Entropy Stable Method for the Euler Equations Revisited: Central Differencing via Entropy Splitting and SBP

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Abstract The two decades old high order central differencing via entropy splitting and summation-by-parts (SBP) difference boundary closure of Olsson & Oliger, Gerritsen & Olsson, and Yee et al. [15, 7, 37] is revisited. The objective of this paper is to prove for the first time that the entropy split scheme is an entropy stable method for central differencing with SBP operators for both periodic and non-periodic boundary conditions for nonlinear Euler equations. Standard high order spatial central differencing as well as high order central spatial DRP (dispersion relation preserving) spatial differencing is part of the entropy stable methodology framework. The proof is to replace the spatial derivatives by summation-by-parts (SBP) difference operators in the entropy split form of the equations using the physical entropy of the Euler equations. The numerical boundary closure follows directly from the SBP operator. No additional numerical boundary procedure is required. In contrast, Tadmor-type entropy conserving schemes [31] using mathematical entropies and more recently in [35], do not naturally come with a numerical boundary closure and a generalized SBP operator has to be developed [18]. Long time integration of 2D and 3D test cases is included to show the comparison of this efficient entropy stable method with the Tadmor-type of entropy conservative methods. Studies also include the comparison among the three skew-symmetric splittings on their nonlinear stability and accuracy performance without added numerical dissipations for smooth flows. These are, namely, entropy splitting, Ducros et al. splitting and the Kennedy & Grubber splitting.

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