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AN UPPER BOUND ON HIGH SPEED SATELLITE COLLISION PROBABILITY WHEN ONLY ONE OBJECT HAS POSITION UNCERTAINTY INFORMATION

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Upper bounds on high speed satellite collision probability, P_C†, have been investigated. Previous methods assume an individual position error covariance matrix is available for each object. The two matrices being combined into a single, relative position error covariance matrix. Components of the combined error covariance are then varied to obtain a maximum P_C. If error covariance information for only one of the two objects was available, either some default shape has been used or nothing could be done. An alternative is presented that uses the known covariance information along with a critical value of the missing covariance to obtain an approximate but potentially useful Pc upper bound.

INTRODUCTION

A standard assumption in the tracked orbital debris collision risk problem is that an individual position error covariance matrix is available for each of the two objects involved in the close approach. The two matrices are combined into a single, relative position error covariance matrix[1]. The combined position error covariance matrix may then be modified to arrive at a maximum P_C. There are various modification schemes along which an upper bound for this high speed collision probability, P_C‡, have been pursued.[1,2] In the collision plane representation of the high speed collision probability problem, the predicted mean miss position in the collision plane is assumed fixed.[3,4] Then the shape (ellipse aspect ratio), the size (scaling of the covariance matrix) or the orientation (rotation of the principal axes) of the combined position error ellipse may be varied to obtain a maximum P_C. However, what is the analyst to do if the position error covariance matrix for one of the two objects is not available?

When error covariance information for one of the objects is not available (usually the debris§ object), the analyst has commonly defaulted to the situation in which only the relative miss position and velocity are known without any corresponding state error covariance information.

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† See the "NOTATION" section for all further notational descriptions.

‡ Throughout this document it is to be understood that collision probability is computed assuming the position errors are normally distributed. This assumption applies to each object individually as well as to the relative position error of one object with respect to the other.

§ In this document the terms "asset" and "debris" will be used exclusively to represent what are also commonly referred to, respectively, as "primary" and "secondary".

The usual methods of finding a maximum P_C do little good as the analyst defaults to the assumption of having no knowledge of the combined, relative position error covariance matrix.

It is reasonable to think that, given the assumption of no combined covariance information, an analyst might still attempt to determine the relative position error covariance matrix that results in an upper bound on the P_C. Without some guidance on limits to the shape, size and orientation of the unknown combined covariance matrix, the limiting case is a degenerate ellipse lying along the relative miss vector in the collision plane. Unless the miss position is exceptionally large or the at-risk object is exceptionally small, this method results in a maximum P_C too large to be of practical use. For example, assuming that the miss distance is equal to the current ISS alert volume along-track (+ or -) distance of 25 kilometers and that the at-risk area has a 70 meter radius, the maximum (degenerate ellipse) P_C is about 0.00136. At 40 kilometers, the maximum P_C would be 0.00085 which is still almost an order of magnitude larger than the ISS maneuver threshold of 0.0001. In fact, a miss distance of almost 340 kilometers is necessary to reduce the maximum P_C associated with this degenerate ellipse to the ISS maneuver threshold value. Such a result may be of no practical value to an analyst. However, it turns out that some improvement may be made with respect to this problem just by realizing that while the position error covariance matrix of one of the objects (usually the debris object) may not be known, the position error covariance matrix of the other object (usually the asset) is almost always available. Making use of the position error covariance information for the one object provides an improvement in finding an upper bound to the P_C. This improvement, in some cases, may offer real utility. Three scenarios in which this method might be useful are i) a well tracked object conjuncting with a poorly tracked object, ii) a commercial or foreign owner/operator object conjuncting with a publicly cataloged resident space object, and iii) a cataloged object conjuncting with a launch trajectory (this would include the so-called COLA Gap as well as the usual Launch COLA).[5,6] In this last case, the launch trajectory might have position error covariance information of dubious quality as compared to that of the trajectory of the cataloged object.

PROPOSED SOLUTION

The approach used to find a maximum P_C when covariance data for one of the objects is missing is straight forward. First, the relative position vector between the two objects and the position error covariance data that is available for the one object are transformed into the collision plane frame*. This is done in the usual manner that the combined, relative position and error covariance data are transformed into that frame for the typical collision risk analysis problem. The position vector transformation is defined by the following relationship:

$$\boldsymbol{r}_{rel} = \boldsymbol{T}_{HC}\boldsymbol{T}_{IH}\boldsymbol{R}_{rel} \tag{1}$$

Note that the inertial miss position at *TCA* is first transformed into the *HNV* frame and then into its two dimensional projection in the collision plane frame. To transform the asset object's 3x3 *uvw* position error covariance matrix into its 2x2 collision plane frame representation, three transformations are required:

$$\boldsymbol{C}_{AC} = \boldsymbol{T}_{HC}\boldsymbol{T}_{IH}\boldsymbol{T}_{AI}\boldsymbol{C}_{AA}\boldsymbol{T}_{AI}^{T}\boldsymbol{T}_{IH}^{T}\boldsymbol{T}_{HC}^{T} \tag{2a}$$

* The transformation between inertial and collision plane frames is not unique. The path shown is just one example.

This is the contribution of the known position error covariance matrix to the calculation of the P_C in the collision plane frame. A similar transformation is possible for the debris object's position error covariance matrix, if it was known, and is presented here for completeness.

$$\boldsymbol{C}_{DC} = \boldsymbol{T}_{HC}\boldsymbol{T}_{IH}\boldsymbol{T}_{DI}\boldsymbol{C}_{DD}\,\boldsymbol{T}_{DI}^{T}\boldsymbol{T}_{IH}^{T}\boldsymbol{T}_{HC}^{T} \tag{2b}$$

The critical value of the contribution of the unknown position error covariance matrix is that variance which makes P_C a maximum. Using simple logic, it is possible to characterize the form of the unknown position error covariance matrix resulting in the maximum P_C. First, in the collision plane frame, adding uncertainty normal to the relative miss vector tends to disperse the position uncertainty away from the asset. To maximize the probability, it follows that it is necessary to minimize the dispersion of the uncertainty in the collision plane frame normal to the relative miss vector. This is accomplished by having the unknown collision plane uncertainty be represented by a degenerate ellipse. This ellipse is oriented along the collision plane relative miss unit vector with its center at the relative miss position of the debris object in the collision plane. The general form of the debris object collision plane position error covariance matrix contribution to the combined error covariance matrix may then be specified as:

$$\boldsymbol{C}_{DC} = V_r\,\boldsymbol{u}_{rel}\,\boldsymbol{u}_{rel}^{T} \tag{3}$$

It should be noted that this is only the collision plane component of the unknown position error covariance matrix. The part of the unknown matrix normal to the collision plane will be discussed later. The combined collision plane relative position error covariance matrix is now given by the sum of the known collision plane position error covariance matrix and the unknown collision plane position error covariance matrix. This result is presented in Equation 4.

$$\boldsymbol{C}_{TC} = \boldsymbol{C}_{AC} + \boldsymbol{C}_{DC} \tag{4}$$

To determine the critical value of V_r that maximizes the probability, the collision probability is first approximated by simply multiplying the collision risk area by the bivariate normal probability density at the origin of the collision plane coordinate frame*. This is presented in Equation 5.

$$P_C \approx A_C \frac{e^{-0.5\boldsymbol{r}_{rel}^{T}\,\boldsymbol{C}_{TC}^{-1}\boldsymbol{r}_{rel}}}{2\pi\sqrt{\left|\boldsymbol{C}_{TC}\right|}} \tag{5}$$

To determine the critical value of V_r, referred to as V_C, Equation 5 is differentiated with respect to V_r. With some simplification, it is possible to solve directly for V_C, the critical value of V_r. This result, in terms of the predicted collision plane miss distance and the sigma level of the collision plane miss with respect to the asset position uncertainty is given by:

$$V_C = r_{mag}^{2}\left(\frac{K_A^2 - 1}{K_A^2}\right) \tag{6}$$

The sigma level of the collision plane miss vector with respect to the asset collision plane position uncertainty is specifically given by:

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* This approximation works well as a proxy for the more formal integral computation of the collision probability. It becomes less effective as the smaller of the two collision plane standard deviations nears or becomes smaller than the collision radius of the combined area of the two objects. Shorter miss distances also do not perform as well.

$$K_A^2 = \boldsymbol{r}_{rel}^T \boldsymbol{C}_{AC}^{-1} \boldsymbol{r}_{rel} \tag{7}$$

Having determined V_C through the use of Equations 6 and 7, Equations 3 and 4 allow for the computation of the total collision plane position error covariance matrix. Using this total position error covariance the corresponding approximate maximum P_C may be computed in the usual manner of Pc computation or the approximation in Equation 5 instead. If the computed maximum P_C is below some risk mitigation threshold (such as a collision avoidance maneuver threshold) this would indicate that the approaching debris object cannot, given the currently available information, present a risk high enough to justify action being taken. Unfortunately, if the computed probability is larger than the mitigation threshold value, no conclusion may be reached about the advisability of action being taken. That is, this method can never show that a risk mitigation action is necessary only that such an action is not necessary.

Inspection of Equation 6 shows that if K_A is less than 1 there is no acceptable solution for V_C. This is the case because V_C would no longer be positive semi-definite. The direct implication here is that if K_A is less than or equal to 1, the maximum probability is obtained by ignoring the contribution of the debris object and using only the position uncertainty of the asset in the calculation of the collision probability. If such a case does arise and the calculated probability, using only the asset error covariance matrix, is below the action threshold then this would indicate that risk of a collision is too low to justify a risk mitigation maneuver. In cases where this condition exists but the asset-only collision probability is at, or just above the action threshold, it might be argued that any realistic debris state uncertainty would drive the collision risk below the threshold and so no action need be taken.

A side result to the above solution is the value of the sigma level associated with the predicted miss using the combined position uncertainty resulting in the maximum collision probability. This K is defined in Equation 8 with $\boldsymbol{C}_{DC}$ being replaced using Equations 3 and 6.

$$K^2 = \boldsymbol{r}_{rel}^T \left(\boldsymbol{C}_{AC} + \boldsymbol{C}_{DC}\right)^{-1} \boldsymbol{r}_{rel} \tag{8}$$

By back substitution it is determined that the right hand side of Equation 8 is equal to 1:

$$\{K^2 : V_r = V_C\} = 1 \tag{9a}$$

And so Equation 5 becomes:

$$P_C \approx A_C \frac{e^{-1/2}}{2\pi\sqrt{\left|\boldsymbol{C}_{TC}\right|}} \tag{9b}$$

Another side result is possible by generalizing Equation 6. For an arbitrary but positive value of V_r, the relationship between V_r and K from Equation 8 is given by Equation 10.

$$V_r = r_{mag}^2 \left(\frac{K_A^2 - K^2}{K_A^2 K^2} \right) \tag{10}$$

A general comment is in order. The smaller the value of the known collision plane position error uncertainty normal to the relative miss vector, the potentially less useful this method happens to be. In principal, if the known uncertainty were small enough to ignore then this method behaves effectively the same as the case in which no position uncertainty information for either object is available. The power of this method in the more general case is that a certain amount of the position uncertainty is already spread out away from the at-risk collision area and therefore this maximum collision probability is a more useful bound. Lastly, it should be kept in mind that the actual size, shape and orientation of the collision plan projection of the asset's position error covariance matrix at TCA is highly dependent upon the close approach geometry. Side approaches will have a dominant role played by the along track uncertainty and less by the radial uncertainty which could be orders of magnitude less than the along track value. In this situation the technique may work well for some geometries that have relative large radial misses. A head on approach would instead see almost no along track effect. Instead, the radial and cross track values, being smaller and of approximately the same order of magnitude, dominate the process. In the head on case then, it might be anticipated that the technique is of less frequent use.

An Example of the Basic Method

Now presented is a basic example of the method. This example has the asset covariance matrix and miss position data already expressed in the collision plane frame. For convenience, the asset collision plane position error covariance matrix is chosen to be diagonal. This is no true loss of generality as it only represents a rotation into the principal axis frame of the asset collision plane covariance matrix. The debris object's collision plane miss position with respect to the asset, the corresponding unit vector and the asset only covariance matrix are:

$$\boldsymbol{r}_{rel} = [1000 \quad 200]^T \tag{11a}$$

$$\boldsymbol{u}_{rel} = \begin{bmatrix} 5/\sqrt{26} & 1/\sqrt{26} \end{bmatrix}^T \tag{11b}$$

$$\boldsymbol{C}_{AC} = \begin{bmatrix} 722500 & 0 \\ 0 & 2500 \end{bmatrix} \tag{11c}$$

(Units have been omitted here. All that is required is that the length unit be consistent across the position vector, the error covariance matrix and the asset/debris collision area.)

The asset only error covariance matrix 1-sigma ellipse, centered at the collision plane miss position, defined by the matrix in 11c is shown in Figure 1.

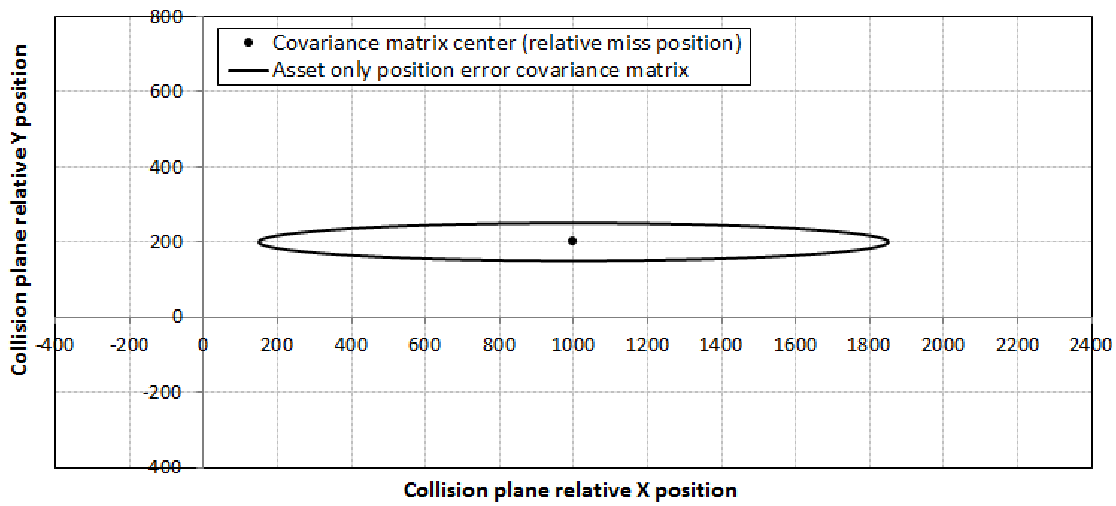

Figure 1: Asset only position error covariance matrix as a 1- sigma ellipse

Substituting the data in Equations 11a and 11c into Equation 7 yields:

$$K_A^2 = \frac{5024}{289} \tag{12}$$

The result of Equation 12 and the miss position data in Equation 11a may then be substituted into Equation 6 to give:

$$V_C = \frac{153887500}{157} \approx 9.8 \cdot 10^5 \tag{13}$$

Now the values in Equations 13, 11b, 11c and 3 (with $V_r = V_C$) may be substituted into Equation 4 to give the critical value of the combined critical collision plane position error covariance matrix (C_{TC}) that will yield the maximum collision probability, Equation 14. This matrix, as a 1-sigma ellipse representation, is shown in Figure 2.

$$C_{TC} = \begin{bmatrix} 26140125/157 & 29593750/157 \\ 29593750/157 & 6311250/157 \end{bmatrix} \tag{14}$$

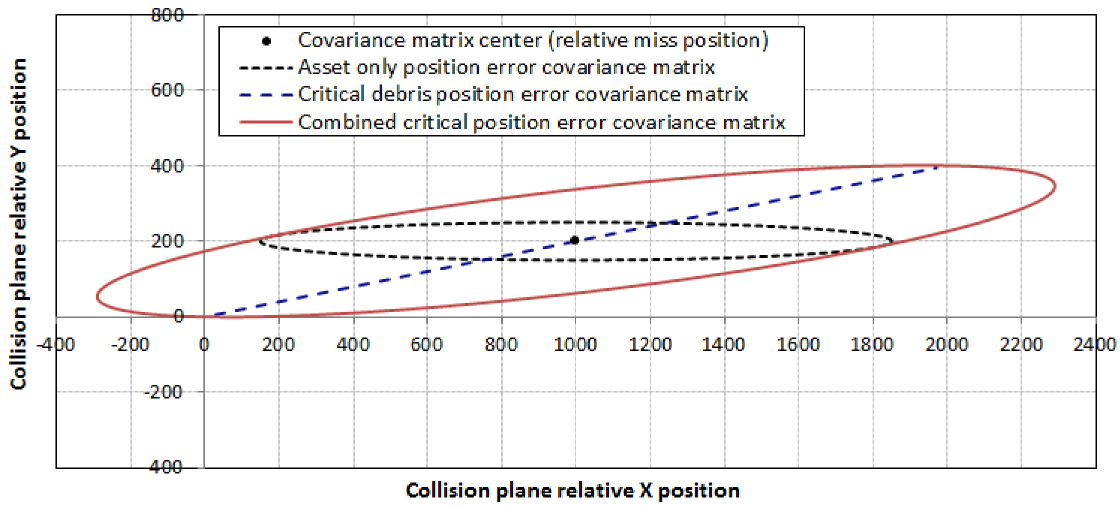

Figure 2: Combined, critical position error covariance matrix as a 1-sigma ellipse

The approximate maximum value of the collision probability associated with this critical collision plane error covariance matrix may be determined by substituting the appropriate data into Equation 9b. The result is shown in Equation 15a.

$$P_C \approx \frac{A_C e^{-1/2}}{2\pi(10000\sqrt{314})} \tag{15a}$$

If the collision risk area is defined as a circle with radius R_{HB} then P_C becomes:

$$P_C \approx \frac{R_{HB}^2 e^{-1/2}}{20000\sqrt{314}} \approx 1.71 \cdot 10^{-6} R_{HB}^2 \tag{15b}$$

Comments on Applying the Basic Method

Investigation of the result described by Equation 6 may or may not provide a definitive answer to the question of whether or not a risk mitigation maneuver is required. If it is clear from the result (by way of the probability calculation showing a maximum P_C less than the action threshold) that no risk mitigation action is necessary, then a useful final result has been obtained. However, as already indicated, if the maximum probability associated with V_C does violate an action threshold, it cannot be concluded that some type of risk mitigation action is absolutely necessary. Table 1 below illustrates, for four different values of R_{HB}, various possible outcomes using the result in Equation 15b.

Table 1. Object size and mitigation outcome examples

R_{HB}	P_C	10^{-3} Threshold	10^{-4} Threshold
5	0.000043	No action required	No action required
10	0.000171	No action required	Unknown
20	0.000685	No action required	Unknown
25	0.001070	Unknown	Unknown

While using the approximate expression, the results in the table above give a good estimate of whether or not a collision avoidance maneuver may be avoided ("No action required") versus the alternative condition ("Unknown") which does not give any additional information on the necessity of a maneuver. The largest R_{HB} value has a probability just slightly above the 10^{-3} threshold. This might lean the analyst toward thinking "No action required" might be acceptable. However, suppose for the unknown debris collision plane covariance, an additional uncertainty normal to the miss vector is added. It turns out (stated without proof) the size of the resulting collision plane covariance matrix for the debris object that would give a P_C of 10^{-3} is about the same size as the matrix of the asset. That would probably not be an ignorable situation. Therefore, the stated condition of "Unknown" would need to stand unless some other condition existed that justified confidence in the maximum P_C being less than the required threshold.

Offered here, simply as a conjecture, is a path to other potential constraints which might further limit the possible range of values of V_r and thus, perhaps, V_C. Recall that the degenerate variance V_r lies only along the collision plane miss vector. The debris object collision plane position error covariance therefore ignores the component normal to the collision plane miss vector in the collision plane. However, this debris object degenerate position uncertainty covariance matrix can have a component along the relative velocity vector which is perpendicular to the collision plane. There also may be some level of correlation between the debris position uncertainty along the relative velocity direction and the degenerate component in the collision plane along the relative miss vector. This out-of-plane element of the debris object position error uncertainty does not affect the outcome of the previous maximum P_C related variance derivation. The out-of-plane component is therefore completely arbitrary except in as how it might be logically or practically bounded. In any case, the complete representation, in the *HNV* frame, of the debris object's critical position error covariance is presented in Equation 16.

$$\boldsymbol{C}_{DH} = \begin{bmatrix} V_r \boldsymbol{u}_{rel} \boldsymbol{u}_{rel}^T & \boldsymbol{C}_{rv} \\ \boldsymbol{C}_{rv}^T & \boldsymbol{C}_{vv} \end{bmatrix} \tag{16}$$

The $\boldsymbol{C}_{DH}$ matrix may be back-transformed into the debris object's *uvw* frame, yielding a hypothetical form of $\boldsymbol{C}_{DD}$. The transformed matrix will display the representation of the critical collision plane position error variance, V_C, as well as the arbitrary components, ***Cvv*** and ***Crv***, in the debris *uvw* frame. It may be that an examination of $\boldsymbol{C}_{DD}$ will indicate some character that is not realistic. For example, suppose one or more elements of $\boldsymbol{C}_{DD}$ (assuming arbitrary but otherwise plausible values for ***Cvv*** and ***Crv***) were unrealistic. This might happen if an out-of-plane or radial component were unrealistically large, particularly when compared to the along track uncertainty. If it is possible to say, with some reasonable degree of confidence, that a specific component of $\boldsymbol{C}_{DD}$ is too large by some amount then this might indicate a reasonable reduction in the size of V_C so as to not exceed the presumed bound on the component of $\boldsymbol{C}_{DD}$ under investigation. Any reduction in V_C would result in a reduction in the collision probability since the original value of V_C is associated with a maximum collision probability. As noted earlier, a variation on this is when the asset only probability is above the action threshold. It may be that reasonable lower limits on the components of the unknown error covariance matrix, along with the recomputed V_C, indicate that the true uncertainty may be large enough to prevent the P_C from being above the action threshold.

Additional, Possible Applications

The application scenarios i, ii and iii noted in the last paragraph of the Introduction, along with the example of the basic method above, are all direct applications of the method to the standard close approach event. This type event is generally characterized by state estimates for two resident space objects that result in a close approach at a specific time in the future. Scenarios i and ii fall into this category. Scenario iii may be interpreted differently in that the Launch COLA is based on a proposed launch time and trajectory. This then is a screening problem in which, given a resident space object catalog, a proposed trajectory is screened, or cleared, against the objects it might pass near along the trajectory. Scenario iii, though clearly a screening problem, is more akin to the usual close approach collision avoidance event. This is the case due to the fact that the risk evaluations are effectively single events at some launch commit time. That is, once the launch commences the vehicle is on its way and further close approach analyses are difficult at best until the newly launched object has a well-established orbit of its own. This situation is also true for the more common on-orbit maneuver screening situation. Both have in common that they involve the near term clearing of a proposed trajectory against a resident population of space objects.

The more typical on-orbit screening problem consists of repeated and continuing close approach evaluations against the same set of objects. These objects will be following natural ballistic trajectories whose estimates are updated as both the asset and other cataloged objects have their orbits updated following additional tracking opportunities. Such standard on-orbit screening my use either a bounding physical volume about the asset (thus requiring only knowledge of nominal trajectories) or the screening may involve anticipated contact between position error covariance matrices (i.e. "covariance" screening) at a selected sigma level when state error covariance matrices exist for the asset and any object it approaches. On-orbit screening is intended to not only recognize near-term close approach collision risks but also to identify those objects that might present a future risk after additional tracking data is included in their trajectory estimates. The two types of screening then are different in some fashion. It is the case though that the "missing covariance" technique may offer a similar approach for each that involves screening to a selected collision risk level.

Consider Equation 9b. This equation relates the known collision plane miss position and asset position covariance matrix, by way of the critical covariance for the debris object, to the maximum collision probability. Suppose, rather than determining the value of P_C, the inverse problem is considered. That is, a screening risk level of collision probability is chosen and then the locus of points in the collision plane frame that corresponds to that P_S is determined. If during screening, a predicted miss lies outside of the locus then that close approach cannot present a collision risk greater than the selected P_S. However, just as with the "regular" close approach problem, if the miss position lies within the locus then no statement can be made as to the actual collision risk presented by that object.

The equation describing the locus of points is greatly reduced when the simplified algorithm for the collision probability, as in Equation 5, is used. After back substitution into Equation 9b, a rearrangement leads to the simplified form which is just the equation of an ellipse. This ellipse is that of the known collision plane covariance matrix centered on the asset. The size of the ellipse is easily computed from known values. This result is shown in Equation 17 and is then followed by the screening locus equation, Equation 18.

$$K_S^2 = \frac{A_C^2 e^{-1}}{4\pi^2 \left|\boldsymbol{C}_S\right| P_S^2} \tag{17}$$

$$\boldsymbol{r}_S^T \boldsymbol{C}_S^{-1} \boldsymbol{r}_S = K_S^2 \tag{18}$$

It should be kept in mind that the same general guidelines as to when this "missing covariance" technique is useful for the standard close approach problem will also apply to this screening application of the technique. That is, the size of the known position covariance matrix, the close approach direction and the actual collision plane frame miss position all play a roll. Certain combinations of these factors give useful results while others may not.

SUMMARY

Presented in this document is a method by which an approximate upper bound on the collision probability between two orbital objects may be computed under the condition that the position uncertainty of only one of the objects is known at the time of their closest approach. This method is less arbitrary than other approaches in finding a maximum collision probability and has some, though not guaranteed, ability to provide a useful, unambiguous upper limit on the collision probability. The method also gives a direct indication of when the one known position uncertainty is sufficient to compute a maximum probability. Additional position uncertainty in such cases would only reduce the maximum collision probability. Lastly, by transforming the critical (maximum probability related) unknown position error uncertainty back into the reference frame of the object with this unknown position uncertainty, it may be possible to put further constraints on the value of the critical position error uncertainty. Any such constraint could result in a reduction in the critical value of the unknown position error uncertainty leading to a reduction in the value of the maximum collision probability, possibly below any specified action threshold. Lastly, other applications of the technique for launch, maneuver and on-orbit screening are also presented.

NOTATION

A_C = Combined collision risk area, accounts for sizes of both asset and debris objects, (length^2).
$\boldsymbol{C}_{AA}$ = Asset object 3x3 position error covariance matrix in the asset uvw frame, (length^2).
$\boldsymbol{C}_{AC}$ = Asset object 2x2 position error covariance matrix partition in the collision plane frame, (length^2).
$\boldsymbol{C}_{DD}$ = Debris object 3x3 position error covariance matrix in the debris uvw frame, (length^2).
$\boldsymbol{C}_{DH}$ = Debris object 3x3 position error covariance matrix in the HNV frame, (length^2).
$\boldsymbol{C}_{DC}$ = Debris object 2x2 position error covariance matrix partition in the collision plane frame, (length^2).
$\boldsymbol{C}_{rv}$ = Debris object 2x1 matrix partition representing the position covariance of the debris object between the relative velocity component and the two components in the collision plane, represented in the *HNV* frame and with respect to the collision plane's normal, (length^2).
$\boldsymbol{C}_{TC}$ = Total, combined 2x2 position error covariance matrix in the collision plane frame, (length^2).
$\boldsymbol{C}_s$ = Collision plane frame 2x2 position error covariance matrix used for screening which may be from either the asset or debris, (length^2).
$\boldsymbol{C}_{vv}$ = Position uncertainty variance of the debris object along the relative velocity direction, represented in the *HNV* frame as along the normal to the collision plane, (length^2).
HNV = Horizontal-near vertical reference frame. Defined by the directions of the cross product of the inertial relative velocity vector and the radius vector of the asset and the cross product of the unit vector of this direction with the relative velocity vector. The first direction is a true horizontal direction and the second is almost nearly vertical, (none).
K = Mahalanobis distance or sigma level (used in this paper) of an error vector with respect to its error covariance matrix that describes the level of uncertainty of the vector error, (unitless).
K_A = Value of K computed by using only the known asset object position error covariance matrix and the collision plane relative miss vector, (unitless).
K_S = Value of K used for covariance based, close approach screening, (unitless).
P_C = Probability of collision or, more typically, the probability that a debris object will violate some boundary surface surrounding the center of mass of the asset, (unitless).
P_s = Value of the probability of collision used for screening (unitless).
R_{HB} = Hard body radius, the combined asset and debris radius defining the collision risk area, (length).
$\boldsymbol{R}_{rel}$ = Debris with respect to asset-relative position vector (3 element column) at *TCA*, represented in the inertial reference frame, (length).
r_{mag} = Magnitude of the collision plane representation of the relative position vector at *TCA*, (length).
$\boldsymbol{r}_{rel}$ = Debris with respect to asset relative position vector (2 element column) at *TCA*, represented in the collision plane frame, (length).
$\boldsymbol{r}_s$ = Locus of relative position vectors (2 element column) at a *TCA* (represented in the collision plane frame) used in covariance based screening, (length).
$\boldsymbol{T}_{AI}$ = Asset state *uvw* frame to inertial frame 3x3 transformation matrix, (unitless).
$\boldsymbol{T}_{DI}$ = Debris state *uvw* frame to inertial frame 3x3 transformation matrix, (unitless).
$\boldsymbol{T}_{HC}$ = *HNV* frame to collision plane frame 3x2 partitioning matrix, (unitless).
$\boldsymbol{T}_{IH}$ = Inertial frame to *HNV* frame 3x3 transformation matrix, (unitless).
TCA = Time of closest approach between the asset and debris objects, (time).
uvw = Local inertial coordinate frame (*u* up, *v* forward and *w* along angular momentum), (none).
$\boldsymbol{u}_{rel}$ = Unit vector (2 element column) along the collision plane relative miss vector, $\boldsymbol{r}_{rel}$, (unitless).
V_r = Debris object collision plane position error variance along the collision plane relative miss vector direction, (length).
V_C = Critical value of V_r resulting in an approximate maximum collision probability, (length).

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