise,  $A_{ij} = 0$  (for undirected graphs). For example, for the 5-agent system shown in Figure 1,

$$D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

The Laplacian matrix  $L$ , which plays a central role in the dynamics of the consensus protocol, is defined as

$$L = D - A \quad (4)$$

For the example system in Figure 1,

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \quad (5)$$

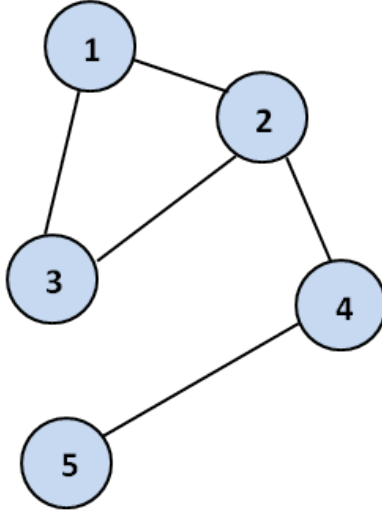


Figure 1. Multi-agent system

For undirected connected graphs, the Laplacian matrix  $L$  is symmetric positive semidefinite and has a single zero eigenvalue [2]. Also all the row sums are zero, i.e.,  $\mathbf{1} = [1, 1, \dots, 1]^T$  is an eigenvector corresponding to the zero eigenvalue.

The aggregated (summed) relative position and velocity for agent  $i$ ,  $y_{pi}$  and  $y_{ri}$ , are given by

$$y_{pi} = \sum_{j \in \mathcal{N}_i} (\xi_i - \xi_j), \quad y_{ri} = \sum_{j \in \mathcal{N}_i} (\dot{\xi}_i - \dot{\xi}_j) \quad (6)$$

where  $\mathcal{N}_i$  denotes the set of neighbors of agent  $i$ . For the example system in Figure 1,  $\mathcal{N}_1 = \{2, 3\}$ ,  $\mathcal{N}_2 = \{1, 3, 4\}$ , etc.

Suppose the leader is given a constant velocity command  $v_D$ , such that the leader's desired position satisfies

$$\dot{\xi}_{1D} = v_D. \quad (7)$$

The objective of the system is to form and maintain a fixed formation  $\mathcal{D}$  starting from any initial states of the agents for any given constant velocity command. (Note that the followers' desired velocity is also the same, i.e.,  $\dot{\xi}_{iD} = v_D \forall i \in [1, \dots, N]$ . Thus,  $v_D$  is the desired formation velocity, which is assumed to be piecewise constant (in time).

Consider the consensus control law for the follower agent  $i$  ( $i = 2, \dots, N$ ):

$$u_i = - \sum_{j \in \mathcal{N}_i} k_p (\xi_i - \xi_j) + k_r (\dot{\xi}_i - \dot{\xi}_j) - k_p (\xi_{iD} - \xi_{jD}) - k_r (\dot{\xi}_{iD} - \dot{\xi}_{jD}) \quad (8)$$

$$= -[k_p y_{pi} + k_r y_{ri}] + k_p \sum_{j \in \mathcal{N}_i} d_{ij}, \quad \text{for } i = 2, \dots, N \quad (9)$$

where  $k_p$   $k_r$  are positive scalars. (Note that the last term in Eq. (8) vanishes because  $\dot{\xi}_{iD} = v_D \forall i$ ).

The leader's control law is given by

$$u_1 = u_{tr} - \sum_{j \in \mathcal{N}_1} k_p(\xi_1 - \xi_j) + k_r(\dot{\xi}_1 - \dot{\xi}_j) - k_p d_{1j} \quad (10)$$

where  $u_{tr}$  is the tracking component of the leader's control law which is given by

$$u_{tr} = -[\kappa_p(\xi_1 - \xi_{1D}) + \kappa_r(\dot{\xi}_1 - v_D)] \quad (11)$$

where  $\kappa_p, \kappa_r$  are positive constants. Suppose the tracking control law gains in Eq. (11) are given by

$$\kappa_p = \gamma k_p; \quad \kappa_r = \gamma k_r \quad (12)$$

where  $\gamma$  is a positive scalar. Define the matrix  $\mathcal{L}$  as

$$\begin{aligned} \mathcal{L}_{11} &= L_{11} + \gamma \\ \mathcal{L}_{ij} &= L_{ij} \text{ for all other } i, j. \end{aligned} \quad (13)$$

where  $L$  denotes the Laplacian matrix. It is shown next that  $\mathcal{L}$  is positive definite.

**Proposition 1.** The matrix  $\mathcal{L}$  is positive definite.

**Proof-** Since  $L$  is positive semidefinite,  $\mathcal{L}$  is at least positive semidefinite. Also, since  $L$  has a single zero eigenvalue with a corresponding eigenvector  $\delta \mathbf{1}$  (where  $\delta$  is any non-zero scalar),  $z^T L z \geq 0 \forall z \in \mathbb{R}^N$ , and  $z^T L z = 0$  only for  $z = \delta \mathbf{1}$ . Since  $(\delta \mathbf{1})^T \mathcal{L}(\delta \mathbf{1}) = \gamma \delta^2 > 0$ ,  $\mathcal{L}$  is positive definite.  $\square$

## 2.1 Formation Stability

Define

$$X = \xi - \xi_D \quad (14)$$

Note that  $\xi_D$  is time-varying in view of (7). The agent dynamics can be written as:

$$\ddot{X} = u \quad (15)$$

The control input from Eqs. (10) and (9) can be written as

$$u = -k_p \mathcal{L} X - k_r \mathcal{L} \dot{X} \quad (16)$$

and the closed-loop dynamics are given by

$$\ddot{X} = -k_p \mathcal{L} X - k_r \mathcal{L} \dot{X} \quad (17)$$

The asymptotic stability of the closed-loop system in (17) can be readily established by considering the Lyapunov function

$$V = \dot{X}^T \dot{X} + k_p X^T \mathcal{L} X \quad (18)$$

which, after differentiation with respect to  $t$  and simplification, yields

$$\dot{V} = -2k_r \dot{X}^T \mathcal{L} \dot{X} \quad (19)$$

Thus  $\dot{V}$  is negative semidefinite. Furthermore,  $\dot{V} \equiv 0$  implies  $\dot{X} = 0$ , which in turn implies (from (17)) that  $X = 0$ . Therefore, by LaSalle's invariance principle, the system in (17) is asymptotically stable, i.e., the consensus-based leader-follower control law in (9), (10) provides asymptotic formation convergence ( $\forall i, j, [\xi_i(t) - \xi_j(t)] \rightarrow d_{ij}$ , and  $\dot{\xi}_i \rightarrow v_D$ , as  $t \rightarrow \infty$ ) for any initial conditions for any constant velocity command  $v_D$ .



### 2.1.1 Remarks

1. The agents' dynamics consist of double integrators, which represents, for example, mass-normalized dynamics of point masses.
2. The control laws for each follower agent only needs the knowledge of the relative positions and velocities of its neighbors (with respect to itself), and the knowledge of the formation geometry  $\{d_{ij}\}$  of its neighbors. The leader's control law additionally needs the desired leader position and velocity.
3. It is assumed in this paper that the position and rate gains are the same for all the agents. It is possible to use different gains for each agent, but would require some additional conditions for stability.

## 3 Stability in the Presence of Actuator Nonlinearities

In practice, it is important to consider the fact that the actuators have limited control authority. In [4], a control law that uses an inherently bounded control input was presented for the double integrator consensus problem. A different approach is used in this paper to show that the control law given in Section 2 preserves global asymptotic stability in spite of saturation type (and more general types of) actuator nonlinearities.

Suppose each agent's actuator has a continuous, monotonic non-decreasing,  $(0, \infty)$ -sector nonlinearity, which results in the actual inputs  $u_{ai}$ :

$$u_{ai} = \phi_i(u_i), \quad i = 1, \dots, N \quad (20)$$

where

$$\sigma \phi_i(\sigma) > 0 \quad \forall \sigma \in \mathfrak{R}, \sigma \neq 0, \text{ and } \phi(0) = 0 \quad (21)$$

Figure 2 shows an example of a  $(0, \infty)$ -sector monotonic non-decreasing nonlinearity.

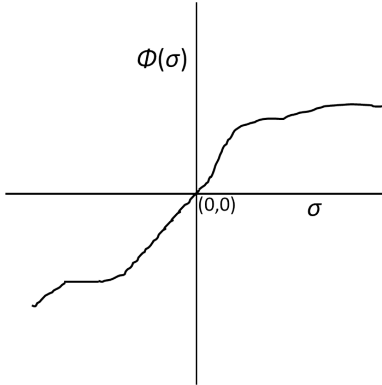


Figure 2.  $(0, \infty)$ -sector monotonic non-decreasing nonlinearity

Then the closed-loop system is given by

$$\ddot{X} = \phi(-k_p w_p - k_r w_r) \quad (22)$$

where  $\phi(\cdot)$  denotes the vector of nonlinearities (assumed to be continuous single-valued functions), and

$$w_p = \mathcal{L}X, \quad w_r = \mathcal{L}\dot{X} \quad (23)$$

The following theorem shows that the closed-loop system is globally asymptotically stable in the presence of  $(0, \infty)$ -sector monotonic non-decreasing actuator nonlinearities.

**Theorem 1-** The closed-loop system (22) is globally asymptotically stable if the actuator nonlinearities  $\phi_i(\cdot)$  are monotonic non-decreasing and belong to the  $(0, \infty)$ -sector.

**Proof-** Denoting

$$\psi_i(\sigma) = -\phi_i(-\sigma), \quad i \dots, N \quad (24)$$

it can be shown that the nonlinear functions  $\psi_i(\cdot)$  are also in the  $(0, \infty)$ -sector and monotonic non-decreasing. The closed-loop system can be expressed as

$$\ddot{X} = -\psi(k_p w_p + k_r w_r) \quad (25)$$

Consider the Lyapunov function

$$V = \dot{X}^T \mathcal{L}\dot{X} + \frac{2}{k_p} \sum_{i=1}^N \int_0^{k_p w_{pi}} \psi_i(\sigma) d\sigma \quad (26)$$

$V$  is non-negative  $\forall X \in \mathfrak{R}^N$ , and  $V = 0$  only when  $\dot{X} = 0$  and  $w_p = 0$ , i.e.,  $X = 0$  (from (23)), therefore  $V$  is positive definite. It is also radially unbounded. Upon differentiating with respect to time  $t$  and using (23),

$$\begin{aligned} \dot{V} &= -2\dot{X}^T \mathcal{L}\psi(k_p w_p + k_r w_r) + 2 \sum_{i=1}^N \dot{w}_{pi} \psi_i(k_p w_{pi}) \\ &= -2 \frac{1}{k_r} (k_r w_r)^T [\psi(k_p w_p + k_r w_r) - \psi(k_p w_p)] \end{aligned} \quad (27)$$

Since  $\psi_i(\cdot)$  are monotonic nondecreasing,  $\dot{V} \leq 0$ . Also,  $\dot{V} \equiv 0$  would imply that  $w_{ri} = 0 \forall i$ , or  $k_{pi} w_{pi}$  and  $(k_{pi} w_{pi} + k_{ri} w_{ri})$  are both in a flat non-zero region of the graph of each actuator nonlinearity  $\psi_i \forall t$ . The former situation ( $w_r = 0$ ) would imply (from (23)) that  $\dot{X} = \ddot{X} = 0$ , and therefore (from (25) and (21)) that  $w_p = 0$  and  $X = 0$  (from (23)). From (25), the latter situation can occur only when  $\dot{X}$  (and therefore  $V$ ) grows unbounded, which is not possible because  $\dot{V} \leq 0$ . Therefore  $\dot{V}$  cannot be zero along any non-zero trajectories, and the system is globally asymptotically stable.  $\square$

## 4 Multi-Dimensional Motion

The results presented in Sections 2 and 3 for 1-d motion can be readily extended to the planar and 3-dimensional cases (or generally for the  $n$ -dimensional case for abstract networks). For the  $n$ -dimensional case, noting that each  $\xi_i$  is  $n$ -dimensional,

$X$  and  $u$  in Eq. (15) are  $(N.n)$ -dimensional, as are  $w_p$  and  $w_r$ . The control input of (16) can be written as

$$u = -k_p[\mathcal{L} \otimes I_n]w_p - k_r[\mathcal{L} \otimes I_n]w_r \quad (28)$$

Alternatively, it may be more intuitive to express the position vector  $X$  partitioned by the  $n$  axes. When  $n = 3$ , define

$$\chi = [\chi_x^T, \chi_y^T, \chi_z^T]^T \quad (29)$$

where

$$\chi_x = [X_{1x}, X_{2x}, \dots, X_{Nx}]^T \quad (30)$$

(similar for y- and z-axes). With  $w_p$ ,  $w_r$ ,  $u$  re-ordered similarly as  $\omega_p$ ,  $\omega_r$ ,  $v$  the closed-loop equations for each axis can be written separately as

$$\ddot{\chi}_i = v_i = -k_{pi}\omega_{pi} - k_{ri}\omega_{ri}, \quad i = x, y, z. \quad (31)$$

Then the stability results of Sections 2 and 3 carry over in a straightforward manner. (Note that a different  $k_p$ ,  $k_r$ , and  $\gamma$  can be chosen for each axis).

#### 4.1 Effect of Rotation

In 2-d and 3-d motion, it is often important to change the formation's orientation to match the orientation of the velocity vector  $v_D$ . Suppose the formation's orientation is initially aligned with the velocity vector  $v_{D0}$ , and a new formation velocity vector  $v_{D1}$  is desired, which has a different magnitude as well as orientation, expressed by Euler rotation angles  $\phi, \theta, \psi$  about the  $x, y, z$  axes, i.e.,

$$v_{D1} = T_3(\psi)T_2(\theta)T_1(\phi)v_{D0} + \delta v \quad (32)$$

where  $T_i$ s are the Euler rotation matrices and  $\delta v \in \mathfrak{R}^3$ . If all the motion is expressed in the new coordinate system, the formation geometry  $\mathcal{D}$  remains unchanged. Assuming each vehicle has the knowledge of the new coordinate system (i.e., the direction of the velocity command)  $(\phi, \theta, \psi)$  and that the vehicles can change their individual orientation to match the new coordinate system, the vehicles' relative positions and velocities are measured in the new coordinate system and the control laws would remain unchanged.

## 5 Example

For the 5-agent system in Figure 1, suppose it is required to form and maintain a V-shaped formation with Agent 1 as the leader, as shown in Figure 3. The motion is assumed to be planar, and the agents are initially at rest and dispersed at random locations. Suppose a velocity command  $v_D = (1, -2)$  is given to the leader. Using the control laws (9), (10) with arbitrarily chosen gains  $k_{px} = k_{py} = 10$ ,  $k_{rx} = k_{ry} = 5$ , and  $\gamma = 1$ , it can be verified that all closed-loop eigenvalues have negative real parts. The resulting trajectories are shown in Figure 4 from  $t = 0s$  to  $t = 10s$ , which indicate that the desired formation shape and velocity are mostly attained by  $t = 6.5s$ , and fully attained and maintained by  $t = 10s$  in the presence of constant velocity command.

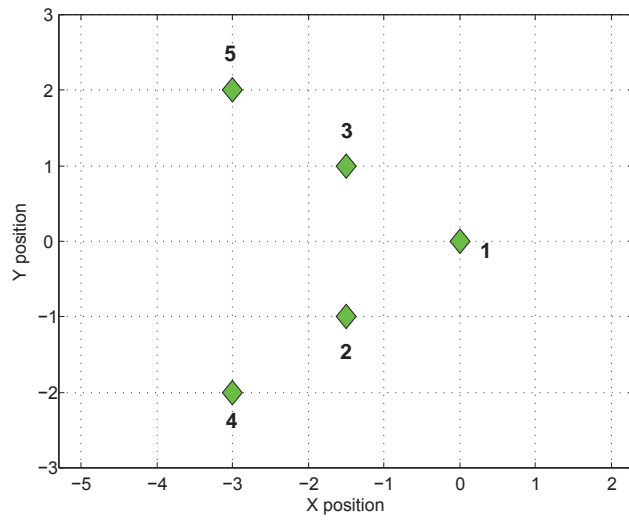


Figure 3. Desired formation shape

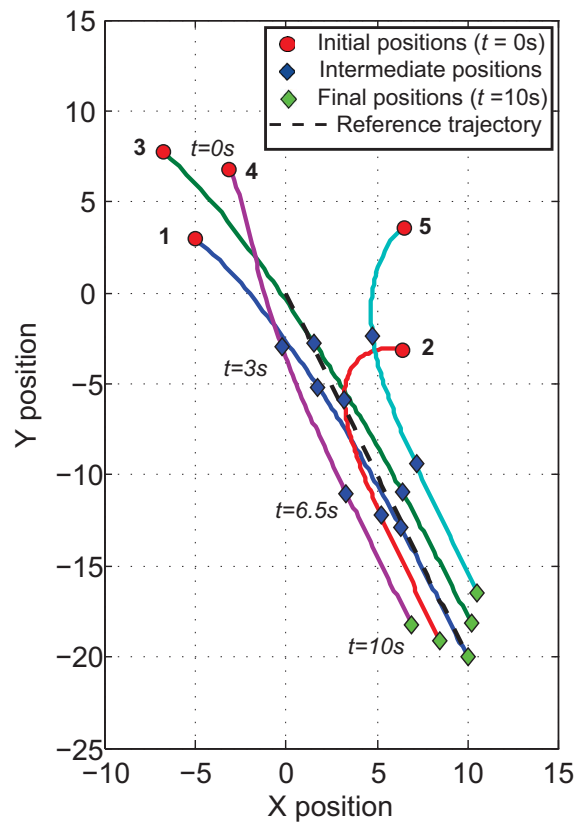


Figure 4. Agent trajectories

## 6 Concluding Remarks

A consensus-based formation control scheme was presented for multi-agent systems consisting of agents described by double integrator dynamics. The information topology of the system was assumed to consist of a connected undirected graph, and a leader-follower architecture was used wherein the leader is required to follow a piecewise constant velocity command. It was shown that the control scheme provides asymptotic stability, i.e., the formation shape is asymptotically attained and maintained, and the formation velocity asymptotically approaches the desired velocity. It was also shown that global asymptotic stability is preserved in the presence of  $(0, \infty)$ -sector monotonic non-decreasing actuator nonlinearities, which include saturation. An important application of multi-agent systems theory is the multi-vehicle cooperative control problem for autonomous vehicles. Considerable further work needs to be done in this area to realistically address this problem and to find practical solutions. In the example problem presented, the controller gains were chosen arbitrarily. It would be highly desirable to investigate techniques for designing controller gains to optimize a given performance index. In addition, optimal design of the communication structure (e.g., selecting the most effective edges in the information topology graph) should be investigated. Furthermore, the control laws and the stability results need to be substantially extended, first to agents represented by inertial masses with translation-rotation coupling, and then to general uncertain dynamic systems, both linear and nonlinear, such as aircraft. An important topic that was not addressed in this paper is collision avoidance strategies, which should be built into the control laws. It is also necessary to develop reliable simulation-based methods and experimental techniques to demonstrate, validate, and verify the control schemes.

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