An Upper Bound on Orbital Debris Collision Probability When Only One Object has Position Uncertainty Information

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Background: High speed close approach problem

- The high-speed, close approach problem
  - Two satellites with known state and state uncertainty information pass near each other
  - State and state uncertainty data are combined to give relative position and relative position uncertainty at the time of closest approach
  - Position and position uncertainty information are transformed into a close approach reference frame
  - Based on an assumed hard body size, a collision risk (Pc) is computed using the relative data.
- Mathematics of the problem
  - Assume normally distributed uncertainties
  - 5 degrees of freedom in the final close approach frame problem
    - 2 position coordinates: x and y
    - 3 covariance matrix parameters: σ_x, σ_y, and ρ

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Background: Maximum Pc problem

- The maximum Pc problem is generally concerned with finding the largest Pc that can exist for a given miss position
- Various versions of this maximum Pc problem exist:
  - Sigma scaling alters the size of the x and y sigmas while leaving the correlation unchanged
  - The correlation may be changed while keeping the sigmas fixed
  - The covariance matrix size and shape may be fixed while the covariance is rotated to maximize Pc - all three covariance parameters are altered in a coordinated way
  - The covariance may be generalized to a circle and sized to determine a maximum Pc
  - Varying all three parameters freely to obtain a maximum Pc results in a degenerate ellipse lying along the miss vector

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Background: Maximum Pc problem

- Such maximum Pc variations are based on having a known estimated miss position but unreliable position uncertainties
- They deal with close approach frame position uncertainty as three final parameters that have, in a sense, been disassociated from their original sources – the two objects involved in the close approach
- It is often the case that in many active satellite close approach events the asset (primary) and the debris (secondary) have different position uncertainty qualities
  - Asset may be well tracked and so has good position uncertainty
  - Debris not well tracked and so has poor position uncertainty
  - Debris may not have any position uncertainty available at all

Background: Maximum Pc problem

- When poor position covariance information for one or both objects exists one or more of the maximum Pc methods may have been tried to bound the collision risk
- The question arises about what to do for that case in which no position uncertainty information exists (or its quality is too poor for use) for the debris but reliable position uncertainty information does exist for the asset
- An analyst might default to the conclusion that it is not possible to determine a Pc and so have no basis at all for any operational decision
- It is possible to use the available data in a more helpful way

Maximum Pc with missing covariance data

- Recall that only state vectors are used to define the close approach frame
- The asset covariance matrix data may still be transformed into the close approach frame
- The missing covariance of the debris must be modeled in some way
  - Maybe as three unknown parameters with constraints
  - There is a better, more direct way

Maximum Pc with missing covariance data

- The covariance matrix describes the probability density dispersion of the unknown state
- Any variation of the covariance matrix that pushes probability away from the hard body figure decreases the collision risk
- Any variation of the covariance matrix that pushes probability toward the hard body figure increases the collision risk
- The asset covariance matrix is given so its contribution to the Pc is effectively constrains the maximum Pc
- The debris covariance matrix contribution to the Pc will be maximized if it is modeled as a degenerate ellipse lying along the miss vector

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Maximum Pc with missing covariance data

- Covariance matrix relationships

$$\boldsymbol{C}_{TC} = \boldsymbol{C}_{AC} + \boldsymbol{C}_{DC} \qquad \boldsymbol{C}_{DC} = V_r \, \boldsymbol{u}_{rel} \, \boldsymbol{u}_{rel}^T$$

- Simplified probability

$$P_C \approx A_C \frac{e^{-0.5 \boldsymbol{r}_{rel}^T \boldsymbol{C}_{TC}^{-1} \boldsymbol{r}_{rel}}}{2\pi \sqrt{\left|\boldsymbol{C}_{TC}\right|}}$$

- Critical values

$$V_{r\_critical} = V_C = r_{mag}^2 \left(\frac{K_A^2 - 1}{K_A^2} \right) \qquad K_A^2 = \boldsymbol{r}_{rel}^T \boldsymbol{C}_{AC}^{-1} \boldsymbol{r}_{rel}$$

- Maximum Pc

$$P_C \approx A_C \frac{e^{-1/2}}{2\pi \sqrt{\left|\boldsymbol{C}_{AC} + V_C \, \boldsymbol{u}_{rel} \, \boldsymbol{u}_{rel}^T\right|}}$$

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Missing covariance matrix example

- Miss position and unit vector:

$$\boldsymbol{r}_{rel} = [1000 \quad 200]^T \qquad \boldsymbol{u}_{rel} = \left[{5}/{\sqrt{26}} \quad {1}/{\sqrt{26}} \right]^T$$

- Asset close approach (or collision plane) frame position error covariance matrix:

$$\boldsymbol{C}_{AC} = \begin{bmatrix} 722500 & 0 \\ 0 & 2500 \end{bmatrix}$$

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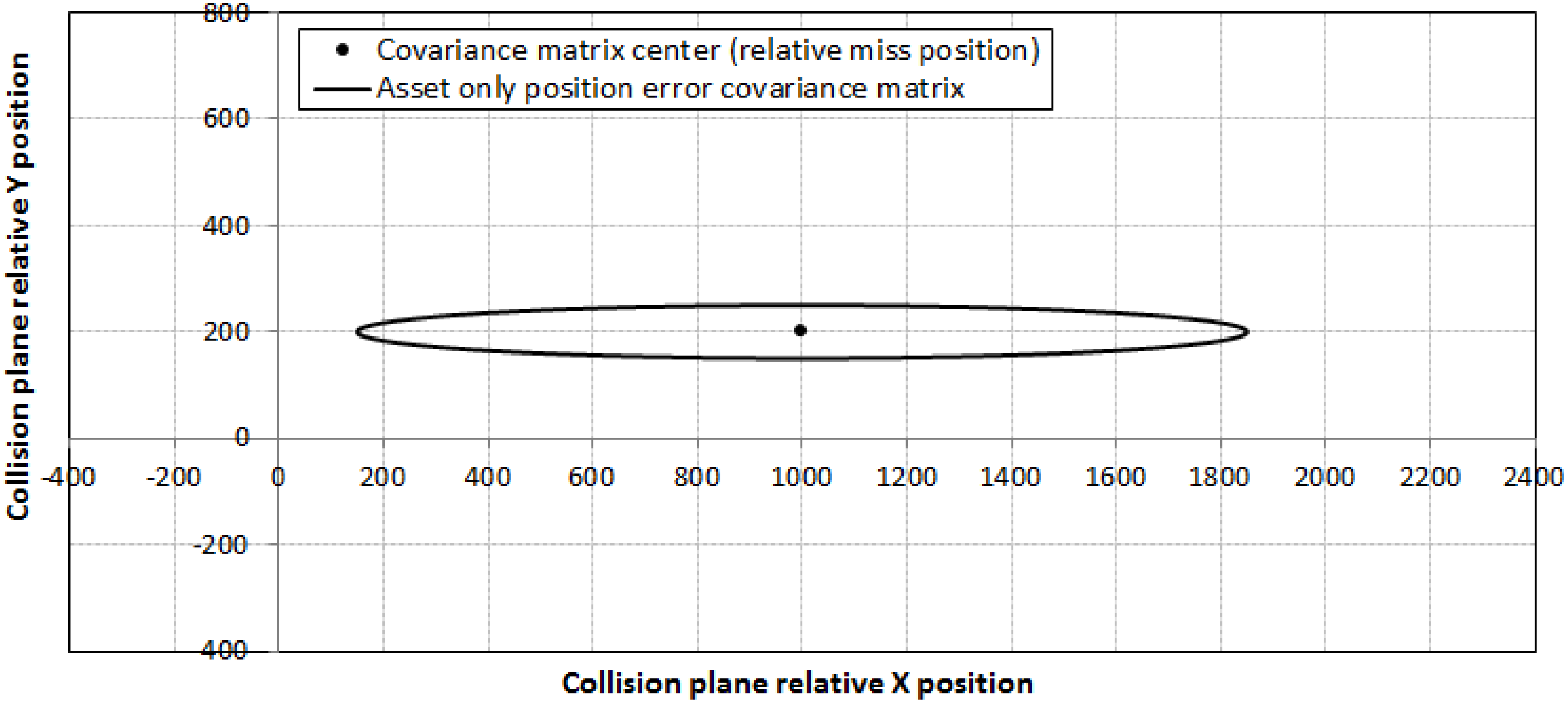
Missing covariance matrix example

Asset only error covariance 1-sigma ellipse



Missing covariance matrix example

- Critical value of the missing covariance:

$$V_C = \frac{153887500}{157} \approx 9.8 \cdot 10^5$$

- Total, close approach frame, position error covariance matrix:

$$C_{TC} = \begin{bmatrix} 261401250/157 & 29593750/157 \\ 29593750/157 & 6311250/157 \end{bmatrix}$$

$$C_{TC} \approx \begin{bmatrix} 1664976.1 & 188495.2 \\ 188495.2 & 40199.0 \end{bmatrix}$$

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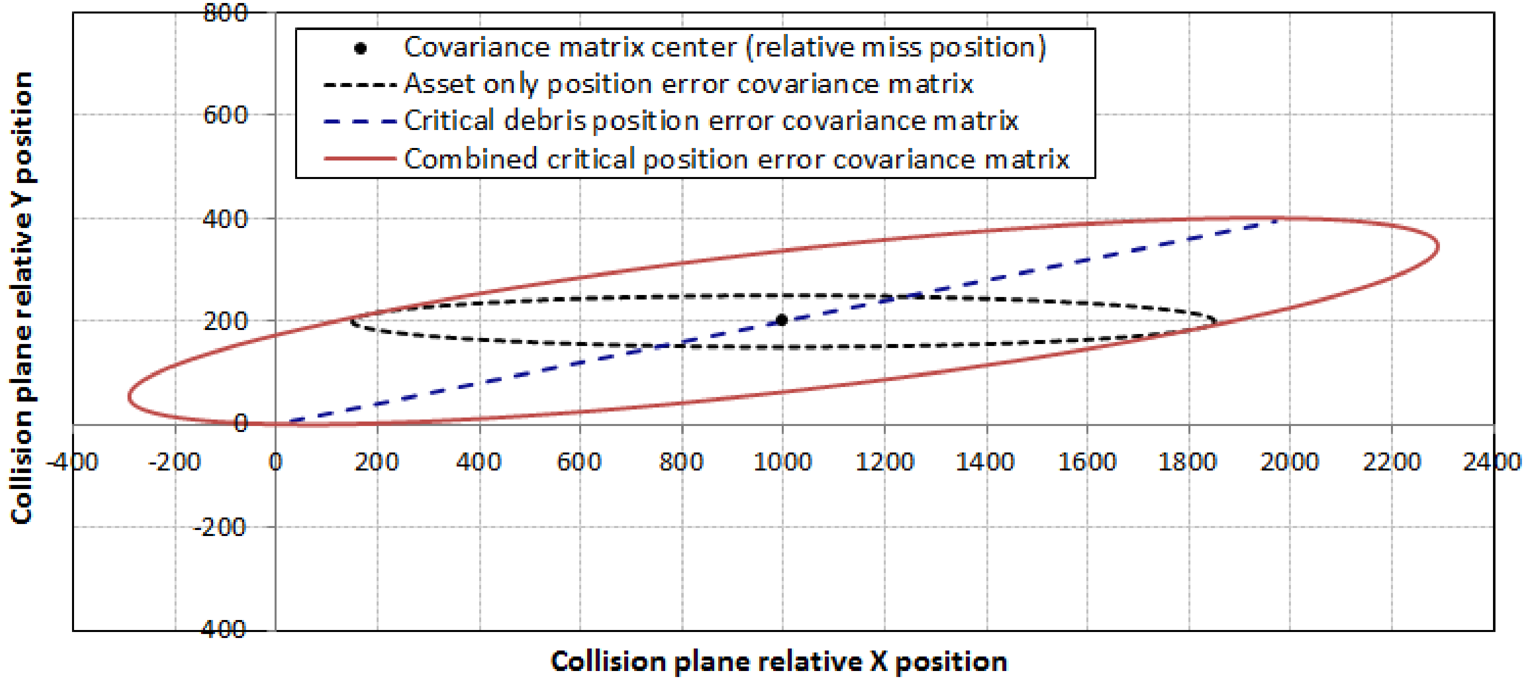
Missing covariance matrix example

Critical covariance 1-sigma ellipse for the maximum Pc



Missing covariance matrix example

- Calculated maximum Pc:

$$P_C \approx \frac{R_{HB}^2 e^{-1/2}}{20000\sqrt{314}} \approx 1.71 \cdot 10^{-6} R_{HB}^2$$

- As compared to:
  - Circular uncertainty: $P_{circ} \approx 3.54 \cdot 10^{-7} R_{HB}^2$
  - Degenerate ellipse: $P_{degen} \approx 4.75 \cdot 10^{-4} R_{HB}$

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Missing covariance matrix discussion

- The technique does not guarantee a high collision risk if the computed Pc is above some threshold, only that there is no high collision risk if the maximum is below that threshold
- The technique does not guarantee that the critical variance is realistic with respect to covariances of typical estimated orbits
  - No discussion about the unknown position uncertainty normal to the close approach plane (see paper)
  - It is possible to map the critical variance and unknown plane-normal uncertainty back into the debris object's uvw frame for examination
  - Rational guidelines may allow a reduction in the magnitude of the critical variance and therefore lower the maximum Pc
- The technique is sensitive to the size/shape of the collision plane frame projection of the known position error covariance matrix (more uncertainty normal to the miss vector is better)

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Missing covariance matrix discussion

- The technique may be applied to variations on the close approach problem that require single pass evaluations:
  - Maneuver screening (asset covariance unknown after a maneuver)
  - Launch COLA and COLA Gap problems (launch covariance unknown)
  - General collision avoidance (CA) against a state-only catalog
- What about probability based CA screening against a catalog?
  - CA screening identifies near term collision threat objects (yes, maybe)
  - CA screening also identifies objects worthy of additional tracking that may remain or may become collision threat objects (no, the technique eliminates objects from interest and cannot guarantee they will not become of interest in the future)

Missing covariance matrix summary

- The technique offers potential for a real improvement in dealing with missing and poor quality position error covariance matrices
- The technique may be applied to variations on the close approach problem
- Along with the approximate Pc calculation, the technique offers a very simple way to estimate collision risk in problems which previously may have been difficult to deal with