NASA Computational Case Study
Where is My Moon?

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1   Introduction

A *moon* or *natural satellite* is a celestial body that orbits a planet or planet-like body, called its *primary*. There are two classes of satellites. *Regular* satellites are those that rotate around their primary body in nearly circular and low inclination close-to-the-planet orbits, and are believed to be created by the same processes that their primaries were created by. *Irregular* satellites have large, inclined, eccentric orbits, and are believed to be fragmented from *asteroids*, small bodies in the solar system without an atmosphere that rotate the Sun and are too small to be called a planet. Therefore, we can learn about origin and evolution of various planets and our solar system by finding and studying moons of planets and planet-like bodies called *dwarf* planets, and asteroids.

Searching for moons around planets has a long history. For example, it was in 1878 when Asaph Hall discovered two moons of Mars, Phobos and Deimos, with the 26-inch Refractor Telescope at the Naval Observatory in Washington D.C. [9,10] (Visitors may schedule a tour of the Naval Observatory and see this telescope). With advances in imaging instruments and technologies, there has been an increase in irregular satellite search and discoveries around various planetary bodies such as Mars, Uranus, and Neptune [13–15]. More recently, two new moons of Pluto were discovered using observations of the Hubble Space Telescope in 2005 [16,19]. In fact, that made Pluto lose its status as a planet and be classified as a dwarf planet [6]. A planet needs to rotate around the Sun, have enough gravity to form a spherical shape and to clear its neighborhood [6]. In other words, it should be the dominant gravitational body in its orbit. This was no longer true for Pluto after the discovery of its two new moons [6]. Any object, like Pluto, that meets the first two requirements but not the third one is called a dwarf planet.

2   Problem Description

In this case study we learn where and how astronomers look for moons, or perform *satellite search*. Moons or satellites of a celestial body are confined to a region of that body's gravitational influence. Scientists approximate the region where the gravitational force of the *primary* body (e.g. Earth, Mars,...) dominates that of the more massive yet more distant body (e.g. Sun), as a spherical region called the *Hill sphere* [13–15]. When a celestial body, which we refer to as the primary, with mass m_p rotates around a more massive body with mass M, the radius of the Hill Sphere for the primary body is calculated as

$$r_H \;=\; a_p \left(\frac{m_p}{3M} \right)^{\frac{1}{3}}, \tag{1}$$

where a_p is the semi-major axis of the primary's orbit going around the more massive body (e.g. Earth-Sun, Mars-Sun,... distance). Astronomers often describe distances to an object from an observer's point of view in terms of the angular field of view occupied at those distances. For very distant targets this angle is very small, and its value in radians is equal to its tangent. Figure 1 demonstrates this concept for calculating the Hill Sphere of Mars. The radius of the Hill Sphere, R_H, can be projected on the *celestial sphere*, an imaginary sphere with Earth at its center and a desirable radius (in our example equivalent to the Earth-Mars distance, d). After this projection, R_H is approximated by the arc length it occupies on this celestial sphere, which in radians is equal to its field of view angle, θ, from an observer's point of view from Earth.
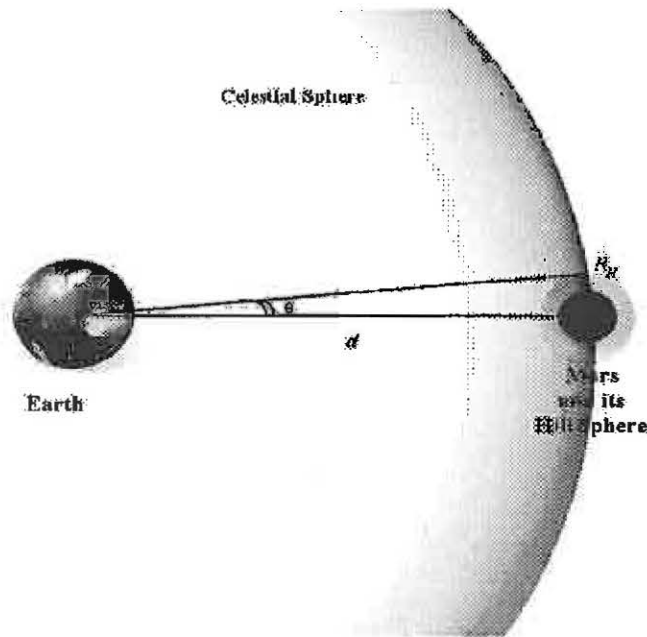

Figure 1: Distance and angular relationship of the Mars Hill Sphere.

Question 1: Considering a mass for Mars of 0.64×10^{24} Kg and a solar mass of 1.99×10^{30} Kg, and a Mars semi-major axis of 1.53 AU, what would be the radius of Mars' Hill Sphere in kilometers?
Hint 1: An Astronomical Unit, AU, is the average distance of Earth from the Sun and is about 1.49×10^8 kilometers.

Question 2: Derive an equation for the relationship between distances and small angles in arcsecond. Using this relationship, how many arcseconds is the radius of the Mars Hill Sphere, R_H? How many degrees is the same radius?
Hint 1: In astronomy often times distances are described in terms of angles. When this is done, it is implicit that this distance is at a very far location from an observer's point of view (on Earth in our example) and can be approximated by its projection on a celestial sphere.
Hint 2: For very small angles, like θ in Figure 1, $\theta \approx \tan(\theta)$. What is the relationship between θ and R_H?
Hint 3: There are 60 minutes (or arcminutes) and 60 seconds (or arcseconds) (60' 60") in every degree. How many arcseconds are there in a circle?

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Question 3: Considering a mass for Pluto of 0.0125×10^{24} Kg and a semi-major axis for Pluto's orbit around the Earth of 5906.38×10^6 kilometers, what is the radius of Pluto's Hill Sphere in kilometers? How does Pluto's Hill Sphere compare to that of Mars? What does that imply?

Hint 1: Use Earth-Sun distance of 1 AU and Mars-Sun distance of 1.524 AU to calculate the Earth-Mars distance.

Let's start searching for satellites of Pluto using images of Pluto provided by the Hubble Space Telescope [12].

Activity 1: Read image file j91601m7q_drz.fits. Read its header information and report the observation date and time, exposure time in seconds, size and data type of the stored image. Display the image.

Hint 1: Flexible Image Transport System (FITS) is the standard astronomical data format [20]. Various tools are available for reading and writing this format at [7]. In particular, you can view a FITS image and its header using SAOimage software [17]. Matlab also supports this format (see 'readfits' and 'fitsinfo' commands or related open source .m files available at Mathworks [2] for more information).

Hint 2: Display the image in log scale. How differently would it appear when you apply histogram matching first?

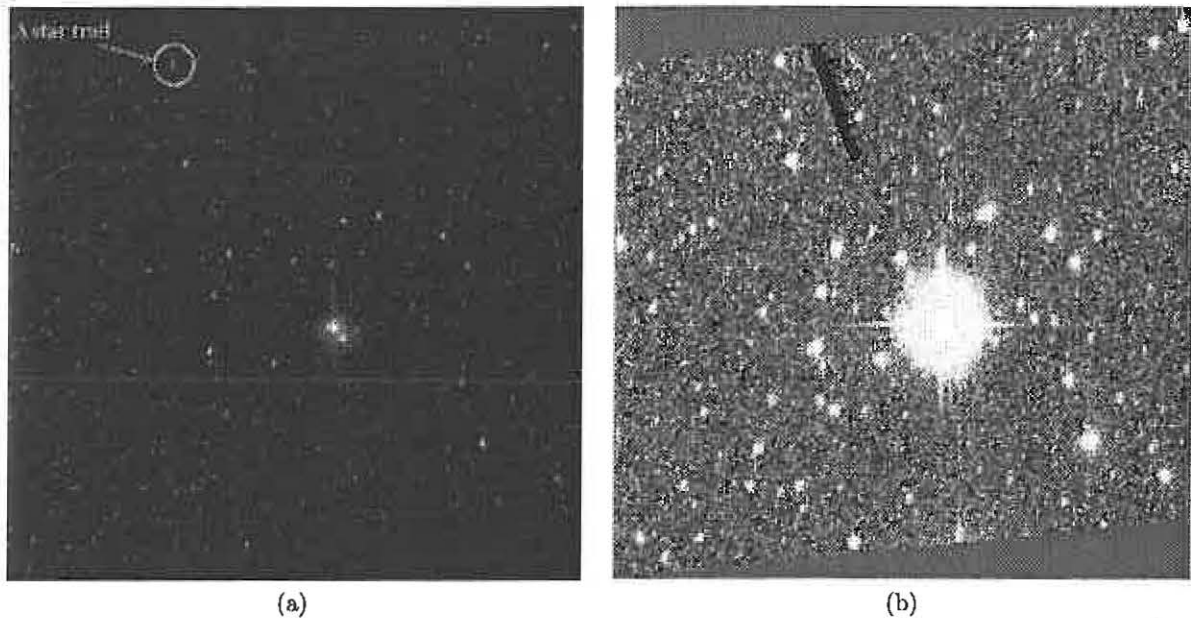(a)   (b)

Figure 2: (a) Image j91601m7q_drz.fits in log scale with a marked star trail, (b) Image j91601m7q_drz.fits after histogram matching.

Figure 2 demonstrates what you should see when you complete Activity 1 (without the red marks and text). This image, and others that you will work with in this case study, are Hubble Space Telescope observations of Pluto made on February 15, 2006, and were retrieved from the MAST [3] archive.

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Activity 2: Repeat Activity 1 for image files named j91601m9q_drz.fits, j91601mcq_drz.fits, and j91601meq_drz.fits. What time was each observation made? How long was the exposure time of each observation?

The Hubble Space Telescope is in orbit around Earth and is continuously moving. While the camera was pointing towards Pluto, images are not perfectly aligned to each other due to vibrations on the spacecraft collectively called *jitter*. As a consequence, a pixel with the same coordinates in each frame does not correspond to the same location in the sky. Therefore, we need to perform *image registration* on different frames. Image registration refers to finding the transformation(s) that would align two or more images of the same object or scene that are obtained at different times, from different viewpoints or sensors [11]. We need to apply the necessary translation, scaling, or rotation to images such that their corresponding pixels represent the same physical location and have the same spatial resolution. The image to which all other images should be aligned is called the *reference* image. Other images are considered *input* images. Then, for each point $P_i = (x_i, y_i)$ in an input image , the objective of any image registration algorithm is to find a transformation matrix T, such that after being applied to input points P_i they are aligned with their corresponding points in the reference image, $P_r = (x_r, y_r)$ [11].

$$\begin{pmatrix} x_r \\ y_r \\ 1 \end{pmatrix} = \begin{pmatrix} s\cos(\theta) & s\sin(\theta) & t_x \\ -s\sin(\theta) & s\cos(\theta) & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} . \tag{2}$$

There are many different algorithms for finding a transformation matrix that could register one image to another [11]. As you can see from Equation 2, the key for finding such a transformation is knowing pairs of corresponding input and reference point coordinates, often referred to as *control points* or *feature points*. There are several approaches for finding such control points. In *supervised* or *manual* image registration, the user's knowledge of data or image is used. Users manually select feature points and provide them as input to the registration algorithm. In *unsupervised* or *automatic* image registration algorithms, these points are selected by an algorithm based on different criteria. One of the most common approaches in such algorithms is based on finding the peak location of the autocorrelation of the two images. Often times a hybrid approach is employed: the user selects some feature points and then an automatic algorithm limits the search area around those points for fine tuning the selected corresponding points.

Suppose two points A, B in the sky have the coordinates shown in Table 2.

Filename	Coordinates	
	A	B
j91601m7q_drz.fits	(738, 490)	(764, 468)
j91601m9q_drz.fits	(744, 492)	(769, 470)
j91601mcq_drz.fits	(740, 496)	(766, 472)
j91601meq_drz.fits	(734, 494)	(760, 470)

Table 1: Coordinates of two points in different frames.

Activity 3: Consider j91601m7q_drz.fits to be the reference image. Find the linear transformation that aligns each input image to the reference image. Apply the registration to each frame.
Hint: We can solve for the transformation matrix, using Eq. 2, without solving for s and θ separately.
Note: From this point on, we work with registered frames.

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Hubble observations of Pluto were made while the telescope was tracking Pluto. Therefore, successive images of Pluto would have Pluto and any of its moons be almost in the same position in images and the stars would be moving, when viewed in sequential order (e.g. in a movie).

Question 4: If we added images of the consecutive observations made from Pluto, how would you expect the stars to appear? How about moons? How would you expect the stars and moons to appear in the integrated image, if Hubble was pointing to a fixed location in the sky instead of tracking Pluto?

Activity 4: Add the provided image frames and display the result. How does the result match your expectations of the outcome? How does it differ?

One of the main obstacles in dealing with astronomical images is the abundance of *cosmic rays*, high energy particles coming from sources far away in the galaxy. They usually appear as random noise in astronomical images and distort the quality of images.

Question 5: Having multiple measurements of the same area in the sky, how do you suggest cleaning the images to remove the cosmic rays? Does this approach remove moons and stars as well? Explain why. What would be the ideal outcome?

Activity 5: Consider a stack of registered image frames. First, apply a one-dimensional median filter along the frames' direction to the stack of images. Then, add all frames. Save and visualize the results in a .jpg file. How does this result compare with those you obtained from the previous Activity? Can you improve the results?
Hint 1: How many observations of each pixel do you have? This gives you the dimension of the points you are working with.
Hint 2: What are the range of possible median filter lengths that you can use? Experiment with each possible filter length. Which one would yield better results?
Hint 3: Replace each pixel with the return value of the median filter along the frames' direction. How do you choose input values for the median filter function applied to each pixel? What would you handle image boundaries?

A more statistically robust approach for removing cosmic rays, and also in this case star trails, is to apply the *resistant mean* algorithm, also referred to as *trimmed mean* algorithm. Details of this algorithm are available at [8] in the commercial IDL language [1], often used by NASA scientists. The algorithm works as follows. For a given input vector y with *npts* number of elements and a given cutoff factor c, compute *res_mean*:

1. $ymed = \texttt{median}(y)$.

2. $abs\_dev = \text{abs}(y - ymed)$.

3. $med\_abs\_dev = \texttt{median}(abs\_dev)/0.6745$.

4. if $med\_abs\_dev \leq 1E-24$ then $med\_abs\_dev = \text{mean}(abs\_dev)/0.8$

5. $cutoff = c * med\_abs\_dev$;

6. for $i = \{1...npts\}$, if $\_abs\_dev(i) \leq cutoff$ then add i to *good_indicies* vector.

7. Let *num_good* be the number of elements stored in *good_indices*.

8. Let *good_points* be those elements in y whose index appears in *good_indices*.

9. $res\_mean = \text{mean}(good\_points)$.

So far, we have calculated an initial estimate of the res_mean value by truncation of y values on a normal distribution. We can refine this value a bit more by compensating for the effects of this truncation.

10. $sigma\_good = \sqrt{\sum_{i=1}^{num\_good} (good\_points - res\_mean)^2 / num\_good}$

11. $sc = \text{floor}(\max(c, 1.0))$

12. if $(sc \leq 4.5)$ then $sigma\_good = sigma\_good / (-0.1398 + 0.8896sc - 0.2300sc^2 + 0.0196sc^3)$

13. $cutoff = c * sigma\_good$

14. Repeat steps 6-9, return res_mean.

Activity 6: Repeat Activity 5, except this time instead of a median filter calculate the resistant mean of the values. Save and visualize the results in a .jpg file. How do results from Activity 5 and 6 compare with each other? Which one looks cleaner?
Hint 1: Apply a cutoff factor of 3 or more (Recommended factor: 4).

You have now finished processing the data. Next, we need to review the final products of Activity 5 or 6 (whichever is cleaner) to identify potential moons. Of course, not every remaining point or feature is necessarily a moon. Some bright stars, galaxies, and even cosmic rays might still appear. Astronomers consider several criteria for identifying a potential moon:

1. It should reside within the primary's Hill sphere.
2. It should not be listed as a star or galaxy in any of the known star catalogues.
3. For high resolution data like those obtained from Hubble, it should have a point spread function. That is, it should not appear as a spike only in one pixel. Furthermore, it should have the correct point spread function (PSF), one matching that of its observing camera.
4. It should appear in all consecutive frames and be observed more than once.
5. It should move in an orbit over time and reappear and be observed in the same location again.

Usually the first three criteria can be verified with a given data set, like the ones you worked with. The fourth criterion requires consecutive observations of the same area, which we have. Potential moons that meet the first four criteria are identified. Then, follow-up observations are made at a later time, after the moon orbits its primary, to verify the discovery. Let's see if we can identify moons that meet the first four requirements.

Question 8: What is the radius of Pluto's Hill sphere in pixels for images you worked with? That is, how many pixels away from the center of Pluto should you search? Are all the pixels in the image within Pluto's Hill sphere?
Hint 1: What was the Hill sphere of Pluto in kilometers or degrees? Use your response to Question 3.
Hint 2: What is the pixel resolution of the images you worked with in kilometers? How about arcseconds?
Hint 3: Use the header files to identify the HST camera and instrument that made the observations. Then, refer to the HST Data Handbook, to find the pixel resolution in arcseconds [18]. Convert this resolution to kilometers if needed, using the same relationships that helped you answer Question 2.

After restricting the search to those pixels that are within the Hill Sphere and identifying moon candidates, we have to eliminate any candidate that is a star or any candidate that cannot be a moon. A star catalogue of the same area of the sky can help with this task. However, for these data sets, we can identify the stars without catalogue entries.

Activity 7: Eliminate any moon candidate that is a star from your final product.
Hint 1: Revisit registered image frames, and locate the moon candidates in them. Are they trailed, similar to the star trail marked in Figure 2-a, in individual frames? If yes, what does it imply?
Hint 2: We are working with long exposure HST images while it was tracking Pluto. Recall your response to Question 4 and results from Activity 4.

Activity 8: Do the remaining moon candidates appear in all frames? Do they appear as point sources, or do they have a PSF, one matching or close to that of the HST's instrument that made the observation?
Hint 1: Refer to the HST Data Handbook, to find the PSF of the instrument that made the observation [18].

Activity 9: Declare any moons you have discovered for Pluto!
Hint 2: Do they pass moon criteria 1-4?

Pluto's largest moon, Charon, was discovered in 1978. Two of the smaller moons of Pluto, Nix and Hydra, were discovered using data provided by the Hubble Space Telescope in May 2005 [16, 19]! Then, more Hubble observations were made in 2006 to verify those discoveries and study the newly discovered outer moons in more detail (verifying criterion number 5). You worked with a subset of those 2006 Hubble images of Pluto in this case study. In fact, Figure 3, showing Pluto and three of its moons, was made by processing the same images you worked with! And just as we were finalizing this case study, Pluto's fourth moon was discovered in July 2011 [5]!

It could have been you discovering moons of Pluto! You can continue searching for new moons of various bodies using NASA's publicly available data. Some of these data archives are Planetary Data System (PDS) [4] and the Multimission Archive at STScI (MAST) [3]. The Dawn mission approached the second largest asteroid in the asteroid belt between Mars and Jupiter, Vesta, in summer 2011. The Dawn mission scientists and engineers were primarily interested in finding moons of Vesta for spacecraft safety. Astronomers have scientific interests for this search, such as learning about origins and evolution of our solar system. Its data will be available on PDS. Maybe by the time you are reading this, a moon of Vesta will have been discovered. If that is the case, you know how it worked! You can join other astronomers around the world searching for moon of Vesta!
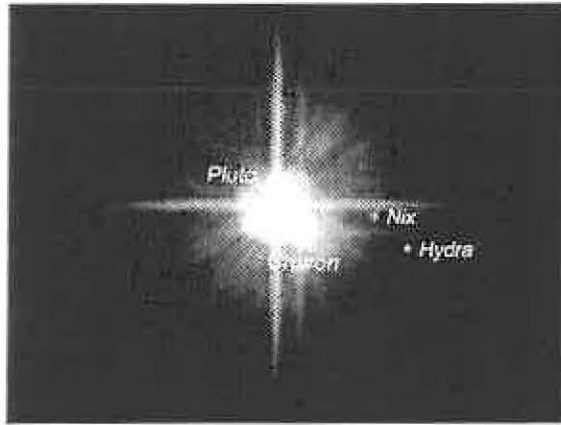

Figure 3: Pluto and its moons from observations made by the Hubble Space Telescope. Image Credit: Stern *et. al* [16] North direction is towards the right, and East is towards the top of the image.

Challenge 1: Can you use a different filter or approach for removing cosmic rays and the stars such that

only Pluto and its three moons remain in the final product? Document your computational steps and visualize your result in a .jpg file.

Challenge 2: In our case study, Hubble was tracking Pluto, which resulted in Pluto and its moons appearing in almost the same location (slight movements exist due to spacecraft jitter) in consecutive frames while stars moved. It often can be the case that the spacecraft would instead point into a fixed direction in the sky instead of tracking an object. In that case, stars would appear in the same location and planets and moons would move from frame to frame. How would the satellite search algorithm be different in that case? In fact, that would be the case for the Dawn mission when searching for moons of *Vesta*, the brightest asteroid, in July 2011. You can obtain Dawn data of Vesta on Planetary Data System (PDS) [4]. Which images, from which Dawn instrument, will you choose to work with? How does your result look after every step of your algorithm? Do you think you have found some moon candidates? Can you rule out the possibility of them being a star or a galaxy?
Note: Compare the pixel resolution of Dawn data with HST data. What does it imply? Can you expect any moon to have a point spread function?

3 Acknowledgements

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