Development of a Phasor Diagram Creator to Visualize the Piston and Displacer Forces in an Advanced Stirling Convertor

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Abstract. The steady state, nearly sinusoidal behavior of the components in a Free Piston Stirling Engine allows for visualization of the forces in the system using phasor diagrams. Based on Newton's second law, $F=ma$, any phasor diagrams modeling a given component in a system should close if all of the acting forces have been considered. Since the Advanced Stirling Radioisotope Generator (ASRG), currently being developed for future NASA deep space missions, is made up of such nearly sinusoidally oscillating components, its phasor diagrams would also be expected to close. A graphical user interface (GUI) has been written in MATLAB by taking user input data, passing it to Sage, a 1-D thermodynamic modeling program used to model the Stirling convertor, running Sage and then automatically plotting the phasor diagrams. Using this software tool, the effect of varying different Sage inputs on the phasor diagrams was determined. The parameters varied were piston amplitude, hot end temperature, cold end temperature, operating frequency, and displacer spring constant. By using these phasor diagrams, better insight can be gained as to why the convertor operates the way that it does.

Keywords: Advanced Stirling Radioisotope Generator, Stirling convertor, Sage, phasors, dynamic analysis

INTRODUCTION

The Advanced Stirling Radioisotope Generator (ASRG) is a radioisotope power system being developed for future deep space missions. One of the main components of the ASRG is the Advanced Stirling Convertor (ASC), developed by Sunpower, Inc. The ASC is a free-piston Stirling engine that uses the heat input from a General Purpose Heat Source (GPHS) to heat the working fluid, helium, and through the Stirling cycle convert heat energy to electrical power. The piston is put into motion by the alternator, and the movement of the piston generates a pressure wave in the working fluid. However the displacer and piston are not oscillating in phase, and so while the piston will generates the pressure wave, the motion of the displacer and the shuttling of gas through the heat exchangers and regenerator determines the phase shift of the pressure wave, resulting in net positive work on the piston. The piston is connected to an alternator, so that once the system is started, the movement of the piston will generate electrical power. Both the displacer and the piston are connected to different springs (a mechanical spring is connected to the displacer and gas springs act on the piston), which provide restoring forces to each component. The alternator provides additional spring force to the piston, in addition to damping. As a result, all of the components in the system oscillate at the same frequency, but they are not all in the same phase (Walker and Senft, 1985). Therefore, it is useful to represent the forces on the piston and displacer as phasors, which only require a magnitude and phase angle shift. If all of the forces on the displacer and piston are accounted for, then the phasor diagram should form a closed polygon, graphically describing Newton's second law that $F=ma$.
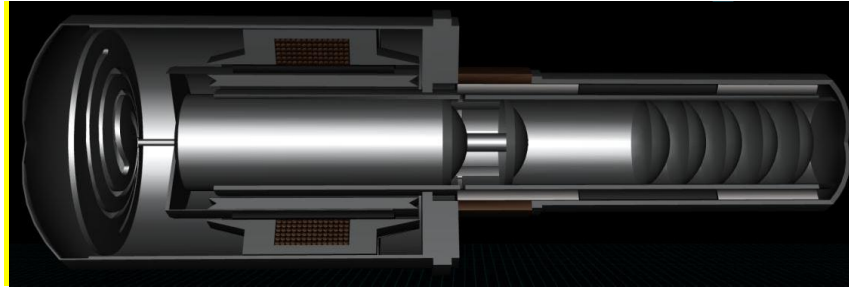
FIGURE 1. Cut-away of the ASC.

SOFTWARE DEVELOPED

The phasor plotting tool incorporates a user friendly graphical user interface (GUI) in MATLAB that is able to take input parameters for operating frequency (Hz), heat input from the GPHS (W) or hot-end temperature (K), cold-end temperature (K), displacer spring constant (N/m), and either a user-defined mean pressure (Pa) or a mean pressure can be calculated by the software as a function of hot-end temperature. These parameters are then sent by MATLAB through a .dll file to be used as inputs for Sage, a 1-D thermodynamic modeling software developed by Gedeon Associates, which runs the model until a converging solution is found (Gedeon, 1995). MATLAB then pulls the desired outputs from Sage and uses them to calculate the forces on the displacer and piston.
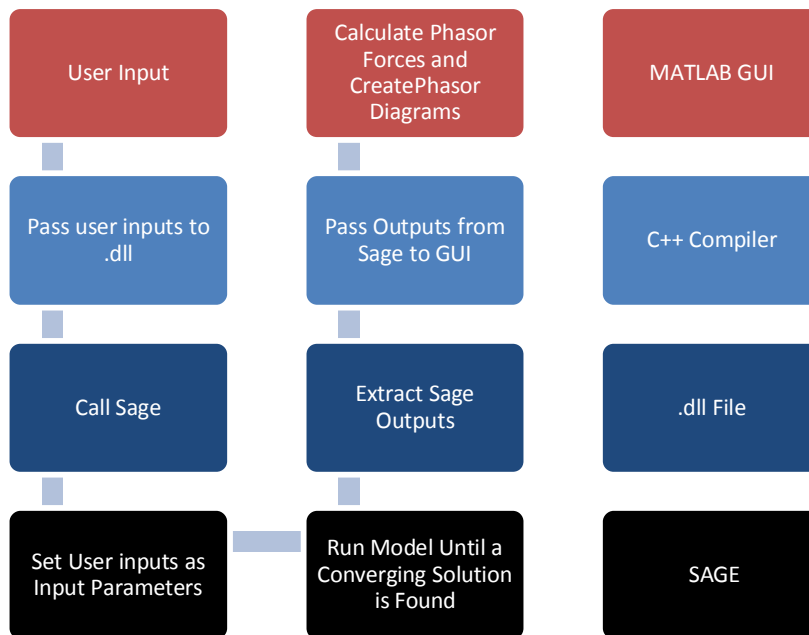

FIGURE 2. Schematic of MATLAB GUI operation with Sage.

Displacer with Separated Pressure Forces

There are two ways to represent the forces acting on the displacer in the ASC. One method is to look at the force on the displacer from the gas in the expansion space and the gas from the compression space. Although both spaces are connected through the regenerator and are parts of the same volume, the gases are actually represented by slightly different pressure waves (different in both magnitude and phase shift). This is the method used by Sage to model displacer forces.

The area on the side of the displacer facing the expansion space is greater than the area on the compression side, since the compression side has a displacer rod attached to the mechanical planar spring. Since P=F/A, a larger area

and a constant pressure will result in a larger force. The other forces acting on the displacer are a spring force and an inertial force. The inertial force is the force required to stop the motion of the displacer and is equal to ma. The spring force is a restoring force that should always be in phase with the inertial force and $180°$ to the displacer's position phasor. There is also a bounce spring force acting on the displacer through the displacer rod. However both the displacer rod area and the bounce space pressure variation are small and so that phasor's magnitude is a small fraction of the other forces and can often be ignored in analysis.

$$F_{EXP.D} = (P_{EXP})(A_{POS}) \tag{1}$$
$$F_{COM.D} = (P_{COM})(A_{NEG}) \tag{2}$$
$$F_{BS.D} = (P_{BS})(A_{ROD}) \tag{3}$$
$$F_S = -(K)(X_{DISP}) \tag{4}$$
$$F_{DISP} = (m_{DISP})(a_{DISP}) \tag{5}$$

Displacer with Pressure Drop

The other method of interpreting the forces on the displacer is to assume a uniform pressure wave in the working space volume (both the compression and expansion spaces), and then determine the pressure drop across the regenerator, heater, and cooler as a separate DeltaP force. The DeltaP phasor describes the work being done by the displacer to shuttle the gas back and forth between the hot and cold ends of the working space. This is the method used by NASA GRC's Stirling convertor system dynamic model (SDM) to determine displacer forces (Lewandowski and Regan, 2004). The DeltaP phasor diagram should have the same inertial, bounce spring, and mechanical spring force phasors as the Separated Pressure Forces Phasor Diagram described previously.

$$F_{P.D} = (P_{COM})(A_{ROD}) \tag{6}$$
$$F_{\Delta P.D} = (P_{Drop})(A_{ROD}) \tag{7}$$
$$F_{BS.D} = (P_{BS})(A_{ROD}) \tag{8}$$
$$F_S = -(K)(X_{DISP}) \tag{9}$$
$$F_{DISP} = (m_{DISP})(a_{DISP}) \tag{10}$$

Piston

The pressure wave in the compression space imparts a driving force on the piston. Since the piston is connected to an alternator, there is also a force that the alternator applies to the piston to keep the entire system in its oscillating state, a force which generally contains both spring and damping components. The bounce spring also imparts a small restoring force to the piston in the same way the mechanical spring does with the displacer. Finally, there is an inertia force on the piston which is especially important because of the connection between the alternator and the piston. The piston's inertia phasor, in addition to the alternator phasor (a direct output of Sage) are necessary inputs into the electrical components of the ASRG system.

$$F_{COM.P} = (P_{COM})(A_P) \tag{11}$$
$$F_{BS.P} = (P_{BS})(A_P) \tag{12}$$
$$F_P = (m_P)(a_P) \tag{13}$$

GRAPHICAL USER INTERFACE

The graphical user interface designed provides a user friendly method to run the relatively complex Sage modeling software to generate phasor diagrams based on a specified set of inputs. Additionally, the user may specify the error tolerance, the file location of the model to be run, export collected data into Excel, and plot multiple phasor diagrams onto one plot in order to compare the effect of varying certain parameters. The user also has a choice of whether to fix the mean pressure or whether the pressure should be automatically calculated as a function of hot-end temperature, which was determined using an equation that was derived theoretically and verified by a test of the ASC hardware.
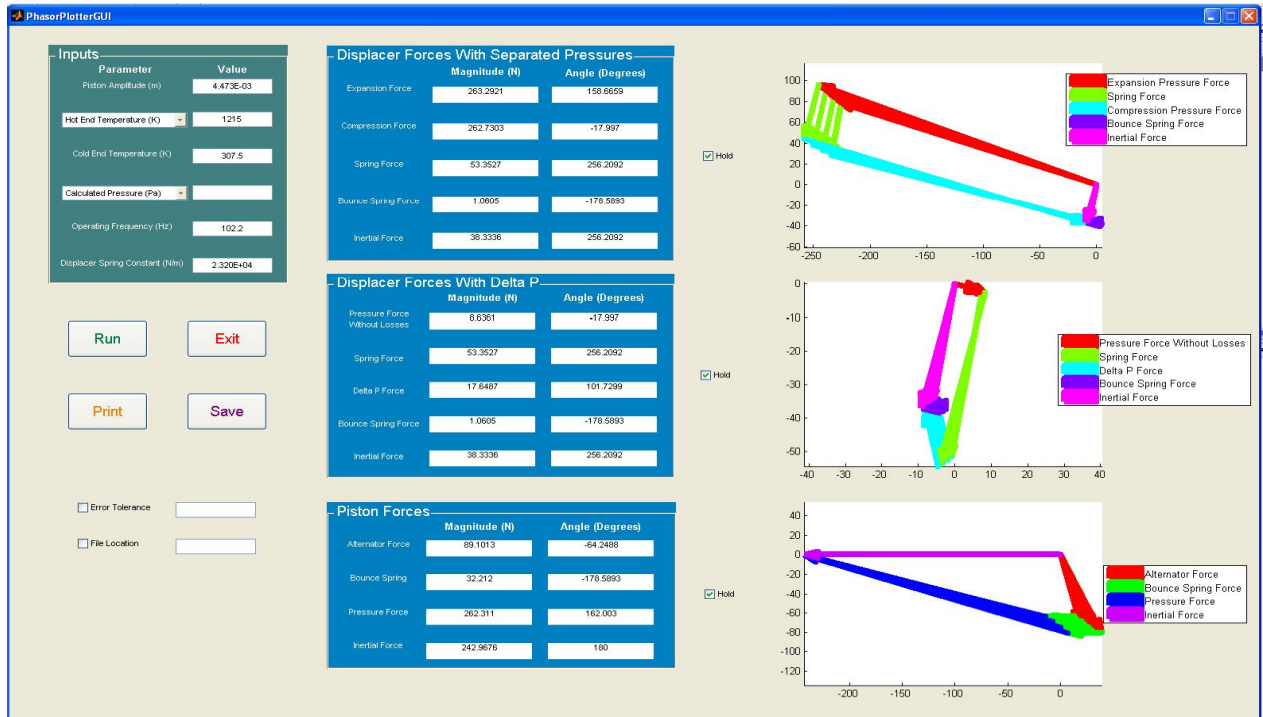
FIGURE 3. A sample run of the MATLAB GUI with Varying Temperature Inputs.

DETERMINING HEAT INPUT

The user also has the option of specifying heat input in watts or hot-end temperature. If the user provides heat input, then the program will actually iterate to find the corresponding hot-end temperature. Iterating is necessary because the heat input from the GPHS for a given insulation environment and hot-end temperature is not necessarily equal to the heat supplied to the ASC as calculated by Sage at that hot-end temperature. The amount of heat into the cycle is determined by subtracting the heat of the GPHS from the heat lost through imperfect insulation.

$$Qin\_net=Qin\_GPHS-Qinsulation\ loss \qquad (14)$$

However, the insulation loss is actually a function of the hot-end temperature, which is a function of the Qin_net, and as mentioned, the Qin_net is affected by the insulation loss.

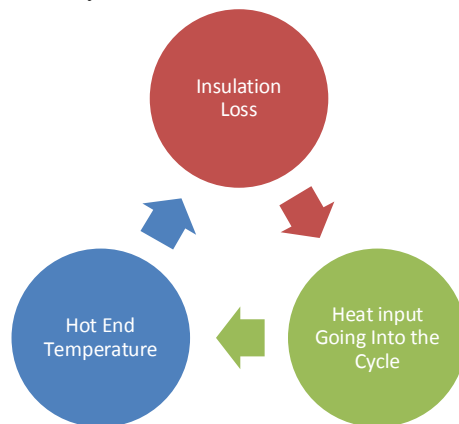**FIGURE 4.** Diagram Description of Heat Input Converger Parameter Relationships.

PHASORS

A useful way to visually represent sinusoidal functions that have the form

$$F(t) = A\cos(\omega t + \varphi) \tag{15}$$

is to plot them as phasors. A phasor is a vector whose magnitude is the amplitude of the cosine function and whose angle from the positive x axis is determined by the function's phase shift from a reference cosine function. If multiple functions all have the same frequency, then their phasors can be plotted on the same set of axes. Since the ASC's components all operate sinusoidally at the same frequency, all of the forces acting on the piston or displacer can be plotted on the same respective set of axes. The reason multiple sinusoidal functions would want to be plotted as phasors on the same axes is that phasors make it easier to add and subtract time varying sinusoidal functions. For example, when trying to verify *F=ma*, all of the force phasors added together should be equivalent to the inertia phasor. To add phasors together, the head of one phasor serves as the tail of the phasor to be added to it, and the angle of both phasors to the reference positive x axis does not change.
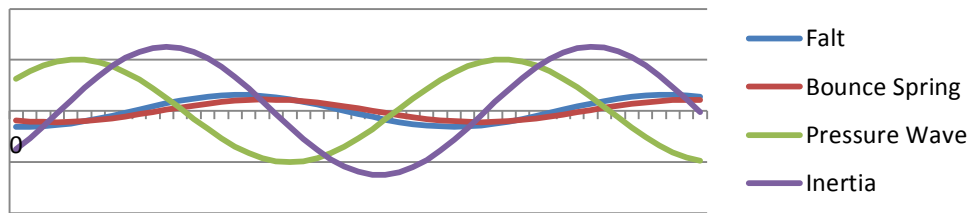

FIGURE 5. Time dependent curves of multiple piston forces.

Sage

Developed by Gedeon Associates, Sage is a modeling software used to accurately model Stirling engines using interconnected components. Using the inputs of hot-end temperature, cold-end temperature, operating frequency, displacer spring constant, mean pressure, and piston amplitude, Sage runs a series of nonlinear differential equations to find a converging solution to the system. User experience with ASC operating conditions is useful to run this model because the model requires the input parameters to be within a certain range for a converging solution to be found. Also, Sage does not automatically change the dependent parameters in certain components. For example setting the hot-end temperature in Sage is not enough; that temperature must also be set for the acceptor temperature, displacer profile, and any other temperature that is a function of hot-end temperature.

For the .dll file to interface with Sage, it is also necessary for all subcomponents to contain different variable names so that the .dll can pull information from the correct data location.

There are currently a number of compatibility issues to deal with when running this GUI. At the moment Sage is a 32-bit program whose dll requires a 32-bit operating system and can only interface with a 32-bit version of MATLAB. It was also found that one of the only compatible C++ compilers was Microsoft Visual C/C++ version 7.1 (also known as Microsoft Visual Studio .Net 2003). Efforts to migrate to 64-bit systems are being pursued.
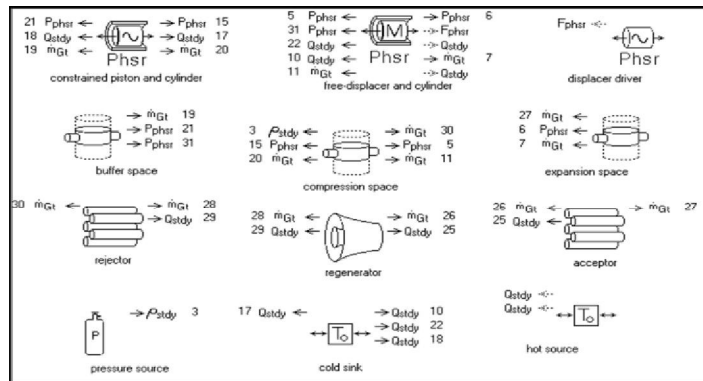

Figure 6. Components used in the ASC Sage model.

PARAMETER VARIATIONS

The effect of varying five parameters was analyzed to find how they change the phasor forces for each component, similar to the approach taken in Shaler and Lewandowski, 2011. The five parameters analyzed were piston amplitude, hot-end temperature, cold-end temperature, operating frequency, and displacer spring constant. The piston amplitudes were varied similarly to the cases done in Lewandowski and Schreiber, 2010. The other parameters where changed at increments so that the high or low range would not cause the Sage modeling software to not converge (although by no means are the values below the limits for the inputs of each parameter).While each parameter was varied at an evenly divisible increment, the other parameters were held constant at the values described in Table 1.

Table 1. Parameters used for Matlab GUI simulation.

	Piston Amplitude (m)	Hot-End Temperature (K)	Cold-End Temperature (K)	Operating Frequency (Hz)	Displacer Spring Constant ($\frac{N}{m^2}$)
Values Held Constant	.004473	1115	307.5	102.2	23200
Variation Range	.004173 to .005073	915-1215	307.5-457	102.2-105.2	22200-25200

Piston Amplitude

The piston amplitude can be constrained in Sage, which is a realistic assumption since the AC Bus voltage can be varied in response to other parameter changes to maintain a given piston amplitude.
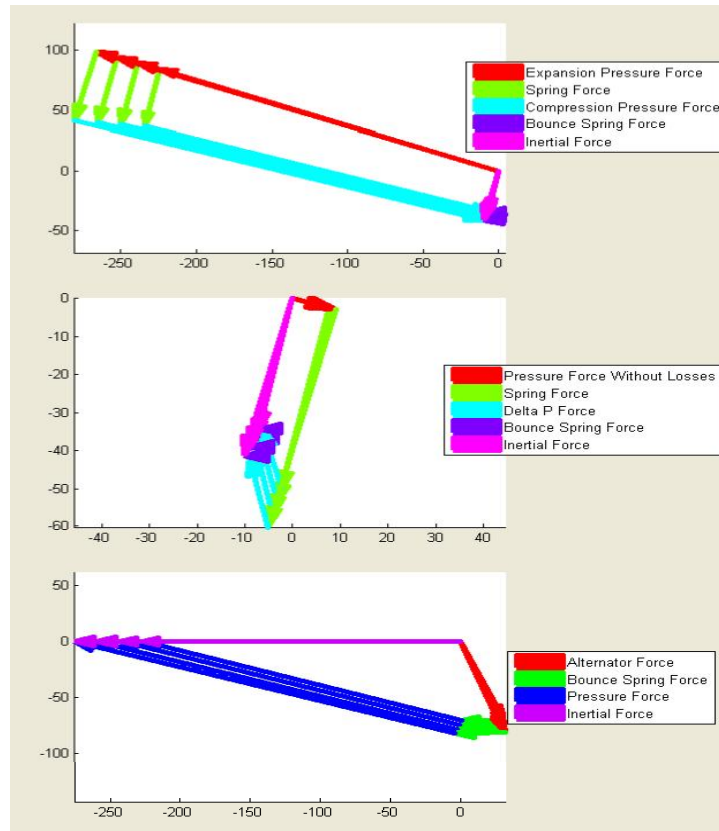

FIGURE 7. The effects of varying piston amplitude on the displacer (top two plots) and piston (bottom).

Varying piston amplitude seems to have the largest effect on the expansion and compression pressure force magnitudes and only a negligible effect on the other phasors. This is expected since a higher amplitude of the piston will compress the gas in each space further, resulting in higher pressure wave magnitude.
Piston amplitude does not appear to have a drastic effect on varying the DeltaP loss. Only very slight angle and magnitude effects can be seen.

Increasing piston amplitude logically increases the magnitude of the inertia of the piston, but also requires an increase in the alternator forcing phasor magnitude. As a result output power is increased.

Generally a restoring spring should have the opposite phase angle as the mass' displacement and the same angle as the mass' acceleration. This is true for the mechanical planar spring attached to the displacer. However, there is a very minor phase difference between the bounce spring force and the inertial force for the piston. This difference is attributable to the fact that there is a phase difference between the piston and the displacer; and more specifically, depending on the point in time, the bounce space gas may be compressed by the piston but is providing a restoring force to the displacer. And because the displacer rod is small, the shift in the force phasor for the bounce spring is also small. Also, thermodynamic losses from compressing and expanding the bounce space gas can shift the angle of the bounce spring phasor.
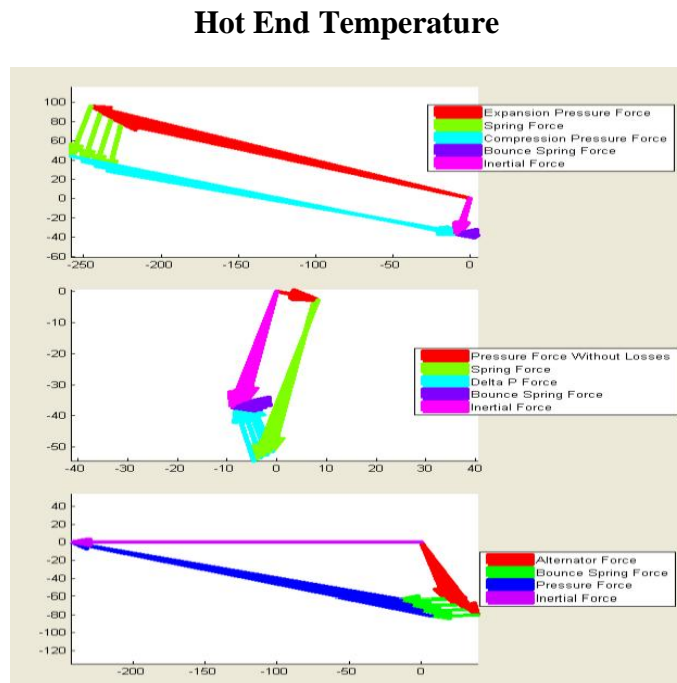
Hot End Temperature

FIGURE 8. The effects of varying hot end temperature on the displacer (top two plots) and piston (bottom).

Increasing hot-end temperature has minor variations in the phase angles of all of the force phasors. There is a small increase of the expansion and compression pressure magnitudes which can be explained by the application of Boyle's Law.

There is a negligible effect on the phasor diagram with DeltaP. The variations shown in the plot are misleading because the axis scale is very small.

Hot-end temperature has the largest effect on the pressure wave and the alternator forcing function. The effect of varying hot-end temperature has the most interesting results with respect to the piston. Changing hot-end temperature has absolutely no effect on the inertial force and a very meager one on the bounce spring force. As mentioned, the bounce spring force is dependent on the motion of the displacer as well as the piston, explaining the slight variation. Also, since the alternator forcing phasor is dependent on the other phasors, the real effect of hot end

temperature is on the pressure force. Interestingly both the pressure forces' magnitude and angle are varied as a function of hot-end temperature and the subsequent result is that as hot-end temperature increases, the alternator forcing angle and the alternator forcing magnitude both increase with respect to the positive x-axis and going counterclockwise.
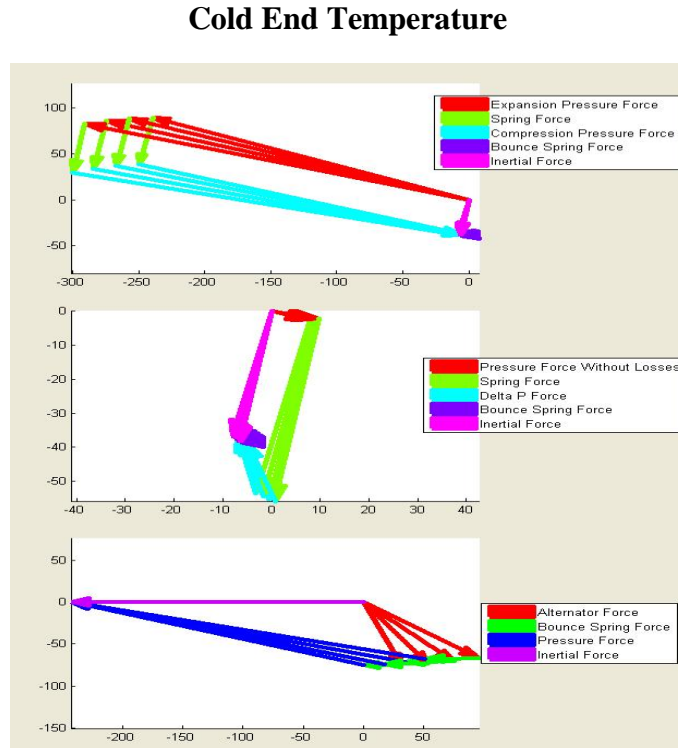
Cold End Temperature

FIGURE 9. The effects of varying cold end temperature on the displacer (top two plots) and piston (bottom).

Varying cold-end temperature has the largest effect on the expansion and compression pressure magnitudes. There are slight phase differences, which appear magnified by the large pressure magnitudes.

Cold-end temperature barely has an effect of any of the DeltaP phasors.

Varying cold-end temperature clearly has a large effect on the phase angles of the alternator forcing function on the piston. Interestingly, the damping component (y component) does not seem to be changing with variations in cold-end temperature. Since the electrical circuit of the ASRG depends on the piston velocity for voltage and the alternator force damping component for current, and the piston is being constrained so that its velocity is not changing, if the alternator force damping component does not change then the convertor's output power is not changing either. If hot-end temperature is fixed, decreasing cold-end temperature should result in greater power because of greater convertor efficiency (Carnot efficiency increases). Although Sage more accurately models Stirling convertor thermodynamics than SDM, the constancy of the damping component suggests that Sage is constraining a variable like the piston amplitude that would not be constrained in the actual system (Regan and Lewandowski, 2005).

Most of the variations in the phasor diagrams are also seen in the discussion of hot-end temperature. These similarities are expected since a Stirling engine depends on two thermal reservoirs and changing the temperature difference from either end will have some similar effects. The major difference however is that changing cold-end temperature has a more noticeable impact on the piston bounce spring magnitude than hot-end temperature did. This is also expected since the bounce space gas volume is an order of magnitude larger than the expansion space and because the bounce space temperature should be close to the cold-end temperature, varying that temperature will

have a large effect on the effective spring constant of the bounce spring. Changing cold-end temperature has a greater effect on the convertor's natural frequency than changing hot-end temperature.

To compare the phasor diagrams of the SDM model from the Sage phasor diagrams, increasing cold-end temperature has a similar effect on the angles and magnitudes of each of the forces (Shaler and Lewandowski, 2011).
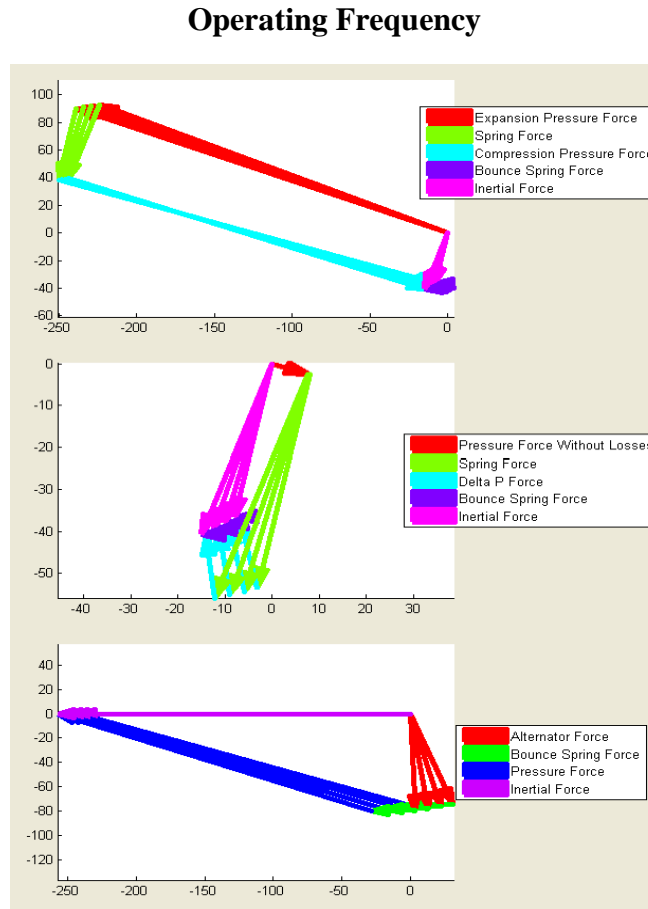
Operating Frequency

FIGURE 10. The effects of varying operating frequency on the displacer (top two plots) and piston (bottom).

Varying operating frequency has the most noticeable (albeit relatively small) effect on the expansion, compression and inertia magnitudes and the spring/inertia phase angles.

A small variation in operating frequency has a noticeable effect on Delta P's magnitude. A 3 Hz change brings down the Magnitude by about 2 Newtons. It is unlikely that such a relationship is linear and minimizing the Delta P loss does not necessarily equate to the overall system being more efficient.

Varying operating frequency has a very noticeable effect on the alternator phase angle, pressure magnitude, and inertia magnitude.

Interestingly, as operating frequency increases, both the expansion pressure angle and compression pressure angle decrease, so both phasors are moving clockwise. As a result, the DeltaP phasor is barely changing as a function of operating frequency. This is a trend also seen in SDM modeling of the ASC (Shaler and Lewandowski, 2011). With all other parameter variations (except for displacer spring constant), the expansion and compression pressure angles have always varied so that there is an effect on DeltaP.
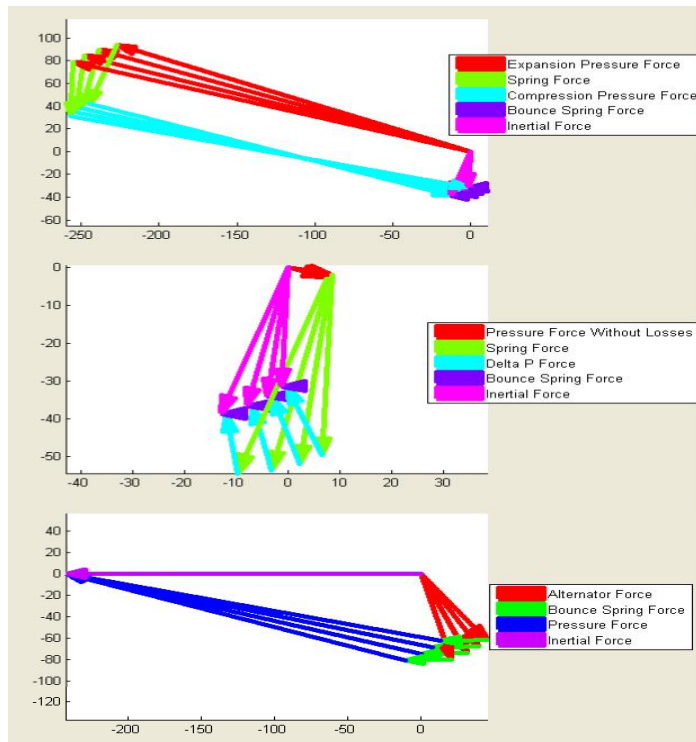
Displacer Spring Constant

FIGURE 11. The effects of varying displacer spring constant on the displacer (top two plots) and piston (bottom).

As with the operating frequency change, the inverse relationship between expansion and compression phasors is also seen.

The direct effect of changing the displacer spring constant is to change the displacer amplitude, as can be seen by the change in the magnitude of the displacer inertial force phasor. This results in the Delta P magnitude and phase shift increasing as the displacer spring constant increases. The change in displacer amplitude also affects output power, as indicated by the change in alternator damping. The alternator phase angle changes, indicating that the amount of spring force added by the alternator to changes due the change in displacer spring constant.

Of all of the parameters varied, displacer spring constant seems to have the largest effect on the damping component of the alternator forcing function. For all of the other parameter changes, the data regressions have all had an R^2 value close to 1 and never less than 0.95. For the spring constant, a quadratic regression was needed or else some of the R^2 values were around 0.92.

IMPROVEMENTS

The software tool has certainly automated the process of generating phasor diagrams from Sage. However, for a holistic understanding of how changing various parameters affect the ASRG, an alternator model needs to be added. Also, different insulation models (air, vacuum, etc.) would be useful to offer explanations to the results of different test situations. The heat input converger also needs improvement in that its converging time is dependent upon the initial guess. Currently, the iteration method to reach a converging solution is rather primitive and the integration of a solver would improve the software's performance. As mentioned previously, the ASC model used by Sage needs to be set up to allow parameters like piston amplitude to change when other parameters like cold-end temperature are changing to make the tool better mimic the effects seen during testing.

CONCLUSIONS

Phasor diagrams are a visually useful method of interpreting steady state sinusoidal behavior. For a system such as the ASC, all of whose components are oscillating nearly sinusoidally, it is especially useful to arrange the forces acting on a component into a phasor diagram, to find the relationship among forces as certain parameters are changed. Of the parameters changed, the inverse relationship of the expansion pressure and compression pressures as functions of displacer spring constant and operating frequency were most interesting. Displacer spring constant also seems to be a useful parameter to change in that it can be used to increase the damping component of the alternator forcing function which could generate more power by the system.

The software used has been successful at providing a user friendly method of varying parameters in Sage, running the model, and plotting the phasor diagrams. In comparison to the phasor diagrams from the SDM model, similar behaviors in the phasor diagrams are seen. However since the Sage model is a more accurate way to model the system than the SDM, better insight can be gained by looking at its phasor diagrams. As the alternator circuit is added to the tool, a more complete picture will be available for how and why the ASC operates the way that it does.

NOMENCLATURE

a_{DISP} = Displacer acceleration (N/m^2)
a_P = Piston acceleration (N/m^2)
A_{NEG} = Negative facing area of displacer (m^2)
A_P = Piston area (m^2)
A_{POS} = Positive facing area of displacer (m^2)
A_{ROD} = Area of displacer rod (m^2)
F_{Alt} = Required alternator forcing function (N)
$F_{BS.D}$ = Bounce spring force on displacer (N)
$F_{BS.P}$ = Piston bounce spring force (N)
$F_{COM.D}$ = Compression force on displacer (N)
$F_{COM.P}$ = Piston pressure force (N)
F_{DISP} = Displacer inertial force (N)
$F_{EXP.D}$ = Expansion force on displacer (N)
F_P = Piston inertial force (N)
$F_{P.D}$ = Displacer pressure force (N)
F_S = Spring force (N)
$F_{\Delta P.D}$ = Displacer DeltaP force (N)
K = Displacer spring constant (N/m)
m_{DISP} = Displacer mass (kg)
m_P = Piston mass (kg)
P = convertor mean pressure (psig)
P_{BS} = Bounce space pressure (Pa)
P_{COM} = Compression pressure (Pa)
P_{Drop} = Pressure drop across displacer (Pa)
P_{EXP} = Expansion pressure (Pa)
Q_{in_net} = Net heat input into the Stirling cycle (W)
Q_{in_GPHS} = heat input from the GPHS (W)
$Q_{insulation\ loss}$ = Heat lost through the insulation (W)
T_{PV} = Pressure vessel temperature (the APDG sets this equal to cold-end temperature plus 8 K) (K)
T_{ACC} = Acceptor temperature = hot-end temperature (K)
X_{DISP} = Displacer position (m)

ACKNOWLEDGEMENTS

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