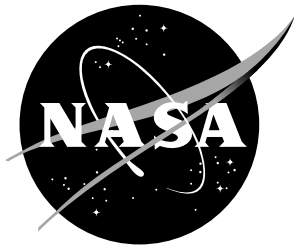


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# Transition Delay in Hypersonic Boundary Layers via Optimal Perturbations

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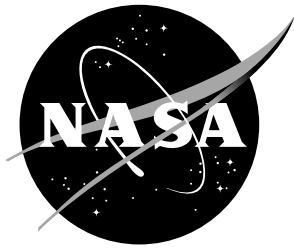
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## Abstract

The effect of nonlinear optimal streaks on disturbance growth in a Mach 6 axisymmetric flow over a  $7^\circ$  half-angle cone is investigated in an effort to expand the range of available techniques for transition control. Plane-marching parabolized stability equations are used to characterize the boundary layer instability in the presence of azimuthally periodic streaks. The streaks are observed to stabilize nominally planar Mack mode instabilities, although oblique Mack mode disturbances are destabilized. Experimentally measured transition onset in the absence of any streaks correlates with an amplification factor of  $N = 6$  for the planar Mack modes. For high enough streak amplitudes, the transition threshold of  $N = 6$  is not reached by the Mack mode instabilities within the length of the cone, but subharmonic first mode instabilities, which are destabilized by the presence of the streaks, reach  $N = 6$  near the end of the cone. These results suggest a passive flow control strategy of using micro vortex generators to induce streaks that would delay transition in hypersonic boundary layers.

## 1 Introduction

Laminar-turbulent transition of boundary layer flows can have a strong impact on the performance of hypersonic vehicles because of its influence on the surface skin friction and aerodynamic heating. Therefore, the prediction and control of transition onset and the associated variation in aerothermodynamic parameters in high-speed flows is a key issue for optimizing the performance of the next-generation aerospace vehicles.

Under low levels of background disturbances, transition is initiated by the exponential amplification of linearly unstable eigenmodes, i.e., modal instabilities of the laminar boundary layer. In two-dimensional boundary layers, different instability mechanisms dominate the exponential growth phase depending on the flight speed. Planar, i.e., two-dimensional, Tollmien-Schlichting (TS) waves are the most unstable in the incompressible regime, whereas oblique first mode instabilities correspond to the most amplified disturbances in supersonic boundary layers. The hypersonic regime is again dominated by the growth of planar acoustic waves of the second mode, i.e., Mack mode type [1]. In the presence of sufficiently strong external disturbances in the form of either freestream turbulence (FST) or three-dimensional wall roughness, streamwise streaks involving alternately low and high streamwise velocity have been observed to appear in incompressible boundary layers [2]. Further research in the incompressible regime has shown that high amplitude streaks can become unstable to shear layer instabilities that lead to a form of “bypass transition” [3]. When the streak amplitudes are low enough to avoid these instabilities, i.e., when the background disturbance level is moderate, the streaks can actually reduce the growth of the TS waves as documented in both theoretical and experimental studies [4-6]. The stabilizing effect of stationary streaks in low-speed boundary layers has been utilized in passive flow control strategies to demonstrate

delayed onset of transition by using micro vortex generators (MVG) along the body surface [7, 8].

Despite the numerous research efforts focused on the tripping of hypersonic boundary layer flows by using roughness elements, there have been a few experimental and numerical studies citing a delay in transition under certain circumstances. Most of these studies used two-dimensional roughness elements. James [9] used fin-stabilized hollow tube models in free flight with a screw-thread type of distributed two-dimensional roughness. He found that for a given freestream Mach number between the range of 2.8 to 7, there exists an optimum roughness height for transition delay. Fujii [10] studied the effects of two-dimensional roughness by using a  $5^\circ$  half-angle sharp cone at a freestream Mach number of 7.1. He also observed transition delay for certain conditions when the wavelength of the wavy wall roughness was comparable to that of the Mack mode instabilities. More recently, Font et al. [11, 12] performed numerical and experimental studies, respectively, that were focused on the effect of two-dimensional surface roughness on the stability of a hypersonic boundary layer at a freestream Mach number of 6. The experiments [11] used a flared cone with strips of roughness in the Boeing/AFOSR Mach 6 Quiet Tunnel and supported the numerical predictions indicating a stabilizing influence on the amplification of Mack mode disturbances [12]. In particular, these studies showed that the most dominant Mack mode instability could be suppressed via judicious placement of the roughness elements along the surface of the cone. Among the limited experimental evidence of delayed transition in a hypersonic boundary layer in the presence of three-dimensional roughness elements is the study by Holloway & Sterrett [13], who used a single row of spherical roughness elements partially recessed within a flat plate model in the NASA Langley 20-Inch Mach 6 tunnel. Data for multiple Mach numbers at the boundary layer edge were obtained by varying the plate mounting angle. They found that, for cases with the smallest roughness diameters, transition was delayed for Mach numbers larger than 3.7, which approximately corresponds to the lower bound for second mode dominance over first mode instabilities in a flat plate boundary layer at typical wind tunnel conditions. Therefore, their results are suggestive of stabilizing influence of roughness induced streaks on Mack mode waves. When the roughness height becomes sufficiently large, the streaks can develop high-frequency instabilities that can lead to earlier transition [14] as found by Holloway & Sterrett [13].

Theoretical studies of the interaction between stationary disturbances and Mack mode instabilities in hypersonic boundary layers have been initiated in recent work. Li et al. [15] studied the interaction of Goertler vortices with Mack mode instabilities on a flared cone, demonstrating a possible route to transition via this interaction. Li et al. [16] studied the secondary instability of crossflow vortices in a hypersonic cone at angle of attack and found that nonlinearly saturated crossflow vortices destabilize the Mack modes. Also, after the completion of this work, the authors became aware of the recent publication by Ren et al. [17], who studied the stabilizing effect of weakly nonlinear suboptimal streaks and Goertler vortices on the planar first mode and Mack mode instabilities. They documented a slight reduction in the logarithmic amplification factor, i.e.,  $N$ -factor, relative to the baseline, zero-streaks case for both

a flat plate boundary layer with suboptimal streaks ( $\Delta N \approx 0.2$ ) and a concave plate with Goertler vortices.

The development of roughness-induced streaks is strongly dependent on the details of roughness element shape, height, and spanwise or azimuthal spacing. A conceptually simple model that can characterize as well as provide an upper bound on the transient algebraic growth and subsequent slow decay of boundary layer streaks due to arbitrary initial disturbances corresponds to the optimal growth theory; see Ref. [18] for a review. The transient growth arises as a result of the non-normality of disturbance equations, and the optimal growth theory seeks to maximize the disturbance growth between a selected pair of streamwise locations. Regardless of the flow Mach number, the disturbances experiencing the highest magnitude of transient growth have been found to be stationary streaks that arise from initial perturbations that correspond to streamwise vortices. The instabilities of optimal streaks with finite initial amplitudes in supersonic and hypersonic boundary layers has been addressed in recent work [19,20]; however, the effect of lower amplitude, i.e., stable or at most weakly unstable streaks, on the growth of Mack mode instabilities has not been studied as yet. The present work seeks to bridge this gap with the goal of developing a more thorough knowledge base for transition prediction in the presence of stationary streaks and potentially expand the range of available techniques for transition control at hypersonic edge Mach numbers.

To that end, we study the effect of a periodic array of finite-amplitude streaks on the dominant instability waves in axisymmetric or two-dimensional boundary layers at hypersonic Mach numbers, i.e., the Mack mode instabilities. Figure 1 shows a schematic of the flow configuration considered in this work. The geometry is a  $7^\circ$  half-angle circular cone with  $r_n = 0.126$  mm nose radius and  $L = 0.305$  m length. The freestream parameters ( $M = 6$ ,  $Re' = 18 \times 10^6/\text{m}$ ,  $T_\infty = 60.98$  K) are selected to match the flow conditions of a previous experiment in the VKI H3 hypersonic tunnel [21]. Experimental measurements and theoretical predictions based on quasi-parallel, linear stability theory (LST) and the nonparallel, parabolized stability equations (PSE) have confirmed that laminar-turbulent transition in this flow is driven by the modal growth of planar Mack mode instabilities [21]. The array of actuators shown in figure 1 is purely notional, as the analysis presented herein is based on boundary layer streaks resulting from the transient growth of an optimal initial perturbation.

## 2 Theory

Transient growth analysis is performed using the linear PSE as explained in [22,23]. In the PSE context, the perturbations have the form

$$\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \hat{\mathbf{q}}(\xi, \eta) \exp \left[ i \left( \int_{\xi_0}^{\xi} \alpha(\xi') d\xi' + m\zeta - \omega t \right) \right]. \quad (1)$$

The suitably nondimensionalized, orthogonal, curvilinear coordinate system  $(\xi, \eta, \zeta)$  denotes streamwise, wall-normal, and azimuthal coordinates and  $(u, v, w)$  represent the corresponding velocity components. Density and temperature are denoted by  $\rho$

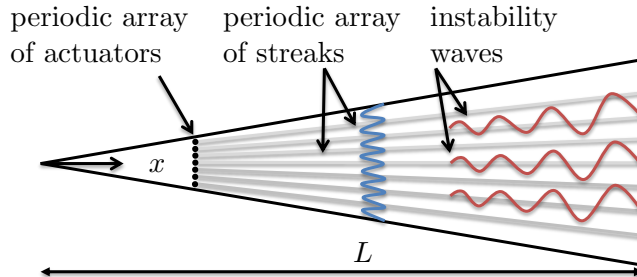


Figure 1. Sketch (side view) of the cone illustrating the present conceptual configuration. The wake of the periodic array of actuators generate the periodic array of streaks that modulate the instability waves.

and  $T$ . The Cartesian coordinates are represented by  $(x, y, z)$ . The vector of perturbation fluid variables is  $\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = (\tilde{\rho}, \tilde{u}, \tilde{v}, \tilde{w}, \tilde{T})^T$  and the vector of amplitude functions is  $\hat{\mathbf{q}}(\xi, \eta) = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T})^T$ . The streamwise and azimuthal wavenumbers are  $\alpha$  and  $m$ , respectively; and  $\omega$  is the angular frequency of the perturbation.

The nonlinear evolution of the stationary, finite-amplitude streaks is solved using an implicit formulation of the nonlinear plane-marching PSE [19, 24]. Subsequently, the linear form of the plane-marching PSE is used to study the linear, nonparallel stability characteristics of the modified basic state corresponding to the sum of the circular cone boundary layer and the finite-amplitude optimal disturbance. The initial disturbance profiles for the plane-marching PSE are obtained using a partial-differential-equation (PDE) based two-dimensional eigenvalue problem (EVP). In the plane-marching PSE context, the perturbations to the streak have the form

$$\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \hat{\mathbf{q}}(\xi, \eta, \zeta) \exp \left[ i \left( \int_{\xi_0}^{\xi} \alpha(\xi') d\xi' - \omega t \right) \right]. \quad (2)$$

### 3 Results

For the present case, we consider an initial disturbance location of  $x_0/L = 0.2$  and a final location of  $x_1/L = 0.4$ . The optimal azimuthal wavenumber that leads to a maximum energy gain,  $G = E(x_1)/E(x_0)$ , where  $E$  denotes Mack's energy norm [1], is found to be  $m = 50$ . For these parameters, the optimal energy gain in the limit of infinitesimal streak amplitudes is  $G = 4,017$ . The range  $[x_0, x_1]$  has been chosen to obtain appreciable streak amplitudes over a majority of the cone length as shown in figure 2, where  $As_u(\xi) = [\max_{\eta, \zeta}(\tilde{u}) - \min_{\eta, \zeta}(\tilde{u})]/2$ . The streak amplitude parameter  $A$  corresponds to the maximum streak amplitude  $As_u$  achieved by a linear perturbation with the same initial amplitude, which is given by  $A \times \sqrt{E_{lin, A=1}}$ , where  $E_{lin, A=1} = 6.01 \times 10^{-3}$ . Thus, the amplitude parameter  $A$  provides a convenient measure of the initial disturbance amplitude. As seen in figure 2, the nonlinear effects reduce the streak amplitude; and hence, for any given case,  $\max(As_u) < A$ . This maximum moves progressively upstream as the amplitude parameter  $A$  is increased. The modified basic state at  $x/L = 0.5$  is also shown in figure 2 for  $A = 0.05, 0.10$ , and  $0.20$ . The streak pattern becomes increasingly



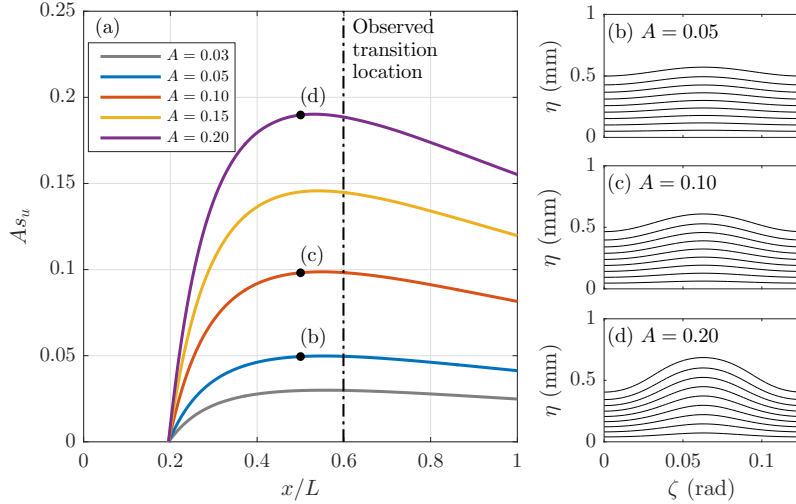


Figure 2. (a) Evolution of streak amplitudes, and isolines of streamwise velocity,  $\bar{u} = 0 : 0.1 : 0.9$ , in the crossplane at  $x/L = 0.5$  for (b)  $A = 0.05$ , (c)  $A = 0.10$ , and (d)  $A = 0.20$ .

evident as  $A$  becomes larger, exhibiting stronger crests and valleys in the azimuthal distribution of boundary layer thickness.

The instability characteristics of the modified, streaky boundary layer flow are examined next. Figure 3 shows the frequency dependence of spatial growth rates at a fixed axial location of  $x/L = 0.5$  as computed using quasi-parallel PDE-based EVP. Results are plotted for three different families of modes: mode  $MM_0$  reduces to a 2D Mack mode disturbance in the limit of  $A \rightarrow 0$ , whereas modes  $MM_{1,V}$  and  $MM_{1,S}$  correspond to oblique Mack mode disturbances (with fundamental azimuthal wavelength equal to streak spacing) of varicose and sinuous type, respectively. Mode shapes for each family at frequencies corresponding to peak local growth rate are shown in figures 3(b)–(d) for a streak amplitude of  $A = 0.10$ . Figure 3(a) shows a progressive reduction in the peak growth rate of  $MM_0$  modes with increasing streak amplitude, although the rate of decrease becomes smaller at higher values of  $A$ . The growth rate curves are displaced toward lower frequencies because the  $MM_0$  mode shape concentrates on the crests of the modified flow (i.e., regions of increased boundary layer thickness) as shown by figure 3(b). Also, figure 3(a) shows the opposite effect for the  $MM_{1,V}$  modes, which are strongest within the valleys of the modified basic state (figure 3(c)). The growth rates of the  $MM_{1,V}$  modes increase with streak amplitude up to  $A = 0.10$ , and then decrease at higher  $A$ . In this case, the peak growth frequencies increase with  $A$ . The upper neutral frequency also increases with  $A$  while the lower neutral frequency remains relatively unchanged, leading to a higher bandwidth of unstable modes at larger  $A$ . In contrast, the streaks have a stabilizing influence on the  $MM_{1,S}$  modes for all  $A$  (figure 3(a)), and similar to the  $MM_0$  modes, their peak growth frequency is reduced as a consequence of their mode shapes distribution that peaks in the neighborhood of the crests (figure 3(d)).

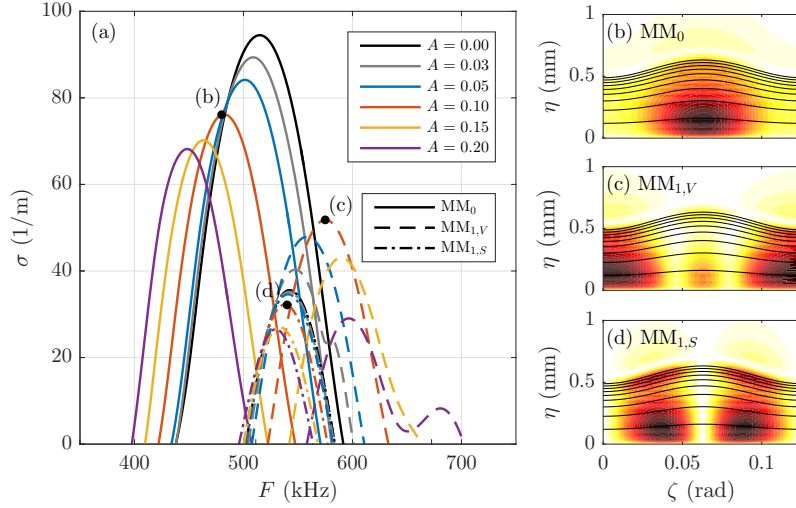


Figure 3. (a) Spatial growth rates ( $\sigma = -\alpha_i$ ) of planar Mack modes ( $MM_0$ ) and oblique Mack modes with varicose ( $MM_{1,V}$ ) and sinuous ( $MM_{1,S}$ ) mode shapes for selected streak amplitudes at  $x/L = 0.5$ . Also, isocontours of streamwise velocity magnitude for  $A = 0.10$  and frequencies (b)  $F = 480$  kHz for  $MM_0$ , (c)  $F = 575$  kHz for  $MM_{1,V}$ , and (d)  $F = 540$  kHz for  $MM_{1,S}$ . The color map of (b,c,d) varies from  $|\hat{u}| = 0$  (light yellow) to  $|\hat{u}| = 1$  (dark red). The isolines of basic state massflux,  $\bar{\rho}\bar{u} = 0 : 0.1 : 0.9$ , are added for reference.

To characterize the overall effect of streaks on the amplification of a Mack mode disturbance, we now examine the spatial evolution of a fixed frequency  $MM_0$  mode using the plane-marching PSE. The logarithmic amplification ratio based on the energy norm,

$$N = - \int_{x_{lb}}^x \alpha_i(x') dx' + 1/2 \log \left[ \hat{E}(x) / \hat{E}(x_{lb}) \right], \quad (3)$$

relative to the location  $x_{lb}$  where the disturbance first becomes unstable, is used as a measure of the disturbance amplification. Figure 4(a) illustrates the  $N$ -factor evolution of the  $MM_0$  mode with frequency  $F = 550$  kHz for the unperturbed basic state ( $A = 0.00$ ) and the streaky flow with  $A = 0.10$ , together with the mode shape at three axial positions ( $x/L = 0.40, 0.55$ , and  $0.70$  for the  $A = 0.10$  case in figures 4(b)–(d), respectively). The maximum value of the  $N$ -factor is  $N = 4.8$  for the perturbed case and  $N = 6$  for the unperturbed case. Furthermore, the location of peak  $N$ -factor is moved downstream from  $x/L = 0.6$  to  $x/L = 0.7$ . The  $N$ -factor evolution for the perturbed case also indicates a different growth rate behavior in comparison with the unperturbed case. As the streak amplitude increases to its maximum value near  $x/L = 0.55$  (figure 2(a)), the  $N$ -factor for the  $A = 0.10$  case progressively deviates from that in the unperturbed case because of a lower amplification rate; however, following a local  $N$ -factor peak near  $x/L = 0.55$ , there is an additional region of amplification that causes the  $N$ -factor to increase again to reach its overall maximum near  $x/L = 0.7$ . The accompanying evolution of the mode shape in figures 4(b)–(d) shows that the peak of the fluctuation is initially located in

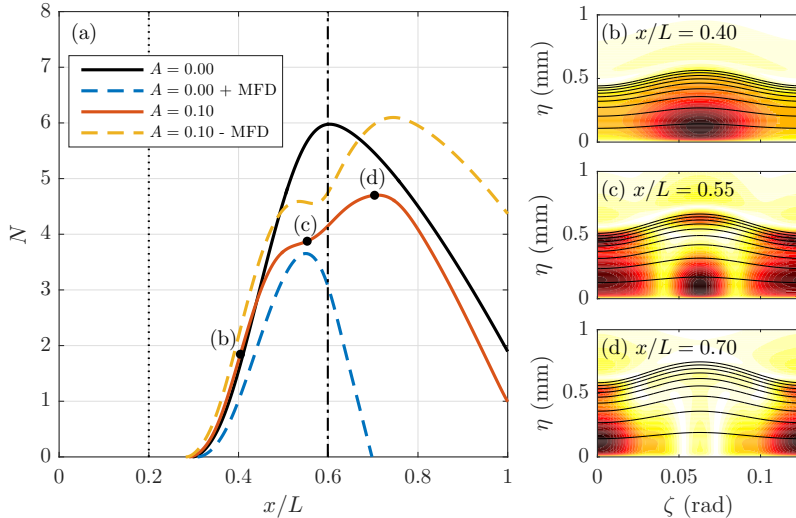


Figure 4. (a) Evolution of  $N$ -factors with frequency  $F = 550$  kHz for the unperturbed basic state ( $A = 0.00$ ), the unperturbed basic state plus the MFD of the  $A = 0.10$  perturbation ( $A = 0.00 + \text{MFD}$ ), the perturbed basic state ( $A = 0.10$ ), and the perturbed basic state without the MFD ( $A = 0.10 - \text{MFD}$ ). The vertical dotted line denotes the initial streak location and the dash-dotted line denotes the experimentally observed transition location. Also, isocontours of streamwise velocity magnitude for  $A = 0.10$  and positions (b)  $x/L = 0.40$ , (c)  $x/L = 0.55$ , and (d)  $x/L = 0.70$ . The color map of (b,c,d) same as figures 3(b)–3(d). The isolines of basic state massflux,  $\bar{\rho}\bar{u} = 0 : 0.1 : 0.9$ , are added for reference.

the crest of the modified flow and then develops a secondary peak in the valley region that eventually becomes the location of dominant fluctuations. Comparison of this mode shape evolution with the local results at  $x/L = 0.5$  (figure 3) reveals that the  $\text{MM}_0$  mode gradually evolves into the  $\text{MM}_{1,V}$  mode downstream. The dashed lines in figure 4(a) indicate the  $N$ -factor evolution for the same frequency using two “artificial” basic states: a two-dimensional basic state corresponding to the spanwise average of the  $A = 0.10$  flow, which corresponds to the unperturbed flow ( $A = 0.00$ ) plus the mean flow distortion (MFD) due to the streak, and the perturbed flow with  $A = 0.10$  minus the MFD of the perturbation. These extra cases are introduced to understand the primary mechanism for the effect of the streak on the reduced amplification of the  $\text{MM}_0$  mode. By comparing the  $N$ -factor of the first extra case ( $A = 0.00 + \text{MFD}$ ) with that for the unperturbed flow ( $A = 0.00$ ), we can see that the MFD has a strong stabilizing influence on the  $\text{MM}_0$  mode. The  $N$ -factor evolution for the second “artificial” case ( $A = 0.10 - \text{MFD}$ ) indicates a longer region of amplification relative to the baseline case ( $A = 0.00$ ), revealing the previously discussed downstream shift of the  $N$ -factor maximum for the total perturbed flow ( $A = 0.10$ ) to be the result of the azimuthal gradients of the modified flow.

The overall effect of the streaks is summarized in figure 5, where the  $N$ -factor envelope of the  $\text{MM}_0$  modes is plotted for each selected value of  $A$ . The primary focus of this work corresponds to the stabilizing effect of streaks on Mack mode

disturbances, which have been shown to cause transition in the present flow configuration [21]. For the conditions of the experiment [21], transition onset in the unperturbed cone boundary layer was measured to occur near  $x_{tr}/L = 0.6$ , where the peak  $N$ -factor of the  $MM_0$  modes is  $N = 6$ . Selecting this value as the transition threshold, figure 5 shows how the transition onset due to  $MM_0$  modes would be displaced downstream by the introduction of the optimal streaks. For the highest streak amplitude considered herein ( $A = 0.20$ ), the  $MM_0$  modes cannot reach the threshold  $N$ -factor over the entire length of the cone.

Due to the myriad paths to transition, however, the reduced growth of Mack modes becomes a “necessary” but not “sufficient” condition for delaying the onset of transition. In particular, oblique first mode disturbances may come into play at the typical conditions of wind tunnel experiments at Mach 6. Indeed, calculations show that oblique sinuous first mode disturbances with a subharmonic wavenumber of one half the streak wavenumber (denoted as  $FM_{1/2,S}$ ) are destabilized by the streaks and, therefore, may reduce the extent of transition delay. Therefore,  $N$ -factor envelopes for the  $FM_{1/2,S}$  modes at the selected streak amplitudes are also shown in figure 5. Observe that for streak amplitudes up to  $A = 0.10$ , the  $N$ -factor envelope of the  $MM_0$  modes lies above the envelope of the  $FM_{1/2,S}$  modes. But for  $A = 0.15$  and  $A = 0.20$ , the  $FM_{1/2,S}$  modes eventually overtake the  $MM_0$  modes. Between the latter two cases, the threshold  $N$ -factor of  $N = 6$  is reached further upstream for  $A = 0.20$  ( $x_{tr}/L = 0.92$ ), so the optimal streak amplitude for the present flow configuration is close to  $A = 0.15$ , which reaches  $N = 6$  at  $x_{tr}/L = 0.96$ , yielding a 60% increase in the length of the laminar flow relative to the unperturbed case.

We note that for streak amplitudes sufficiently higher than  $A = 0.20$ , the subharmonic first mode disturbances evolve into highly unstable streak instabilities that eventually reverse the transition delay and become the agent for bypass transition [19, 20]. Of course, due to the lower surface temperature at typical flight conditions, the role of these first mode waves diminishes in flight, implying that the streaks are likely to be even more effective in delaying transition on flight vehicles. Nonetheless, the computations presented in this work suggest that the effect of streaks on the onset of transition can be verified even under wind tunnel conditions. Prior to that, however, a thorough parameter study is desirable to address the effects of detuned disturbances and of potentially suboptimal streak profiles that are more readily realizable via the available set of actuation techniques. A parameter study for additional streak wavenumbers and excitation locations would also help with identifying the optimal flow control settings to maximize the transition delay for a given flow configuration.

## 4 Conclusions

The present results have demonstrated that finite-amplitude optimal growth streaks in a Mach 6 axisymmetric flow over a cone reduce the peak amplification of boundary layer instabilities, suggesting a delay in the onset of laminar turbulent transition. For large streak amplitudes, the transition threshold is not reached by originally-

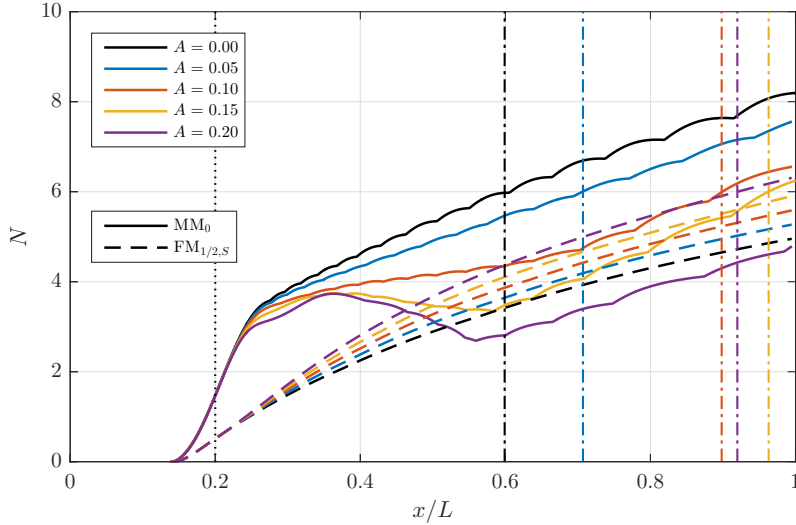


Figure 5. N-factor envelopes for nominally 2D Mack mode disturbances (mode  $MM_0$ , solid lines) with a frequency range  $F \in [350, 800]$  kHz and sinuous modes originating from first mode waves with one half the azimuthal wavenumber of underlying streaks (mode  $FM_{1/2,S}$ , dashed lines) with a frequency range  $F \in [50, 150]$  kHz. The vertical dotted line denotes the initial streak location, the black dash-dotted line denotes the experimentally observed transition location, and the colored dash-dotted lines denote the location where  $N = 6$  is reached for each corresponding streak amplitude.

dominant Mack mode instabilities, but subharmonic first mode waves destabilized by the streaks can limit the extent of transition delay. Results suggest a passive flow control strategy for transition delay in hypersonic flows and provide a framework for testing the strategy in a wind tunnel experiment.

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