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User’s Manual: Routines for Radiative Heat Transfer and Thermometry

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July 2016

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Abstract

Determining the intensity and spectral distribution of radiation emanating from a heated surface has applications in many areas of science and engineering. Areas of research in which the quantification of spectral radiation is used routinely include thermal radiation heat transfer, infrared signature analysis, and radiation thermometry. In the analysis of radiation, it is helpful to be able to predict the radiative intensity and the spectral distribution of the emitted energy. Presented in this report is a set of routines written in Microsoft Visual Basic for Applications® (VBA) (Microsoft Corporation, Redmond, Washington) and incorporating functions specific to Microsoft Excel® (Microsoft Corporation, Redmond, Washington) that are useful for predicting the radiative behavior of heated surfaces. These routines include functions for calculating quantities of primary importance to engineers and scientists. In addition, the routines also provide the capability to use such information to determine surface temperatures from spectral intensities and for calculating the sensitivity of the surface temperature measurements to unknowns in the input parameters.

Nomenclature

c	speed of light in a vacuum, 2.99792458 × 10^8 m/s
C_1	Planck's first constant, $2\bar{h}c^2$ = 1.191043 × 10^8 W-μm^4/m^2-sr
C_2	Planck's second constant, $\bar{h}c/k$ = 14,387.77 μm-K
C_3	Wien's displacement law constant, 2,897.77 μm-K
C_4	Constant giving maximum spectral intensity at peak wavelength, $C_1/C_3^5\left(e^{C_2/C_3}-1\right)$ = 4.09567 × 10^{-12} W/m^2-μm-sr-K^5
$D_i(\lambda)$	detector spectral response function for detector i
$e_b(T)$	total blackbody emissive power, W/m^2
$e(T)$	total emissive power, W/m^2
$\bar{h}$	Planck's constant, 6.626070040 × 10^{-34} J-s
$i_{b,\lambda}(\lambda,T)$	spectral emissive intensity of a perfect blackbody at wavelength λ and temperature T, W/m^2-sr-μm
$i_\lambda(\lambda,T)$	spectral emissive intensity of a non-blackbody at wavelength λ and temperature T, W/m^2-sr-μm
$i_b(T)$	total emissive intensity of a blackbody at temperature T, W/m^2-sr
$i(T)$	total emissive intensity of a non-blackbody at temperature T, W/m^2-sr
$I(\lambda_l,\lambda_u,T)$	general integrated spectral intensity function from wavelength λ_l to λ_u at temperature T
$I_0(\lambda_l,\lambda_u,T)$	integrated spectral intensity function from wavelength λ_l to λ_u at temperature T, W/m^2-sr
$I_1(\lambda_l,\lambda_u,T)$	integrated spectral intensity derivative with respect to temperature from wavelength λ_l to λ_u at temperature T, W/m^2-sr-K
$I_2(\lambda_l,\lambda_u,T)$	integrated first moment with respect to wavelength of the spectral intensity function from wavelength λ_l to λ_u at temperature T, W-μm/m^2-sr
$I_3(\lambda_l,\lambda_u,T)$	integrated first moment with respect to wavelength of the spectral intensity derivative with respect to temperature from wavelength λ_l to λ_u at temperature T, W-μm/m^2-sr-K
k	Boltzmann constant, 1.38064852 × 10^{-23} J/K
R	universal gas constant, 8.3144598 J/mol-K
T	actual true surface temperature, K

T_λ	measured equivalent blackbody temperature at wavelength λ assuming a perfect emitter, K
$\bar{\varepsilon}$	average emissivity across a waveband
$\varepsilon_\lambda(\lambda, T)$	monochromatic emissivity of a non-blackbody at wavelength λ and temperature T
$\bar{\varepsilon_i}$	wavelength-averaged emissivity for detector i
ε_r	emissivity ratio at two wavelengths λ_1 and λ_2, $\varepsilon_2/\varepsilon_1$
$\bar{\varepsilon_r}$	wavelength averaged emissivity for detector 1 and 2, $\bar{\varepsilon_2}/\bar{\varepsilon_1}$
λ_i	wavelength of detector i
λ_l	lower wavelength on wide-band detector
λ_u	upper wavelength on wide-band detector
λ_{il}	lower wavelength on wide-band detector i
λ_{iu}	upper wavelength on wide-band detector i
Λ	equivalent wavelength, $\lambda_1\lambda_2/(\lambda_2 - \lambda_1)$, μm
σ	Stefan-Boltzmann constant, 5.670367×10^{-8} W/m^2-K^4
Θ	elevation or altitude angle, rad
ϕ	circumferential or azimuthal angle, rad
Ω	solid angle, sr

Subscripts

b	blackbody
i	for detector i
λ	at wavelength λ

Note: Values for physical constants were obtained from ref. 1.

List of Acronyms

InGaAs	indium-gallium-arsenide
IR	infrared
LWIR	long-wavelength infrared
MWIR	medium-wavelength infrared
NASA	National Aeronautics and Space Administration
NESC	NASA Engineering Safety Center
SWIR	short-wavelength infrared
UV	ultraviolet
UVA	A-band ultraviolet
UVB	B-band ultraviolet
UVC	C-band ultraviolet
VBA	Visual Basic for Applications®

Routine Summary

The seven tables presented immediately below summarize each function included in the blackbody function library. The full definition of each function is included in the Section "VBA Routine Summaries."

Narrow-band spectral functions	
Routine	**Description**
bb_ibl	Calculate the blackbody emissive intensity at a given temperature and wavelength.
bb_tibl	Calculate the equivalent blackbody temperature at a specified wavelength for a given monochromatic blackbody emissive intensity.
bb_dibldt	Calculate the derivative of the blackbody emissive intensity with temperature.
bb_dlnibldlnt	Calculate the derivative of the logarithm of the blackbody emissive intensity with respect to the logarithm of temperature at a given temperature and wavelength.
bb_d2ibldt2	Calculate the second derivative of the blackbody emissive intensity with temperature.
bb_dibldl	Calculate the derivative of the blackbody emissive intensity with wavelength.
bb_dlnibldlnl	Calculate the derivative of the logarithm of the blackbody emissive intensity with respect to the logarithm of wavelength at a given temperature and wavelength.
bb_d2ibldl2	Calculate the second derivative of the blackbody emissive intensity with wavelength.
bb_ib	Calculate the total blackbody emissive intensity at a specified temperature.
bb_eb	Calculate the total blackbody emissive power at a specified temperature.
bb_lmax	Calculate the wavelength at the maximum spectral intensity for a given temperature using Wien's displacement law.
bb_imax	Calculate the maximum spectral intensity for a given temperature.
bb_tmax	Calculate the temperature for the maximum spectral intensity.

Spectral method functions	
Routine	**Description**
bb_tsi	Calculate the true temperature for a non-blackbody given the measured equivalent blackbody spectral intensity at a specified wavelength and the spectral emissivity.
bb_tli	Calculate the equivalent blackbody temperature at a specified wavelength for a non-blackbody given the blackbody spectral intensity at the true temperature and the spectral emissivity.
bb_tst	Calculate the true temperature for a non-blackbody given the equivalent blackbody temperature at a specified wavelength and the spectral emissivity.
bb_tstw	Calculate the true temperature for a non-blackbody given the equivalent blackbody temperature at a specified wavelength and the spectral emissivity assuming Wien's approximation is applicable. This routine can also be used to calculate the true temperature using the ratio method with the effective wavelength and the ratio temperature replacing the

	wavelength and blackbody temperature, again assuming Wien's approximation is applicable.
bb_tlt	Calculate the equivalent blackbody temperature at a specified wavelength for a non-blackbody given the true temperature and the spectral emissivity.
bb_dlntdlne	Calculate the sensitivity of the true temperature with the spectral emissivity.
bb_dlntldlne	Calculate the sensitivity of the equivalent blackbody temperature with the spectral emissivity.
bb_dlntdlntl	Calculate the sensitivity of the true temperature with the equivalent blackbody temperature. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the equivalent blackbody temperature.
bb_dlntldlnt	Calculate the sensitivity of the equivalent blackbody temperature with the true temperature. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the equivalent blackbody temperature.
bb_emiss	Calculate the spectral emissivity at a specified wavelength given the equivalent blackbody temperature and the true temperature.
bb_dlne2dlne1	Calculate the sensitivity of the emissivity at a given wavelength (2) with respect to the uncertainty in emissivity at another wavelength (1) which has been used to determine the true temperature. The sensitivity is expressed as the derivative of the logarithm of the emissivity at the second wavelength with respect to the logarithm of the emissivity at the first wavelength.

Narrow-band ratio functions	
Routine	**Description**
bb_erlam	Calculate the effective wavelength for a two-band radiation thermometer.
bb_ertemp	Calculate the ratio temperature for a two-band radiation thermometer.
bb_tratio	Calculate the true temperature for a two-band radiation thermometer given the temperatures at two wavelengths and an emissivity ratio.
bb_emissr	Calculate the effective emissivity ratio for a two-band radiation thermometer given the temperatures at two wavelengths and the true temperature.
bb_dlnerdlnt	Calculate the derivative of the logarithm of the emissivity ratio with respect to the logarithm of the true temperature for a two-band radiation thermometer given the temperatures at two wavelengths and emissivity ratio.
bb_dlnerdlnt1	Calculate the derivative of the logarithm of the emissivity ratio with respect to the logarithm of the equivalent blackbody temperature of the first detector for a two-band radiation thermometer given the given the detector wavelength and temperature.
bb_dlntrdlnt1	Calculate the derivative of the logarithm of the ratio temperature for a two-band radiation thermometer with respect to the logarithm of the first spectral temperature.
bb_dlnlrdlnl1	Calculate the derivative of the logarithm of the equivalent wavelength for a two-band radiation thermometer with respect to the logarithm of the first wavelength.

Wide-band functions	
Routine	**Description**
bb_iibls	Calculate the integrated emissive intensity for a wide- or narrow-band wavelength band using a series solution.
bb_fiibls	Calculate the spectral emissive intensity fraction between zero and specified wavelength using a series solution.
bb_iibl	Calculate the total integrated emissive intensity for a wide- or narrow-band using a quadrature integration scheme. The routine provides the integration of the emissive intensity, derivative of the emissive intensity with temperature, the first moment of the emissive intensity, the first moment of the emissive intensity, or the first moment of the derivative of the emissive intensity with temperature. Optionally, the integrand can be weighted by the emissivity, a detector response function, or both.
bb_tiiibl	Calculate the equivalent blackbody temperature across a waveband for a non-blackbody with an emissivity as a function of wavelength for a specified integrated intensity.
bb_ttiibl	Calculate the equivalent blackbody temperature across a waveband for a non-blackbody with an emissivity as a function of wavelength for a specified temperature.

Wide-band ratio functions	
Routine	**Description**
bb_itratio	Calculate the true temperature for a two-band radiation thermometer given the temperatures at the two bands and an emissivity ratio.
bb_iemissr	Calculate the effective emissivity ratio from a two-band radiation thermometer given the temperatures for the two bands and the true temperature.
bb_dlniedlnt	Calculate the derivative of the logarithm of the emissivity ratio with respect to the logarithm of the true temperature for a two-band radiation thermometer.
bb_dlniedlnt1	Calculate the derivative of the logarithm of the emissivity ratio with respect to the logarithm of the first band temperature for a two-band radiation thermometer.

Constant functions (values from ref. 1)	
Routine	**Description**
bb_C1	Return the value of C_1= 1.191043 × 10^8 W-μm^4/m^2-sr
bb_C2	Return the value of C_2= 14,387.77 μm-K
bb_C3	Return the value of C_3 = 2,897.77 μm-K
bb_C4	Return the value of C_4 = 4.09567 × 10^{-12} W/m^2-sr-μm
bb_SIGMA	Return the value of the Stephan-Boltzmann constant, σ = 5.670367 × 10^{-8} W/m^2-K^4
bb_RCONST	Return the value of the universal gas constant R = 8.3144598 J/mol-K
bb_CLIGHT	Return the value of the speed of light in a vacuum c = 2.99792458 × 10^8 m/s
bb_HBAR	Return the value of Planck's constant $\bar{h}$ = 6.62607004 × 10^{-34} J/s
bb_KBOLTZ	Return the value of the Boltzmann constant k =1.38064852 × 10^{-23} J/K

Configuration control functions	
Routine	**Description**
bb_version	Returns the version of the blackbody function library as a string.

Introduction

Determining the intensity and spectral distribution of radiation emanating from a heated surface has applications in many areas of science and engineering. Areas of research in which the quantification of spectral radiation is routinely used include thermal radiation heat transfer, infrared signature analysis, and radiative thermometry.[1] In these applications, the radiation is predominantly emitted across the entire electromagnetic spectrum (continuum radiation) as opposed to radiation emitted at specific wavelengths due to molecular or electronic transitions (line radiation).

In the analysis of radiation, it is helpful to be able to predict the heat transfer rate radiative intensity and the spectral distribution of the emitted energy. Presented in this report is a set of routines written in Microsoft Visual Basic for Applications® (VBA) (Microsoft Corporation, Redmond, Washington) and incorporating functions specific to Microsoft Excel® (Microsoft Corporation, Redmond, Washington) that are useful for predicting the behavior of heated surfaces. These routines include functions for calculating engineering quantities of primary importance to engineers and scientists. In addition, the routines also provide the capability to use such information to determine surface temperatures from spectral intensities and for calculating the sensitivity of these temperature measurements to unknowns in the input parameters.

Theoretical Background

This section presents the basic theory underlying radiative thermometry. The basic equations governing the emission of continuum radiation from a heated surface are presented first, followed by the derivation of the applicable equations for three common thermometry methods: 1) spectral method, 2) ratio method, and 3) multispectral method. The equations derived here are the basis for the routines described in the subsequent sections.

Basic Equations

Every surface emits radiation when heated. The emission of radiant energy from an ideal heated emitter (known as a black surface or a blackbody is governed by the Planck equation (ref. 1). The Planck equation relates the emitted radiation intensity for a perfect emitter $i_{b,\lambda}(\lambda, T)$, in power per unit area of emitting surface per unit solid angle Ω and per unit wavelength λ, to the wavelength and the surface temperature T, as shown in equation (1).

$$i_{b,\lambda}(\lambda, T) = \frac{1}{\lambda^5} \cdot \frac{C_1}{(\,e^{C_2/\lambda T} - 1)} \tag{1}$$

In equation (1), C_1 and C_2 are Planck's first and second constants 1.191043 × 10^8 W-μm^4/m^2-sr and 2,897.77 μm-K, respectively. A graphical representation of equation (1) is shown in figure 1.

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[1] Radiative thermometry is the technique for determining the temperature of a surface or a volume by measuring the electromagnetic radiation it emits.

Figure 1 illustrates that for a given temperature, the emitted intensity is distributed across an infinite range of wavelengths and for the high temperatures of interest to engineering (0 to 6,000 K), falls predominantly within the visible and infrared wavelength ranges. This range of electromagnetic energy, commonly referred to as thermal radiation, is divided into several subranges, as shown in figure 2.

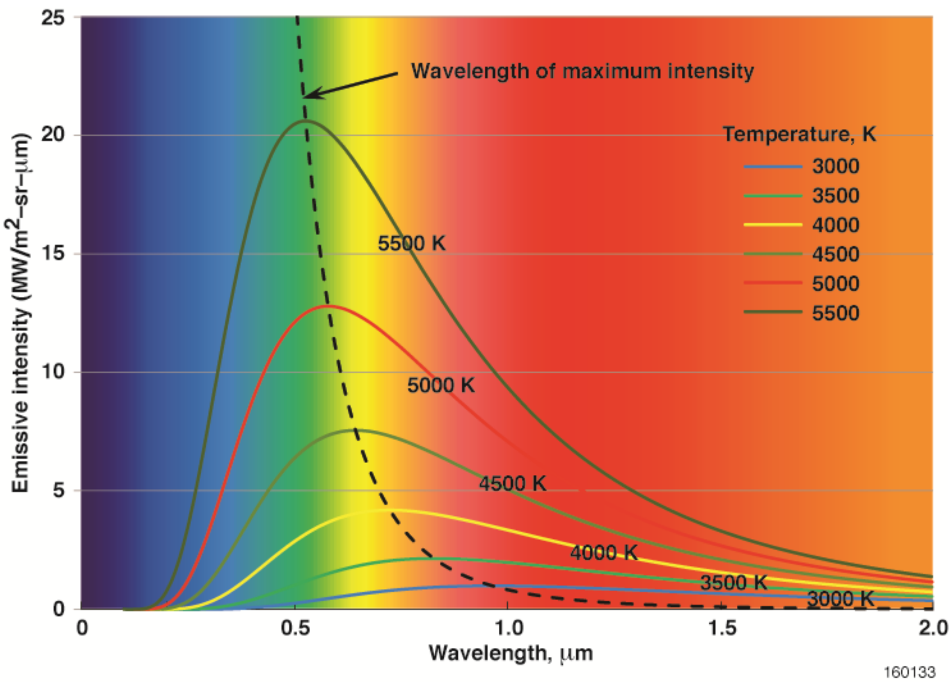

Figure 1. Emissive intensity versus wavelength calculated from the Planck distribution equation. The wavelength at the maximum spectral intensity calculated from Wien's displacement law is also shown.

The Planck equation has the following three important properties, as shown in figure 1:

- The radiant intensity increases with increasing temperature at all wavelengths and for all temperatures (that is, $di_{b,\lambda}/dT$ is always greater than zero).
- For any given temperature, the radiant intensity reaches a maximum at a specific wavelength, and for radiant intensities below the maximum there are two wavelengths at which the radiant intensity is equal.
- The wavelength at the maximum radiant intensity decreases as the temperature increases (Wien's displacement law).

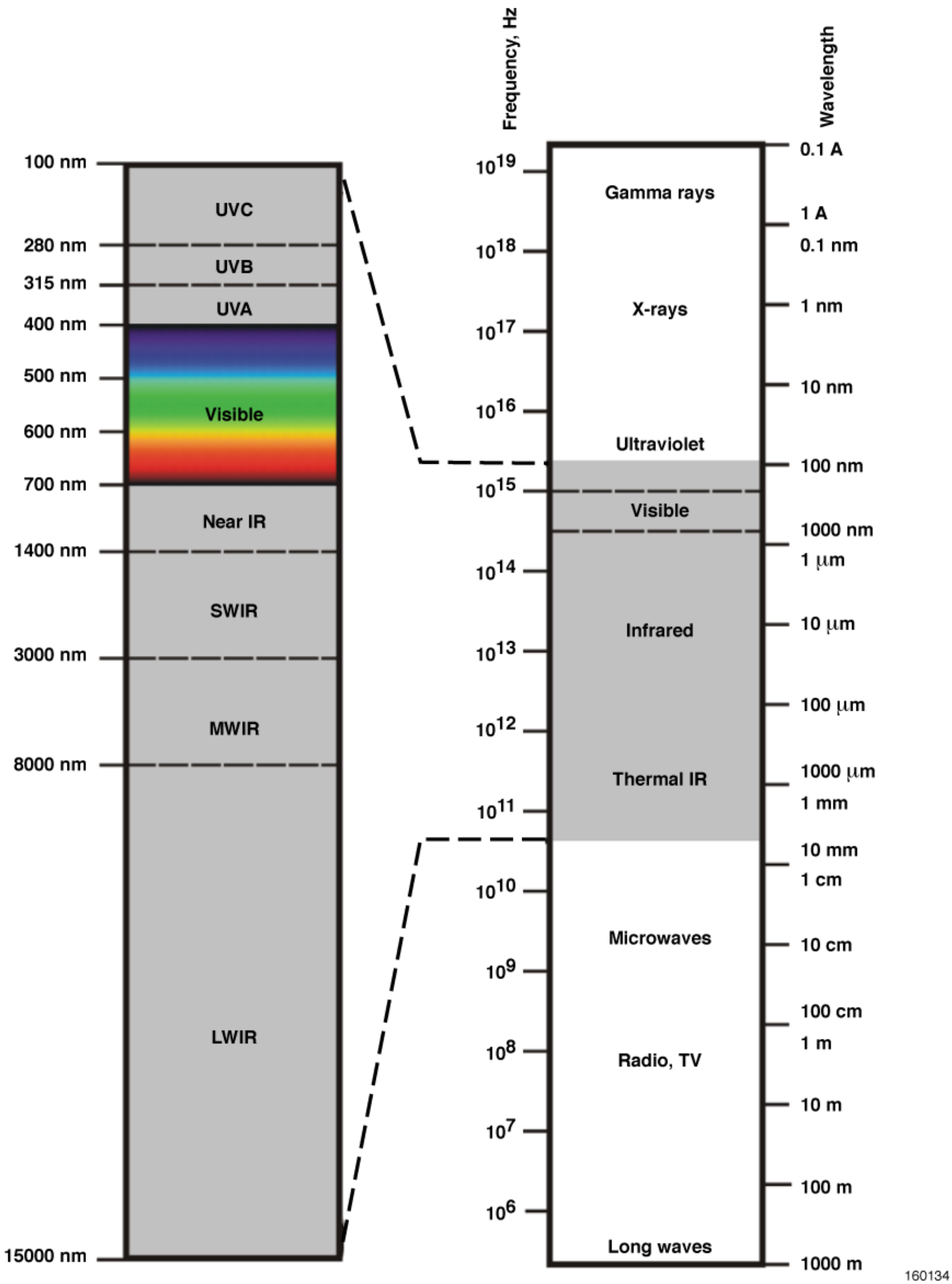

Figure 2. Electromagnetic spectrum showing ranges applicable to radiative thermometry.

The inversion of Equation 1 provides the black body temperature T for a given spectral intensity $i_{b,\lambda}$ at wavelength λ and is given in equation (2):

$$T = \frac{C_2}{\lambda} \cdot \frac{1}{\left(\ln \frac{C_1}{\lambda^5 i_{b,\lambda}} + 1\right)} \quad (2)$$

The wavelength of maximum spectral intensity λ_{max} is given by Wien's displacement law, as shown in equation (3):

$$T\lambda_{max} = C_3 = 2897.77 \text{ μm-K} \quad (3)$$

and varies from approximately 10 μm at room temperature to 0.7 μm at 6,000 K. According to equations (1) and (2), the maximum spectral intensity $i_{b,\lambda_{max}}$ at λ_{max} is equal to (eq. (4)).

$$i_{b,\lambda_{max}} = \frac{T^5}{C_3^5} \cdot \frac{C_1}{(e^{C_2/C_3} - 1)} = C_4 T^5 \quad (4)$$

Real materials do not act as perfect emitters, but instead emit at a rate less than a perfect emitter. The ratio of the intensity emitted by a real surface to that of a perfect emitter (blackbody) is (eq. (5)):

$$\varepsilon_\lambda = i_\lambda(\lambda, T)/i_{b,\lambda}(\lambda, T) \quad (5)$$

and defines the spectral emissivity ε_λ. The spectral emissivity is temperature-, wavelength-, and angle-dependent, and can vary substantially as the surface conditions change. In the work described in this report, emissivity is assumed to be independent of angle.

The derivative of the Planck equation (eq. (1)) with temperature is presented in equation (6):

$$\frac{di_{b,\lambda}}{dT} = \frac{C_2}{\lambda^6 T^2} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} \quad (6)$$

while the derivative with wavelength is presented in equation (7).

$$\frac{di_{b,\lambda}}{d\lambda} = \frac{1}{\lambda^6} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} \left(\frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 5 \right) \quad (7)$$

The derivatives expressed in equations (6) and (7) are useful in determining the uncertainty in radiative measurements. The derivatives can be more usefully represented in terms of sensitivities, which equal the fractional change in the intensity with fractional change in temperature or wavelength, as shown in equations (8) and (9):

$$\frac{T}{i_{b,\lambda}} \frac{di_{b,\lambda}}{dT} = \frac{d \ln i_{b,\lambda}}{d \ln T} = \frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} \quad (8)$$

$$\frac{\lambda}{i_{b,\lambda}}\frac{di_{b,\lambda}}{d\lambda} = \frac{d\ln i_{b,\lambda}}{d\ln\lambda} = \frac{C_2}{\lambda T}\cdot\frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T}-1)} - 5 \quad (9)$$

Derivatives discussed later in this report will be presented predominantly in this form.

The second derivatives of the Planck equation with respect to wavelength and temperature are useful for numerical analysis involving solutions for derivative values. The second derivative of intensity with temperature is identified in equation (10).

$$\frac{d^2 i_{b,\lambda}}{dT^2} = \frac{C_2}{\lambda^6 T^3}\cdot\frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T}-1)^2}\cdot\left[\frac{C_2}{\lambda T}\cdot\left(\frac{2\,e^{C_2/\lambda T}}{(e^{C_2/\lambda T}-1)} - 1\right) - 2\right] \quad (10)$$

The second derivative with respect to wavelength is given in equation (11).

$$\frac{d^2 i_{b,\lambda}}{d\lambda^2} = \frac{C_1}{\lambda^7}\cdot\frac{1}{(e^{C_2/\lambda T}-1)}\cdot\left[\frac{C_2}{\lambda T}\cdot\frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T}-1)}\cdot\left(\frac{2C_2}{\lambda T}\,\frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T}-1)} - 12 - \frac{C_2}{\lambda T}\right) + 30\right] \quad (11)$$

Often the cumulative intensity across a wavelength band is of interest, because many practical detectors measure across a wide spectral band. The cumulative intensity can be determined by integrating the Planck equation across the band bounded by the lower and upper wavelengths with the solution shown in equation (12).

$$\int_{\lambda_l}^{\lambda_u} i_{b,\lambda}(\lambda,\,T)\,d\lambda = \int_{\lambda_l}^{\lambda_u}\frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)}\,d\lambda = I(\lambda_l,\lambda_u,T) \quad (12)$$

No closed-form solution to equation (12) exists, however, so that the integration must be performed numerically, for example, by polynomial formulae (for example, Simpson's rule); Gaussian quadrature; or series solution.

A useful expression for the fraction of emitted intensity between zero and a specified wavelength is illustrated in eqs. (13) and (14) (ref. 2):

$$F_{0\to\lambda T} = \frac{15}{\pi^4}\sum_{n-1}^{\infty}\frac{e^{-n\xi}}{n}\left(\xi^3 + 3\frac{\xi^2}{n} + 6\frac{\xi}{n^2} + 6\frac{1}{n^3}\right) \quad (13)$$

where $\xi = \frac{C_2}{\lambda T}$ and $F_{0\to\lambda T} = \frac{e_{0\to\lambda T}}{\sigma T^4}$ (14)

so that the fractional intensity between two wavelengths λ_1 and λ_2 is as shown in equation (15).

$$F_{\lambda_l T\to\lambda_u T} = F_{0\to\lambda_u T} - F_{0\to\lambda_l T} \quad (15)$$

Equation (14) is included in the blackbody function library and within a higher level routine that integrates the energy across a finite energy band with an arbitrary starting wavelength. The blackbody function library described herein includes a VBA function to integrate the basic Planck equation (eq. 1) using Gaussian quadrature (eq. (16)).

$$I_0(\lambda_l,\lambda_u,T)=\int_{\lambda_l}^{\lambda_u} D(\lambda)\varepsilon_\lambda(\lambda)\,i_{b,\lambda}\,d\lambda=\int_{\lambda_l}^{\lambda_u} D(\lambda)\varepsilon_\lambda(\lambda)\frac{1}{\lambda^5}\cdot\frac{C_1}{(e^{C_2/\lambda T}-1)}d\lambda, \tag{16}$$

The library function also includes the ability to integrate three additional integrands derived from the Planck equation, as seen in equations (17)-(19).

1) The derivative of the Planck equation with temperature (eq. (5)):

$$I_1(\lambda_l,\lambda_u,T)=\int_{\lambda_l}^{\lambda_u} D(\lambda)\varepsilon_\lambda(\lambda)\frac{di_{b,\lambda}}{dT}d\lambda=\int_{\lambda_l}^{\lambda_u} D(\lambda)\varepsilon_\lambda(\lambda)\frac{C_2}{\lambda^6T^2}\cdot\frac{C_1e^{C_2/\lambda T}}{(e^{C_2/\lambda T}-1)^2}d\lambda \tag{17}$$

2) The first moment with respect to wavelength of the Planck equation:

$$I_2(\lambda_l,\lambda_u,T)=\int_{\lambda_l}^{\lambda_u} D(\lambda)\varepsilon_\lambda(\lambda)\,i_{b,\lambda}\lambda d\lambda=\int_{\lambda_l}^{\lambda_u} D(\lambda)\varepsilon_\lambda(\lambda)\frac{1}{\lambda^4}\cdot\frac{C_1}{(e^{C_2/\lambda T}-1)}d\lambda \tag{18}$$

3) The first moment with respect to wavelength of the derivative of the Planck equation with temperature:

$$I_3(\lambda_l,\lambda_u,T)=\int_{\lambda_l}^{\lambda_u} D(\lambda)\varepsilon_\lambda(\lambda)\frac{di_{b,\lambda}}{dT}\lambda d\lambda=\int_{\lambda_l}^{\lambda_u} D(\lambda)\varepsilon_\lambda(\lambda)\frac{C_2}{\lambda^5T^2}\cdot\frac{C_1e^{C_2/\lambda T}}{(e^{C_2/\lambda T}-1)^2}d\lambda \tag{19}$$

Equations (16)-(19) also incorporate weighting response functions for the detector sensitivity $D(\lambda)$ and the surface emissivity $\varepsilon_\lambda(\lambda)$. The inclusion of the response functions accounts for the wavelength varying response of a detector or the surface emissivity, if required.

Integrating over all wavelengths for a perfect emitter provides the total intensity of the emitted radiation and is shown in equation (20).

$$\int_0^\infty \frac{1}{\lambda^5}\cdot\frac{C_1}{(e^{C_2/\lambda T}-1)}d\lambda=i_b=\frac{\sigma}{\pi}T^4 \tag{20}$$

The quantity σ is the Stefan-Boltzmann constant and is equal to 5.6704×10^{-8} W/m^2-K^4.

To calculate the radiant power of a planar surface requires the integration of an arbitrary emitted ray into a hemispherical cap over the surface, as shown in figure 3 and equation (21) (ref. 3):

$$e_{b,\lambda}(\lambda,\,T)=i_{b,\lambda}(\lambda,\,T)\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\cos\theta\sin\theta d\theta d\phi=\pi i_{b\lambda}(\lambda,\,T) \tag{21}$$

The quantity $e_{b,\lambda}(\lambda,\,T)$ is known as the spectral blackbody emissive power. The integrated spectral emissive intensity across all wavelengths is the total emissive power and is given by equation (22):

$$e_b=\pi\int_0^\infty i_{b,\lambda}(\lambda,\,T)d\lambda=\sigma T^4 \tag{22}$$

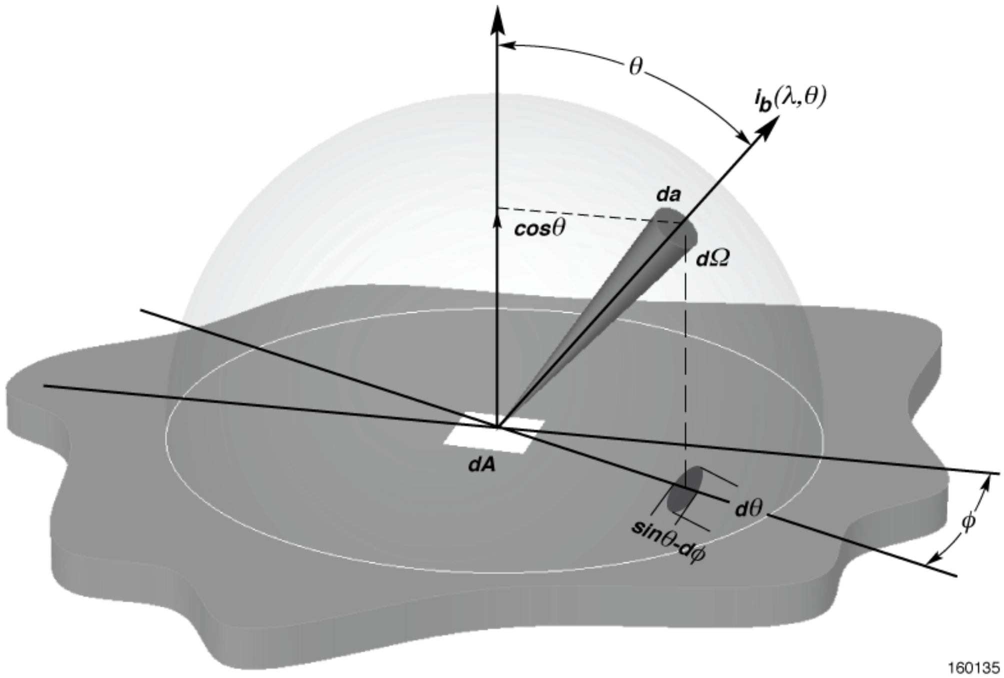

Figure 3. Integration of the spectral intensity through a cap to determine the emissive power.

The basic challenge in radiation thermometry, aside from actually measuring the emitted radiant intensity, is reducing the data to meaningful temperature and emissivity values. For any real surface, the emitted spectral intensity depends on the surface temperature and the spectral emissivity. The intensity measured from a calibrated detector can be readily converted to temperature using unity emissivity and the calibrated response function of signal voltage versus temperature. This temperature, known as the "brightness temperature" or "equivalent blackbody temperature", does not correspond to the actual or "true" surface temperature, since the surface of a real emitting object never behaves as a perfect emitter.

The brightness temperature or equivalent blackbody temperature corresponds to the lower limit on the true or actual surface temperature, since by definition, the emissivity of all real surfaces must be less than or equal to one. Using the definition of spectral emissivity in equation (4) along with the Planck equation from equation (1), the general expression for the conversion of measured spectral intensity to true surface temperature for a non-blackbody is presented in equation (23).

$$T = \frac{C_2}{\lambda} \cdot \frac{1}{\left(\ln \frac{\varepsilon_\lambda C_1}{\lambda^5 i_\lambda} + 1\right)} \tag{23}$$

Equivalently, the intensity measurement can be related to an equivalent blackbody temperature at the specified wavelength, so the surface temperature can be determined from equation (24).

$$T = \frac{C_2}{\lambda} \cdot \frac{1}{\ln[\varepsilon_\lambda (e^{C_2/\lambda T_\lambda} - 1) + 1]} \tag{24}$$

If the value of $\exp(C_2/\lambda T_\lambda)$ is much greater than 1 (Wien's approximation, ref. 3), then equation (24) can be simplified to the form shown in equation (25).

$$\frac{1}{T} \doteq \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda \tag{25}$$

It is not possible to decouple or separate the effect of temperature and emissivity from only a single intensity measurement. Two or more measurements must be analyzed together to give both temperature and emissivity results. The methods used to decouple the effect of temperature and emissivity from the measured radiometric signals have been the subject of extensive analysis. Several methods are available, each utilizing different assumptions and requiring different levels of effort. Three of the current methods that have been used to reduce data taken from measurements are discussed: the spectral method; the ratio method; and multispectral methods (ref. 4 and 5).

Spectral Method

In the spectral method, the temperature of the emitting surface is inferred from the measured spectral intensity or equivalent blackbody temperature at a single wavelength by assuming a value for the surface emissivity using equation (24) or (25). Equations (24) and (25) can be used in principle to calculate the surface temperature exactly if the assumed emissivity is known precisely.

Generally, however, the emissivity is not known exactly, so there will be some uncertainty in the emissivity value, which will result in a corresponding error in the calculated temperature. The sensitivity of the inferred true temperature due to uncertainties in the assumed emissivity can be computed by differentiating equation (24) with respect to the surface emissivity to obtain equation (26).

$$\frac{d \ln T}{d \ln \varepsilon_\lambda} = -\frac{\lambda T}{C_2} \cdot \frac{\left(e^{C_2/\lambda T} - 1\right)}{e^{C_2/\lambda T}} \tag{26}$$

If the wavelength is short, such that the quantity $\exp(C_2/\lambda T)$ is much greater than 1 (Wien's approximation), the sensitivity of equation (26) simplifies to equation (27).

$$\frac{d \ln T}{d \ln \varepsilon_\lambda} \doteq -\frac{\lambda T}{C_2} \tag{27}$$

Equation 27 shows that the magnitude of the uncertainty in the inferred true surface temperature decreases with decreasing wavelength and that short wavelength detectors have a lower uncertainty than longer wavelength detectors when used to determine the surface temperature.

Once the corrected blackbody temperature is determined, the resulting surface emissivities at longer wavelengths can be computed from the basic definition of emissivity (eq. (5)) and the Planck equation (eq. (1)), as shown in equation (28).

$$\varepsilon_\lambda = \frac{\left(e^{C_2/\lambda T} - 1\right)}{\left(e^{C_2/\lambda T_\lambda} - 1\right)} \tag{28}$$

Equations (26) and (28) can be differentiated to obtain an expression for the uncertainty in the calculated emissivities at other wavelengths, ε_2, with respect to the uncertainty in the emissivity at the reference wavelength ε_1, as in equation (29).

$$\frac{d \ln \varepsilon_2}{d \ln \varepsilon_1} = \frac{\varepsilon_1}{\varepsilon_2}\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{\lambda_1}{\lambda_2} \cdot \frac{\left(e^{-C_2/\lambda_1 T} - 1\right)}{\left(e^{-C_2/\lambda_2 T} - 1\right)} \tag{29}$$

If Wien's approximation is appropriate for both wavelengths, then equation (29) reduces to equation (30):

$$\frac{d \ln \varepsilon_2}{d \ln \varepsilon_1} = \frac{\varepsilon_1}{\varepsilon_2}\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{\lambda_1}{\lambda_2} \tag{30}$$

In the simplest terms, when the exponential terms are small the resulting uncertainty in the calculated emissivity at wavelength λ_2 from a temperature measurement at wavelength λ_1 is attenuated by the ratio λ_1/λ_2. The resulting uncertainty in the emissivity wavelength λ_2 is the ratio of the two measurement wavelengths multiplied by the relative uncertainty in the assumed emissivity at wavelength λ_1.

The equation (30) example indicates that the error in the temperature measurement is minimized as the detector wavelength decreases, while the uncertainty in the emissivity decreases as the wavelength increases. This reasoning suggests that to minimize the error in the temperature measurement, a detector with a very short wavelength should be selected, while to achieve a precise emissivity measurement, a detector with a very long wavelength should be selected.

There are, however, limitations on this method. When measuring temperature, the available detector signal decreases rapidly as the wavelength decreases. Therefore, there is a practical lower limit to the wavelength based on detector sensitivity, even though the theoretical uncertainty always decreases with wavelength. At the other extreme, while the relative error in the calculated emissivity does indeed decrease with increasing wavelength, the emissive properties of many materials vary significantly over a wide wavelength band. Thus, assurance should be made that the longer wavelength detectors are chosen in the spectral region wherein the emissivity is desired, or at the least, in a region wherein the emissivity is expected to be close to the emissivity of the desired wavelength.

For a given emissivity, the sensitivity of the true surface with respect to the equivalent blackbody temperature is as given by equation (31).

$$\frac{d \ln T}{d \ln T_\lambda} = \varepsilon_\lambda \frac{T}{T_\lambda} e^{\frac{C_2}{\lambda}\left(\frac{1}{T_\lambda} - \frac{1}{T}\right)} \tag{31}$$

Additionally, the effects of signal-to-noise ratios propagate in the same manner as those of the emissivities. The measured signal being directly proportional to the emissivity, the signal sensitivity is equal to the calculated emissivity sensitivity. That is, uncertainties in the measured detector signals affect the resulting temperature and computed emissivities at the long wavelengths in the same way as the assumed emissivity at the short wavelength.

Ratio Method

The ratio method attempts to simultaneously determine a single temperature and emissivity ratio that satisfies the measurements from two detectors at two independent wavelengths. The expression for the ratio method is obtained from the definition of the spectral emissivity in equation (5) for two wavelengths. Taking the ratio of the two emissivities at the two wavelengths results in the following relationship between the two equivalent blackbody temperatures T_{λ_1} and T_{λ_2} and the true surface temperature T (eq. (32)):

$$\frac{\left(e^{C_2/\lambda_2 T_{\lambda_2}} - 1\right)}{\left(e^{C_2/\lambda_2 T} - 1\right)} \cdot \frac{\left(e^{C_2/\lambda_1 T} - 1\right)}{\left(e^{C_2/\lambda_1 T_{\lambda_1}} - 1\right)} - \varepsilon_r = 0 \qquad (32)$$

where (eq. (33)):

$$\varepsilon_r = \frac{\varepsilon_1}{\varepsilon_2} \qquad (33)$$

Given the two measured equivalent blackbody temperatures T_{λ_1} and T_{λ_2} at wavelengths λ_1 and λ_2, the surface temperature is found that satisfies equation (32).

There is no analytic solution to equation (32), so the solution must be found numerically by iteration. If, however, Wien's approximation is applicable, then equation (32) can be reduced to equation (34):

$$\frac{1}{T} = \frac{1}{T_r} + \frac{\Lambda}{C_2} \ln \varepsilon_r \qquad (34)$$

where T_r is the ratio temperature given by equation (35):

$$\frac{1}{T_r} = \Lambda \left[\frac{1}{\lambda_1 T_{\lambda_1}} - \frac{1}{\lambda_2 T_{\lambda_2}} \right] \qquad (35$$

and Λ is an effective wavelength (eq. (36)).

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \qquad (36)$$

The ratio temperature is the temperature of an equivalent blackbody having the same ratio of spectral intensities at two specified wavelengths as that of the target material.

Equation (34) is the same form as equation (25) for the spectral method except that the emissivity ratio is used instead of the spectral emissivity ratio, the ratio temperature replaces the equivalent blackbody temperature, and the effective wavelength replaces the wavelength.

The uncertainty in the surface temperature with respect to uncertainty in the emissivity ratio can be determined by the reciprocal of equation (37).

$$\frac{d \ln \varepsilon_r}{d \ln T} = \frac{1}{T} \cdot \frac{\left(e^{C_2/\lambda_2 T} - 1\right)}{\left(e^{C_2/\lambda_1 T} - 1\right)} \cdot \left[\left(\frac{C_2}{\lambda_2 T}\right) \frac{e^{C_2/\lambda_2 T}\left(e^{C_2/\lambda_1 T} - 1\right)}{\left(e^{C_2/\lambda_2 T} - 1\right)^2} - \left(\frac{C_2}{\lambda_1 T}\right) \frac{e^{C_2/\lambda_1 T}}{\left(e^{C_2/\lambda_2 T} - 1\right)} \right] \qquad (37)$$

If Wien's approximation is applicable, then the sensitivity can be approximated by the analog of equation (26), seen as equation (38).

$$\frac{d \ln T}{d \ln \varepsilon_r} \doteq - \frac{\Lambda T}{C_2} \qquad (38)$$

Note that the equivalent wavelength Λ can be many times larger than either of the two individual wavelengths if the two wavelengths are closely spaced. The key concern is whether the decrease in emissivity uncertainty is more significant than the increase in the effective wavelength as the wavelengths are chosen closer together. Otherwise, the large increase in effective wavelength can render the ratio method highly inaccurate. Conversely, the advantage of the ratio method is that it may be possible to more accurately estimate the emissivity ratio over a small wavelength difference, and this method may reduce the uncertainty in the temperature measurement overall.

Many commercial systems that claim to measure "true" temperature contain two independent detectors operating at two distinct wavelengths between 0.5 and 1.0 μm. The devices also include a "ratio" setting that adjusts the measurements for variations in the assumed emissivity ratio of the surface. Obviously such a device can produce an accurate reading only if the emissivity ratio at the two wavelengths approaches a well-characterized value.

Once the surface temperature is determined, the emissivities at the two wavelengths can be calculated using equation (28), similar to the spectral method.

Other useful derivatives that quantify error analysis include the sensitivity of the emissivity ratio to the first equivalent blackbody temperature T_{λ_1} shown in equation (39):

$$\frac{d\ln\varepsilon_r}{d\ln T_{\lambda_1}} = \frac{C_2}{\lambda_1 T}\cdot\frac{e^{C_2/\lambda_1 T_{\lambda_1}}}{e^{C_2/\lambda_1 T_{\lambda_1}}-1} \tag{39}$$

and the derivative of the logarithm of the ratio temperature T_r with respect to the logarithm of the first equivalent blackbody temperature T_{λ_1} shown in equation (40):

$$\frac{d\ln T_r}{d\ln T_{\lambda_1}} = \frac{\Lambda}{\lambda_1}\frac{T_r}{T_{\lambda_1}} \tag{40}$$

and the derivative of the logarithm of the equivalent wavelength Λ with respect to the logarithm of the wavelength of the first detector $\lambda_1$for a two-band radiation thermometer shown in equation (41).

$$\frac{d\ln\Lambda}{d\ln\lambda_1} = 1 + \frac{\lambda_1}{\lambda_2-\lambda_1} \tag{41}$$

Multispectral Methods

Multispectral methods are an extension of the ratio method; measurements from a larger number of detectors are used. Multiple measurements are then used to perform a best fit to single temperature and wavelength relationship. The emissivity relationship can be a constant value or some other more complex function, such as a polynomial. A least-squares minimization of the data is then performed to determine the temperature and emissivity relationship.

In addition to providing a continuous emissivity across a wide wavelength range, another advantage of multispectral methods is that the large number of detectors and measurements reduces the noise and averages errors inherent in measurements. This method requires more complex hardware, however, and the use of multiple detectors increases the data collection and processing requirements. One shortcoming of the method is that it may not provide an identifiably unique solution unless a large number of measurements is made.

Although the routines described herein do not include any specific capability directly applicable to analyzing multi-spectral measurements, the routines along with the general capabilities of Microsoft Excel® can be used to implement this method. Example 12, located in the Example section demonstrates one variation of this method.

Wide-Band Detectors

A wide-band detector is one that covers a range of wavelengths and the range is sufficiently broad that spectral intensity cannot be assumed to be constant over the wavelength band. Additionally, for certain detectors, the sensitivity of the detector is not constant with wavelength, and, therefore, the detector signal will represent a weighted average across the waveband. The averaged signal can be represented mathematically as an integral over the waveband consisting of the product of the Planck equation, the detector response function, and, for a non-blackbody, the spectral emissivity.

For the given true surface temperature T, the equivalent blackbody temperatures T_{λ_i} that a given detector should measure can be calculated by solving equation (42).

$$\int_{\lambda_{il}}^{\lambda_{iu}} \varepsilon_{\lambda_i}(\lambda)\, D_i(\lambda)\, i_{b,\lambda}(T)\, d\lambda = \int_{\lambda_{il}}^{\lambda_{iu}} D_i(\lambda)\, i_{b,\lambda}\left(T_{\lambda_i}\right) d\lambda \tag{42}$$

The equivalent blackbody temperature provides the same detector signal from a blackbody source at the equivalent blackbody temperature as the real surface at the given true temperature.

Following the same concept as the ratio method for a set of narrow-band detectors, an equivalent method can be employed, except that measurements at the discrete wavelengths are replaced by integrated measurements across the bands, as seen in equation (43).

$$\bar{\varepsilon_r} = \frac{\bar{\varepsilon_1}}{\bar{\varepsilon_2}} = \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}\left(\lambda, T_{\lambda_1}\right) d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}(\lambda, T) d\lambda} \cdot \frac{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}(\lambda, T)\, d\lambda}{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}\left(\lambda, T_{\lambda_2}\right) d\lambda} \tag{43}$$

Here, $\bar{\varepsilon_r}$ is a weighted average emissivity ratio. Again, since the response of most detectors is not uniform across the band, the integrals include the detector response function $D_i(\lambda)$, which accounts for this variation. Similar to the wavelength ratio for discrete wavelengths, equation (42) is solved by iteration for the true temperature.

The sensitivity of the emissivity ratio to the true surface temperature is therefore as shown in equation (44):

$$\frac{d \ln \bar{\varepsilon_r}}{d \ln T} = T \frac{\varepsilon_2}{\varepsilon_1} \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}\left(\lambda, T_{\lambda_1}\right) d\lambda}{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}\left(\lambda, T_{\lambda_2}\right) d\lambda} \cdot$$

$$\left[\frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda} d\lambda \cdot \int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\;\; di_{b,\lambda}/dT\, d\lambda - \int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda} d\lambda \cdot \int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\;\; di_{b,\lambda}/dT\, d\lambda}{\left(\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda} d\lambda\right)^2}\right] \tag{44}$$

and the sensitivity of the emissivity ratio to the first equivalent blackbody temperature is as shown in equation (45).

$$\frac{d \ln \overline{\varepsilon_r}}{d \ln T_{\lambda_1}} = T_{\lambda_1} \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, di_{b,\lambda}(T_{\lambda_1})/dT_1\, d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}(T_{\lambda_1})\, d\lambda} \tag{45}$$

Alternatively, wideband measurements are sometimes converted to equivalent narrow-band measurements assuming that the discrete wavelength is selected at some intermediate point (usually the mean wavelength) in the band. The determination of the equivalent surface temperature and emissivities then reduces to solving an equivalent form of equation (31), seen here as equation (46):

$$\overline{\varepsilon_r} = \frac{\left(e^{-C_2/\overline{\lambda_2}T_{\lambda_2}} - 1\right)}{\left(e^{-C_2/\overline{\lambda_2}T} - 1\right)} \cdot \frac{\left(e^{-C_2/\overline{\lambda_1}T} - 1\right)}{\left(e^{-C_2/\overline{\lambda_1}T_{\lambda_1}} - 1\right)} \tag{46}$$

where $\overline{\lambda_1}$and $\overline{\lambda_2}$ are some equivalent wavelengths.

Note that the accuracy of this method can be highly variable and especially inaccurate if the detector bandwidths are wide or are widely separated in wavelength. The derived emissivities and surface temperatures can result in an ambiguous interpretation for the following reasons: 1) the detector response function varies with detector type; 2) the detector response function is often asymmetrical around the equivalent wavelength; or 3) the Planck equation is also asymmetric around the equivalent wavelength. The asymmetrical shape of the detector response and Planck function combined with a spectrally non-uniform emissivity can result in distortions that cannot be corrected through calibrations. For these reasons, the use of wideband detectors is highly discouraged unless there are other more compelling reasons for them to be used. Example 13 in the Examples portion of the document will demonstrate this point.

VBA Routine Summaries

The following section contains detailed descriptions of the individual blackbody function library functions and the calling conventions. For each routine, a description is given of the calling parameters and their type. The return value is also described. Note that each function only returns one value.

For all functions, a value of zero is returned if there is a calculation error or if one of the parameters is out of range. Typically, the input values for all temperatures, spectral intensities, and emissivities must be greater than zero.

Basic Single Wavelength Functions

Function bb_ibl(lambda As Double, temp As Double) As Double

Calculate the blackbody emissive intensity $i_{b,\lambda}(\lambda, T)$ at a given temperature T and wavelength λ (eq. (1)).

$$i_{b,\lambda}(\lambda, T) = \frac{1}{\lambda^5} \cdot \frac{C_1}{\left(e^{C_2/\lambda T} - 1\right)} \tag{1}$$

Description of Parameters:

lambda	*wavelength* λ*,* μ*m*
temp	*temperature* T*, K*

Returns:

Monochromatic blackbody emissive intensity for a given wavelength and temperature $i_{b,\lambda}(\lambda, T)$ in W/m^2-sr-μm.

Function bb_tibl(lambda As Double, ibl As Double) As Double

Calculate the equivalent blackbody temperature T at a specified wavelength λ for a given monochromatic blackbody emissive intensity $i_{b,\lambda}$ (eq. (2)).

$$T = \frac{C_2}{\lambda} \cdot \frac{1}{\ln\left(\frac{C_1}{\lambda^5 i_{b,\lambda}} + 1\right)} \tag{2}$$

Description of Parameters:

lambda	*wavelength* λ*,* μ*m*
ibl	*blackbody emissive intensity* $i_{b,\lambda}$*, W/m*^{2}*-sr-*μ*m*

Returns:

Equivalent blackbody temperature T in K.

Function bb_dibldt(lambda As Double, temp As Double) As Double

Calculate the derivative of the blackbody emissive intensity with respect to temperature $di_{b,\lambda}/dT$ at a given temperature T and wavelength λ (eq. (6)).

$$\frac{di_{b,\lambda}}{dT} = \frac{C_2}{\lambda^6 T^2} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} \tag{6}$$

Description of Parameters:

lambda	*wavelength* λ*,* μ*m*
temp	*temperature* T*, K*

Returns:

The derivative of the monochromatic blackbody emissive intensity with temperature $di_{b,\lambda}/dT$ in W/m^2-sr-μm-K.

Function bb_dlnibldlnt(lambda As Double, temp As Double) As Double

Calculate the derivative of the logarithm of the blackbody emissive intensity with respect to the logarithm of temperature $d\ln i_{b,\lambda}/d\ln T$ at a given temperature T and wavelength λ (eq. (8)).

$$\frac{d\ln i_{b,\lambda}}{d\ln T} = \frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} \tag{8}$$

Description of Parameters:

lambda	*wavelength λ, μm*
temp	*temperature T, K*

Returns:

The derivative of the logarithm of the monochromatic blackbody emissive intensity with the logarithm of temperature $d\ln i_{b,\lambda}/d\ln T$.

Function bb_dibldl(lambda As Double, temp As Double) As Double

Calculate the derivative of the blackbody emissive intensity with respect to wavelength $di_{b,\lambda}/d\lambda$ at a given temperature T and wavelength λ (eq. (7)).

$$\frac{di_{b,\lambda}}{d\lambda} = \frac{1}{\lambda^6} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} \cdot \left(\frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 5\right) \tag{7}$$

Description of Parameters:

lambda	*wavelength λ, μm*
temp	*temperature T, K*

Returns:

The derivative of the monochromatic blackbody emissive intensity with wavelength $di_{b,\lambda}/d\ln\lambda$ in W/m^2-sr-μm^2.

Function bb_dlnibldlnl(lambda As Double, temp As Double) As Double

Calculate the derivative of the logarithm of the blackbody emissive intensity with respect to the logarithm of wavelength $d\ln i_{b,\lambda}/d\ln\lambda$ at a given temperature T and wavelength λ (eq. (9)).

$$\frac{d\ln i_{b,\lambda}}{d\ln\lambda} = \frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 5 \tag{9}$$

Description of Parameters:

lambda	*wavelength λ, μm*
temp	*temperature T, K*

Returns:

The derivative of the logarithm of the monochromatic blackbody emissive intensity with the logarithm of the wavelength $d \ln i_{b,\lambda} / d \ln \lambda$.

Function bb_d2ibldt2(lambda As Double, temp As Double) As Double

Calculate the second derivative of the blackbody emissive intensity with respect to temperature $d^2 i_{b,\lambda}/dT^2$ at a given temperature T and wavelength λ (eq. (10)).

$$\frac{d^2 i_{b,\lambda}}{dT^2} = \frac{C_2}{\lambda^6 T^3} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} \cdot \left[\frac{C_2}{\lambda T} \cdot \left(\frac{2\, e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 1\right) - 2\right] \tag{10}$$

Description of Parameters:

lambda	*wavelength λ, μm*
temp	*temperature T, K*

Returns:

The second derivative of the monochromatic blackbody emissive intensity with temperature $d^2 i_{b,\lambda}/dT^2$ in W/m^2-sr-μm-K^2.

Function bb_d2ibldl2(lambda As Double, temp As Double) As Double

Calculate the second derivative with respect to wavelength of the blackbody emissive intensity $d^2 i_{b,\lambda}/d\lambda^2$ at a given temperature T and wavelength λ (eq. (11)).

$$\frac{d^2 i_{b,\lambda}}{d\lambda^2} = \frac{C_1}{\lambda^7} \cdot \frac{1}{(e^{C_2/\lambda T} - 1)} \cdot \left[\frac{C_2}{\lambda T} \cdot \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} \cdot \left(\frac{2C_2}{\lambda T}\, \frac{e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)} - 12 - \frac{C_2}{\lambda T}\right) + 30\right] \tag{11}$$

Description of Parameters:

lambda	*wavelength λ, μm*
temp	*temperature T, K*

Returns:

The second derivative of the monochromatic blackbody emissive intensity with wavelength in $d^2 i_{b,\lambda}/d\lambda^2$ in W/m^2-sr-μm^3.

Function bb_ib(temp As Double) As Double

Calculate the total blackbody emissive intensity i_b in at a specified temperature T (eq. (20)).

$$i_b = \frac{\sigma}{\pi} T^4 \tag{20}$$

Description of Parameters:

temp *temperature T, K*

Returns:

The total blackbody emissive intensity i_b in W/m^2-sr.

Function bb_eb(temp As Double) As Double

Calculate the total blackbody emissive power e_b in at a specified temperature T (eq. (22)).

$$e_b = \sigma T^4 \tag{22}$$

Description of Parameters:

temp *temperature T, K*

Returns:

The total blackbody emissive power e_b in W/m^2.

Function bb_lmax(temp As Double)

Calculate the wavelength at the maximum spectral intensity λ_{max} for a given temperature T using Wien's displacement law where (eq. (3)).

$$T\lambda_{max} = C_3 = 2897.77 \text{ μm-K} \tag{3}$$

Description of Parameters:

temp *temperature T, K*

Returns:

Wavelength at maximum spectral intensity λ_{max} in μm.

Function bb_imax(temp As Double)

Calculate the maximum spectral intensity $i_{b,\lambda_{max}}$ for a given temperature T. The maximum spectral intensity occurs at the wavelength λ_{max} according to Wien's displacement law where (eq. (3)):

$$T\lambda_{max} = C_3 \tag{3}$$

and the maximum spectral intensity is therefore (eq. (4)).

$$i_{b,\lambda_{max}} = \frac{T^5}{C_3^5} \cdot \frac{C_1}{(e^{C_2/C_3} - 1)} = C_4 T^5 \quad (4)$$

Description of Parameters:

temp	*temperature T, K*

Returns:

Maximum spectral intensity $i_{b,\lambda_{max}}$ in W/m^2-sr-μm.

Function bb_tmax(ibl As Double)

Calculate the temperature T at the maximum spectral intensity. The maximum spectral intensity occurs at the wavelength λ_{max} according to Wien's displacement law where (eq. (3)):

$$T\lambda_{max} = C_3 \quad (3)$$

and the maximum spectral intensity is therefore (eq. (4)).

$$i_{b,\lambda_{max}} = \frac{T^5}{C_3^5} \cdot \frac{C_1}{(e^{C_2/C_3} - 1)} = C_4 T^5 \quad (4)$$

The temperature at the maximum spectral intensity is (eq. (47)).

$$T = \left(\frac{i_{b,\lambda_{max}}}{C_4}\right)^{1/5} \quad (47)$$

Description of Parameters:

ibl	*maximum spectral intensity at the given temperature $i_{b,\lambda_{max}}$ in W/m^2-sr-μm*

Returns:

Temperature at the maximum spectral intensity T in K.

Spectral Method Functions

Function bb_tsi(lambda As Double, ilambda As Double, emiss As Double) As Double

Calculate the true surface temperature T for a non-black surface given the measured spectral intensity i_λ at a specified wavelength and the spectral emissivity (eq. (23)):

$$T = \frac{C_2}{\lambda} \cdot \frac{1}{\left(\ln \frac{\varepsilon_\lambda C_1}{\lambda^5 i_\lambda} + 1\right)} \quad (23)$$

Description of Parameters:

lambda	*wavelength λ, μm*
ilambda	*measured spectral intensity i_λ, W/m^2-sr-μm*
emiss	*spectral emissivity ε_λ*

Returns:

The true surface temperature T in K.

Function bb_tli(lambda As Double, ibl As Double, emiss As Double) As Double

Calculate the equivalent blackbody temperature T_λ at a specified wavelength λ for a non-blackbody given the blackbody spectral intensity at the true surface temperature $i_{b,\lambda}$ and the spectral emissivity ε_λ (eq. (48)).

$$T_\lambda = \frac{C_2}{\lambda} \cdot \frac{1}{\left(\ln \frac{C_1}{\lambda^5 \varepsilon_\lambda i_{b,\lambda}} + 1\right)} \tag{48}$$

Description of Parameters:

lambda	*wavelength λ, μm*
ibl	*blackbody spectral intensity at true surface temperature $i_{b,\lambda}$, W/m^2-sr-μm*
Emiss	*spectral emissivity ε_λ*

Returns:

The effective blackbody temperature at a specific wavelength T_λ in K.

Function bb_tst(lambda As Double, tlambda As Double, emiss As Double) As Double

Calculate the true temperature T for a non-blackbody given the equivalent blackbody temperature T_λ at a specified wavelength λ and the spectral emissivity ε_λ (eq. (24)).

$$T = \frac{C_2}{\lambda} \cdot \frac{1}{\ln[\varepsilon_\lambda (e^{C_2/\lambda T_\lambda} - 1) + 1]} \tag{24}$$

Description of Parameters:

lambda	*wavelength λ, μm*
tlambda	*equivalent blackbody temperature at the given wavelength T_λ, K*
emiss	*spectral emissivity ε_λ*

Returns:

The true surface temperature T in K.

Function bb_tstw(lambda As Double, tlambda As Double, emiss As Double) As Double

Calculate the true temperature T for a non-blackbody given the equivalent blackbody temperature T_λ at a specified wavelength λ and the spectral emissivity ε_λ assuming Wien's approximation is applicable (eq. (25)).

$$\frac{1}{T} = \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda \qquad (25)$$

This routine also can be used to calculate the true temperature using the ratio method according to equation (34):

$$\frac{1}{T} = \frac{1}{T_r} + \frac{\Lambda}{C_2} \ln \varepsilon_r \qquad (34)$$

where T_r is the ratio temperature given by equation (35):

$$\frac{1}{T_r} = \Lambda \left[\frac{1}{\lambda_1 T_{\lambda_1}} - \frac{1}{\lambda_2 T_{\lambda_2}} \right] \qquad (35)$$

and Λ is an effective wavelength, shown in equation (36).

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \qquad (36)$$

Description of Parameters:

lambda	*wavelength λ or effective wavelength Λ, μm*
tlambda	*equivalent blackbody temperature at the given wavelength T_λ or ratio temperature T_r, K*
emiss	*spectral emissivity ε_r*

Returns:

The true temperature T in K.

Function bb_tlt(lambda As Double, temp As Double, emiss As Double) As Double

Calculate the equivalent blackbody temperature T_λ at a specified wavelength λ for a non-blackbody given the true temperature T and the spectral emissivity ε_λ (eq. (49)).

$$T_\lambda = \frac{C_2}{\lambda} \cdot \frac{1}{\ln[1/\varepsilon_\lambda (e^{C_2/\lambda T} - 1) + 1]} \qquad (49)$$

Description of Parameters:

lambda	*wavelength λ, μm*
temp	*true temperature T, K*
emiss	*spectral emissivity ε_λ*

Returns:

The equivalent blackbody temperature T_λ in K.

Function bb_dlntdlne(lambda As Double, temp As Double) As Double

Calculate the sensitivity of the true temperature T with the spectral emissivity ε_λ. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the spectral emissivity and is expressed by equation (26).

$$\frac{d\ln T}{d\ln \varepsilon_\lambda} = -\frac{\lambda T}{C_2}\cdot\frac{\left(e^{C_2/\lambda T}-1\right)}{e^{C_2/\lambda T}} \quad (26)$$

Description of Parameters:

lambda	*wavelength λ, μm*
tlambda	*equivalent blackbody temperature at the given wavelength T, K*

Returns:

The derivative of the logarithm of the temperature with the logarithm of the spectral emissivity $d\ln T/d\ln \varepsilon_\lambda$.

Function bb_dlntldlne(lambda As Double, tlambda As Double) As Double

Calculate the sensitivity of the equivalent blackbody temperature T_λ with the spectral emissivity ε_λ. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the spectral emissivity and is expressed by equation (50):

$$\frac{d\ln T_\lambda}{d\ln \varepsilon_\lambda} = \frac{\lambda T_\lambda}{C_2}\cdot\frac{\left(e^{C_2/\lambda T_\lambda}-1\right)}{e^{C_2/\lambda T_\lambda}} \quad (50)$$

Description of Parameters:

lambda	*wavelength λ, μm*
tlambda	*equivalent blackbody temperature at the given wavelength T, K*

Returns:

The derivative of the logarithm of the equivalent blackbody temperature with the logarithm of the spectral emissivity $d\ln T_\lambda/d\ln \varepsilon_\lambda$.

Function bb_dlntdlntl(temp As Double, tlambda As Double, emiss As Double) As Double

Calculate the sensitivity of the true temperature T to the equivalent blackbody temperature T_λ. The sensitivity is expressed as the derivative of the logarithm of the true temperature with respect to the logarithm of the equivalent blackbody temperature as shown in equation (31):

$$\frac{d\ln T}{d\ln T_\lambda} = \varepsilon_\lambda \frac{T}{T_\lambda} e^{\frac{C_2}{\lambda}\left(\frac{1}{T_\lambda} - \frac{1}{T}\right)} \tag{31}$$

Description of Parameters:

temp	*true temperature T, K*
tlambda	*equivalent blackbody temperature at the given wavelength T_λ, K*
emiss	*spectral emissivity ε_λ*

Returns:

The derivative of the logarithm of the true temperature with respect to the logarithm of the equivalent blackbody temperature $d\ln T/d\ln T_\lambda$.

Function bb_dlntldlnt(temp As Double, tlambda As Double, emiss As Double) As Double

Calculate the sensitivity of the equivalent blackbody temperature T_λ with the respect to the true temperature T. The sensitivity is expressed as the derivative of the logarithm equivalent blackbody temperature with respect to the logarithm of the true temperature (eq. (51)):

$$\frac{d\ln T_\lambda}{d\ln T} = \frac{1}{\varepsilon_\lambda} \frac{T_\lambda}{T} e^{\frac{C_2}{\lambda}\left(\frac{1}{T} - \frac{1}{T_\lambda}\right)} \tag{51}$$

Description of Parameters:

temp	*true temperature T, K*
tlambda	*equivalent blackbody temperature at the given wavelength T_λ, K*
emiss	*spectral emissivity ε_λ*

Returns:

The derivative of the logarithm of the equivalent blackbody temperature with respect to the logarithm of the true temperature $d\ln T_\lambda/d\ln T$.

Function bb_emiss(lambda As Double, tlambda As Double, temp As Double) as Double

Calculate the spectral emissivity at a specified wavelength ε_λ given the equivalent blackbody temperature and the true temperature T (eq. (28)):

$$\varepsilon_\lambda = \frac{\left(e^{C_2/\lambda T} - 1\right)}{\left(e^{C_2/\lambda T_\lambda} - 1\right)} \tag{28}$$

Description of Parameters:

lambda	*wavelength λ, μm*
tlambda	*equivalent blackbody temperature at the given wavelength T_λ, K*
temp	*true temperature T, K*

Returns:

The spectral emissivity at the given wavelength ε_λ.

Function bb_dlne2dlne1(lambda1 As Double, lambda2 As Double, temp As Double) As Double

Calculate the sensitivity of the emissivity at a given wavelength ε_2 with respect to the uncertainty in emissivity at another wavelength ε_1 which has been used to determine the true temperature T. The sensitivity is expressed as the derivative of the logarithm of the emissivity at the second wavelength with respect to the logarithm of the emissivity at the first wavelength (eq. (29)).

$$\frac{d\ln\varepsilon_2}{d\ln\varepsilon_1} = \frac{\varepsilon_1}{\varepsilon_2}\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{\lambda_1}{\lambda_2}\cdot\frac{\left(e^{-C_2/\lambda_1 T} - 1\right)}{\left(e^{-C_2/\lambda_2 T} - 1\right)} \tag{29}$$

If Wien's approximation is appropriate for both wavelengths, then equation (31) reduces to equation (30).

$$\frac{d\ln\varepsilon_2}{d\ln\varepsilon_1} = \frac{\varepsilon_1}{\varepsilon_2}\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{\lambda_1}{\lambda_2} \tag{30}$$

Description of Parameters:

lambda1	*first detector wavelength λ_1, μm*
lambda2	*second detector wavelength λ_2, μm*
temp	*true temperature T, K*

Returns:

The sensitivity of emissivity 2 ε_2 with respect to emissivity 1 ε_1 or $d\ln\varepsilon_2/d\ln\varepsilon_1$.

Ratio Method Functions

Function bb_erlam(lambda1 As Double, lambda2 As Double) As Double

Calculate the effective wavelength Λ for a two-band radiation thermometer given the wavelengths of the two detectors $\lambda_1$and λ_2 (eq. (36)).

$$\Lambda = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} \tag{36}$$

Note that it is assumed that the second wavelength is greater than the first which will give positive values for the effective wavelength. If the two wavelengths are the same, then the result is undefined and the function returns zero.

Description of Parameters:

lambda1	*first wavelength λ_1, μm*
lambda2	*second wavelength λ_2, μm*

Returns:

The effective wavelength Λ in μm.

Function bb_ertemp(lambda1 As Double, lambda2 As Double, temp1 As Double, temp2 As Double) as Double

Calculate the ratio temperature T_r for a two-band radiation thermometer given the two equivalent blackbody temperatures $T_{\lambda_1}$and T_{λ_2} at the two detector wavelengths $\lambda_1$and λ_2.
The definition for the ratio temperature T_r is seen in equation (35).

$$\frac{1}{T_r} = \Lambda\left[\frac{1}{\lambda_1 T_{\lambda_1}} - \frac{1}{\lambda_2 T_{\lambda_2}}\right] \tag{35}$$

Description of Parameters:

lambda1	*first wavelength λ_1, μm*
lambda2	*second wavelength λ_2, μm*
temp1	*equivalent blackbody temperature at first wavelength T_{λ_1}, K*
temp2	*equivalent blackbody temperature at second wavelength T_{λ_2}, K*

Returns:

The effective ratio temperature T_r in K.

Function bb_tratio(lambda1 As Double, lambda2 As Double, temp1 As Double, temp2 As Double, emissr As Double, temp0 As Double, tol As Double, nmax As Long) As Double

Calculate the true temperature T for a two-band radiation thermometer given the equivalent blackbody temperatures $T_{\lambda_1}$and T_{λ_2} at the two detector wavelengths λ_1, λ_2 and an emissivity ratio ε_r.
The true temperature satisfies equation (32):

$$\frac{\left(e^{C_2/\lambda_2 T_{\lambda_2}} - 1\right)}{\left(e^{C_2/\lambda_2 T} - 1\right)} \cdot \frac{\left(e^{C_2/\lambda_1 T} - 1\right)}{\left(e^{C_2/\lambda_1 T_{\lambda_1}} - 1\right)} - \varepsilon_r = 0 \tag{32}$$

The solution cannot be represented analytically but can only be solved numerically by iteration. The true temperature is determined by a Newton-Raphson iteration scheme.

Note that if the solution returns zero, then the iteration has probably failed. Using a better value for the starting guess, increasing the maximum number of iterations, or increasing the tolerance will most likely result in convergence.

Description of Parameters:

lambda1	*first detector wavelength λ_1, μm*
lambda2	*second detector wavelength λ_2, μm*
temp1	*equivalent blackbody temperature of first detector T_{λ_1}, K*
temp2	*equivalent blackbody temperature of second detector T_{λ_2}, K*
emissr	*emissivity ratio ε_r*
temp0	*starting guess on true temperature, K*
tol	*tolerance on temperature* *> 0 relative tolerance* *< 0 absolute tolerance, K*
nmax	*maximum number of iterations*

Returns:

The true temperature T in K.

Function bb_emissr(lambda1 As Double, lambda2 As Double, temp1 As Double, temp2 As Double, temp As Double) As Double

Calculate the effective emissivity ratio ε_r from a two-band radiation thermometer given the equivalent blackbody temperatures T_{λ_1} and T_{λ_2} at the two detector wavelengths λ_1, λ_2 and the true temperature T (eq. (52) derived from eq. (32)):

$$\varepsilon_r = \frac{\left(e^{C_2/\lambda_2 T_{\lambda_2}} - 1\right)}{\left(e^{C_2/\lambda_2 T} - 1\right)} \cdot \frac{\left(e^{C_2/\lambda_1 T} - 1\right)}{\left(e^{C_2/\lambda_1 T_{\lambda_1}} - 1\right)} \qquad (52)$$

Description of Parameters:

lambda1	*first detector wavelength λ_1, μm*
lambda2	*second detector wavelength λ_2, μm*
temp1	*equivalent blackbody temperature of first detector T_{λ_1}, K*
temp2	*equivalent blackbody temperature of second detector T_{λ_2}, K*
temp	*true temperature T, K*

Returns:

The emissivity ratio ε_r from the two-band temperatures.

Function bb_dlnerdlnt(lambda1 As Double, lambda2 As Double, temp As Double) As Double

Calculate the derivative of the logarithm of the emissivity ratio ε_r with respect to the logarithm of the true temperature T for a two-band radiation thermometer given the temperatures at two wavelengths and true temperature (eq. (37)):

$$\frac{d\ln\varepsilon_r}{d\ln T} = \frac{1}{T}\cdot\frac{\left(e^{C_2/\lambda_2 T}-1\right)}{\left(e^{C_2/\lambda_1 T}-1\right)}\cdot\left[\left(\frac{C_2}{\lambda_2 T}\right)e^{\frac{C_2}{\lambda_2 T}}\frac{\left(e^{C_2/\lambda_1 T}-1\right)}{\left(e^{C_2/\lambda_2 T}-1\right)^2}-\left(\frac{C_2}{\lambda_1 T}\right)\frac{e^{C_2/\lambda_1 T}}{\left(e^{C_2/\lambda_2 T}-1\right)}\right] \quad (37)$$

Description of Parameters:

lambda1	*first detector wavelength λ_1, μm*
lambda2	*second detector wavelength λ_2, μm*
temp	*true temperature T, K*

Returns:

The derivative of the logarithm of the emissivity ratio with respect to the logarithm of the true temperature $d\ln\varepsilon_r/d\ln T$.

Function bb_dlnerdlnt1(lambda1 As Double, temp1 As Double) As Double

Calculate the derivative of the logarithm of the emissivity ratio ε_r with respect to the logarithm of the equivalent blackbody temperature of the first detector T_1 for a two-band radiation thermometer given the given the detector wavelength and equivalent blackbody temperature (eq. (39)):

$$\frac{d\ln\varepsilon_r}{d\ln T_{\lambda_1}} = \frac{C_2}{\lambda_1 T}\cdot\frac{e^{C_2/\lambda_1 T_{\lambda_1}}}{e^{C_2/\lambda_1 T_{\lambda_1}}-1} \quad (39)$$

Description of Parameters:

lambda1	*first detector wavelength λ_1, μm*
temp1	*equivalent blackbody temperature of first detector T_{λ_1}, K*

Returns:

The derivative of the logarithm of the emissivity ratio with respect to the logarithm of the equivalent blackbody temperature of the first detector $d\ln\varepsilon_r/d\ln T_1$.

Function bb_dlntrdlnt1(lambda1 As Double, lambda2 As Double, temp1 As Double, temp2 As Double) as Double

Calculate the derivative of the logarithm of the ratio temperature T_r with respect to the logarithm of the equivalent blackbody temperature of the first detector T_1 for a two-band radiation thermometer as shown in equation (40):

$$\frac{d \ln T_r}{d \ln T_{\lambda_1}} = \frac{\Lambda}{\lambda_1} \frac{T_r}{T_{\lambda_1}} \tag{40}$$

where T_r is defined as such in equation (35):

$$\frac{1}{T_r} = \Lambda \left[\frac{1}{\lambda_1 T_{\lambda_1}} - \frac{1}{\lambda_2 T_{\lambda_2}} \right] \tag{35}$$

and the equivalent wavelength is shown in equation (36).

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \tag{36}$$

Description of Parameters:

lambda1	*first detector wavelength* λ_1, μ*m*
lambda2	*second detector wavelength* λ_2, μ*m*
temp1	*equivalent blackbody temperature of first detector* T_{λ_1}, *K*
temp2	*equivalent blackbody temperature of second detector* T_{λ_2}, *K*

Returns:

The derivative of the logarithm of the ratio temperature with respect to the logarithm of the first temperature $d \ln \varepsilon_r / d \ln T_r$.

Function bb_dlnlrdlnl1(lambda1 As Double, lambda2 As Double) as Double

Calculate the derivative of the logarithm of the equivalent wavelength Λ with respect to the logarithm of the wavelength of the first detector λ_1 for a two-band radiation thermometer (eq. (41)):

$$\frac{d \ln \Lambda}{d \ln \lambda_1} = 1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} \tag{41}$$

in which the equivalent wavelength Λ is defined in equation (36).

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \tag{36}$$

Description of Parameters:

lambda1	*first detector wavelength* λ_1, μ*m*
lambda2	*second detector wavelength* λ_2, μ*m*

Returns:

The derivative of the logarithm of the equivalent wavelength with respect to the logarithm of the first detector wavelength $d \ln \Lambda / d \ln \lambda_1$.

Wide-Band Functions

Function iibls(llow As Double, lhigh As Double, temp As Double, tol As Double, nmax As Long) As Double

Calculate the integrated emissive intensity for a wide or narrow-band using a series solution as described in ref. 3. For a wide-band calculation, the upper and lower wavelengths are specified. For a narrow-band calculation, the same value is input for the upper and lower wavelengths.

For a wide-band, the calculated integral is shown in equation (12):

$$I(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} \frac{1}{\lambda^5} \frac{C_1}{e^{C_2/\lambda T} - 1} d\lambda \qquad (12)$$

which can be written as equation (53):

$$I(\lambda_l, \lambda_u, T) = \int_0^{\lambda_u} \frac{1}{\lambda^5} \frac{C_1}{e^{C_2/\lambda T} - 1} d\lambda - \int_0^{\lambda_l} \frac{1}{\lambda^5} \frac{C_1}{e^{C_2/\lambda T} - 1} d\lambda \qquad (53)$$

or using equation 14 and 15 as equation (54).

$$\sigma T^4 \left(F_{0 \to \lambda_u T} - F_{0 \to \lambda_l T} \right) \qquad (54)$$

Now $F_{0 \to \lambda T}$ can be represented by an integrated series as shown in equation in (13):

$$F_{0 \to \lambda T} = \frac{15}{\pi^4} \sum_{n-1}^{\infty} \frac{e^{-n\xi}}{n} \left(\xi^3 + 3\frac{\xi^2}{n} + 6\frac{\xi}{n^2} + 6\frac{1}{n^3} \right) \qquad (13)$$

where (eq. (14)):

$$\xi = \frac{C_2}{\lambda T} \qquad (14)$$

For a narrow-band, where $\lambda_1 = \lambda_2 = \lambda$, no integration is performed, but the spectral radiant intensity at the single wavelength is returned (eq. (1)):

$$i_{b,\lambda}(\lambda, T) = \frac{1}{\lambda^5} \frac{C_1}{(e^{C_2/\lambda T} - 1)} \qquad (1)$$

Description of Parameters:

llow	*lower wavelength λ_l, μm*
lhigh	*upper wavelength λ_u, μm*
temp	*Temperature T, K*
tol	*tolerance on series term*
	> 0 absolute tolerance
	< 0 relative tolerance
nmax	*maximum number of series terms*

Note that the integration is carried out until the value of the series term falls below the tolerance value or until the maximum number of terms is reached.

Returns:

The integrated emissive intensity I in W/m^2-sr between two wavelengths for a given temperature if different values are given for the upper and lower wavelengths. If the same value is input for the upper and lower wavelength, then the spectral emissive intensity i_λ in W/m^2-sr-μm is computed.

Function bb_fiibls(lamt As Double, tol As Double, nmax As Long) As Double

Calculate the spectral emissive intensity fraction between zero and specified wavelength $F_{0\rightarrow\lambda T}$ using a series solution as described in ref. 3.

The fraction is calculated using a series solution according to equation (13):

$$F_{0\rightarrow\lambda T} = \frac{15}{\pi^4}\sum_{n-1}^{\infty}\frac{e^{-n\xi}}{n}\left(\xi^3 + 3\frac{\xi^2}{n} + 6\frac{\xi}{n^2} + 6\frac{1}{n^3}\right) \tag{13}$$

where (eq. (14)):

$$\xi = \frac{C_2}{\lambda T} \tag{14}$$

Description of Parameters:

lamt	*product of temperature and wavelength λT, μm –K*
tol	*tolerance on series term*
	> 0 absolute tolerance
	< 0 relative tolerance
nmax	*maximum number of series terms*

Note that the integration is carried out until the value of the series term falls below the tolerance value or until the maximum number of terms is reached.

Returns:

The fraction of emissive intensity between zero and a specified wavelength at a given temperature $F_{0\rightarrow\lambda T}$.

Function bb_iibl(DRange As range, ERange As range, llow As Double, lhigh As Double, temp As Double, tol As Double, maxdbl As Long, flag As Long) As Double

Calculate the total integrated emissive intensity $I(\lambda_l, \lambda_u, T)$ for a wide- or narrow-width band using a quadrature integration scheme. For a wide-band calculation, the upper and lower wavelengths are specified. For a narrow-band calculation, the same value is input for the upper and lower wavelengths. This routine also allows the spectral distribution to be weighted by a detector response function $D(\lambda)$ and a spectral emissivity $\varepsilon_\lambda(\lambda)$. These weighting functions are input in tabular forms as two dimensional Excel® Ranges.

For options 0 through 3, the equation evaluated is the basic Planck equation (eq. (16)).

$$I_0(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon_\lambda(\lambda)\, i_{b,\lambda} d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon_\lambda(\lambda) \frac{1}{\lambda^5} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} d\lambda \qquad (16)$$

For options 4 through 7, the equation evaluated is the derivative of the Planck equation with temperature (eq. (17)).

$$I_1(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon_\lambda(\lambda) \frac{di_{b,\lambda}}{dT} d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon_\lambda(\lambda) \frac{C_2}{\lambda^6 T^2} \cdot \frac{C_1 e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} d\lambda \qquad (17)$$

For options 10 through 13, the equation evaluated is the first moment with respect to wavelength of the Planck equation (eq. (18)).

$$I_2(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon_\lambda(\lambda)\, i_{b,\lambda}\, \lambda\, d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon_\lambda(\lambda) \frac{1}{\lambda^4} \cdot \frac{C_1}{(e^{C_2/\lambda T} - 1)} d\lambda \qquad (18)$$

Finally, for options 14 through 17, the equation (eq. (19)) evaluated is the first moment with respect to wavelength of the derivative of the Planck equation with temperature:

$$I_3(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon_\lambda(\lambda) \frac{di_{b,\lambda}}{dT} \lambda\, d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon_\lambda(\lambda) \frac{C_2}{\lambda^5 T^2} \cdot \frac{C_1 \lambda e^{C_2/\lambda T}}{(e^{C_2/\lambda T} - 1)^2} d\lambda \qquad (19)$$

Description of Parameters:

DRange *Excel Range object containing the wavelength and detector response function $D(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*

ERange *Excel Range object containing the wavelength and spectral emissivity function $\varepsilon(\lambda)$. The wavelength is contained in the first column of the range variable and the spectral emissivity is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*

llow	*lower wavelength λ_l, μm*	
lhigh	*upper wavelength λ_u, μm*	
temp	*Temperature T, K*	
tol	*tolerance on integration* *> 0 absolute tolerance* *< 0 relative tolerance*	
maxdbl	*maximum number of interval doublings in integration*	
flag	*flag that determines the integrand function*	
	= 0	*Blackbody spectral intensity function only*
	= 1	*Blackbody spectral intensity function multiplied by spectral detector response function*
	= 2	*Blackbody spectral intensity function multiplied by spectral emissivity*
	= 3	*Blackbody spectral intensity function, detector response, and spectral emissivity spectral intensity function multiplied together*
	= 4	*Derivative of blackbody spectral intensity function with temperature only*
	= 5	*Derivative of blackbody spectral intensity function with temperature multiplied by spectral detector response function*
	= 6	*Derivative of blackbody spectral intensity function with temperature multiplied by spectral emissivity*
	= 7	*Derivative of blackbody spectral intensity function with temperature, detector response, and spectral emissivity function multiplied together*
	= 8	*Reserved*
	= 9	*Reserved*
	= 10	*First moment of blackbody spectral intensity function only*
	= 11	*First moment of blackbody spectral intensity function multiplied by spectral detector response function*
	= 12	*First moment of blackbody spectral intensity function multiplied by spectral emissivity*
	= 13	*First moment of blackbody spectral intensity function, detector response, and spectral emissivity function multiplied together*
	= 14	*First moment of derivative of blackbody spectral intensity function with temperature only*
	= 15	*First moment of derivative of blackbody spectral intensity function with temperature multiplied by spectral detector response function*
	= 16	*First moment of derivative of blackbody spectral intensity function with temperature multiplied by spectral emissivity*
	= 17	*First moment of derivative of blackbody spectral intensity function with temperature, detector response, and spectral emissivity function multiplied together*

Returns:

The detector and/or emissive weighted average of the integrand which can be: 1) wide-band emissive intensity in W/m^2-sr; 2) derivative of the weighted average wide-band emissive intensity with temperature in W/m^2-sr-K; 3) first moment with respect to wavelength of the Planck equation in W-μm/m^2-sr; or 4) first moment with respect to wavelength of the derivative of the Planck equation with temperature in function in W-μm/m^2-sr-K. If different values are given for the upper and lower wavelengths, the integral is computed. If the same value is input for the upper and lower wavelength, then the value of the integrand at the particular wavelength is computed.

Function bb_tiiibl(DRange As range, ERange As range, lambda1 As Double, lambda2 As Double, iibln As Double, temp0 As Double, toli As Double, toln As Double, nmaxdbl As Long, nmax As Long) As Double

Calculate the equivalent blackbody temperature T_λ across a waveband for a non-blackbody with a spectral emissivity as a function of wavelength $\varepsilon(\lambda)$ given the total integrated intensity across the waveband $I(\lambda_l, \lambda_u, T)$ and detector response as a function of wavelength $D(\lambda)$. The blackbody temperature is calculated iteratively using a Newton-Raphson iteration scheme that solves equation (55).

$$\int_{\lambda_l}^{\lambda_u} D(\lambda)\, \varepsilon(\lambda)\, i_{b,\lambda}(\lambda, T)\, d\lambda = I(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, i_{b,\lambda}(\lambda, T_\lambda)\, d\lambda \qquad (55)$$

Note that if the solution returns zero, then the iteration has probably failed. Using a better value for the starting guess, increasing the maximum number of iterations, or increasing the tolerance will most likely result in convergence.

Description of Parameters:

DRange	*Excel Range object containing the wavelength and detector response function $D(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
ERange	*Excel Range object containing the wavelength and spectral emissivity function $\varepsilon(\lambda)$. The wavelength is contained in the first column of the range variable and the spectral emissivity is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
llow	*lower wavelength λ_l, μm*
lhigh	*upper wavelength λ_u, μm*
iibln	*integrated intensity at the true temperature, W/m^2*
temp0	*starting guess on equivalent blackbody temperature, K*
toli	*tolerance on integration* *> 0 relative tolerance* *< 0 absolute tolerance, K*
toln	*tolerance on Newton-Raphson temperature iteration* *>0 relative tolerance* *< 0 absolute tolerance, K*
nmaxdbl	*maximum number of doublings during integration*
nmax	*maximum number of iterations*

Returns:

The equivalent blackbody temperature T_λ in K.

Function bb_ttiibl(DRange As range, ERange As range, lambda1 As Double, lambda2 As Double, temp As Double, temp0 As Double, toli As Double, toln As Double, nmaxdbl As Long, nmax As Long) As Double

Calculate the equivalent blackbody temperature T_λ across a waveband for a non-blackbody with an emissivity as a function of wavelength $\varepsilon_\lambda(\lambda)$ given the true temperature T and detector response as a function of wavelength $D(\lambda)$. The blackbody temperature is calculated iteratively using a Newton-Raphson iteration scheme that solves equation (56).

$$\int_{\lambda_l}^{\lambda_u} D(\lambda)\,\varepsilon_\lambda(\lambda)\,i_{b,\lambda}(\lambda,T)\,d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda)\,i_{b,\lambda}(\lambda,T_\lambda)\,d\lambda \qquad (56)$$

Note that if the solution returns zero, then the iteration has probably failed. Using a better value for the starting guess, increasing the maximum number of iterations, or increasing the tolerance will most likely result in convergence.

Description of Parameters:

DRange	*Excel Range object containing the wavelength and detector response function $D(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
ERange	*Excel Range object containing the wavelength and spectral emissivity function $\varepsilon(\lambda)$. The wavelength is contained in the first column of the range variable and the spectral emissivity is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
llow	*lower wavelength λ_l, μm*
lhigh	*upper wavelength λ_u, μm*
temp	*true temperature T, K*
temp0	*starting guess on equivalent blackbody temperature, K*
toli	*tolerance on integration* *> 0 relative tolerance* *< 0 absolute tolerance, K*
toln	*tolerance on Newton-Raphson temperature iteration* *> 0 relative tolerance* *< 0 absolute tolerance, K*
nmaxdbl	*maximum number of doublings during integration*
nmax	*maximum number of iterations*

Returns:

The equivalent blackbody temperature T_λ in K.

Wide-Band Ratio Functions

Function bb_itratio(DRange1 As range, DRange2 As range, lambda1l As Double, lambda1u As Double, lambda2l As Double, lambda2u As Double, temp1 As Double, temp2 As Double, emissr As Double, temp0 As Double, toli As Double, toln As Double, nmaxdbl As Long, nmax As Long) As Double

Calculate the true temperature T for a wide, two-band, radiation thermometer given the equivalent blackbody temperatures for the two detectors T_{λ_1} and T_{λ_2} and a band-averaged emissivity ratio $\bar{\varepsilon_r}$. The true temperature satisfies equation (43):

$$\bar{\varepsilon_r} = \frac{\bar{\varepsilon_1}}{\bar{\varepsilon_2}} = \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}\left(\lambda, T_{\lambda_1}\right) d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}(\lambda, T) d\lambda} \cdot \frac{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}(\lambda, T)\, d\lambda}{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}\left(\lambda, T_{\lambda_2}\right) d\lambda} \tag{43}$$

where $D_1(\lambda)$ and $D_2(\lambda)$ are the detector response functions that are functions of wavelength. The true temperature is calculated iteratively using a Newton-Raphson iteration scheme.

Note that if the solution returns zero, then the iteration has probably failed. Using a better value for the starting guess, increasing the maximum number of iterations, or increasing the tolerance will most likely result in convergence.

Description of Parameters:

DRange	*Excel Range object containing the wavelength and detector response function $D(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
ERange	*Excel Range object containing the wavelength and spectral emissivity function $\varepsilon(\lambda)$. The wavelength is contained in the first column of the range variable and the spectral emissivity is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
lambda1l	*lower wavelength for detector 1 λ_{1l}, μm*
lambda1u	*upper wavelength for detector 1 λ_{1u}, μm*
lambda2l	*lower wavelength for detector 2 λ_{2l}, μm*
lambda2u	*upper wavelength for detector 2 λ_{2u}, μm*
temp1	*equivalent blackbody temperature for detector 1 T_{λ_1}*
temp2	*equivalent blackbody temperature for detector 2 T_{λ_2}*
emissr	*emissivity ratio $\bar{\varepsilon_r}$*
temp0	*starting guess on true temperature, K*
toli	*tolerance on temperature integration* *> 0 relative tolerance* *< 0 absolute tolerance, K*

toln	*tolerance on Newton-Raphson integration* *> 0 relative tolerance* *< 0 absolute tolerance, K*
nmaxdb	*maximum number of integration doublings*
nmax	*maximum number of iterations*

Returns:

The true temperature T in K.

Function bb_iemissr(DRange1 As range, DRange2 As range, lambda1l As Double, lambda1u As Double, lambda2l As Double, lambda2u As Double, temp1 As Double, temp2 As Double, temp As Double, tol As Double, nmaxdbl As Long) As Double

Calculate the effective emissivity ratio $\overline{\varepsilon_r}$ from a wide, two-band radiation thermometer given the equivalent blackbody temperatures for the two detectors T_{λ_1} and T_{λ_2} and the true temperature T.

The emissivity ratio is calculated from equation (43).

$$\overline{\varepsilon_r} = \frac{\overline{\varepsilon_1}}{\overline{\varepsilon_2}} = \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}\left(T_{\lambda_1}\right) d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}(T) d\lambda} \cdot \frac{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}(T)\, d\lambda}{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}\left(T_{\lambda_2}\right) d\lambda} \tag{43}$$

Description of Parameters:

DRange1	*Excel Range object containing the wavelength and detector response function for detector 1 $D_1(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
DRange2	*Excel Range object containing the wavelength and detector response function for detector 2 $D_2(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
lambda1l	*lower wavelength for detector 1 λ_{1l}, μm*
lambda1u	*upper wavelength for detector 1 λ_{1u}, μm*
lambda2l	*lower wavelength for detector 2 λ_{2l}, μm*
lambda2u	*upper wavelength for detector 2 λ_{2u}, μm*
temp1	*equivalent blackbody temperature for detector 1 T_{λ_1}*
temp2	*equivalent blackbody temperature for detector 2 T_{λ_2}*
temp	*true temperature T, K*
tol	*tolerance on temperature integration* *> 0 relative tolerance* *< 0 absolute tolerance, K*
nmaxdbl	*maximum number of doublings*

Returns:

The effective emissivity ratio $\bar{\varepsilon_r}$.

Function bb_dlniedlnt(DRange1 As range, DRange2 As range, lambda1l As Double, lambda1u As Double, lambda2l As Double, lambda2u As Double, temp1 As Double, temp2 As Double, temp As Double, tol As Double, nmaxdbl As Long) As Double

Calculate the derivative of the logarithm of the emissivity ratio $\bar{\varepsilon_r}$ with respect to the true temperature T for a wide, two-band radiation thermometer (eq. (44)):

$$\frac{d\ln\bar{\varepsilon_r}}{d\ln T} = T\,\frac{\varepsilon_2}{\varepsilon_1}\frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\; i_{b,\lambda}\left(T_{\lambda_1}\right) d\lambda}{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\; i_{b,\lambda}\left(T_{\lambda_2}\right) d\lambda}\cdot$$

$$\left[\frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\; i_{b,\lambda}(T)d\lambda\cdot\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\;\; di_{b,\lambda}(T)/dT\, d\lambda - \int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\; i_{b,\lambda}(T)d\lambda\cdot\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\;\; di_{b,\lambda}(T)/dT\, d\lambda}{\left(\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\; i_{b,\lambda}(T)d\lambda\right)^2}\right] \tag{44}$$

Description of Parameters:

DRange1	*Excel Range object containing the wavelength and detector response function for deector 1 $D_1(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
DRange2	*Excel Range object containing the wavelength and detector response function for detector 2 $D_2(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
lambda1l	*lower wavelength for detector 1 λ_{1l}, μm*
lambda1u	*upper wavelength for detector 1 λ_{1u}, μm*
lambda2l	*lower wavelength for detector 2 λ_{2l}, μm*
lambda2u	*upper wavelength for detector 2 λ_{2u}, μm*
temp1	*equivalent blackbody temperature for detector 1 T_{λ_1}*
temp2	*equivalent blackbody temperature for detector 2 T_{λ_2}*
temp	*true temperature T, K*
tol	*tolerance on temperature integration* *> 0 relative tolerance* *< 0 absolute tolerance, K*
nmaxdbl	*maximum number of doublings*

Returns:

The derivative of the logarithm of the emissivity ratio with respect to the true temperature for a two-band radiation thermometer $d \ln \bar{\varepsilon_r} / d \ln T$.

Function bb_dlniedlnt1(DRange1 As range, lambda1l As Double, lambda1u As Double, temp1 As Double, tol As Double, nmaxdbl As Long) As Double

Calculate the derivative of the logarithm of the emissivity ratio $\bar{\varepsilon_r}$ with respect to the first band temperature T_{λ_1} for a two-band radiation thermometer (eq. (45)).

$$\frac{d \ln \bar{\varepsilon_r}}{d \ln T_{\lambda_1}} = T_{\lambda_1} \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, di_{b,\lambda}\left(T_{\lambda_1}\right)/dT_1\, d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}\left(T_{\lambda_1}\right) d\lambda} \tag{45}$$

Description of Parameters:

DRange1	*Excel Range object containing the wavelength and detector response function for detector 1 $D_1(\lambda)$. The wavelength is contained in the first column and the detector response value is contained in the last column. In the integration, values for the detector response function are interpolated linearly from the table of data. Note that the wavelengths in the table must be in ascending order and duplicate wavelength entries are not allowed.*
lambda1l	*lower wavelength for detector 1 λ_{1l}, μm*
lambda1u	*upper wavelength for detector 1 λ_{1u}, μm*
temp1	*equivalent blackbody temperature for detector 1 T_{λ_1}*
tol	*tolerance on temperature integration* *> 0 relative tolerance* *< 0 absolute tolerance, K*
nmaxdbl	*maximum number of doublings*

Returns:

The derivative of the logarithm of the emissivity ratio with respect to the first band temperature $\ln \bar{\varepsilon_r} / d \ln T_{\lambda_1}$.

Constant Functions

Note: Values for physical constants were obtained from ref. 1.

Function bb_C1() As Double

Returns the value of C_1= 1.191043 × 10^8 W-μm^4/m^2-sr.

Function bb_C2() As Double

Returns the value of C_2= 14,387.77 μm-K.

Function bb_C3() As Double

Returns the value of C_3 = 2,897.77 μm-K.

Function bb_C4() As Double

Returns the value of C_4 = 4.09567 × 10^{-12} W/m^2-μm-sr-K^5.

Function bb_SIGMA() As Double

Returns the value of the Stephan-Boltzmann constant σ = 5.670367 × 10^{-8} W/m^2-K^4.

Function bb_RCONST() As Double

Returns the value of the universal gas constant R = 8.3144598 J/mol-K.

Function bb_CLIGHT() As Double

Returns the value of the speed of light in a vacuum c = 2.99792458 × 10^8 m/s.

Function bb_HBAR() As Double

Returns the value of Planck's constant $\bar{h}$ = 6.62607004 × 10^{-34} J/s.

Function bb_KBOLTZ() As Double

Returns the value of the Boltzmann constant k = 1.38064852 × 10^{-23} J/K.

Configuration Control Functions

Function bb_version() As String

Returns the version of the blackbody function library as a string.

Examples

This section contains example problems using the routines defined previously. Each example contains a brief problem statement followed by a reference to the applicable equations, VBA routine names, and the numerical solution.

Example 1

Example 1: What is the emitted spectral intensity for a blackbody at 3,000 K measured with a detector at a wavelength of 0.5 μm?

Answer 1: The spectral intensity is calculated from equation (1) using routine bb_ibl, as shown in equation (57):

$$i_{b,\lambda}(\lambda,T) = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)} = \frac{1.191043 \times 10^8}{0.5^5\left(e^{14,387.75/0.5\cdot 3000}-1\right)} = 2.60 \times\ 10^5 \text{ W/m}^2\text{-sr-}\mu\text{m} \tag{57}$$

Example 2

Example 2: What is the emitted power across a 10-nm (0.01-μm) waveband centered at 0.5 μm and at 3,000 K?

Answer 2: Since the bandwidth is small, the total power can be evaluated using the spectral intensity at the given wavelength calculated using routine bb_ibl and then multiplied by the wavelength band, as shown in equation (58):

$$\begin{aligned} I_{b,\lambda}(\lambda_l, \lambda_u, T) &\cong i_{b,\lambda}\left(\lambda_l,\ T\right) \cdot \Delta\lambda = 2.60 \times\ 10^5 \text{ W/m}^2\text{-sr-}\mu\text{m}\cdot 0.01\ \mu\text{m} \\ &= 2.60 \times\ 10^3 \text{ W/m}^2\text{-sr} \end{aligned} \tag{58}$$

Example 3

Example 3: What is the total emitted flux for a body at 1,500 K? What is the emitted flux between the wavelengths of 1 to 3 μm?

Answer 3: The total emitted flux is given by equation (15) and calculated using bb_eb, as shown in equation (59):

$$e_b = \sigma T^4 = 5.67 \times 10^{-8} \cdot 1{,}500^4 = 2.87\ \times\ 10^5 \text{ W/m}^2 \tag{59}$$

The wavelength-temperature products for 1 and 3 μm at 1,500 K are (eq. (60)):

$$\lambda_l T = 1{,}500\ \mu\text{m} - \text{K and } \lambda_u T = 4{,}500\ \mu\text{m} - \text{K} \tag{60}$$

Therefore, using the series solution described in equations (5) and (6) and the routine bb_fiibls, the fraction of flux is as shown in equation (61)):

$$F_{0\to\lambda_l T}\ = 0.01285 \text{ and } F_{0\to\lambda_u T} =\ 0.56430 \tag{61}$$

So, (eq. (62)):

$$e_{\lambda_{1l}T\to\lambda_u T} = F_{\lambda_l T\to\lambda_u T} \cdot \sigma T^4 = \left(F_{0\to\lambda_u T} - F_{0\to\lambda_l T}\right) \cdot \sigma T^4 \tag{62}$$

and the flux in band is (eq. (63)):

$$= (0.56430 - 0.01285) \cdot 2.87\ \times\ 10^5 \text{ W/m}^2 = 1.58 \times 10^5 \text{ W/m}^2 \tag{63}$$

A graphical representation of the integration is shown in figure 4.

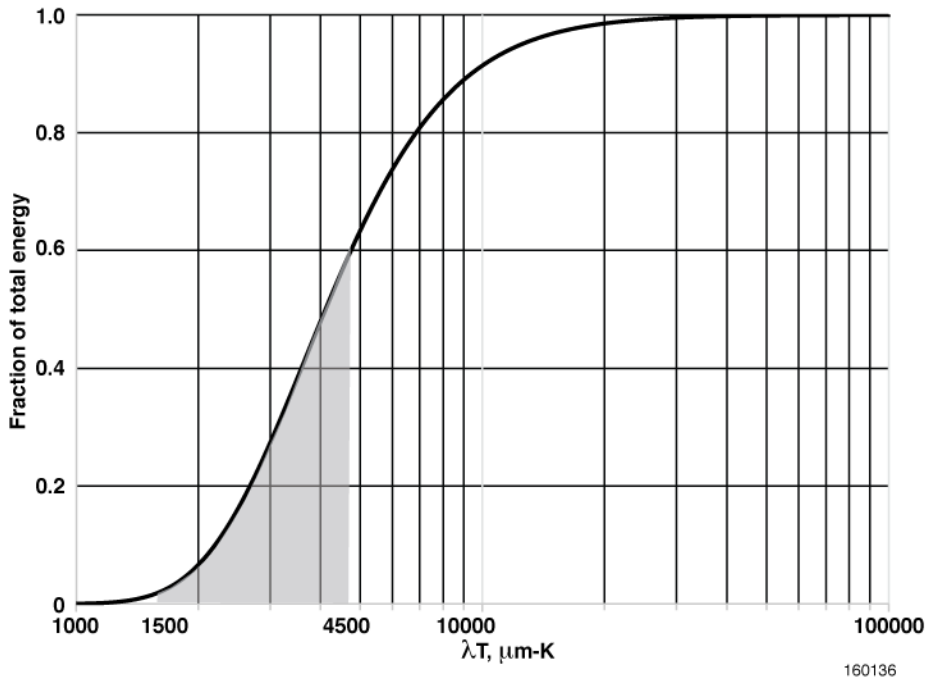

Figure 4. Integration of the Planck equation over the range of the wavelength temperature product of 1,500 to 4,500 μm-K.

Alternatively, the solution could be found directly using routines bb_iibls or bb_iibl.

Example 4

Example 4: What is the true temperature when a surface is measured with a 0.5-μm wavelength detector and indicates an equivalent blackbody temperature of 3,820 K? Assume an emissivity of 0.8.

Answer 4: The wavelength temperature product is 3,820 K times 0.5 μm or 1,910 μm-K. This product is much less than C_2 or 14,388 μm-K. So, Wien's approximation is valid (as discussed in the Basic Equations subsection of the Background section) and equation (24) using routine bb_tstw can be used, as shown in equations (64) and (65):

$$\frac{1}{T} \doteq \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda = \frac{1}{3{,}820\ K} + \frac{0.5\ \mu\mathrm{m}}{14{,}388\ \mu\mathrm{m\text{-}K}} \ln(0.8) \tag{64}$$

$$T = 3{,}937\ \mathrm{K} \tag{65}$$

Example 5

Example 5: Repeat Example 4, using an 8-μm detector. What is the true temperature?

Answer 5: The wavelength temperature product is 3,820 K times 8 μm or 30,560 μm-K. This product is greater than C_2 or 14,388 μm-K. So, the expression derived from the full Planck equation (eq. (24)) must be used, using routine bb_tst as shown in equation (66):

$$T = \frac{C_2/\lambda}{\ln\left(\varepsilon_\lambda \cdot \left(e^{C_2/\lambda T_\lambda} - 1\right) + 1\right)} \quad (24)$$

$$T = \frac{14{,}388\ \mu\text{m-K}/8\ \mu\text{m}}{\ln\left(0.8 \cdot \left(e^{14{,}388\ \mu\text{m-K}/8\ \mu\text{m}\ 3820\ \text{K}} - 1\right) + 1\right)} = 4{,}579\ \text{K} \quad (66)$$

If Wien's approximation had been used, the result obtained for the calculated true temperature would have been over 7,200 K (and inaccurate by more than 2,600 K).

Example 6

Example 6: If the spectral emissivity of graphitic material at 0.53 μm is estimated to be 0.8 with an estimated uncertainty of 20%, what is the estimated uncertainty in the temperature if a detector at this wavelength indicates an equivalent blackbody temperature of 2,950 K?

Answer 6: The wavelength temperature product is 2,950 K times 0.53 μm or 1,563.5 μm-K. This product is much less than C_2 or 14,388 μm-K. Thus, Wien's approximation is valid and equation (24) and routine bb_tstw can be used as shown in equations (67)-(69):

$$\frac{1}{T} \doteq \frac{1}{T_\lambda} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda = \frac{1}{2{,}950\ K} + \frac{0.52\ \mu\text{m}}{14{,}388\ \mu\text{m-K}} \ln(0.8) = 0.0003308 \quad (67)$$

$$T = 3{,}023\ \text{K} \quad (68)$$

$$\frac{d \ln T}{d \ln \varepsilon_{\lambda_i}} = -\frac{\lambda T}{C_2} = -\frac{0.53\mu\text{m} \cdot 3{,}023\ \text{K}}{14{,}388\ \mu\text{m-K}} = -11\% \quad (69)$$

With the emissivity uncertainty estimated to be 20%, the resulting uncertainty in the true temperature is 20% times -11%, or -2.2%. For a temperature of 3,023 K, the absolute uncertainty is -67 K.

Example 7

Example 7: Repeat example 6 for a detector with a wavelength of 5.8 μm.

Answer 7: The wavelength temperature product is 2,950 K times 5.8 μm or 17,110 μm-K. This product is greater than C_2 or 14,388 μm-K, so, Wien's approximation is not valid and the full Planck equation (equation (1)) and routine bb_tst must be used.

The estimated "true" temperature with the nominal emissivity is calculated from equation (24) and routine bb_tst, as shown in equation (70):

$$T = \frac{14{,}388\ \mu\text{m-K}/5.8\ \mu\text{m}}{\ln\left(0.8 \cdot \left(e^{14{,}388\ \mu\text{m-K}/5.8\ \mu\text{m} \cdot 2{,}950\ \text{K}} - 1\right) + 1\right)} = 3{,}445\ \text{K} \quad (70)$$

The estimated uncertainty in the emissivity is determined from equation (25) and routine dlntdlne and is shown in equation (71):

$$\frac{d\ln T}{d\ln\varepsilon_{\lambda_i}} = -\frac{\lambda_i T}{C_2}\cdot\frac{e^{C_2/\lambda_i T_{\lambda_i}}-1}{e^{C_2/\lambda_i T_{\lambda_i}}} = \frac{5.8\mu\text{m}\ \cdot 3{,}445\ \text{K}}{14{,}388\ \mu\text{m-K}}\cdot\frac{e^{5.8\mu\text{m}\ \cdot 3{,}445\ \text{K}/14{,}388\ \mu\text{m-K}}-1}{e^{5.8\mu\text{m}\ \cdot 3{,}445\ \text{K}/14{,}388\ \mu\text{m-K}}} = -71\% \quad (71)$$

With the emissivity uncertainty estimated to be 20%, the resulting uncertainty in temperature is 20% times 71%, or 14%. For a temperature of 3,445 K, the absolute uncertainty is approximately 491 K!

Example 8

Example 8: A 0.53-μm wavelength detector is used to determine the true temperature. The measured equivalent blackbody temperature from this detector is 1,950 K and the spectral emissivity is estimated to be 0.6 with an estimated uncertainty of 25%. What are the inferred emissivities and uncertainties at 1.0 μm and 5.8 μm if detectors at both wavelengths indicate equivalent blackbody temperatures of 1,740 K?

Answer 8: The true temperature using equation (24) and routine bb_tst is obtained as shown in equation (72):

$$T = \frac{14{,}388\ \mu\text{m-K}/0.53\ \mu\text{m}}{\ln\left(0.8\cdot\left(e^{14{,}388\ \mu\text{m-K}/0.53\ \mu\text{m}\cdot\ 1{,}950\ \text{K}}-1\right)+1\right)} = 2{,}024\ \text{K} \quad (72)$$

The emissivity at 1.0 μm using equation (28) and routine bb_emiss is obtained as shown in equation (73):

$$\varepsilon_{\lambda_2} = \frac{e^{14{,}388\ \mu\text{m}-\text{K}/1.0\ \mu\text{m}\cdot 2{,}024\ \text{K}}-1}{e^{14{,}388\ \mu\text{m}-\text{K}/1.0\ \mu\text{m}\cdot 1{,}740\ \text{K}}-1} = 0.313 \quad (73)$$

The sensitivity to the emissivity at wavelength 2 based on the assumed emissivity at wavelength 1 using equation (28) and routine bb_dlne2dlne1 is obtained as shown in equation (74):

$$\frac{d\ln\varepsilon_{\lambda_2}}{d\ln\varepsilon_{\lambda_1}} = \frac{0.53\ \mu\text{m}}{1.0\ \mu\text{m}}\cdot\frac{e^{-14{,}388\ \mu\text{m}-\text{K}/0.53\ \mu\text{m}\cdot 2{,}024\ \text{K}}-1}{e^{-14{,}388\ \mu\text{m}-\text{K}/1.0\ \mu\text{m}\cdot 2{,}024\ \text{K}}-1} = 0.530 \quad (74)$$

The relative uncertainty in the emissivity is obtained as shown in equation (75):

$$\frac{\Delta\varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \frac{d\ln\varepsilon_{\lambda_2}}{d\ln\varepsilon_{\lambda_1}}\cdot\frac{\Delta\varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = 0.530\cdot 25\% = 13.3\% \quad (75)$$

The emissivity at 5.8 μm using routine bb_emiss is obtained as shown in equation (76):

$$\varepsilon_{\lambda_2} = \frac{e^{14{,}388\ \mu\text{m}-\text{K}/5.8\ \mu\text{m}-2{,}204\ \text{K}}-1}{e^{14{,}388\ \mu\text{m}-\text{K}/5.8\ \mu\text{m}-1{,}740\ \text{K}}-1} = 0.761 \quad (76)$$

The sensitivity to the emissivity at wavelength 2 based on the assumed emissivity at wavelength 1 using routine bb_dlne2dlne1 is obtained as shown in equation (77):

$$\frac{d\ln\varepsilon_{\lambda_2}}{d\ln\varepsilon_{\lambda_1}} = \frac{0.53\mu\text{m}}{5.8\mu\text{m}} \cdot \frac{e^{-14{,}388\ \mu\text{m}-\text{K}/0.53\mu\text{m}\cdot 2{,}024\ \text{K}} - 1}{e^{-14{,}388\ \mu\text{m}-\text{K}/5.8\mu\text{m}\cdot 2{,}024\ \text{K}} - 1} = 0.129 \quad (77)$$

Finally, the relative uncertainty in the emissivity is obtained as shown in equation (78):

$$\frac{\Delta\varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \frac{d\ln\varepsilon_{\lambda_2}}{d\ln\varepsilon_{\lambda_1}} \cdot \frac{\Delta\varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = 0.129 \cdot 25\% = 3.2\% \quad (78)$$

Note that the emissivity uncertainty at 5.8 μm is four times less than at 1 μm.

Example 9

Example 9: For a two-band radiation thermometer with detector wavelengths at 0.5 and 0.6 μm, what is the true temperature and the spectral emissivities when the measured equivalent blackbody temperatures at the two wavelengths are 2,800 and 2,750 K, respectively and an emissivity ratio of 0.9 is assumed?

Answer 9: In this case, Wien's approximation is valid, so that using the approximate method from equation (33) and calculating the effective wavelength Λ from equation (36) using routine bb_erlam is as presented in equation (79):

$$\Lambda = \frac{0.5\ \mu\text{m} \cdot 0.6\ \mu\text{m}}{0.6\ \mu\text{m} - 0.5\ \mu\text{m}} = 3.0\ \mu\text{m} \quad (79)$$

Using equation (34), the ratio temperature T_r from routine bb_ertemp is as presented in equation (80):

$$T_r = \left(\frac{3.0\mu\text{m}}{0.5\ \mu m \cdot 2{,}800\ \text{K}} - \frac{3.0\ \mu m}{0.6\ \mu m \cdot 2{,}750\ \text{K}}\right)^{-1} = 3{,}080\ \text{K} \quad (80)$$

As well, using equation (33) and routine bb_tstw, the true temperature is as presented in equation (81):

$$T = \left(\frac{1}{3{,}080\ \text{K}} - \frac{3.0\ \mu m}{14{,}388\ \mu m-\text{K}} \cdot \ln 0.9\right)^{-1} = 3{,}304\ \text{K} \quad (81)$$

`

Knowing the individual measured detector temperatures and the true temperature, the emissivities are determined using equation (29) and routine bb_emiss as shown in equations (82) and (83):

$$\varepsilon_{\lambda_1} = \frac{\left(e^{14{,}388\ \mu\text{m}-\text{K}/0.5\ \mu\text{m}\cdot 3{,}304\ \text{K}} - 1\right)}{\left(e^{14{,}388\ \mu\text{m}-\text{K}/0.5\ \mu\text{m}\cdot 2{,}800\ \text{K}} - 1\right)} = 0.209 \quad (82)$$

$$\varepsilon_{\lambda_2} = \frac{\left(e^{14{,}388\ \mu\text{m}-\text{K}/0.6\ \mu\text{m}\cdot 3{,}304\ \text{K}} - 1\right)}{\left(e^{14{,}388\ \mu\text{m}-\text{K}/0.6\ \mu\text{m}\cdot 2{,}750\ \text{K}} - 1\right)} = 0.232 \quad (83)$$

It is seen that $\varepsilon_{\lambda_1}/\varepsilon_{\lambda_2} = 0.9$, which is consistent with the initial assumption.

Example 10

Example 10: For the same conditions as used in Example 9, what would the error in the true temperature be if an assumed emissivity ratio of 1.0 was used but the actual ratio was 0.9?

Answer 10: This problem can be solved two ways: directly or using sensitivities. Sensitivities are presented first.

In the sensitivities case, Wien's approximation is valid, since both of the wavelength-temperature products are small than C_2. The sensitivity from equation (27) is as shown in equation (84):

$$\frac{d\ln T}{d\ln \varepsilon_{\lambda_i}} = -\frac{3.0\ \mu\text{m} \cdot 3{,}304\ \text{K}}{14{,}388\ \mu\text{m–K}} = -0.688 \tag{84}$$

The estimated error in temperature is as shown in equation (85):

$$-0.688\ \cdot 0.1 = -6.9\% \text{ or } -228\ \text{K} \tag{85}$$

The difference can also be calculated directly, rather than by using sensitivities.

For an emissivity of 1.0, the true temperature is simply the ratio temperature, or 3,080 K. The error, therefore, is as obtained by equation (86):

$$3080\ \text{K}\ - 3{,}304\ \text{K} = 224\ \text{K} \tag{86}$$

This result is almost exactly equal to the linear approximation that was obtained using sensitivities.

Example 11

Example 11: For a two-band radiation thermometer with detector wavelengths at 4.0 and 8.0 μm, what is the error introduced in the temperature measurement when an emissivity ratio of 1.0 is assumed, when in fact the emissivity ratio of the sample has a ratio of 0.9? Assume the measured spectral temperatures at the two wavelengths are 2,800 and 2,750 K, as in Example 9.

Answer 11: Since the products of T_λ and λ for both wavelengths are not less than C_2 or $14{,}388\ \mu$m-K, the full Planck equation must be used. Similar to Example 10, this problem can be solved two ways: directly or by using sensitivities. Again, the solution using sensitivities is presented first. The true temperature is calculated using equation (31) and routine bb_tratio as shown in equation (87):

$$\frac{\left(e^{C_2/\lambda_2 T_2} - 1\right)}{\left(e^{C_2/\lambda_2 T} - 1\right)} \cdot \frac{\left(e^{C_2/\lambda_1 T} - 1\right)}{\left(e^{C_2/\lambda_1 T_1} - 1\right)} - \varepsilon_r = 0 \tag{87}$$

The calculated values for true temperatures are for the two emissivity ratios presented as equations (88) and (89):

$$\varepsilon_r = 0.9,\ T = 4{,}118\ \text{K} \tag{88}$$

$$\varepsilon_r = 1.0,\ T = 2{,}974\ \mathrm{K} \tag{89}$$

The difference is -1,144 K!

This example points out the large error using long wavelengths and emphasizes the need to use short wavelengths whenever possible.

The sensitivity of the emissivity ratio to true temperature is (equation (37) as shown in equation (90):

$$\frac{d\ln\varepsilon_r}{d\ln T} = \frac{1}{T}\cdot\frac{\left(e^{C_2/\lambda_2 T}-1\right)}{\left(e^{C_2/\lambda_1 T}-1\right)} \cdot\left[\left(\frac{C_2}{\lambda_2 T}\right) e^{\frac{C_2}{\lambda_2 T}}\frac{\left(e^{C_2/\lambda_1 T}-1\right)}{\left(e^{C_2/\lambda_2 T}-1\right)^2} - \left(\frac{C_2}{\lambda_1 T}\right)\frac{e^{C_2/\lambda_1 T}}{\left(e^{C_2/\lambda_2 T}-1\right)}\right] \tag{90}$$

The temperature change is so great that linear sensitivities extrapolated from one point will not be accurate, so instead an average of temperatures and emissivity is used.

The calculated sensitivities are as obtained in equations (81) and (92):

$$\varepsilon_r = 0.9,\ \frac{d\ln T}{d\ln\varepsilon_r} = -3.77 \tag{91}$$

$$\varepsilon_r = 1.0,\ \frac{d\ln T}{d\ln\varepsilon_r} = -2.56 \tag{92}$$

The temperature error can be estimated using a centered average of the temperature and sensitivity, as shown in equation (93):

$$\Delta\mathrm{T} = -\left(4{,}118\ \mathrm{K} + 2{,}974\ \mathrm{K}\right)\cdot\frac{(3.77+2.56)}{4}\cdot 0.1 = -1{,}122\ \mathrm{K} \tag{93}$$

Example 12

Example 12: Given the data presented in figure 5 from a 32-channel silicon array detector, calculate the spectral emissivity and true temperature if the spectral emissivity is assumed to follow a second-order polynomial with wavelength.

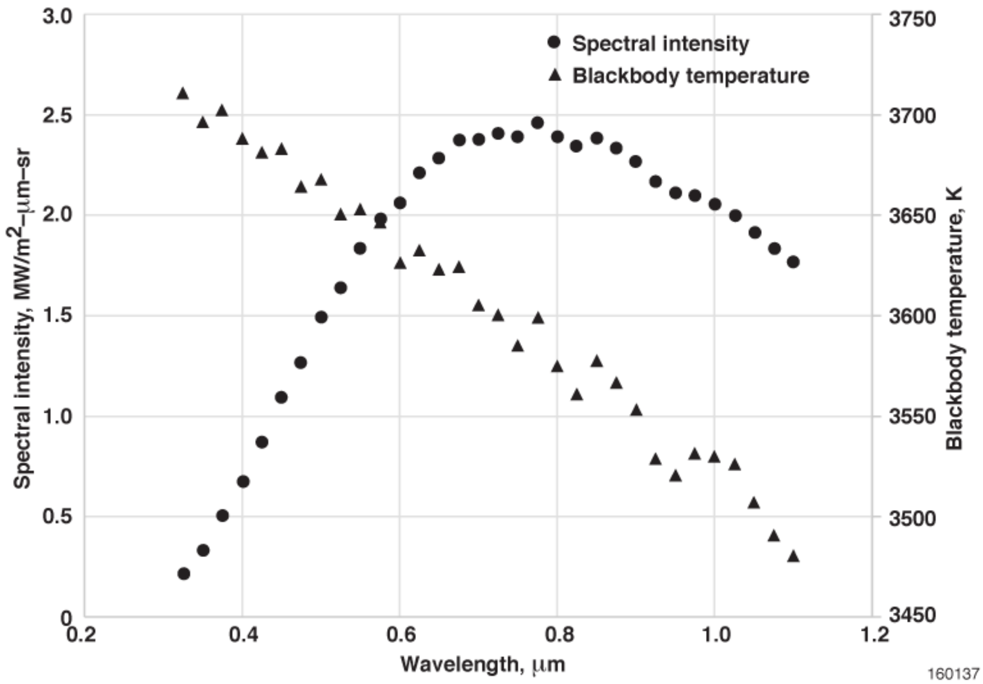

Figure 5. Spectral intensity versus wavelength for a 32-detector measurement.

Answer 12: The solution requires that values be found for T, a_0, a_1, and a_2 that minimize the function f (eq. (94)):

$$f = \sum_{i=1}^{n} \left(\varepsilon_{\lambda_i} \cdot i_{b,\lambda_i}(T) - i_{\lambda_i}\left(T_{\lambda_i}\right) \right)^2 \tag{94}$$

where equation (95) is shown as:

$$\varepsilon_\lambda = a_2 \lambda^2 + a_1 \lambda + a_0 \tag{95}$$

Using routine bb_ibl to calculate $i_{b,\lambda_i}(T)$ along with the multi-variable minimization capability “Solver” in Excel®, the solution is as shown in equations (96) and (97):

$$T = 3{,}802 \text{ K} \tag{96}$$

$$a_2 = \text{-}0.06222,\ a_1 = 0.0610, \text{ and } a_0 = 0.7347 \tag{97}$$

The solution is shown in figure 6.

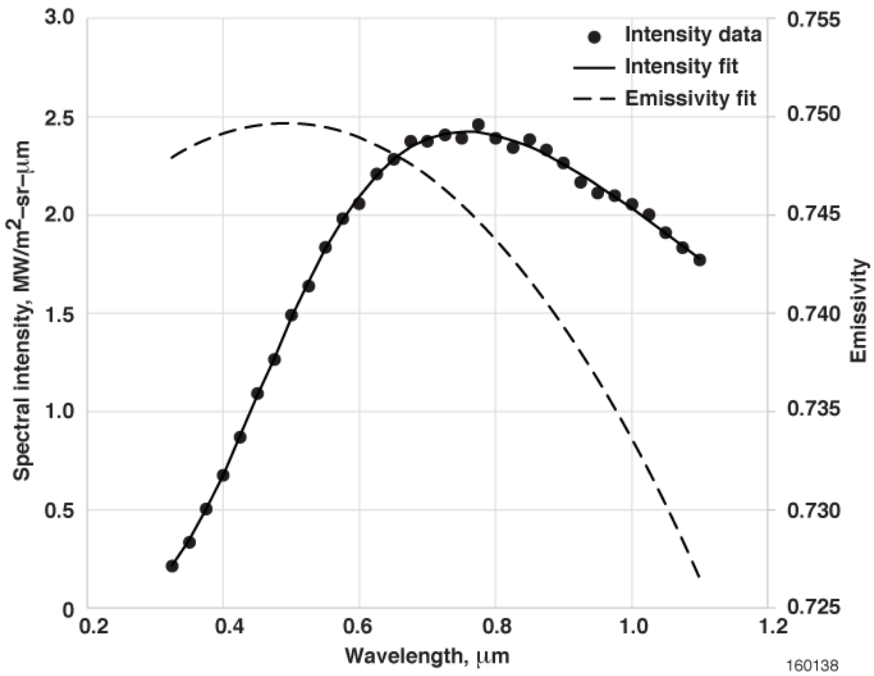

Figure 6. Calculated best-fit spectral intensity and emissivity using least squares minimization of 32-detector measurement.

Example 13

Example 13: What is the non-linearity introduced by averaging the response of a detector covering the range of 1 to 4 μm at temperatures in between 600 to 1,100 K? Assume a uniform spectral response function.

Answer 13: The problem requires a comparison (eq. (98)):

$$I(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} i_{b,\lambda}(\lambda, T) d\lambda \tag{98}$$

using routine bb_iibl for each temperature T to the quantity calculated at the average wavelength (eq. (99)):

$$I(\bar{\lambda}, \Delta\lambda, T) = i_{b,\lambda}(\bar{\lambda}, T)\Delta\lambda \tag{99}$$

using routine bb_ibl, where (eq. (100):

$$\bar{\lambda} = \frac{1}{2}(\lambda_l + \lambda_u) \text{ and } \Delta\lambda = \lambda_u - \lambda_l \tag{100}$$

so that $\bar{\lambda}$ = 2.5 μm and Δλ = 3.0 μm. The comparison is shown in figure 7.

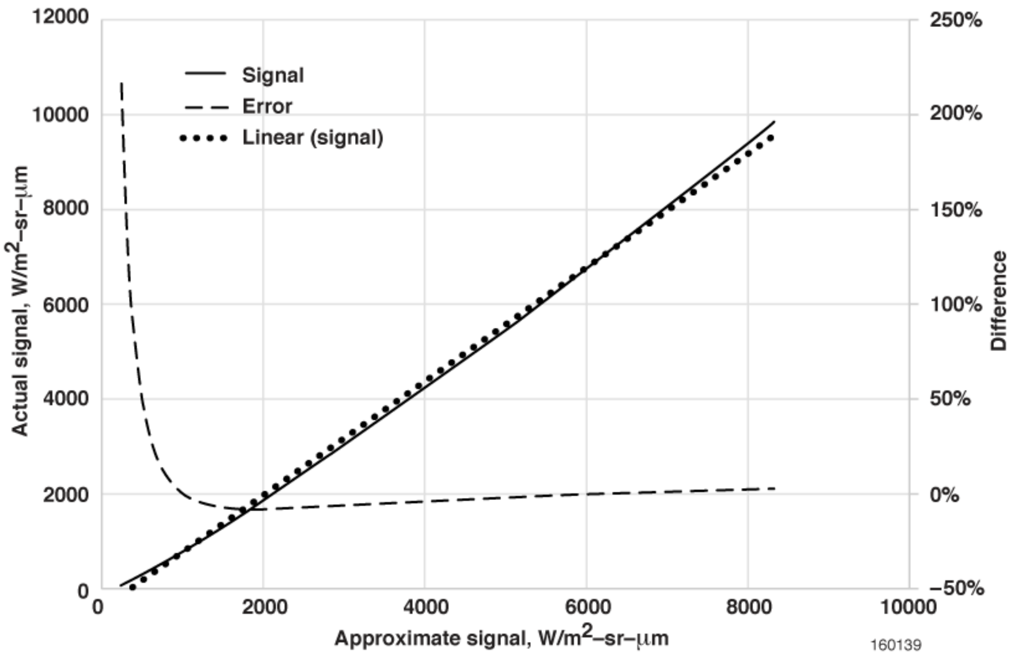

Figure 7. Comparison of signals calculated with exact and approximate methods for a wide-band detector with wavelengths between 1 and 4 μm and a uniform detector response across the wavelength band.

Note the non-linearity in the calibration and the high error at low intensities as discussed in the section pertaining to wide-band detectors, found in the Basic Equations Chapter.

Example 14

Example 14: What is the non-linearity introduced by averaging the response of a detector covering the range of 0.25 to 1.05 μm at temperatures in between 2,000 to 3,000 K? Assume a linearly increasing detector spectral response function, as shown in figure 8.

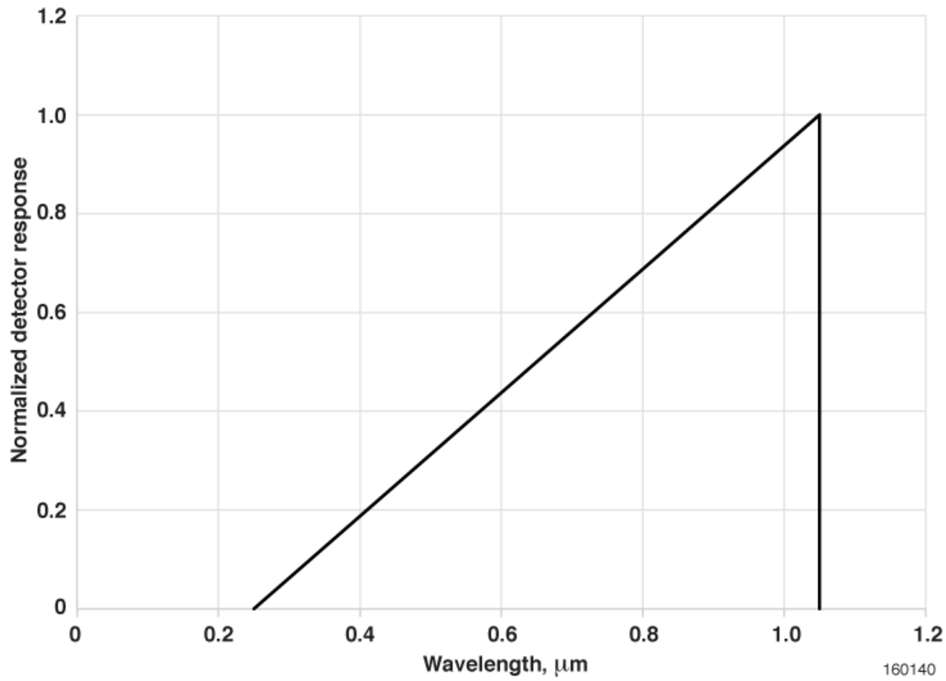

Figure 8. Assumed detector response function for a 0.25- to 1.05-µm detector showing a linearly increasing detector response across the wavelength band.

Answer 14: The problem requires that we compare the quantity calculated by the integral in eq. 101:

$$I(\lambda_l, \lambda_u, T) = \int_{\lambda_l}^{\lambda_u} D(\lambda) \cdot i_{b,\lambda}(\lambda,\, T) d\lambda \tag{101}$$

using routine bb_iibl for each temperature T, to the quantity calculated at the average wavelength by eq. 102:

$$I(\bar{\lambda}, \Delta\lambda, T) = D(\bar{\lambda}) \cdot i_{b,\lambda}(\bar{\lambda},\, T)\Delta\lambda \tag{102}$$

using routine bb_ibl, where (eq. (103)):

$$\bar{\lambda} = \frac{1}{2}(\lambda_l + \lambda_u) \text{ and } \Delta\lambda = \lambda_u - \lambda_l \tag{103}$$

such that $\bar{\lambda}$ = 0.65 µm and $\Delta\lambda$ = 0.8 µm. The comparison is shown in figure 9.

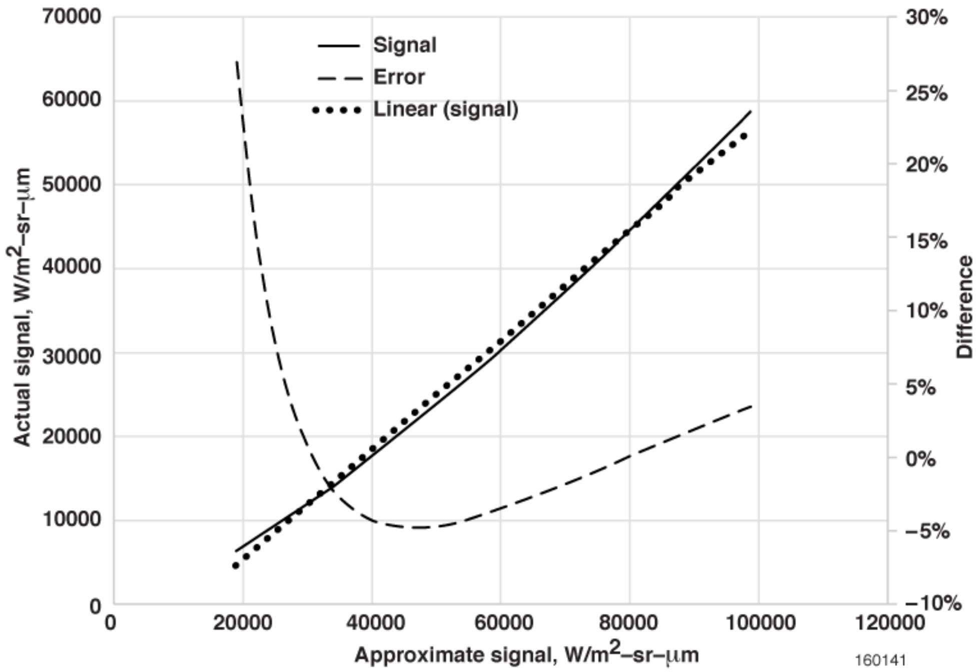

Figure 9. Comparison of signals calculated with exact and approximate methods for a wide-band detector with wavelengths between 0.25 to 1.05 μm and a linearly increasing detector response across the wavelength band.

The result is similar to Example 13. Again note the non-linearity in the calibration and the high error at low intensities.

Example 15

Example 15: A commercial Silicon/InGaAs sandwich detector has the detector response shown in figure 10. If the Silicon detector measures 3,013 K and the InGaAs measures 2,909 K equivalent blackbody temperatures, what are the average spectral emissivities for the two wavelengths and the true temperature, assuming an emissivity ratio of 1.0?

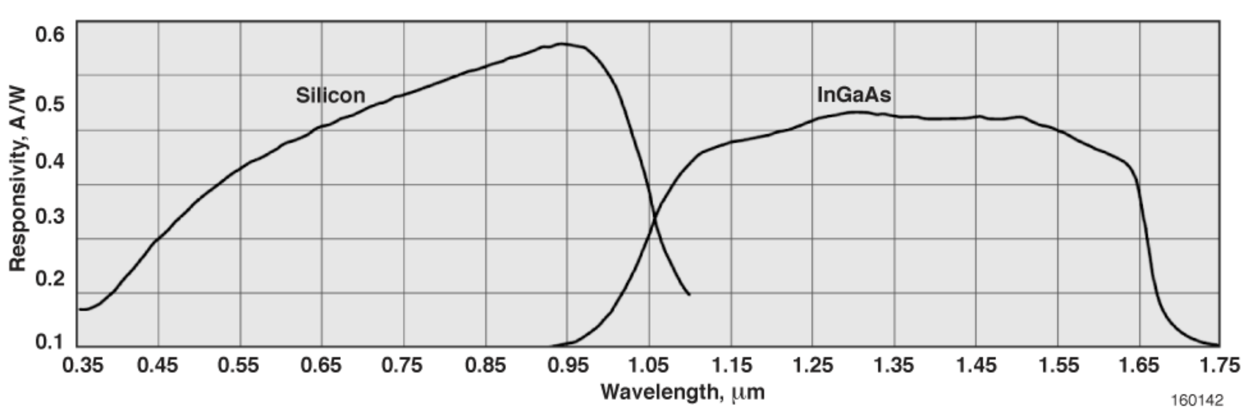

Figure 10. Detector response function for a Silicon/InGaAs sandwich detector.

Answer 15: The detector response function is modeled by the line segments shown in figure 11.

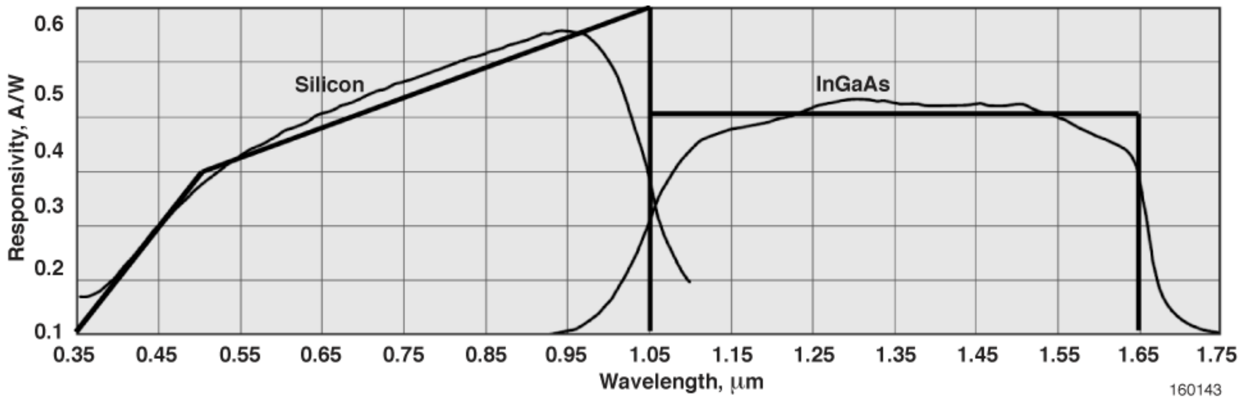

Figure 11. Detector response function for a Silicon/InGaAs sandwich detector modeled as a series of straight lines.

To ensure that the problem is self-consistent, already calculated are the equivalent blackbody temperatures T_λ from integration of the true temperature T across the wavelength band. The calculation uses the detector response functions as shown previously with an emissivity $\bar{\varepsilon}$ uniform over the wavelength range and equal to 0.7 from equation (104):

$$\bar{\varepsilon}\int_{\lambda_l}^{\lambda_u} D(\lambda)\, i_{b,\lambda}(T)\, d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, i_{b,\lambda}(T_\lambda)\, d\lambda \qquad (104)$$

for $T = 3{,}200$ K, T_{λ_1} = 3,013 K and $T_{\lambda_2} = 2.909$ K. The problem requires solving equation (43) for a wide-band, or equation (46) for an equivalent narrow-band, with $\bar{\varepsilon_r} = 0.9$ and T_1 = 3,013 K and T_2 = 2,909 K, as shown in equations (105) and (106):

$$\bar{\varepsilon_r} = \frac{\bar{\varepsilon_1}}{\bar{\varepsilon_2}} = \frac{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}\left(T_{\lambda_1}\right) d\lambda}{\int_{\lambda_{1l}}^{\lambda_{1u}} D_1(\lambda)\, i_{b,\lambda}(T) d\lambda} \cdot \frac{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}(T)\, d\lambda}{\int_{\lambda_{2l}}^{\lambda_{2u}} D_2(\lambda)\, i_{b,\lambda}\left(T_{\lambda_2}\right) d\lambda} \qquad (105)$$

$$\bar{\varepsilon_r} = \frac{\left(e^{-C_2/\overline{\lambda_2}T_{\lambda_2}} - 1\right)}{\left(e^{-C_2/\overline{\lambda_2}T} - 1\right)} \cdot \frac{\left(e^{-C_2/\overline{\lambda_1}T} - 1\right)}{\left(e^{-C_2/\overline{\lambda_1}T_{\lambda_1}} - 1\right)} \qquad (106)$$

The results are summarized in table 1.

Table 1. Solution results for Example 14 with $\overline{\varepsilon_r}$ 0.9.

Parameter	Integrated band	Averaged band	Difference, %
T, K	3,200	3,134	2
$\overline{\varepsilon_1}$	0.70	0.77	10
$\overline{\varepsilon_2}$	0.70	0.76	9

Example 16

Example 16: Consider a dual-band instrument with spectral bands covering 1 to 4.5 μm and 2 to 13.5 μm. The detector response functions are shown in figure 12. The average detector wavelengths are 2.75 and 7 μm. Using the average detector wavelengths, what is the true temperature and what is the inferred emissivity assuming an emissivity ratio of 1?

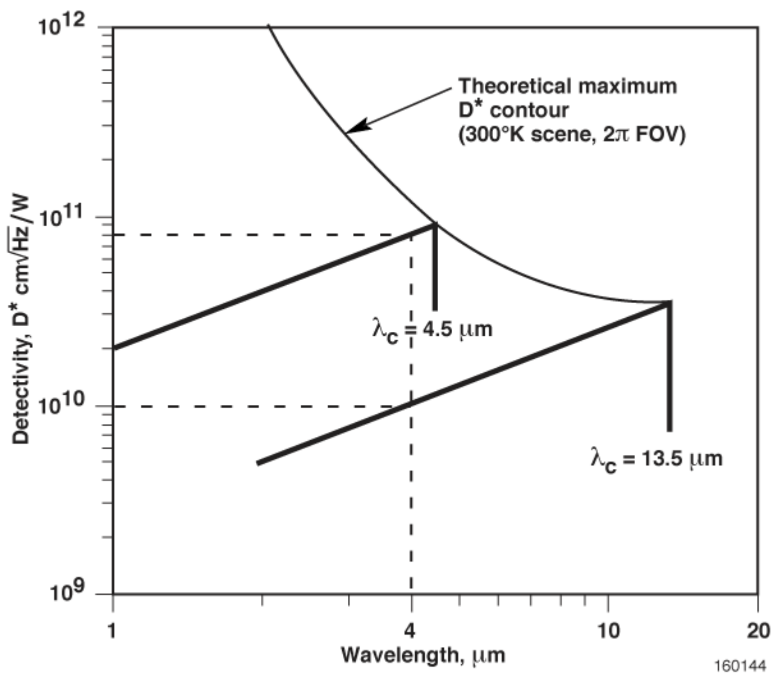

Figure 12. Detector response function two infrared detectors.

Answer 16: The example assumes a gray material, which means that the spectral emissivity is independent of wavelength. Assume that the emissivity is equal to 0.65, as shown in figure 13, and that the true temperature is 3,000 K.

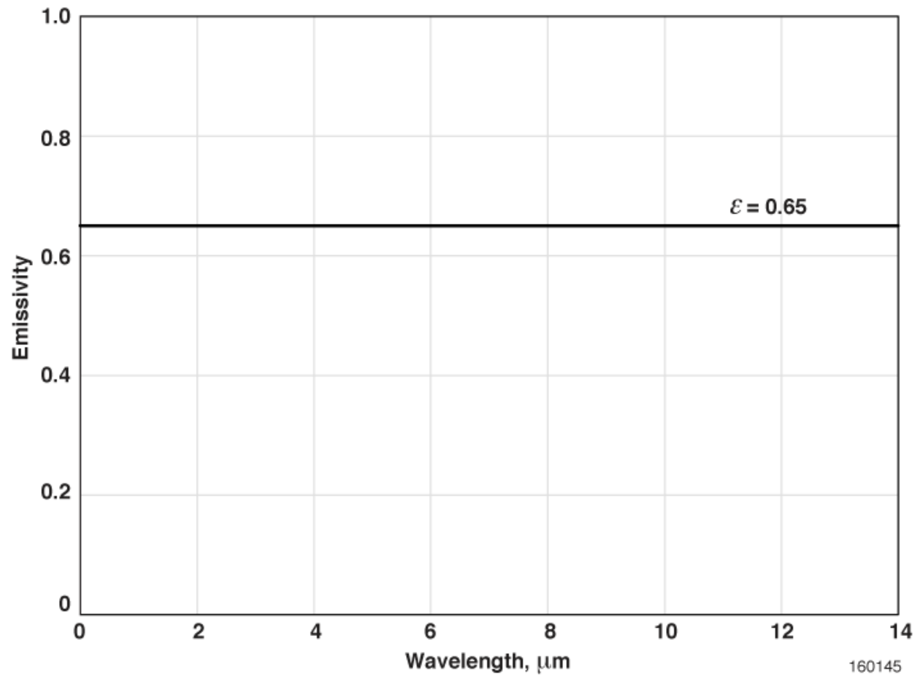

Figure 13. Assumed spectral emissivity as a function of wavelength for Example16.

Using the function bb_tiibl, calculate what the measured equivalent blackbody temperatures T_λ would be by solving equation (107) given the detector response functions $D(\lambda)$, the emissivity ε (independent of wavelength), and the spectral blackbody radiant intensity.

$$\bar{\varepsilon}\int_{\lambda_l}^{\lambda_u} D(\lambda)\, i_{b,\lambda}(T)\, d\lambda = \int_{\lambda_l}^{\lambda_u} D(\lambda)\, i_{b,\lambda}(T_\lambda)\, d\lambda \tag{107}$$

The calculated blackbody temperatures for detectors 1 and 2 are 2,589 and 2,426 K, respectively.

Using the average wavelengths, calculate the equivalent blackbody temperature T and emissivity for an emissivity ratio of 1.0 from the two detector equivalent blackbody temperatures T_1 and T_2. This problem is solved using the function bb_tratio, shown in equation (108):

$$\frac{\left(e^{-C_2/\lambda_2 T_{\lambda_2}}-1\right)}{\left(e^{-C_2/\lambda_2 T}-1\right)}\cdot\frac{\left(e^{-C_2/\lambda_1 T}-1\right)}{\left(e^{-C_2/\lambda_1 T_{\lambda_1}}-1\right)}-\varepsilon_r = 0 \tag{108}$$

For ε_r = 1, the result is (eq. (109)):

$$T = 2{,}879 \text{ K and } \bar{\varepsilon} = 0.79 \tag{109}$$

versus the correct values of T = 3,000 K and ε = 0.65, a -4% and 18% error, respectively.

This solution demonstrates the uncertainty introduced using wide-bandwidth detectors when assuming a single, average wavelength for each detector.

Example 17

Example 17: The final wide-band detector problem demonstrates not only the complexity in reducing data from wide-band detectors, but also the difficulty in interpreting the data.

First, consider a dual-band instrument with spectral bands covering 1 to 4.5 μm and 2 to 13.5 μm. The applicable detector response functions are shown in figure 12.

The material to be measured has a spectral emissivity decreasing with wavelength as shown in figure 14. The true temperature is assumed to be 3,000 K. Using equation (41) for the given detector response functions and the spectral emissivity relationship, the calculated equivalent blackbody temperatures for detectors 1 and 2 are 2,941 and 2,841 K, respectively.

If an emissivity ratio equal to the values at the center wavelength is used, what is the calculated true temperature and the individual spectral emissivities? The exact emissivity behavior with wavelength will generally be unknown, so repeat the problem when the emissivity ratio is naturally assumed to be equal 1.

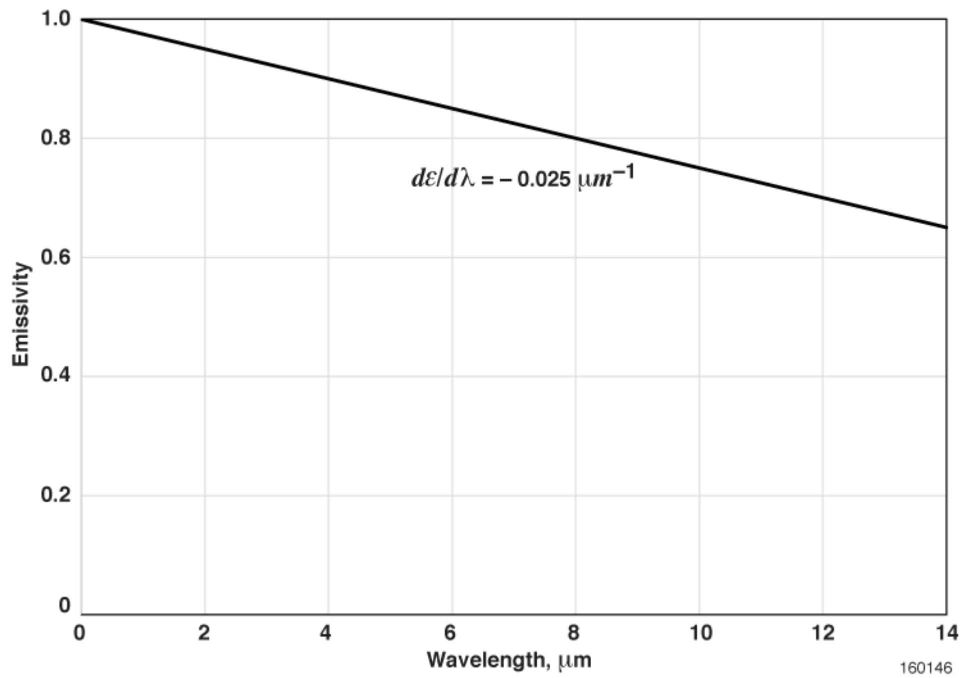

Figure 14. Assumed spectral emissivity as a function of wavelength for Example 17.

Answer 17: The centers of the two bands are 2.75 and 7.75 μm. The emissivities at the center wavelengths from figure 14 are 0.931 and 0.806, resulting in an emissivity ratio of 1.155. Using equation (45) and routine bb_tratio for an equivalent narrow-band calculation, the calculated true temperature is 2,677 K with emissivities for detector 1 and detector 2 being 1.29 and 1.12, respectively. Neither the temperature nor the calculated emissivities are close to that of the actual conditions and material properties.

Using the equivalent wideband equation (equation (43)) and routine bb_itratio with an emissivity ratio $\bar{\varepsilon}_r$ equal to 1.16 results in a calculated true temperature of 2,706 K and emissivities for detector 1 and 2 of 1.27 and 1.10. While the temperature is closer, the accuracy is still poor and the emissivities are quite different and non-physical (both greater than 1).

If an emissivity ratio $\bar{\varepsilon_r}$ equal to 1 is used, then the calculated true temperature is 3,179 K for the wideband calculation with resulting emissivities for the two detectors equal to 0.81. The calculated true temperature is also quite far from the true value. As can be seen, over such a wide range of wavelengths, it is difficult to choose the proper emissivity ratio since the wavelength dependence on spectral emissivity generally is not known.

Table 2. Solution results for Example 17.

	$\bar{\varepsilon_r}$ =1.0		$\bar{\varepsilon_r}$ =1.13	
Parameter	Integrated band	Averaged band	Integrated band	Averaged band
T, K	3,179	3,127	2,706	2,622
$\bar{\varepsilon_1}$	0.81	0.88	1.27	1.29
$\bar{\varepsilon_2}$	0.81	0.88	1.10	1.12

Using wide-band detectors centered at long wavelengths and that span a wide range of wavelengths do not give realistic results for both emissivity and temperature for any assumption.

Example 18

Example 18: A blackbody source is estimated to have an uncertainty of 10 K and is used to perform a single-point calibration on two detectors, 1) a short-wavelength, 0.5-μm detector and 2) a longer-wavelength 3-μm detector. An unknown material is then measured at the same temperature with these detectors. The equivalent blackbody temperatures of the material from the two detectors are measured to be 1,600 and 1,500 K, respectively. What is the value and the uncertainty in the derived emissivity at 3 μm, assuming the material has an emissivity of 0.8 ±0.1 at 0.5 μm?

Answer 18: The equivalent blackbody temperature from the measurement of the 0.5-μm detector is for T_{λ_1} = 1,600K and ε_{λ_1}= 0.8 using equation (23) and routine bb_tst is as shown in equation (110):

$$T = \frac{C_2}{\lambda_1} \cdot \frac{1}{\ln\left[\varepsilon_{\lambda_1}\left(e^{\frac{C_2}{\lambda T_{\lambda_1}}} - 1\right) + 1\right]} = 1{,}620 \text{ K} \tag{110}$$

The calculated emissivity at 3 μm using routine bb_emiss is as shown in equation (111):

$$\varepsilon_\lambda = \frac{\left(e^{C_2/\lambda T_\lambda} - 1\right)}{\left(e^{C_2/\lambda T} - 1\right)} = 0.78 \tag{111}$$

Assuming the temperature errors in the two calibration measurements are uncorrelated and random, the total uncertainty in the emissivity will be as shown in equation (112):

$$\Delta\varepsilon_{\lambda_2} = \sqrt{\left(\left(\frac{\partial\varepsilon_{\lambda_2}}{\partial T_{\lambda_1}}\right)\Delta T_{\lambda_1}\right)^2 + \left(\left(\frac{\partial\varepsilon_{\lambda_2}}{\partial T_{\lambda_2}}\right)\Delta T_{\lambda_2}\right)^2 + \left(\left(\frac{\partial\varepsilon_{\lambda_2}}{\partial\varepsilon_{\lambda_1}}\right)\Delta\varepsilon_{\lambda_1}\right)^2} \qquad (112)$$

where $\Delta T_{\lambda_1} = 10\mathrm{K}$ and $\Delta\varepsilon_{\lambda_1} = 0.1$; or, in terms of sensitivities (eq. (113)):

$$\frac{\Delta\varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{\left(\left(\frac{\partial \ln\varepsilon_{\lambda_2}}{\partial \ln\varepsilon_{\lambda_1}}\right)\left(\frac{\partial \ln\varepsilon_{\lambda_1}}{\partial \ln T_{\lambda_1}}\right)\frac{\Delta T_{\lambda_1}}{T_{\lambda_1}}\right)^2 + \left(\left(\frac{\partial \ln\varepsilon_{\lambda_2}}{\partial \ln T_{\lambda_2}}\right)\frac{\Delta T_{\lambda_2}}{T_{\lambda_2}}\right)^2 + \left(\left(\frac{\partial \ln\varepsilon_{\lambda_2}}{\partial \ln\varepsilon_{\lambda_1}}\right)\frac{\Delta\varepsilon_{\lambda_1}}{\varepsilon_{\lambda_1}}\right)^2} \qquad (113)$$

Evaluating terms gives the following calculations, seen in equations (114)-(118):

$$\frac{d\ln\varepsilon_{\lambda_1}}{d\ln T_{\lambda_1}} = \frac{C_2}{\lambda_1 T_{\lambda_1}} \cdot \frac{e^{C_2/\lambda_1 T_{\lambda_1}}}{\left(e^{C_2/\lambda_1 T_{\lambda_1}} - 1\right)} = 18.0 \qquad (114)$$

$$\frac{d\ln\varepsilon_{\lambda_2}}{d\ln T_{\lambda_2}} = \frac{C_2}{\lambda_2 T_{\lambda_2}} \cdot \frac{e^{\frac{C_2}{\lambda_2 T_2}}}{\left(e^{C_2/\lambda_2 T_{\lambda_2}} - 1\right)} = 3.33 \qquad (115)$$

$$\frac{d\ln\varepsilon_{\lambda_2}}{d\ln\varepsilon_{\lambda_1}} = \frac{\lambda_1}{\lambda_2} \cdot \frac{\left(e^{-C_2/\lambda_1 T} - 1\right)}{\left(e^{-C_2/\lambda_2 T} - 1\right)} = 0.176 \qquad (116)$$

$$\frac{\Delta\varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{\left(0.176 \cdot 18.0 \cdot \frac{10}{1{,}600}\right)^2 + \left(3.33 \cdot \frac{10}{1{,}500}\right)^2 + \left(0.176 \cdot \frac{0.1}{0.8}\right)^2} \qquad (117)$$

$$\frac{\Delta\varepsilon_{\lambda_2}}{\varepsilon_{\lambda_2}} = \sqrt{3.94 \times 10^{-4} + 4.94 \times 10^{-4} + 4.83 \times 10^{-4}} = 3.7\% \qquad (118)$$

and, therefore, (eqs. (119) and (120)):

$$\Delta\varepsilon_{\lambda_2} = 0.78 \cdot 0.037 = 0.029 \quad (119)$$

$$\varepsilon_{\lambda_2} = 0.78 \pm 0.029 \quad (120)$$

Note that the uncertainty resulting from the calibration is on the same order as the estimation of the emissivity at 0.5 μm, 0.8 ±0.1.

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