Analysis and Characterization of Damage Utilizing an Orthotropic Generalized Composite Material Model Suitable for Use in Impact Problems

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ABSTRACT

The need for accurate material models to simulate the deformation, damage and failure of polymer matrix composites under impact conditions is becoming critical as these materials are gaining increased usage in the aerospace and automotive communities. In order to address a series of issues identified by the aerospace community as being desirable to include in a next generation composite impact model, an orthotropic, macroscopic constitutive model incorporating both plasticity and damage suitable for implementation within the commercial LS-DYNA computer code is being developed. The plasticity model is based on extending the Tsai-Wu composite failure model into a strain hardening-based orthotropic plasticity model with a non-associative flow rule. The evolution of the yield surface is determined based on tabulated stress-strain curves in the various normal and shear directions and is tracked using the effective plastic strain. To compute the evolution of damage, a strain equivalent semi-coupled formulation is used in which a load in one direction results in a stiffness reduction in multiple material coordinate directions. A detailed analysis is carried out to ensure that the strain equivalence assumption is appropriate for the derived plasticity and damage formulations that are employed in the current model. Procedures to develop the appropriate input curves for the damage model are presented and the process required to develop an appropriate characterization test matrix is discussed.

INTRODUCTION

As composite materials are gaining increased use in aircraft components where impact resistance under high energy impact conditions is important (such as the turbine engine fan case), the need for accurate material models to simulate the deformation, damage and failure response of polymer matrix composites under impact conditions is becoming more critical. Within commercially available transient dynamic finite element codes such as LS-DYNA (Hallquist, 2013), there are several material models currently available for application to the analysis of composites. The available models include relatively simple equations where criteria related to ratios of stresses to failure strengths are used to signify failure. More sophisticated sets of material models, based on continuum damage mechanics approaches (such as Matzenmiller et al (1995)), are also available where the initiation and accumulation of damage is assumed to be the primary driver of any nonlinearity in the composite response. While these material models have been utilized with some level of success in modeling the nonlinear and impact response of polymer composites, there are some areas where the predictive capability can be improved. Most importantly, the existing models often require correlation based on structural level impact tests, which significantly limits the use of these methods as predictive tools. Furthermore, the current models generally assume that the nonlinear response of the composite can be modeled either by using a deformation based plasticity approach (such as in Sun and Chen (1989)) or by a continuum damage mechanics approach (such as in Matzenmiller et al (1995)). By using a plasticity based model, the nonlinear unloading and strain softening observed in actual composites (Barbero, 2013) cannot be simulated. However, by using a continuum damage mechanics based model, the rate dependence in the material response, which is often observed in composites under high strain rate conditions (Gilat et al 2002), is difficult to incorporate in a theoretically consistent manner. Furthermore, a continuum damage mechanics approach cannot fully account for the significant nonlinearity that is observed in the shear stress-strain response (Daniel and Ishai, 2006). Therefore, a modeling approach in which a plasticity based deformation model is combined with a damage model (specifically designed to account for the nonlinear unloading and strain softening observed after the peak stress) can provide some advantages. The input to current material models currently generally consists of point-wise properties (such as a specified failure stress or failure strain) that lead to curve fit approximations to the material stress-strain curves. This type of approach leads either to models with only a few parameters, which provide a crude approximation at best to the actual stress-strain curve, or to models with many parameters which require a large number of complex tests to characterize. An improved approach would be to use tabulated data, in which the material stress-strain curves are explicitly entered into the model in a discretized form. The discretized data, obtained from a well-defined straightforward set of experiments, would allow the complete stress-strain response of the material to be accurately defined. In addition, while many of the existing models are designed to be used with two-dimensional shell elements, to properly capture the through-thickness response of the material, which may be significant in impact applications, a fully three-dimensional formulation suitable for use with solid elements would be desirable.

To begin to address these needs, a new composite material model is being developed and implemented for use within LS-DYNA. The material model is meant to be a fully generalized model suitable for use with any composite architecture (unidirectional, laminated or textile). For the deformation model, the commonly used Tsai-Wu composite failure criteria (Daniel and Ishai, 2006) has been generalized and extended to a strain-hardening plasticity model with a quadratic yield function and a non-associative flow rule. For the damage model, a strain equivalent formulation has been developed, which allows the plasticity and damage calculations to be uncoupled, and thus allows the plasticity calculations to take place in the effective (undamaged) stress space. In traditional damage mechanics models such as the one developed by Matzenmiller et al (1995), a load in a particular coordinate direction is assumed to result in a stiffness reduction only in the direction of the applied load. However, as will be described in more detail later in this paper, in the current model a semi-coupled formulation is developed in which a load in one direction results in a stiffness reduction in all of the coordinate directions.

In the following sections of this paper, a summary of the rate-independent deformation model is presented. Next, the strain equivalent semi-coupled damage model is discussed, along with the procedures that need to be used to properly characterize the damage model. Finally, a detailed discussion of the suitability of the strain equivalence assumption for the damage model is presented.

DEFORMATION MODEL

A quadratic three-dimensional orthotropic yield function based on the Tsai-Wu failure model is specified as follows, where 1, 2, and 3 refer to the principal material directions:

$$f(\sigma) = -1 + \begin{pmatrix} F_1 & F_2 & F_3 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{pmatrix} + \begin{pmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{23} & \sigma_{31} \end{pmatrix} \begin{pmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ F_{12} & F_{22} & F_{23} & 0 & 0 & 0 \\ F_{13} & F_{23} & F_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & F_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & F_{66} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{pmatrix} \tag{1}$$

In the yield function, σ_{ij} represents the stresses and F_{ij} and F_k are coefficients that vary based on the current values of the yield stresses in the various coordinate directions. By allowing the coefficients to vary, the yield surface evolution and hardening in each of the material directions can be precisely defined. The values of the normal and shear coefficients can be determined by simplifying the yield function for the case of unidirectional tensile and compressive loading in each of the coordinate directions along with shear tests in each of the shear directions, with results as shown below:

$$
\begin{aligned}
F_1 &= \frac{1}{\sigma_{11}^T} - \frac{1}{\sigma_{11}^C} \qquad F_{11} = \frac{1}{\sigma_{11}^T \sigma_{11}^C} \qquad F_{44} = \frac{1}{\sigma_{12}^2} \\
F_2 &= \frac{1}{\sigma_{22}^T} - \frac{1}{\sigma_{22}^C} \qquad F_{22} = \frac{1}{\sigma_{22}^T \sigma_{22}^C} \qquad F_{55} = \frac{1}{\sigma_{23}^2} \\
F_3 &= \frac{1}{\sigma_{33}^T} - \frac{1}{\sigma_{33}^C} \qquad F_{33} = \frac{1}{\sigma_{33}^T \sigma_{33}^C} \qquad F_{66} = \frac{1}{\sigma_{31}^2}
\end{aligned}
\tag{2}
$$

In the above equation, the stresses are the current value of the yield stresses in the normal and shear directions (determined using procedures to be discussed below), where the superscript T indicates the tensile yield stress, and the superscript C indicates the absolute value of the compressive yield stress. To determine the values of the off-axis coefficients (which are required to capture the stress interaction effects), the results from 45° off-axis tests in the various coordinate directions can be used. An important point to note is that due to experimental or numerical variability, or alternatively just due to the fundamental behavior of the material, computing the off-diagonal terms of the yield function in this manner may result in a yield function that is not convex (which is a requirement for plasticity theory (Khan and Huang, 1995)). As a result, to satisfy the requirements of the chosen yield function, the off-diagonal terms may need to be adjusted based on the values of the other coefficients in the yield function in order to ensure convexity of the yield surface.

A non-associative flow rule is used to compute the evolution of the components of plastic strain. The plastic potential for the flow rule is shown below:

$$
h = \left(\boldsymbol{\sigma}^T \mathbf{H} \boldsymbol{\sigma}\right)^{0.5} \tag{3}
$$

where $\boldsymbol{\sigma}$ is a vector containing the current values of the stresses and the $\mathbf{H}$ matrix is composed of a set of independent coefficients, assumed to remain constant, written as follows:

$$
\mathbf{H} = \begin{bmatrix}
H_{11} & H_{12} & H_{13} & 0 & 0 & 0 \\
H_{12} & H_{22} & H_{23} & 0 & 0 & 0 \\
H_{13} & H_{23} & H_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & H_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & H_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & H_{66}
\end{bmatrix} \tag{4}
$$

The values of the coefficients are computed based on average plastic Poisson's ratios (Goldberg et al, 2014). The plastic potential function in Equation (3) is used in a flow law to compute the components of the plastic strain rate, where the usual normality hypothesis from classical plasticity (Khan and Huang, 1995) is assumed to apply and the variable, $\dot{\lambda}$, is a scalar plastic multiplier:

$$\dot{\boldsymbol{\varepsilon}}^{p} = \dot{\lambda}\frac{\partial h}{\partial \boldsymbol{\sigma}} \tag{5}$$

Given the flow law, the principal of the equivalence of plastic work (Khan and Huang, 1995) can be used to determine expressions for the effective stress and effective plastic strain. By following this procedure, one can conclude that the plastic potential function h can be defined as the effective stress and the plastic multiplier can be defined as the effective plastic strain rate. As will be discussed below, the evolution of the effective plastic strain rate ($\dot{\lambda}$) is computed in the material model, and is used in combination with the derivative of the plastic potential function to compute the components of the plastic strain rate tensor.

To compute the current value of the yield stresses needed for the yield function, the common practice in plasticity constitutive equations is to use analytical functions to define the evolution of the stresses as a function of the components of plastic strain (or the effective plastic strain). Alternatively, in the developed model tabulated stress-strain curves are used to track the yield stress evolution. The user is required to input twelve stress versus plastic strain curves in a tabulated, discretized form. Specifically, the required curves include uniaxial tension curves in each of the normal directions (1,2,3), uniaxial compression curves in each of the normal directions (1,2,3), shear stress-strain curves in each of the shear directions (1-2, 2-3 and 3-1), and 45 degree off-axis tension curves in each of the 1-2, 2-3 and 3-1 planes. The 45 degree curves are required in order to properly capture the stress interaction effects. By utilizing tabulated stress-strain curves to track the evolution of the deformation response, the experimental stress-strain response of the material can be captured to a much higher degree of accuracy than would be possible by using an analytical function and the relevant failure stresses (and strains) to approximate the stress-strain curves. While some slight interpolation is still required between the discretized points, by assuming a sufficient level of discretization, the actual stress-strain response can be approximated to a much finer level of accuracy. The required stress-strain data can be obtained either from actual experimental test results, or by appropriate numerical experiments utilizing stand-alone codes. Currently, only static test data is considered. Future efforts will involve adding strain rate and temperature dependent effects to the computations. To track the evolution of the deformation response along each of the stress-strain curves, the effective plastic strain is chosen to be the tracking parameter. Using a numerical procedure based on the radial return method (Khan and Huang, 1995) in combination with an iterative approach, the effective plastic strain is computed for each time/load step. The stresses for each of the tabulated input curves corresponding to the current value of the effective plastic strain are then used to compute the yield function coefficients.

DAMAGE MODEL OVERVIEW

The deformation portion of the material model provides the majority of the capability of the model to simulate the nonlinear stress-strain response of the composite. However, in order to capture the nonlinear unloading and local softening of the stress-strain response often observed in composites (Barbero, 2013), a complementary damage law is required. In the damage law formulation, strain equivalence is assumed, in which for every time step the total, elastic and plastic strains in the actual and effective stress spaces are the same (Lemaitre and Desmorat, 2005). The utilization of strain equivalence permits the plasticity and damage calculations to be uncoupled, as all of the plasticity computations can take place in the effective (undamaged) space.

In the damage model, the actual stresses are related to the effective (undamaged) stresses by use of a damage tensor **M**:

$$\boldsymbol{\sigma} = \mathbf{M}\boldsymbol{\sigma}_{eff} \tag{6}$$

The effective stress rate tensor can be related to the total and plastic strain rate tensors by use of the standard elasto-plastic constitutive equation:

$$\dot{\boldsymbol{\sigma}}_{eff} = \mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{p}\right) \tag{7}$$

where **C** is the standard elastic stiffness matrix and the actual total and plastic strain rate tensors are used due to the strain equivalence assumption.

DEFINITION AND CHARACTERIZATION OF DAMAGE TENSOR

As specified in Equation (6), the effective and actual stresses are related through a damage tensor. Given the usual assumption that the actual stress tensor and the effective stress tensor are symmetric, Equation (6) can be rewritten in the following form, where the damage tensor **M** is assumed to have a maximum of 36 independent components:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{pmatrix} = \left[\mathbf{M}\right] \begin{pmatrix} \sigma_{11}^{eff} \\ \sigma_{22}^{eff} \\ \sigma_{33}^{eff} \\ \sigma_{12}^{eff} \\ \sigma_{23}^{eff} \\ \sigma_{31}^{eff} \end{pmatrix} \tag{8}$$

In many damage mechanics models for composites, for example the models discussed in Barbero (2013) and Matzenmiller et al (1995), the damage tensor is assumed to be diagonal or manipulated to be a diagonal tensor, leading to the following form:

$$[\mathbf{M}] = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix} \tag{9}$$

The implication of a diagonal damage tensor is that loading the composite in a particular coordinate direction only leads to a stiffness reduction in the direction of the load due to the formation of matrix cracks perpendicular to the direction of the load. However, several recent experimental studies (Ogasawara et al, 2005, Salavatian and Smith, 2014, Salem and Wilmoth, unpublished data, 2015) have shown that in actual composites, particularly those with complex fiber architectures, a load in one coordinate direction can lead to stiffness reductions in multiple coordinate directions.

One approach to incorporating the coupling of damage modes would be to use a non-diagonal damage tensor, such as the one shown below for the case of plane stress:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{pmatrix} \sigma_{11}^{eff} \\ \sigma_{22}^{eff} \\ \sigma_{12}^{eff} \end{pmatrix} \tag{10}$$

However, while this formulation would allow for directional coupling, it would have the side effect of a unidirectional load in the actual stress space resulting in a multiaxial load in the effective undamaged space. For the strain equivalent combined plasticity damage formulation envisioned for this model, this would be an undesirable side effect as the plasticity calculations could be adversely affected due to the introduction of nonphysical stresses.

To avoid the undesired stress coupling, a diagonal damage tensor is required. However, to account for the damage interaction in at least a semi-coupled sense, each term in the diagonal damage matrix should be a function of the plastic strains in each of the normal and shear coordinate directions, as follows for the example of the M_{11} term for the plane stress case:

$$M_{11} = M_{11}\left(\varepsilon_{11}^{p}, \varepsilon_{22}^{p}, \varepsilon_{12}^{p}\right) \tag{11}$$

Note that plastic strains are chosen as the "tracking parameter" due to the fact that, within the context of the developed formulation, the material nonlinearity during loading is simulated by use of a plasticity based model. The plastic strains therefore track the current state of load and deformation in the material.

To explain this concept of damage coupling further, assume a load is applied in the 1 direction to an undamaged specimen, with an original area A_{11} perpendicular to the 1 axis and an original area A_{22} perpendicular to the 2 axis. The undamaged modulus in the 1 direction is E_{11} and the undamaged modulus in the 2 direction is E_{22}. The stress-strain response of the material is assumed to become nonlinear (represented in the current model by the accumulation of plastic strain) and damage is assumed to occur. The original specimen is unloaded and reloaded elastically in the 1 direction. Due to the damage, the reloaded specimen has a reduced area in the 1 direction of A_{11}^{d11} due to the fact that the composite damage reduces the effective area of the composite that can carry load, and a reduced modulus in the 1 direction of E_{11}^{d11}. The reduced area and modulus are a function of the damage induced by the loading and resulting nonlinear deformation in the 1 direction (reflected as plastic strain) as follows:

$$
\begin{aligned}
E_{11}^{d11} &= \left(1 - d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)E_{11} \\
A_{11}^{d11} &= \left(1 - d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)A_{11}
\end{aligned}
\qquad (12)
$$

where d_{11}^{11} is the damage in the 1 direction due to a load in the 1 direction. Alternatively, if the damaged specimen was reloaded elastically in the 2 direction, due to the assumed damage coupling resulting from the load in the 1 direction, the reloaded specimen would have a reduced area in the 2 direction of A_{22}^{d11} and a reduced modulus in the 2 direction of E_{22}^{d11}. The reduced area and modulus are again functions of the damage induced by the load and resulting nonlinear deformation in the 1 direction as follows:

$$
\begin{aligned}
E_{22}^{d11} &= \left(1 - d_{11}^{22}\left(\varepsilon_{11}^{p}\right)\right)E_{22} \\
A_{22}^{d11} &= \left(1 - d_{11}^{22}\left(\varepsilon_{11}^{p}\right)\right)A_{22}
\end{aligned}
\qquad (13)
$$

where d_{11}^{22} is the damage in the 2 direction due to a load in the 1 direction. Similar arguments can be made and equations developed for the situation where the original specimen is loaded in the 2 direction.

For the case of multiaxial loading, the semi-coupled formulation needs to account for the fact that as the load is applied in a particular coordinate direction, the loads are acting on damaged areas due to the loads in the other coordinate directions, and the load in a particular direction is just adding to the damaged area. For example, if one loaded the material in the 2 direction first, the reduced area in the 1 direction would be equal to A_{11}^{d22} and the reduced modulus in the 1 direction would be equal to E_{11}^{d22}. If one would then subsequently load the material in the 1 direction, the baseline area in the 1 direction would not equal the original area A_{11}, but the reduced area A_{11}^{d22}. Likewise, the baseline modulus in the 1 direction would not be equal to the original modulus E_{11}, but instead the reduced modulus E_{11}^{d22}. Therefore, the loading in the 1 direction would result in the following further reduction in the area and modulus in the 1 direction:

$$\begin{aligned} E_{11}^{d11} &= \left(1-d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)E_{11}^{d22} = \left(1-d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)\left(1-d_{22}^{11}\left(\varepsilon_{22}^{p}\right)\right)E_{11} \\ A_{11}^{d11} &= \left(1-d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)A_{11}^{d22} = \left(1-d_{11}^{11}\left(\varepsilon_{11}^{p}\right)\right)\left(1-d_{22}^{11}\left(\varepsilon_{22}^{p}\right)\right)A_{11} \end{aligned} \tag{14}$$

These results suggest that the relation between the actual stress and the effective stress should be based on a multiplicative combination of the damage terms as opposed to an additive combination of the damage terms. For example, for the case of plane stress, the relation between the actual and effective stresses could be expressed as follows:

$$\begin{aligned} \sigma_{11} &= \left(1-d_{11}^{11}\right)\left(1-d_{22}^{11}\right)\left(1-d_{12}^{11}\right)\sigma_{11}^{eff} \\ \sigma_{22} &= \left(1-d_{11}^{22}\right)\left(1-d_{22}^{22}\right)\left(1-d_{12}^{22}\right)\sigma_{22}^{eff} \\ \sigma_{12} &= \left(1-d_{11}^{12}\right)\left(1-d_{22}^{12}\right)\left(1-d_{12}^{12}\right)\sigma_{12}^{eff} \end{aligned} \tag{15}$$

where for each of the damage terms the subscript indicates the direction of the load which initiates the particular increment of damage and the superscript indicates the direction in which the damage takes place. Note that for the full three-dimensional case the stress in a particular coordinate direction is a function of the damage due to loading in all of the coordinate directions (1, 2, 3, 12, 31 and 23). By using a polynomial to describe the damage, the coupled terms represent the reduction to the degree of damage resulting from the fact that in a multiaxial loading case the area reductions are combined.

There are two primary items needed for model characterization and input for the damage portion of the material model. First, the values of the various damage parameter terms d_{ij}^{kl} need to be defined in a tabulated manner as a function of the effective plastic strain. Similar to the deformation model, the values of the damage parameters are defined in a tabulated, discretized form in order to reflect the actual material behavior in the most accurate manner possible. The values are tabulated as a function of the effective plastic strain in order to provide a unified framework to simultaneously track the evolution of multiple damage parameters under multiaxial loading conditions. As mentioned above, since in the context of the current model the plastic strains are used to represent the nonlinear deformation of the material, using the effective plastic strain as an equivalent parameter to track the damage parameter evolution should be reasonable. Note that for the case of uniaxial loading the effective plastic strain equals the uniaxial plastic strain, which maintains consistency with the formulation described above. In addition to characterizing the damage parameters, the various input stress-strain curves need to be converted into plots of effective (undamaged) stress versus effective plastic strain in order to carry out the calculations required by the deformation (plasticity) model. As an example of how this process could be carried out, assume that a material is loaded unidirectionally in the 1 direction. At multiple points, once the actual stress-strain curve has become nonlinear, the total strain (ε_{11}), actual stress (σ_{11}), and average local, damaged modulus E_{11}^{d11} can be measured. Assuming that the original, undamaged modulus E_{11} is known, since the damage in the 1 direction is assumed to be only due to load in the 1 direction (due to the uniaxial load), the damage parameters and effective stress in the 1 direction can be computed at a particular point along the stress-strain curve as follows:

$$1-d_{11}^{11}=\frac{E_{11}^{d11}}{E_{11}}$$
$$M_{11}=1-d_{11}^{11}$$
$$\sigma_{11}^{eff}=\frac{\sigma_{11}}{M_{11}} \tag{16}$$
$$\varepsilon_{11}^{p}=\varepsilon_{11}-\frac{\sigma_{11}^{eff}}{E_{xx}}$$

These values need to be determined at multiple points, representing different values of plastic strain, in order to fully characterize the evolution of damage as the plastic strain increases.

An example of this process is shown in Figure 1. Assume the material is loaded in the 1 direction. As shown in the figure, as the material is loaded the stress-strain response becomes nonlinear. To characterize the damage parameters, at points 1, 3 and 2i-1 (representing different values of plastic strain) the material is unloaded to zero stress. The average unloading modulus is then determined at points 2, 4 and 2i. These values are used in the calculations specified in Equation (16).

With this information, an effective stress versus plastic strain $\left(\varepsilon_{11}^{p}\right)$ plot can be generated. From this plot, the effective plastic strain corresponding to the plastic strain in the 1 direction at any particular point can be determined by using the equations shown below, which are based on applying the principal of the equivalence of plastic work (Khan and Huang, 1995) in combination with Equation (3), simplifying the expressions for the case of unidirectional loading in the 1 direction (Goldberg et al, 2014):

$$h=\sqrt{H_{11}\left(\sigma_{11}^{eff}\right)^{2}}$$
$$\varepsilon_{e}^{p}=\int\frac{\sigma_{11}^{eff}\,d\varepsilon_{11}^{p}}{h} \tag{17}$$

where ε_{e}^{p} is the effective plastic strain and $d\varepsilon_{11}^{p}$ is the increment of plastic strain in the 1 direction. From this data, plots of the effective stress in the 1 direction versus the effective plastic strain as well as plots of the damage parameter d_{11}^{11} as a function of the effective plastic strain can be generated. By measuring the damaged modulus in the other coordinate directions at each of the measured values of plastic strain in the 1 direction, the value of the damage parameters $d_{11}^{22}, d_{11}^{12}, d_{11}^{33}$, etc. can be determined as a function of the plastic strain in the 1 direction, and thus as a function of the effective plastic strain. An example of this process is shown in Figure 2. In this example, assume that at point 2 in Figure 1 the material is then reloaded in the elastic range in the 2 direction. The resulting stress-strain curve is shown in Figure 2. The modulus E_{22}^{d11} can then be determined, which can be used to determine the value of d_{11}^{22} for the particular value of plastic strain in the 1 direction. A similar process would need to be carried out by loading the material elastically in the other coordinate directions at point 2 in Figure 1, and by repeating the entire process at the various points where the material is unloaded (such as points 4 and 2i in Figure 1). To determine the remaining required damage terms, the process shown in Figure 1 and Figure 2 would need to be repeated by the loading the material in the other coordinate directions.

To convert the 45º off-axis stress-strain curves into plots of the effective (undamaged) stress versus effective plastic strain, the total and plastic strain (permanent strain after unload) in the structural axis x direction needs to be measured at multiple points along the stress-strain curve. Given the undamaged modulus E_{xx}, and utilizing the strain equivalence hypothesis, the effective stress in the structural axis system x direction can be computed as follows:

$$\sigma_{xx}^{eff} = E_{xx}\left(\varepsilon_{xx} - \varepsilon_{xx}^{p}\right) \tag{18}$$

Given the effective stress in the structural axis system, the effective stresses in the material axis system can be generated by use of stress transformation equations. Using the material axis system stresses, the plastic potential function and effective plastic strain corresponding to each value of plastic strain can be determined using the standard stress transformation equations for the case of 45º off-axis loading and the principal of the equivalence of plastic work in combination with Equation (3) as shown below (Goldberg et al, 2014):

$$\begin{aligned}
\sigma_{11}^{eff} &= 0.5\sigma_{xx}^{eff} \\
\sigma_{22}^{eff} &= 0.5\sigma_{xx}^{eff} \\
\sigma_{12}^{eff} &= -0.5\tilde{\sigma}_{xx} \\
h &= 0.5\sigma_{xx}^{eff}\sqrt{H_{11} + H_{22} + 2H_{12} + H_{44}} \\
\varepsilon_{e}^{p} &= \int \frac{\sigma_{xx}^{eff}\, d\varepsilon_{xx}^{p}}{h}
\end{aligned} \tag{19}$$

Ongoing efforts will involve developing and carrying out an appropriate experimental test matrix to characterize and validate the model for a series of representative composite materials.

VALIDATION OF STRAIN EQUIVALENCE ASSUMPTION

As discussed in previous sections of this paper, employing the strain equivalence assumption specifies that the effective stresses result in the same deformations in the effective, undamaged material as would be caused by applying the actual stresses on the damaged material. In this way, the yield function, flow rule and constitutive equation, specified in Equations (1), (5) and (7), respectively, can be written in terms of the effective stresses and applied in effective stress space. The use of effective stresses allows the plasticity calculations to be uncoupled from the damage law.

For the strain equivalence formulation to be valid, one requirement is that the derivative of the plastic potential function (written in terms of the effective stresses) with respect to the effective stresses must equal the derivative of a damaged plastic potential (written in terms of the actual stresses) with respect to the actual stresses. This concept is expressed mathematically below:

$$\frac{\partial h(\boldsymbol{\sigma}_{eff})}{\partial \boldsymbol{\sigma}_{eff}} = \frac{\partial h(\mathbf{M}^{-1}\boldsymbol{\sigma})}{\partial \mathbf{M}^{-1}\boldsymbol{\sigma}} = \frac{\partial h_d(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} \tag{20}$$

Therefore, for the strain equivalence assumption to be valid, an essential requirement involves identifying an appropriate damaged version of the plastic potential function (in terms of the actual stresses as opposed to the effective stresses) such that the same strain state results whether the material is loaded in the actual or effective stress space. In a similar vein, damaged versions of the stiffness matrix and yield function, in terms of the actual stresses and utilized in the damaged space, also need to be identified.

To develop the damaged elastic stiffness matrix, Equation (7) is substituted into the time derivative of Equation (6), leading to the following expressions:

$$\begin{aligned} \dot{\boldsymbol{\sigma}} &= \mathbf{M}\dot{\boldsymbol{\sigma}}_{eff} + \dot{\mathbf{M}}\boldsymbol{\sigma}_{eff} \\ \dot{\boldsymbol{\sigma}} &= \mathbf{MC}(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{p}) + \dot{\mathbf{M}}\mathbf{M}^{-1}\boldsymbol{\sigma} \end{aligned} \tag{21}$$

For the case of elastic only loading the plastic strain rate and the time derivative of the damage tensor are both set equal to zero, leading to the following relation between the actual stress rate and the total strain rate:

$$\begin{aligned} \dot{\boldsymbol{\sigma}} &= \mathbf{MC}\dot{\boldsymbol{\varepsilon}} \\ \dot{\boldsymbol{\varepsilon}} &= (\mathbf{MC})^{-1}\dot{\boldsymbol{\sigma}} \end{aligned} \tag{22}$$

For the case of elastic only loading, Equation (7) can be simplified, leading to the following relation between the effective stress rate and the total strain rate:

$$\begin{aligned} \dot{\boldsymbol{\sigma}}_{\mathbf{eff}} &= \mathbf{C}\dot{\boldsymbol{\varepsilon}} \\ \dot{\boldsymbol{\varepsilon}} &= (\mathbf{C})^{-1}\dot{\boldsymbol{\sigma}}_{\mathbf{eff}} \end{aligned} \tag{23}$$

For the strain equivalence assumption to be valid, the elastic strains resulting from loading in the damaged configuration must be identical to the elastic strains resulting from loading in the effective (undamaged) configuration. Therefore, by comparing Equation (22) to Equation (23), the damaged elastic stiffness matrix, $\mathbf{C}_d$, can be defined as follows:

$$\mathbf{C_d} = \mathbf{MC} \tag{24}$$

To develop the damaged version of the yield function, the yield function shown in Equation (1) can be written in a quadratic form in terms of the effective stresses and converted to be in terms of the actual stresses (using Equation (6)). From there, a damaged yield function, f_d, in terms of the actual stresses can be defined as shown below:

$$\begin{aligned}
f &= \mathbf{f}^T \boldsymbol{\sigma}_{eff} + \boldsymbol{\sigma}_{eff}^T \mathbf{F} \boldsymbol{\sigma}_{eff} - 1 \le 0 \\
f &= \mathbf{f}^T \mathbf{M}^{-1} \boldsymbol{\sigma} + \boldsymbol{\sigma}^T \mathbf{M}^{-T} \mathbf{F} \mathbf{M}^{-1} \boldsymbol{\sigma} - 1 \le 0 \\
f_d &= \mathbf{f}_d^T \boldsymbol{\sigma} + \boldsymbol{\sigma}^T \mathbf{F}_d \boldsymbol{\sigma} - 1 \le 0 \\
\mathbf{f}_d &= \mathbf{M}^{-T} \mathbf{f} \\
\mathbf{F}_d &= \mathbf{M}^{-T} \mathbf{F} \mathbf{M}^{-1}
\end{aligned} \tag{25}$$

where the vector **f** and the matrix **F** are defined as follows:

$$\begin{aligned}
\mathbf{f} &= \begin{pmatrix} F_1 & F_2 & F_3 & 0 & 0 & 0 \end{pmatrix} \\
\mathbf{F} &= \begin{pmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ F_{12} & F_{22} & F_{23} & 0 & 0 & 0 \\ F_{13} & F_{23} & F_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & F_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & F_{66} \end{pmatrix}
\end{aligned} \tag{26}$$

To determine the damaged form of the plastic potential function, first the definition of the plastic potential function given in Equation (2) is written in terms of effective stresses and differentiated with respect to the effective stresses, resulting in the following expression:

$$h = \left(\boldsymbol{\sigma}_{eff}^T \mathbf{H} \boldsymbol{\sigma}_{eff}\right)^{0.5} \Rightarrow \frac{\partial h_{eff}}{\partial \boldsymbol{\sigma}_{eff}} = \frac{1}{2h} 2\mathbf{H}\boldsymbol{\sigma}_{eff} = \frac{1}{\left(\boldsymbol{\sigma}^T \mathbf{M}^{-T} \mathbf{H} \mathbf{M}^{-1} \boldsymbol{\sigma}\right)^{0.5}} \mathbf{H}\left[\mathbf{M}^{-1}\boldsymbol{\sigma}\right] \tag{27}$$

where the effective stresses have been converted back into the actual stresses and the **H** matrix is as defined earlier in Equation (4).

By applying the results of Equation (27) in Equation (5), the plastic strain rate tensor in the damaged state can be defined and a damaged version of the **H** matrix defined in Equation (4) can be specified:

$$\begin{aligned}\dot{\boldsymbol{\varepsilon}}_p &= \frac{\dot{\lambda}}{2h} 2\mathbf{H}\boldsymbol{\sigma}_{eff} = \frac{\dot{\lambda}}{\left(\boldsymbol{\sigma}^T \mathbf{M}^{-T} \mathbf{H} \mathbf{M}^{-1} \boldsymbol{\sigma}\right)^{0.5}} \mathbf{H}\left[\mathbf{M}^{-1}\boldsymbol{\sigma}\right] = \dot{\lambda}\mathbf{H}_d \boldsymbol{\sigma} \\ \mathbf{H}_d &= \frac{1}{\left(\boldsymbol{\sigma}^T \mathbf{M}^{-T} \mathbf{H} \mathbf{M}^{-1} \boldsymbol{\sigma}\right)^{0.5}} \mathbf{H}\left[\mathbf{M}^{-1}\right]\end{aligned} \tag{28}$$

For the strain equivalence assumption to be valid for the given plastic potential function, based on the expression shown in Equation (20) a damaged version of the plastic potential function h_d needs to be defined such that the following expression is true:

$$\frac{\partial h_d(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} = \frac{1}{\left(\boldsymbol{\sigma}^T \mathbf{M}^{-T} \mathbf{H} \mathbf{M}^{-1} \boldsymbol{\sigma}\right)^{0.5}} \mathbf{H}\left[\mathbf{M}^{-1}\boldsymbol{\sigma}\right] \tag{29}$$

One possible form for the plastic potential function in terms of the actual stresses is defined as follows. By taking the derivative of the proposed function with respect to the actual stresses, the required expression (shown in Equation (29)) is obtained:, and thus the strain equivalence assumption can be employed:

$$\begin{aligned}h_d &= \mathbf{M}^T \left(\boldsymbol{\sigma}^T \mathbf{M}^{-T} \mathbf{H} \mathbf{M}^{-1} \boldsymbol{\sigma}\right)^{0.5} \\ \frac{\partial h_d}{\partial \boldsymbol{\sigma}} &= \mathbf{M}^T \frac{1}{2\left(\boldsymbol{\sigma}^T \mathbf{M}^{-T} \mathbf{H} \mathbf{M}^{-1} \boldsymbol{\sigma}\right)^{0.5}} 2\mathbf{M}^{-T}\mathbf{H}\left[\mathbf{M}^{-1}\boldsymbol{\sigma}\right] \\ \frac{\partial h_d}{\partial \boldsymbol{\sigma}} &= \frac{1}{\left(\boldsymbol{\sigma}^T \mathbf{M}^{-T} \mathbf{H} \mathbf{M}^{-1} \boldsymbol{\sigma}\right)^{0.5}} \mathbf{H}\left[\mathbf{M}^{-1}\boldsymbol{\sigma}\right]\end{aligned} \tag{30}$$

This result demonstrates that an appropriate damaged plastic potential function can be derived. Since an appropriate damaged stiffness matrix, yield function and plastic potential function can be defined in the actual (damaged) stress space, the requirements for the strain equivalence assumption to be valid are therefore established.

Given the identification of an appropriate elastic stiffness matrix, yield function and plastic potential function in both the effective and damaged configurations, the full elasto-plastic deformation law for the undamaged state can be specified:

$$
\begin{aligned}
&\dot{\boldsymbol{\sigma}}_{eff} = \mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{p}\right) \\
&\dot{\boldsymbol{\varepsilon}}_{p} = \dot{\lambda}\frac{\partial h\left(\boldsymbol{\sigma}_{eff}\right)}{\partial \boldsymbol{\sigma}_{eff}} \\
&f\left(\boldsymbol{\sigma}_{eff}, \mathbf{q}\right) \leq 0 \\
&\dot{\mathbf{q}} = \dot{\lambda}\frac{\partial \mathbf{q}}{\partial \lambda} \\
&\dot{\lambda} \geq 0
\end{aligned}
\tag{31}
$$

where the **q** vector is the vector of the yield stresses in the various coordinate directions in the undamaged (effective) configuration and all of the other terms are as identified earlier. Likewise, the elasto-plastic law in the damaged configuration can also be defined:

$$
\begin{aligned}
&\boldsymbol{\sigma} = \mathbf{M}\boldsymbol{\sigma}_{eff} \\
&\dot{\boldsymbol{\sigma}} = \mathbf{M}\mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{p}\right) + \dot{\mathbf{M}}\mathbf{M}^{-1}\boldsymbol{\sigma} = \mathbf{C}_{d}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{p}\right) + \dot{\mathbf{M}}\mathbf{M}^{-1}\boldsymbol{\sigma} \\
&\dot{\boldsymbol{\varepsilon}}_{p} = \dot{\gamma}\frac{\partial h\left(\mathbf{M}^{-1}\boldsymbol{\sigma}\right)}{\partial \mathbf{M}^{-1}\boldsymbol{\sigma}} = \dot{\gamma}\frac{\partial h_{d}\left(\boldsymbol{\sigma}\right)}{\partial \boldsymbol{\sigma}} \\
&f\left(\mathbf{M}^{-1}\boldsymbol{\sigma}, \mathbf{q}\right) = f_{d}\left(\boldsymbol{\sigma}, \mathbf{q}\right) \leq 0 \\
&\dot{\mathbf{q}} = \dot{\gamma}\frac{\partial \mathbf{q}}{\partial \gamma} \\
&\dot{\gamma} \geq 0
\end{aligned}
\tag{32}
$$

where $\dot{\gamma}$ is the effective plastic strain rate in the damaged configuration and all of the other terms are as defined earlier. To demonstrate that the strain equivalence assumption is valid for the current model, the effective plastic strain rate $\dot{\gamma}$ in the damaged configuration must be shown to be equal to the effective plastic strain rate $\dot{\lambda}$ in the undamaged state.

To carry out this proof, first the consistency condition from classical plasticity theory (Khan and Huang, 1995) is applied in combination with Equation (31) in the undamaged effective state as follows:

$$
\begin{aligned}
&\dot{f} = \frac{\partial f}{\partial \boldsymbol{\sigma}_{eff}}\dot{\boldsymbol{\sigma}}_{eff} + \frac{\partial f}{\partial \mathbf{q}}\dot{\mathbf{q}} = 0 \\
&\frac{\partial f}{\partial \boldsymbol{\sigma}_{eff}}\mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{p}\right) + \frac{\partial f}{\partial \mathbf{q}}\dot{\lambda}\frac{\partial \mathbf{q}}{\partial \lambda} = 0 \\
&\frac{\partial f}{\partial \boldsymbol{\sigma}_{eff}}\mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\lambda}\frac{\partial h\left(\boldsymbol{\sigma}_{eff}\right)}{\partial \boldsymbol{\sigma}_{eff}}\right) + \frac{\partial f}{\partial \mathbf{q}}\dot{\lambda}\frac{\partial \mathbf{q}}{\partial \lambda} = 0
\end{aligned}
\tag{33}
$$

In the damaged configuration, the damage matrix can be assumed to be a function of the effective plastic strain rate. With this assumption, and by combining several of the expressions in Equation (32), the stress rate expression in the damaged configuration can be rewritten as follows:

$$\begin{aligned} &\dot{\mathbf{M}} = \frac{\partial \mathbf{M}}{\partial \gamma}\dot{\gamma} \\ &\dot{\boldsymbol{\sigma}} = \mathbf{MC}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\gamma}\frac{\partial h\left(\mathbf{M}^{-1}\boldsymbol{\sigma}\right)}{\partial \mathbf{M}^{-1}\boldsymbol{\sigma}}\right) + \frac{\partial \mathbf{M}}{\partial \gamma}\dot{\gamma}\mathbf{M}^{-1}\boldsymbol{\sigma} \end{aligned} \tag{34}$$

By applying the plastic consistency condition to the damaged yield function, substituting in the stress rate expression shown in Equation (34), applying the relations defined in Equation (32), and simplifying, the following expressions can be determined:

$$\begin{aligned} &\dot{f} = \frac{\partial f}{\partial \mathbf{M}^{-1}\boldsymbol{\sigma}}\left[\mathbf{M}^{-1}\dot{\boldsymbol{\sigma}} + \frac{\partial \dot{\mathbf{M}}^{-1}}{\partial \gamma}\dot{\gamma}\boldsymbol{\sigma}\right] + \frac{\partial f}{\partial \mathbf{q}}\dot{\mathbf{q}} = 0 \\ &\frac{\partial f}{\partial \mathbf{M}^{-1}\boldsymbol{\sigma}}\left\{\mathbf{M}^{-1}\left[\mathbf{MC}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\gamma}\frac{\partial h\left(\mathbf{M}^{-1}\boldsymbol{\sigma}\right)}{\partial \mathbf{M}^{-1}\boldsymbol{\sigma}}\right) + \frac{\partial \mathbf{M}}{\partial \gamma}\dot{\gamma}\mathbf{M}^{-1}\boldsymbol{\sigma}\right] - \mathbf{M}^{-2}\frac{\partial \mathbf{M}}{\partial \gamma}\dot{\gamma}\boldsymbol{\sigma}\right\} + \frac{\partial f}{\partial \mathbf{q}}\dot{\gamma}\frac{\partial \mathbf{q}}{\partial \gamma} = 0 \\ &\frac{\partial f}{\partial \mathbf{M}^{-1}\boldsymbol{\sigma}}\mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\gamma}\frac{\partial h\left(\mathbf{M}^{-1}\boldsymbol{\sigma}\right)}{\partial \mathbf{M}^{-1}\boldsymbol{\sigma}}\right) + \frac{\partial f}{\partial \mathbf{q}}\dot{\gamma}\frac{\partial \mathbf{q}}{\partial \gamma} = 0 \end{aligned} \tag{35}$$

By comparing the last expression in Equation (35) to the last expression in Equation (33), applying the relation between the actual and effective stress defined in Equation (6) and considering the relations between the derivatives of the plastic potential function defined in Equation (20), the conclusion can be made that the effective plastic strain rates in the damaged and undamaged configurations must be equal. Therefore, the final conclusion can be reached that for the plasticity and damage model developed for this work the strain equivalence assumption is not only appropriate but is required.

CONCLUSIONS

A generalized composite model suitable for use in polymer composite impact simulations has been developed. The model utilizes a plasticity based deformation model based on generalizing the Tsai-Wu failure criteria. A strain equivalent damage model has also been developed in which loading the material in a particular coordinate direction can lead to damage in multiple coordinate directions. A detailed examination of the damage model has demonstrated that the strain equivalence formulation is appropriate for the developed damage model. Procedures have also been developed to appropriately characterize the damage model.

Ongoing efforts will include developing the detailed numerical algorithms to implement the developed deformation and damage model into the LS-DYNA computer code. Methods to model failure and element removal will also be developed and implemented into LS-DYNA. An extensive set of verification and validation studies will be undertaken in order to fully exercise the developed model.

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REFERENCES

Barbero, E.J. (2013). *Finite Element Analysis of Composite Materials Using ABAQUS.* CRC Press, Boca Raton, FL.

Daniel, I.M. and Ishai, O. (2006). *Engineering Mechanics of Composite Materials Second Edition*. Oxford University Press, New York.

Gilat, A., Goldberg, R.K., and Roberts, G.D. (2002). “Experimental study of strain-rate-dependent behavior of carbon/epoxy composite.” *Comp. Sci. Technol.,* 62,1469–1476.

Goldberg, R., Carney, K., DuBois, P., Hoffarth, C., Harrington, J., Rajan, S., and Blankenhorn, G. (2014). “Theoretical Development of an Orthotropic Elasto-Plastic Generalized Composite Model.” *NASA/TM-2014-218347*, National Aeronautics and Space Administration, Washington, DC.

Hallquist, J (2013). *LS-DYNA Keyword User's Manual, Version 970,* Livermore Software Technology Corporation, Livermore, CA.

Khan, A.S. and Huang, S. (1995). *Continuum Theory of Plasticity*. John Wiley and Sons, New York, NY.

Lemaitre, J, and Desmorat, R. (2005). *Engineering Damage Mechanics: Ductile, Creep and Brittle Failures*. Springer, Berlin.

Matzenmiller, A., Lubliner, J., and Taylor, R.L. (1995). “A constitutive model for anisotropic damage in fiber composites.” *Mechanics of Materials*, 20, 125-152.

Ogasawara, O, T., Ishikawa, T., Yokozeki,, T., Shiraishi, T. and Watanabe, N. (2005). “Effect of on-axis tensile loading on shear properties of an orthogonal 3D woven SiC/SiC composite.” *Comp. Sci. Technol.*, 65, 2541-2549.

Sun, C.T. and Chen, J.L. (1989). “A Simple Flow Rule for Characterizing Nonlinear Behavior of Fiber Composites.” *Journal of Composite Materials*, 23, 1009-1020.

Salavatian, M. and Smith, L.V. (2014). “The effect of transverse damage on the shear response of fiber reinforced laminates,” *Comp. Sci. Technol.*, 95, 44-49.

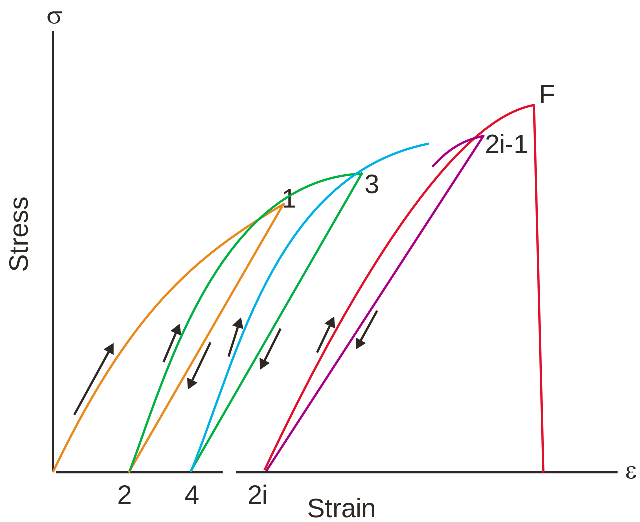

Figure 1: Load-Unload-Reload tests required

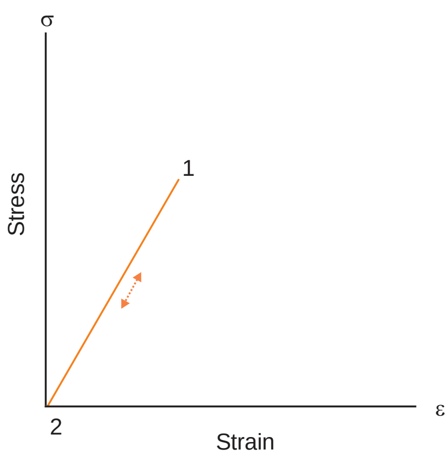

Figure 2: Elastic Reload Test