

# Scaling relation for occulter manufacturing errors

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## ABSTRACT

An external occulter is a spacecraft flown along the line-of-sight of a space telescope to suppress starlight and enable high-contrast direct imaging of exoplanets. The shape of an external occulter must be specially designed to optimally suppress starlight and deviations from the ideal shape due to manufacturing errors can result in loss of suppression in the shadow. Due to the long separation distances and large dimensions involved for a space occulter, laboratory testing is conducted with scaled versions of occulters etched on silicon wafers. Using numerical simulations for a flight Fresnel occulter design, we show how the suppression performance of an occulter mask scales with the available propagation distance for expected random manufacturing defects along the edge of the occulter petal. We derive an analytical model for predicting performance due to such manufacturing defects across the petal edges of an occulter mask and compare this with the numerical simulations. We discuss the scaling of an extended occulter testbed.

**Keywords:** External Occulters, Starshades, High Contrast Imaging, Scaling, Scalar Diffraction, Optical Verification

## 1. INTRODUCTION

With over 4000 exoplanet candidates and over 1000 confirmed exoplanets using principally indirect detection methods, direct imaging allowing for characterization and spectral measurements of their reflected light is of great interest as the next major step in exoplanet science. Direct imaging for an Earth-like planet is a challenging task because of the ten order of magnitude contrast ratio between the bright star and its dim rocky planetary companion and due to the small angular separation that covers the habitable zone.

An external *occulter*, also called a starshade, is a mission concept that involves a specially-shaped spacecraft flying in formation with the space telescope. The occulter is placed along the line-of-sight of the telescope and target star and suppresses most of the stellar flux before it reaches the telescope pupil allowing for the high-contrast imaging of exoplanets. The edge of the occulter has to be designed to minimize diffraction and create a shadow over a region that will cover the telescope's diameter and an additional alignment margin for the formation flight. Optimization methods can be used to suppress starlight to the necessary factor of  $10^{10}$  contrast for imaging an exo-Earth, with the size of the occulter, observation wavelengths, and manufacturing limitations incorporated as optimization constraints. Proposed designs range from flagship-class such as THEIA<sup>1</sup> to smaller missions such as the recent EXO-S mission design.<sup>2</sup>

Experimental verification of occulter designs is an important part of the technological development for an occulter mission. Full-scale petals have been built and assembled; their individual edge shape and assembled configuration has been precisely and repeatedly measured to predict the optical performance based on existing manufacturing capabilities.<sup>3</sup> However, the performance prediction is based on the same scalar diffraction models used to design the occulter shape. Because a full-scale occulter cannot be tested on the ground, it is therefore important to experimentally verify the performance of scaled occulters in the laboratory to validate the scalar diffraction optical models used for their design and performance prediction.

These scaled occulter designs are etched onto silicon wafers. However, the etching process introduces deviations from the ideal specified shape. We have previously developed a two-dimensional diffraction model and

Table 1: Summary of operational Fresnel numbers for different occulter designs.

Design	Occulter Radius, $R$	Separation, $z$	Wavelength, $\lambda$	Fresnel Number
THEIA	20m	55,000 km	600 nm	12.1
O3	17m	19,500 km	600 nm	24.7
Current Experiment	188 m	97,000 km	600 nm	607.3
Extended Experiment	21.9 m	55,000 km	600 nm	14.5

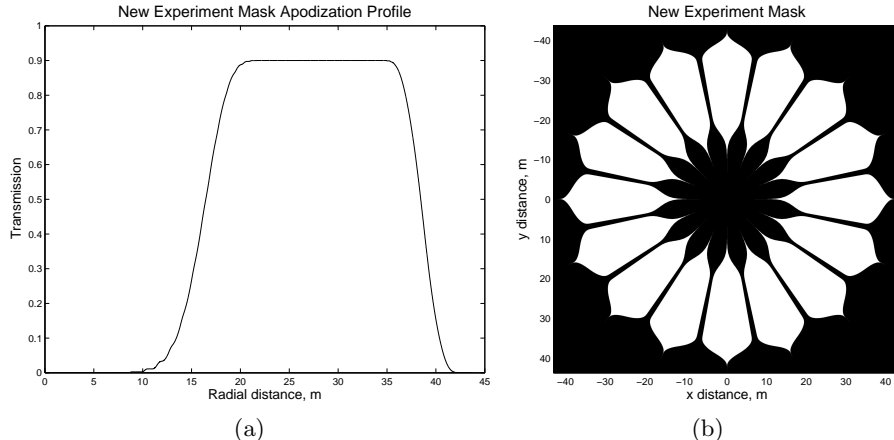


Figure 1: (a) Designed apodization profile including outer ring and struts (b) Binary realization of mask profile.

used it to identify the edge manufacturing accuracy as the limiting factor in the existing occulter experiment at Princeton which was scaled to fit a total available 10-m propagation distance.<sup>4</sup> We use the same numerical optical model to predict the suppression performance of an occulter mask designed at a flight Fresnel number when re-scaled for a longer propagation distance. We also develop a power-law analytical model for predicting performance due to such random manufacturing defects across the petal edges of an occulter mask and compare this with the numerical simulations. We use these results to inform the design of an extended occulter testbed.

The paper is organized as follows. In Section 2, we discuss the flight occulter design and its scaling to laboratory dimensions. In Section 3, we summarize the two-dimensional diffraction model used for numerical simulations and the edge errors introduced. In Section 4, we present simulation results of suppression performance as a function of occulter manufacturing deviation. We also rescale the testbed at different propagation distances. In Section 5, we develop an analytical power law model for performance due to edge defects, and compare it to the simulation results. We provide thoughts for the extended occulter experiment in Section 6.

## 2. OCCULTER DESIGN AND SCALING

Optimization methods can be used to design an occulter apodization profile that creates a dark shadow downstream from the occulter,<sup>5</sup> and can then be turned into a binary occulter profile by petalization.<sup>6</sup> The linear programming optimization process used is described in more detail in previous papers.<sup>7,8</sup>

In the existing occulter testbed at Princeton, the occulter mask is oversized compared to a realistic design that would be flown in space. The rationale to oversize the design arose due to the limited propagation distance available in the occulter testbed – an oversized design means that when scaled to laboratory size the occulter can fill the silicon wafer, and fixed manufacturing errors on the  $\mu\text{m}$  level represent a smaller percentage of that mask. Here, we explore scaling of a realistic occulter design for a testbed with a longer propagation distance so that a realistic space design can be experimentally tested. In Table 1 we compare the Fresnel number ( $\frac{R^2}{\lambda z}$ ) for the inner occulter mask used in the current and proposed laboratory experiments with two design reference

Table 2: Summary of final scaled occulter parameters for a testbed with a 76 m total propagation distance available and a design reference of 100 m total propagation distance.

Parameter	Space Design	Collimated Scale		Diverging Scale	
Separation distance, $z$	55,000 km	38 m	50 m	38 m	50 m
Distance scale, $a$	1	1200	1050	1200	1050
Source distance, $h$	$\infty$	$\infty$	$\infty$	38 m	50 m
Divergence scale, $\gamma$	1	1	1	1.4	1.4
Inner radius, $R_{\text{inn}}$	21.9 m	18.2 mm	20.8 mm	12.8 mm	14.7 mm
Outer radius, $R_{\text{out}}$	43.7 m	36.4 mm	41.7 mm	25.8 mm	29.4 mm
Dark shadow radius, $\rho_{\text{dark}}$	2.6 m	2.2 mm	2.5 mm	3.1 mm	3.5 mm
Outer shadow radius, $\rho_{\text{out}}$	43.7 m	36.4 mm	41.7 mm	51.5 mm	57.8 mm
Telescope diameter, $D$	4 m	3.3 mm	3.8 mm	3.3 mm	3.8 mm

missions at 600 nm: THEIA<sup>1</sup> is a flagship-class mission designed for a 4-m telescope, and O3<sup>9</sup> is a probe-class mission designed for a smaller 1.1 m telescope.

Our laboratory design uses an outer ring to minimize diffraction from the finite size of the input beam and laboratory dimensions, a set of support struts that can be shown to theoretically not introduce diffractive structure across the optimized dark hole portion of the shadow, and the usage of an expanding beam to mitigate surface feature effects of any optics upstream from the occulter. The occulter is designed for a plane input wave at space dimensions. Two scalings are subsequently applied:<sup>8</sup> first a scaling to laboratory separation, and second a scaling to account for the expanding input beam. In Figure 1 we show the occulter design we use throughout this paper. Figure 1(a) shows the combined transmission profile for an optimized inner occulter with an optimized outer ring and supporting struts. The optimization of this particular design is described in more detail elsewhere.<sup>10</sup> The corresponding binary mask realization with 16-petals is shown in Figure 1(b). Both the apodization profile and the binary mask are shown at designed space dimensions.

For the extension of the existing testbed, we have investigated total propagation distances available in the range of 10 m to 140 m. We use a 100 m total propagation distance as a baseline for simulations. The final selected testbed location will have 76 m total propagation distance available, and the scaling of the testbed parameters from design to these testbed sizes are summarized in Table 2. Additionally, the diameter of the occulter mask is maximized for the diverging scaling corresponding to  $h = z$ ; the occulter mask is placed halfway between the artificial diverging point source and the telescope location.

### 3. NUMERICAL MODEL

The oversized nature of the Princeton experiment allows spatially resolving the source of scattered light, and thus it is easier to disambiguate the sources of error. We have developed an optical propagation approach that modeled edge defects, and employed this model to verify the limitations of the existing occulter experiment at Princeton.<sup>4</sup> This propagation model uses two-dimensional Fresnel propagation via Matrix Fourier Transforms.<sup>11</sup> Thus, we can directly model almost any type of non-idealization including edge manufacturing defects. This approach is fairly computationally expensive, and to make this approach numerically tractable we introduce anti-aliasing at the edges of the occulter mask in order to represent a higher-resolution mask.

We refer to Figure 2 to describe the propagations between the different planes involved in the occulter testbed. We define four planes of propagation: P0 is the plane of the pinhole source ( $2M \times 2N$  in physical size), P1 is the plane of the occulter mask ( $2U \times 2V$  in physical size), P2 is the pupil plane where the telescope aperture lies and where we measure the *suppression* of the shadow ( $2X \times 2Y$  in physical size), and lastly P3 is the focal plane of the telescope and the plane in which we measure the *contrast* of the point spread function. Thus, to obtain the PSF of the occulter testbed we need three separate Fresnel propagations across distances  $h$ ,  $z$ , and  $s$  respectively between these four planes.

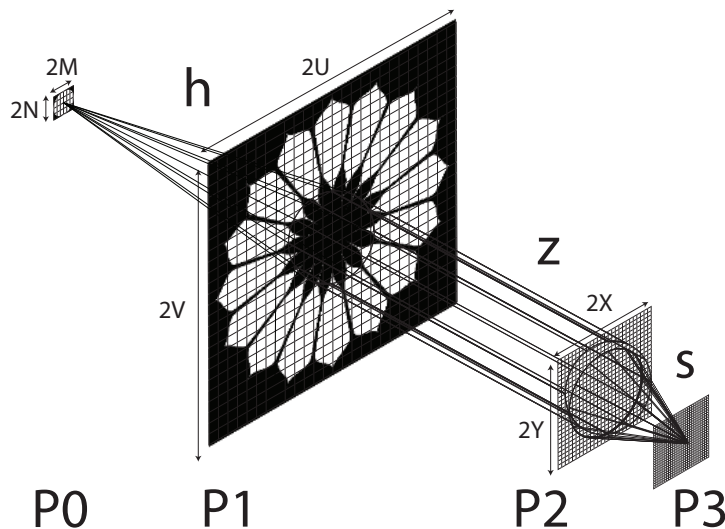


Figure 2: Optical propagation planes for two-dimensional diffraction model.

We use a two-dimensional mask model across P1. The mask model has  $n \times n$  samples, and it is allowed to take on gray values at the petal edges. The gray value is computed by  $g \times g$  anti-aliasing. Therefore, a mask model with  $(n, g)$  and with radius  $R$  will therefore be representative of the resulting feature size given by  $\delta R = 2R/n/g$ .

To simulate manufacturing defects, we isolate all the gray pixels along a petal contour and add or subtract to the gray area the corresponding area of a set of circular defects with varying diameters. A set of circular defect diameters representing the edge perturbation along one petal is generated as white noise with a specified RMS.

#### 4. SIMULATION RESULTS

The ideal performance of the occulter mask assuming a perfect realization using sixteen petals can be computed using the Jacobi-Anger expansion of the one-dimensional Fresnel propagation of the occulter mask's transmission profile following the same procedure as described in previous work.<sup>4</sup> The effect of finite feature sizes can be computed using the two-dimensional mask model without introducing any circular defects along the edges. In Table 3, we have listed several mask models, their feature sizes both at 100 m propagation distance laboratory dimensions and at space scale. Using the two dimensional Fresnel propagation equations, we compute the suppression at the pupil plane across the telescope aperture and the corresponding contrast in the annular working region for the different manufacturing feature accuracies. For comparison, the first experimental results obtained in the Princeton testbed were limited by edge features and manufacturing defects on the order of  $5\mu\text{m}$ . Currently, a typical occulter shape etched on silicon can maintain  $0.5\mu\text{m}$  feature accuracy, and improvements in the manufacturing process are expected to generate an improved feature accuracy of  $0.2\mu\text{m}$ . The final listing of the table presents the corresponding ideal performance assuming no manufacturing errors. These results demonstrate monotonic worsening of performance from the ideal case as the feature size is increased. In Figure 3, we show the output corresponding to a feature size of  $0.1\mu\text{m}$  at both the pupil plane (P2) in Figure 3(a) and at the image plane (P3) in Figure 3(b). We present azimuthally averaged cross-sections as the feature size is increased for both suppression and contrast in Figures 3(c) and 3(d) respectively.

The most computationally expensive part of this numerical validation process for different feature sizes is the creation of the mask models. Since the suppression and contrast performance worsens monotonically with increasing feature sizes, we can compute estimates of the performance at intermediate feature sizes by applying a piecewise spline interpolation between the computed suppression and contrast results.

We can use these interpolated results to obtain an estimate of the performance of the occulter experiment as the total propagation distance is modified resulting in size changes of the occulter mask while manufacturing

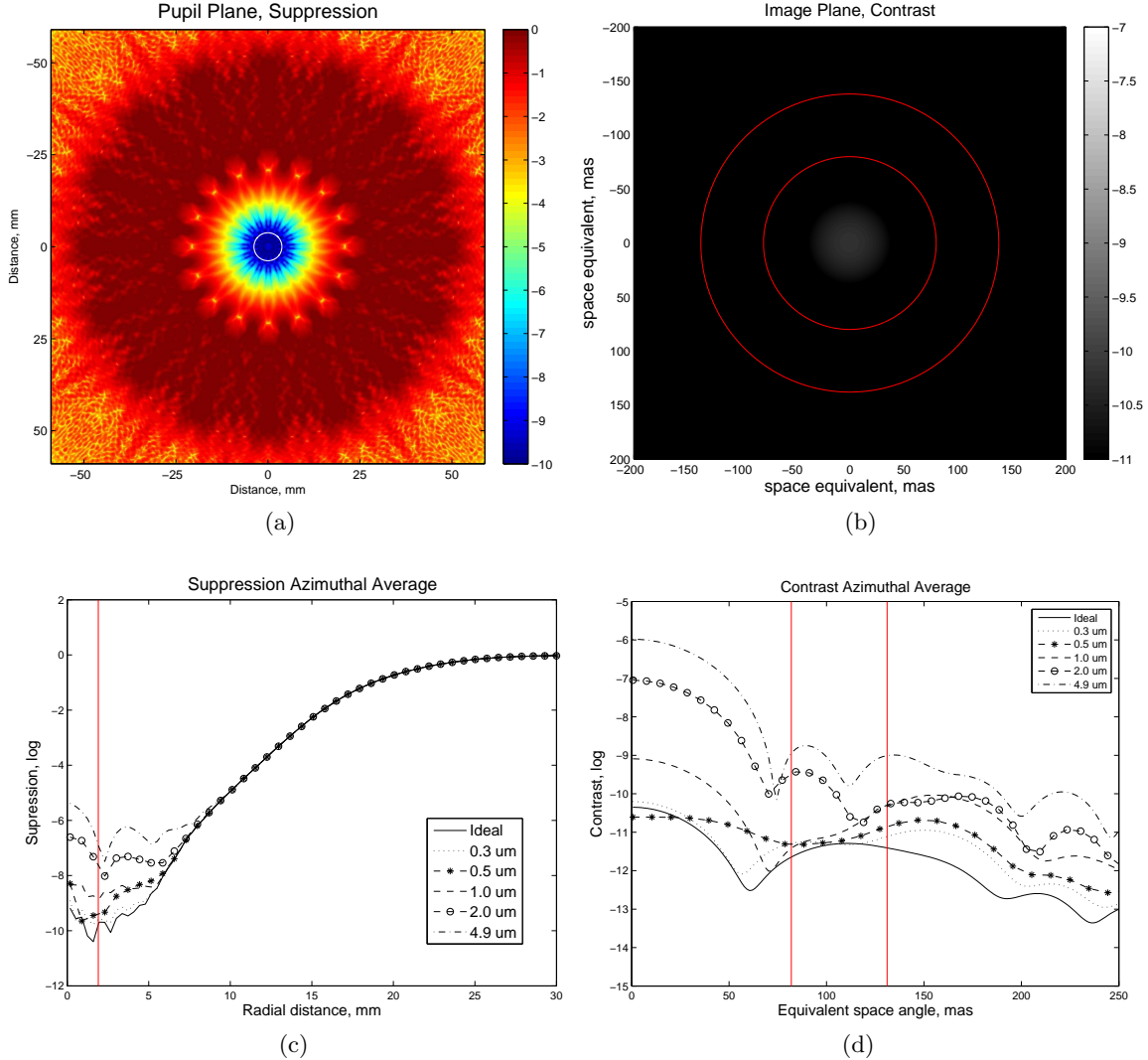


Figure 3: Model simulation results scaled for total propagation distance of 100-m and sensitivity for increasing feature sizes. Sample output for highest-accuracy model with feature size of  $0.1 \mu\text{m}$  featuring near-ideal performance: (a) Suppression at the laboratory pupil plane, and (b) contrast at the laboratory image plane. Comparison for increasing feature sizes: (c) Azimuthally averaged suppression across pupil plane with solid red line indicating extent of telescope aperture across which mean suppression is reported. Red line indicates extent of telescope aperture. (d) Azimuthally averaged contrast across image plane. Annular working zone is indicated by solid red line with inner working angle defined by inner occulter's petal tips and the outer working angle the outer ring's inner tips.

Table 3: Suppression and contrast performance for different manufacturing feature accuracies corresponding to 100 m total propagation distance for a monochromatic diverging input beam at 633 nm.

Lab Feature Size $\delta R, \mu\text{m}$	Mask Model $n \times n$	Mask Model $g \times g$	Space Scale Feature mm	Suppression $\text{Log}_{10}$	Contrast $\text{Log}_{10}$
9.8	6000	1	16.7	-5.36	-8.04
7.4	4000	2	12.5	-5.52	-8.44
4.9	6000	2	8.3	-5.89	-9.06
2.0	6000	5	3.3	-6.97	-9.83
1.4	6000	7	2.4	-7.90	-10.4
1.0	6000	10	1.7	-8.44	-10.8
0.5	12000	10	0.8	-9.27	-11.2
0.3	6000	40	0.4	-9.49	-11.3
0.1	6000	100	0.2	-9.77	-11.4
Ideal	N/A	N/A	Ideal	-9.82	-11.4

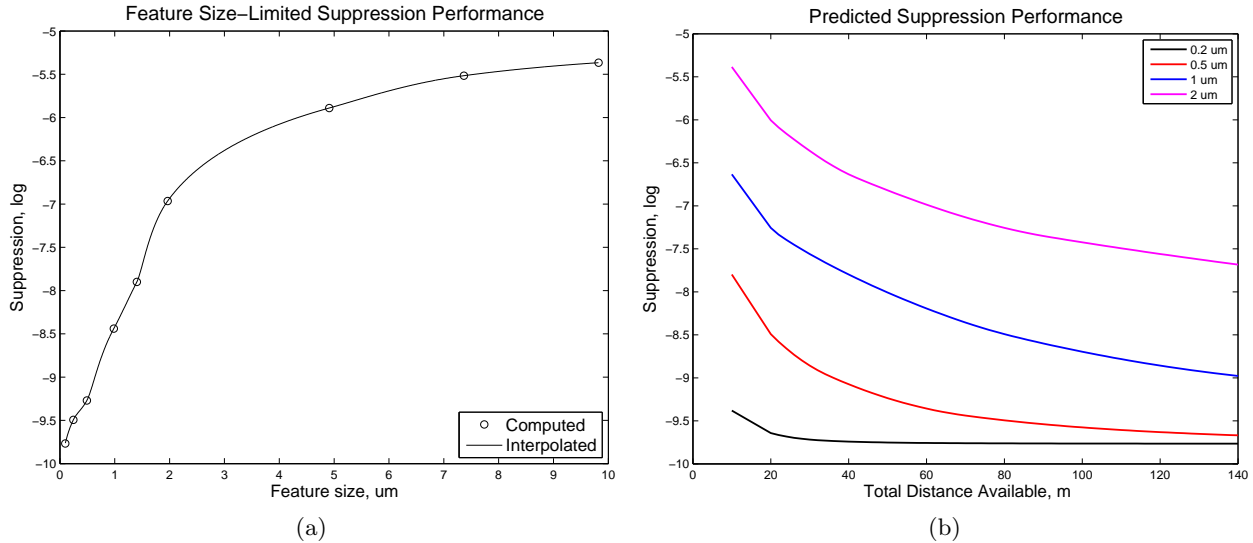


Figure 4: Suppression performance from computed manufacturing feature accuracy: (a) Computed suppression at different manufacturing feature size and interpolation for other feature sizes (b) Predicted suppression performance as a function of total propagation distance available for fixed manufacturing feature sizes. Dashed lines indicate 76-m and 100-m propagation distances which match the selected testbed size and the simulation reference respectively.

feature size is maintained fixed. The relative size of the fixed feature sizes decreases as the total propagation distance and corresponding occulter mask radius increases, and, conversely the relative size increases as the mask shrinks when a smaller propagation distance is available. This process is applied for the suppression performance in Figure 4. In Figure 4(a), the computed mean suppression results across the telescope aperture are denoted by open circles corresponding to the mask models listed in Table 3. The mask models give us the computed feature sizes and we apply an interpolation to obtain the suppression curve at intermediate feature sizes. Based on the suppression curve as a function of feature size, we estimate the effect of changing the total propagation distance in Figure 4(b) while maintaining a fixed manufacturing feature size.

For an aggressive 0.2  $\mu\text{m}$  and 0.5  $\mu\text{m}$  feature sizes, the suppression performance is better than  $10^{-9}$  for all testbed sizes greater than 65 m. For the 0.2  $\mu\text{m}$  feature size the considered testbed size of 76 m corresponds to a suppression performance of  $10^{-9.7}$  which is only a factor of 1.3 short off the ideal suppression.

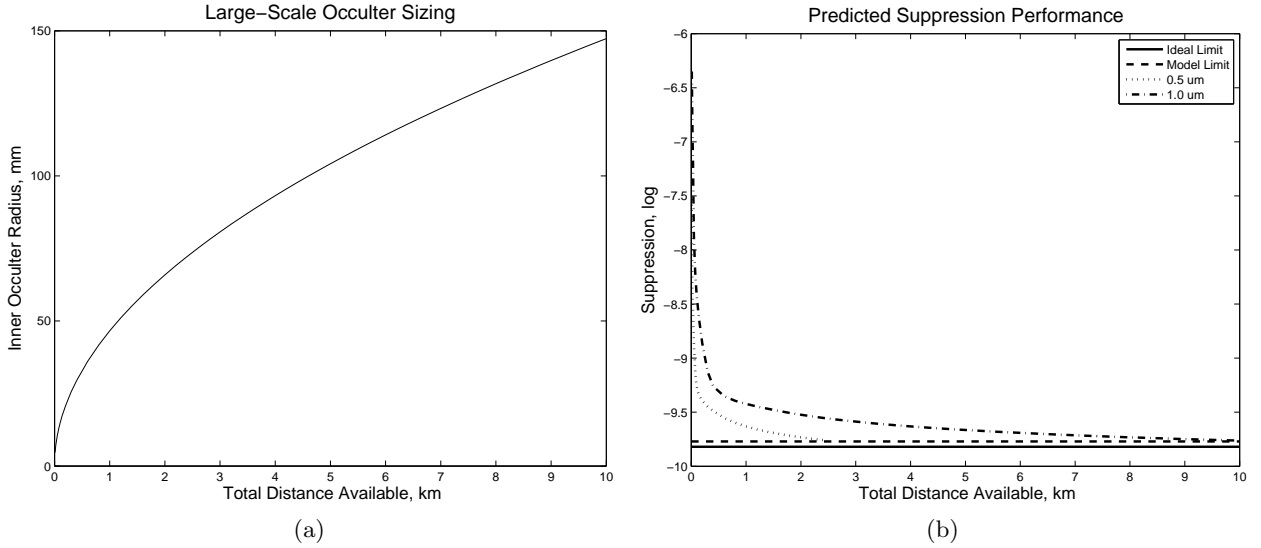


Figure 5: Large-scale occulter sizing and suppression performance (a) Radius of inner occulter sized under same diverging beam scaling with large-scale distance available (b) Expected suppression performance for large-scale occulter and comparison to ideal and model limits.

For all the propagation distances and manufacturing feature accuracies greater than  $0.5 \mu\text{m}$  considered, the suppression performance is limited by the feature accuracy at more than half an order of magnitude from the ideal case. In Figure 5, we therefore consider large-scale sizing of the occulter similar for longer-distance field tests. The radial size of the occulter scaled at km separations is shown in Figure 5(a). The scaled suppression performance at these separations is shown for high-resolution feature sizes in Figure 5(b), and indeed the expected performance if manufacturing feature sizes can be kept constant better approaches the ideal case. If  $0.5 \mu\text{m}$  can be maintained over a  $\approx 100 \text{ mm}$  mask diameter,  $\approx 2.5 \text{ km}$  scaling would be sufficient to achieve near ideal performance. However, it will be necessary to enclose the experiment in a tube that similarly scales in size, with a minimum diameter of  $0.7 \text{ m}$  at the  $100 \text{ m}$  propagation distance baseline scaling up to  $5 \text{ m}$  at  $2.5 \text{ km}$ .<sup>10</sup>

These deterministic models are symmetrical, except for quantization errors arising from translation of a sixteen-fold circularly symmetric pattern to a rectangular grid, and therefore represent an optimistic bound on performance. We now consider the effect on performance due to the loss of symmetry that arises from perturbations at the mask edges that model manufacturing defects, that is differences from the prescribed mask edges. For example, consider the degradation in suppression and contrast performance in Figure 6(a) and Figure 6(b) respectively due to introduction of  $2 \mu\text{m}$  RMS edge errors. Performance with such stochastic manufacturing defects added represents a more conservative bound on performance. In Figure 6, we show the newly computed conservative, stochastic performance bound. Due to the stochastic nature, we perform ten trials for each considered edge defect diameter and report the mean suppression across the dark hole in Figure 6(c) and the mean contrast in Figure 6(d) across the annular working region. The piecewise spline interpolation is shown for intermediate defect sizes. In Figure 6(d), we plot the previously shown suppression performance curves in Figure 4 (b) corresponding to fixed feature sizes as the scaling of the mask is changed as a function of the total propagation distance available. In dashed lines we show the performance degradation due to the introduction of edge defects with the corresponding specified RMS levels.

When edge perturbations are considered, the suppression performance degrades. For example at  $0.2 \mu\text{m}$  the suppression which was  $10^{-9.7}$  for the non-perturbed case is now  $10^{-9.5}$ . This is a factor of 2 short off the ideal. For a testbed of  $76 \text{ m}$  propagation distance, the  $0.5 \mu\text{m}$  is still expected to be better than  $10^{-9}$ . Therefore, as long as feature accuracy and edge deviations can be maintained at the desired  $0.5 \mu\text{m}$  level of better, a very deep suppression level can be verified.

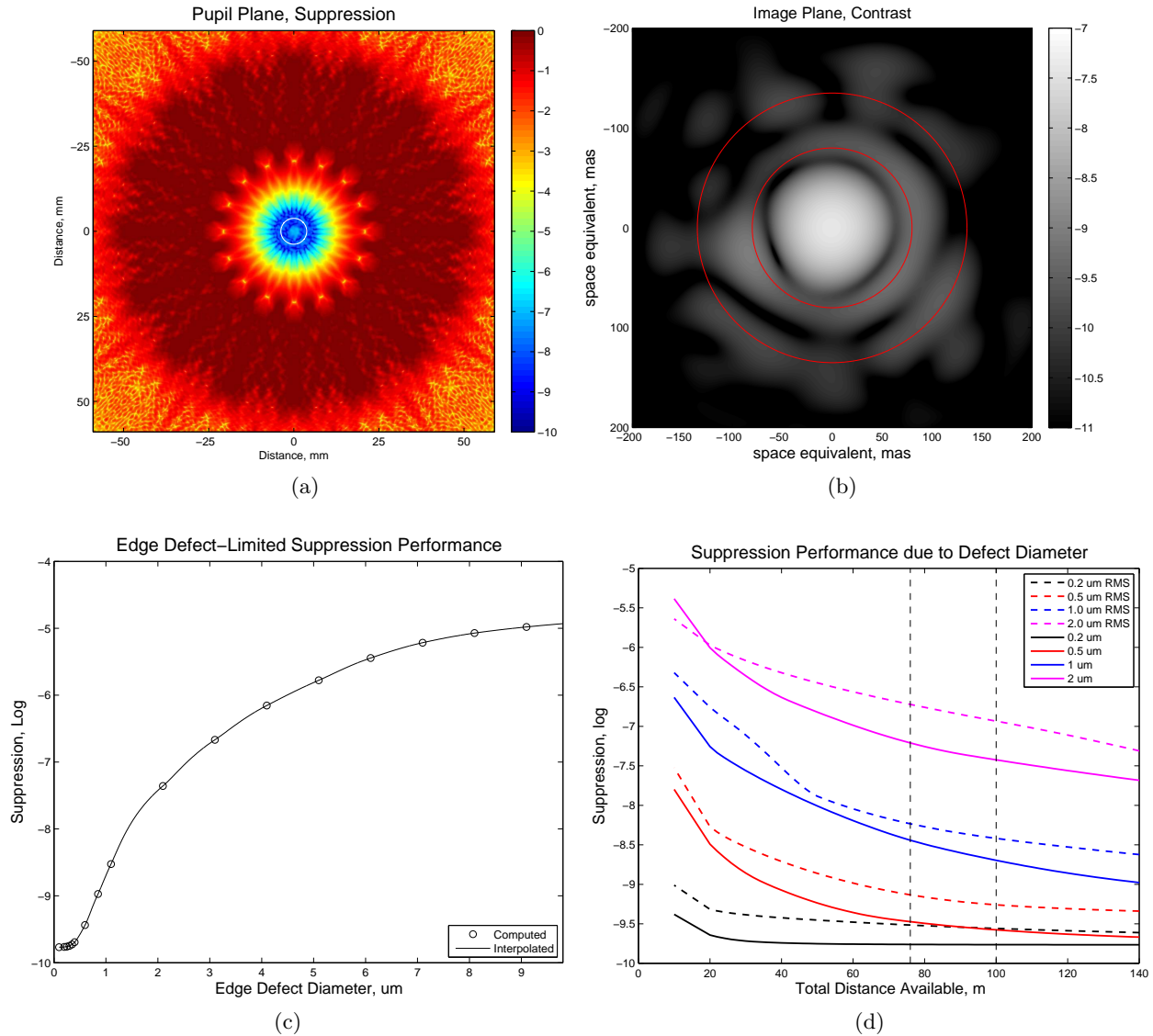


Figure 6: Performance of mask with white noise perturbations injected at the petal edges. Injected perturbations correspond to certain specified defect diameter RMS. Sample output showing performance degradation due to 2  $\mu\text{m}$  RMS edge defects: (a) Suppression at the laboratory pupil plane, and (b) contrast at the laboratory image plane (c) average suppression level from 10 trials corresponding to increasing edge defect diameter RMS and interpolation for intermediate defect diameters. (d) Suppression curves for different fixed feature sizes (solid lines) and edge defect diameter RMS (dashed lines) for varying total propagation distance available.



## 5. ANALYTICAL MODEL

We have developed a simple analytical model to corroborate the general behavior of the more complex numerical model. The model computes the ratio of the scatter at the pupil plane from a defect with dimension  $d$  in the mask plane to the light level at the pupil plane if the mask were not present. It then adds up the contributions of many such defects distributed along the edges of the mask, ignoring the coherent interference terms because we are only interested in the average suppression at the pupil plane.

The model is laid out in Figure 7. A point source in plane P0 illuminates the mask plane P1 distance  $h$  away with an irradiance of  $P \text{ W/m}^2$ . At the pupil plane P2, a distance  $z$  further downstream, the irradiance is reduced by the ratio of areas to  $P(h/(h+z))^2 \text{ W/m}^2$ . Here, we have assumed that  $h$  and  $z$  are much larger than the transverse dimensions of the mask and pupil planes. A defect in P1, e.g., a hole or extra material near the edge of the mask, transmits or blocks the light, scattering power  $Pd^2$  through the plane. For defects larger than the wavelength, the light diverges over a solid angle of  $(\lambda/d)^2 \text{ sr}$ . The irradiance at plane P2 is then  $Pd^2/(z\lambda/d)^2 \text{ W/m}^2$ . Finally, if the integrated length of the mask edge is  $L$ , then there are  $N=L/d$  potential defects along the edge. The ratio of the pupil plane energy received from the defects to the energy received from the point source is the suppression limit, given by:

$$R = \frac{Ld^3(h+z)^2}{(\lambda hz)^2}, \quad d \gg \lambda \quad (1)$$

For the mask located midway between the point source and the pupil plane,  $h = z$  and the expression simplifies to:

$$R = \frac{4Ld^3}{(\lambda h)^2}, \quad d \gg \lambda. \quad (2)$$

When the defects are sub-wavelength,  $d \ll \lambda$ , the divergence angle is independent of diameter and light is scattered over a solid angle of  $\lambda \text{ sr}$ . In this case, the suppression limit is given by

$$R = \frac{Ld(h+z)^2}{\pi(hz)^2}, \quad d \ll \lambda \quad (3)$$

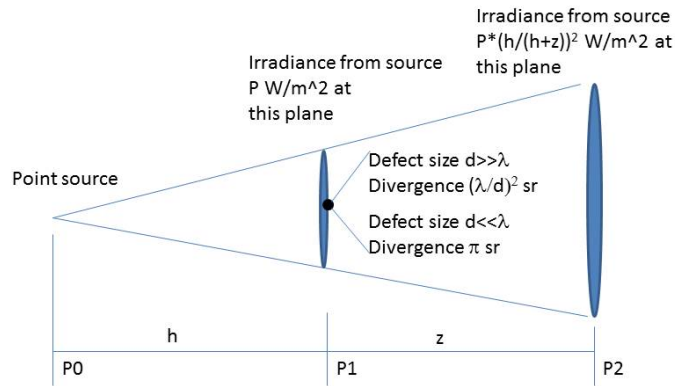


Figure 7: Schematic of analytical model for mask-defect scattering analysis.

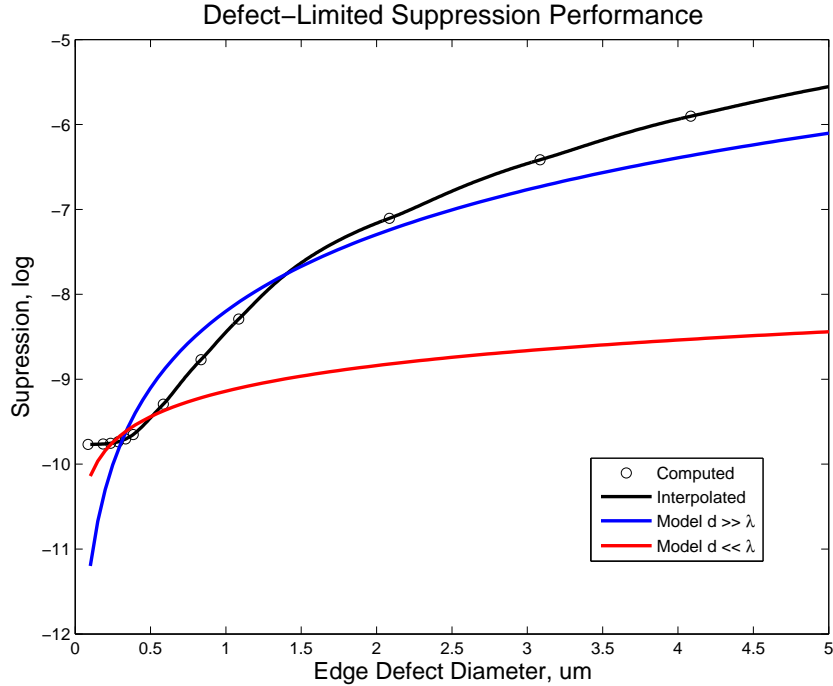


Figure 8: Comparison of simulated defect-limited suppression performance with analytical model for the 76-m propagation testbed at 633 nm.

which reduces to

$$R = \frac{4Ld}{\pi h^2} \quad (4)$$

when the mask is midway between P0 and P2.

In Figure 8, we have plotted the result of the numerical simulation including the edge-perturbation modeling the effect of circular edge defects. We also plot the expected suppression curves corresponding to the analytical model for both defects smaller and greater than the wavelength  $\lambda$ . The curve of the numerical simulation flattens around 600 nm which is the simulated wavelength as predicted by the analytical model; we observe a transition region around 1  $\mu\text{m}$  where the numerical result is between the two model curves. For very small defect sizes, the suppression approaches the actual diffraction-limit of the occulter design.

## 6. CONCLUSIONS

In this paper, we have used the optical model previously introduced and verified on the propagation distance-limited testbed at Princeton<sup>4</sup> to determine the expected performance for an occulter mask scaled for a longer propagation-distance testbed featuring the same type of edge errors that have limited the performance of the current experiment. This numerical simulation has been used to show the sensitivity to total propagation distance available and it was shown that a testbed scaled at 76-m and with mask designs specified at sub- $\mu\text{m}$  will achieve a suppression level better than  $10^{-8}$ . For an improved manufacturing process with 0.2 – 0.5  $\mu\text{m}$  features the achievable suppression performance is better than  $10^{-9}$ . These correspond to contrast levels better than  $10^{-10}$  consistent with the level required for an occulter mission capable of imaging exo-Earths.

We have also developed an analytical model to predict the suppression limitation due edge defects at the occulter edges and compared this model with the numerical simulations. The analytical model can be easily used to examine trends and predict scaled performance of an occulter mask for different manufacturing accuracies without requiring a complicated numerical simulation.

In this investigation we have maintained the same occulter design to study the effect of the total available distance for the propagation testbed and to choose a location for the expanded testbed. Additional performance improvements can be achieved compared to what we have presented here in part by optimizing the occulter mask to minimize the perimeter of the occulter mask.<sup>12</sup>

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