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PEM Fuel Cell MODEL for Conceptual Design of Hydrogen eVTOL Aircraft

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1 Abstract

A model and software for design and analysis of a Proton Exchange Membrane fuel cell (PEMFC) system are developed for hydrogen eVTOL aircraft. Examples are provided of stacks designed for 80 kWe and 500 kWe net electrical power. Examples are provided of eVTOL designed for 250 and 400 lb payload. The trade-offs included stack characteristics, hydrogen storage characteristics, and aircraft payload and range. The objectives were to identify the key technology drivers of a hydrogen rotorcraft, establish technology targets for a viable aircraft, and recommend research to address the fundamental pre-competitive barriers. The current U.S. infrastructure on hydrogen informed the targets and recommendations. The key conclusion is that the advances made in cell electrochemical power density over the last decade might allow a PEMFC system to meet, or even beat, piston engine powered light-utility rotorcraft. The development must focus on the key drivers of a hydrogen eVTOL. The drivers are ultra-light stack cooling and short-term hydrogen storage. A stack system of net electrical power $100 - 150$ kWe with specific power **1.1 kWe/kg** including air, cooling, and electrical subsystems, and a tank storage of **15% weight fraction** hydrogen are the minimum targets to meet the performance of a modern piston-engine rotorcraft, with a range of 180 nautical miles, payload of 400 lb, and gross weight of about 1400 lb. This is defined as the objective aircraft. If only one target is met, an aircraft of half the range could be produced, with a 30% greater gross take-off weight. This is defined as an intermediate aircraft. Because definitive conclusions are premature without weights and loads data on a flight-worthy stack system, it is recommended that a fuel cell powered hydrogen eVTOL demonstrator be built and flown. The existing hydrogen infrastructure for cars provide ample opportunity to create a pilot program. About 12 metric tons of retail hydrogen are available for cars every day in California, which is less than 0.05% of the yearly hydrogen production in the U.S.. A hypothetical fleet of 100 aircraft, operating 3 flights a day, would increase the demand by 2.25 tons per day and require 140 MWh of renewable electricity for green hydrogen.

2 Introduction

This report describes a model and software for design and analysis of a Proton Exchange Membrane (also known as Polymer Electrolyte Membrane) fuel cell (PEM FC) system. The software sizes a PEMFC system to meet certain specified requirements (design) and then predicts its performance for a set of conditions (analysis). The software is meant to support the conceptual design and technology assessment of a PEMFC powered hydrogen electric-Vertical Take off and Landing aircraft.

An electric-Vertical Take off and Landing aircraft (eVTOL) is defined as a vertical lift aircraft propelled by electric power and capable of carrying people. A hydrogen eVTOL (H2eVTOL) derives its electrical power from a hydrogen PEMFC. The abbreviation VTOL is used synonymously with rotorcraft. The notations H2, H and h2 are used as shorthands for hydrogen. The words fuel cell, PEM FC, and PEMFC are used interchangeably.

A PEMFC system consists of a fuel stack, ancillary subsystems, and hydrogen storage. The software facilitates the description and analysis of a typical general-purpose PEMFC system, while retaining the ability to assess new and advanced concepts. The software implements low-fidelity phenomenological models based on test data and look-up tables. For ancillary subsystems, device level models are used which can be calibrated to test data.

The models were developed for eVTOL sizing and analysis reported in Datta and Johnson 2012 and 2014, Ng and Datta 2019, and Ng, Patil and Datta 2021. They are consolidated here. The models for stack weight and cooling system are expanded. Cell characteristics are updated with recent advances reported by the U.S. Department of Energy Hydrogen Program (DOE HP).

A section on rotorcraft sizing is included, with examples to illustrate how the fuel cell models enter and affect sizing. The rotorcraft sizing model is kept simple so it can be reproduced easily and might serve as a common-basis for technology assessment by fuel cell, hydrogen, and rotorcraft specialists. The examples are chosen so that operations are feasible within the current U.S. hydrogen infrastructure. The sizing methodology is generic however, and can be easily extended to larger aircraft.

Datta and Johnson 2012, 2014 made assessments of battery and hydrogen eVTOL using DOE HP technology of the 2000s and gave a set of technology targets for practical eVTOL. The fuel cell targets have now been met by the DOE HP. The advances have been scaled up to 80 kW power, tested, peer-reviewed, and published, see Ahluwalia, Wang and Steinbach 2016 and Thompson et al 2018. The reported cell characteristics have nearly reached the pure-oxygen limit. The objective of this report is to assess the implications of these advances on a conceptual hydrogen eVTOL, identify the key technology drivers for a viable aircraft, and establish clear technology targets for the future.

Unique opportunities of growth in PEMFC and hydrogen infrastructure may be afforded by eVTOL. The usual reservations about PEMFC, such as the lack of regenerative braking and heavy hydrogen storage, do not apply to eVTOL. First, there is no regenerative braking in VTOL, so batteries provide no special advantage. Second, very short duration on-board storage is needed (1-3 hrs), so lighter tanks are conceivable. The problem of low pressures at high altitudes, encountered in an airliner, is also bypassed; rotorcraft are by design low altitude aircraft. The requirement for quiet flight in urban / civil missions eliminates the hydrogen combustion engine as an option. Combustion also produces NOx, a criteria air pollutant. Thus PEMFC is the natural choice for clean and quiet hydrogen eVTOL.

Background

There is a vast literature on fuel cells. Modern texts such as Barbir 2005, Spiegel 2007, O'Hayre, Cha, Colella and Prinz 2009 and Dicks and Rand 2018, cover them comprehensively. Many peer-reviewed journals publish fuel cell research (Table 1), including *Fuel Cells from Fundamentals to Systems* (Wiley) and *Int. J. of Hydrogen Energy* (Elsevier), that are dedicated exclusively to the topic. The papers mainly report advances in material science and electrochemistry. Some that are relevant to aeronautics are also published in various AIAA journals, but they are few. Only a single paper appears in the *Journal of the American Helicopter Society.* Fuel cells are not mature yet to be a viable means of aircraft propulsion.

Advanced Materials	Fuel Cells from Fundamentals to Systems
Advanced Energy Materials	Int J of Energy Research
Angewandte Chemie (in German)	Int J of Hydrogen Energy
Computers and Chemical Engineering	J of Power Sources
Chemical Engineering Science	J of Electrochemical Society (ECS)
Chemical Reviews	J of Materials Chemistry (A, B, C)
ChemSusChem	J of Physical Chemistry (A, B, C)
Electrochimica Acta	J of Membrane Science
Energies	Nature Materials
Energy and Fuels	Nature Energy
Energy and Environmental Science	Renewable and Sustainable Energy Reviews
Environmental Science and Technology	World Electric Vehicle Journal

Table 1: **Some journals that publish fuel cell research.**

Figure 1 highlights some of the interesting past and present applications of fuel cells in aerospace.

There is a long history of fuel cells in space. The first fuel cell powered spacecraft was the Gemini V launched in 1965. The PEMFC system was developed by General Electric beginning 1962. Its legacy continues to this day; see Burke 2003 and Burke 2012 for reviews of hydrogen and fuel cell programs at NASA. Space fuel cells must use pure oxygen and are often needed in a regenerative (reverse) mode where electricity (from solar panels) is used to produce hydrogen and oxygen from water. The power required is usually low, around 1-10 kW, and the consideration of weight less important than in aviation. Reliable operation over extreme duration (years and perhaps decades) is more crucial.

The modern PEMFC matured in ground and automobile sectors. These PEMFC extract oxygen from air and operate in the normal (or forward) mode to generate electricity while producing water from hydrogen and oxygen. The electricity drives a motor. The power output of automobile fuel cells is of the order of 100 kW. The U.S. Department of Energy began its Hydrogen Program in 2002, pursuant to the Hydrogen Future Act of 1996 and the Matsunaga Hydrogen RD&D Program Act of 1990. The legislative mandate of the Hydrogen Program covered industrial, residential, transportation, and utility applications. By transportation, only ground vehicles were meant. The Founding Agreement — FreedomCAR in 2002, was a Fuel Partnership between DOE and the U.S. Council of Automotive Research. Even though the Hydrogen Program cited aerospace experience in its mandate as context for accelerating wider application of hydrogen technologies, aviation always remained outside its scope. The Year-2020 DOE Hydrogen Program Plan (released 12 November 2020) mentions aviation in passing as one of many *Other emerging opportunities* (pg. 29) and lists a program called REEACH (pg. 40) but otherwise places no emphasis on aviation. Thus, the progress achieved in the Hydrogen Program generally remains outside the purview of aviation experts. A few isolated attempts from outside are made from time to time, such as a conversion study of Boeing 787 at Sandia Labs (Pratt et al 2011) or a conversion study of Robinson R22 at NASA Ames (Datta and Johnson 2014), but they are limited to paper studies.

Inspired by space and ground applications, experimental airplanes have been flown with PEMFC. The

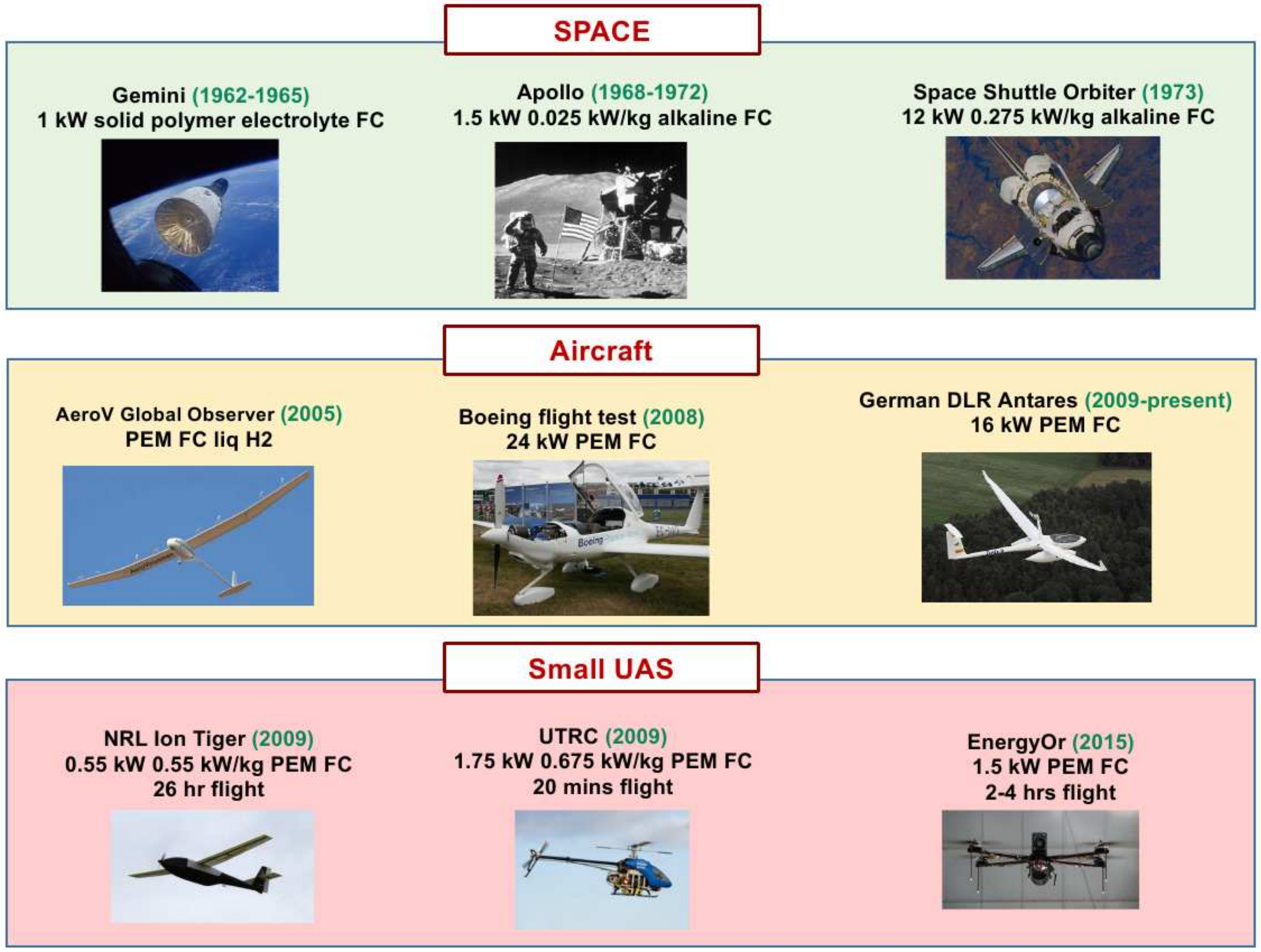

SPACE
Gemini (1962-1965)
1 kW solid polymer electrolyte FC
Apollo (1968-1972)
1.5 kW 0.025 kW/kg alkaline FC
Space Shuttle Orbiter (1973)
12 kW 0.275 kW/kg alkaline FC
Aircraft
AeroV Global Observer (2005)
PEM FC liq H2
Boeing flight test (2008)
24 kW PEM FC
German DLR Antares (2009-present)
16 kW PEM FC
Small UAS
NRL Ion Tiger (2009)
0.55 kW 0.55 kW/kg PEM FC
26 hr flight
UTRC (2009)
1.75 kW 0.675 kW/kg PEM FC
20 mins flight
EnergyOr (2015)
1.5 kW PEM FC
2-4 hrs flight

Figure 1: **Fuel cells in aerospace. Space applications use pure oxygen fuel cells. Aircraft use air breathing fuel cells.**

unmanned Global Observer prototype (15.24 m wing span, 1/3-scale) by AeroVironment is the earliest documented fuel-cell powered aircraft that flew successfully (an earlier attempt by a similar aircraft designated HP-03 ended in crash on its first flight on 26 June 2003; the crash was due to aeroelastic problems not the fuel cells, see NASA-NOAA crash report by Noll et al 2004). The Global Observer used PEMFC with liquid hydrogen as fuel (the HP-03 was originally meant to produce hydrogen on board using a regenerative fuel cell, but the actual aircraft was built with a normal fuel cell to bypass problems encountered during design of the regenerative system). The Global Observer prototype flew on 26 May 2005 at the U.S. Army's Yuma Proving Grounds in Arizona. Details remain unpublished. Since then, full-scale Global Observers (53 m wing span) have made many successful flights (the first prototype crashed 18 hrs into its 9th flight), but very little technical information is available in the public domain. A number of manned airplanes have also been flight tested since then. These include the single-occupant/pilot Boeing (Spain) Fuel Cell airplane (Lapena-Rey et al 2010), the single-occupant/pilot DLR Antares (Kallo et al 2015) and the four-occupant DLR Hy4 (unpublished, but some information can be found in Flade et al 2016). Gaseous hydrogen was used as fuel for both. The Boeing (Spain) and DLR programs are well documented, but weight information is unavailable. Only the Naval Research Laboratory Ion Tiger Program, which flew a small 0.55 kW unmanned airplane on 0.5 kg of liquid hydrogen, documented weights methodically (across several publications between 2011-2014, see Stroman et al 2014 and Swider-Lyons et al 2014 for final numbers). Most of these efforts use derivatives of automobile fuel cells.

Inspired by airplanes, small experimental rotorcraft have been flown with PEMFC. A battery powered small kit helicopter Maxi-Joker 2 was converted to a fuel cell helicopter (2 m single main rotor diameter, 10 kg gross take-off weight, and 4 kg empty weight) by United Technologies Research Corporation (UTRC) (Moffitt and Zaffou 2012). It flew on 11 October 2009 at East Hartford Campus of UTRC. Some weight information is available. The power density of the stack was 0.65 kW/kg, stack including ancillaries was 0.5 kW/kg, the stack weight was 3.4 kg, and the maximum power output was 1.75 kW. Gaseous hydrogen was stored at 289 bar, in a 2.3% weight fraction tank. The PEMFC was custom made, the tank was a commercial unit. Since then, commercial rotary-wing drones have made their debut in the UAS market and demonstrated extreme endurance compared to batteries. Little to no information on weight breakdown is published.

The weights that are available in public are plotted in Fig 2. Stack power is plotted versus weight. Only Toyota, Honda, Ion Tiger, and the UTRC helicopter data are obtained from peer-reviewed sources. Most sources provide only the stack weight. Some provide weights of ancillary subsystems but do not specify what they are. Whether the power reported is gross or net electrical is unspecified. The commercial rotary-wing drones report total weight including hydrogen. Fewer details are available for the under-water systems. Out of this incomplete and often inconsistent information, no useful trends or sensible weight models can be generated.

A number of previous NASA reports describe PEMFC weight models for special-purpose applications. These include, Burke 1999, on regenerative PEMFC performed in the context of AeroVironment's high-altitude unmanned solar Centurion airplane (a precursor to the Helios series culminating in the HP-03), Colozza 2002, on hydrogen storage weights for aircraft applications, and Colozza and Jakupca 2019, on thermal sizing for ground-based systems on Mars.

The objective of this work is a general-purpose method with well-defined models for weights. Users with access to measured weights should be able to calibrate the models and make reliable assessment of more advanced concepts. The models are simple, as appropriate for a conceptual design environment, yet rooted in test data, geometry and materials. Only PEMFC is considered as it is the lightest of all fuel cells (highest specific power, kW/kg) and at present the only path to clean, renewable, emissions-free flight. Only air-breathing PEMFC is considered, and only the normal or forward operation is modeled, as appropriate for eVTOL.

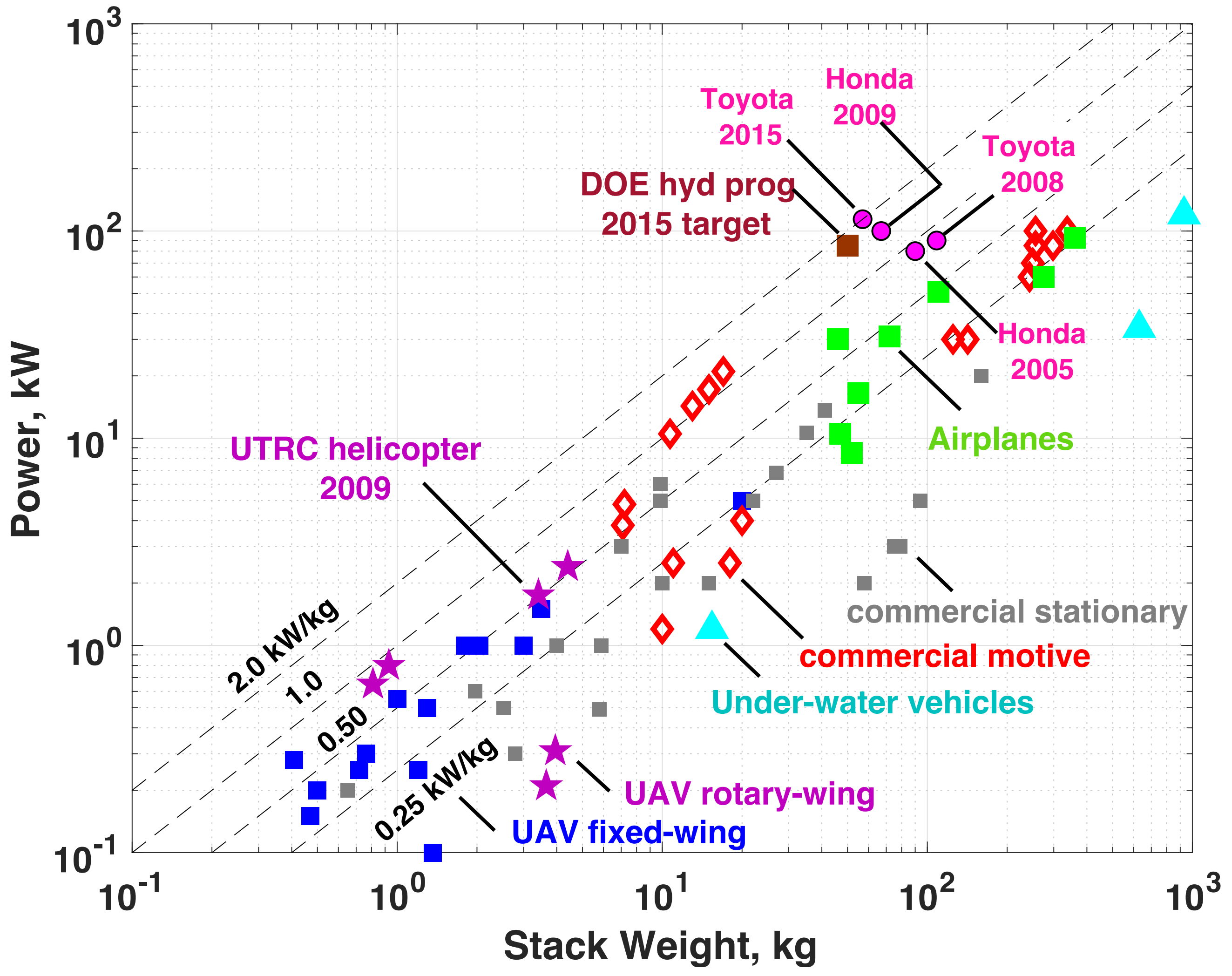


Figure 2: **State-of-the-art power and weight of PEMFC systems of < 500 kW.**

Units and Standard Conventions

Throughout the report, SI units are used unless otherwise mentioned. Non-SI units are used for some components where they are the norm to maintain uniformity in literature. The pertinent conversions are described below.

Energy :

1 Watt-hr (Wh) = 3600 Joule (J)

1 kilo Watt-hr (kWh) = 3.6 Mega-Joule (MJ)

Pressure :

1 atm = 101, 325 Pascal (Pa) = 1.01325 bar = 14.6959 lbf/inch^{-2} (psi)

1 bar = 0.1 Mega-Pascal (MPa)

Temperature :

1° Celsius (C) = 1 Kelvin (K) − 273.15

Volume :

1 Liter (L) = 0.001 m^3

1 U.S. gallon (gal) = 3.7854 L

1 cubic ft/min (CFM) = 14.72 × 10^{-4} m^3/s = 1.472 L/s

Throughout the report, ISA and STP-NIST standard conventions are used (Table 2). The STP-IUPAC is also used in fuel cell literature. It is identical to STP-NIST except for a small difference in temperature.

Standard	T	T	p	p	p
K	$°C$	Kelvin	bar	kPa	atm
ISA/SL	15	288.15	1.01325	101.325	1
STP-NIST	25	298.15	1	100	0.9869
STP-IUPAC	0	273.15	1	100	0.9869

Table 2: **Standard conditions.**

The Proton Exchange Membrane Fuel Cell (PEMFC)

A Proton Exchange Membrane (also known as Polymer Electrolyte Membrane) fuel cell (PEMFC) is shown in Fig 3. The basic operation is as follows.

Hydrogen flows into the anode electrode and decomposes into protons and electrons.

$H_2 \rightarrow 2H^+ + 2e^-$ Anode, release of electrons, oxidation

The electrons pass through an external circuit which is how current is generated. The protons pass through an electrolyte membrane.

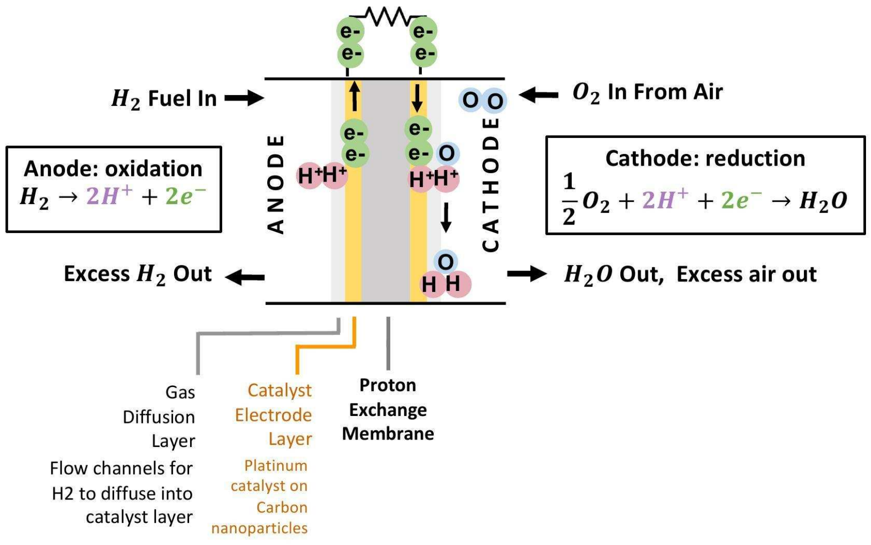
Air flows into the cathode electrode and the oxygen combines with protons and electrons to produce water (vapor or liquid).

${}^1/{}_2 O_2 + 2e^- + 2H^+ \rightarrow H_2O$ Cathode, capture of electrons, reduction

Nitrogen and other gases simply pass through without reaction.

Overall, a fuel cell extracts energy released during the following reaction as charge with a voltage

$$H_2 + {}^1/{}_2\, O_2 \rightarrow H_2O$$



H_2 Fuel In
O_2 In From Air
Anode: oxidation
$H_2 \rightarrow 2H^+ + 2e^-$
Cathode: reduction
$\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O$
ANODE
CATHODE
Excess H_2 Out
H_2O Out, Excess air out
Gas Diffusion Layer
Flow channels for H2 to diffuse into catalyst layer
Catalyst Electrode Layer
Platinum catalyst on Carbon nanoparticles
Proton Exchange Membrane

Figure 3: **A proton exchange membrane fuel cell.**

Hydrogen and air enter a fuel cell through channels built into what are called bipolar plates. The bipolar plates can also contain channels for coolant. The channels have intricate design. The bipolar plates are the heaviest parts of a fuel cell and can contribute up to 80% of its weight.

The anode and cathode electrode each consist of two parts: a gas diffusion layer (GDL) and a catalyst layer (Anode or Cathode catalyst layers (ACL or CCL)). The hydrogen and air diffuse through these layers. The GDL is typically made of carbon paper or cloth, carbon powder, and polytetrafluoroethylene (PTFE). Teflon is a brand name of PTFE made by Chemours, a spin-off of DuPont (DuPont discovered the compound in 1938). The catalyst layer is typically made of Platinum on Carbon (Pt/C) powder and an ionomer. Nafion, a brand name for sulfonated tetrafluoroethylene, is a typical ionomer (also discovered by DuPont, in late 1960s). In its simplest form, the electrode is just a carbon cloth coated with Pt/C on one side. The typical catalyst loading varies from $0.1 - 0.4$ milligram of Pt/cm^2. The catalyst is responsible for breaking down hydrogen into protons and electrons. Both the GDL and ACL/CCL are porous and partially hydrated structures. The dispersed carbon particles have a typical diameter of 0.7 μm whereas the Pt nanoparticles parched on top of carbon have typical diameters of $3 - 5$ nm. Large pores in between the particles form ducts for hydrogen and oxygen to diffuse from the bipolar plates to the respective catalyst layers. The smaller pores form ducts for water. Once split, hydrogen passes its electrons to Platinum and protons to the electrolyte membrane. The membrane is also a polymer, typically Nafion. When sufficiently hydrated it is an excellent conductor of protons. The membrane and the electrodes together are called the Membrane Electrode Assembly (MEA).

The electrons flowing through the outer circuit is captured back into the cathode catalyst layer (CCL) by Platinum. At the cathode, oxygen diffuses through the GDL to react with the protons coming from the membrane and electrons from Platinum to produce water (vapor or liquid) and heat. So the cathode side generates water and heat. Platinum ensures water, not hydrogen peroxide, a toxic gas, is produced. Platinum makes fuel cells expensive.

It is crucial that the membrane is sufficiently hydrated for proton conduction. Because water and heat are generated at the cathode, the membrane on the cathode side tends to saturate, while the membrane on the anode side tends to dehydrate. This reduces the conductivity of the protons. To limit this problem, the cell is fed with humidified gases at temperatures higher than the cell temperature. However, too much humidification — meant for the membrane — can flood the electrodes and prevent gas transport to the membrane/catalyst interface. Balanced humidification is critical — too little or too much are both detrimental.

Unlike a combustion engine, a fuel cell has no exhaust gases to carry heat out, so the heat must be removed entirely by other means. The low temperature operation of the stack (65-85°C) makes it harder for coolants to carry heat out; the small temperature difference between stack and the ambient also require a large radiator. Radiators are heavy, and a major overhead in high power stacks. Cooling can be a major barrier in aviation unless lighter weight methods are devised.

The PEMFC System

Cells connected in parallel form a PEMFC stack. Ancillary subsystems are needed to operate the stack. The stack and the ancillary subsystems form the PEMFC stack system. The stack system, and the fuel tank and fuel delivery subsystem form the total PEMFC system.

The PEMFC system model is sketched in Fig 4. The architecture mimics a modern automobile PEMFC system. The subsystems are listed below.

1. Stack
2. Air management system
3. Low temperature cooling (LTC) system
4. High temperature cooling (HTC) system
5. Water management system
6. Electrical system
7. Hydrogen tank and delivery system.

The stack and the tank are the principal subsystems. Not all ancillary subsystems are needed to model every PEMFC system.

The design method sizes the PEMFC system. The block diagram for sizing is shown in Fig 5 (top). A design power P_D is specified and the stack is sized to generate this power. The net electrical power P_{De} is the power generated minus the power to operate the ancillary subsystems which is called the balance of plant P_{BOP}.

$$P_{De} = P_D - P_{BOP}$$

The design method iterates the sizing method to compensate for the balance of plant and achieve a specified net electrical power. The block diagram for design is shown in Fig 5 (bottom).

The analysis method predicts the performance of a PEMFC system after it is designed under various off design conditions. The block diagram of analysis is shown in Fig 6. The current is specified and the voltage and net electrical power are calculated. Only steady-state current is considered. Ng and Datta 2019 measured the stack transients and found them fast (similar to a Li ion battery) and first order (resistive and capacitive behavior). The system transients are therefore determined by air and cooling system transients. These are discipline dependent (car or aircraft) and in absence of data left out of scope.

Cost analysis is left out of scope. Cost analysis for fuel cell cars are published by the DOD HP, see for example Thompson et al 2018 for a recent analysis for high-volume manufacturing scenarios. Cost analysis of fuel cell eVTOL may be premature before an airworthy stack system is built and tested and a clear understanding of parts and supply chain is acquired.

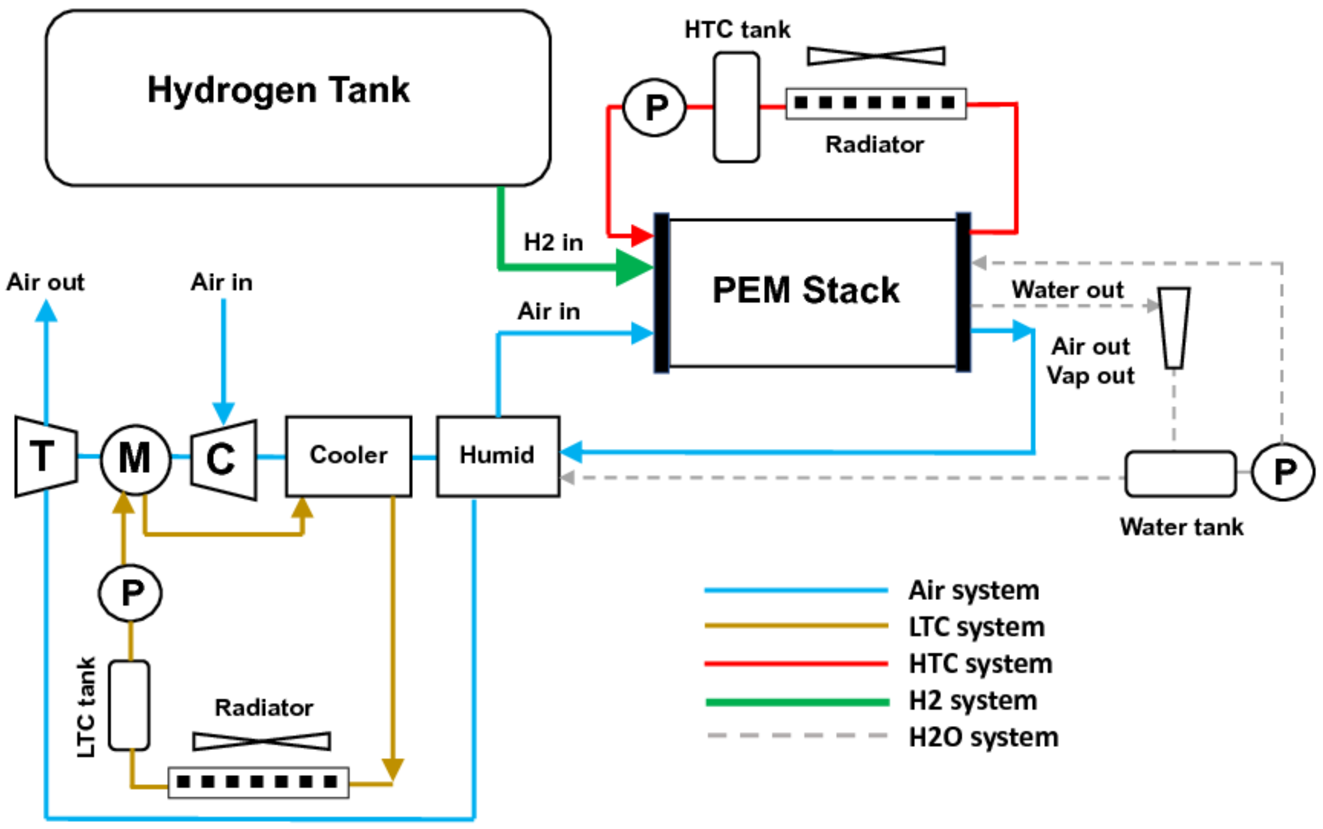
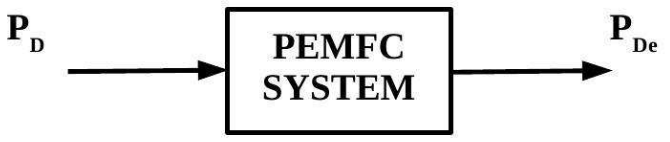


Figure 4: **A PEMFC system. The subsystems shown are air, low temperature cooling (LTC), stack, high temperature cooling (HTC), and water (H20). The lines and arrows show the flow of air, coolant, and water. The electrical subsystem (not shown) covers transmission and distribution of power from the stack to the drive as well as controllers and power to each subsystem. P=pump. M=motor. T=turbine. C=compressor. Humid=humidifier.**



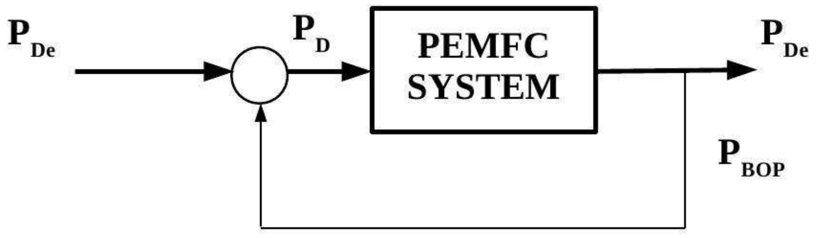


Figure 5: **Design block diagrams.**

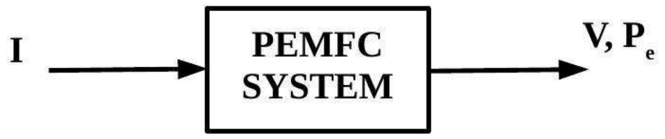


Figure 6: **Analysis block diagram.**

Outline of Report

In the sections that follow, the existing U.S. hydrogen infrastructure is described first. It sets the context for design. The design and analysis steps are outlined in Sections 4 and 5 respectively. The theory behind the steps are described in Section 6. Sections 7 and 8 are dedicated respectively to power consumption and weight models. Section 9 outlines the software. Section 10 describes the inputs and outputs. Example fuel cell designs are described in Section 11 (Yr-2000 cell technology, 80 kWe stack), Section 12 (modern cells, 80 kWe stacks) and Section 13 (modern cells, 500 kWe stacks). The 80 kWe models can be used for cars. The 500 kWe models are meant for aviation. The examples are templates to be calibrated with test data when they become available. Section 14 describes a simplified rotorcraft sizing method which is used in examples later to illustrates how the fuel cell models enter and affect sizing. The method is kept deliberately simple so it can be reproduced by non-rotorcraft specialists. Three examples are given: 1) a single-occupant conceptual NASA electric quad-rotor, 2) a two-occupant conceptual Maryland electric quad-rotor, and 3) a two-occupant main rotor-tail rotor helicopter calibrated loosely to a R22 Beta II weight, range, and payload. The third example allows technology targets to be calibrated for a certified airframe. Section 15 gives summary and conclusions. Section 16 gives the technology drivers, and requirements and makes recommendations for a hydrogen eVTOL pilot program. The software is appended at the end.

3 Hydrogen Infrastructure in the U.S.

This section describes the hydrogen infrastructure in the U.S., from supply, to retail to codes and standards.

Infrastructure informed design, particularly tank and aircraft size, and the scale of examples.

In 2020, 9 million metric tons of hydrogen were produced in the U.S., and 87 million globally.

Globally, hydrogen is produced from various sources (Table 3). They are color coded accordingly, although there is no formal or universal convention. About 95% of hydrogen is produced from fossil fuels. This hydrogen is coded grey or brown. The production process emits 9–12 tons of CO and CO_2 per ton of hydrogen from grey to brown. When backed by carbon capture and storage it is coded blue. The energy content of hydrogen is 39 kWh/kg. The production efficiency is around 90% which means 43 kWh/kg of energy is spent to produce this hydrogen. About 4% of hydrogen is produced from water electrolysis. When the electricity is derived from wind or solar, the hydrogen is coded green. When the electricity is from nuclear, it is coded purple. About 250 megawatts (MW) are presently dedicated to producing green hydrogen. The production efficiency is around 70% which means 55 kWh/kg of energy is spent to produce green hydrogen.

% Produced	Source	Process	Color
48%	Natural gas	1. Steam reformation	grey
		If backed by carbon capture & storage	blue
		2. Methane pyrolysis with	turquoise
		no green house gas emissions	
30%	Petroleum	Steam reformation	grey
18%	Coal	Coal gasification or carbonisation	brown
4%	Renewable	Water electrolysis with electricity	green
		Electricity from nuclear power	purple

Table 3: **Sources of hydrogen.**

The price of grey-brown hydrogen is U.S. $1-3/kg. The price of green hydrogen varies widely. Often $5-19/kg is quoted but that number is the price of electricity. In the U.S., the average price of electricity is 13 cents/kWh, but variation across states, Hawaii-34, California-22, Maryland-13, to Washington-9.5 cents/kWh, gives the range. Then there is the cost of transportation (except green hydrogen produced on-site) and compression or liquefaction.

Globally, hydrogen has a vast industrial usage (Table 4). Only a tiny fraction is used in transportation. A significant amount is used in oil refining. So even though hydrogen is produced largely from fossil fuels, fossil fuels are also produced using hydrogen.

% Use	Industry
25%	Oil refining through hydrodesulfurization and hydrocracking
	Hydrodesulfurization — removal of Sulfur from gasoline, jet fuel, kerosene etc
	Hydrocracking — conversion of heavy fuel into diesel, jet fuel, kerosene etc
55%	Ammonia production through Haber-Bosch
	Haber-Bosch — artificial fixation of nitrogen with hydrogen
10%	Methanol production
10%	Other, such as HCl production, hydrogenation, coolant, transportation

Table 4: **Usage of hydrogen.**

Hydrogen is transported through pipelines and trucks. According to the U.S. DOE Office of Energy Efficiency and Renewable Energy, there are 1600 miles of hydrogen pipelines in the U.S.. They are owned by merchant hydrogen producers, and located near large users such as oil refineries and ammonia plants, most of which are in the Gulf Coast region. Beyond pipelines, hydrogen is distributed through trucks that haul trailers stacked with long pressure-tight steel cylinders of gaseous hydrogen (CGH_2) called tube trailers. The cylinders are limited to 250 bar pressure, per U.S. Department of Transportation regulation, and carry typically $380 - 420$ kg of hydrogen. The capacity of the truck is limited by the weight of steel cylinders. Specially insulated, cryogenic ($-253°$C), tanker trucks haul liquid hydrogen (LH_2). Liquefaction can consume up to 30% of hydrogen energy. In addition, some hydrogen can evaporate (boil-off), more so if the tank surface to volume ratio is high as in small trucks. Typical tanks can carry from 1,500–25,000 gallons (400–6,500 kg, using density of 70 kg/m^3) of liquid hydrogen at 1 bar. Thus, liquid trucks are economical for longer distances and larger amounts of hydrogen, for rocket launches for example. Hydrogen is then dispensed into compressed gas cylinders at the retail stations or poured directly as liquid. Compared to the 1600 miles of hydrogen pipeline, there are 3 million miles of natural gas pipelines, as of December 3, 2020, per U.S. Energy Information Administration. A study on how hydrogen might be blended into existing natural gas pipelines can be found in the NREL report by Melaina, Antonia and Penev 2013.

The Alternative Fuels Data Center, under the Office of Energy Efficiency and Renewable Energy, U.S. DOE, maintains a data base on fleet, fuels and fueling stations in the U.S. and Canada. According to this database, as of June 2021, there are 68 open hydrogen stations in the U.S.; 19 are private and 49 are public retail outlets. These provide fuel to the 10,665 fuel cell cars, sold and leased, and 48 fuel cell buses currently in operation. Almost all outlets provide compressed gas at 350 or 700 bar.

State	Fueling station	City	Pressure bar
Arizona	Nel hydrogen-Nikola, 4141 E Broadway Rd	Phoenix	350
California	SunLine Transit Agency, 32-505 Harry Oliver Trail	Thousand Palms	350
California	Honda MC Formula Station, 1900 Harpers Way	Torrance	700
California	AC Transit, Oakland, 1100 Seminary Ave	Oakland	350
California	AC Transit, Emeryville, 1172 45th St	Emeryville	350
California	Orange County Trans Authority, 4301 MacAurthur Blvd	Santa Ana	350
Colorado	DOE/NREL, 15013 Denver West Pkwy	Golden	350, 700
Connecticut	Tri-Gen, 539 Technology Park Dr	Torrington	350
Delaware	Air Liquide, 200 Gbc Dr	Newark	unknown
Hawaii	Blue Planet Research, 71-1645 Mamalahoa Hwy 2	Kailua-Kona	350
Massachusetts	Bus Transit Auth., Charlestown Garage, 21 Arlington Ave	Boston	350
Massachusetts	Greentown Labs, 444 Somerville Ave	Somerville	700
Michigan	Ford Lab, Miller Rd and S Dix St	Dearborn	350, 700
Michigan	Bus Transportation Authority, 5051 S Dort Hwy	Grand Blanc	350
New York	Hempstead - Dept of Cons & Waterways, 1401 Lido Blvd	Point Lookout	350
Ohio	Regional Transit Auth., Stark Area, 1600 Gateway Blvd SE	Canton	350
Ohio	DOD/Defense Supply Center	Columbus	unknown
Virginia	DOD/Fleet Service Center, 7847 Senate St, Bldg 126	Richmond	unknown
Washington	Army/Ft Lewis, McKinley Ave & Garfield St	Ft Lewis	unknown

Table 5: **Private outlets of hydrogen in the U.S..**

The 19 private outlets are listed in Table 5 along with their tank pressure capability. Of the 49 public retail outlets, 48 are in California. The only other outlet is in Honolulu, Hawaii, a Toyota Mirai dealership in 2850 Pukoloa St run by Servo. It produces 20 kg of hydrogen per day and allows up to 4 car fill-ups at pressures of 350 and 700 bar. The California Air Resources Board (CARB) maintains a yearly count of fleet and fueling stations in California (see CARB 2020). The count for Yr-2019 was reported as 43 by CARB on July 3, 2020. The projection for Yr-2020 at the time was 58. The unofficial count as of July 2021 by the

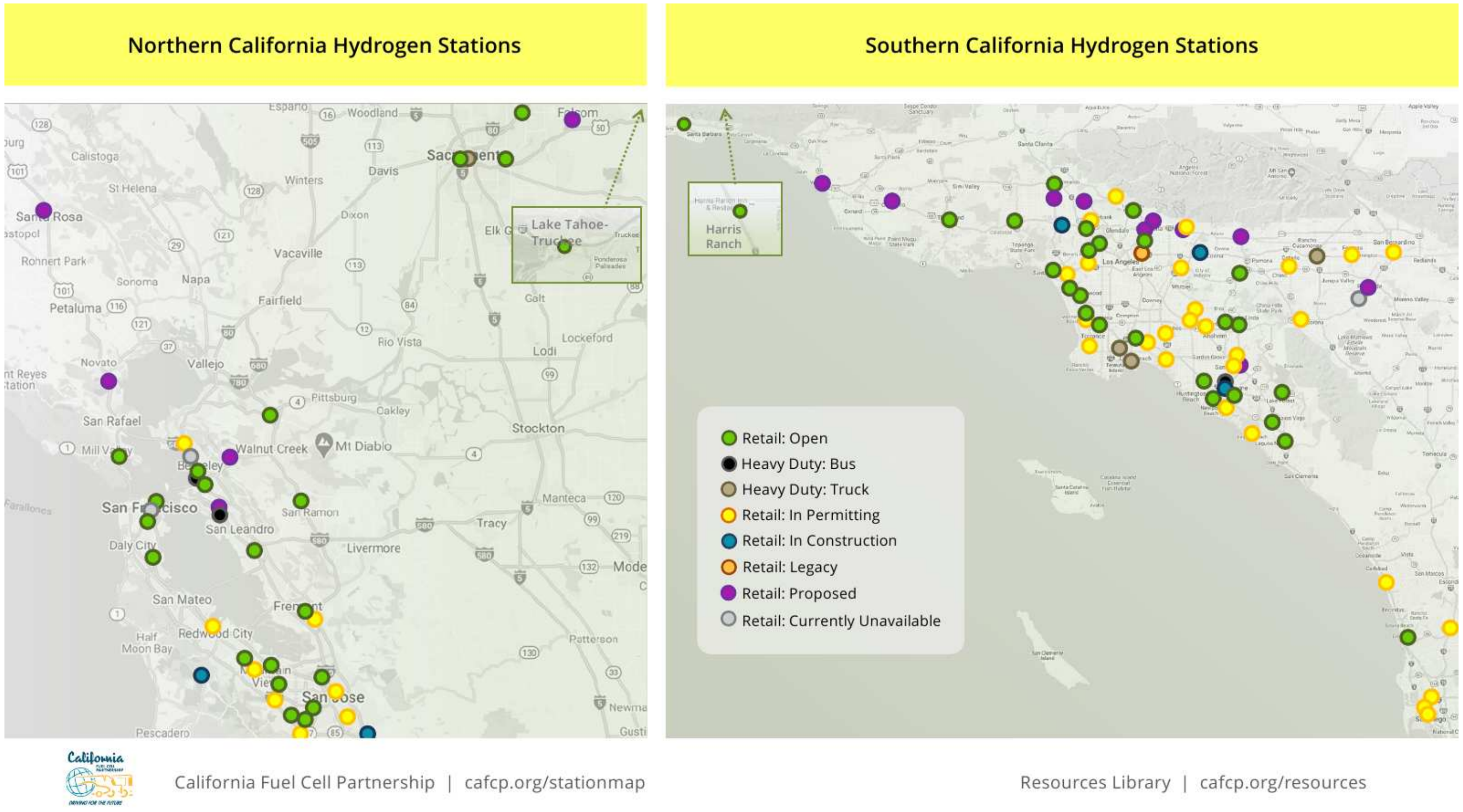


Figure 7: **California public outlets of hydrogen as of June 2021; retail open–47, retail in permitting phase–35, retail only for heavy duty bus–3, and retail only for heavy duty truck–4.**

California Fuel Cell Partnership (a non-government entity) is 49. Figure 7, taken from their report, shows the location of these 49 stations. 47 are currently open. The stations are all in the north (21) or south of the state (26). There are 35 additional stations that are completed but awaiting permit, 8 in the north and 27 in the south. The three identified for heavy duty buses appear under private fueling station in Table 5. The four identified for heavy duty trucks are unaccounted for in Table 5. Overall, the data from various sources are consistent.

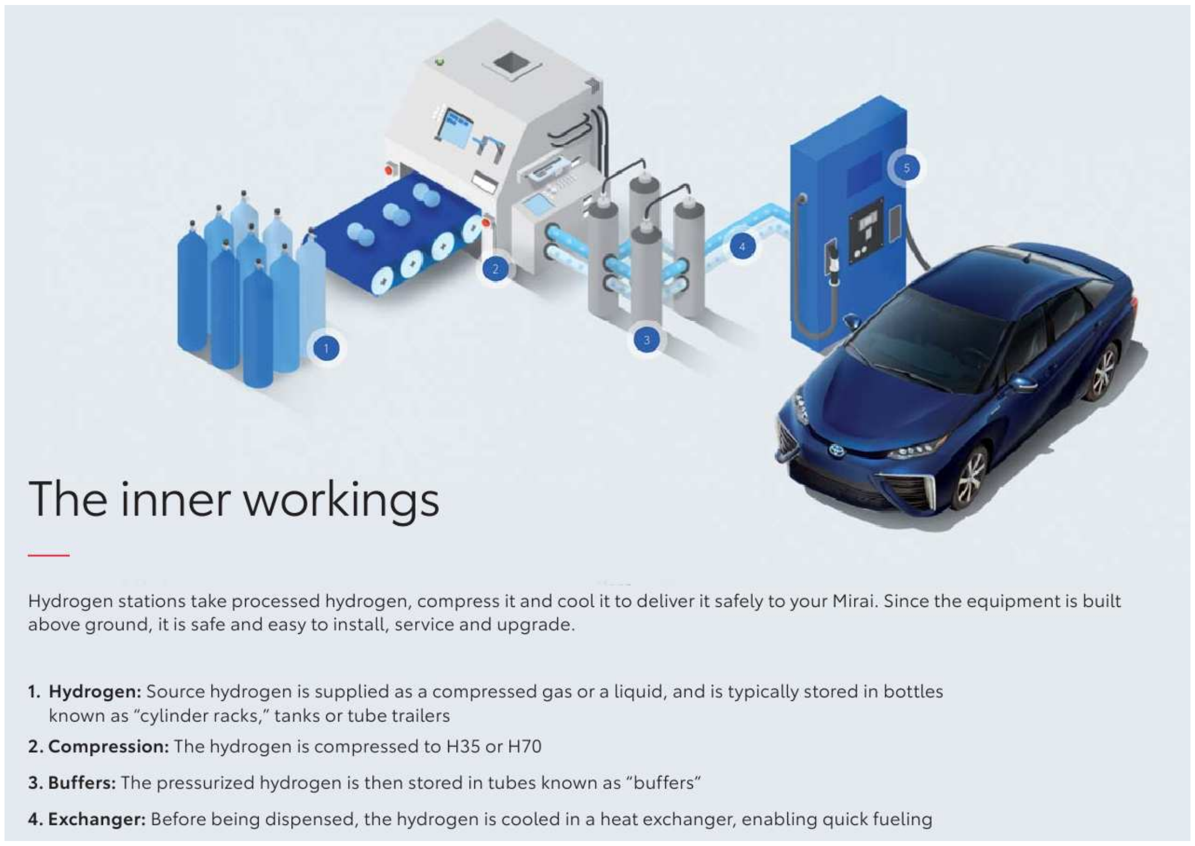
The fuel retail statistics of California is compiled in Table 6. The fossil fuel data is from California Energy Commission Annual Retail Fuel Outlet Reporting (CEC-A15) published online. The latest available data is for Yr-2019. Sales are reported, in volume (Million gallons) per year. There is no reporting requirement for hydrogen, so sales numbers are not available. But the capacity of the stations are reported. This number is often combined with projections for stations about to come online. The data from CARB 2020 Annual Report (CARB 2020) is assumed to be the authentic source. It reports Yr-2019 capacity as 11,972 kg/day, which converted to /year is given in Table 6. Since volume is a function of pressure, capacity is reported in mass (Million kg) not volume. 4.3 Million kg is 0.0043 Million tons $\approx$ 0.048% of annual U.S. hydrogen production. For Yr-2020 and 2021, the CARB 2020 report gives only projected capacities. Un-official counts of actual amounts dispensed (from a CARB official) are 2.1 Million kg in 2020 and 3.5 Million kg so far in 2021, numbers that are possibly lower due to Covid-19.

Fuel	Number of Stations	Amount of H2
Fossil Fuel		Sales (Million U.S. gallons/yr)
Gasoline	8,269	13,473
Diesel	4,958	1,559
Propane	301	3
E-85 Gas	181	37
Natural Gas	121	18
Total	10,449	15,090
Hydrogen		Capacity (Million kg/yr)
Retail: 2019	43	4.3
Retail: 2020	48	2.1
Retail: 2021 (until June)	47	3.5
Total	84	

Table 6: **Retail fuel outlet statistics for the U.S. state of California; some outlets sell multiple fuel types so the number of stations may not add up to the total. E-85 is 83% ethanol and 17% gasoline in California.**

The 700 bar gaseous storage is the current standard for on-board tanks in fuel cell cars. The 350 bar storage is used for buses and fork-lifts. Protocols for fueling are defined by the SAE J2601 document. The J2601 is a family of protocols applicable to two fuel delivery pressures: 35 MPa (350 bar, $\approx$5000 psi, designation H35) and 70 MPa (700 bar, $\approx$10,000 psi, designation H70), three fuel delivery temperatures: -40°C, -30°C and -20°C (designation T40, T30, and T20 respectively) and two storage volume categories, one from 49.7 to 248.6 L (for H35 and H70) and another for 248.6 L and above (for H70 only). All cars sold in California are designed to follow this protocol (see CARB 2020 for details). The general arrangement in a typical car re-fueling station is shown in Fig 8, taken from a Toyota Mirai Yr-2018 brochure.

As hydrogen is pumped into the tank, its internal temperature rises rapidly. Nominal mass flow rates are 10-30 gram/s (the advertised re-fuel time for cars is 3 mins for 5 kg hydrogen which gives a rate of 28 g/s) at the end of which the internal tank temperature reaches $60 - 85$°C. The rate is limited by maximum



The inner workings

Hydrogen stations take processed hydrogen, compress it and cool it to deliver it safely to your Mirai. Since the equipment is built above ground, it is safe and easy to install, service and upgrade.

1. **Hydrogen:** Source hydrogen is supplied as a compressed gas or a liquid, and is typically stored in bottles known as "cylinder racks," tanks or tube trailers
2. **Compression:** The hydrogen is compressed to H35 or H70
3. **Buffers:** The pressurized hydrogen is then stored in tubes known as "buffers"
4. **Exchanger:** Before being dispensed, the hydrogen is cooled in a heat exchanger, enabling quick fueling

Figure 8: **Toyota Mirai refueling diagram taken from the Yr-2018 sales brochure).**

temperature of 85°C to prevent material degradation and over pressure greater than 125% of 700 bar. After fill-in, the tank settles to its nominal working temperature of $15-20$°C. The lower the delivery temperature, more the fill-in. The current DOE stipulation is 15°C. The lowest delivery temperature of -40°C can produce 99% fill-in whereas -20°C is likely to produce about 94% fill-in. The faster the rate, the lower the fill-in, with the penalty diminishing at lower delivery temperatures.

The impact on design is that the on-board tanks must be sized for 700 bar and $15-20$°C to merge with the current infrastructure. An assumption of 100% fill-in is reasonable for sizing. Re-fueling rates of 10-15 g/s can be assumed.

Energy is spent to compress hydrogen. About 15% of hydrogen energy is needed to compress to 700 bar (adiabatic limit) and about 30% to liquefy it. So the total energy spent from fossil fuel to the tank including the 90% production efficiency are about $(1/0.9+0.15)\times 39 = 49$ kWh/kg for 700 bar and $(1/0.9+0.30)\times 39 = 55$ kWh/kg for liquid. The numbers for green hydrogen including its 70% production efficiency are $(1/0.7+0.15)\times 39 = 62$ kWh/kg for 700 bar and $(1/0.7+0.30)\times 39 = 67$ kWh/kg for liquid respectively.

Codes and Standards

Because of the vast industrial usage, there are a large number of U.S. codes and standards on hydrogen, covering pipelines, storage, transportation, and fire protection. Some have recently been established for fuel cell cars. A few have ventured into unmanned and even manned aircraft. Most of these are incomplete drafts, and none applicable directly to rotorcraft. So none informed the design and analysis method in this report. But the existing standards are important guide rails for innovation as well as indicators of the issues and challenges that await aviation. Therefore, for completeness, some of the relevant U.S. standards are listed in Table 7 with limited description. They are available through the corresponding societies: American Bureau of Shipping (ABS), American Society of Mechanical Engineers (ASME), American Society of Testing and Materials (ASTM), Compressed Gas Association (CGA), American National Standards Institute (ANSI), Canadian Standards Association (CSA), National Fire Protection Association (NFPA), and Society of Automotive Engineers (SAE).

Code/Standard	Description
ABS	Fuel Cell Power Systems for Marine and Offshore
ASME B31.12	H2 piping & pipelines
ASME STP-PT-006	Design guide for H2 piping & pipelines
ASME BPVC	Section VIII construction of pressure vessels
	Division 3 (pressure $>$ 10,000 psi), Article KD-10 (gaseous H2)
	Code Case 2579 — Composite tanks for gaseous H2
	Code Case 2569 — SA-372 Steel tanks for high-pressure gaseous H2
	Code Case 2563 — Al Allow 6061 for high-pressure gaseous H2
	Section X construction of fiber-reinforced plastic pressure vehicles
	Section XII transportation tanks
ASTM	Various testing methods
	D7606-17, D7634-10, D7649-19, D7650-13, D7651-17, D7652-11,
	D7653-18, D7675-15, D7676-18, D7892-15, D7941/7941M-14
	Test method for CO chemisorption: WK17123
	Design of Fuel Cells for Unmanned Aircraft: WK60937
CGA	Various piping, inspection, leak detection, and qualification methods
	Publications C6.4, C21, G5.3-5.6, H1-H5, H12-H15, Hxxxx
	Publications P6, P12, P28, P41, PS31, PS33, PS46, PS48
ANSI/CSA	CHMC1, CHMC2, FC1, FC3, FC5, FC6, HGV2, HGV3.1, HGV4.1-4.10
CSA	HGV19880-3: Gaseous H2 fueling
	HPRD1
	H2 powered industrial forklifts: HPIT1, HPIT2
	Disposal of H2 fuel containers: SPE 2.3.1
NFPA	2, 55, 70 Article 692, 110 Appendix A-3-1.4, 853, 855
SAE AIR 6464	H2 Fuel Cell Aircraft Fuel Cell Safety
SAE AS 6865	Installation of Fuel Cell Systems in Large Civil Aircraft
SAE	Terminology: J2574, J2760
	RP for measuring emissions and fuel consumption of gaseous H2 vehicles: J2572
	RP for measuring fuel consumption of hybrid heavy-duty gaseous H2 vehicles: J2601
	Fueling protocols and connection devices:
	J2601, J2601/2-/4, J2719, J2719/1, J2799, J3219
	Performance Test Procedures: J2615-J2617
	Recycling: J2594
	Safety: J2578, 2579, 2760, 2990/1, 3089
State Standards	
California	California Fuel Standard: SAE J2719:201109
	H2 Station Permitting Guidebook
Michigan	Handling gaseous and liquefied H2 systems: R 29 7001 - R 29 7126
South Carolina	Hydrogen Permitting Act for H2 and Fuel Cells: H3835

Continued on next page

Table 7 – *Continued from previous page*

Code/Standard	Description
U.S. Federal Standards	
Dept of Commerce	NIST H2 Gas measuring devices
	NIST Regulations — Retail Sales of H2 Fuel Standard Specification
	NIST Regulations — Retail Sales of H2 Fuel
Dept of Energy	H2 and Fuel Cells Permitting Guide: Module 1, Module 2
	H2 Fuel Quality Specifications for PEMFC in Road Vehicles
Dept of Labor	Hydrogen safety: OSHA 29 CFR1910.103
Dept of Transportation	Transporting Micro Fuel Cells on Passenger Aircraft

Table 7: U.S. codes and standards for H2 and Fuel Cell systems.
WK=work item under development. RP=recommended practice.

There are many hydrogen standards outside the U.S.; national standards by Japan (Japanese Standards Association, JIS), China (Guobiao Standards, GB for mandatory and GB/T for recommended), Korea (KS, adapted from French IEC), Taiwan (CNS, based on ISO), and Australia (AS ISO, based on ISO); European standards by the European Community Directive (EC), Economic Commission for Europe for the United Nations (UN/ECE), CEN-CENELEC, European Industrial Gases Association (EIGA), the European Organization for Civil Aviation Equipment (EUROCEA), and the Det Norske Veritas (DNV) (formerly Germanischer Lloyd and Det Norske Veritas); and international standards by the International Standards Organization (ISO), International Electrotechnical Commission (IEC, encompassing the Austrian, British, and German standards), International Maritime Organization (IMO), International Organization for Legal Metrology (IOML), and the United Nations Work Party 29 on Global Regulations on Pollution and Environment Global Technical Regulations (GTR) of hydrogen vehicles.

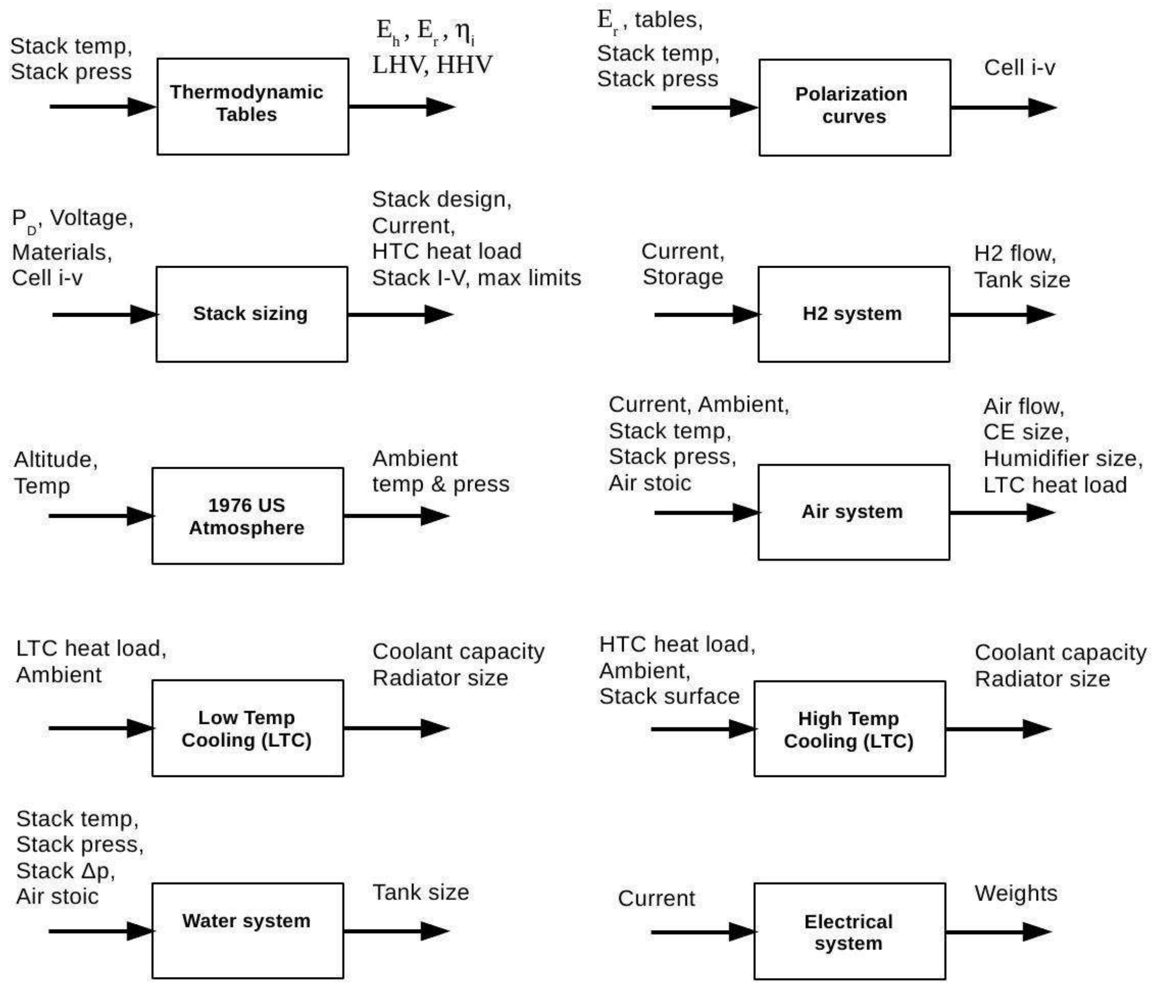
The American Institute of Aeronautics and Astronautics (AIAA) has a generic guidance document on hydrogen safety, ANSI/AIAA G095A-2017.

In summary, about 12,000 kg/day of retail hydrogen is available for 10,000 fuel cell cars at 49 stations in the U.S.. All but one are in the state of California, concentrated in its northern and southern regions. This amount is less than 0.05% of the annual hydrogen production in the U.S.. To conform to existing infrastructure and standards of re-fueling, tanks must be sized for gaseous hydrogen at 700 bar 15-20°C.

4 Design Method

Design sizes the fuel stack first, followed by the hydrogen system, air system, cooling systems, water system, and the electrical system. Sizing means predicting weights and volumes to meet certain specified requirements. The requirements are to deliver a specified power P_D (design power) at minimum weight or at a specified stack efficiency. An outline of the method is given here. The details of each step are given in the Theory section.

Figure 9 shows the functional blocks of design. The same blocks are also used for analysis.



Stack temp, Stack press
Thermodynamic Tables
E_h, E_r, η_i LHV, HHV
E_r, tables, Stack temp, Stack press
Polarization curves
Cell i-v
P_D, Voltage, Materials, Cell i-v
Stack sizing
Stack design, Current, HTC heat load Stack I-V, max limits
Current, Storage
H2 system
H2 flow, Tank size
Altitude, Temp
1976 US Atmosphere
Ambient temp & press
Current, Ambient, Stack temp, Stack press, Air stoic
Air system
Air flow, CE size, Humidifier size, LTC heat load
LTC heat load, Ambient
Low Temp Cooling (LTC)
Coolant capacity Radiator size
HTC heat load, Ambient, Stack surface
High Temp Cooling (LTC)
Coolant capacity Radiator size
Stack temp, Stack press, Stack Δp, Air stoic
Water system
Tank size
Current
Electrical system
Weights

Figure 9: **Functional blocks of design and analysis.**

The reversible cell voltages E_h and E_r are calculated from the cell reaction: $H_2 + (1/2)\ O_2 \rightarrow H_2O$ at a specified stack temperature T_S and pressure p_S. E_h represents the total energy released by the reaction. E_r is a smaller amount that is actually converted to voltage. The ideal thermodynamic efficiency is: $\eta_r = E_r/E_h$. The energies are higher when the product water is in liquid form (higher heating value (HHV)) than in vapor form (lower heating value (LHV)). In vapor form, the latent heat of vaporization is not released.

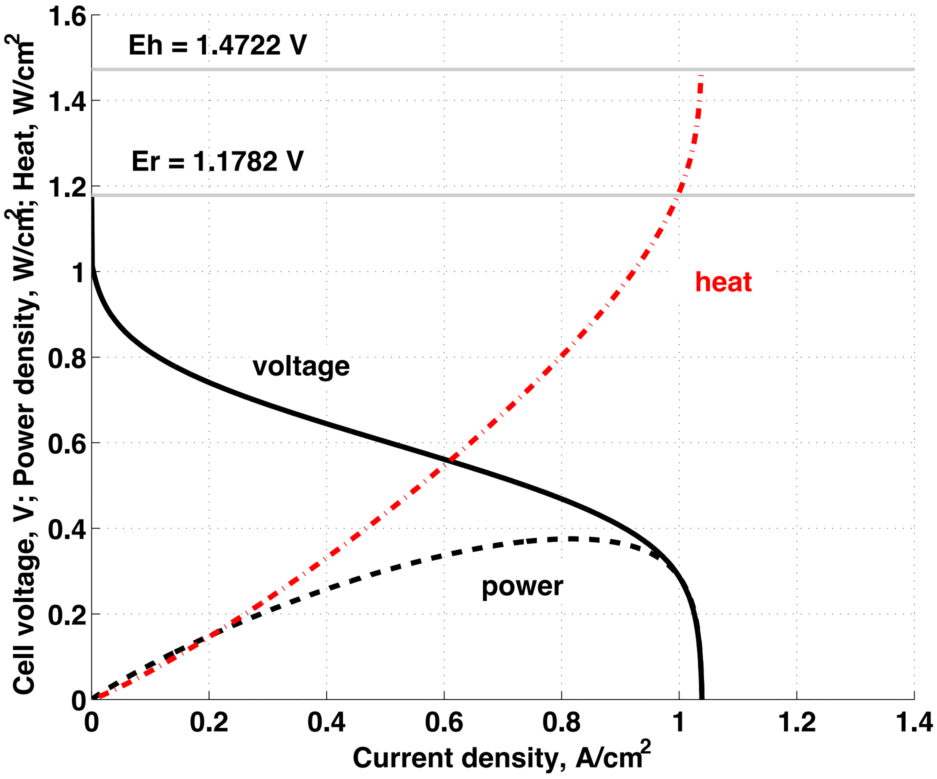


Figure 10: **Typical characteristics of a PEM fuel cell. Voltage, power, and heating plotted versus current density ($i-v$, $i-p$, $i-q$). E_h and E_r correspond to higher heating values for cell reaction at 2.5 atm air pressure and 80°C temperature.**

E_r is the open circuit voltage. As current is drawn, the cell voltage drops.

The variation of cell voltage with current is the $i-v$ curve. It is the characteristic curve of a fuel cell. Figure 10 shows a typical curve. The current is normalized by the area of the membrane so it is current density (typical unit is Ampere/cm^2). The corresponding power-current characteristic curve is $i-p$ where $p = i\,v$ is the power density (Watt/cm^2). The corresponding heat-current characteristic curve is $i-q$ where $q = i\,(E_h - v)$ is the heat released (Watt/cm^2). In the present model, the $i-v$ curve is constructed from test data (or predictions from higher-fidelity analysis) as a model fit using underlying thermodynamic constants. The variation of the constants with stack temperature, pressure, and cathode and anode relative humidity can be specified. The thermodynamic nature of the underlying constants allow sensible technology exploration.

At any point on the curve, the practical thermodynamic efficiency of the cell is $\eta = v/E_h$. The higher heating value of E_h gives the proper efficiency, even when the product water is in vapor form, as it is the true fraction of chemical energy extracted from hydrogen. The lower heating value of E_h skews the efficiency to a higher value. This might be appropriate when comparing efficiencies with combustion engines where the product water is always in vapor form.

The $i-v$ curve is the basis for design. The design decision is to select a point on the $i-v$ curve.

Any point on the curve can be selected and the stack sized to deliver a specified power at a specified voltage. If the specified power is P, and the selected power density is p, then the membrane area must be P/p since p is the power produced per unit area. The volume and weight will be proportional to this area. Any specified voltage V can be supplied by splitting the membrane area into n_c pieces (cells) and arranging them in parallel (stack) and make the cell voltages add up to $V = n_c\, v$. Thus, stack voltage V only determines the number of cells in a stack: $n_c = V/v$ (rounded to integer), not the weight. The membrane area of each cell is called the active area: $A_c = P/(n_c\, p)$. Higher the voltage larger the number of cells and smaller the area. Thus, voltage determines the aspect ratio of the stack. Substituting $p = i\, v$ gives $A_c = P/(n_c\, v\, i) = P/(V\, i)$ and since P/V is the stack current I, the active area $A_c = I/i$. Because the cells are in series the same current $I = i\, A_c$ flows through all cells. The characteristic curve $i - p$ has a peak power p_{max} ($\approx 0.4 - 0.8$ Watt/cm^2 depending on technology level). Selecting this point produces a minimum weight stack. But this point occurs at a high current density ($\approx 0.6 - 1.2$ Ampere/cm^2 depending on technology level) where the cell voltage is very low ($\approx 0.3 - 0.5$ V) and hence cell efficiency v/E_h is very low. Low efficiency means more hydrogen to deliver the same power. Therefore the criteria for selecting a point on the $i - v$ curve is not the stack weight alone but also the weight of hydrogen needed to accomplish the mission. This is where mission enters the calculation and the weight of hydrogen storage makes this entry quite dramatic.

The following targets can be specified for selecting the cell design point:

1) minimize stack weight (selects point of peak power p_{max}),

2) specify cell efficiency η (cell voltage $v = \eta\, E_h$ is a fall out), or

3) specify cell voltage v (cell efficiency $\eta = v/E_h$ is a fall out).

Let the selected cell design point be: i_c, v_c and p_c, where $p_c = i_c\, v_c$. The number of cells n_c, the cell active area A_c, and the cell efficiency η_D for a specified design power P_D and voltage V_D (or equivalently specified current $I_D = P_D/V_D$) are then the following.

$$n_c = \frac{V_D}{v_c} \qquad A_c = \frac{P_D}{n_c\, p_c} = \frac{I_D}{i_c} \qquad \eta_D = \frac{v_c}{E_h} \tag{1}$$

The rest of the design falls out of these key variables.

The size of the stack: weight, volume, and aspect ratio are estimated from A_c and n_c.

Once sized, the current, voltage, power, and efficiency at any current density i are the following.

$$I = i\, A_c \qquad V = n_c\, v \qquad P = V\, I = A_c\, n_c\, p \qquad \eta = \frac{v}{E_h} \tag{2}$$

These are the stack $I - V$ and $I - P$ characteristics. The maximum power available is $P_{\text{max}} = A_c\, n_c\, p_{\text{max}}$ where p_{max} is the maximum cell power density obtained at i_{pmax}. Note that i_{pmax} is not the maximum cell current density i_{max}. The current at maximum power is $I_{\text{pmax}} = i_{\text{pmax}}\, A_c$. So at maximum power the following relations hold.

$$\begin{aligned} I_{\text{pmax}} &= i_{\text{pmax}}\, A_c \\ V_{\text{pmax}} &= n_c\, v_{\text{pmax}} \\ P_{\text{max}} &= V_{\text{pmax}}\, I_{\text{pmax}} = A_c\, n_c\, p_{\text{max}} \\ \eta|_{\text{pmax}} &= v_{\text{pmax}}/E_h \end{aligned} \tag{3}$$

The hydrogen (fuel) mass flow rate $\dot{w}_H$ (kg/s) is calculated from the current I_D for design flow rate and current I_{pmax} for maximum power flow rate. The weight of hydrogen carried on-board can be specified and the time of operation calculated, or, the time of operation specified and weight of hydrogen calculated. The tank weight is calculated from a specified tank weight fraction (weight of hydrogen ÷ weight of hydrogen plus weight of tank). The tank volume is calculated from density of hydrogen and a specified volume fraction (volume of hydrogen ÷ volume of tank). The density depends on specified full-tank pressure and temperature if stored as gas.

The oxygen mass flow rate $\dot{w}_O$ (kg/s) is calculated similarly. The air mass flow rate is obtained from the mole fraction of oxygen in air $x_O = 0.2095$. The mass flow rate supplied $\dot{w}_{\text{Air in}}$ is higher by a specified factor ($\approx 1.5 - 2.5$). The air mass flow rate at the outlet $\dot{w}_{\text{Air out}}$ is the supply minus oxygen. The air supply must be pressurized to the stack pressure p_S. A blower can be used up to ≈ 30 kPa, a pump up to ≈ 100 kPa, but for higher pressures a compressor is needed. Compressed air can reach high temperatures, and if the temperature rises above the stack temperature T_S the air has to be cooled. This is the task of the low temperature cooling system (a low temperature coolant circulates in a loop). For pressurized stacks, the air at the outlet is at a high pressure, and some power can be recovered from the air with a turbine. At automobile power levels ($\approx$ 100 kW) axial-flow turbines called expanders are used, and the compressor-expander module (CEM) forms a single unit.

The water generation rate $\dot{w}_W$ is the sum of hydrogen and oxygen rates. It can be in a combined vapor and liquid form. The vapor and liquid fractions are calculated based on the stack pressure drop Δp_S and the saturated vapor pressure of water at the stack output temperature. The stack pressure drop is specified (typically $\approx$ 30 kPa). It is assumed the stack output temperature is maintained at the stack temperature T_S by the stack cooling system. The saturated vapor pressure of water is then a function of the stack temperature. The stack exit pressure $p_S - \Delta p_S$ is the saturated vapor pressure plus the pressure of dry air. So the pressure of dry air can be calculated by subtracting the saturated vapor pressure. The humidity ratio h is calculated from the saturated vapor pressure and the pressure of dry air. The mass flow rate of vapor is the humidity ratio times the mass flow rate of air at the outlet: $\dot{w}_{W\ \text{vap}} = h\,\dot{w}_{\text{Air out}}$. If the vapor rate is greater than water generation rate it means a humidifier will be needed or the cathode will dry up. If this condition occurs, the vapor rate is made equal to the water generation rate. If the vapor rate is lower than water generation rate, then an external humidifier will not be needed. The water generation minus the vapor rate is the liquid water rate. A specified fraction of the liquid water can be stored. This simplified model is adequate for basic sizing.

The stack heating at design power is found from the voltage loss in each cell, number of cells, and the current: $Q = (E_h - v_c)\,n_c\,A_c\,i_c = (1/\eta_D - 1)\,P_D$. The option to use either the higher or lower value of E_h is available for when the product water is entirely in vapor form and the latent heat of vaporization is not released. Stack cooling is the task of the high temperature cooling system (a high temperature coolant circulates in a loop). The main difference from the low temperature coolant loop is a larger radiator operating at a higher temperature.

The fuel, air, heating, water, and electrical subsystems have additional weight fraction allocations for regulators, filters, plumbing, and power. Regulators cover mechanical valves, ejectors, and mixers to electronic switches, diodes and converters. Filters covers demisters and separators. Plumbing covers mechanical pipes, connectors, cooling lines and electrical wires. Power covers pumps, fans, motors, electronic controllers and batteries wherever needed.

The power to operate the subsystems is the balance of plant P_{BOP}. The net useful power is reported in kW-electrical or kWe.

$$P_{De} = P_D - P_{BOP} \tag{4}$$

The system efficiency is

$$\eta_D = \frac{v}{E_h}\,\frac{P_{De}}{P_D} \tag{5}$$

The fraction $P_{De}/P_D \approx 0.86 - 0.90$ for a modern system of 80 kWe.

The specific power of the stack is P_D/W (kW/kg) where W is the weight of the stack, the specific power of the stack system is P_{De}/W (kWe/kg) where W is the weight of the entire stack system including all ancillary subsystems but without the fuel and tank, and the specific fuel consumption (SFC) is $\dot{w}_H/P_{De}$ (kg/kWe-hr).

5 Analysis Method

Analysis predicts the performance of a PEMFC after it is designed for a set of conditions. Performance covers electrical outputs, efficiency, balance of plant, heating and the net useful power. An outline of the method is given here. The details of each step are given in the Theory section.

The same functional blocks in Fig 9 are used, only in a different sequence.

The stack current I is specified. The current is assumed steady. Transients are important for detailed design of the stack and power system, so their neglect is a simplifying assumption.

Under design conditions, the performance follows Eq 2. Under off-design conditions, the characteristic curve $i - v$ must be re-calculated. So the airflow is calculated first, which is the only change from the design method. The hydrogen (fuel), oxygen, and air mass flow rates are calculated from the specified current. The compressor power is calculated from the air mass flow rate and the ambient pressure. If it exceeds the maximum compressor power then the stack pressure cannot be maintained. The new stack pressure is calculated assuming the compressor is at maximum power. The result is detrimental, either due to rise in compressor power or deterioration of the cell voltage. This loss in power is the principal altitude effect in the model.

The stack temperature is maintained, the heating is adjusted accordingly. If the ambient temperature is below the design value, the compressor output temperature is closer to the stack temperature, which means the inlet cooling (low temperature cooling system) takes less power. If there is no compressor, the inlet air must be heated instead, but this heat is taken from the stack, so the stack cooling (high temperature cooling system) takes less power. The result is beneficial, due to ease of cooling.

The anode and cathode relative humidity are specified.

Once the new $i - v$ is known, the remaining steps are identical to the design method.

6 Theory

This section describes the steps outlined in the design and analysis methods. It covers E_h and E_r, the $i-v$ curve, hydrogen flow, hydrogen energy, and air flow.

6.1 E_h and E_r

The enthalpy h and entropy s of hydrogen, oxygen, and water are calculated at the stack temperature and standard pressure (1 bar) using the Shomate equations (Shomate, 1954)

$$\begin{aligned} h =& At + Bt^2/2 + Ct^3/3 + Dt^4/4 - E/t + F - H \\ s =& A\ln(t) + Bt + Ct^2/2 + Dt^3/3 - E/(2t^2) + G \end{aligned} \tag{6}$$

where $t = T/1000$, T is the stack temperature in Kelvin (K), and $A-H$ are constants obtained from the National Institute of Standards and Technology (NIST) Standard Reference Database (SRD) 69 (Chase 1998). The equations give enthalpy in kJ/mol and entropy in J/mol-K.

For the fuel cell reaction: H_2 + (1/2) $O_2 \rightarrow H_2O$ (liquid), the heat of reaction is the difference in enthalpy of formation of the products and reactants: $\Delta h = h_{W\text{liq}} - h_H - (1/2)h_O$. The equation is exothermic so Δh is negative. The ideal reversible cell voltage is calculated by equating its magnitude to electrical energy (following Nernst): $-\Delta h = q\,E_h$ where E_h is the ideal reversible cell voltage, q is the charge transferred by all the electrons released by a mole of hydrogen, and the negative sign gives the magnitude of Δh. Two electrons ($N = 2$) are released by a molecule of hydrogen from the anode half reaction (oxidation: $H_2 \rightarrow 2\ H^+ + 2\ e^-$) and one mole contains $N_A = 6.02214076 \times 10^{23}$ molecules (Avogadro's number), thus $N\,N_A$ electrons are released by a mole of hydrogen. The charge of one electron is $q_e = 1.6022 \times 10^{-19}$ Coulomb. Thus $q = N\,N_A\,q_e$. The product $N_A\,q_e$ is the charge of one mole of electrons called Faraday's constant F. $F = 96485$ Coulomb/mole. Thus $q = N\,F$, and the ideal reversible cell voltage is

$$E_h = -\frac{\Delta h}{N\,F} \tag{7}$$

In this equation, Δh is in Joules/mole (note: the Shomate equation gives h in kJ/mole so it should be multiplied with 1000 for consistent unit), $N = 2$, and $F = 96485$ Coulomb/mole.

For example, at STP, $T = 298.15$ K (25° C), $h_H = 0$, $h_O = 0$, $h_{W\text{liq}} = 286$ J/mole (water in liquid form) or $h_{W\text{vap}} = 242$ kJ/mole (water in vapor form). So $\Delta h = -286$ kJ/mole and $E_h = 1.48$ V (liquid) or $\Delta h = -242$ kJ/mole and $E_h = 1.25$ V (vapor). The enthalpy changes are called the higher heating value (HHV) and lower heating value (LHV) of hydrogen; loosely, the voltages $E_h = 1.48$ V and 1.25 V are called the HHV and LHV of hydrogen.

When the product water is in liquid form, more heat is released, hence the heat of reaction is called the higher heating value (HHV) or gross calorific value (GCV). When the product water is in the vapor form, the latent heat of vaporization is lost, so less heat is released and hence the heat of reaction is called the lower heating value (LHV) or the net calorific value (NCV).

$$\begin{aligned} &H_2 + (1/2)O_2 \rightarrow H_2O \text{ liquid} + 286 \text{ kJ/mole} \\ &H_2 + (1/2)O_2 \rightarrow H_2O \text{ vapor} + 242 \text{ kJ/mole} \end{aligned} \tag{8}$$

The higher heating value (HHV) is the thermodynamically correct heating value as it accounts for all the energy that is contained in the fuel. It is the heating value used throughout this report.

The heating value changes with temperature. For example, at $T = 353.15$ K (80° C), the HHV $\Delta h = -284$ kJ/mole and $E_h = 1.472$ V.

The heat of reaction is the internal energy contained in the fuel. This heat cannot be entirely converted to useful work. There are irreversible losses due to entropy. The part that can be converted is the Gibbs free energy. The change in entropy for the cell reaction: H_2 + (1/2) $O_2 \rightarrow H_2O$ (liquid) is $\Delta s = s_{W\text{liq}} - s_H - (1/2)s_O$. The Gibbs free energy is then $\Delta g = \Delta h - T\,\Delta s$. At a non-standard pressure P and impure reactants (air instead of oxygen) the expression is modified by the concentration and stoichiometric coefficients of the species.

$$\Delta g = \Delta h - T\,\Delta s + R\,T\,\ln\frac{p_W}{p_H\,p_O^S} \tag{9}$$

The logarithmic term is non-dimensional, with the numerator and denominator representing activities of the product and reactant species. The activities are partial pressures in atmosphere (without units) raised to a power of their stoichiometry. Thus $p_W = 1$, $p_H = 1$, and $p_O = x_O \times P$ where $x_O = 0.2095$ is the mole fraction of oxygen in air and P is the stack pressure (pressure of air supply to cathode) in atmosphere. $S = 1/2$ is the number of molecules of oxygen per molecule of fuel from the cathode half reaction (reduction: (1/2) O_2 + 2 e^- + 2 $H^+ \rightarrow H_2$ O). $R = 8.3144621$ J/mol-K is the universal gas constant. T is temperature in K. Converting the Gibbs free energy to electrical energy (following Nernst) produces the true reversible cell voltage

$$E_r = -\frac{\Delta g}{N\,F} \tag{10}$$

This is the Nernst equation. E_r is also called the Nernst potential.

The ideal thermodynamic efficiency is

$$\eta_i = \frac{\Delta g}{\Delta h} = \frac{E_r}{E_h} \tag{11}$$

where Δh and E_h correspond to the HHV of hydrogen and equal 286 kJ/mole and 1.48 V respectively.

For example, for $T = 298.15$ K (25° C), $P = 1$ atm, and $x_O = 1$ (pure oxygen), $\Delta g = -237,141$ J/mole and $E_r = 1.229$ V. The ideal thermodynamic efficiency $\eta_i = 1.229/1.48 = 0.83$. For $T = 353.15$ K (80° C), $P = 3$ atm, and $x_O = 0.2095$ (air), $\Delta g = -227,629$ J/mole and $E_r = 1.180$ V. The ideal thermodynamic efficiency $\eta_i = 1.180/1.472 = 0.80$.

6.2 The $i - v$ Curve

The operating cell voltage v is a function of the current density i (A/cm^2) and is equal to the reversible cell voltage or the Nernst potential E_r minus three losses: activation loss (due to reaction kinetics), ohmic loss (from ionic and electronic resistance), and concentration loss (due to mass transport). Normally, a leakage current i_{leak} is needed to fully model the behavior. The following model is used (O'Hayre, Cha, Colella, Prinz, 2009).

$$\begin{aligned} v = E_r - &\eta_{\text{act}} - \eta_{\text{ohmic}} - \eta_{\text{conc}} \\ &\eta_{\text{act}} = \left[a_A + b_A \ln\left(i + i_{\text{leak}}\right)\right] + \left[a_C + b_C \ln\left(i + i_{\text{leak}}\right)\right] \\ &\eta_{\text{ohmic}} = i\ \text{ASR}_\Omega \\ &\eta_{\text{conc}} = C\ \ln\frac{i_L}{i_L - \left(i + i_{\text{leak}}\right)} \end{aligned} \tag{12}$$

The true efficiency is defined as

$$\eta = \frac{v}{E_h} = \eta_i \, \frac{v}{E_r} = \eta_i \, \eta_r \tag{13}$$

where the voltage efficiency η_r is defined as

$$\eta_r = \frac{v}{E_r} \tag{14}$$

The true efficiency is the fraction of fuel energy converted to voltage whereas the voltage efficiency is the fraction of maximum possible voltage.

The total current produced at the electrodes is $i + i_{\text{leak}}$, of which i_{leak} is lost in side reactions and i is the useful current that flows through the cell. The leakage current i_{leak} reduces E_r to the measured open circuit voltage $v(0)$

$$v(0) = E_r - (a_A + b_A \ln i_{\text{leak}}) - (a_C + b_C \ln i_{\text{leak}}) - C \ln \frac{i_L}{i_L - i_{\text{leak}}} \tag{15}$$

Typically $i_{\text{leak}} \approx 0.01 - 0.3$ A/cm^2. i_L is the limiting current, the maximum current beyond which reactant concentration would fall to zero. Thus $i + i_{\text{leak}} < i_L$ which gives the maximum current for which the model is valid: $i < i_L - i_{\text{leak}}$. Beyond this current the concentration loss is undefined. The concentration loss is significant only when i approaches $i_L - i_{\text{leak}}$ when it rises abruptly. Under nominal operations, this loss is insignificant. Typically $i_L \approx 1 - 2$ A/cm^2.

The true and voltage efficiencies including the leakage current are the following.

$$\eta = \frac{v}{E_h} \, \frac{i}{i + i_{\text{leak}}} \qquad\qquad \eta_r = \frac{v}{E_r} \, \frac{i}{i + i_{\text{leak}}} \tag{16}$$

They are zero when the current is zero, but undefined if the leakage current is also zero. The correction is significant only at low currents.

The ohmic loss is proportional to i (not $i + i_{\text{leak}}$) as it is the current that flows through the cell.

The activation loss represent kinetic losses at the electrodes and the two constants a and b are related to two other constants i_0 and α that are more fundamental.

$$a_A = -\frac{R\,T}{\alpha_A \, n_A \, F} \ln i_{0A} \qquad\qquad b_A = \frac{R\,T}{\alpha_A \, n_A \, F} \tag{17}$$

for the anode and

$$a_C = -\frac{R\,T}{\alpha_C \, n_C \, F} \ln i_{0C} \qquad\qquad b_C = \frac{R\,T}{\alpha_C \, n_C \, F} \tag{18}$$

for the cathode. This form is the Tafel approximation of the Butler-Volmer equation and is valid only for $i >> i_0$. However it is a good approximation for PEMFC and adequate for its normal operating currents.

There are eight fitting constants in total: $\alpha_A, i_{0A}, \alpha_C, i_{0C}$ (unit-less), C (volt), ASR_Ω (area specific resistance in Ohm.cm^2), i_L (limiting current, A/cm^2), and i_{leak} (leakage current in A/cm^2). The temperature T is the stack temperature (in K), $R = 8.3144621$ J/mol.K, $n_A = 2$ is the number of moles of electron released at the anode per mole of hydrogen, $n_C = 4$ is the number of moles of electron captured at the cathode per mole of oxygen, and $F = 96485$ Faraday's constant in Coulombs/mole.

The constants are obtained by curve fitting through measured $i - v$ data (or perhaps predictions from a higher-fidelity analysis). The $i - v$ data is sensitive to pressure, temperature, and anode and cathode relative humidity. The effect of pressure is important and the most significant on α_C, ASR$_\Omega$, and i_L. These are modeled by table look-up.

The effect of temperature and relative humidity can be significant on ASR_Ω and i_L. These are modeled as polynomial fits.Temperature corrections relative to the reference values are introduced in the form

$$\Delta = k_1 \, t + k_2 \, t^2 \tag{19}$$

where $t = T_S - T_{ref}$ is the stack temperature deviation from a reference value and k_1 and k_2 are obtained from test data. The reference temperature and the constants k can be different for ΔASR_Ω and Δi_L.

Anode relative humidity arh and cathode relative humidity crh corrections relative to the reference values are introduced in the form

$$\begin{aligned}\Delta =& k_{1a}\, a \,+\, k_{2a}\, a^2 + \\ & k_{1a1c}\, a\, c \,+\, k_{2a1c}\, a^2\, c \,+\, k_{2a2c}\, a^2\, c^2 \,+\, k_{1a2c}\, a\, c^2 + \\ & k_{2c}\, c^2 \,+\, k_{1c}\, c\end{aligned} \tag{20}$$

where $a = arh - arh_{ref}$ and $c = crh - crh_{ref}$ are deviations from the reference values (arh and crh are in percentages) and the constants k are obtained from test data. The reference humidity and the constants k are different for ΔASR_Ω and Δi_L.

The characteristic curves for a typical fuel cell are shown in Figs 11 and 12 (data from Yan, Toghiani and Wu 2006, and Yan, Toghiani and Causey 2006). The cell test conditions were 80° C, 100% anode and cathode relative humidity, H2/Air stoichiometry of 1.2/2, 25 cm^2 area, 1 mg/cm^2 Pt loading. The cell pressure was varied. Model parameters for this cell are given in Table 8.

The characteristic curves for an advanced automotive fuel cell are shown in Figs 13 and 14 (data from Ahluwalia, Wang and Steinbach 2016). The cell test conditions were 80° C, 100% anode and cathode relative humidity, H2/Air stoichiometry of 2/2, 50 cm^2 area, 0.05-0.15 mg/cm^2 Pt loading. The cell pressure was varied. Model parameters for this cell are given in Table 9. Based on the parameters, the activation constant α_C is higher implying lesser kinematic loss at the cathode (Eq 18). The limiting current i_L is also significantly higher. These produce significantly higher voltages and power densities. They are now equal or nearly equal to values possible with pure-oxygen fuel cells.

Pressure	1 atm	2 atm	3 atm	4 atm
E_r, V	1.1713	1.1765	1.1797	1.1819
i_{0C}	0.0001	0.0001	0.0001	0.0001
α_C	0.15	0.155	0.155	0.16
i_{0A}	0.1	0.1	0.1	0.1
α_A	0.5	0.5	0.5	0.5
ASR$_\Omega$, Ω cm^2	0.12	0.115	0.1	0.06
i_L, A/cm^2	0.73	1.0	1.1	1.23
i_{leak}, A/cm^2	0.01	0.01	0.01	0.01
C, V	0.08	0.08	0.08	0.12

Table 8: **Model parameters for a baseline cell; 80°C.**

There is very limited data below stack pressure of 1 atmosphere. Measurements on a 12 kW fuel cell at 750 bar (0.74 atm; equivalent to 9,500-ft ISA/SL altitude) reported a drop of about 5.3% in gross power and efficiency (Werner et al 2015) as long as humidity and stoichiometry were well controlled. This amounts to a correction of about 0.56% per 1000 ft, close to the 0.5% assumed earlier in Datta and Johnson 2014.

6.3 H$_2$ flow

Hydrogen flows into the anode. The mass flow needed is obtained from the anode half reaction.

$$H_2 \rightarrow 2H^+ + 2e^- \qquad \text{Anode, release of electrons, oxidation} \tag{21}$$

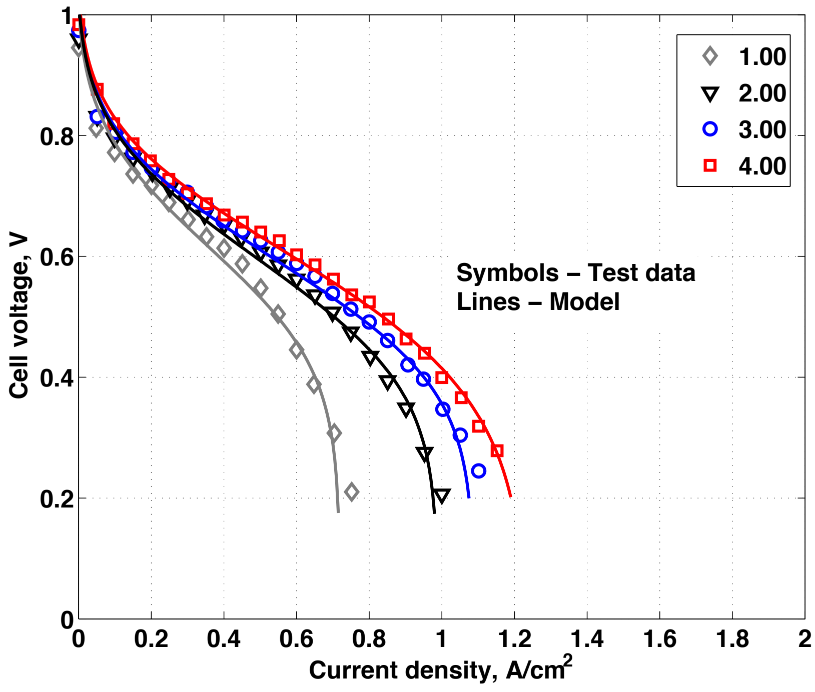


Figure 11: **Current density-voltage (*i*-*v*) characteristics of a baseline PEM FC at various stack pressures in atm; 80°C.**

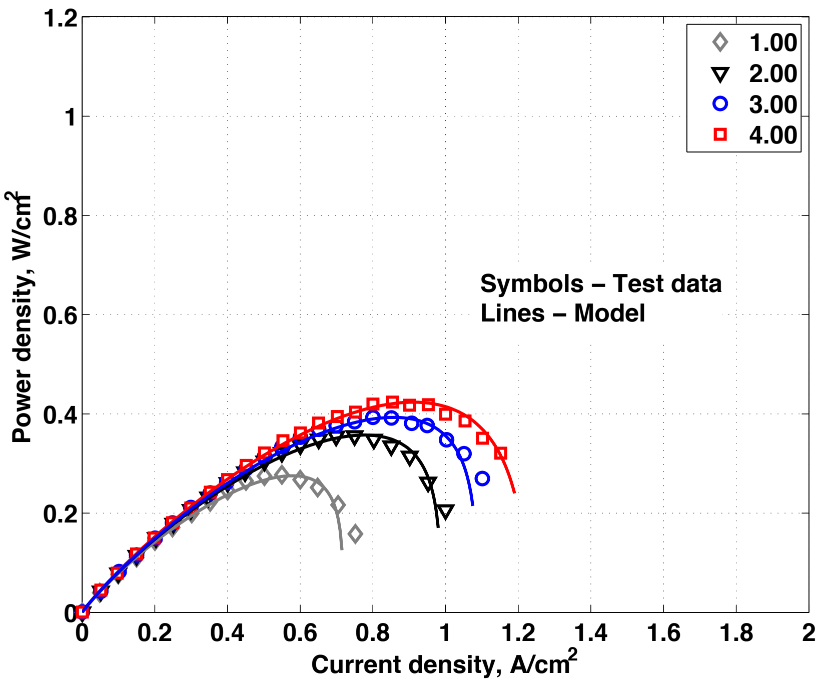


Figure 12: **Power density-voltage (*i*-*p*) characteristics of a baseline PEM FC at various stack pressures in atm; 80°C.**

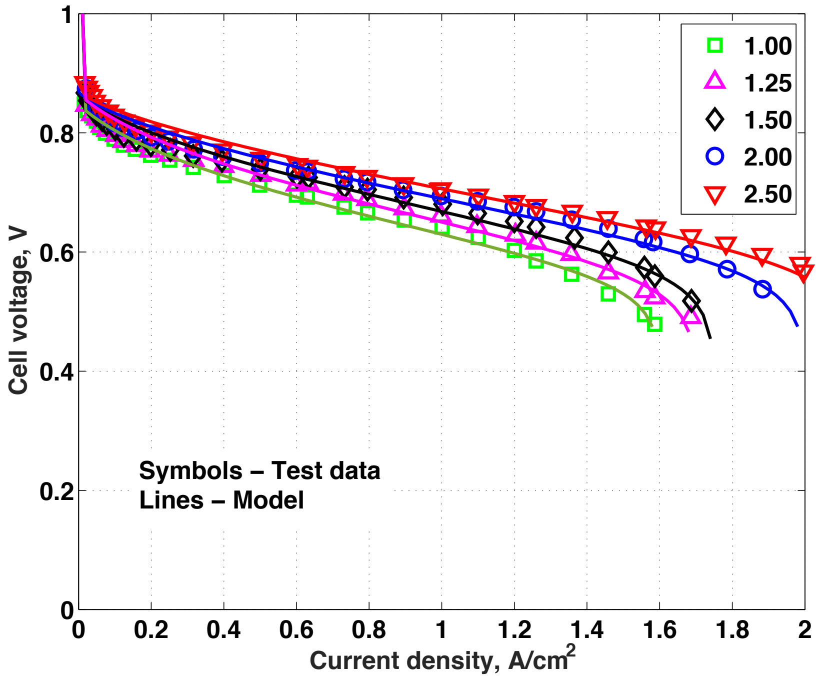


Figure 13: **Current density-voltage (i-v) characteristics of a modern PEM FC at various stack pressures in atm; 80°C.**

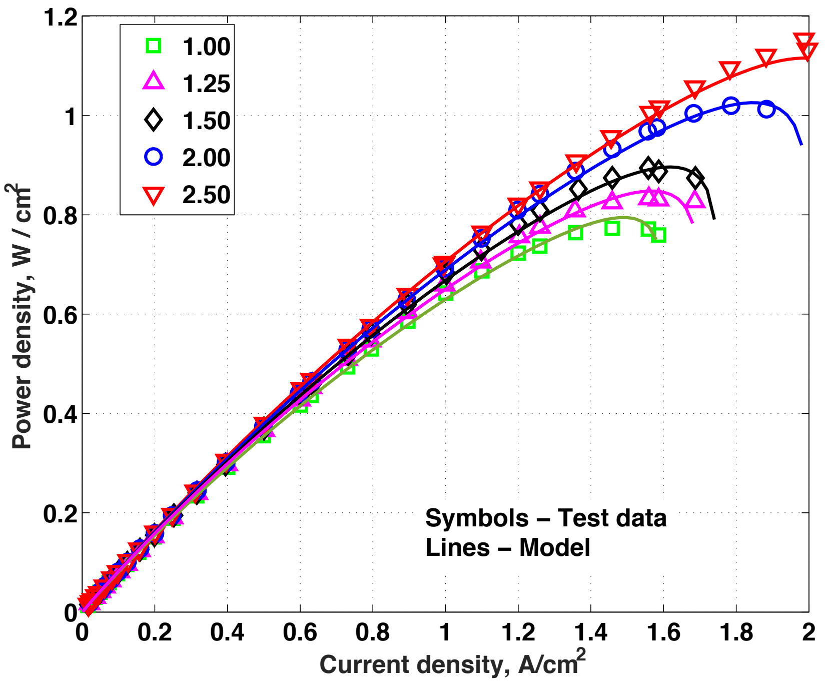


Figure 14: **Power density-voltage (i-p) characteristics of a modern PEM FC at various stack pressures in atm; 80°C.**

Pressure	1 atm	1.25 atm	1.5 atm	2 atm	2.5 atm
E_r, V	1.1713	1.1729	1.1743	1.1765	1.1782
i_{0C}	0.0001	0.0001	0.0001	0.0001	0.0001
α_C	0.18	0.19	0.20	0.21	0.22
i_{0A}	0.1	0.1	0.1	0.1	0.1
α_A	0.5	0.5	0.5	0.5	0.5
ASR$_\Omega$, Ω cm^2	0.04	0.04	0.04	0.04	0.04
i_L, A/cm^2	1.75	1.85	1.95	2.25	2.45
i_{leak}, A/cm^2	0.15	0.15	0.20	0.25	0.30
C, V	0.03	0.035	0.035	0.35	0.35

Table 9: **Model parameters for a modern cell; temperature 80°C.**

Let S_H moles of hydrogen produce N moles of electrons. Here $S_H = 1$ and $N = 2$.

Since a mole of electrons carry a charge of F Coulomb ($F = 96485$ Coulomb/mole is the Faraday's constant), N moles of electrons carry NF Coulomb. So S_H moles of hydrogen produce NF Coulomb. If one mole of hydrogen weigh m_H kg, S_H moles of hydrogen weigh $S_H m_H$ kg. So $S_H m_H$ kg of hydrogen produce NF Coulomb. Or 1 Coulomb is produced by $S_H m_H/NF$ kg of hydrogen. So I Ampere (Coulomb/s) is produced by $IS_H m_H/NF$ kg/s of hydrogen. The current I must be produced in every cell so the total consumption is found by multiplying the number of cells n_c.

Thus, the mass flow rate of hydrogen is

$$\dot{w}_H = S_H \frac{m_H}{N\,F} I\, n_c \tag{22}$$

where $\dot{w}_H$ is the mass flow in kg/s, n_c is the number of cells, I is the current in Ampere, $N = 2$, $F = 96485$ Coulomb/mole, $m_H = 2.01588 \times 10^{-3}$ kg/mole is the molecular weight of hydrogen and $S_H = 1$.

If supply rate is greater than consumption, S_H is multiplied with a utilization factor f_H.

$$\dot{w}_{H\ \text{in}} = f_H\, S_H \frac{m_H}{N\,F} I\, n_c \tag{23}$$

where $f_H \approx 1$ is desired but can be up to 2.

From Eq (2), $I\, n_c = P\, n_c/V = P/v$, so it can also be written as

$$\dot{w}_{H\ \text{in}} = f_H\, S_H \frac{m_H}{N\,F} \frac{P}{v} \tag{24}$$

which shows it is proportional to power but varies inversely with cell voltage. A higher cell voltage gives lower fuel flow for the same power.

At the design point: $I = I_D$, $P = P_D$, $v = v_c$.

At maximum power: $I = I_{\text{pmax}}$, $P = P_{\text{max}}$, $v = v_{\text{pmax}}$.

6.4 H$_2$ Energy and Storage

Hydrogen is the first element in the periodic table (official NIST version, SP 966, August 2019). It has the minimum number of protons in its nucleus, 1, and has the lowest atomic weight 1.00794×10^{-3} kg/mole (the molecular weight m_H is twice this value).

Hydrogen makes up 73% of the mass of the visible universe (25% is helium and 2% everything else) but only 0.14% of Earth's crust. More than 99% of Earth's crust is made of oxygen 46.1%, silicon 28.2%, aluminum 8.23%, iron 5.63%, calcium 4.15%, sodium 2.36%, magnesium 2.33%, potassium 2.09%, and titanium 0.565% (CRC Handbook, Haynes 2016-2017). Hydrogen comes next at $0.14 - 0.15\%$. It is as abundant as carbon, which is placed between $0.02 - 0.15\%$ by various sources, and an order of magnitude more abundant than lithium $0.0017 - 0.002\%$.

The triple point of hydrogen, where three phases solid-liquid-gas are in thermodynamic equilibrium, occurs at a temperature of 13.96 K ($-259.19°$ C) and pressure of 0.0712 atm (0.0721 bar or 0.00721 MPa). The critical point, where two phases liquid-gas are in thermodynamic equilibrium, occurs at a temperature of 33.18 K ($-239.97°$ C) and pressure of 12.83 atm (13 bar or 1.3 MPa). The density of hydrogen at the critical point is only 31 kg/m^3 (0.031 kg/L). At a lower pressure of 1 atm (0.987 bar or 0.0987 MPa) hydrogen attains a fully liquid state at a temperature of 20.28 K ($-252.87°$ C), which is almost as low as the triple point. But the density is almost twice that of the density at critical point and equal to 70.99 kg/m^3 (0.07099 kg/L). This density is used for sizing liquid hydrogen tanks. The temperature is cryogenic, defined as < 93 K ($-180°$ C), but the pressure is near 1 atm.

For gaseous storage, the pressure must be very high. The density is calculated as a function of temperature and pressure using the compressibility equation from Lemmon, Huber, and Leachman 2008 (basis for SAE Standard J2572, Recommended Practice for measuring fuel consumption and range of fuel cell and hybrid fuel cell Vehicles fueled by compressed gaseous hydrogen). The compressibility factor Z varies with equilibrium temperature and pressure in the form of a truncated virial equation.

$$Z = 1 + \sum_{i=1}^{9} a_i \, p^{c_i} \left(\frac{100}{T}\right)^{b_i} \tag{25}$$

Here p is the pressure in MPa, T is temperature in K, and the constants are given in Table 10. The density is

$$\rho = m_H \, \frac{p \times 10^6}{Z\,R\,T} \tag{26}$$

where $m_H = 2.01588 \times 10^{-3}$ kg/mole is the molecular weight of hydrogen and the factor 10^6 converts the pressure p from MPa to Pa. The significance of the virial equation is that by measuring pressure and temperature inside the tank the amount of hydrogen can be inferred. Hydrogen deviates from ideal gas at low temperatures and at high pressures.

i	a_i	b_i	c_i
1	0.05888460	1.325	1.0
2	-0.06136111	1.87	1.0
3	-0.002650473	2.5	2.0
4	0.002731125	2.8	2.0
5	0.001802374	2.938	2.42
6	-0.001150707	3.14	2.63
7	0.9588528×10^{-4}	3.37	3.0
8	-0.1109040×10^{-6}	3.75	4.0
9	0.1264403×10^{-9}	4.0	5.0

Table 10: **Constants associated with the density equation of hydrogen.**

The calculated density of gaseous hydrogen for a range of temperatures and pressures are shown in Table 11. The density is very low under normal conditions ($\approx$ 0.08 kg/m^3), but at high pressures of $700 - 800$ bar and low temperatures near $-150°$ C, it can reach up to $67 - 70$ kg/m^3, close to the density of liquid hydrogen, 70.99 kg/m^3. The variation of density with temperature and pressure are shown in Fig 15 and Fig 16 respectively. These are obtained using Eq 26.

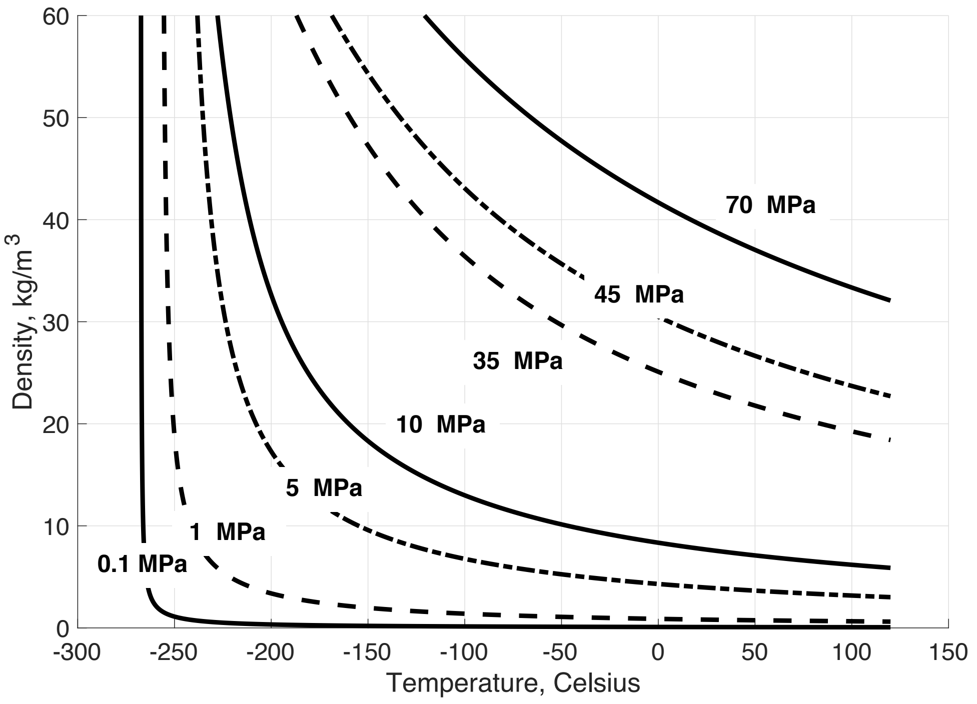


Figure 15: **Hydrogen density versus temperature.** 1 **MPa** = 10 **bars** = 9.87 **atm.**

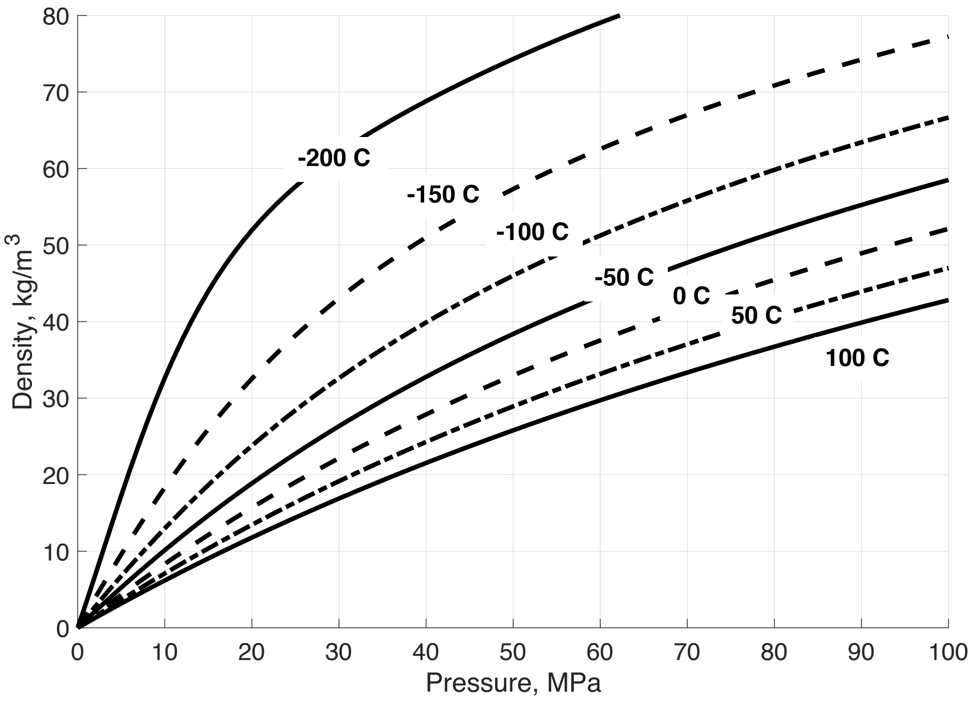


Figure 16: **Hydrogen density versus pressure.** 1 **MPa** = 10 **bars** = 9.87 **atm.**

T K	T $°C$	p MPa	p bar	Z	ρ kg/m^3
288.15	15	0.101325	1.01325	1.00060847	0.0852
298.15	25			1.00059670	0.0823
123.15	-150	35	350	1.45787964	47.27
173.15	-100			1.34577496	36.42
223.15	-50			1.28161475	29.67
273.15	0			1.23794514	25.10
288.15	15			1.22728706	23.99
293.15	20			1.22394613	23.65
123.15	-150	70	700	2.05722315	66.99
173.15	-100			1.75662614	55.80
223.15	-50			1.59397850	47.71
273.15	0			1.49044155	41.69
288.15	15			1.46616598	40.17
293.15	20			1.46608910	40.17
123.15	-150	80	800	2.22310657	70.84
173.15	-100			1.87250052	59.82
223.15	-50			1.68245728	51.66
273.15	0			1.56192187	45.46
288.15	15			1.53365948	43.88
293.15	20			1.52489131	43.38

Table 11: **Compressibility Z and density ρ of gaseous hydrogen.**

The energy stored is the higher heating value (HHV) of hydrogen. At STP, it is 286 kJ/mole from the change in enthalpy for the cell reaction: H_2 + (1/2) $O_2 \rightarrow H_2O$ (liquid) at $T = 298.15$ K where the product water is in liquid form. Converting mole to mass using the molecular weight of hydrogen $m_H = 2.01588 \times 10^{-3}$ kg/mole, gives 142 MJ/kg (or 39.4 kWh/kg). This is the specific energy (energy per unit mass of the fuel) at STP. The lower heating value (LHV) of hydrogen is 242 kJ/mole at STP from the change in enthalpy for the cell reaction: H_2 + (1/2) $O_2 \rightarrow H_2O$ (vapor) at $T = 298.15$ K where the product water is now in vapor form. Converting mole to mass gives 120 MJ/kg (or 33.3 kWh/kg). The energy available for voltage is lower, and is equal to the Gibbs free energy, which depends on additional conditions of the reaction namely, the pressure and mole fraction of oxygen. At pressure of $P = 1$ atm, and mole fraction $x_O = 1$ (pure oxygen), the HHV gives Gibbs free energy of $\Delta g = -237$ kJ/mole, which when converted to mass gives 117.64 MJ/kg (32.7 kWh/kg).

The heating values depend on temperature. At a typical stack temperature of $T = 353.15$ K (80° C), the HHV is 141 MJ/kg (39 kWh/kg). The corresponding available energy for voltage, the Gibbs free energy, at $P = 1$ atm and mole fraction $x_O = 1$ (pure oxygen), is 113.25 MJ/kg (31.5 kWh/kg). The available energy at $P = 3$ atm and mole fraction $x_O = 1$ (pure oxygen), is 114.06 MJ/kg (31.7 kWh/kg). The energy at $P = 3$ atm and mole fraction $x_O = 0.2095$ (air), is even lower, 112.92 MJ/kg (31.4 kWh/kg). The mole to mass conversion only uses the molecular weight of hydrogen even though oxygen is also a reactant. Thus, the assumption is only the fuel is carried, the oxygen is supplied from outside air.

The density ρ (mass per volume) multiplied by specific energy (energy per mass) gives energy density ED (energy per volume) of the fuel. The specific energy is the heating value Δh. Common units of ED are

MJ/L and kWh/gal.

$$\begin{aligned} ED \text{ in MJ/L } &= \frac{\rho}{1000} \times \Delta h \text{ in MJ/kg} \\ ED \text{ in kWh/gal } &= 1.0515 \times ED \text{ in MJ/L} \end{aligned} \tag{27}$$

where ρ is density in kg/m^3 and Δh is the heating value.

The specific energy of hydrogen at 80° C is the HHV 141 MJ/kg. The energy density depends on storage pressure and temperature. At pressures of 350, 700, and 800 bar and tank temperature of 15°C the energy densities would be 3.4, 5.7, and 6.2 MJ/L (or 3.6, 6.0, and 6.5 kWh/gal) (using Eq 27 and ρ from Table 11). The numbers can improve if low temperature storage becomes viable. At −150°C for example, the energy densities would be 6.7, 9.4, and 10.0 MJ/L (or 7.0, 9.9, and 10.5 kWh/gal). If stored as liquid hydrogen (−253°C), the energy density would be 10.0 MJ/L (10.5 kWh/gal).

The specific energy available for voltage, the Gibbs energy, is a lower 113.25 MJ/kg. The balance is lost as heat. Then at pressures of 350, 700, and 800 bar and temperature of 15°C, the available energy densities are only 2.7, 4.5, and 5.0 MJ/L (or 2.9, 4.8, and 5.2 kWh/gal). For liquid hydrogen, it would be 8.0 MJ/L or (8.5 kWh/gal). The ratio of ideal to available specific energy is included in the ideal thermodynamic efficiency of the stack η_i (Eq 11) and consequently in its operating thermodynamic efficiency η (Eq 16). Note that the stack efficiency and energy density should be quoted with consistent heating values. Here the HHV is used for both. Quoting stack efficiency with LHV (to show high efficiency) and energy density with HHV (to show high energy density) will produce inconsistent results.

Pressure bar	T °C	ρ_{h2} kg/m^3	KGE gal	GGE gal
350	15	23.99	10.7	9.4
700	15	40.17	6.4	5.6
800	15	43.88	5.8	5.1
350	0	25.10	10.2	9.0
700	0	41.69	6.1	5.4
800	0	45.46	5.6	5.0
350	-150	47.27	5.4	4.8
700	-150	66.99	3.8	3.4
800	-150	70.84	3.6	3.2
0.987 liq	-253	70.99	3.6	3.2

Table 12: **Kerosene and gasoline gallon equivalents of one U.S. gallon hydrogen.**

The energy density of hydrogen is compared to another fuel using fuel gallon equivalent. A fuel gallon equivalent is simply gallons of hydrogen which store the same energy as a gallon of the other fuel.

Consider gasoline. Assume the ideal specific energy of gasoline is 44 MJ/kg (varies from $36 - 49$ MJ/kg). If the density is $\rho \approx 722.13$ kg/m^3 the energy density becomes 31.77 MJ/L or 33.41 kWh/gal. This is the EPA standard for gasoline.

Consider jet fuel (kerosene). Assume the ideal specific energy of jet fuel is 43 MJ/kg. If the density is $\rho \approx 840$ kg/m^3 the energy density becomes 36.12 MJ/L or 37.98 kWh/gal.

For combustion fuels, the lower heating values (LHV) are quoted since the product water is always in vapor form and the latent heat of vaporization is always lost.

The energy stored in a gallon of gasoline (31.77 MJ or 33.41 kWh) is stored in 9.3, 5.6, and 5.1 gallons of hydrogen if stored respectively as 350, 700, and 800 bar compressed gas at 15°C temperature (33.41

kWh of energy ÷ 3.6, 6.0, and 6.5 kWh/gal of hydrogen); or in 3.2 gallons of hydrogen if stored as liquid (33.41 ÷ 10.5). These are the gasoline gallon equivalents (GGE). Similarly, the energy stored in a gallon of kerosene (36.12 MJ/L or 37.98 kWh/gal) is stored in 10.7, 6.4, or 5.8 gallons of hydrogen under the same pressures and temperature (37.98 kWh of energy ÷3.6, 6.0, and 6.5 kWh/gal of hydrogen); or in 3.6 gallons of hydrogen when stored as liquid. This would be the kerosene gallon equivalent (KGE).

So a fuel gallon equivalent (FGE) is gallons of hydrogen given by

$$\text{FGE} \;= \frac{ED_{\text{fuel}} \times 1 \text{ gallon}}{ED_{\text{h2}}} = \frac{\rho_{\text{fuel}}}{\rho_{\text{h2}}} \frac{\Delta h_{\text{fuel}}}{\Delta h_{\text{h2}}} \times 1 \text{ gallon} \tag{28}$$

where ρ is density and Δh is the specific energy or heating value.

Table 12 lists the kerosene and gasoline gallon equivalents. The numbers are generated using: Δh_{h2} = 141 MJ/kg (HHV of hydrogen at 80° C), ρ_{ker} = 840 kg/m^3 and Δh_{ker} = 43 MJ/kg for kerosene, and ρ_{gas} = 722.13 kg/m^3 and Δh_{gas} = 44 MJ/kg for gasoline. Only ρ_{h2} varies with pressure and temperature (third column). If stored at 700 bar and 15° C, 6.4 gallons of hydrogen are needed for every gallon of kerosene, and 5.6 gallons of hydrogen for every gallon of gasoline.

The true fuel gallon equivalent (FGE true) is found by including engine efficiency in the definition.

$$\text{FGE true} \;= \frac{\rho_{\text{fuel}}}{\rho_{\text{h2}}} \frac{\Delta h_{\text{fuel}}}{\Delta h_{\text{h2}}} \frac{\eta_{\text{fuel}}}{\eta_{\text{h2}}} \times 1 \text{ gallon} \tag{29}$$

Figure 17 shows commercial and experimental automobile tank data for 350 bar, 450 bar, 700 bar and liquid storage for 5 − 10 kg of hydrogen. The weight fraction is defined as weight of hydrogen at full tank ÷ weight of tank system including hydrogen. A weight fraction of 5% means 5 kg of hydrogen needs 100 kg of tank system including the hydrogen. This data was collected in 2014 and some products no longer exist. The average still remains at 5.5% or so, for 5 kg of storage, due to the requirement to prevent boil-off over days and weeks. The fraction is expected to improve with higher amounts of storage. Figure 18 gives extreme examples. But it is really short-term storage, where boil-off can be allowed, that might enable higher fractions. The Navy Ion Tiger demonstrated 13% with only 0.5 kg of hydrogen (liquid) using the short term nature of storage. There is a gap in test data for the amount of hydrogen envisioned for eVTOL (10 − 20 kg) and the amount of storage time envisioned for eVTOL (1 − 3 hr) for a definitive conclusion.

6.5 Air flow

Air flows into the cathode and carries with it oxygen needed for the reaction. Let the mass flow rate of air be $\dot{w}_{\text{Air in}}$ kg/s and the oxygen needed be $\dot{w}_O$ kg/s. Non-reacting gases (mainly nitrogen) and excess oxygen leave as exhaust. Let this rate be $\dot{w}_{\text{Air out}}$. Water is generated inside the stack at a rate of $\dot{w}_W = \dot{w}_O + \dot{w}_H$. This water also leaves as exhaust.

The cathode should be humidified, but not flooded. Water and humidity management in the cathode is complex. In general, the total water exhaust contains the product water generated and water transported from anode to cathode through the membrane. A part of the water generated can be humidified and passed back into the anode and cathode to keep the electrodes and membranes hydrated. Then, water vapor also enters the cathode. Water production, transport, and balance are affected by intricate mechanisms inside a fuel cell. Modeling these mechanisms are important for stack design but not aircraft design. In the present model, only the product water is considered. So the model is simple; the input to the cathode is $\dot{w}_{\text{Air in}}$ and the outputs are $\dot{w}_{\text{Air out}}$ and $\dot{w}_W$. The product water is either vapor or liquid, if the output air is saturated with vapor. Regardless of simplicity, the mass flow rate across the stack must be conserved.

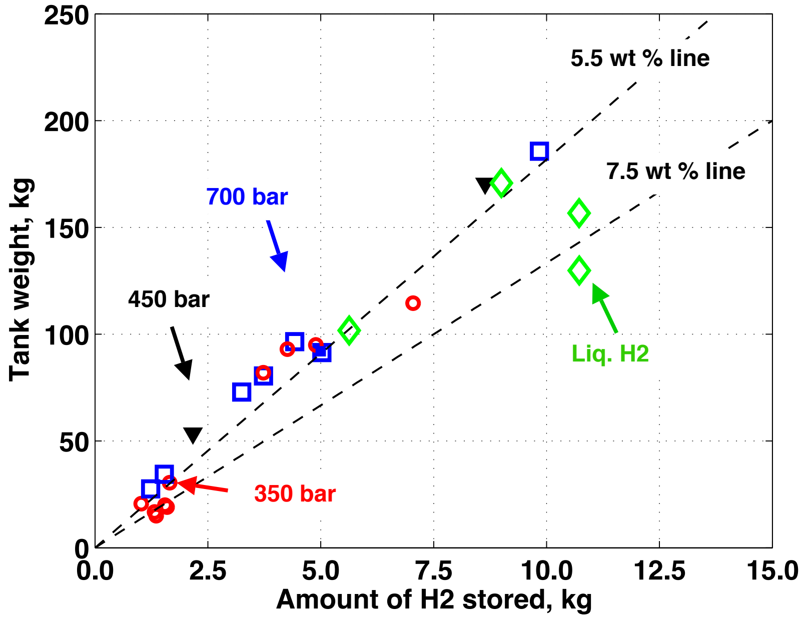


Figure 17: **Hydrogen storage in automobile tanks. 5.5 to 7.5% weight fraction.**

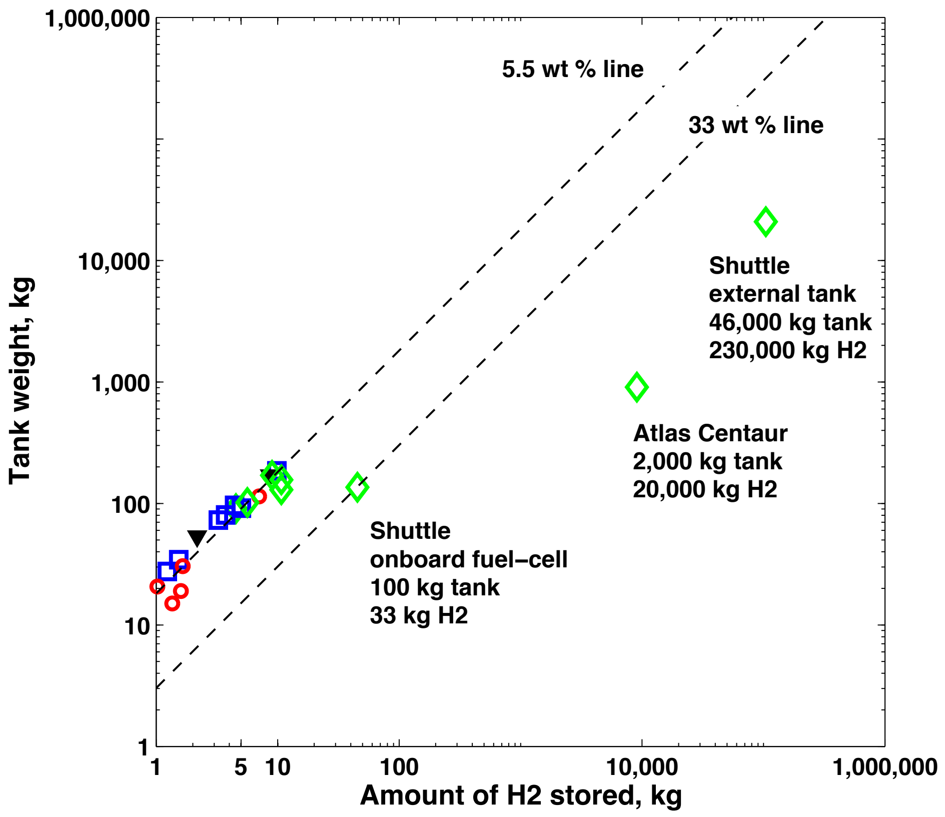


Figure 18: **Hydrogen storage in automobile (5.5%) versus space system tanks ($\geq$ 33%).**

The conservation of mass across the stack is described as follows.

$$\begin{aligned}\text{(input) } \dot{w}_H + \dot{w}_{\text{Air in}} &= \dot{w}_H + \dot{w}_O + \dot{w}_{\text{Air out}} \\ &= \dot{w}_W + \dot{w}_{\text{Air out}} \\ &= \dot{w}_{W\text{ liq}} + \dot{w}_{W\text{ vap}} + \dot{w}_{\text{Air out}} \text{ (output)}\end{aligned} \tag{30}$$

The mass flow rate of air is calculated from the mass flow rate of oxygen. The mass flow rate of oxygen is obtained from the cathode half reaction

$$\tfrac{1}{2}\,O_2 + 2e^- + 2H^+ \rightarrow H_2O \qquad \text{Cathode, capture of electrons, reduction} \tag{31}$$

Let S_O moles of oxygen capture the N moles of electrons in the cathode arriving from the anode half reaction. Here $S_O = 1/2$ and $N = 2$. (One mole of oxygen captures $N/S_O = n_C = 4$ moles of electrons at the cathode which was used during the construction of the $i - v$ curve earlier.)

Since a mole of electron carries a charge of F Coulomb ($F = 96485$ Coulomb/mole is the Faraday's constant), N moles of electrons carry NF Coulomb. So S_O moles of oxygen are needed to capture NF Coulomb. If one mole of oxygen weigh m_O kg, S_O moles of oxygen weigh $S_O m_O$ kg. So $S_O m_O$ kg of oxygen is needed to capture NF Coulomb. Or 1 Coulomb is captured by $S_O m_O/NF$ kg of oxygen. So I Ampere (Coulomb/s) is captured by $IS_O m_O/NF$ kg/s of oxygen. The current I must be captured in every cell so the total consumption is found by multiplying the number of cells n_c.

Thus, the mass flow rate of oxygen is

$$\dot{w}_O = S_O \frac{m_O}{N\,F}\,I\,n_c \tag{32}$$

where $\dot{w}_O$ is the mass flow in kg/s, n_c is the number of cells, I is the current in Ampere, $N = 2$, $F = 96485$ Coulomb/mole, $m_O = 31.9988 \times 10^{-3}$ kg/mole is the molecular weight of oxygen, and $S_O = 1/2$.

The mass flow rate of air is then

$$\dot{w}_{\text{Air}} = S_O \frac{m_A}{x_O\,N\,F}\,I\,n_c \tag{33}$$

where $\dot{w}_{\text{Air}}$ is the mass flow in kg/s and $x_O = 0.2095$ is the mole fraction of oxygen in air.

If supply rate is greater than consumption S_O is multiplied with an utilization factor f_A.

$$\dot{w}_{\text{Air in}} = f_A\,S_O \frac{m_A}{x_O\,N\,F}\,I\,n_c \tag{34}$$

Typically, $f_A \approx 1.5 - 2.5$ for PEMFC.

From Eq (2), $I\,n_c = P\,n_c/V = P/v$, so it can also be written as

$$\dot{w}_{\text{Air in}} = f_A\,S_O \frac{m_A}{x_O\,N\,F}\,\frac{P}{v} \tag{35}$$

which shows it is proportional to power but varies inversely with cell voltage. A higher cell voltage requires lower air flow for the same power.

The volume flow rate of air is

$$\dot{v}_{\text{Air in}} = \frac{\dot{w}_{\text{Air in}}}{m_A}\,\frac{R\,T}{p} \tag{36}$$

where $\dot{v}_{\text{Air in}}$ is the volume flow of air in m^3/s, p is stack pressure in Pa, $R = 8.3144621$ J/mol.K is the universal gas constant, T is the stack temperature in K, $m_A = 28.9655 \times 10^{-3}$ kg/mole is the molecular weight of dry air, and $\dot{w}_{\text{Air in}}$ is the mass flow in kg/s. The fraction $R/m_A = 287$ J/kg-K is the specific gas constant of dry air.

The volume flow rate in cubic feet per minute (CFM) is often used.

$$\text{CFM} = 4.72 \times 10^{-4} \ \times \ \dot{v}_{\text{Air in}} \tag{37}$$

At the design point, $I = I_D$, $P = P_D$, $v = v_c$.
At the maximum power point: $I = I_{\text{pmax}}$, $P = P_{\text{max}}$, $v = v_{\text{pmax}}$.
The air flow out is air flow in minus the oxygen consumed.

$$\dot{w}_{\text{Air out}} = \dot{w}_{\text{Air in}} - \dot{w}_O \tag{38}$$

The product water is the hydrogen plus oxygen consumed.

$$\dot{w}_W = \dot{w}_H + \dot{w}_O \tag{39}$$

The vapor and liquid content can be estimated from the humidity ratio. The saturation pressure of water vapor, p_{Ws}, is calculated using the empirical Arden Buck equations (Buck 1981; revised 1996).

$$\begin{aligned} \alpha &= \left(18.678 - \frac{{}^\circ C}{234.5}\right)\left(\frac{{}^\circ C}{257.14 +{}^\circ C}\right) \\ p_{Ws} &= 611.21 \ e^{\alpha} \end{aligned} \tag{40}$$

where °C is the stack output temperature and p_{Ws} is the pressure in Pa.
The maximum saturation humidity ratio is

$$h_s = \frac{m_W}{m_A} \ \frac{p_{Ws}}{p - p_{Ws}} = 0.622 \ \frac{p_{Ws}}{p - p_{Ws}} \tag{41}$$

where $m_W = 18.0153 \times 10^{-3}$ kg/mole is the molecular mass of water, $m_A = 28.965 \times 10^{-3}$ kg/mole is the molecular mass of air, and p is the stack output pressure. $p = p_S - \Delta p_S$. p_S is the stack inlet pressure and Δp_S is a specified stack pressure drop. $\Delta p_S \approx 20 - 50$ kPa for a 80 kWe stack.
If $h_s \ \dot{w}_{\text{Air out}} \geqslant \dot{w}_W$, the product water is in vapor form.
Then

$$\begin{aligned} \dot{w}_{W \text{ vap}} &= \dot{w}_W \\ \dot{w}_{W \text{ liq}} &= 0 \end{aligned} \tag{42}$$

In this case less water is produced than what the air can hold. A cathode humidifier will be needed.
If $h_s \ \dot{w}_{\text{Air out}} < \dot{w}_W$, the product water is partially liquid after air saturation with vapor.
Then

$$\begin{aligned} \dot{w}_{W \text{ vap}} &= h_s \ \dot{w}_{\text{Air out}} \\ \dot{w}_{W \text{ liq}} &= \dot{w}_W - \dot{w}_{W \text{ vap}} \end{aligned} \tag{43}$$

In this case more water is produced than what the output air can hold. A cathode humidifier will not be needed. A prescribed fraction f of $\dot{w}_{W \text{ liq}}$ is stored on-board.

7 Balance of Plant

The power to operate the subsystems is the balance of plant (BOP) P_{BOP}. This section describes the calculation of power for air supply, hydrogen supply, cooling systems, water system, and the electrical system.

Power generated (kW) minus BOP is the net electrical power (kWe). Normally, air supply consumes the most power, but power is also needed to supply fuel, circulate coolants, pump water, and operate the controller and other power electronics.

$$P_{BOP} = P_{\text{Air}} + P_{\text{Fuel}} + P_{\text{Ltc}} + P_{\text{Htc}} + P_{\text{Water}} + P_{\text{Elec}} \tag{44}$$

Power is calculated in Watts in the following form.

$$P = dP + P_{\text{model}} \tag{45}$$

where P_{model} is the model prediction and dP an user specified increment. Because there is no historical data base, the model uses elementary theory and scaling laws of the form $k\,X^e$ where X is the principal scaling variable and k and e are calibration constants from data or higher fidelity analysis. dP can be used to specify losses directly with $k = 0$.

Air supply

The air supply loss is from a blower or pump or a compressor-expander.

The power of a blower or pump is calculated as

$$\begin{aligned} P_{\text{blower}} &= \Delta p\ \dot{v}_{\text{Air in}}/\eta \\ P_{\text{blower max}} &= \Delta p\ \dot{v}_{\text{Air in max}}/\eta \end{aligned} \tag{46}$$

where $\Delta p = p_S - p_a$ is the difference between the stack pressure and atmospheric pressure in Pa, $\dot{v}$ is the volume flow rate in m^3/s (Eq 36), and η is the efficiency. A blower can be used up to $\Delta p \approx 30$ kPa, a pump up to ≈ 100 kPa (stationary), but for higher pressures a compressor is needed.

The power of a compressor is

$$\begin{aligned} P_{\text{comp}} &= C_p\ \dot{w}_{\text{Air in}}\ \Delta T \\ P_{\text{comp, max}} &= C_p\ \dot{w}_{\text{Air in max}}\ \Delta T \end{aligned} \tag{47}$$

where C_p is specific heat of air in J/kg-K, $\dot{w}$ is the inlet air flow in kg/s, and ΔT is the rise in temperature in the compressor in K. If the input and output temperatures and pressures are T_1 and P_1, and T_2 and P_2 respectively, then the ideal (minimum) power is obtained for isentropic compression when $T_2 = T_1\,(P_2/P_1)^{(\gamma-1)/\gamma}$ where γ is the ratio of specific heat. An efficiency η accounts for non-ideal compression and other mechanical losses. Typically $\eta \approx 0.7 - 0.8$.

$$\Delta T = \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] T_1\ \frac{1}{\eta} \tag{48}$$

The input temperature and pressure are assumed to be atmospheric values. The output pressure P_2 is the stack pressure p_S. C_p and γ are calculated at the input ambient temperature for dry air. Humidification, if it is required, is carried out after compression on the hot air.

The power from a turbine (or expander) is

$$\begin{aligned} P_{\text{turb}} &= C_p\ \dot{w}_{\text{Air out}}\ \Delta T \\ P_{\text{turb, max}} &= C_p\ \dot{w}_{\text{Air out, max}}\ \Delta T \end{aligned} \tag{49}$$

where C_p is specific heat of air in J/kg-K, $\dot{w}$ is the outlet air flow in kg/s, and ΔT is the drop in temperature in the turbine in K. If the input and output temperatures and pressures are T_1 and P_1 and T_2 and P_2 respectively then the ideal (maximum) power is recovered for isentropic expansion when $T_2 = T_1\,(P_2/P_1)^{(\gamma-1)/\gamma}$ where γ is the ratio of specific heat. So the ideal expression for ΔT is the same as in Eq 47 only the efficiency η is now in the numerator and limits the drop. Typically $\eta \approx 0.6 - 0.7$.

$$\Delta T = \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\ T_1\ \eta \tag{50}$$

The turbine input temperature and pressure are the stack output temperature (assumed equal to the stack temperature T_S) and the stack output pressure $p_S - \Delta p_S$. The turbine output temperature and pressure are atmospheric values. C_p and γ are calculated at the stack temperature. The outlet air has a different composition from the inlet air (less oxygen, more vapor) but this change has a small effect on the C_p and γ in a fuel cell since other gases pass through without reaction. So the dry air values are still used. The power from the turbine should be negative signifying power generation.

The combined power for the compressor-turbine (or expander) module is

$$P_{CEM} = P_{\text{comp}} + P_{\text{turb}} \tag{51}$$

The compressor is sized to the maximum power given by Eq 47 and 48. If the required power is higher during flight, the compressor will be unable to maintain the stack pressure to its design value. Then the new stack pressure is found from

$$\Delta T = \frac{P_{\text{comp, max}}}{C_P\ \dot{w}_{\text{Air in}}} \qquad\qquad p_S = P_1\ \left(\frac{\eta\,\Delta T}{T_1} + 1\right)^{\frac{\gamma}{\gamma-1}} \tag{52}$$

This condition can arise if altitude exceeds design altitude.

Finally

$$\begin{aligned} &P_{\text{Air}} = P_{\text{blower}} \\ &\text{or} \\ &P_{\text{Air}} = P_{CEM} \end{aligned} \tag{53}$$

H_2 system

The power required to circulate hydrogen (fuel) is calculated as

$$P_{\text{Fuel}} = dP + k\,\dot{w}^e \tag{54}$$

where $\dot{w} = \dot{w}_{H\ \text{in}}$ is the mass flow rate of hydrogen in kg/s from Eq 23 or Eq 24.

High temperature cooling system

The high temperature cooling system refers to the stack cooling system. The nomenclature follows the DOE HP. The nomenclature is generic since it is not essential a liquid cooling system be part of the model. Normally, a liquid cooling system is needed for stacks of power > 10 kW. The distinction between high and low temperature coolants is also formal since often the same coolant is used to circulate in both systems — the stack and the air. Other nomenclatures are also used in literature such as the inner and outer cooling systems. The model developed is conceptual and broadly applicable to all.

The Heat Ventilation and Air Conditioning (HVAC) system of the vehicle is ignored so refrigerant and condensers are not included in the model. It is assumed these are already dealt as in any conventional aircraft with cabin air and not strictly a part of the propulsion system.

The power required to circulate the coolant is calculated as

$$P_{\text{Htc}} = dP + k\,Q^e\,\dot{w}^f \tag{55}$$

where Q in Watts is the net heat to be rejected and $\dot{w} = \dot{w}_c$ in kg/s is the coolant mass flow rate. For air cooled systems, there is no coolant, and the exponent $f = 0$. The Q and $\dot{w}$ are calculated as follows.

The total stack heat is

$$Q_{\text{stack}} = (E_h - v)\,i\,A\,n_c = (E_h - v)\,i\,\frac{P}{p} = (E_h - v)\,\frac{P}{v} = \left({}^{1}\!/_{\eta} - 1\right)\,P \tag{56}$$

where E_h is the higher heating value and $\eta = v/E_h$. The higher heating value provides a conservative estimate by including conditions under which liquid water forms in the stack releasing the latent heat of condensation. This estimate can be too conservative, in which case the lower heating value can be used.

The maximum heat is at the maximum power

$$Q_{\text{stack max}} = (E_h - v_{\text{pmax}})\,i_{\text{pmax}}\,A\,n_c = \left({}^{1}\!/_{\eta|_{\text{pmax}}} - 1\right)\,P_{\text{max}} \tag{57}$$

where $\eta|_{\text{pmax}} = v_{\text{pmax}}/E_h$. If the maximum heat is deemed too high, the design heat can be used for sizing. Then the maximum power is not continuous power and must be rated.

If there is no compressor, a part of the stack heat can be used to heat the inlet air.

$$Q_{\text{in}} = C_p\ \dot{w}_{\text{Air in}}\ \Delta T \tag{58}$$

where C_p is the specific heat of air J/kg-K at the ambient temperature, $\dot{w}$ is the mass flow rate of air in kg/s (Eq 34), and ΔT is the difference in temperature between the stack and the ambient.

A part of the heat is carried out by the air

$$Q_{\text{out}} = C_p\ \dot{w}_{\text{Air out}}\ \Delta T \tag{59}$$

where C_p is the specific heat of air J/kg-K at the stack temperature, $\dot{w}$ is the mass flow rate of air in kg/s (Eq 38), and ΔT is the difference in temperature between the stack and the ambient. The input stoichiometry ($f_A\,S_O$ in Eq 34) is a key variable that affects this heat.

A part of the heat is dissipated.

The dissipated heat can be specified as a fraction of the total stack heat

$$Q_d = f_d\,Q_{\text{stack}} \tag{60}$$

or calculated from natural convection (Q_c) and radiation (Q_r) losses from the stack exposed surface area.

$$\begin{aligned} Q_d &= Q_c + Q_r \\ Q_c &= A_s\,h\,(T_S - T_a) \\ Q_r &= A_s\,\epsilon\,\sigma\,\left(T_S^4 - T_a^4\right) \end{aligned} \tag{61}$$

The area A_s is the stack exposed surface area specified as a fraction of stack active area.

$$A_s = f_s \, n_c \, A_c \tag{62}$$

The coefficient h is the overall convection heat transfer coefficient in W/m^{2}K, T_S is the stack temperature in K, T_a is the ambient temperature in K as seen by the stack (atmospheric ambient or the enclosure), ϵ is the emissivity of the stack (non-dimensional ≈ 0.8), and $\sigma = 5.670367 \times 10^{-8}$ W/m^{2}K^4 is the Stefan-Boltzmann constant. The coefficient h is difficult to calculate and assumed to be an input from a more detailed heat analysis that considers air flow over the stack. It is typically in the range $10 - 100$ W/m^{2}K. If these details are not available the fraction f_d can be specified instead. For very small stacks ($\approx 10 - 50$ W) f_s can be so high that cooling is not a problem at all, and a heater might be needed instead to raise the stack temperature.

The net heat to be rejected is the balance

$$Q = Q_{\text{stack}} - Q_{\text{in}} - Q_{\text{out}} - Q_d \tag{63}$$

The cooling system is a pumped loop circulating a coolant which collects the heat (by flowing through the cells or by other means) to a radiator which rejects it to the atmosphere.

The mass flow rate of the coolant is found from

$$Q = C_p \, \dot{w}_{\text{C}} \, \Delta T_c \tag{64}$$

where C_p is the specific heat of the coolant in J/kg-K at the coolant temperature, $\dot{w}_c$ is the mass flow rate of the coolant in kg/s, and ΔT_c is the rise in coolant temperature as it absorbs stack heat. $\Delta T_c < 15°$ C. The coolant inlet and outlet temperatures are then assumed to be $T_S - \Delta T_c$ and T_S respectively.

The difference in temperature between the stack coolant outlet temperature (assumed equal to the stack temperature T_S) and the ambient temperature of the radiator T_a is an important parameter ΔT. The ambient temperature is the maximum temperature for which the system is still be able to reject the heat. The ratio $Q/\Delta T$ is a heat rejection parameter that has been found useful in automobile fuel cells to optimize stack temperature and efficiency. $Q/\Delta T < 1.45$ kW/°C is a target for automobile stacks of 80-kWe with ambient heat rejection temperature of $T_a = 40$°C. In the present model, T_a is specified and $Q/\Delta T$ is calculated.

Medium	Specific heat kJ/kg-K	Density kg/m^3	Thermal conductivity W/m K	Boiling point °C	Freezing point °C
Water	4.18	995	0.65	100	0
50-50 Water/Ethylene Glycol	3.28	1082	0.40	107	-37
50-50 Water/Propylene Glycol	3.56	1010	0.36	106	-45

Table 13: **Properties of typical coolants.**

Table 13 shows typical liquid coolants. The cooling channels are usually built into the bipolar plates of the cells. A deionizer is used to prevent the coolant from becoming conductive due to ion contamination from the bipolar plates. An electrically conductive coolant leads to high leakage current which reduces cell efficiency. Antioxidants are also used for the same purpose. In addition, corrosion inhibitors are also used. So the properties are modified from the nominal water/Glycol values in Table 13 and should be specified accordingly.

The radiator rejects heat by convection (Q_c) and radiation (Q_r) from the radiator surface area.

$$\begin{aligned} &Q = Q_c + Q_r \\ &Q_c = A_r\, h\, (T_r - T_a) \\ &Q_r = A_r\, \epsilon\, \sigma\, \left(T_r^4 - T_a^4\right) \end{aligned} \tag{65}$$

The area A_r is the radiator surface area in m^2 that is to be calculated. The coefficient h is the overall convection heat transfer coefficient in W/m^{2}K, T_r is a specified radiator temperature in K, ϵ is the emissivity of the stack (non-dimensional), and $\sigma = 5.670367 \times 10^{-8}$ W/m^{2}K^4 is the Stefan-Boltzmann constant. The coefficient h is difficult to calculate and assumed to be an input from a more detailed heat analysis that considers air flow over the radiator. The coefficient $h = \mathrm{Nu}\, k/D$ where Nu is the Nusselt number (non-dimensional), k is the thermal conductivity in W/m K, and D is the hydraulic diameter. Nu depends on natural or forced convection, laminar or turbulent flow, and the Prandtl, Reynolds, and Rayleigh numbers (Nu = 1 for pure conduction, ≈ 10 for laminar flow, and $\approx 100 - 1000$ for turbulent flow). Other inputs are the radiator temperature T_r (≈ 100°C), ϵ ($\approx 0.7 - 0.8$), and the ambient temperature T_a for which the system should still be able to reject heat. The final output is the radiator area A_r from which the weight and volume can be calculated.

Low temperature cooling

The power required to circulate the coolant is calculated as

$$P_{\mathrm{Ltc}} = dP + k\, Q^e\, \dot{w}^f \tag{66}$$

where Q in Watts is the heat to be rejected and $\dot{w} = \dot{w}_c$ in kg/m is the coolant mass flow rate. For air cooled systems (no coolant) the exponent $f = 0$. The heat Q and coolant mass flow rate $\dot{w}$ are calculated as follows.

$$Q = C_p\; \dot{w}_{\mathrm{Air\ in}}\; \Delta T \tag{67}$$

where C_p is the specific heat of air J/kg-K at the compressor output temperature, $\dot{w}$ is the mass flow rate of air in kg/s (Eq 34), and ΔT is the difference in temperature between the compressor output and the stack temperature in K.

The cooling system is a pumped loop circulating a coolant which collects the heat (by flowing through the cells or by other means) to a radiator which rejects it to the atmosphere.

The mass flow rate of the coolant is found from

$$Q = C_p\; \dot{w}_c\; \Delta T_c \tag{68}$$

where C_p is the specific heat of the coolant J/kg-K at the coolant temperature, $\dot{w}_c$ is the mass flow rate of the coolant in kg/s, and ΔT_c is the rise in coolant temperature. $\Delta T_c < 15°$ C.

The heat rejection in the radiator follows the same theory as high temperature cooling. If the same radiator is used, only one should be modeled by using $k = 0$ for the other.

Water system

The power required to circulate the water is

$$P_{\mathrm{Water}} = dP + k\, \dot{w}^e \tag{69}$$

where $\dot{w} = \dot{w}_{W\ \mathrm{liq}}$ is the maximum liquid water rate in kg/m (Eq 43 or, for a conservative estimate, Eq 39).

Electrical system

The power required for the electrical system is

$$P_{\mathrm{Elec}} = dP + k\,P^e \tag{70}$$

where $P = P_{\mathrm{max}}$ is the maximum stack power in Watts.

8 Weights

This section describes the calculation of weights, although by weight, really mass is meant.

The total weight consists of the weight of the stack, air supply, hydrogen (fuel), tank, cooling systems, water system, and the electrical system.

$$W = W_{\text{Stack}} + W_{\text{Air}} + W_{\text{Fuel}} + W_{\text{Tank}} + W_{\text{Ltc}} + W_{\text{Htc}} + W_{\text{Water}} + W_{\text{Elec}} \tag{71}$$

Weight (mass) is calculated in kg in the following form.

$$W = dW + W_{\text{model}} \tag{72}$$

where W_{model} is the model prediction and dW is an user specified increment. Because there is no historical data base, the models use elementary theory and scaling laws of the form $k\,X^e$ where X is the principal variable and k and e are calibration constants from data or higher fidelity analysis. dW can be used to specify weights directly with $k = 0$.

Volume is calculated in m^3 in the same manner

$$S = dS + S_{\text{model}} \tag{73}$$

Stack

If the stack power is P, and the cell power density is p, the membrane area must be P/p since p is the power produced per unit area. If the weight is assumed to be proportional to this area: $W \propto P/p$, then the constant of proportionality is a ratio of cell power density and stack power to weight.

$$\kappa = \frac{p}{P/W} \tag{74}$$

This ratio is a text-book design factor. It has dimensions of mass per area and unit of kg/m^2. It varies only with materials and mechanical construction of the stack. It ranges from $4 - 20$ kg/m^2 with lower values for better stacks. As an example, if the design factor is 5 kg/m^2, then cells of power density $p_c = 0.5$ W/cm^2 (= 5 kW/m^2) would produce a stack of $P/W = 1.0$ kW/kg. If superior cells of $p_c = 1$ W/cm^2 (= 10 kW/m^2) can be found, a lighter stack of $P/W = 2.0$ kW/kg can be built.

The stack power $P = A_c\, n_c\, p$ from Eq 2, hence the design factor is

$$\kappa = \frac{W}{n_c\, A_c} \tag{75}$$

which is how it is calculated.

The weight of a stack is calculated part by part. Figure 19 gives a diagram of the main parts of a typical fuel cell which is the basis for the present model. For detailed topology and fabrication processes and materials of modern cells, see De Las Heras et al 2018.

The weight of the proton exchange membrane (PEM) is

$$W_{\text{pem}} = A_c\, t_{\text{pem}}\, \rho_{\text{pem}} \tag{76}$$

where W is the weight in kg, A_c is the cell active area in m^2 (the area covered by the catalysts), ρ is density in kg/m^3, and t is thickness in m.

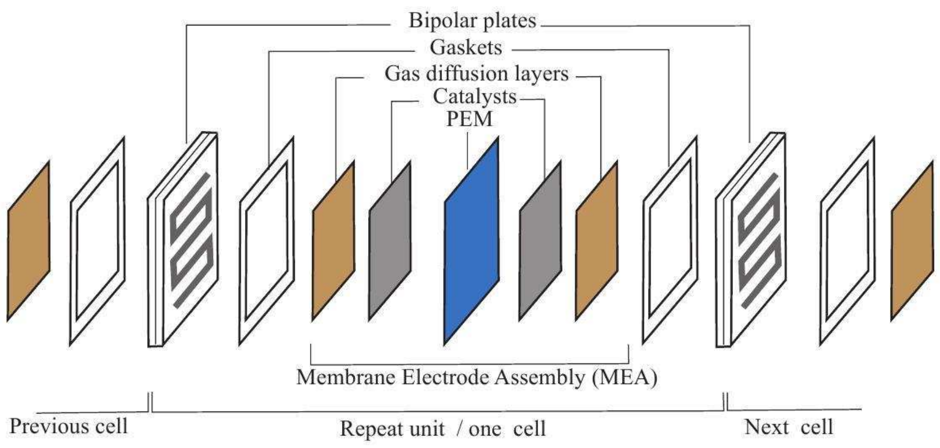


Figure 19: **Parts of a typical fuel cell.**

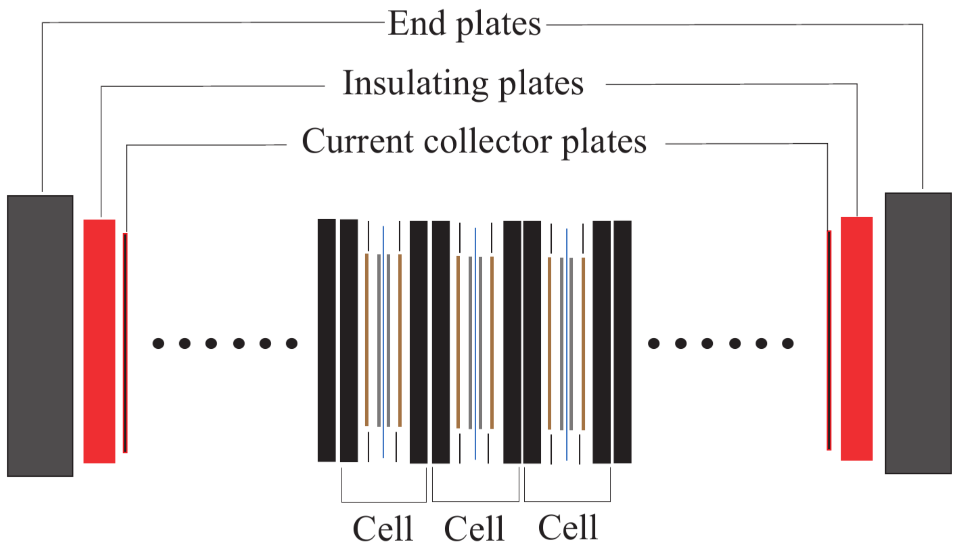


Figure 20: **Parts of a typical fuel stack.**

The membrane electrode assembly (MEA) is the membrane sandwiched between a pair of gas diffusion layers (GDL) and then sealed with gaskets. The GDL have the same area as the active area. The sealing gaskets occupy a small fraction of this area given by the factor k. The weight of the MEA is

$$W_{\text{mea}} = W_{\text{pem}} + 2\, A_c\, t_{\text{gdl}}\, \rho_{\text{gdl}} + 2\, k_{\text{seal}}\, A_c\, t_{\text{seal}}\, \rho_{\text{seal}} \tag{77}$$

The cell is the MEA sandwiched between a pair of bipolar plates. The bipolar plates contain channels through which reactants (air on cathode and hydrogen on anode side) and coolants flow. The channels can be of many designs. The non-dimensional porosity ϕ accounts for them at a gross level. The weight of the cell is

$$W_{\text{cell}} = W_{\text{mea}} + 2\, k_{\text{bp}}\, A_c\, t_{\text{bp}}\, \rho_{\text{bp}}\, \phi_{\text{bp}} \tag{78}$$

The unit is the collection of cells

$$W_{\text{unit}} = n_c\, W_{\text{cell}} \tag{79}$$

The stack is the unit sandwiched between current collectors, insulating layers, and end plates, all tied together by a fastener. Figure 20 shows the parts of a typical stack. The stack weight is

$$W_{\text{Stack}} = W_{\text{unit}} + \sum\ 2\, k\, A_c\, t\, \rho\ + W_{\text{fas}} + W_{\text{other}} \tag{80}$$

where the summation is over the collector plates (cp), insulating plates (ip), and end-plates (ep) with their respective area fraction, thickness, and density. The weight of the fastener is an input since they vary widely from basic steel bolts and clamps to lighter composite wraps. The other weights cover for missing items in the model.

In general, the weight is proportional of the active area times the number of cells, plus an overhead. Since $A_c = P/(n_c\, p)$, the weight is simply proportional to the design power P divided by the cell power density p.

$$W_{\text{Stack}} = k_0 + k_1\, \frac{P}{p} \tag{81}$$

The constants k_0 and k_1 vary only with construction technology and materials.

The length of the stack is

$$L_{\text{stack}} = t_c\ + \sum\ 2\, t \tag{82}$$

where the summation is over the gdl, seal, bp, cp, ip, and ep. The thickness and length are in m.

The cross-sectional area of the stack (m^2) is the area of the end-plate.

$$A_{\text{stack}} = A_{\text{ep}} = k_{\text{ep}}\ A_c \tag{83}$$

The volume of the stack (m^3) is

$$S_{\text{stack}} = L_{\text{stack}}\, A_{\text{stack}} \tag{84}$$

An equivalent cell thickness (m) is calculated by dividing stack length with number of cells.

$$t_{\text{cell}} = \frac{L_{\text{stack}}}{n_c} \tag{85}$$

An equivalent cell density (kg/m^3) is calculated by specifying a porosity ϕ_{stack} (non-dimensional)

$$\rho_{\text{cell}} = \frac{W_{\text{stack}} - W_{\text{fas}} - W_{\text{other}}}{L_{\text{stack}}\, A_c\, \phi_{\text{stack}}} \tag{86}$$

The equivalent cell thickness and density make up the simplest weight model

$$W_{\text{Stack}} = \rho_{\text{cell}}\, t_{\text{cell}}\, n_c\, A_c\, \phi_{\text{stack}} + W_{\text{fas}} + W_{\text{other}} \tag{87}$$

For this model $L_{\text{stack}} = t_{\text{cell}}\, n_c$ and $A_{\text{stack}} = A_c$.

Either Eq 80 or Eq 87 can be used to calculate the stack weight.

The design factor is then found using Eq 75 and Eq 81

$$\kappa = \frac{W_{\text{Stack}}}{n_c\, A_c} = \frac{k_0}{P/p} + k_1 \tag{88}$$

where k_0 is from the overhead of W_{fas} and W_{other}. It can dominate small low power stacks < 1 kW, and produce a poor (high) design factor even with good electrochemistry.

Fuel system

The fuel system supplies hydrogen to the stack. The weight model consists of hydrogen and the tank.

$$W_{\text{Fuel}} = W_{\text{h2}} + W_{\text{tank}} \tag{89}$$

There is no real model of the tank, just specified fractions, which are adequate for the conceptual nature of the design.

The gravimetric weight fraction of hydrogen storage w_f is defined as the weight of a full tank of hydrogen divided by the weight of hydrogen and tank system combined.

$$w_f = \frac{W_{\text{h2}}}{W_{\text{h2}} + W_{\text{tank}}} \qquad\qquad \text{wt } \% = w_f \times 100 \tag{90}$$

Thus the weight of hydrogen and tank system is

$$W_{\text{h2}} + W_{\text{tank}} = \frac{W_{\text{h2}}}{w_f} \tag{91}$$

and the tank alone is

$$W_{\text{tank}} = \frac{1 - w_f}{w_f}\, W_{\text{h2}} \tag{92}$$

The system gravimetric capacity or net specific energy of storage is

$$\Delta h_{\text{h2 tank}} = w_f\, \Delta h_{\text{h2}} \tag{93}$$

The specific energy of hydrogen is $\Delta h_{\text{h2}} = 141$ MJ/kg (HHV) or 120 MJ/kg (LHV). But 1 kg of hydrogen requires $1/w_f$ kg of tank system. Thus the net specific energy is $w_f \times 141$ (HHV) or $w_f \times 120$ (LHV). For 6 wt% the net specific energy is 8.5 MJ/kg (HHV) or 7.2 MJ/kg (LHV), or 2.35 kWh/kg (HHV) or 2 kWh/kg (LHV). This value should not be compared with batteries; the weight of the fuel stack should be included for any comparison since batteries combine the engine and the fuel. In fact a battery also carries its oxidizer which contributes to both its high compactness and heavy weight.

A volume fraction is defined similarly, except the tank volume automatically includes the hydrogen.

$$s_f = \frac{S_{\text{h2}}}{S_{\text{tank}}} \tag{94}$$

Thus the tank volume is

$$S_{\text{tank}} = \frac{S_{\text{h2}}}{s_f} \tag{95}$$

The volume of hydrogen is related to the hydrogen weight and density at full tank.

$$S_{\text{tank}} = \frac{W_{\text{h2}}}{\rho_{\text{h2}}\, s_f} \tag{96}$$

A tank volume overhead factor can also be defined for volume

$$s_{ov} = \frac{S_{\text{tank}} - S_{\text{h2}}}{S_{\text{tank}}} = 1 - s_f \tag{97}$$

Thus alternatively, the tank volume is

$$S_{\text{tank}} = \frac{W_{h2}}{\rho_{\text{h2}}\, (1 - s_{ov})} \tag{98}$$

The weight of hydrogen divided by the volume of the tank is the storage density.

$$\rho_{\text{h2 store}} = \frac{W_{\text{h2}}}{S_{\text{tank}}} = \rho_{\text{h2}}\, s_f = \rho_{\text{h2}}\, (1 - s_{ov}) \tag{99}$$

The storage density is also called the volumetric weight fraction in literature.

The weight fraction w_f and storage density $\rho_{\text{h2 store}}$ are typical model inputs. The storage density makes it easy and intuitive to calculate the tank volume S_{tank} from weight of hydrogen W_{h2}. But the storage density must be lower than hydrogen density otherwise non-physical results are produced. Since the hydrogen density is a function of temperature and pressure, this is not easy to track. The tank overhead factor s_{ov} is a safer and the preferred input. Results are always physically meaningful for $s_{ov} > 0$.

Air supply system

The air supply to the stack consists of a blower (or compressor and turbine) and a humidifier. Provisions are also kept for filters, regulators, power supply, and plumbing. A power supply might be needed for humidification or for cold-start.

$$\begin{aligned} W_{\text{Air}} =& W_{\text{blower}} + W_{\text{comp}} + W_{\text{turb}} + W_{\text{humidifier}} + \\ & W_{\text{filter}} + W_{\text{regulator}} + W_{\text{power}} + W_{\text{plumbing}} \end{aligned} \tag{100}$$

The weights and volumes of the blower, compressor, and turbine are scaled to the maximum mass flow rate (kg/s) and power with specified constants and exponents.

$$W \text{ or } S = k\, \dot{w}^e\, P^f \tag{101}$$

where k, e, and f are input separately for each component, $\dot{w}$ is the maximum mass flow rate (kg/s) and P is the maximum power (W).

For the blower or the compressor, $\dot{w}$ is maximum mass flow rate of air input; for the turbine, $\dot{w}$ is the maximum mass flow rate of air output.

$$\begin{aligned} &\dot{w} = \dot{w}_{\text{Air in, max}} && \text{blower or compressor} \\ &\dot{w} = \dot{w}_{\text{Air out, max}} && \text{turbine} \end{aligned} \tag{102}$$

For stacks of power < 100 kW, an axial-flow turbine (expander) is typically integrated into a single compressor-expander unit. Then only one sizing model can be used.

The weight of the humidifier, cooler, filter, regulator, and power are simply fractions of total weight of the air supply system

$$W = dW + f\, W_{\text{Air}} \tag{103}$$

The weight of the plumbing is given by

$$W = dW + wl \tag{104}$$

where w is the weight per length (kg/m) and l is the length of plumbing (m).

High temperature cooling system

A high temperature cooling system cools the stack. The weight model consists of a radiator and coolant. A cooler is not needed as the purpose is to simply remove the heat. Provisions are kept for filters, regulators, power supply (to circulate the coolant), and plumbing.

$$\begin{aligned} W_{\text{Htc}} = & W_{\text{coolant}} + W_{\text{radiator}} + \\ & W_{\text{filter}} + W_{\text{regulator}} + W_{\text{power}} + W_{\text{plumbing}} \end{aligned} \tag{105}$$

The weight of the coolant is proportional to the heat rejected or coolant mass flow rate.

$$W_{\text{coolant}} = dW + k\, Q^e\, \dot{w}_c^f \tag{106}$$

where W is the weight in kg, dW is an increment, k, e and f are input parameters, Q is heating in Watt from Eq 63, and $\dot{w}_c$ is the coolant mass flow from Eq 64. For $e = 0$ and $f = 1$, k is time in seconds gives total mass of coolant $k\,\dot{w}$ in kg.

The weight of the radiator is scaled to the heat rejected or the radiator area.

$$W_{\text{radiator}} = dW + k\, Q^e\, A_r^f \tag{107}$$

where W is the weight in kg, dW is an increment, k and e are input parameters, and Q is heating in W from Eq 63, and A_r is the radiator area in m^2 from Eq 65. If there is no radiator $f = 0$. If there is a radiator $f = 1$ and k is the radiator weight per area in kg/m^2. Typical values can range from $3 - 8$ kg/m^2.

The volume of the radiator is accounted for in the same manner as the weight

$$S_{\text{radiator}} = dS + k\, Q^e\, A_r^f \tag{108}$$

where S is volume in m^3, dS is an increment, and k and e are input parameters. If there is no radiator $f = 0$. If there is a radiator $f = 1$, and k is the radiator thickness in m.

The radiator can be a very heavy component and a major weight overhead. Determining the factors require detailed design that takes advantage of the application environment.

The weight of the filter, regulator, and power are simply input fractions of total weight of the cooling system

$$W = dW + f\, W_{\text{Htc}} \tag{109}$$

The weight of the plumbing is given by

$$W = dW + wl \tag{110}$$

where w is the weight per length (kg/m) and l is the length of plumbing (m).

Low temperature cooling system

If a compressor is used, and the compression ratio is high, the output air is at a high temperature. A low temperature cooling system is required to cool the air before it enters the stack. The weight model consists of a cooler, coolant, and a radiator. Provisions are kept for filters, regulators, power supply (to circulate the coolant), and plumbing.

$$\begin{aligned} W_{\text{Ltc}} =& W_{\text{cooler}} + W_{\text{coolant}} + W_{\text{radiator}} + \\ & W_{\text{filter}} + W_{\text{regulator}} + W_{\text{power}} + W_{\text{plumbing}} \end{aligned} \tag{111}$$

The weight of the cooler is scaled to the heat rejected.

$$W_{\text{cooler}} = dW + k\,Q^e \tag{112}$$

where W is weight in kg, dW is an increment, k is an input parameter, and Q is heating from Eq 67.

The weight of the coolant is scaled to the heat rejected or the coolant mass flow rate.

$$W_{\text{coolant}} = dW + k\ Q^e\,\dot{w}_c^f \tag{113}$$

where W is the weight in kg, dW is a prescribed weight, k, e and f are input parameters, Q is heating from Eq 67 and $\dot{w}_c$ is the coolant mass flow from Eq 68. For $e = 0$ and $f = 1$, k is time in seconds gives total mass of coolant $k\,\dot{w}$ in kg.

The weight of the radiator is scaled to the heat rejected or the area.

$$W_{\text{radiator}} = dW + k\,Q^e\,A_r^f \tag{114}$$

where W is the weight in kg, dW is a prescribed weight, k, e and f are input parameters, and Q is heating in W from Eq 67, and A_r is the radiator area in m^2. If there is no radiator $f = 0$. If there is a radiator $f = 1$ and k is the radiator weight per area in kg/m^2. Typical values can range from $3 - 8$ kg/m^2.

The volume of the radiator is accounted for in the same manner as the weight

$$S_{\text{radiator}} = dS + k\,Q^e\,A_r^f \tag{115}$$

where S is volume in m^3, dS is a prescribed volume in m^3, and k, e, and f are input parameters. If there is no radiator $f = 0$. If there is a radiator $f = 1$ and k is the radiator thickness in m.

The weight of the filter, regulator, and power are prescribed as fractions of total weight of the cooling system

$$W = dW + f\,W_{\text{ltc}} \tag{116}$$

The weight of the plumbing is given by

$$W = dW + wl \tag{117}$$

where w is the weight per length (kg/m) and l is the length of plumbing (m).

Water system

The water system model is just a storage tank with provisions for filters, regulators, power supply (to pump), and plumbing.

$$\begin{aligned} W_{\text{Water}} =& W_{\text{tank}} + \\ & W_{\text{filter}} + W_{\text{regulator}} + W_{\text{power}} + W_{\text{plumbing}} \end{aligned} \tag{118}$$

The weight of water stored is

$$W = f\,t\,\dot{w}_{W\mathrm{liq}} \tag{119}$$

where f is the fraction of water produced by the stack over a time t at a rate $\dot{w}$. The fraction f and time t are specified inputs.

The weight and volume of the tank are

$$\begin{aligned} W_{\mathrm{tank}} &= (1 + w_{ov})\; W \\ S_{\mathrm{tank}} &= (1 + s_{ov})\; W/\rho \end{aligned} \tag{120}$$

where the fractions w_{ov} and s_{ov} are specified and ρ is the density of water.

The weight of the filter, regulator, and power are input fractions of total weight of the water system.

$$W = dW\ +\ f\,W_{\mathrm{Water}} \tag{121}$$

The weight of the plumbing is given by

$$W = dW\ +\ wl \tag{122}$$

where W is the weight per length (kg/m) and l is the length of plumbing (m).

Electrical system

The electrical system model consists of a controller with provisions for power electronics and plumbing (wiring).

$$W_{\mathrm{Elec}} = W_{\mathrm{controller}}\ +\ W_{\mathrm{power}}\ +\ W_{\mathrm{plumbing}} \tag{123}$$

The weight of the controller is scaled to the maximum stack current.

$$W_{\mathrm{controller}} = dW\ +\ k\,I^{e} \tag{124}$$

where dW, k and e are inputs and $I = I_{\mathrm{pmax}}$.

The weight of power electronics is simply input fraction of total weight of the electrical system.

$$W_{\mathrm{power}} = dW\ +\ f\,W_{\mathrm{Elec}} \tag{125}$$

The weight of the plumbing is given by

$$W = dW\ +\ wl \tag{126}$$

where w is the weight per length (kg/m) and l is the length of wiring (m).

9 Software Overview

The software is written in MATLAB.

The *src* directory contains the following source files.

createiv.m	Creates $i - v$.
DATA_gptX.m	Gas phase thermochemistry data. X=H2, O2, H2Ol (liq water) and H2Ov (vapor).
DATA_DryAirCp.m	Air C_p and γ.
h2density.m	Density of compressed hydrogen.
ISAtmosphere.m	Atmospheric model up to 11 km.
ivpolarization.m	Uses $i - v$. Applies corrections.
pemfc.m	Designs system for a specified power kW.
pemfcdesign.m	Designs system for a specified net power kWe. Iterates *pemfc.m.*
pemfcstatic.m	Analyzes a design.
watervp.m	Saturated vapor pressure of water.

The following run directories are provided.
run Baseline 80kW
run Modern 80kW
run Modern 80kW liqcool
run Modern 500kW liqcool
run Modern 500kW liqcool P1.0
Each *run* directory contain the following files.

Input-output file	
fcvariables.m	Lists all inputs and outputs. Outputs set to zero.
Drivers for design	
drivercreativ.m	Fits model to $i - v$ data.
driverpemfc.m	Executes pemfc.m
driverpemfcdesign.m	Executes pemfcdesign.m
driverpemfcPvsW.m	Generates design power versus weight.
driverpemfcragone.m	Generates Ragone plot.
Drivers for analysis	
driverpemfcenginechartvsH.m	For a fixed design, generates power, SFC, and H2 flow versus altitude.
driverpemfcenginechartvsP.m	For a fixed design, generates SFC and H2 flow versus net power.

All PEMFC design variables are contained in a single fuel cell data structure FCST. All PEMFC analysis variables are contained in a single fuel cell operation structure FCOP. The structures are contained in a single file *fcvariables.m.* Only this file and the two structures are needed for integration with an aircraft design code.

The codes are listed at the end of the report and also available upon request from Aeromechanics Branch, NASA-ARC or the Alfred Gessow Rotorcraft Center, University of Maryland.

10 List of inputs and outputs

The inputs and outputs are listed in the file *fcvariables.m*. The outputs are initialized to zero. The variables are in SI units unless appended otherwise; for example, POWER_kW is in kilo-Watts.

The primary inputs for stack weight and size are listed in Table 14. The design power P_D is the primary input (POWERdsg_kW). The power option ioptp=1 produces a stack with the design power as the maximum power. The maximum power is continuous power. The cell efficiency η_c and voltage v_c fall out. This option produces the minimum stack weight and volume. The option ioptp=2 produces a stack that generates the design power at a specified cell efficiency η_c (etac). The cell voltage v_c and maximum power fall out. The power option ioptp=3 produces a stack that generates the design power at a specified cell voltage v_c (vc). The cell efficiency and the maximum power fall out. For options ioptp=2 and 3, whether maximum power is continuous power, depends on the inputs to the cooling system. The stack voltage at design power V_D does not affect weight or volume but the aspect ratio; higher voltage require more cells and a longer stack but have lower current and a smaller cross-section.

variables	definition	units
POWERdsg_kW	design power	kW
vc	cell voltage at design power	V
etac	cell efficiency at design power	
ioptp	input options for design power:	
	1 POWERdsg_kW is max power; etac and vc fall out	
	2 etac; vc and max power fall out	
	3 vc; etac and max power fall out	V
Voltage	stack voltage at design power	V

Table 14: **Primary inputs for stack sizing.**

The inputs for environment are listed in Table 15. Only pressure and temperature are needed for the fuel cell. The altitude input is limited to 11 km.

The inputs for cell $i - v$ data are listed in Table 16. The option ioptiv=1 uses data from the table ivdatainp. The row number for maximum power ipmaxinp is needed for this option. Alternatively, the data can be created using the variables listed in the table. The ideal reversible cell voltage E_r can be input directly, or calculated inside if the input is set to zero.

The inputs for stack weights are listed in Table 17. A pem sandwiched between two gdl, two gaskets, and two bipolar plates constitute one pem cell. All cells together constitute the unit. The unit sandwiched between two current collector plates, insulating plates, and two end plates constitute the stack. The fastener can be bolts and nuts or specially designed clamps and straps. The bipolar plate contains channels for fuel and air flow, accounted for by the porosity.

The inputs for the fuel (hydrogen) system are listed in Table 18.

The inputs for the air supply system are listed in Table 19.

The inputs for the high temperature cooling system are listed in Table 20.

The inputs for the low temperature cooling system are listed in Table 21. They are similar to the high temperature system except the stack dissipation inputs are replaced with cooler inputs.

The inputs for the water system are listed in Table 22.

variables	definition	units
iatm	input options for environment	
	0 ISA	
	1 ISA + temperature correction	
	2 input pressure and temperatureC	
OneAtm	Atmospheric pressure	Pa
pressure	pressure at design altitude	Pa
temperatureC	temperature at design altitude	$^\circ$C
altitudeDesign	design altitude; used for iatm 0 or 1	m

Table 15: **Inputs for environment.**

The inputs for the electrical system are listed in Table 23.
Other inputs are listed in Table 24.

variables	definition	units
TstackC	stack operating temperature	$^\circ$C
Pstack	stack operating pressure	Pa
xO2	mole fraction of oxygen in supply	0.2095
ioptiv	input option for ivdata	
	1 use input i-v table	
	2 create i-v table	
	3 create i-v table from PMAT	
ioptiv=1		
ivdatainp(:,3)	i-v table	
	col 1 cell current density i	A/cm^2
	col 2 cell voltage v	V
	col 3 cell power density p	W/cm^2
ipmaxinp	row of maximum power in ivdatainp	
ioptiv=2		
Er	Reversible cell voltage (open circuit voltage)	V
jL	limiting cell current density	A/cm^2
jLeak	parasitic current loss at electrodes	A/cm^2
j0A	anode constant for activation loss	
alphaA	anode constant for activation loss	
j0C	cathode constant for activation loss	
alphaC	cathode constant for activation loss	
ASR	area specific resistance for ohmic loss	Ohm.cm^2
C	constant for concentration loss	V
ioptiv=3		
PMAT(10, :)	row 1: pressures	Atm
	rows 2-10: 9 constants jL to C for each pressure	
ioptiv=2, 3		
arh, crh	anode and cathode relative humidity	%
deltai	Δi in created i-v table	A/cm^2
ASRt	temperature in $^\circ$C for ASR0	
ASRtk1, ASRtk2	constants for temperature correction	
jLt	temperature in $^\circ$C for jL0	
jLtk1, jLtk2	constants for temperature correction	
ASR0arh, ASR0crh	reference ASR for anode and cathode rel humidity	
ASRrhk1a–ASRrhk1c	constants for rel humidity correction	
jL0arh, jL0crh	reference jL for anode and cathode rel humidity	
jLrhk1a–jLrhk1c	constants for rel humidity correction	
icorr	current correction (1/0 for yes/no)	
TFi	tech factor; i = TFi $\times i$	
vcorr	voltage correction (1/0 for yes/no)	
vrate1000ft	Δv per 1000-ft altitude gain	
vdropaboveft	altitude above which Δv applied	ft

Table 16: **Inputs for polarization curve i-v.**

variables	definition	units
ioptsw	input option for stack weight	
	0 simple	
	1 detailed	
TFwstack	tech factor, weight	
TFsstack	tech factor, volume	
dWwstack	correction weight	kg
dSsstack	correction volume	m^3
ioptsw=0		
tcell	thickness of a cell	m
dcell	density of stack	kg/m^3
porosity	porosity of stack	fraction
ioptsw=1		
dendplate	density of end-plate (2 per stack)	kg/m^3
dinsulplate	density of insulator plate (2 per stack)	kg/m^3
dcollplate	density of current collector (2 per stack)	kg/m^3
dpem	density of proton exchange membrane	kg/m^3
dgdl	density of gas diffusion layer (2 per pem)	kg/m^3
dbipolarplate	density of bipolar plate electrodes (2 per pem)	kg/m^3
dgasket	density of sealing gasket (2 per pem)	kg/m^3
Aendplatefrac	area of end plate fraction active area	
tendplatecons	thickness of end plate, constant	m
tendplatefrac	thickness of end plate, fraction thickness of unit	
Ainsulplate	area of insulating plate, fraction active area	
tinsulplatecons	thickness of insulating plate, constant	m
tinsulplatefrac	thickness of insulating plate, fraction thickness of unit	
Acollplatefrac	area of collector plate, fraction active area	
tcollplatecons	thickness of collector plate, constant	m
tcollplatefrac	thickness of collector plate, fraction thickness of unit	
Wfastener	weight of fastener	kg
tpem	thickness of pem	m
tgdl	thickness of gas diffusion layer	m
tbipolarplate	thickness of bipolarplate	m
pbipolarplate	porosity of bipolarplate	
Abipolarplatefrac	area of bipolarplate, fraction active area	
tgasket	thickness of gasket	m
Agasketfrac	area of gasket, fraction active area	
Wstackother	other weights	kg

Table 17: **Inputs for stack weight.**

variables	definition	units
fH2utilization	H2 utilization factor ($\approx 1 - 1.05$)	
h2tankwfg	wt fraction gravimetric	kg/kg
	= h2 weight ÷ (h2+tank) weight	
itanktype	type of tank volume model	
	1 input wt fraction volumetric	
	2 liquid hydrogen	
	3 compressed gas	
<u>itanktype=1</u>		
h2tankwfs	weight fraction volumetric $\rho_{\text{h2 store}}$	kg/m^3
	= h2 weight ÷ tank volume (storage density)	
<u>itanktype=2, 3</u>		
h2tankSov	tank volume overhead s_{ov}	
	= (vol tank - vol h2) ÷ vol tank	
<u>itanktype=3</u>		
h2tankTC	compressed gas temperature	$^\circ$C
h2tankBar	compressed gas pressure	bar
ioptf	fuel option for energy calculation	
	1 input weight of hydrogen	
	2 input time	
<u>ioptf=1</u>		
Whydrogen	weight of hydrogen	kg
<u>ioptf=2</u>		
Endurance_min	duration of design power	min
FGErho	base fuel density	kg/m^3
FGEMJkg	base fuel Δh	MJ/kg
FGEeta	base fuel engine efficiency	
P0	power, prescribed	W
k, e	power, constant and exponent	
W0	wt, prescribed, for filter, regulator, power	kg
fW	wt, fraction system weight	

Table 18: **Inputs for hydrogen system. The base fuel inputs are used only for gallon equivalents.**

variables	definition	units
fAirFlowRate	air utilization factor (≈ 2.5)	
DPstack	Stack pressure drop	Pa
iBlower	1/0 for yes/no	
iCompressor	1/0 for yes/no	
iTurbine	1/0 for yes/no; turned 0 for iCompressor=0	
	blower, compressor, and turbine	
Eta	Efficiency	
W0	wt, prescribed	kg
Wtf	wt, technology factor	
Wk, Wfe, Wpe	wt, constant and flow and power exponents	
S0	vol, prescribed	m^3
Stf	vol, technology factor	
Sk, Sfe, Spe	vol, constant and flow and power exponents	
	filter, regulator, power, humidifier	
W0	wt, prescribed	kg
fW	wt, fraction system weight	
PlumbW0	wt, prescribed	
PlumbWl	length of plumbing	m
PlumbWw	weight per length	kg/m

Table 19: **Inputs for air-supply system.**

variables	definition	units
iHV	Heating values used for sizing	
	0-LHV, 1-HHV	
iHTR	Power level used in sizing	
	0-design power, 1-max power	
coolantCp	Cp of coolant fluid	J/kg-K
coolantdT	Coolant rise in temperature	K
W0coolant	wt, constant, coolant	kg
W0k	wt, constant of flow	sec
	radiator	
W0	wt, prescribed	kg
Wk, We, Wf	wt, constant, heat, and area exponents	
	Wk = 0 zero area	
	Wf = 0 unknown area	
	Wf = 1, We = 0, calculated area, Wk = kg/m^2	
S0	vol, prescribed	m^3
Sk, Se, Sf	vol, constant, heat, and area exponents	
	Sk = 0 zero area	
	Sf = 0 unknown area	
	Sf = 1, We = 0, calculated area, Sk = thickness in m	
	radiator area	
h	radiator heat transfer coefficient	W/m^2/K
Tr	radiator temperature	K
Ta	ambient temperature for cooling	K
epr	radiator emissivity	
	stack dissipation	
fd	fraction of stack heat	
hs	stack heat transfer coefficient	W/m^2/K
Tas	ambient temperature for cooling	K
eps	stack emissivity	
fs	stack surface area, fraction active area	
	filter, regulator, power	
W0	wt, prescribed	kg
fW	wt, fraction system weight	
PlumbW0	wt, prescribed	
PlumbWl	length of plumbing	m
PlumbWw	weight per length	kg/m
P0	power, prescribed	W
k, e, f	power, constant, heat and coolant flow exponent	

Table 20: **Inputs for the high temperature cooling system.**

variables	definition	units
iLTR	Power level used in sizing	
	0-design power, 1-max power	
coolantCp	Cp of coolant fluid	J/kg-K
coolantdT	Coolant ambient temperature	K
W0coolant	wt, constant, coolant	kg
W0k	wt, constant of flow	sec
	radiator	
W0	wt, prescribed	kg
Wk, We, Wf	wt, constant, heat, and area exponents	
	Wk = 0 zero area	
	Wf = 0 unknown area	
	Wf = 1, We = 0, calculated area, Wk = kg/m^2	
S0	vol, prescribed	m^3
Sk, Se, Sf	vol, constant, heat, and area exponents	
	Sk = 0 zero area	
	Sf = 0 unknown area	
	Sf = 1, We = 0, calculated area, Sk = thickness in m	
	radiator area	
h	radiator heat transfer coefficient	W/m^2/K
Tr	radiator temperature	K
Ta	ambient temperature for cooling	K
eps	radiator emissivity	
	cooler	
W0	wt, prescribed	kg
Wk, We	wt, constant and heat exponents	
	filter, regulator, power	
W0	wt, prescribed	kg
fW	wt, fraction system weight	
PlumbW0	wt, prescribed	
PlumbWl	length of plumbing	m
PlumbWw	weight per length	kg/m
P0	power, prescribed	W
k, e, f	power, constant, heat and coolant flow exponent	

Table 21: **Inputs for the low temperature cooling system.**

variables	definition	units
rhowater	density of water	kg/m^3
waterstoragefrac	frac of produced water stored	
waterstoragehour	for how long	hr
watertankSov	tank volume overhead	
watertankWov	tank weight overhead	
	filter, regulator, power	
W0	wt, prescribed	kg
fW	wt, fraction system weight	
PlumbW0	wt, prescribed	
PlumbWl	length of plumbing	m
PlumbWw	weight per length	kg/m
P0	power, prescribed	W
k, e	power, constant and exponent	

Table 22: **Inputs for water storage system.**

variables	definition	units
	controller	
W0	wt, prescribed	kg
Wk, We	wt, constant and current exponent	
	regulator, power	
W0	wt, prescribed	kg
fW	wt, fraction system weight	
PlumbW0	wt, prescribed	
PlumbWl	length of wiring	m
PlumbWwk	weight per length constant	
PlumbWwe	weight per length current exponent	
P0	power, prescribed	W
k, e	power, constant and exponent	

Table 23: **Inputs for electrical systems.**

variables	definition	units
	unforeseen overheads	
fovpower	power, fraction total power	
fovweight	weight, fraction total weight	
fovvolume	volume, fraction total volume	
iplotVIP	plot stack i-v-p 1/0 yes/no	
iprntSCR	screen output 1/0 yes/no	

Table 24: **Other inputs.**

The outputs of design are shown in Table 25. All variables are within the data structure FCST. For example the variable ivdata is really FCST.ivdata. The design outputs are produced by the driver *driverpemfc.m* and *driverpemfcdesign.m*.

The outputs of analysis have the same variable names and are not repeated. But they are fewer in number as the stack is already designed. They are within the data structure FCOP. The analysis outputs are produced by the driver *pemfcstatic.m*. Analysis should be performed only after design. See Examples in later sections.

variables	definition	units
Cell/stack performance		
ivdata(:,3)	cell $i - v - p$ table used	
	col 1: i, current density	A/cm^2
	col 2: v, voltage	V
	col 3: p, power density	W/cm^2
Eh	E_h, ideal reversible cell voltage (enthalpy)	V
Er	E_r, true reversible cell voltage (Gibbs)	V
Aactive_cm2	A_c, cell active area	cm^2
Ncell	n_c, number of cells	
Istackdsg	stack current at design power	A
Vstackdsg	stack voltage at design power	V
Istackmax	stack current max	A
Vstackmax	stack voltage max	V
IstackPmax	stack current at max power	A
VstackPmax	stack voltage at max power	V
vcdsg	v_c, cell voltage, design	V
icdsg	i_c, cell current density, design	A/m^2
pcdsg	p_c, cell power density, design	W/m^2
etadsgHHV	η, true efficiency, design power with HHV E_h	
etadsgLHV	η, true efficiency, design power with LHV E_h	
etamaxHHV	η, true efficiency, max power with HHV E_h	
etamaxLHV	η, true efficiency, max power with LHV E_h	
etavdsgHHV	η_r, voltage efficiency, design power with HHV E_r	
etavdsgLHV	η_r, voltage efficiency, design power with LHV E_r	
Cell/stack size		
Sstack	stack volume	m^3
Lstack	stack length	m
Astack	stack cross-sectional area	m^2
Wstack	stack weight data structure	
.total	weight, total	kg
	total = end + insulating + collector + unit	
.endplate	weight, end plate	kg
.insulplate	weight, insulating plate	kg
.collplate	weight, collector plate	kg
.unit	weight, unit	kg
	unit = number of cells $\times$ weight of each cell	
.cell	weight, cell	kg
	cell = mea + bipolar plates	
.bipolarplate	weight, bipolar plates	kg

Continued on next page

Table 25 – *Continued from previous page*

variables	definition	units
.mea	weight, membrane electrode assembly	kg
	mea = pem + gdl + gasket	
.pem	weight, membrane	kg
.gdl	weight, gas diffusion layers	kg
.gasket	weight, gaskets	kg
.other	weight, other	kg
Wstackk0	k_0, design factor constant	kg
Wstackk1	k_1, design factor constant	kg/m^2
DFstack	design factor, p_c/(P/Wstack)	kg/m^2
SP	specific power, design power	kW/kg
SPmax	specific power, max power	kW/kg
Air system		
Sblower	volume, blower	m^3
Scomp	volume, compressor	m^3
Sturb	volume, turbine	m^3
Pblower_kW	power, blower, design	kW
Pblowermax_kW	power, blower, max	kW
Pcomp_kW	power, compressor, design	kW
Pcompmax_kW	power, compressor, max	kW
Pturb_kW	power, turbine, design	kW
Pturbmax_kW	power, turbine, max	kW
Pair_kW	power, total, design	kW
Pairmax_kW	power, total, max	kW
Wair	weight	kg
.total	total	kg
.blower	blower	kg
.comp	compressor	kg
.turb	turbine	kg
.precooler	precooler	kg
.humidifier	humidifier	kg
.filter	filter	kg
.plumbing	plumbing	kg
.power	power components	kg
.regulator	regulators	kg
MassAirrate_kgs	air mass flow, design power	kg/s
MassAirrate_max_kgs	air mass flow, max power	kg/s
MassAirrate_Ls	air volume flow, design power	L/s
MassAirrate_max_Ls	air volume flow, max power	L/s
MassAirrate_out_kgs	air mass flow out, design power	kg/s
MassAirrate_out_max_kgs	air mass flow out, max power	kg/s
Humidity_ratio	humidity ratio out, design power	
MassWaterrate_kgs	water mass flow out, design power	kg/s
MassVaporrate_kgs	water vapor mass flow out, design power	kg/s
MassLiquidrate_kgs	water liquid mass flow out, design power	kg/s

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Table 25 – *Continued from previous page*

variables	definition	units
Fuel/H2 system		
Sh2tank	volume of H2 tank	m^3
Wfuel	weight of H2 fuel system	kg
.total	weight, total	kg
.H2	weight, H2	kg
.tank	weight, tank	kg
.filter	weight, filters	kg
.plumbing	weight, plumbing	kg
.power	weight, power components	kg
.regulator	weight, regulators	kg
Pfuel_kW	power, fuel system, design power	kW
Pfuelmax_kW	power, fuel system, max power	kW
MassH2rate_kgs	H2 mass flow, design power	kg/s
MassH2rate_Pmax_kgs	H2 mass flow, max power	kg/s
SFCPdsg_kgkWh	SFC, design power	kg/kW-hr
SFCPmax_kgkWh	SFC, max power	kg/kW-hr
Low temp cooling (ltc)		
Wlt	weight of ltc system	kg
.total	weight, total	kg
.filter	weight, filters	kg
.plumbing	weight, plumbing	kg
.regulator	weight, regulators	kg
.coolant	weight, coolant liquid	kg
.radiator	weight, radiator	kg
.power	weight, power components	kg
Plt_kW	power, ltc, design power	kW
Pltmax_kW	power, ltc, max power	kW
Sltradiator	volume, radiator	m^3
Altradiator	area, radiator	m^2
HEATINGlt_kW	heat load	kW
HEATINGlt_max_kW	heat load max	kW
Cflowdsglt	coolant flow, design power	kg/s
Cflowmaxlt	coolant flow, max power	kg/s
Cflowlt	coolant flow used for ltc sizing = Cflowdsglt for input iLTR=0	kg/s
High temp cooling (htc)		
Wht	weight of htc system	kg
.total	weight, total	kg
.filter	weight, filters	kg
.plumbing	weight, plumbing	kg
.regulator	weight, regulators	kg
.coolant	weight, coolant liquid	kg
.radiator	weight, radiator	kg
.power	weight, power components	kg

Continued on next page

Table 25 – *Continued from previous page*

variables	definition	units
Pht_kW	power, htc, design power	kW
Phtmax_kW	power, htc, max power	kW
Shtradiator	volume, radiator	m^3
Ahtradiator	area, radiator	m^2
HEATINGstack_kW	heat generated, design power	kW
HEATINGairin_kW	lost to heat inlet air	kW
HEATINGairout_kW	lost through exhaust	kW
HEATINGdiscon_kW	dissipated by convection	kW
HEATINGdisrad_kW	dissipated by radiation	kW
HEATINGhtc_kW	heat load at design power	kW
HEATINGstack_max_kW	heat generated, max power	kW
HEATINGairin_max_kW	lost to heat inlet air, max power	kW
HEATINGairout_max_kW	lost through exhaust, at max power	kW
HEATINGhtc_max_kW	heat load at max power	kW
Cflowdsght	coolant flow, design power	kg/s
Cflowmaxht	coolant flow, max power	kg/s
Cflowht	coolant flow used for htc sizing	kg/s
	= Cflowdsght for input iHTR=0	
Water		
Swatertank	volume of water tank	m^3
Wwater	weight of water system	kg
.total	weight, total	kg
.tank	weight, tank	kg
.plumbing	weight, plumbing	kg
.power	weight, power components	kg
.regulator	weight, regulators	kg
Pwater_kW	power, water sys, design power	kW
Pwatermax_kW	power, water sys, design power	kW
MassTankrate_kgs	tank accumulation rate	kg/s
Electrical		
Welec	weight of electrical system	kg
.total	weight, total	kg
.controller	weight, fuel cell controller	kg
.power	weight, batteries and power components	kg
.plumbing	weight, transmission and distribution	kg
Pelec_kW	power, electrical, design power	kW
Pelecmax_kW	power, electrical, max power	kW
Stack + BOP		
Sstacksys	volume, stack system	m^3
Wstacksys	weight, stack system	kg
SPower_Pdsg_Stacksys_kWkg	specific gross power, design	kW/kg
SPower_Pdsg_Stacksys_kWekg	specific net power, design	kWe/kg
SPower_Pmax_Stacksys_kWkg	specific gross power, max	kW/kg

Continued on next page

Table 25 – *Continued from previous page*

variables	definition	units
SPower_Pmax_Stacksys_kWekg	specific net power, max	kWe/kg
DFstacksys	design factor, p_c/(P/Wstacksys)	kg/m^2
Total System		
Stotal	volume, total system	m^3
Wtotal	weight, total system	kg
POWERdsg_kWe	net electrical design power	kWe
POWERmax_kW	gross max power	kW
POWERmax_kWe	net electrical max power	kWe
EtadsgHHV	true efficiency at design power using HHV	
EtadsgLHV	true efficiency at design power using LHV	
EtamaxHHV	true efficiency at max power using HHV	
EtamaxLHV	true efficiency at max power using LHV	
FGEHHV	Fuel gallon equivalent using HHV (chemical)	gal
FGELHV	Fuel gallon equivalent using HHV (chemical)	gal
FGEHHVtrue	chemical × efficiency	gal
FGELHVtrue	chemical × efficiency	gal
SFCPdsg_kgkWhe	SFC, at net electrical design power	kg/kWe-hr
SFCPmax_kgkWhe	SFC, at net electrical max power	kg/kWe-hr
ENERGY_Pdsg_kWh	energy capacity, at design power	kWh
ENERGY_Pdsg_kWhe	net electrical energy at design power	kWe-hr
ENERGY_Pmax_kWhe	net electrical energy at max power	kWe-hr
ENERGY_Pmax_kWh	gross energy capacity at max power	kW-hr
SPower_Pdsg_kWkg	specific gross power	kW/kg
SPower_Pdsg_kWekg	specific net power	kWe/kg
SPower_Pmax_kWkg	specific gross max power	kW/kg
SPower_Pmax_kWekg	specific net max power	kWe/kg
EDensity_Pdsg_kWhL	energy density at gross design power	kW-hr/L
EDensity_Pdsg_kWheL	energy density at net design power	kWe-hr/L
EDensity_Pmax_kWhL	energy density at gross max power	kW-hr/L
EDensity_Pmax_kWheL	energy density at net max power	kWe-hr/L
SEnergy_Pdsg_kWhkg	specific energy at gross design power	kW-hr/kg
SEnergy_Pdsg_kWhekg	specific energy at net design power	kWe-hr/kg
SEnergy_Pmax_kWhkg	specific energy at gross max power	kW-hr/kg
SEnergy_Pmax_kWhekg	specific energy at net max power	kWe-hr/kg

Table 25: Outputs of stack design. All variables are within the data structure FCST. Design and max power refer to conditions when stack generates design and maximum power. L=Liter. gal=U.S. gallon. HHV=Higher Heating Value of H2. LHV=Lower Heating Value of H2.

The variables required to design the rotorcraft are acquired by executing the *driverpemfc.m* and *pemfc-static.m* over a range of inputs. These variables and corresponding drivers are listed in Table 26. The driver inputs over write *fcvariables.m*.

variables	definition	units
driverpemfcPvsW.m		
FCST.		
StackWvsP(:,2)	gross power, stack weight	kW, kg
StackSystemWvsPe(:,3)	net power, net power max, stack sys weight	kWe, kWe, kg
	stack sys = total system - fuel - tank	
driverpemfcWvsH.m		
FCST.		
StackSystemWvsH(:,2)	altitude, stack sys weight	m, kg
driverpemfcenginechartvsH.m		
FCOP.		
Pevs1000ft(:,5)	col 1: altitude	1000-ft
	col 2, 3: power, net power	kW, kWe
	col 4, 5: SFC, fuel flow	kg/kW-hr, kg/hr
driverpemfcenginechartvsP.m		
FCOP.		
H2flowvsP(:,2)	net power, fuel flow	kWe, kg/hr
SFCvsP(:,2)	net power, SFC	kWe, kg/kW-hr
SFCvsPefrac(:,2)	power fraction, SFC	non-dim, kg/kW-hr
	power fraction = net power ÷ net design power	

Table 26: **Drivers and outputs for rotorcraft design.**

11 Example: Baseline 80 kW

A hypothetical PEMFC system of net power 80 kWe (kW-electrical) is sized with cell characteristics given in Figs 11 and 12 (data from Yan, Toghiani and Wu 2006). These are representative of Yr-2000 technology and considered the baseline.

The model constants for $i-v$ curves for pressures 1 to 4 atm were given in Table 8. Temperature and humidity corrections were extracted for area specific resistance and the limiting current about the reference temperature 80°C (353.15 K) and reference humidity 100%. In absence of data, the same corrections were assumed to apply to all pressures. The symbols have usual meanings and units (see Section on Theory/The $i-v$ Curve).

The temperature corrections are the following.

$$\Delta ASR_{\Omega} = 0.002\,(T-353.15) + 0.0006\,(T-353.15)^2$$

$$\Delta i_L = -0.003\,(T-353.15) - 0.0015\,(T-353.15)^2$$

The humidity corrections are the following.

$$a = arh - 100$$

$$c = arh - 100$$

$$\Delta ASR_{\Omega} = -0.005a + 2\times10^{-6}\,a\,c^2 + 7\times10^{-5}\,c^2 + 0.0021\,c$$

$$\Delta i_L = -1\times10^{-4}\,c^2 - 0.0076\,c$$

The example case is given in the run directory *run Baseline 80kW*.

The stack pressure is specified as 2.5 atm. So the 2 and 3 atm model constants in Table 8 are interpolated linearly for cell $i-v$. The stack temperature is specified as 80°C which is the temperature of the dataset.

The driver *drivercreateiv.m* is used to calculate the model constants. The $i-v$ data is available for $1, 2, 3,$ and 4 atm pressures. The constants in Table 8 produced good fits.

Figure 21 shows the voltage data at 2 and 3 atm and the model fit for 2.5 atm. The cell power density is $p = i\,v$, and the heat density is $q = i\,(E_h - v)$. The true efficiency and the voltage efficiency (Eq 16) are shown in Fig 22. There is nominally no maximum efficiency point, the peak is merely an artifact of the leakage current. Higher efficiencies are obtained at lower currents (lower power) and lower efficiencies at higher currents (higher power). So the maximum power density generally occurs at low efficiency.

The driver *drivercreateiv.m* outputs an $i-v-p$ data table and a variable ipmax which is the row number of maximum power. The resolution Δi is set by the variable deltai (deltai = 0.01 is used). The $i-v-p$ table and ipmax are input in *fcvariables.m* as ivdatainp and ipmaxinp.

The model fit is input in *fcvariables.m.* For option ioptiv=1, the data table $i-v-p$ is input as ivdatainp along with the row number of maximum power ipmaxinp; for ioptiv=2, the 9 constants are input; and for ioptiv=3, a matrix PMAT is input which contains the 9 constants for each pressure.

There can be a loss in cell voltage from cell-level to the stack-level. The variable deltav accounts for this loss. A typical value for a 80 kWe stack is 0.01 V. There can be a drop in cell voltage with altitude when a compressor is not used. A typical value is 0.56% per 1000-ft. A base altitude can be input above which the correction is applied. The variable vcorr determines whether these corrections are applied. Here vcorr=0 so no corrections are applied.

vcorr = 0

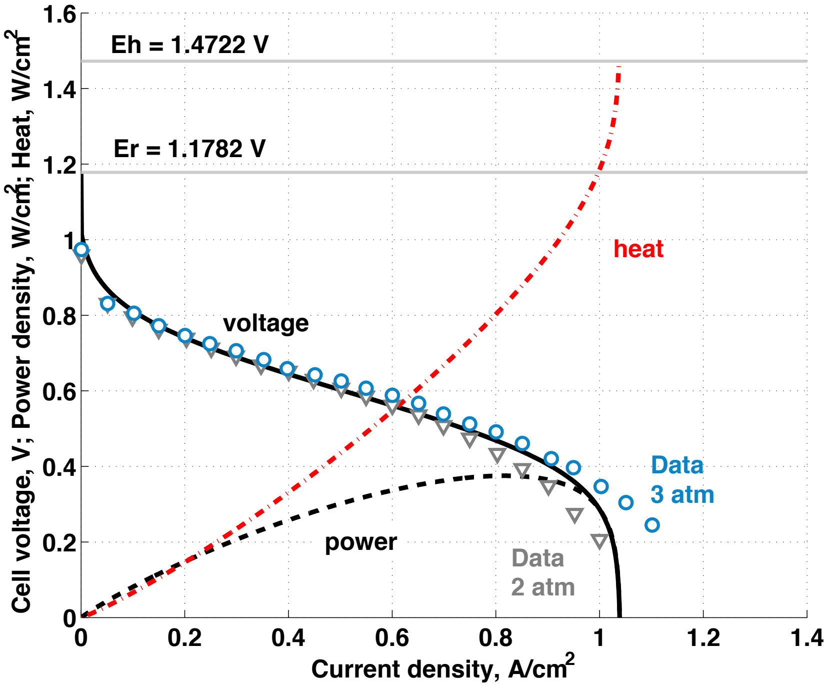

Figure 21: **Cell voltage, power, and heating versus current of baseline cells representative of Yr-2000 technology. Symbols are data at 2 and 3 atm. Lines are models at 2.5 atm. 80°C. 100% anode and cathode relative humidity.**

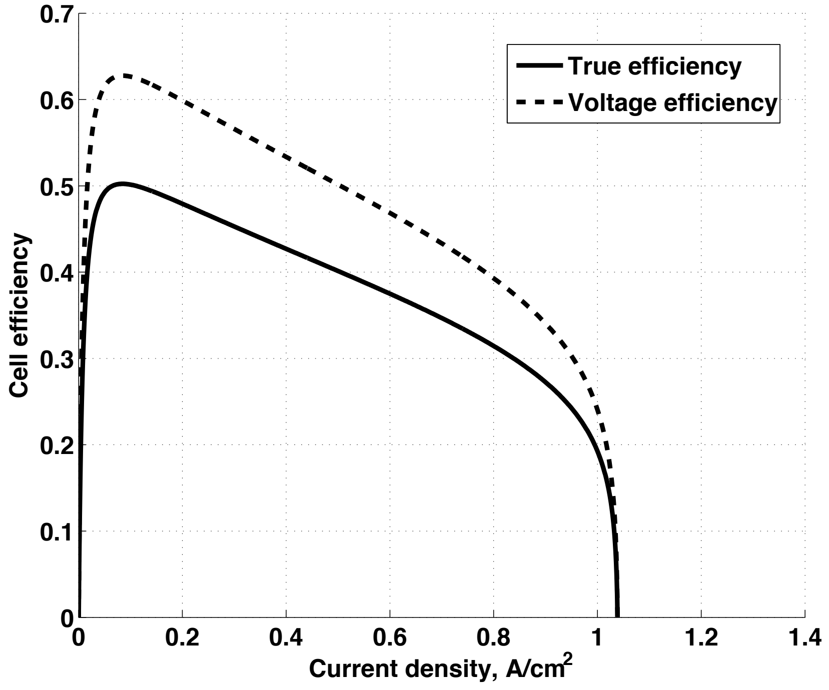

Figure 22: **Cell efficiency of baseline cells at 2.5 atm, 80°C.**

deltav = -0.01
vrate1000ft = -0.0056
vdropaboveft = 1000
The stack is built of conventional materials.

The end plate is Aluminum (ρ = 1600 kg/m^3), the insulation plate is polyoxy-methylene (ρ = 1420 kg/m^3), the collector plate is Copper (ρ = 8940 kg/m^3), the membrane is Nafion (ρ = 500 kg/m^3), the gas diffusion layer and catalyst layer are a single carbon cloth coated with Pt/C on one side (ρ = 440 kg/m^3), the bipolarplate is graphite (ρ = 2160 kg/m^3), and the gasket is rubber (ρ = 1000 kg/m^3). These inputs can be changed based on an actual stack.

A technology factor can be specified.
TFwstack = 1.0
The design environment is SL/ISA. This impacts the balance of plant.

The driver *driverpemfc.m* sizes a stack for a gross power given by the input POWERdsg_kW. The net electric power is lower due to the balance of plant. The option ioptp=1 produces a stack of minimum weight. For this option, the cell operates at the maximum power density, hence at a low voltage (Fig 21), and therefore at a poor efficiency (Fig 22) (efficiency is $\approx v/1.4722$). The option ioptp=3, with voltage specified as vc=0.6, produces a stack of high efficiency. For this option, the maximum power is higher than the design power.

The cooling systems can be sized either to design power or to maximum power. In this example, it is sized to the design power.
iLTR = 0
iHTR = 0
The high temperature cooling system is the stack cooling system. The heat load can be specified either for LHV (product water in vapor form, E_h = 1.256 V) or HHV (product water in liquid form, E_h = 1.4722 V). Less cooling is needed when the water is in vapor form as the heat of vaporization is not released. In this example, the LHV is used.
iHV = 0
Hydrogen is stored at 5.7% weight fraction as compressed gas at 700 bar 15°C. For option ioptf=1, the amount of hydrogen is specified. In this example 5 kg is assumed.
H2wfrac = 0.057
Whydrogen = 5
h2tankBar = 700
h2tankTC = 15
Table 27 shows two stack designs — minimum weight and high efficiency (based on input voltage vc selected from Fig 22). In the table, only the variables that change from one column to the next are shown, an empty column means a repeated value. The outputs are saved in files *B80minwgtDESIGN.tex* and *B80highetaDESIGN.tex*.

In the minimum weight design, the design power 80 kWe is the maximum power and the maximum continuous power (MCP). In the high efficiency design, the design power 80 kWe is the maximum continous power (MCP), but it is not the maximum power. The maximum power is 95 kWe. But since the stack cooling system is sized to the design power, this is not continuous power. 95 kWe is 1.188 × 80 kWe which is close to the typical intermediate rated power factor for helicopters (IRP=1.185) but short of the maximum rated power factor (MRP=1.269) and the contingency rated power factor (CRP=1.33). These margins can be provided if the stack is sized to higher efficiencies (by increasing input voltage vc) but at the cost of higher weight.

In summary, baseline cells can provide typical power densities $p_c \approx 0.30 - 0.37$ W/cm^2 at efficiencies of $\eta \approx 0.3 - 0.4$. Specific power of the stack can be $0.8 - 1.0$ kW/kg with a charitable design factor of 3.5 kg/m^2. But specific power of the stack system is what is more important. This is driven by the weight of the

Component	Variables	Unit	B80 min weight	B80 high η
Cell	v_c	V	0.458	0.601
	p_c	W/cm^2	0.375	0.303
	η_c, η_v		0.31, 0.38,	0.40, 0.50
	Weight	kg	0.15	0.23
Stack	No of cells		546	416
	Active area	cm^2	470	731
	Weight	kg	90	107
	Sp power	kW/kg	1.07	0.86
	Design fac	kg/m^2	3.52	
Air	Air flow	kg/s	0.19	0.14
	Power	kW	12.7	9.3
	Weight	kg	17.5	20.8
HT cool	Q gen	kW	170	102
	Q load	kW	153	87
	Q/ΔT	kW/°C	2.3	1.3
	Power	kW	1.5	0.87
	Weight	kg	1.7	0.99
LT cool	Q gen	kW	9.6	6.9
	Power	kW	0.10	0.07
	Weight	kg	1.0	0.74
Electrical	Power	kW	1.9	1.8
	Weight	kg	4.8	7.5
Stack system				
	P, P net	kW, kWe	96, 80	
	Pmax, Pmax net	kW, kWe	96, 80	114, 95
	Weight	kg	116	137
	Sp Power	kWe/kg	0.69	0.58
Fuel	H2 flow	kg/s	0.0022	0.0016
	H2 stored	kg	5	
	Tank wt%		5.7	
	Total Weight	kg	90	
Total system				
	Endurance	min	38	51
	Energy	kWhe	50	68
	Weight	kg	205	227
	Sp Power	kWe/kg	0.39	0.35
	Sp Energy	kWe-hr/kg	0.24	0.30
	Energy density	kWe/L	0.21	0.29
	η		0.26	0.35
	GGE	U.S. gal	5.5	4.0

Table 27: **80 kWe PEMFC designs with baseline cells. SL/ISA. Stack: 250V, 2.5 atm, 80°C; tank: 700 bar, 15°C; cell: E_h, E_r: 1.47, 1.18 V (HHV) and 1.26, 1.17 (LHV). No water tank. η_c, η_v: true (HHV) and voltage efficiencies of cell; η: efficiency of the system (HHV). Gallon gasoline equivalent (GGE) based on combustion efficiency 0.25.**

balance of plant subsystems, particularly the air and the cooling systems. The stack system specific power is placed around 0.6 – 0.7 kWe/kg in this example.

Comparison with batteries must include the hydrogen and tank weight. In this example, with 5 kg hydrogen storage, the specific energy and power are around 240 – 300 Wh/kg and 0.39 – 0.35 kW/kg respectively, where the net electrical energy and power are used. The specific power is equivalent to battery C-rates of 1.625 —1.167 (390 W/240 Wh — 350 W/300 Wh), where the lower C-rate corresponds to the higher specific energy — a behavior consistent with any electrochemical source. Note that the battery specifications used for comparison should be pack level, available and installed.

12 Example: Modern 80 kW

A hypothetical PEMFC system of net power 80 kWe (kW-electrical) is sized with cell characteristics given in Figs 13 and 14 (Alhuwalia, Wang and Steinbach 2016). These are representative of modern automobile fuel cells. This example can be used as a template for a DOE HP benchmark stack.

Only the cell characteristics are changed from the previous example, so the new results reflect only the impact of advanced electrochemistry.

The model constants for the relevant $i - v$ curves were given in Table 9. In absence of data, the same corrections are retained for temperature and humidity as the baseline cells.

The example cases are given in the run directories *run Modern 80kW* and *run Modern 80kW liqcool*. The liqcool example has additional inputs to size radiators and coolants.

The stack pressure is again specified as 2.5 atm. The model constants need not be interpolated since data is available at this pressure. The stack temperature is again 80°C.

The driver *drivercreateiv.m* is used to calculate the model constants. The $i - v$ data is available for $1, 1.25, 1.5, 2.0$ and 2.5 atm. The constants in Table 9 produced good fits.

Figure 23 shows the data and the model fit. The true efficiency and the voltage efficiency of the cell (Eq 16) are shown in Fig 24. There is truly no maximum efficiency point, the peak efficiency is merely an artifact of the leakage current. The leakage current is quite high in this example, hence the waveform different from the baseline cell, with a clear maximum around $v_c \approx 0.7$ V.

The driver *drivercreateiv.m* outputs an $i-v-p$ data table and a variable ipmax which is the row number of maximum power. The resolution Δi is set by the variable deltai (deltai = 0.01 is used). The $i - v - p$ table and ipmax are input in *fcvariables.m* as ivdatainp and ipmaxinp.

The model fit is input in *fcvariables.m*. For option ioptiv=1, the data table $i-v-p$ is input as ivdatainp along with the row number of maximum power ipmaxinp; for ioptiv=2, the 9 constants are input; and for ioptiv=3, a matrix PMAT is input which contains the 9 constants for each pressure.

The stack is built of same materials as the baseline stack.

The design environment remains SL/ISA.

The driver *driverpemfc.m* sizes a stack for a gross power given by the input POWERdsg_kW. The net electric power is lower due to balance of plant. The option ioptp=1 produces a stack of minimum weight. For this option, the cell at maximum power density hence a low voltage (Fig 23) and therefore a low efficiency (Fig 24) (efficiency is $\approx v/1.4722$). The option ioptp=3, with voltage specified as vc=0.7, produces a stack near maximum efficiency. For this option, the maximum power is also higher than the design power.

The cooling systems are still sized to the design power as the baseline case.

The heat load is specified for LHV as the baseline case.

Hydrogen is stored at 5.7% weight fraction as compressed gas at 700 bar 15°C. For option ioptf=1, the amount of hydrogen is specified. In this example 5 kg is assumed.

H2wfrac = 0.057

Whydrogen = 5

h2tankBar = 700

h2tankTC = 15

Table 28 shows two stack designs (columns 4 and 5) — minimum weight and high efficiency (as determined by the input cell voltage). In the table, only the variables that change from one column to the next are shown, an empty column is a repeat value. The outputs are saved in files *M80minwgtDESIGN.tex* and *M80highetaDESIGN.tex*.

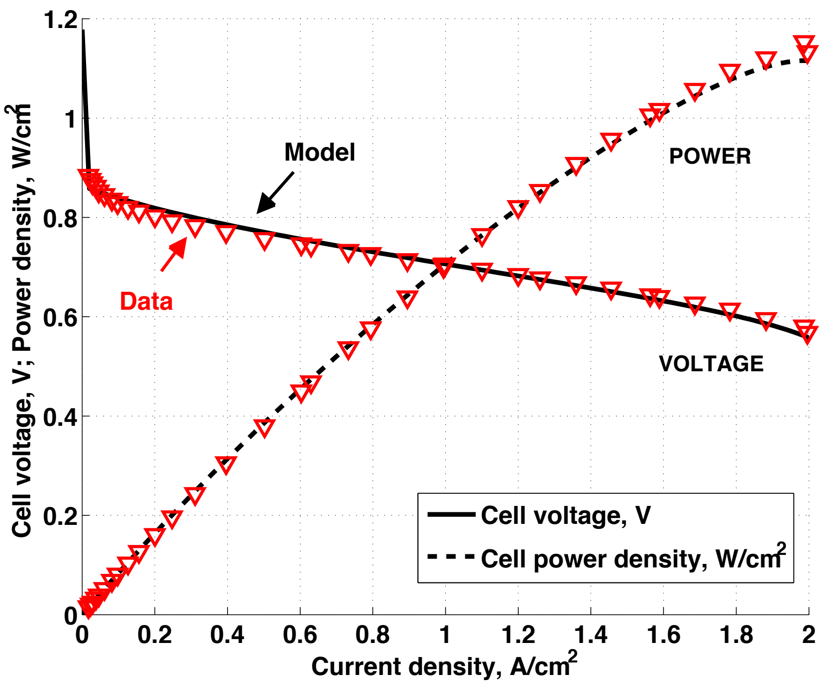

Figure 23: **Cell voltage, power, and heating versus current of modern automobile cells representative of Yr-2020 technology. Symbols are data at 2.5atm. Lines are models. 80°C.**

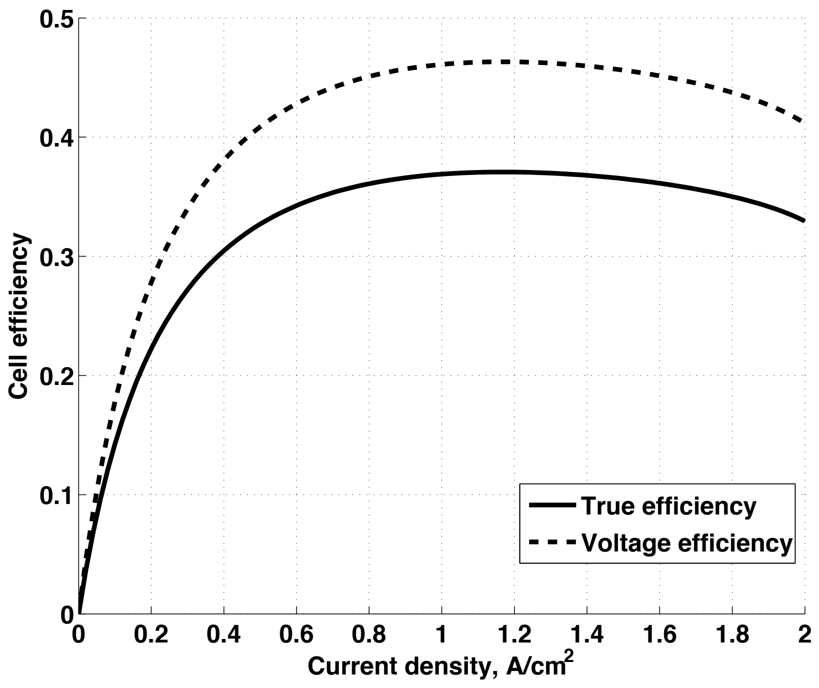

Figure 24: **Cell efficiency of modern cells at 2.5 atm, 80°C.**

Component	Variables	Unit	M80 min weight	M80 max η	M80 max η/liq cool
Cell	v_c	V	0.564	0.700	
	p_c	W/cm^2	1.116	0.734	
	η_c, η_v		0.33, 0.42,	0.37, 0.46	
	Weight	kg	0.06	0.11	
Stack	No of cells		443	357	
	Active area	cm^2	192	355	
	Weight	kg	30	45	
	Sp power	kW/kg	3.2	2.1	
	Design fac	kg/m^2	3.52		
Air	Air flow	kg/s	0.1732	0.1529	
	Power	kW	11.6	10.2	
	Weight	kg	16	24	
HT cool	Q gen	kW	134	95	
	Q load	kW	122	83	
	Q/ΔT	kW/°C	1.9	1.3	
	Power	kW	1.2	0.82	0.59
	Weight	kg	1.4	0.9	44
LT cool	Q gen	kW	8.7	7.7	13
	Power	kW	0.09	0.08	0.06
	Weight	kg	0.9	0.8	6.9
Electrical	Power	kW	1.9	1.9	
	Weight	kg	4.1	7.7	
Stack system					
	P, P net	kW, kWe	95, 80	93, 80	
	Pmax, Pmax net	kW, kWe	95, 80	141, 119	
	Weight	kg	52	78	127
	Sp Power	kWe/kg	1.53	1.03	0.63
Fuel	H2 flow	kg/s	0.002	0.0018	
	H2 stored	kg	5		
	Tank wt%		5.7		
	Tank + H2	kg	90		
Total system					
	Endurance	min	41	47	
	Energy	kWhe	55	62	
	Weight	kg	142	167	216
	Sp Power	kWe/kg	0.56	0.48	0.37
	Sp Energy	kWe-hr/kg	0.39	0.37	0.29
	Energy density	kWe/L	0.3	0.32	0.28
	η		0.28	0.32	
	GGE	U.S. gal	5.0	4.4	

Table 28: **80 kWe PEMFC designs with modern cells. SL/ISA. Stack: 250V, 2.5 atm, 80°C; tank: 700 bar, 15°C; cell: E_h, E_r: 1.47, 1.18 V (HHV) and 1.26, 1.17 (LHV). No water tank. η_c, η_v: true (HHV) and voltage efficiencies of cell; η: efficiency of the system (HHV). Gasoline gallon equivalent (GGE) based on combustion efficiency 0.25. Liquid cooling is sized to design power.**

The high efficiency design mimics the Toyota Mirai loosely, based on the few details that are easily available: cell weight 0.102 kg, number of cells 370, maximum power output 114 kW (kW or kWe unclear), 5 kg hydrogen storage at 700 bar, tank wt% 5.7, and tank internal volume of 122.4 L. The example assumes a storage temperature of 15°C and predicts 126 L.

In the minimum weight design, the design power 80 kWe is the maximum power and the maximum continuous power (MCP). In the high efficiency design, the design power 80 kWe is the maximum continous power (MCP), but it is not the maximum power. The maximum power is 119 kWe. But since the stack cooling system is sized to the design power, this is not continuous power. 119 kWe is 1.475 × 80 kWe which is ample margin to cover all rated power factors — intermediate rated power factor (IRP=1.185), maximum rated power factor (MRP=1.269) and contingency rated power factor (CRP=1.33). These margins can be reduced if the stack is sized to lower efficiencies (input voltage vc) which would also reduce its weight.

In summary, modern cells can provide typical power densities $p_c \approx 0.7 - 1.1$ W/cm^2 at efficiencies of $\eta \approx 0.33 - 0.37$. These power densities are twice as high as the baseline cells. Specific power of the stack can be $2 - 3$ kW/kg with a charitable design factor of 3.5 kg/m^2. Note that the stack design factor remains the same as baseline cells since no change is made to the structure and materials. But specific power of the stack system is what is more important. This value is driven by the weight of the balance of plant components particularly air and heat. The stack system specific power is placed around $1.0 - 1.5$ kWe/kg.

Comparison with batteries must include the hydrogen and tank weight. With modern cells, the specific energy and power are increased to $370 - 390$ Wh/kg and $0.56 - 0.48$ kW/kg respectively, where the net electrical energy and power are used. The specific power is equivalent to battery C-rates of 1.5 — 1.23 continuous, where the lower C-rate corresponds to the higher specific energy — a behavior consistent with any electrochemical source. Note that the battery specifications used for comparison should be pack level, available and installed.

For sizing an aircraft, the following power and fuel flow models are required.

1. Weight versus power (design),
2. Weight versus altitude for a specified power (design),
3. Power versus altitude for a specified engine (analysis),
4. Fuel flow versus power for a specified engine (design and analysis), and
5. Power versus velocity for a specified engine (design and analysis).

There is no model for 5. Power versus velocity. Some variation is expected from the airspeed effect on cooling and perhaps even ram effect on the stack pressure and air density. But there is no measured data to build such a model.

The following energy and electrical models are needed.

1. Ragone plot of specific power versus specific energy (design), and
2. Current, voltage and power range for a specified engine (analysis).

The drivers that generate these models are now briefly described.

The driver *driverpemfcPvsW.m* generates power versus weight. Figure 25 shows the net electrical power output versus the weight of the stack system.

The driver *driverpemfcWvsH.m* calculates the stack system weight versus altitude for a specified net electrical power. Figure 26 shows the variations for two stack designs. The compressor power increases with altitude which requires a heavier compressor (so the stack system weight is increased), while at the same time a larger stack is needed to compensate for this loss (so the stack weight is also increased).

The driver *driverpemfcragone.m* calculates the Ragone plots. Figure 27 shows the Ragone plots for the two designs. A Ragone plot shows the variation of specific power and specific energy for the total system between the two extremes of all stack-no fuel and no stack-all fuel. The left end of the plot approaches all stack-no fuel, so the specific energy approaches zero and specific power to that of the stack system alone (Fig 25). The right end of the plot approaches no stack-all fuel, so the specific energy approaches that of the fuel system alone whereas specific power approaches zero. The variation is a straight line, but curves on a log-log plot which is how it is typically plotted. Since power P is related to energy E through time t, $P\,t = E$, or $\log P = \log E - \log t$. For $t = 1$ hr, $\log P = \log E$; for $t = 1/2$ hr, $\log P = \log E + \log 2$; for $t = 2$ hr, $\log P = \log E - \log 2$; in general then, for $t = 2^n$ hr, $\log P = \log E - n \log 2$, where $n = \ldots, -2, -1, 0, 1, 2 \ldots$ produce parallel lines equi-spaced by $\log 2$. The Ragone plots in Fig 27 show that the high efficiency stack system is lighter for continuous operations longer than 2 hours and that their maximum power is higher than the minimum weight stacks. Thus, the selection of design is governed by the time of continuous operation, and the requirement for intermittent high power. For continuous operation beyond 2 hr, the high efficiency stack will burn less hydrogen and this effect will more than compensate for the stack weight.

Figure 28 shows the dramatic impact of hydrogen storage weight fraction.

Figure 29 shows specific power versus energy density.

Figure 30 shows the impact of the type of hydrogen storage on the energy density. There is not much to be gained by increasing pressure; but lowering the temperature could have a major impact. However, none of these effects are nearly as dramatic as that of a rising weight fraction, as shown in Fig 28.

Once the stack is sized, its operation is determined by the stack current. The stack power and voltage variations with current are shown in Fig 31, stack efficiency in Fig 32, and stack specific fuel consumption (SFC) in Fig 33.

The driver *driverpemfcenginechartvsH.m* generates the engine charts of power versus altitude. Figure 34 shows these variations.

The driver *driverpemfcenginechartvsP.m* generates the engine charts of fuel flow versus power. Figure 36 shows the specific fuel consumption. The effect of the leakage current is quite significant, particularly at low power operation.

The example *run Modern 80kW liqcool* is same as the maximum efficiency stack, but with the cooling systems sized in greater detail. The radiator and the coolant weight are now included in sizing. This stack is shown in column 6 of Table 28.

The low temperature cooling system is sized to maximum power.

iLTR = 1

The high temperature cooling system is sized to the design power.

iHTR = 0

So the stack can be operated at maximum power as long as the high temperature cooling system would allow.

The radiator ambient temperature is assumed as 20°C which is lower than the typical 40°C for automobile fuel cells. The stack ambient temperature is specified as 40°C to allow for temperature rise in its housing. For both the cooling systems, the radiator weight is assumed to be 3.5 kg/m^2. The radiator temperature is T_r = 95°C. The coolant is water with specific heat ratio C_p = 4180 J/kg-K and density ρ = 997 kg/m^3. In an actual stack deionized water/Glycol mixer with other additives would be used and properties specified accordingly. The rise in coolant temperature is input as ΔT_c = 10°C (typically the maximum is 15°C). Coolant volume flow rates of around 70 L/min are predicted which are consistent with a typical automobile fuel cell. Fresh supply for 20 sec continuous is input, which gives a coolant capacity of 23 L. Conventional automobiles typically have $\approx 5 - 18$ L coolant capacity. The data for fuel cell automobiles is not available. The increase in overall weight is very significant with the radiator and the coolant.

In summary, a more detailed model of the cooling system increased the stack system weight very significantly from 78 kg to 128 kg. The specific energy and power for the total system are reduced to 290 Wh/kg and 0.37 kW/kg respectively, for net electrical energy and power. The specific power is equivalent to a battery C-rate of 1.28. The leakage current was found to have a significant effect on the low power operation.

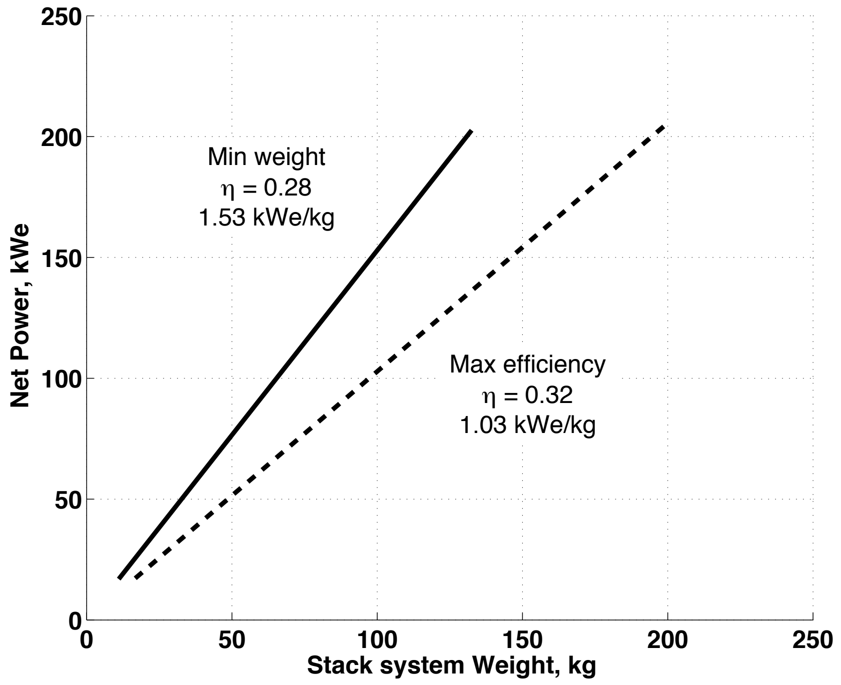

Figure 25: **Stack net electrical power (kWe) versus stack system weight (kg).** η **is system efficiency.**

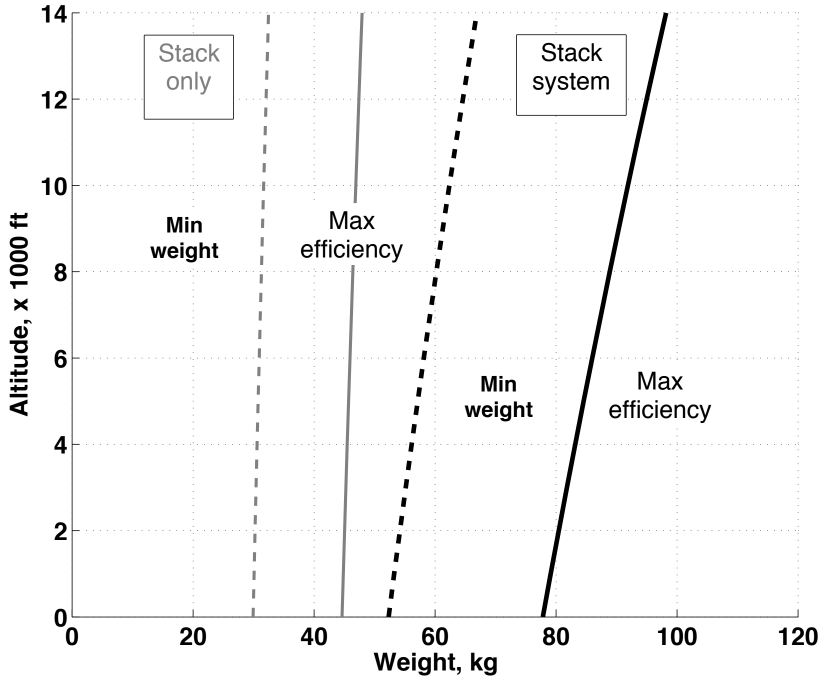

Figure 26: **Stack and stack system weight versus altitude/ISA; net electrical power is 80 kWe.**

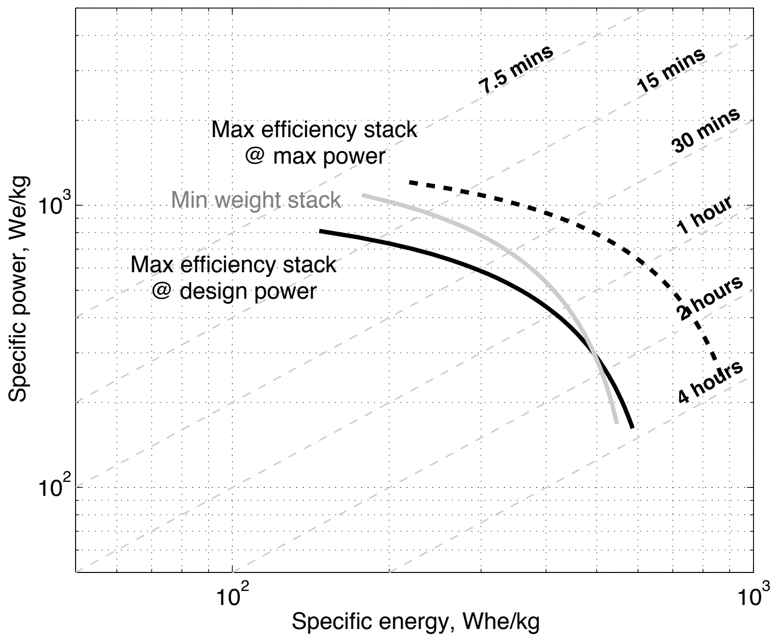

Figure 27: **Ragone plot. Full system specific power versus specific energy. Design power and maximum power shown for the maximum efficiency stack. Design power = maximum power for the minimum weight stack.**

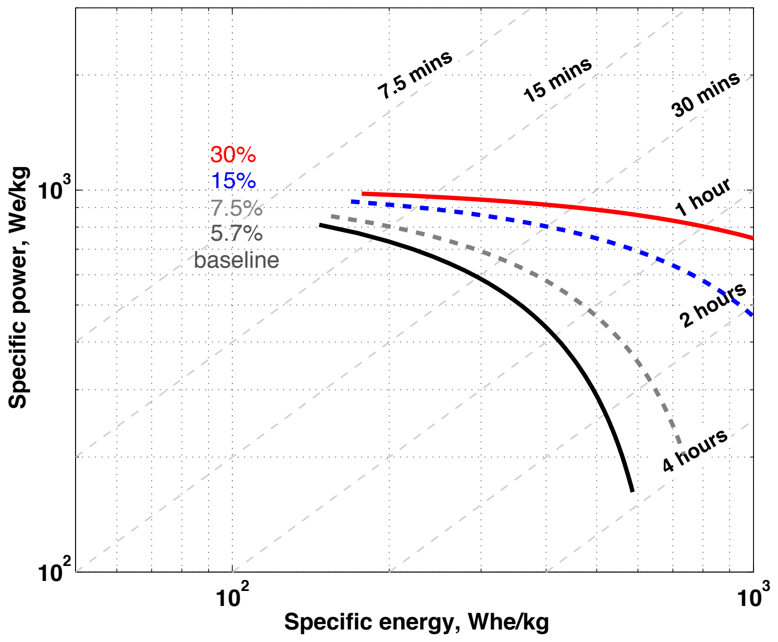

Figure 28: **Ragone plot. Impact of hydrogen storage weight fraction 5.7%–30%. Design power shown for the maximum efficiency stack.**

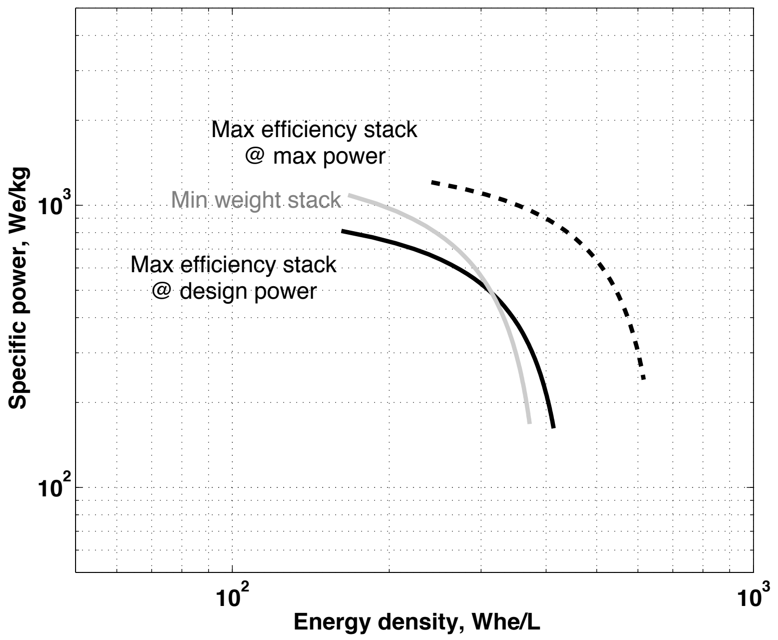

Figure 29: **Specific power versus energy density. 700 bar 15°C compressed gas storage. Design power and maximum power shown for the maximum efficiency stack. Design power = maximum power for the minimum weight stack.**

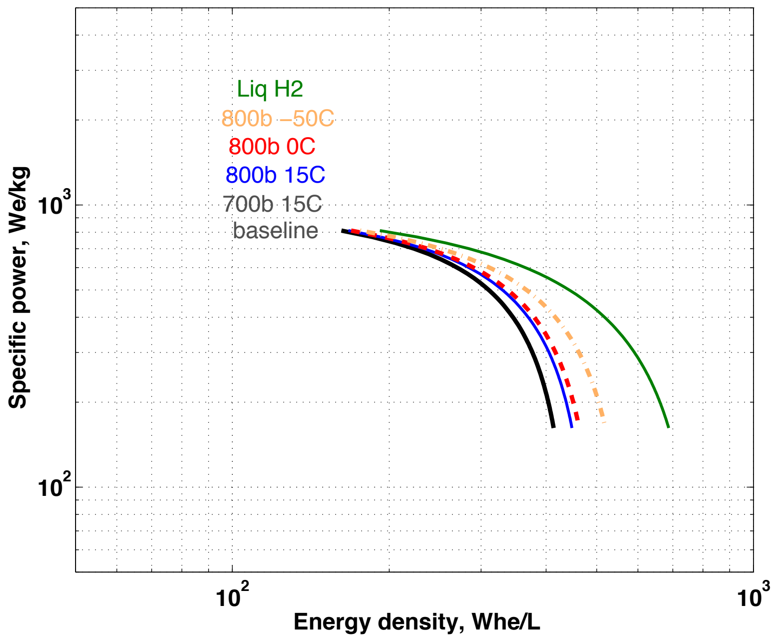

Figure 30: **Specific power versus energy density. Impact of hydrogen storage type. Design power shown for the maximum efficiency stack.**

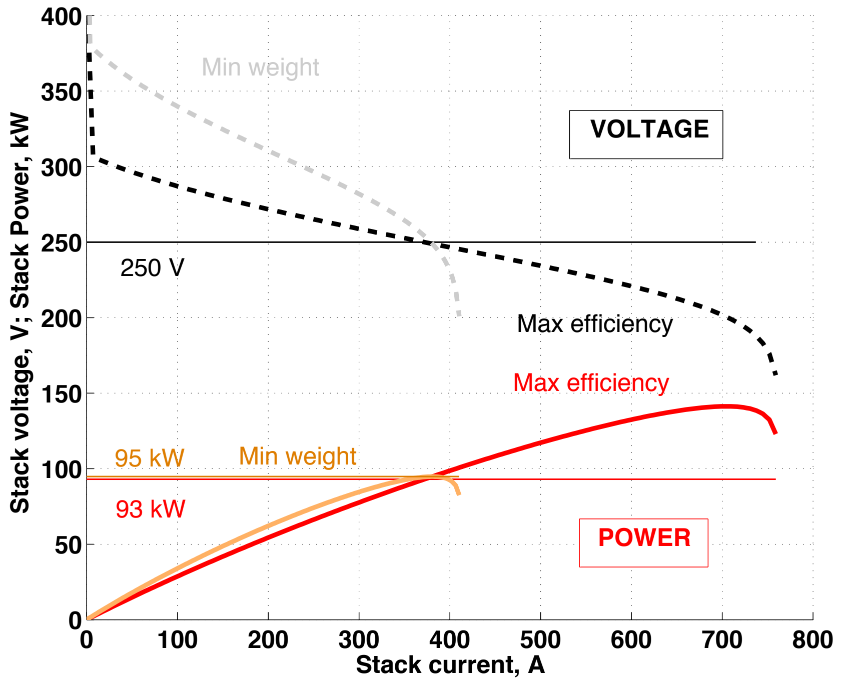

Figure 31: **Operating electrical parameters of 80 kWe stacks.**

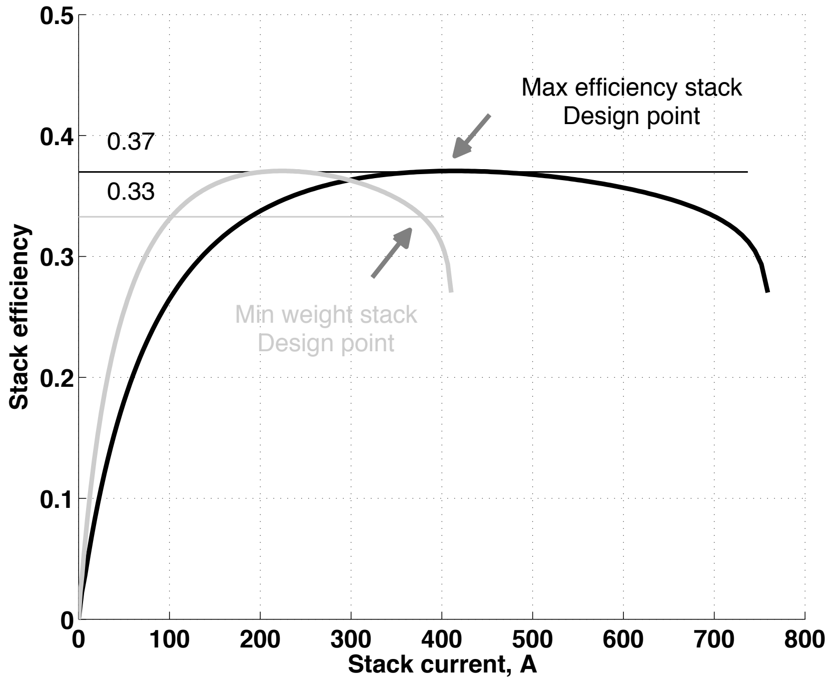

Figure 32: **Operating efficiency of 80 kWe stacks.**

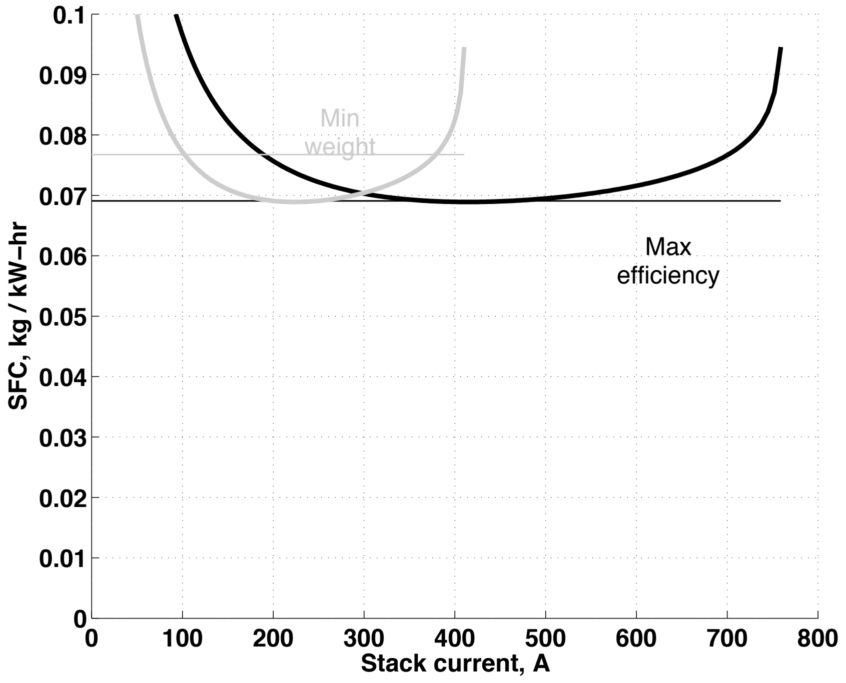

Figure 33: **Operating specific fuel consumption of 80 kWe stacks.**

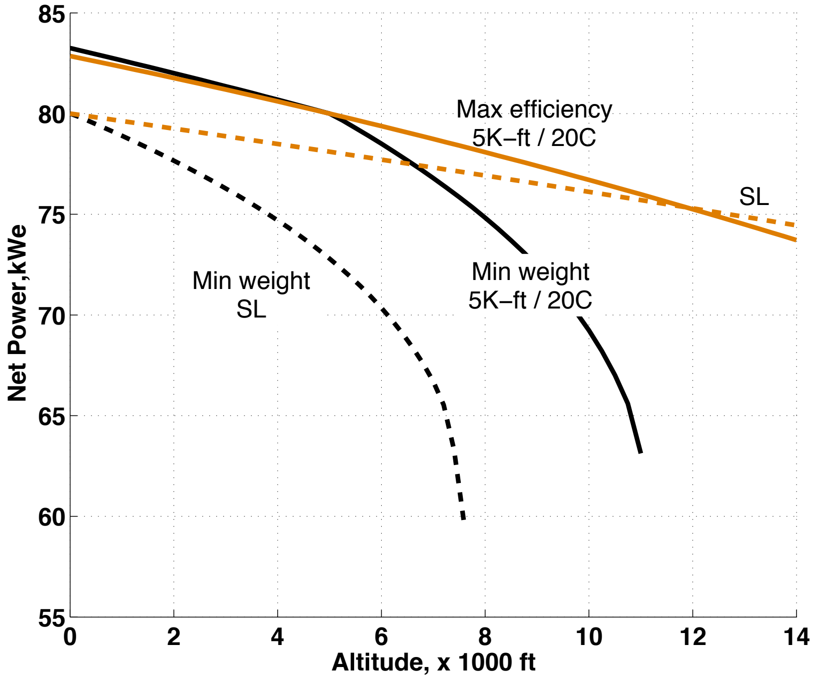

Figure 34: **Operating altitude versus power for 80 kWe stacks. Impact of SL and 5,000-ft / 20°C designs. Fuel flow fixed at design power 80 kWe.**

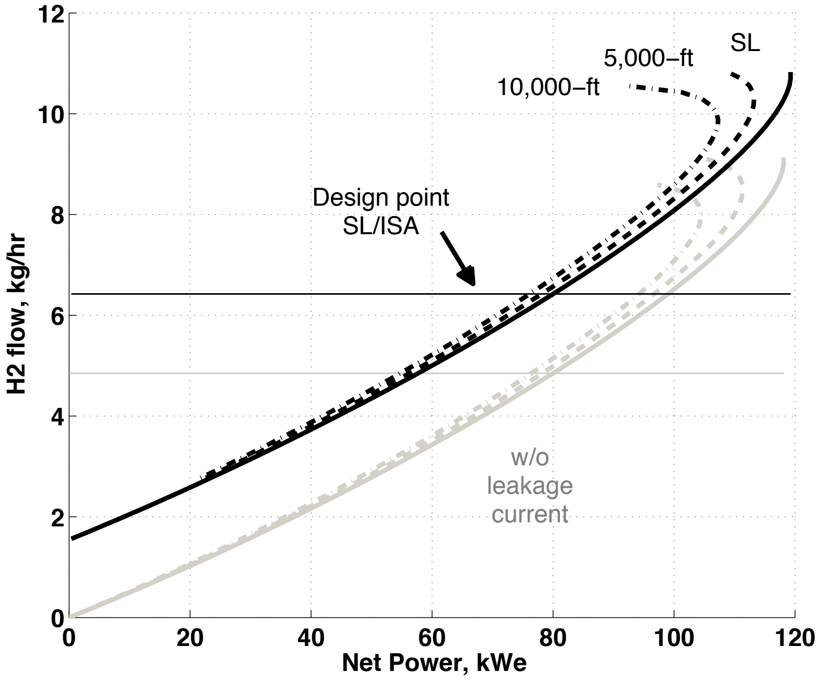

Figure 35: **Operating fuel flow versus power at various altitudes for a ISA/SL design. Impact of leakage current.**

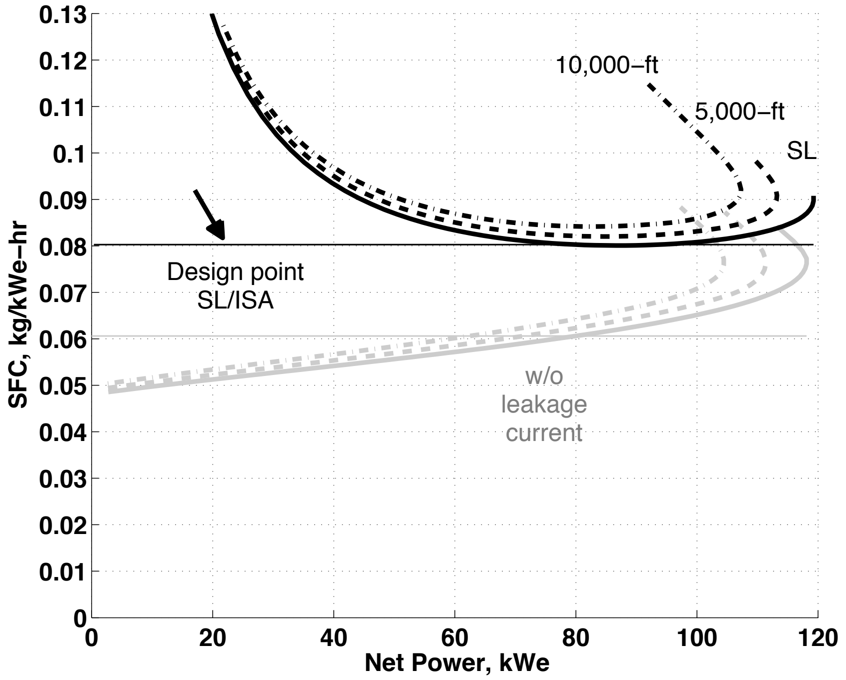

Figure 36: **Operating fuel flow versus power at various altitudes for a ISA/SL design. Impact of leakage current.**

13 Example: Modern 500 kW

The 80 kWe liquid cooled example of the previous section is extended to a hypothetical 500 kWe power. In preparation of rotorcraft sizing, the environment is changed from SL/ISA to 5000-ft / 20°C.

The new environment inputs in *fcvariables.m* are.

iatm = 1

altitudeDesign = 1524

temperatureC = 20

Hydrogen is still stored at 5.7% weight fraction as compressed gas at 700 bar 15° C but the amount of hydrogen is doubled to 10 kg. For option ioptf=1, the amount of hydrogen is specified.

H2wfrac = 0.057

Whydrogen = 10

h2tankBar = 700

h2tankTC = 15

A 500 kW system designed for rotorcraft will likely require many changes that will affect scaling with power. These include the radiator heat transfer coefficients, radiator weight along with support structure, the surface area of the stack after packaging, the type of coolant, and coolant capacity. A realistic exploration of these changes require reliable test data.

eVTOL might allow the elimination of some subsystems. For low altitude flight, a pressurized stack may not be essential, so the compressor-expander can be deleted, and along with it, the low temperature cooling system. Because hydrogen storage is needed for only a short duration ($1 - 3$ hr), high weight fraction tanks ($15 - 30\%$ or higher) that allow boil-off, may be practical. These excursions are easily modeled. Some of them are illustrated below and their impact studied later in rotorcraft sizing.

The same drivers described earlier are used to design the stacks.

The driver *driverpemfcdesign.m* sizes stacks for net electrical power. Table 29 shows four stacks designed for 500 kWe. Two of these are 2.5 atm stacks (columns 4 and 5) — minimum weight and high efficiency (as determined by the input cell voltage from 2.5 atm $i - v$ data), and the other two are 1 atm stacks (columns 6 and 7) — minimum weight and high efficiency (as determined by the input cell voltage from 1 atm $i - v$ data).

The driver *driverpemfcPvsW.m* generates power versus weight.

The relevant output is FCST.StackSystemWvsPe (Table 26) which contains net power, maximum net power, and the stack system weight. Figure 37 shows the variation of weight with power for the minimum weight stack designs. The design power is the maximum power for a minimum weight design. Figure 38 shows the effect of environment on the design. Figure 39 shows the same variation for a maximum efficiency design. Figure 40 shows the effect of environment. The maximum power is higher than the design power for a maximum efficiency stack. But the maximum power is not continuous power since the stack cooling was sized only to the design power. If there is a low and high power requirement, such as cruise and hover, the stack could be sized to the lower power as the design power (stack weight W_L) if the higher power is within the maximum power and the cooling system limits. The next option would be to size it to the higher power as the maximum power (stack weight W_H). The last and the heaviest option would be to size it to the higher power as the design power. The last two options are used later in rotorcraft sizing.

The driver *driverpemfcenginechartvsP.m* generates fuel flow versus power. The relevant outputs are FCOP.H2flowvsPe, FCOP.SFCvsPe and FCOP.SFCvsPefrac (Table 26) which contain respectively the fuel flow versus net power, specific fuel consumption versus net power, and specific fuel consumption versus net power as fraction of net design power.

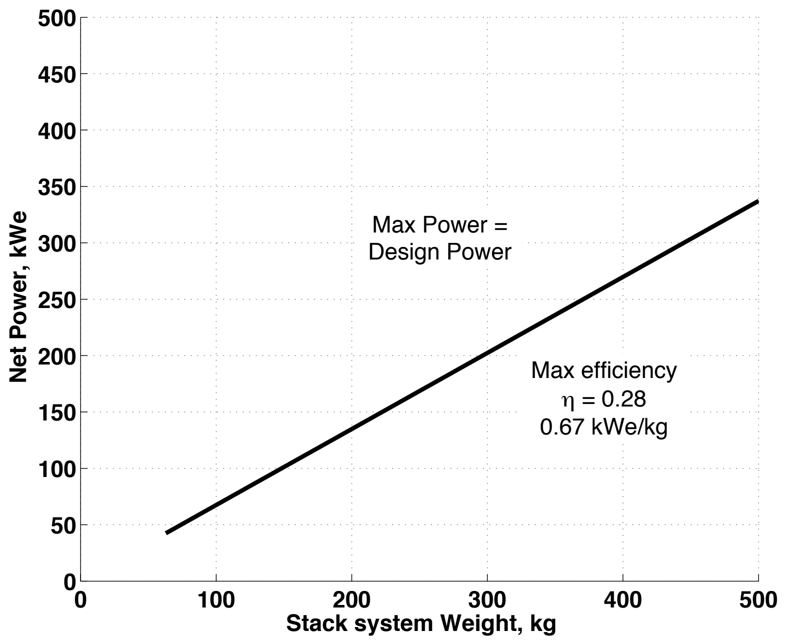

Figure 37: **Stack net electrical power (kWe) versus stack system weight (kg) for a minimum weight stack. Maximum power = design power. 2.5 atm stack. SL/ISA design.**

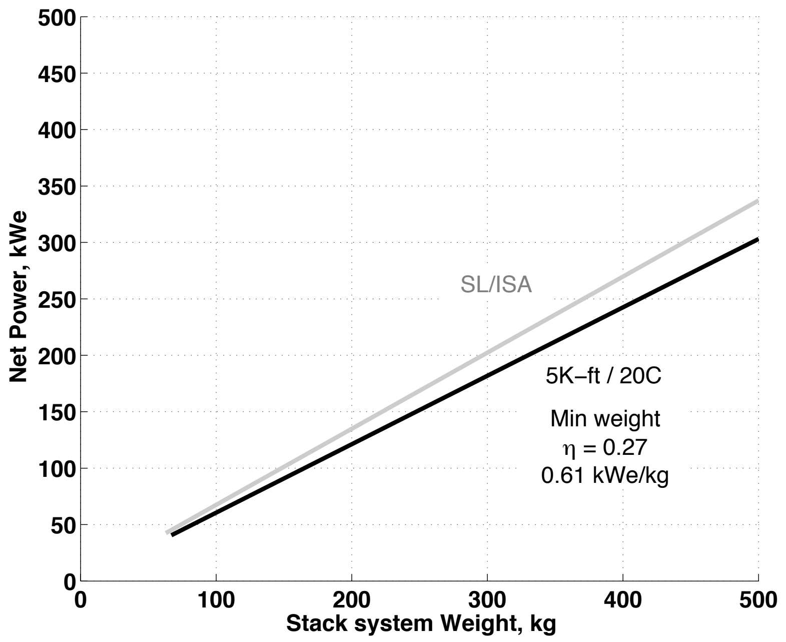

Figure 38: **Stack net electrical power (kWe) versus stack system weight (kg) for a minimum weight stack. Maximum power = design power. 2.5 atm stack. ISA/SL versus 5000-ft/20°C design.**

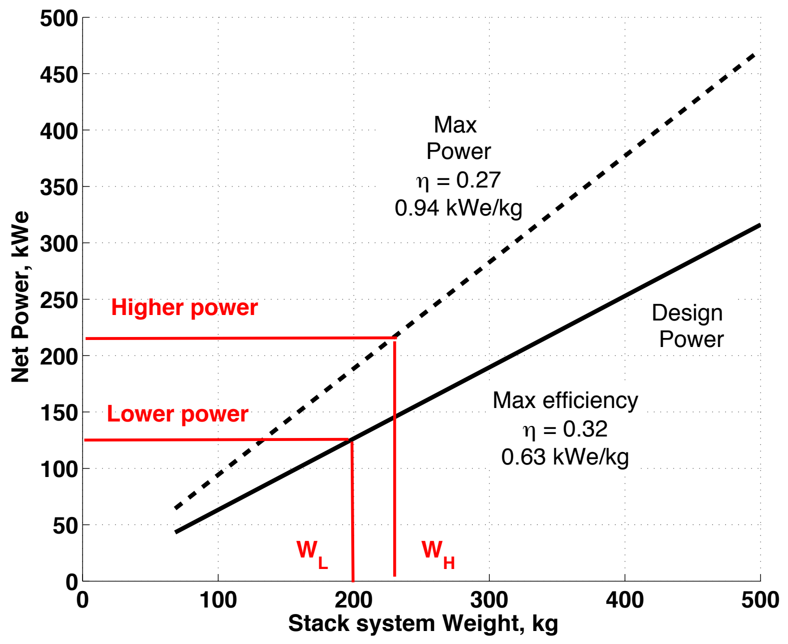

Figure 39: **Stack net electrical power (kWe) versus stack system weight (kg) for a maximum efficiency stack. Maximum power > design power. 2.5 atm stack. SL/ISA design.**

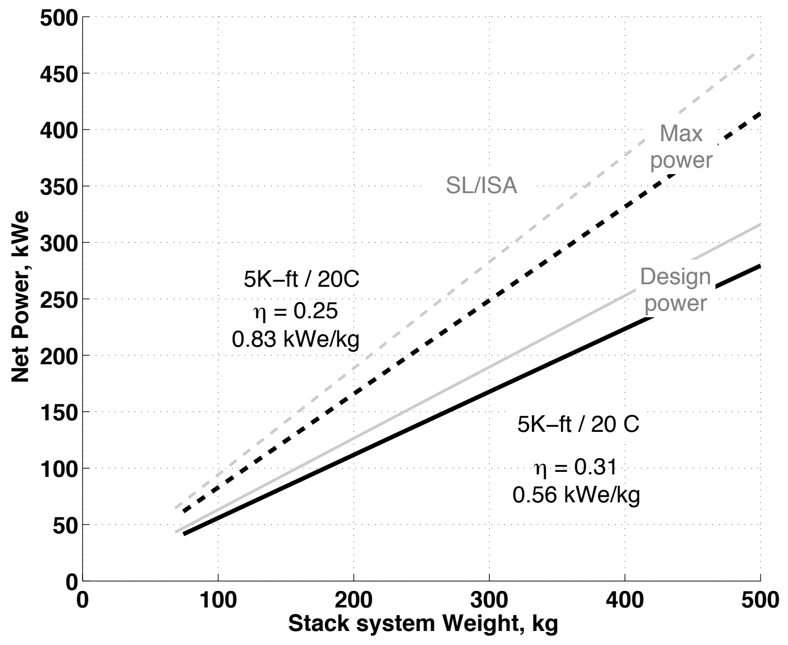

Figure 40: **Stack net electrical power (kWe) versus stack system weight (kg) for a maximum efficiency stack. Maximum power > design power. 2.5 atm stack. SL/ISA versus 5000-ft/20°C design.**

Figure 41 shows the fuel flow for stacks of $100 - 500$ kWe designs. Figure 42 shows the corresponding specific fuel consumption. Figure 43 shows the specific fuel consumption plotted versus fraction of net design power. The variations collapse to one curve. Note that the minimum occurs at a slight offset from 1.0. This is because the maximum efficiency point on the $i - v$ curve occurs in fact at $v_c = 0.688$ V not $v_c = 0.7$ V as input. If re-designed with $v_c = 0.688$ V, the curves will touch the minimum at 1.0.

The two low pressure 1 atm stacks are sized to explore possible avenues of weight reduction. The 1.0 atm data in Fig 13 is used which produce the model constants given in Table 9. The constants can be inserted in the PMAT table with ioptiv = 3 or input directly. The only change in *fcvariables.m* is then

Pstack = 1.0 × 101325

For the maximum efficiency design, *drivercreateiv.m* is used to find the voltage of maximum efficiency.

vc = 0.67

Since the stacks are designed for 5000-ft, the 1 atm stacks also require a compressor. But the compression ratio is small (about 0.83) and the temperature of the compressor output is lower than the stack temperature so the low temperature cooling system can still be avoided. These lead to significant weight reduction.

Although the low pressure stacks have a lower voltage, they also have a lower leakage current, so the cell efficiency ultimately remains the same. The drawback is they have lower power density, and are therefore twice as heavy. However, the overall weights change when the entire stack system is considered. The low pressure stacks need a smaller compressor so weight and balance of plant are both reduced, and the overall system is lighter. The net efficiency is also higher, so the specific fuel consumption in Fig 44 is lower. Figures 45 and 46 show the weights for minimum weight designs. Figures 47 and 48 show the weights for maximum efficiency designs. The penalty in low pressure stacks is in the ceiling due to altitude loss, shown earlier in Fig 34.

In summary, a low pressure stack system might be lighter overall, even though the stack itself is heavier. The air management system can be lighter and the low temperature cooling system can be eliminated. Thus, interesting possibilities exist, but definitive conclusions require a testbed and weight data.

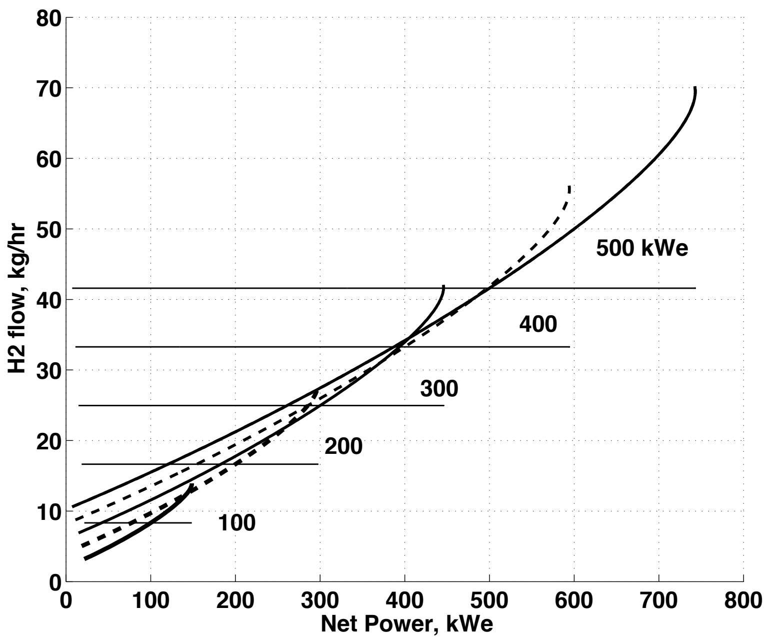

Figure 41: **Operating fuel flow versus power for stacks of various design powers. Horizontal lines mark fuel flow at design power. Maximum efficiency stacks. 5000-ft/20°C designs.**

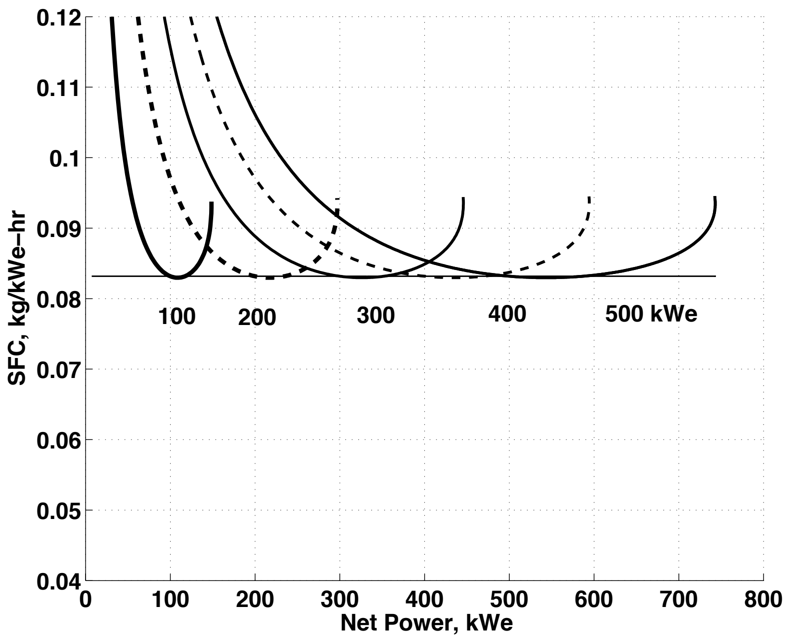

Figure 42: **Operating specific fuel consumption (SFC) versus power for stacks of various design powers. Horizontal line marks SFC at design power. Maximum efficiency stacks. 5000-ft/20°C designs.**

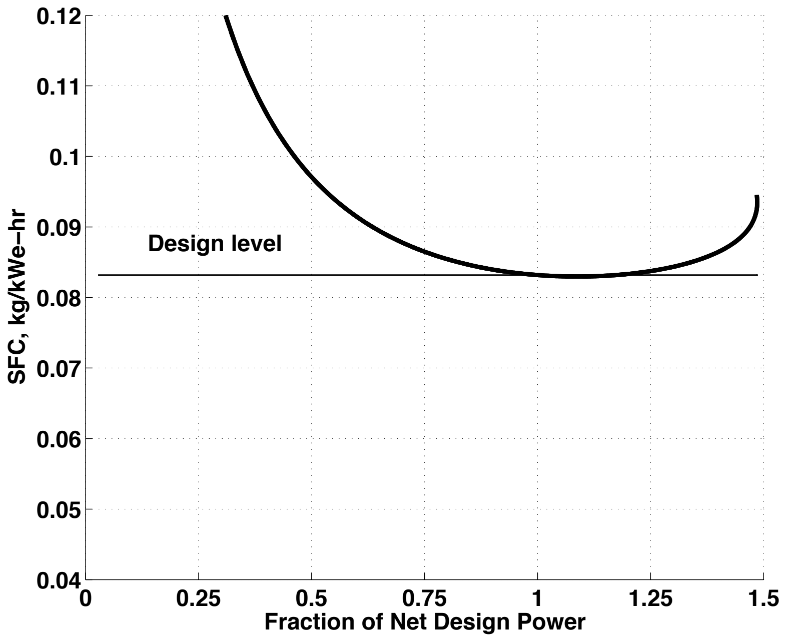

Figure 43: **Operating specific fuel consumption (SFC) versus fraction of design power for stacks of various design powers. Horizontal line marks SFC at design power. Maximum efficiency stacks. 5000-ft/20°C designs.**

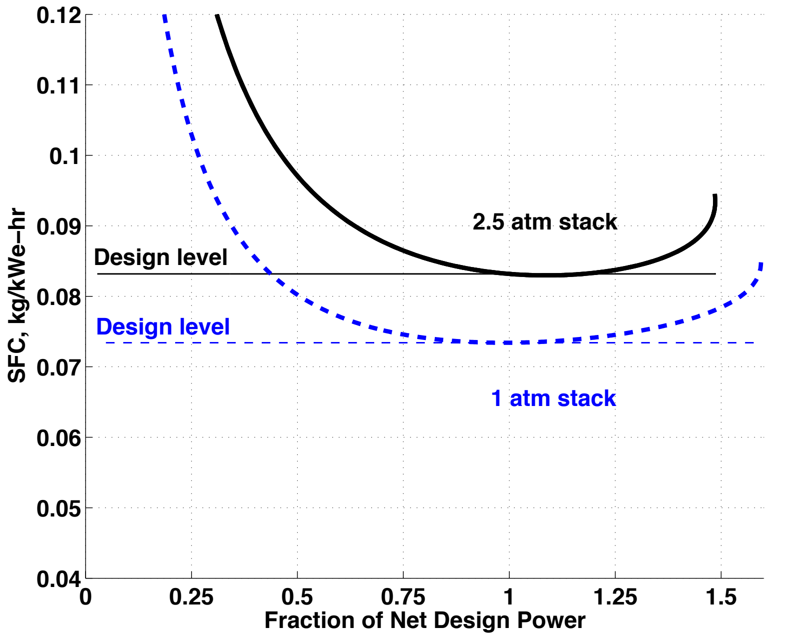

Figure 44: **Operating specific fuel consumption (SFC) versus fraction of design power for stacks of various design powers. Horizontal line marks SFC at design power. 2.5 atm versus 1 atm stacks. 5000-ft/20°C designs.**

Component	Variables	Unit	2.5 atm min weight	2.5 atm max η	1 atm min weight	1 atm max η
Cell	v_c	V	0.564	0.700	0.530	0.67
	p_c	W/cm^2	1.116	0.734	0.794	0.493
	q_c	W/cm^2	1.367	0.582	1.090	0.431
	η_c		0.33	0.37	0.33	0.38
	η_v		0.42	0.46	0.41	0.48
Stack	No of cells		443	357	472	373
	Active area	cm^2	1247	2297	1467	2957
	Q load	kW	795	541	758	498
	Q/ΔT	kW/°C	13.3	9.0	12.6	8.3
	Stack Sp P	kW/kg	3.17	2.08	2.25	1.4
	Design factor	kW/m^2	3.52	3.52	3.52	3.52
BOP						
	Air	kW	97	86	34	29
	LT cool	kW	0.67	0.60	0	0
	HT cool	kW	5.7	3.9	5	3.6
	Elec	kW	12.3	12.0	11	10.9
	Total	kW	161	102	50	43
Weights						
	Stack	kg	194	289	244	389
	Air	kg	130	193	17	27
	LT cool	kg	51	76	0	0
	HT cool	kg	422	287	402	264
	Elec	kg	27	50	23	47
	Total	kg	825	895	687	728
Fuel						
	H2 flow	kg/s	0.0131	0.0116	0.0119	0.0102
	H2 stored	kg	10			
	Tank wt%		5.7	—	same	—
	Tank + H2	kg	175	—	same	—
Total system						
	P, P net	kW, kWe	616, 500	602, 500	550, 500	543, 500
	Pmax, Pmax net	kW, kWe	616, 500	915, 742	550, 500	876, 796
	Stack sys Sp P	kWe/kg	0.61	0.56	0.73	0.69
	Stack sys Sp P	kWe/kg max	0.61	0.83	0.73	1.09
	Sp Energy	kWe-hr/kg	0.130	0.135	0.145	0.163
	η		0.27	0.31	0.30	0.35
	Design factor	kg/m^2	15	11	10	6.6

Table 29: **500 kWe PEMFC designs with modern cells and liquid cooling. 5000-ft/20°C. Stack: 250V, 80°C. No water tank. η_c, η_v: true (HHV) and voltage efficiencies of cell; η: efficiency of the system (HHV) at design power. BOP is at design power. Stack cooling is sized to design power.**

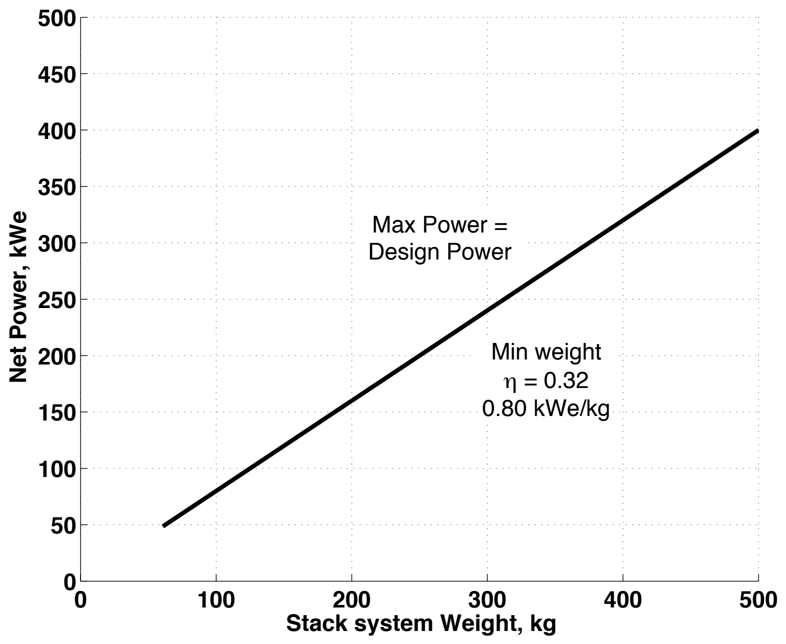

Figure 45: **Stack net electrical power (kWe) versus stack system weight (kg) for a minimum weight stack. Maximum power = design power. 1 atm stack. SL/ISA design.**

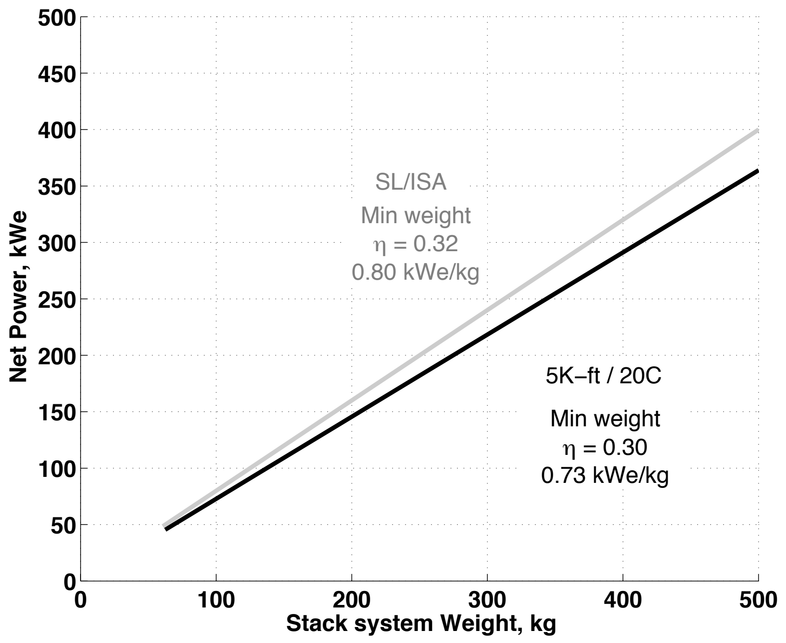

Figure 46: **Stack net electrical power (kWe) versus stack system weight (kg) for a minimum weight stack. Maximum power = design power. 1 atm stack. SL/ISA versus 5000-ft/20°C design.**

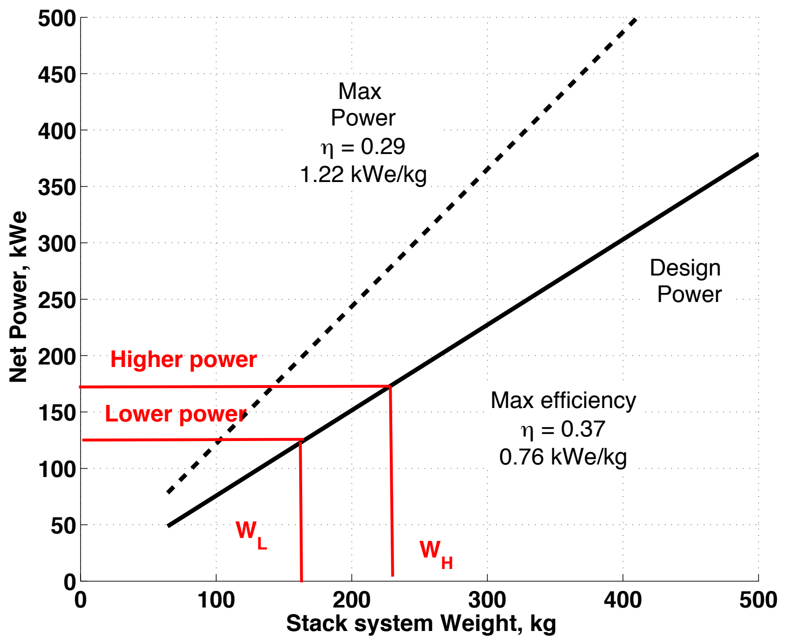

Figure 47: **Stack net electrical power (kWe) versus stack system weight (kg) for a maximum efficiency stack. Maximum power > design power. 1 atm stack. SL/ISA design.**

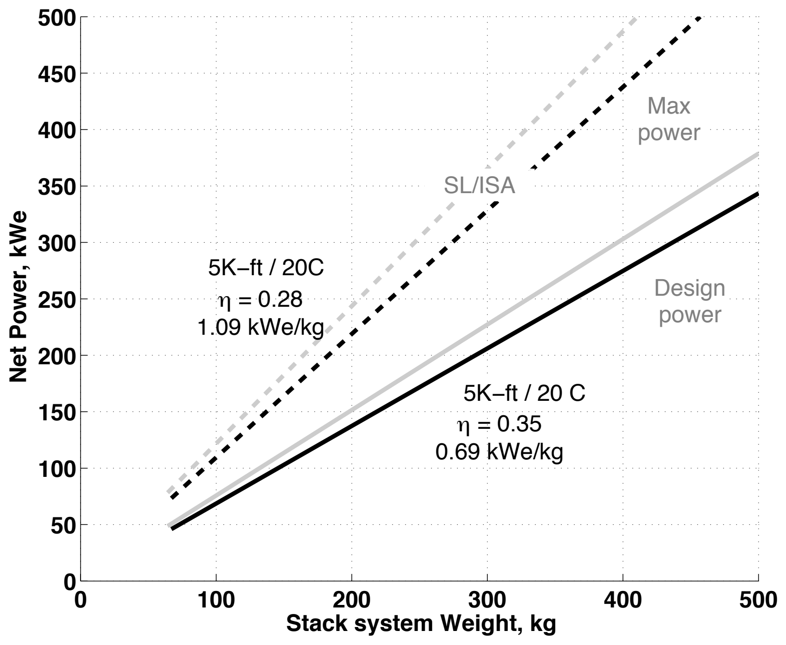

Figure 48: **Stack net electrical power (kWe) versus stack system weight (kg) for a maximum efficiency stack. Maximum power > design power. 1 atm stack. SL/ISA vs 5000-ft/20°C design.**

14 Rotorcraft Sizing

A simple sizing method is presented. The objective is to illustrate how the fuel cell models enter and affect sizing, and how it compares with other powerplants, namely, batteries and piston-engines. The model is kept deliberately simple, with a minimal set of parameters, so it can be reproduced easily and might serve as a common-basis for assessment by fuel cell, hydrogen, and rotorcraft specialists.

The sizing problem is defined as the following.

Given — Mission and payload W_{PAY}.

Find — Design gross weight W and engine power P.

The mission is generally described in segments, with environment, reserves, power factors and flight tasks defined for each segment. For simplicity, only two segments are considered, a hover and a cruise segment.

The principal design variable is the disk loading DL. $DL = W/A$ (lb/ft^2 or N/m^2) where W is the rotorcraft gross take-off weight (lb or N) and A (ft^2 or m^2) is the projected disk area of all lifting rotors that balance the weight. If there are N_R identical rotors of radius R (ft or m), the projected disk area is $A = N_R\,\pi R^2$. Disk loadings of 8 and 14 lb/ft^2 are typical of large helicopters and tiltrotors respectively, whereas $2-4$ lb/ft^2 are typical of small light-utility helicopters.

The sizing method finds the gross weight and engine power for a specified disk loading. The disk loading is then varied to span the design space.

The sizing method begins with an initial estimate of W, calculates aircraft power, calculates the weights of all subsystems based on this power, adds them to obtain a new aircraft weight, and iterates the new weight until convergence. The subsystem models keep account of the running weight of the aircraft due to fuel burn or payload change. Here, weight changes during the mission are ignored.

The hover Figure of Merit FM and cruise lift to drag ratios L/D are required. These are the efficiency metrics in hover and cruise.

FM = $0.7-0.8$ for all rotorcraft. FM < 1.0.

L/D = $4-6$ for helicopters and compounds and $7-9$ for tiltrotors.

If the configuration is known, FM and L/D can be calculated and updated within the weight iteration.

14.1 Basic Sizing

Specify a disk loading DL, hover power factor PF, air density ρ, and cruise speed V.

Assume FM in hover out of ground effect (OGE) and L/D in cruise.

Starting from an initial gross take-off weight W, iterate the following steps to converge.

1. Calculate hover power out of ground effect (OGE) P_H and engine power P

$$\begin{aligned} P_H &= \frac{1}{FM} W \sqrt{\frac{DL}{2\,\rho}} \\ P &= P_H \times PF \end{aligned} \tag{127}$$

   The power factor $PF \geq 1$ is a specified margin.

   The unit of ρ is kg/m^3 if W is in N, or slug/ft^3 if W is in lb.

2. From DL and number of rotors, find rotor radius R.

3. Calculate cruise power P_C at the specified cruise speed V

$$P_C = \frac{W\,V}{L/D} \tag{128}$$

4. Calculate structural weight W_S

$$W_S = f_S\;W \tag{129}$$

The factor f_S is very difficult to determine yet it can make or break a design. This is where full-scale prototype testing becomes essential for a new kind of aircraft. Historical trends of conventional helicopters indicate $f_S = 0.25$ is a good target to aspire (Harris 2012).

5. Calculate power plant weight W_P and fuel weight W_{Fuel}.

The engine models are from Harris 2012 and the motor model from Ng and Datta 2019.

$$\begin{aligned} \text{Turboshaft: } W_P \text{ in lb} &= 1.8\;(P \text{ in hp})^{0.9} \\ \text{Piston: } W_P \text{ in lb} &= 6.5\;(P \text{ in hp})^{0.9} \\ \text{Electric motor: } W_P \text{ in kg} &= 0.4\;(Q \text{ in Nm}/\eta)^{0.71} \end{aligned} \tag{130}$$

where Q is the motor torque and η is motor efficiency.

The fuel weight models are

$$\begin{aligned} \text{Turboshaft: } W_{Fuel} \text{ in lb} &= SFC \text{ in lb/hp-hr } \times E \text{ in hp-hr} \\ \text{Piston: } W_{Fuel} \text{ in lb} &= SFC \text{ in lb/hp-hr } \times E \text{ in hp-hr} \\ \text{Battery: } W_{Batt} \text{ in kg} &= \frac{E_B \text{ in Wh}}{\text{Specific Energy SE in Wh/kg}} \end{aligned} \tag{131}$$

The specific fuel consumption (SFC) varies from turboshaft to piston. E is the mission energy. If SFC varies from hover to cruise then the calculations are performed accordingly using energy in hover and cruise (E_H and E_C). In general, engine charts provide SFC as a function of power.

The battery energy E_B requires calculation of C-rates ζ (1/hr). If h_H and h_C are the hover and cruise times in hours, including reserves, then the energy and required C-rates are

$$\begin{aligned} E_H &= h_H\,P_H \\ E_C &= h_C\,P_C \\ E &= E_H + E_C \\ \zeta_H &= \frac{P_H}{E} = \frac{1}{h_H + (P_C/P_H)\,h_C} \text{ or 0 if } h_H = 0 \\ \zeta_C &= \frac{P_C}{E} = \frac{1}{(P_H/P_C)\,h_H + h_C} \text{ or 0 if } h_C = 0 \end{aligned} \tag{132}$$

If the required C-rates are within the maximum available C-rate ζ_{max} then $E_B = E$. Otherwise $E_B = P/\zeta_{max}$ where P is the higher power. In this case, more energy is carried than needed. For example, if only 5 min of hover is needed, i.e. 1/12 hr, then the required C-rate in hover is $\zeta_H = P_H/E_H = 12$. If the available C-rate is only 6, then $\zeta_H = P_H/E_H = 6$, or E_H =(1/6) P_H, which means the battery must be sized for 1/6 hr of hover, i.e. 10 min, even when only 5 min were needed.

The specific energy SE (Wh/kg) of current Li-ion batteries is approximated by Ng and Datta 2019

$$\text{SE} = \frac{v_c\,C_c}{(0.0075 + 0.024\,C_c)\,f_T} \approx \frac{v_c}{0.024\,f_T} \tag{133}$$

where $v_c = 3.7$ V and C_c is the individual cell capacity in Ah. The technology factor f_T accounts for the available and installed energy at the pack level. $f_T = 1$ places SE around 150 Wh/kg. The battery SE is in general a function of C-rate, particularly for C-rates ≥ 2. Then the battery weight is found piece-wise for hover and cruise.

The PEMFC is an engine and generator combined, that could be considered a part of W_P, but is included in W_{Fuel} to allow for a consistent comparison with batteries. The weight W (kg) and volume S (m^3) are given below.

$$\begin{aligned}\text{PEMFC: } W_{Fuel} &= W_{\text{stack sys}} + W_{\text{tank sys}} \\ W_{\text{stack sys}} &= k_0 + k_1\,(P/p_c) \text{ or simply a function } = f(P) \\ W_{\text{tank sys}} &= W_{\text{h2}}/w_f \\ W_{\text{h2}} &= SFC \text{ in kg/kWe-hr} \times E \text{ in kW-hr or piece-wise when } SFC = f(P) \\ S_{\text{tank sys}} &= \frac{W_{\text{h2}}/\rho_{\text{h2}}}{1 - s_{ov}}\end{aligned} \tag{134}$$

The PEMFC design code supplies the factors k_0 and k_1 and the cell power density p_c (or the table $f(P)$ directly), SFC, and the density ρ of hydrogen with full tank. w_f is the specified tank weight fraction (see Eq 90) and s_{ov} is the specified tank volume fraction (see Eq 97). $w_f = 0.057$ for automobile hydrogen storage and $\rho = 0.042$ kg/m^3 for 700 bar 15°C compressed gas at full tank. The tank volume overhead is assumed to be $s_{ov} = 0.1$.

6. Calculate empty weight W_E.

   Assume weights for vibration and systems are a specified fraction of empty weight.

$$\begin{aligned}W_E &= W_S + W_P + W_{vib} + W_{sys} \\ &= W_S + W_P + f_E\, W_E \\ W_E &= \frac{W_S + W_P}{1 - f_E}\end{aligned} \tag{135}$$

7. Calculate useful weight W_{UL}. Assume the fixed useful weight W_{FUL} (such as the pilot) is included in W_{PAY}. The definition of payload is then the same for manned or unmanned aircraft.

$$\begin{aligned}W_{UL} &= \underbrace{W_{FUL} + W_{PAY}} + W_{Fuel} \\ &= \qquad W_{PAY} \qquad + W_{Fuel}\end{aligned} \tag{136}$$

8. Update the gross take-off weight

$$\begin{aligned}W &= W_E + W_{UL} \\ &= \frac{W_S + W_P}{1 - f_E} + W_{Fuel} + W_{PAY}\end{aligned} \tag{137}$$

   Iterate Steps 1-8 until convergence.

   In summary, the gross take-off weight is

$$\begin{aligned}W &= \underbrace{W_S + W_P + W_{Vib} + W_{Sys}}_{W_E} + \underbrace{W_{FUL} + W_{Fuel} + W_{PAY}}_{W_{UL}}\end{aligned}$$

Also

$$
\begin{aligned}
W &= \underbrace{W_S + W_P + W_{Vib} + W_{Sys} + W_{FUL} + W_{Fuel}} + W_{PAY} \\
&= \qquad\qquad\qquad W_O \qquad\qquad\qquad + W_{PAY}
\end{aligned}
$$

where W_O is the operating weight.

The gross take-off weight W is also denoted by W_{GTO}. W_{GTO} for the primary mission is the design gross weight denoted by W_D.

14.2 Essential Refinements

While the basic sizing gives a crude assessment of whether the aircraft can lift off the ground, for realistic assessment, the following refinements are essential.

1) FM in Step 1 should be calculated and updated every iteration,
2) L/D in Step 3 should be calculated and updated every iteration,
3) Step 4 should include rotor weight with dependence on flapping frequency, and
4) Step 5 should be combined with a gear-box.

Refinement 1: Hover Figure of Merit (FM)

The calculation of FM requires the additional inputs: tip Mach number M_T, solidity σ, an induced power factor κ, an average airfoil lift curve slope $c_{l\alpha}$ and a mean airfoil drag coefficient c_{d0}.

As W is updated (thrust $T = W$), the ideal and actual powers P_h and P_H can be calculated. Their ratio is the FM. The ideal power is the ideal induced power

$$P_h = W \sqrt{\frac{DL}{2\,\rho}} = \frac{W^{1.5}}{\sqrt{2\,\rho\,A}}$$

The actual power is the true induced power $\kappa_h\, P_h$ where κ_h is the induced power factor ($\approx 1.05 - 1.15$) and the profile power from drag of the blades.

$$P_H = \kappa_h\, \frac{W^{1.5}}{\sqrt{2\,\rho\,A}} + \frac{1}{8}\,\sigma\, c_{d0}\,\rho\, A\,(\Omega R)^3$$

Thus

$$FM = \frac{P_h}{P_H} = \frac{\frac{W^{1.5}}{\sqrt{2\,\rho\,A}}}{\kappa_h\,\frac{W^{1.5}}{\sqrt{2\,\rho\,A}} + \frac{1}{8}\,\sigma\, c_{d0}\,\rho\,A\,(\Omega R)^3} \tag{138}$$

The tip speed $\Omega R = M_T\, a_s$ were a_s is the speed of sound. The solidity is the ratio of blade area to disk area $\sigma = A_b/A$. The blade area is written as $A_b = N_b\,\bar{c}\,R$ where N_b is the number of blades and $\bar{c}$ is a mean chord, so $\sigma = N_b\,\bar{c}/\pi R$.

The power loading $PL = W/P_H$ (lb/hp) is then

$$PL = \frac{FM}{PF}\,\sqrt{\frac{2\,\rho}{DL}} \tag{139}$$

Figure 49 shows measured PL versus DL of current rotorcraft. The maximum engine installed power P is used in the plot, so the constant FM curves are really constant FM/PF curves.

Also needed is the thrust coefficient

$$C_T = \frac{T}{\rho\, A\, (\Omega R)^2}$$

and the blade loading C_T/σ. The mean airfoil lift coefficient c_l is related to this parameter

$$c_l = 6\,\frac{C_T}{\sigma}$$

The mean airfoil angle of attack is then $c_l/c_{l\alpha}$. The mean angle of attack must remain below stall for sensible results. Therefore, a maximum C_T/σ is specified (normally ≈ 0.12), and if the updated W exceeds this limit, the solidity σ is increased, by increasing the mean chord of the blade. The wider chord enters the rotor weight model.

Free-wake analysis and measured or 2D-CFD calculated airfoil decks are needed for proper estimates of κ and c_{d0} which are then input into the analysis. Or, Mach-scaled model rotors (same tip Mach number as full-scale) are hover tested to identify these constants.

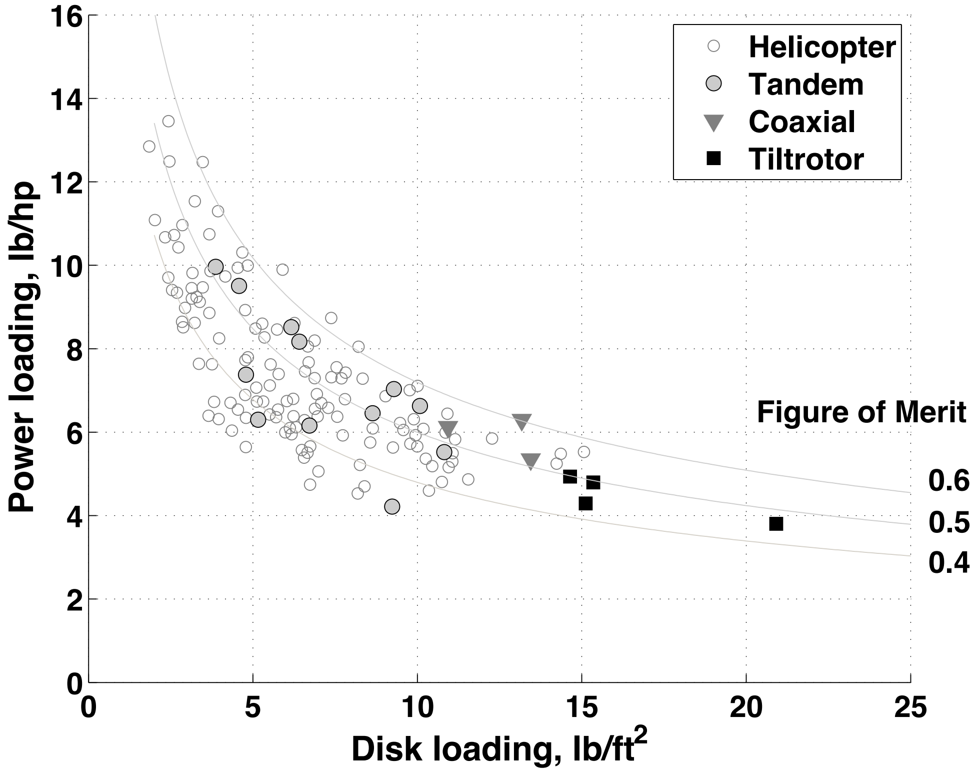

Figure 49: **Power loading versus disk loading of rotorcraft. Engine installed power and take-off weight are used.**

Refinement 2: Cruise Lift-to-Drag (L/D)

The calculation of L/D requires the additional input: the drag of the fuselage without the rotor blades (but including the hub). The fuselage drag D_F is expressed as

$$D_F = {}^1\!/_2\, \rho\, V^2\, F$$

where F is a drag area. The drag area in ft^2 can be approximated for existing rotorcraft by the Harris curve

$$F = f\left(\frac{W \text{ in lb}}{1000}\right)^{2/3}$$

where the factor $f \approx 4.2$ on average for all helicopters from $2,000 - 40,000$ lb, but ≈ 2.6 for a modern civil helicopter with retractable landing gear, and even lower ≈ 1.5 for a tiltrotor.

The power required by a rotorcraft in level equilibrium flight is composed of rotor induced power (to generate lift, subscript i), rotor profile power (to overcome drag of the rotating blades, subscript o) and the propulsive power required to overcome drag (subscript p), $D_F\,V$.

$$P \;=\; P_i \;+\; P_o \;+\; D_F\,V$$

The lift-to-drag ratio is

$$L/D = \frac{W}{P/V} \tag{140}$$

The coefficient of power is

$$C_P = \frac{P}{\rho\,A\,(\Omega R)^3}$$

Helicopter:

For a helicopter, the coefficients of power are the following.

$$\begin{aligned}
C_{Pi} \; &= \kappa_f\,\lambda_i\,C_T \\
&\quad \text{where} \\
&\quad \lambda_i = \frac{C_T}{2\,\sqrt{\mu^2+\lambda^2}} \qquad \text{is the induced inflow} \\
&\qquad \mu = V\,\cos\alpha_S/\Omega R \text{ is the advance ratio} \\
&\qquad \alpha_S \approx \tan^{-1} D_F/W \text{ is the rotor shaft tilt forward} \\
&\qquad \lambda = V\,\sin\alpha_S/\Omega R \,+\, \lambda_i = \mu\,\tan\alpha_S \,+\, \lambda_i \text{ is the total inflow} \\
&\quad C_T = \frac{T}{\rho\,A\,(\Omega R)^2} \text{ where rotor thrust } T \approx \left(D_F^2 \,+\, W^2\right)^{1/2}
\end{aligned} \tag{141}$$

$$C_{Po} \; = {}^1\!/_8\,\sigma\,c_{d0}\,(1+4.65\,\mu^2)$$

$$C_{Pp} \; = \frac{D_F\,V}{\rho\,A\,(\Omega R)^3} = {}^1\!/_2\;\frac{\mu^3}{\cos^3\alpha_S}\,\frac{F}{A} \approx {}^1\!/_2\;\mu^3\,\frac{F}{A} \text{ for } \alpha_S < 10^\circ$$

All together

$$C_P = \kappa_f\,\frac{C_T^2}{2\,\sqrt{\mu^2+\lambda^2}} \,+\, {}^1\!/_8\,\sigma\,c_{d0}\,(1+4.65\,\mu^2) \,+\, {}^1\!/_2\,\mu^3\,\frac{F}{A}$$

Given aircraft weight W, fuselage drag area F, rotor tip speed, solidity, disk area, and airfoil drag, the power can be calculated at any speed. κ_f is the induced power factor in cruise and can be used to account for non-uniform inflow, wake distortion and interference losses. The classical approximation to account for non-uniform inflow is $\kappa_f \approx 1.15$. The effective drag coefficient c_{d0} can be calibrated to higher-fidelity predictions.

The inflow

$$\lambda = \mu \, \tan \alpha_S + \lambda_i \text{ where } \lambda_i = \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}} \text{ is itself a function of } \lambda$$

requires Newton-Raphson iterations to solve, but a simpler approximation $\lambda_i \approx {}^{C_T}/_{2\mu}$ can be used for speeds $\mu > 0.1$ or so. This gives

$$C_P \approx \kappa_f \, \frac{C_T^2}{2\,\mu} + {}^1/_8 \, \sigma \, c_{d0} \, (1 + 4.65\,\mu^2) + {}^1/_2 \, \mu^3 \, \frac{F}{A}$$

Compound Helicopter:

The same Eq 141 apply with the following changes.

For wing/s, the wing drag is added to the fuselage drag and the rotor thrust reduced by a specified wing lift share.

$$\begin{aligned} D_F &= {}^1/_2 \, \rho \, V^2 \left[F + A_W \left(c_{d0W} + \frac{c_{lW}^2}{\pi \, AR \, e} \right) \right] \\ c_{lW} &= \frac{L_W}{{}^1/_2 \, \rho \, V^2 \, A_W} \end{aligned} \tag{142}$$

$$\begin{aligned} T &\approx \left(D_F^2 + (W - L_W)^2 \right)^{1/2} \\ \alpha_S &\approx \tan^{-1} D_F/(W - L_W) \end{aligned}$$

A_W is the total surface area of the wing, c_{d0W} is its zero-lift drag coefficient and the c_{lW}^2 term is its lift-induced drag coefficient. AR is the aspect ratio and e is the Oswald efficiency factor. $e < 1$ and typically ≈ 0.8.

For propeller/s, the propeller profile drag is added and the rotor thrust reduced by a specified propeller thrust share. The propeller profile power P_o = propeller $C_{Po} \times \rho \, A_P \, (\Omega R)_P^3$, where A_P and $(\Omega R)_P$ are the disk area and tip speed of the propeller. The profile power coefficient of Eq 141 can be used or refined to Eq 143 given later for tiltrotor. The rotor thrust is reduced by a specified propeller thrust T_P, so $T = \left((D_F - T_P)^2 + (W - L_W)^2 \right)^{1/2}$ and $\alpha_S \approx \tan^{-1}(D_F - T_P)/(W - L_W)$.

Given the wing properties — area, aspect ratio, drag coefficient, e, and propeller properties — c_{d0}, tip speed, and disk area, and specified lift and thrust shares, D_F and T can be calculated and inserted in Eq 141 to find power.

Tiltrotor:

The tiltrotor is a winged compound with the special capability of airplane mode when the rotors are fully tilted forward.

In hover, $D_F = 0$ and $T = W$. Wing download might increase required thrust by $10 - 30\%$ W.

In airplane mode cruise, $L_W = W$ and $T = D_F$. The wing loading $WL = W/A_W$ (lb/ft^2) is similar to disk loading and a typical value is 70 lb/ft^2. Specifying a design wing-loading determines the wing area. Specifying a design wing-loading along with a corresponding design speed, determines the design wing lift coefficient c_{lW}. Alternatively, specifying a design wing-loading along with a corresponding design lift coefficient c_{lW}, sets the design speed.

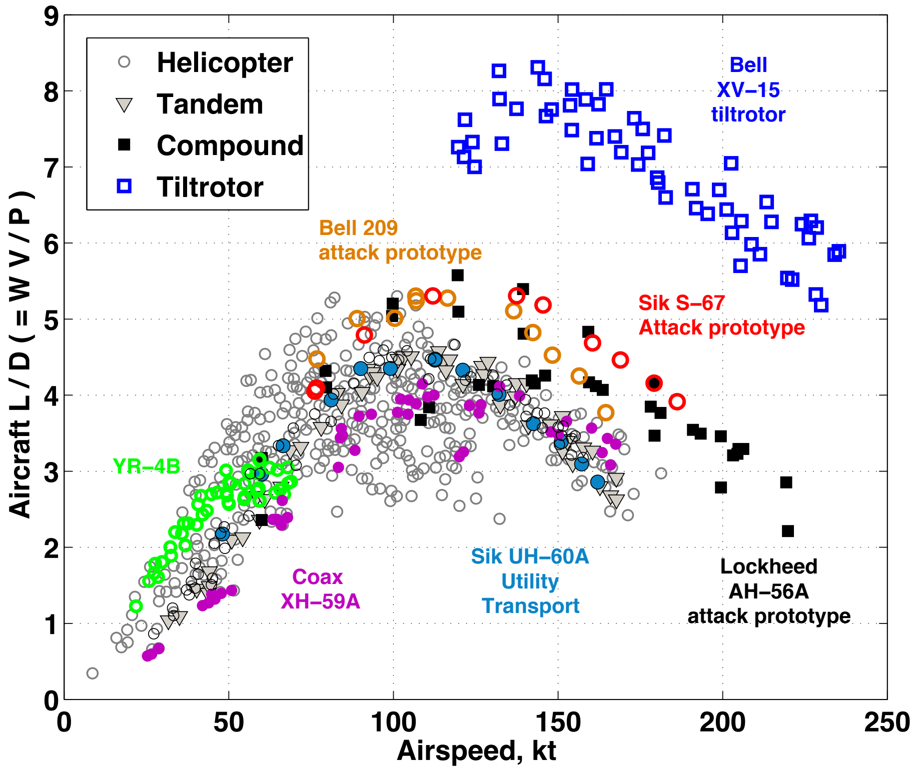

Figure 50: **Measured L/D of various rotorcraft.**

In airplane mode, the coefficients of power are the following.

$$C_{Pi} = \kappa_f \, \lambda_i \, C_T$$

where

$$\lambda_i = -\frac{\lambda_C}{2} + \sqrt{\left(\frac{\lambda_C}{2}\right)^2 + \frac{C_T}{2}} \approx \frac{C_T}{2\,\lambda_C} \text{ and } \lambda_C = \frac{V}{\Omega R} \text{ is the cruise inflow}$$

$$C_T = \frac{T}{\rho \, A \, (\Omega R)^2} \text{ where rotor thrust } T = D_F$$

$$C_{Po} = {}^1\!/_8 \, \sigma \, c_{d0} \, F_P \tag{143}$$

where

$$F_P = \sqrt{1+\lambda_C^2}\,\left(1 + {}^5\!/_2\,\lambda_C^2\right) + {}^3\!/_2\,\lambda_C^4 \ln \frac{1+\sqrt{1+\lambda_C^2}}{\lambda_C}$$

$$C_{Pp} = \frac{D_F \, V}{\rho \, A \, (\Omega R)^3} = {}^1\!/_2\;\lambda_C^3 \left[\frac{F}{A} + \frac{A_W}{A}\left(c_{d0W} + \frac{c_{lW}^2}{\pi \, AR \, e}\right)\right]$$

where

$$c_{lW} = \frac{L_W}{{}^1\!/_2\,\rho\,V^2\,A_W} = \frac{W\,L}{{}^1\!/_2\,\rho\,V^2}$$

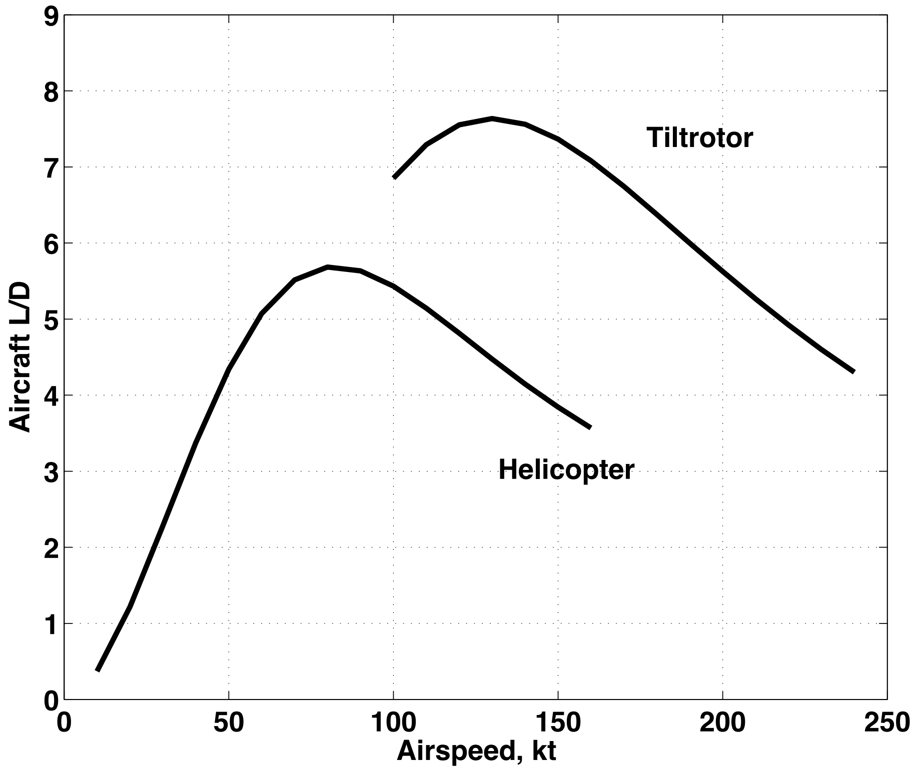

Figure 51: **Calculated L/D of a helicopter and tiltrotor.**

All together

$$C_P = \kappa_f \, \frac{C_{DF}^2}{2\,\lambda_C} \, + \, {}^1\!/_8 \, \sigma \, c_{d0} \, F_P \, + \, {}^1\!/_2 \, \lambda_C^3 \, \left(\frac{F}{A} \, + \, \frac{A_W}{A} \, c_{d0W} \right) + \, {}^1\!/_2 \, \lambda_C^3 \, \frac{A_W}{A} \, \frac{c_{lW}^2}{\pi \, AR \, e}$$

In dimensional form

$$P = \left(\kappa_f \, \frac{C_{DF}^2}{2\,\lambda_C} \, + \, {}^1\!/_8 \, \sigma \, c_{d0} \, F_P \right) \rho \, A \, (\Omega R)^3 \, + \, \frac{1}{2} \, \rho \, V^3 \, (F + A_W \, c_{d0W}) \, + \, \frac{W}{V} \, \frac{2\,W L}{\rho} \, \frac{1}{\pi \, AR \, e}$$

The rotor induced power is negligible in airplane mode as $C_{DF}^2/2\lambda_C \approx 0$.

At speeds slower than the design speed, the lift coefficient of the wing must increase for wing lift to still support the weight, so the aircraft must pitch up. The pitch angle θ also tilts the rotor thrust up so it must now equal $T = D_F/\cos\theta$. The lifting component of this thrust, $L_R = T\,\sin\theta$, reduces the lift required from the wing, so really $L_W = W - L_R$. The lift coefficient can be iterated to find the right pitch angle at which the wing and the rotor lifts balance the weight. Normally for pitch angles $\theta < 10°$ this iteration can be ignored as $L_R << W$. Regardless, the results is that a slight pitch up of the aircraft usually produces at better L/D. The tiltrotor data in Fig 50 (from XV-15) shows this peak around 150 kt.

Example calculations are shown in Fig 51. The examples consist of two aircraft of same gross weight $W = 5000$ lb, flying at SL/ISA with air density $\rho = 0.00237$ slug/ft^3, and speed of sound $a_S = 1100$ ft/s. The rotor properties are the same: tip Mach number $M_T = 0.5$ (lower than the typical ≈ 0.65 to reduce noise), solidity $\sigma = 0.06$, induced power factor $\kappa_f = 1.15$, and airfoil mean profile drag coefficient $c_{d0} = 0.008$. For the helicopter, the Harris drag factor $f = 4.2$ and disk loading $DL = 2.5$ lb/ft^2. For the tiltrotor, the

Harris drag factor $f = 1.5$ and disk loading $DL = 13$ lb/ft^2. Additional inputs for the tiltrotor are the wing properties: design wing loading $WL = 70$ lb/ft^2, design speed $V = 250$ kt, wing mean profile drag coefficient $c_{d0W} = 0.01$, aspect ratio $AR = 8$, Oswald efficiency $e = 0.8$, and airfoil mean lift curve slope $c_{l\alpha} = 5.73$. The pylon is always down. Below 100 kt the wing stalls due to the high pitch of the aircraft; in practice, flaps are deployed to increase lift coefficient, which typically brings down the L/D below a helicopter. The speed of peak L/D is a function of the tip speed (i.e. tip Mach number) either through the advance ratio μ (helicopter) or the inflow ratio λ_C (tiltrotor).

The simple expressions are based on energy balance in equilibrium flight without regard to how the equilibrium is achieved (trim solution) or whether it can in fact be achieved (stall limit). The many underlying assumptions used to arrive at these closed form expressions are too simplistic. Some of them, such as uniform inflow, uniform chord, linear lift, and constant drag can be accounted for by calibrating the coefficients κ, c_d, c_l, e to flight test data. Some, such as, compressibility, stall, 3D effects and interference require higher-fidelity lifting-line analysis. The simple analysis also assumes all N_R rotors operate at the same state.

The measured $L/D = WV/P$ of current rotorcraft is compiled in Fig 50. Compounds buy higher speeds without significant low speed penalty. Tiltrotors buy very high speed with significant low speed penalty. The penalty leads to higher structural weight fraction f_S.

In a tiltrotor, reducing tip speed ΩR in cruise to 50% of hover, reduces power and improves L/D significantly. An electric motor is capable of this reduction with less than 15% loss in efficiency. The turbo-shaft limit is around $15 - 20\%$ reduction in ΩR (the V-22). Ng and Datta 2019 predicts $L/D \approx 10 - 11$ with 50% reduction in tip speed, with a refined trim analysis and airfoil decks, see Fig 52. The 50% reduction was the limit beyond which the rotor stalled and there was no gain. This is an exclusive advantage of electric power and a tiltrotor with high cruise speed can take advantage of it.

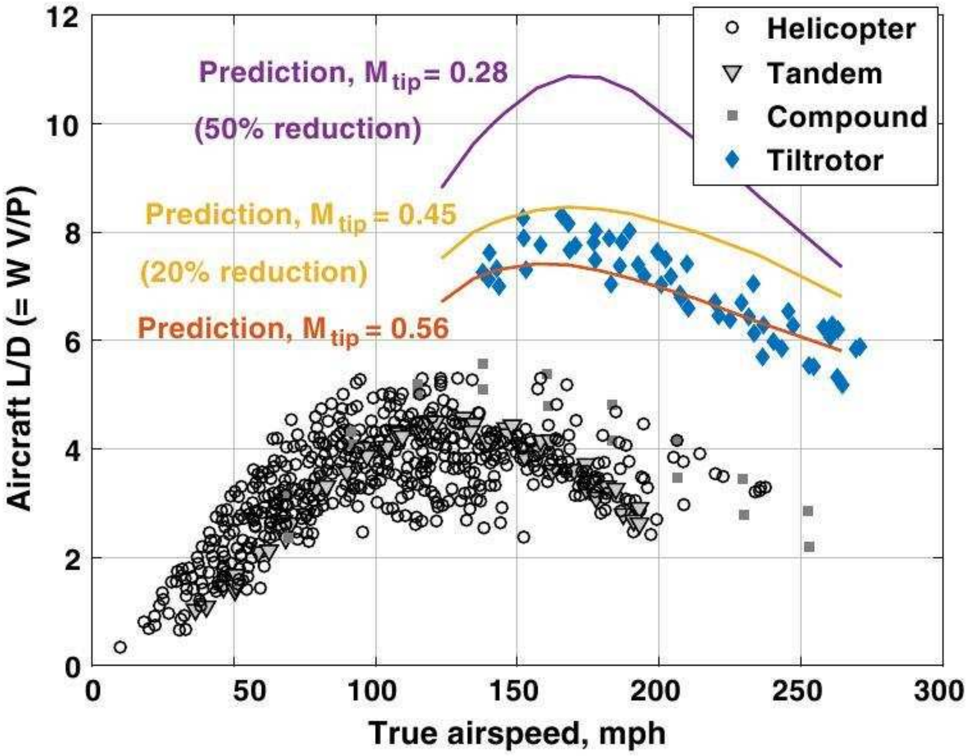

Figure 52: **Predicted L/D of an electric tiltrotor with 50% rotor tip speed reduction in cruise.**

Refinement 3: Rotor Weight

The structural weight calculation in Step 4 should be refined to calculate rotor weights. Even though there are no production eVTOL, there is a rich variety of production rotors, and the historical weights from these rotors are a reliable basis on which to predict rotor weights. Rotor weights vary with rotor radius, chord, and flapping frequency and these variations must be accounted for in the weight iteration.

The weight of each rotor is calculated using the U.S. Army AFDD82 model which calculates the weight of all blades (7.7% average error based on 37 aircraft) and the hub (10.2% average error based on 35 aircraft) in lb.

$$\begin{aligned} W_b &= \chi_b \; 0.02606 \, N_b^{0.6592} \, R^{1.3371} \, c^{0.9959} \, (\Omega R)^{0.6682} \, \nu_\beta^{2.5279} \\ W_h &= \chi_h \; 0.003722 \, N_b^{0.2807} \, R^{1.3371} \, (\Omega R)^{0.4290} \, \nu_h^{2.1414} \, W_b^{0.5505} \\ W_R &= W_b + W_h \end{aligned} \tag{144}$$

where
ν_β in /rev is the blade flapping frequency (rad/s) divided by rotational speed Ω (rad/s),
ν_h in /rev is the hub frequency (= ν_β, or collective frequency for teetering rotors),
N_b is the number of blades,
c and R are the mean chord and radius respectively in ft,
and ΩR is the tip speed in ft/s.
The χ are calibration or technology factors (< 1 for future advanced rotor assumption).
The structural weight W_S in step 4 is now changed to

$$W_S = N_R \, W_R \, + \, f_S \, W$$

where f_s is now the weight fraction of the rest of the aircraft without the rotors.

Refinement 4: Gear-box

Inclusion of a gear-box in Step 5 changes the electric motor equation in Eq 130 and adds a gear-box weight. Gear-boxes are far more torque dense than electric motors. They also absorb the intense rotor vibratory loads that might otherwise break the motors.

The weight of the gear box and rotor shaft is calculated using the U.S. Army AFDD83 model (7.7% average error based on 30 aircraft).

$$\begin{aligned} W_P \text{ in kg} &= 0.4 \, \left(\frac{Q \text{ in Nm}}{GR \;\, \eta_g \, \eta} \right)^{0.71} \\ W_G \text{ in lb} &= 57.72 \, (P \text{ in hp})^{0.8195} \, f_Q^0.068 \, n_g^{0.0663} \, \left(\frac{\text{Motor RPM}}{1000} \right)^{0.037} \, \frac{1}{\text{Rotor RPM}^{0.638}} \end{aligned} \tag{145}$$

where η_g is the gear box efficiency, GR is the gear ratio (Motor RPM/Rotor RPM), f_Q is a constant, and n_g is the number of gear boxes (= 1 for eVTOL for each motor/rotor). The total weight is found by multiplying with the number of rotors N_R. f_Q represents the second rotor torque limit in %. $f_Q = 60$ for twin rotors and 3 for single main-rotor and tail-rotor.

A conservative model for efficiency is used, valid for $GR < 100$.

$$\eta_g = 1 - \frac{GR}{100}$$

These are crude trends and should be used with caution. The gear-ratio can be increased to very high numbers, since there is no mechanical limit on the gear-speed or heat limit on the motor. The motor equation is derived from permanent magnet motors whereas the gear box equation is from conventional turbo-shaft helicopters.

14.3 Example: NASA Electric Quad-rotor

The NASA electric quad-rotor is a battery-powered single-occupant conceptual design by Johnson, Silva and Solis 2018 and Johnson and Silva 2018.

The original work used NASA Design and Analysis of Rotorcraft software with detailed modeling of many subsystems. Here, the method described earlier in the section on Rotorcraft Sizing / Essential Requirements is used. The Figure of Merit is calculated using Eq 138. The helicopter power and L/D are calculated using Eq 140 and 141 respectively. The basic trends can be reproduced with a small number of inputs listed below.

The conditions are 5000-ft ISA + 20°C which give the following.
Air density $\rho = 0.0019438$ slug/ft^3 (1.0018 kg/m^3)
Speed of sound $a_s = 1132$ ft/s (345 m/s)

The sizing mission is given by the following.
Payload = 250 lb (including pilot)
Hover OGE = 4 min
Cruise speed for best range = $V_{br} \approx 70$ knot (36 m/s), so this speed is specified for all designs
Cruise reserve = 20 min

The aircraft inputs are the following.
Disk loading = 2.5 lb/ft^2
Fraction structural weight $f_S = 0.20$
Fraction vibration and systems weight $f_E = 0.15$
Harris drag factor $f = 2.95$
Number of rotors = 4

The rotor inputs are the following.
Number of blades per rotor = 3
Tip Mach number $M_T = 0.4$ (tip speed = 138 m/s)
Solidity = 0.0646
Airfoil mean drag coefficient $c_{d0} = 0.009$
Airfoil mean lift curve slope $c_{l\alpha} = 5.73$ per radian
$\kappa_h = 1.15$
$\kappa_f = 1.15$
Flapping frequency $\nu_\beta = 1.125$ / rev
Hub frequency $\nu_h = \nu_\beta$
Blade tech factor = 0.5
Hub tech factor = 0.5

The power system inputs are the following.
Power factor PF = 1.5
RPM motor = 1000
η Motor = 0.80
Fraction controller and power conditioner weight = 0.5
Fraction cooling system weight = 0.5
$f_Q = 60$
$n_g = 1$
Battery Wh/kg available and installed = 150, 200, 300, 400
Battery Max C-rate = unlimited

Figures 53 and 54 show the gross weight and hover power respectively varying with range for a battery powered aircraft. There is a mis-match in predictions at higher battery specific energies, but they match at the lower levels. The lower levels are closer to the state-of-the-art. Note that the specific energy are at the pack level, installed and available (80% discharge, end-of-life) values.

The batteries are now replaced with PEMFC. A 2.5 atm modern stack sized for maximum efficiency is used. The power versus weight was shown in Fig 40 and specifications for a 500 kWe design given in Table 29 column 5. The tank specifications were 5.7% weight fraction filled to 700 bar 15°C with 10% overhead for structural volume. Figure 55 shows the design gross weight varying with range. If the stack is sized to deliver the maximum aircraft power (1.5× hover power OGE) as design power (cooling system sized to design power) the higher solid line is obtained. If the stack is sized to deliver the maximum aircraft power as maximum power (cooling system sized to design power) then the lower dashed line is obtained. This is the more appropriate design as the power factor of 1.5 places the hover power near and slightly below the design power ensuring continuous hover. This is still an over design as only 4 min of hover is required. The PEMFC solution surpasses the 200 Wh/kg battery solution above a range of 60 nm (produces a lighter and smaller aircraft). The barrier is the 400 L tank volume seen in Fig 56. These solutions are representative of modern automobile fuel cell technology.

In eVTOL, short-term hydrogen storage is envisioned to allow greater hydrogen weight fractions. The dramatic impact of increasing hydrogen weight fractions from 5.7 to 7.5 to 15% is evident in both the figures. At 15% weight fraction, a range of 125 nm is possible with an aircraft of 2750 lb. About 35 lb of hydrogen would be needed to fly this range, so the tank volume remains very high at 450 L. If a 1 atm stack is used instead of a 2.5 atm stack, the predictions change significantly. The power versus weight of a 1 atm stack was shown in Fig 48 and specifications for a 500 kWe design given in Table 29 column 7. Figure 57 and 58 show the gross weight and tank volumes with 1 atm stacks. The PEMFC solution now overtakes the 200 Wh/kg battery solution above a range of 25 nm and the 300 Wh/kg battery solution above 85 nm. The corresponding tank volumes are shown in Fig 58. At 15% weight fraction, a range of 125 nm is now possible with an aircraft of only 2000 lb. About 20 lb of hydrogen would be needed to fly this range, so the tank volume is reduced to only 260 L.

Quantitative conclusions are premature without calibrations from an actual flight test aircraft (or a flight-worthy test bed), but the trends are clear, and they reveal the key technology drivers. These are: 1) high weight fraction hydrogen storage up to at least 15%, and 2) power-to-weight ratios greater than 1.0 kWe/kg for the stack system (stack plus ancillary subsystems). The unique attributes of an eVTOL — short hover, short hydrogen storage (2-3 hrs), and low altitude near 1-atm flight should be leveraged to open opportunities for achieving these targets. The downwash from the rotors should be leveraged to pursue avenues for lighter stack cooling.

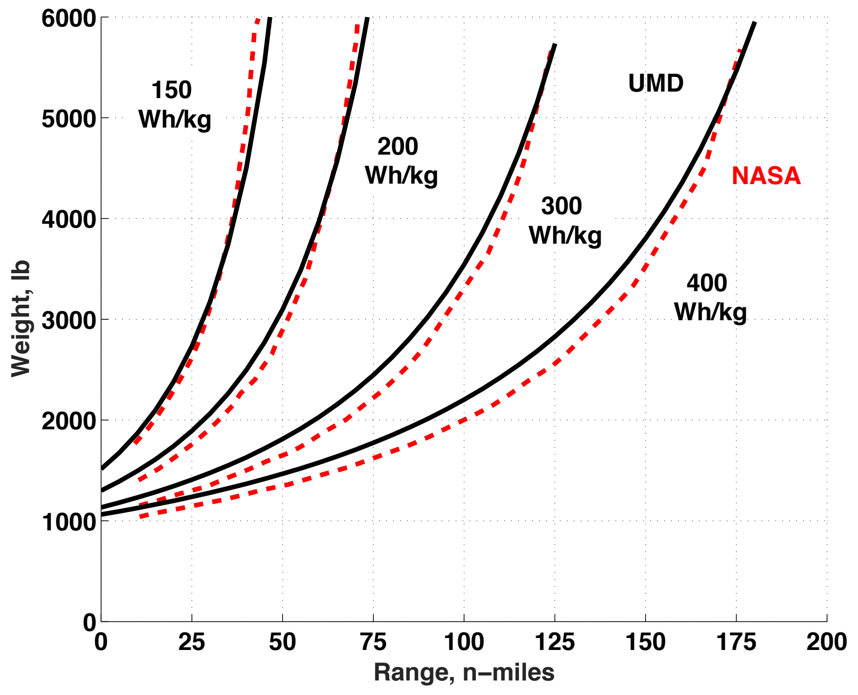

Figure 53: **Gross weight versus range with batteries. Electric quad-rotor. 250 lb payload. Battery specific energy are pack-level available and installed values.**

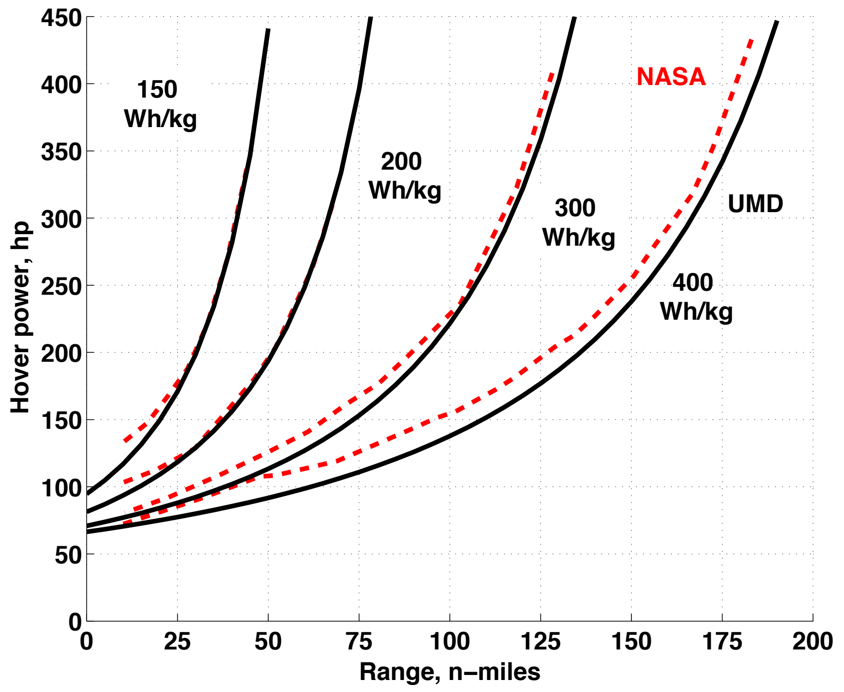

Figure 54: **Hover power versus range with batteries. Electric quad-rotor. 250 lb payload. Battery specific energy are pack-level available and installed values.**

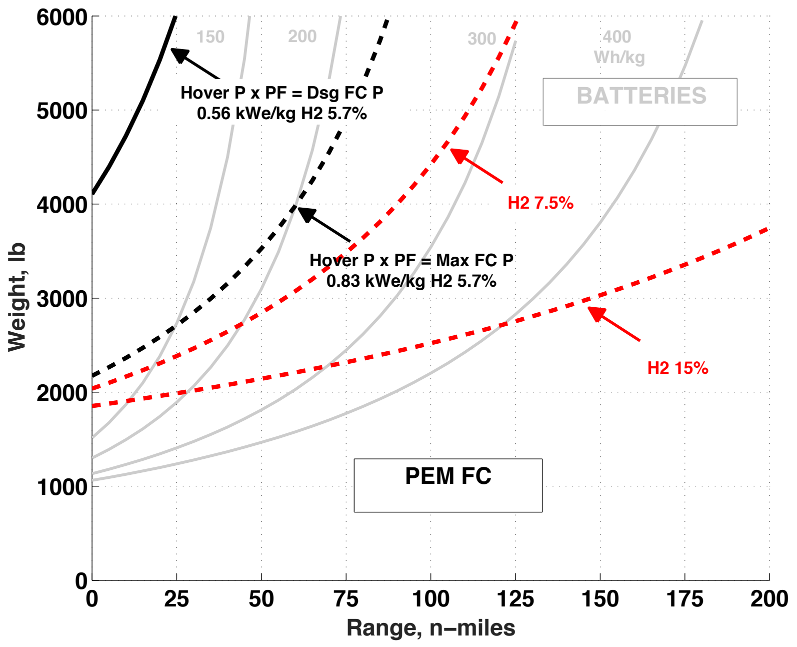

Figure 55: **Gross weight versus range with 2.5 atm PEMFC sized to deliver** $1.5\times$ **hover power as design power (solid line) and maximum power (dashed line). Hydrogen wt% 5.7, 7.5 and 15. Electric quad-rotor. 250 lb payload.**

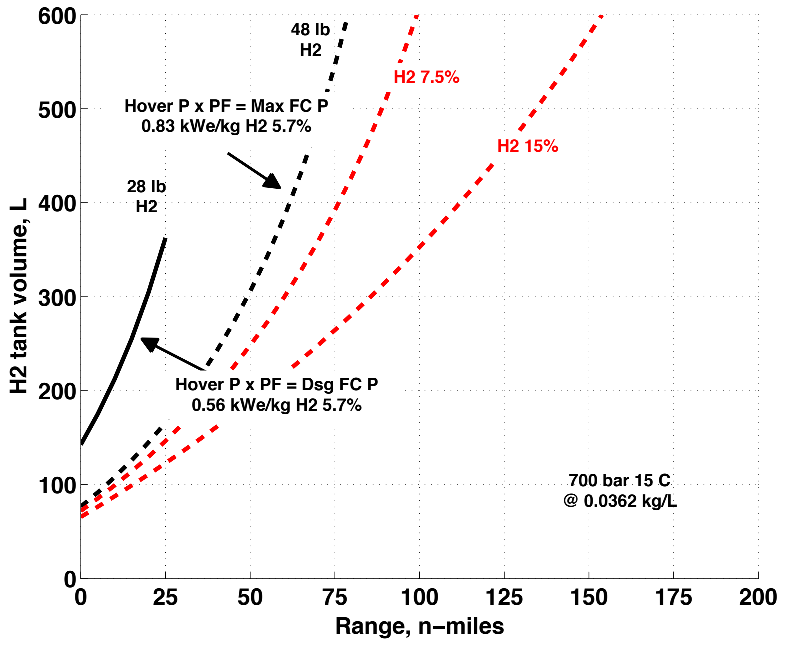

Figure 56: **Tank volume versus range with 2.5 atm PEMFC sized to deliver** $1.5\times$ **hover power as design power (solid line) and maximum power (dashed line). 2.5 atm stacks. 700 bar 15°C tank. Hydrogen wt% 5.7, 7.5 and 15. Electric quad-rotor. 250 lb payload.**

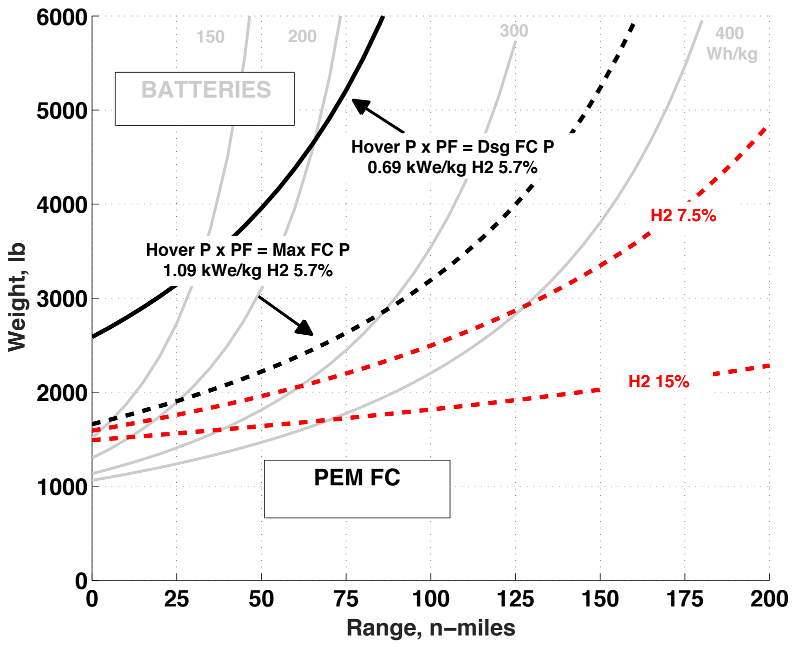

Figure 57: **Tank volume versus range with 1 atm PEMFC sized to deliver** $1.5\times$ **hover power as design power (solid line) and maximum power (dashed line). 700 bar 15°** **C tank. Hydrogen wt% 5.7, 7.5 and 15. Electric quad-rotor. 250 lb payload.**

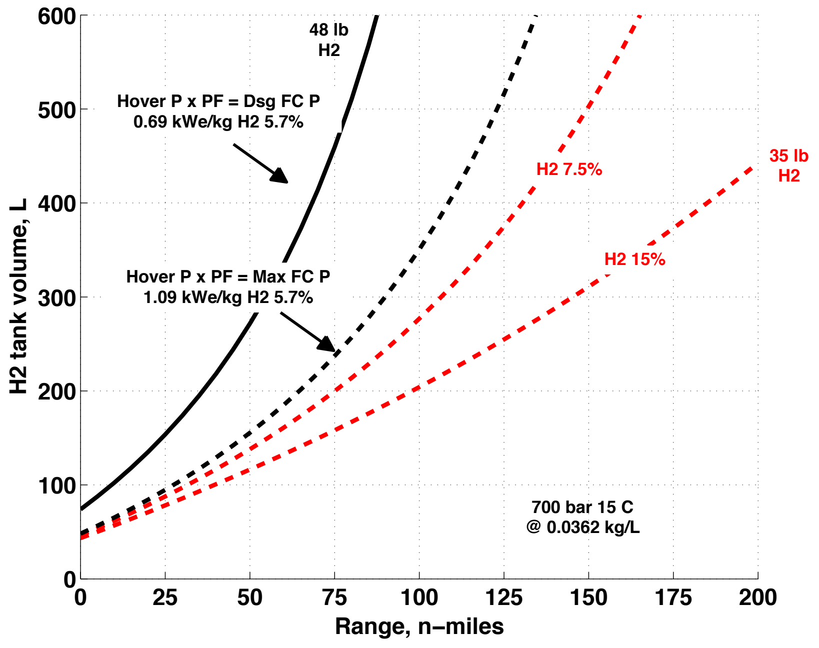

Figure 58: **Tank volume versus range with 1 atm PEMFC sized to deliver** $1.5\times$ **hover power as design power (solid line) and maximum power (dashed line). 700 bar 15°** **C at full tank. Hydrogen wt% 5.7, 7.5 and 15. Electric quad-rotor. 250 lb payload.**

14.4 Example: Maryland Electric Quad-rotor

The Maryland electric quad-rotor is a battery-powered two-occupant conceptual design by Fisler and Datta 2020. It follows the NASA quad-rotor, only carries a higher payload of 400 lb, equal to the payload of the R22 Beta II helicopter.

The method described earlier in the section on Rotorcraft Sizing / Essential Requirements is used. The Figure of Merit is calculated using Eq 138. The helicopter power and L/D are calculated using Eq 140 and 141 respectively. The changes in inputs from the NASA quad-rotor are the payload (400 lb instead of 250 lb), the Harris drag factor f (3.95 instead of 2.95), and a stall limit on the blade loading ($C_T/\sigma \leq 0.12$).

The conditions are 5000-ft ISA + 20°C which give the following.
Air density $\rho = 0.0019438$ slug/ft^3 (1.0018 kg/m^3)
Speed of sound $a_s = 1132$ ft/s (345 m/s)

The sizing mission is given by the following.
Payload = 400 lb (increased)
Hover OGE = 4 mins
Cruise speed = $V_{br} \approx 70$ knot (118 ft/s) so this speed is specified for all designs
Cruise reserve = 20 min

The aircraft inputs are the following.
Disk loading = 2.5 lb/ft^2
Fraction structural weight $f_S = 0.20$
Fraction vibration and systems weight $f_E = 0.15$
Harris drag factor $f = 3.95$ (increased)
Number of rotors = 4

The rotor inputs are the following.
Number of blades per rotor = 3
Tip Mach number $M_T = 0.4$ (tip speed = 138 m/s)
Solidity = 0.0646
Airfoil mean drag coefficient $c_{d0} = 0.0090$
Airfoil mean lift curve slope $c_{l\alpha} = 5.73$ per radian
$\kappa_h = 1.15$
$\kappa_f = 1.15$
Flapping frequency $\nu_\beta = 1.125$ / rev
Hub frequency $\nu_h = \nu_\beta$
Blade tech factor = 0.5
Hub tech factor = 0.5
Maximum $C_T/\sigma = 0.12$ (stall limit)

The power system inputs are the following.
Power factor PF = 1.5
RPM motor = 1000
η Motor = 0.80
Fraction controller and power conditioner weight = 0.5
Fraction cooling system weight = 0.5
$f_Q = 60$
$n_g = 1$

Battery Wh/kg available and installed = 150, 200, 300, 400
Battery Max C-rate = unlimited, 2

Figure 59 shows the gross weight varying with range for a battery powered aircraft. Figure 60 shows the corresponding C-rates. The 150 and 200 Wh/kg values are near the state-of-the-art specific energy at pack level, installed and available (80% discharge, end-of-life). The C-rates required for a range of 25 nm are 2.5C in hover and 1C in cruise. The required C-rates are set by the hover time and range, hence independent of specific energy of the batteries, a requirement that is typically at odds with electrochemistry where the available C-rates typically diminish with specific energy. The plots show, higher energy batteries (200 or 300 Wh/kg) enable greater range for the same aircraft weight, or lighter aircraft for the same range. The latter appears to be the more effective option. If however, higher energy comes at the cost of lower C-rate, flying a greater range might be the only option. These options impact infrastructure. Thus, aircraft and infrastructure are tied to the trajectory of battery technology.

The batteries are now replaced with PEMFC. Figure 61 shows the gross weight versus range with three types of designs — two with 2.5 atm stacks and one with a 1 atm stack. All are sized for maximum efficiency. The tank specifications for all are 5.7% weight fraction filled to 700 bar 15°C with 10% overhead for structural volume. First consider the 2.5 atm stack. If it is sized to deliver the maximum aircraft power (1.5× hover power OGE) as the design power the higher solid line in the farthest corner is obtained. If the stack is sized to deliver the maximum aircraft power as maximum power (cooling system sized to design power) then the lower solid line is obtained. This is the more appropriate design as the power factor of 1.5 places the hover power near or slightly below the design power ensuring continuous hover. If the 1 atm stack is used, the weights are lowered significantly, showing the potential benefits of low altitude flight. The corresponding tank volumes are shown in Fig 62. These solutions are representative of modern automobile fuel cell technology.

For comparison, the piston engine R22 Beta II gross weight (1371 lb), range (180 nm), and fuel tank volumes (100 L), are plotted. As expected, these are located far below the electric power plants.

In eVTOL, short-term hydrogen storage is envisioned to allow greater hydrogen weight fractions. The effect of weight fraction is shown in Fig 63. Increasing weight fraction flattens the curve up to 15%, beyond which there is a diminishing return. Figure 64 shows the effect of stack system power-to-weight. Increasing power-to-weight lowers the curve up to 1.5 kWe/kg beyond which there is a diminishing return. Thus, for a 400 lb payload, 15% weight fraction and 1.5 kWe/kg power-to-weight appear to be effective targets. The remaining difference from the R22 is largely due to the quad-rotor configuration, not the power plant.

The effect of hydrogen weight fraction on tank volume is shown in Fig 65. Increasing weight fraction reduces the gross weight and the amount of hydrogen needed and therefore the volume. At 15%, 44 lb of hydrogen is needed to fly the R22 range, which implies a 550 L tank. Figure 66 shows that storage pressure has a limited impact on tank volume beyond 700 bar. From 700 bar to 800 bar there is a gain of only 50 L. Temperature has a greater impact. From 15°C to −150°C the volume is halved and equals to that of liquid hydrogen. However, these are not available options with the current infrastructure.

Quantitative conclusions are premature without calibrations from an actual flight aircraft (or a flight-worthy test bed), but the trends are clear, and they reveal the key technology drivers. These are refined from the previous section to include: 1) high weight fraction hydrogen storage of at least 15—30%, 2) power-to-weight ratios of at least 1.0—1.5 kWe/kg for the stack system (stack plus ancillary subsystems), and 3) achieve storage tank temperatures of −50°C or lower.

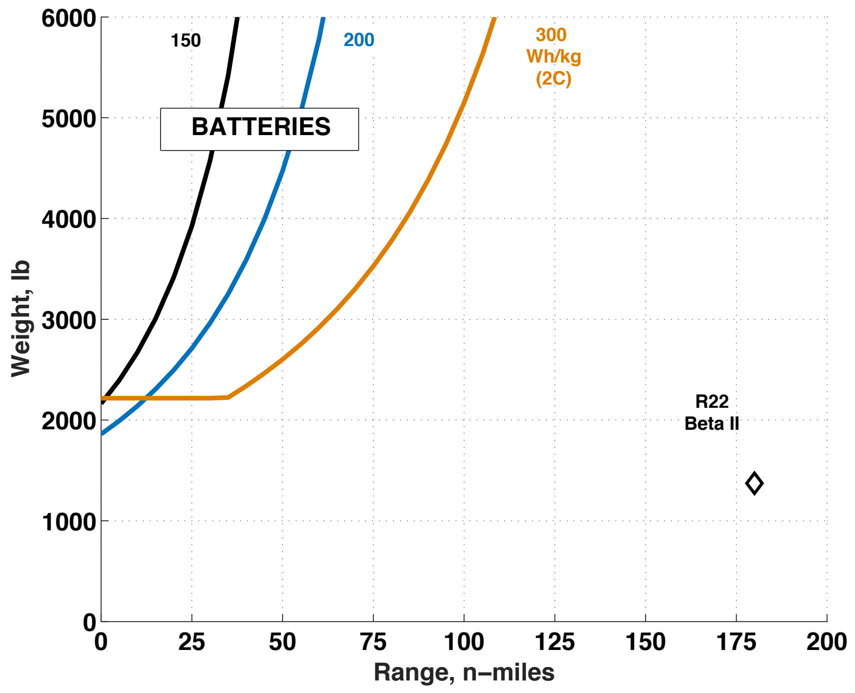

Figure 59: **Gross weight versus range with batteries. Electric quad-rotor. 400 lb payload. Specific energy are pack-level available and installed values.**

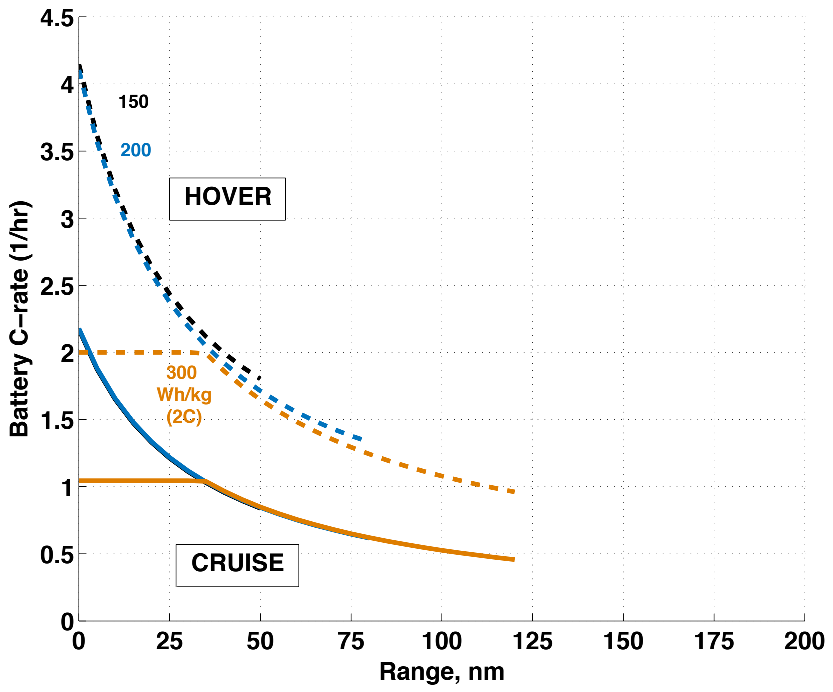

Figure 60: **Required C-rate versus range with batteries. Electric quad-rotor. 400 lb payload. Specific energy are pack-level available and installed values. Dotted: hover. Solid: cruise.**

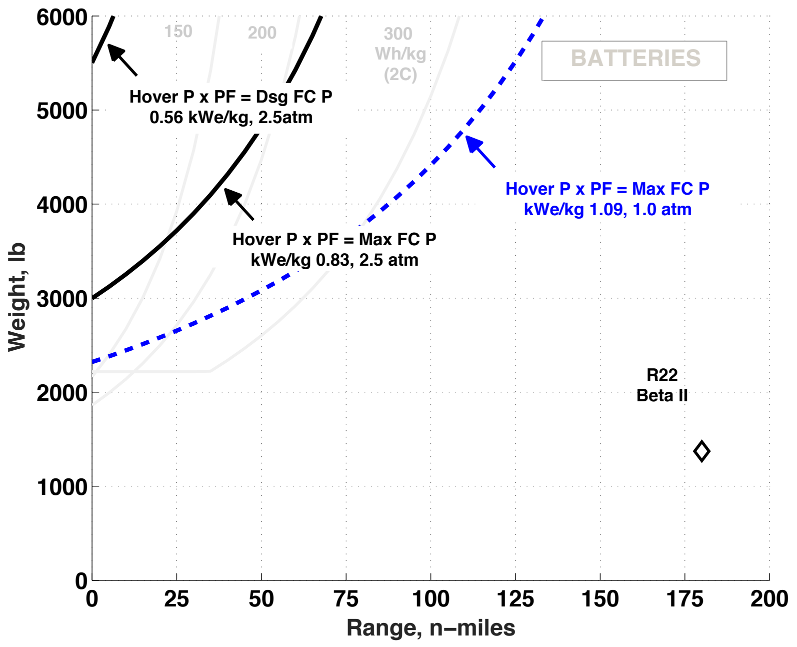

Figure 61: **Gross weight versus range with 2.5 atm and 1 atm PEMFC sized to deliver 1.5× hover power as design power (solid line) or as maximum power (dashed line). Electric quad-rotor. 400 lb payload. Hydrogen wt% 5.7.**

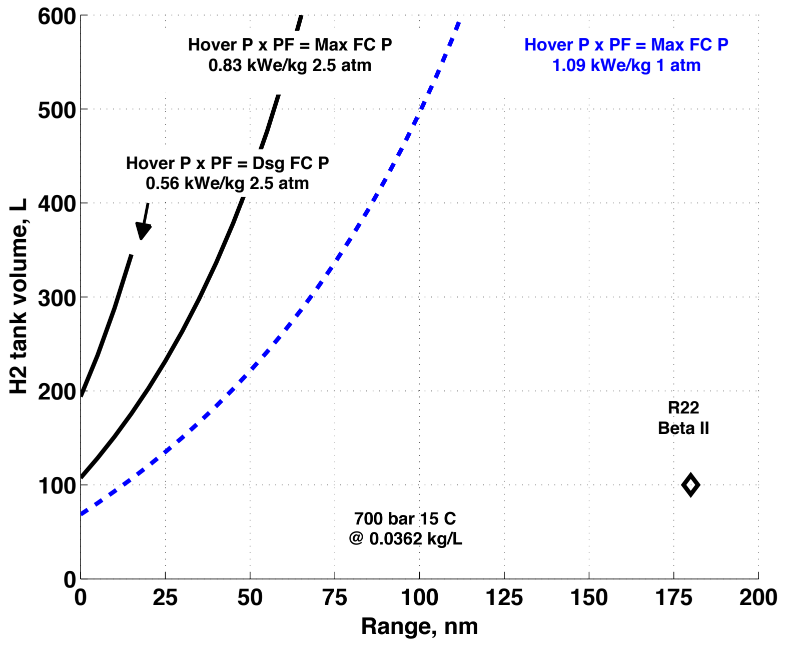

Figure 62: **Tank volume versus range with 2.5 atm and 1 atm PEMFC sized to deliver 1.5× hover power as design power (solid line) or as maximum power (dashed line). Electric quad-rotor. 400 lb payload. Hydrogen wt% 5.7. H2 storage 700 bar 15°C.**

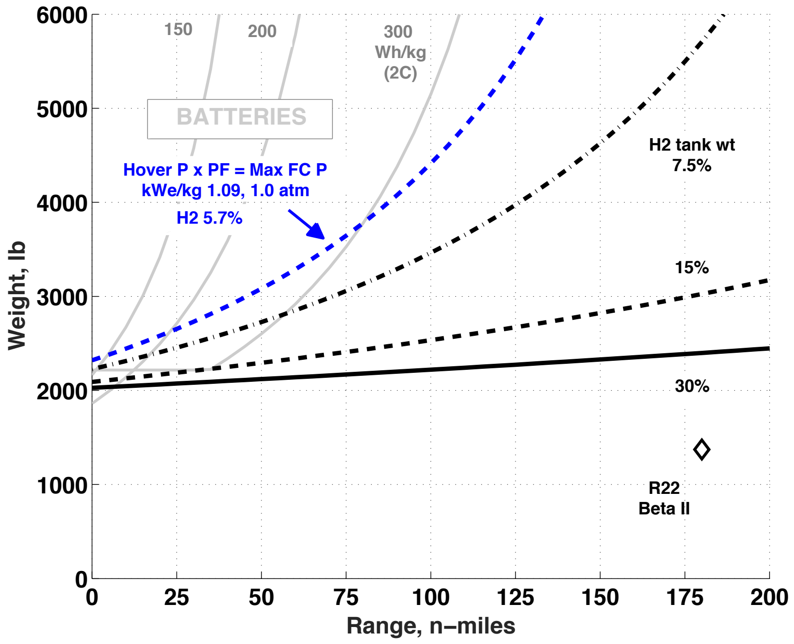

Figure 63: **Gross weight versus range with 1 atm PEMFC sized to deliver** $1.5\times$ **hover power as maximum power. Impact of hydrogen wt%. Electric quad-rotor. 400 lb payload. H2 storage 700 bar 15°C.**

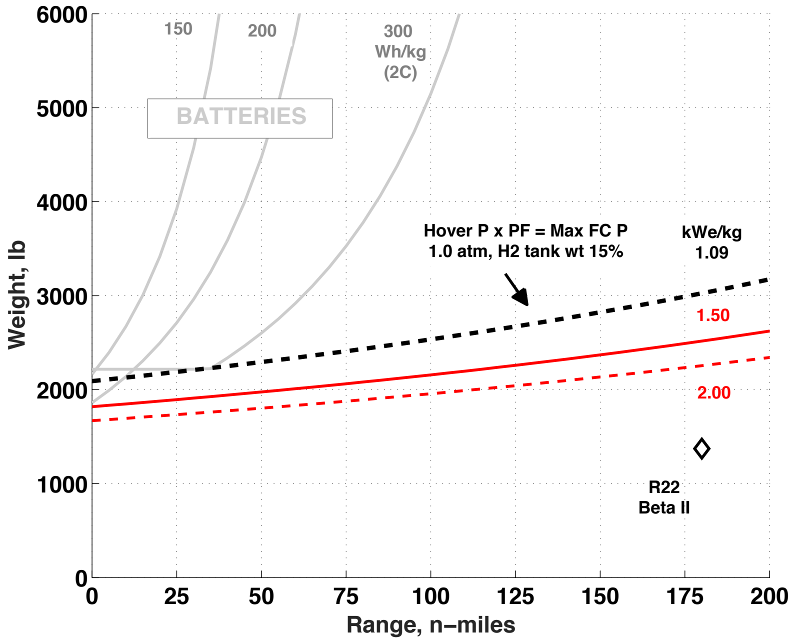

Figure 64: **Gross weight versus range with 1 atm PEMFC sized to deliver** $1.5\times$ **hover power as maximum power. Impact of stack system specific power. Electric quad-rotor. 400 lb payload. Hydrogen wt% 15. H2 storage 700 bar 15°C.**

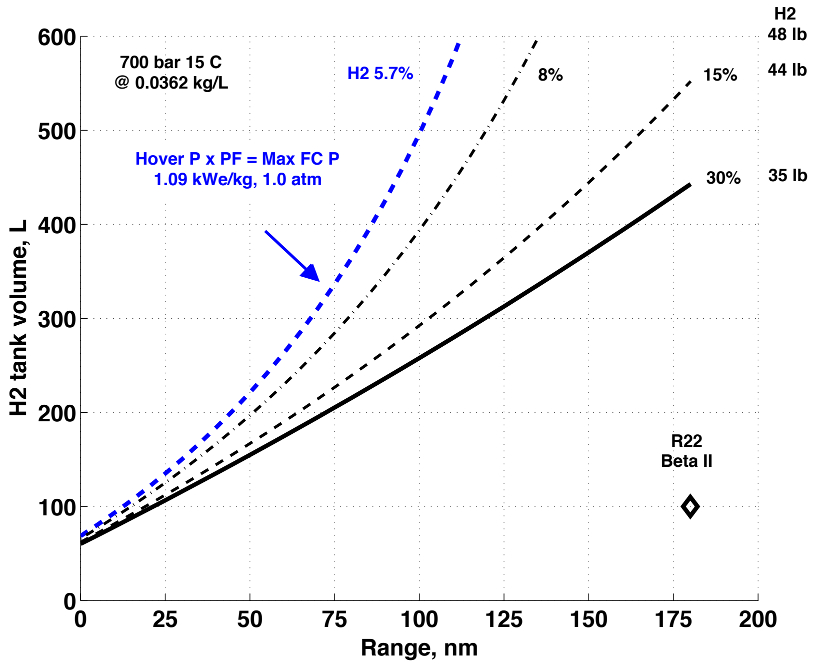

Figure 65: **Tank volume versus range with 1 atm PEMFC sized to deliver** 1.5× **hover power as maximum power. Impact of hydrogen wt%. Electric quad-rotor. 400 lb payload. H2 storage 700 bar 15° C.**

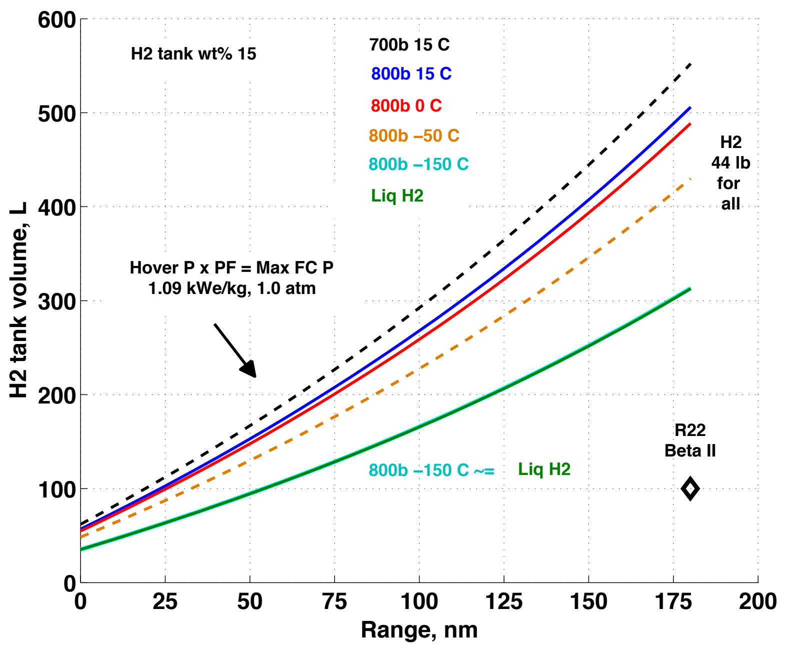

Figure 66: **Tank volume versus range with 1 atm PEMFC sized to deliver** 1.5× **hover power as maximum power. Impact of hydrogen storage type. Hydrogen wt% 15. Electric quad-rotor. 400 lb payload.**

14.5 Example: Single Main-Rotor and Tail-rotor

The gap between the R22 and the quad-rotors in the previous sections were due to the configuration. A single main-rotor and tail-rotor configuration is sized to bridge this gap.

First, a piston engine helicopter is sized to the R22 Beta II-like specifications: payload 400 lb, range 180 nm, and gross weight 1370 lb. Next, the power system is replaced with a PEMFC and an electric motor and gear box. The specific power and hydrogen weight fraction are then varied until the results converge to R22 again. The resulting targets are the benchmarks for achieving the objective aircraft.

The 400 lb payload corresponds to a standard tank. The payload and the 20 min cruise reserve (required for VFR) determine the range. It is not clear what this range is; published values vary between 160 nm to 208 nm. A range of 180 nm is assumed.

The auxiliary tank flies empty for a 400 lb payload. This is useful in the context of hydrogen storage where higher volumes are needed. The standard and the auxiliary tanks have usable volumes of 19.2 and 10.5 U.S. gallons respectively. The 19.2 gallon (73 L) was used in Datta and Johnson 2014. Since then, the R22 Service Bulletin SB-109 on 8 January 2014 (superseded by SB-109A on 15 January 2018) required all-aluminum tanks (new or retrofitted) to be bladder-type tanks no later than 15 January 2020. The new bladder-type tanks have lower usable volumes of 16.9 and 9.4 U.S. gallons (63.9 L and 35.6 L) respectively. So the total volume available for hydrogen storage is assumed to be the sum, $\approx$ 100 L. Since the target payload is 400 lb, the auxiliary tank must be flown empty for a range of 180 nm. For the hydrogen eVTOL, of same payload and range, fuel volume is a fall out, and this volume should be compared with 100 L available from both tanks. The R22 fuel is 100LL Avgas (also identified as 100L overseas), which is a 100 octane, low-lead aviation gasoline of density $\approx$ 800 kg/m^3 and specific energy 43.5 MJ/kg (12.1 kWh/kg).

The main configuration inputs such as disk loading, number of blades, solidity, cruise speed and fuselage drag, are fixed to the R22 values. The mission remains the same as that of the electric quad-rotor. So the resulting design is similar but not identical to the R22.

The Lycoming O-360-J2A piston engine of the R22 has a maximum power of 145 hp (108 kW). It is de-rated to the transmission limit of 97.5 kW. The PEMFC is sized to the maximum power to maintain consistency in power to weight. The hover power is $\approx 82 - 93$ kW, depending on conditions, so a power factor $PF = 1.25$ is used. The engine speed is 2652 RPM. The same speed is specified for the electric motor so that the transmission does not contribute to differences between the power plants. The flapping frequency is assumed as 1.01/rev, and the blade and hub technology factors were calibrated to converge to R22 specifications.

The final R22 inputs are listed below.

The conditions are 5000-ft ISA + 20°C which give the following.
Air density $\rho = 0.0019438$ slug/ft^3 (1.0018 kg/m^3)
Speed of sound $a_s = 1132$ ft/s (345 m/s)

The sizing mission is given by the following.
Payload = 400 lb
Hover OGE = 4 mins
Cruise speed = 80 knot (135 ft/s)
Cruise reserve = 20 min

The aircraft inputs are the following.
Disk loading = 2.75 lb/ft^2
Fraction structural weight $f_S = 0.22$
Fraction vibration and systems weight $f_E = 0.15$
Harris drag factor $f = 2.95$

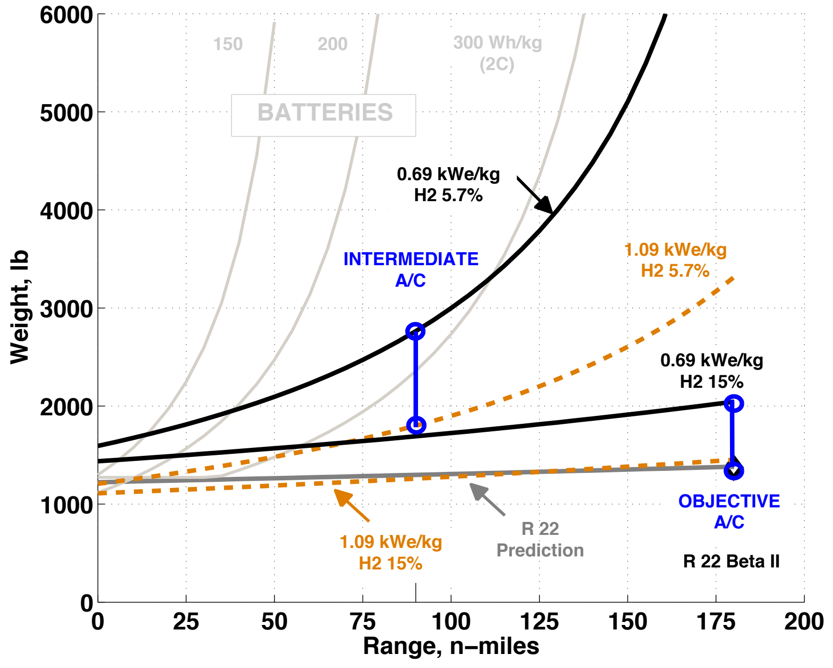

Figure 67: **Gross weight versus range. Piston engine, PEMFC and battery aircraft. Intermediate aircraft: 1/2 range of R22. Objective aircraft: same range and weight of R22. Stack system of 1.09 kWe/kg (including BOP) and H2 weight fraction 15%.**

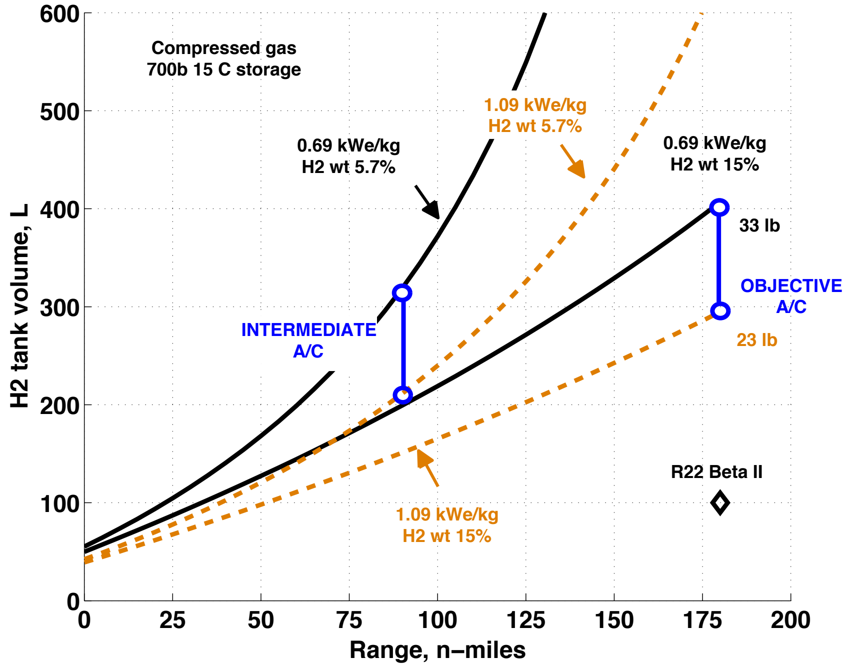

Figure 68: **Tank volume versus range of PEMFC aircraft. Intermediate aircraft: 1/2 range of R22. Objective aircraft: same range and weight of R22.**

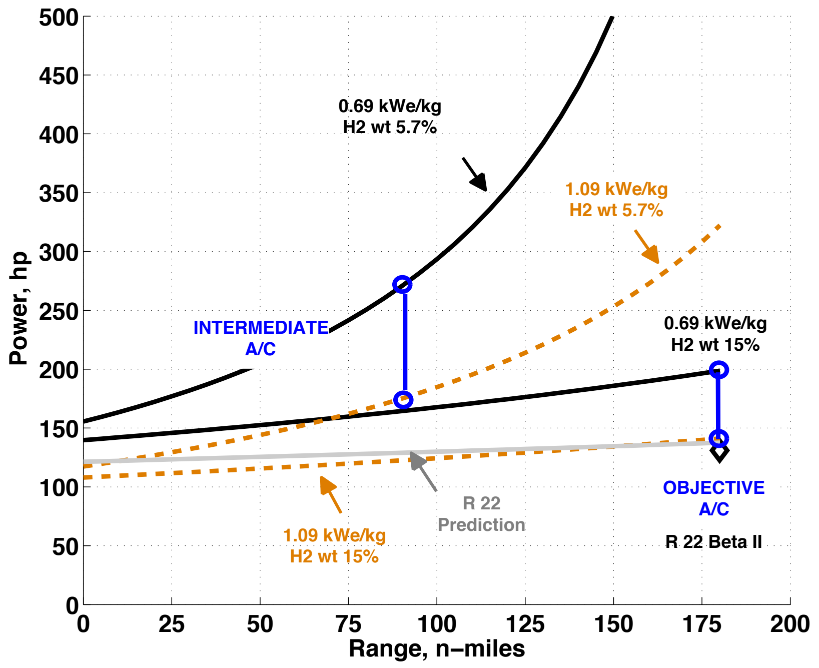

Figure 69: **Installed power versus range. Piston engine, PEMFC and battery aircraft. Intermediate aircraft: 1/2 range of R22. Objective aircraft: same range and weight of R22. Stack system of 1.09 kWe/kg (including BOP) and H2 weight fraction 15%.**

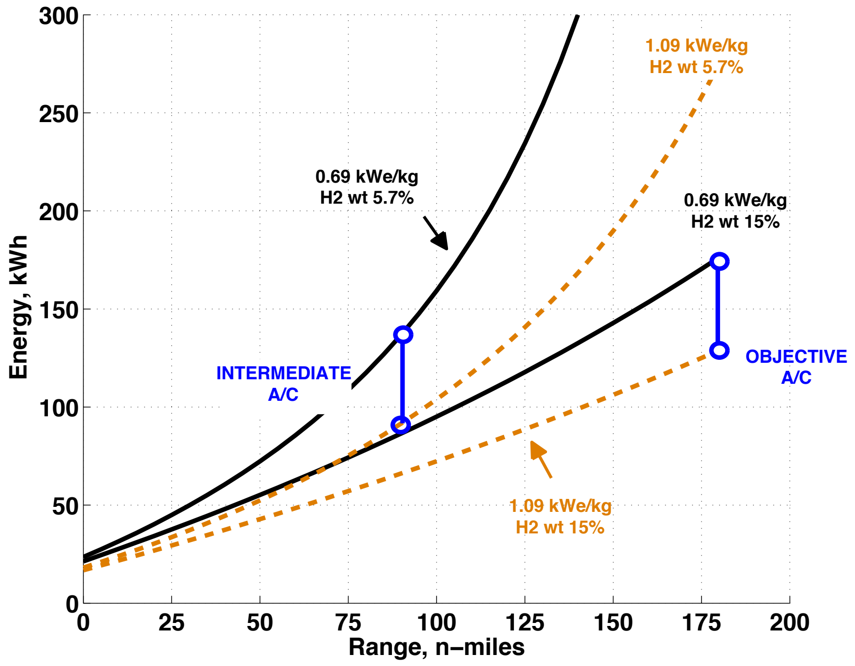

Figure 70: **Energy capacity versus range of PEMFC aircraft. Intermediate aircraft: 1/2 range of R22. Objective aircraft: same range and weight of R22.**

Number of rotors = 1

The rotor inputs are the following.
Number of blades per rotor = 2
Tip Mach number $M_T = 0.6$ (tip speed = 138 m/s)
Solidity = 0.030
Airfoil mean drag coefficient $c_{d0} = 0.01$
Airfoil mean lift curve slope $c_{l\alpha} = 5.73$
$\kappa_h = 1.15$
$\kappa_f = 1.15$
Flapping frequency $\nu_\beta = 1.01$ / rev
Hub frequency $\nu_h = \nu_\beta$
Blade tech factor = 0.5
Hub tech factor = 0.5

The power system inputs are the following.
Power factor PF = 1.25
RPM piston engine = 2652
SFC piston engine = 0.40 lb/hp-hr
Engine tech factor = 0.70
$f_Q = 3$ (for single main-rotor and tail-rotor)

The electric power system inputs are the same as the quad-rotor inputs in the previous section.
Power factor PF = 1.25
RPM motor = 2652
η Motor = 0.80
Fraction controller and power conditioner weight = 0.5
Fraction cooling system weight = 0.5
$n_g = 1$
For batteries, are following inputs are used.
Battery Wh/kg available and installed = 150, 200, 300
Battery Max C-rate = unlimited, 2
For PEMFC, 1 atm stacks are used, sized for maximum efficiency.

Figure 67 shows the design gross weight varying with range. The piston engine aircraft is calibrated to produce the R22 weight and range (R22 prediction line).

The PEMFC design uses a 1 atm stack sized for maximum efficiency. This is the same technology as in the last column of Table 29 and in Fig 48. At 5000 ft/20°C, the power to weight at design power (maximum efficiency $\eta = 0.35$) is 0.69 kWe/kg. The power to weight at maximum power is (lower efficiency $\eta = 0.28$) 1.09 kWe/kg which is 1.58× the design power. The tank specification is 5.7% weight fraction filled to 700 bar 15°C with 10% overhead for structural volume.

If the stack is sized to deliver the aircraft maximum power (1.25× hover power OGE) as the stack design power, the solid black line is obtained. The stack design power is continuous power since the cooling system is sized for it. Half the range of the R22 (90 nm) can be flown with an aircraft twice its weight (2800 lb). The stack system specific power is 0.69 kWe/kg which is the total weight including air, cooling, and electrical systems (everything except the fuel and fuel tank) divided by the net electrical power at design power. But this is an over design since the stack maximum power is $\approx 1.58\times$ the stack design power and this margin remains unused. Since the aircraft is at cruise most of the time and cruise power is only $\approx 0.7 - 0.8\times$ hover power OGE, even the stack design power remains unused most of the time.

Given the short 4 min hover, the stack could be sized to deliver the aircraft maximum power ($1.25\times$ hover power OGE) as the stack maximum power. Then the dashed line is obtained. The stack system specific power is now 1.09 kWe/kg which is the total weight divided by the net electrical power at maximum power. Half the range of the R22 (90 nm) can now be flown with an aircraft of 1800 lb. The aircraft maximum power is no longer continuous, but rated according to stack cooling. Thus, the actual design could lie anywhere along the marked vertical line; the heaviest aircraft if stack cooling is impossible above design power, and the lightest aircraft when it is possible for 4 min hover.

Even if the stack is sized to deliver the aircraft maximum power as the stack design power, the lightest aircraft would still be possible, if hydrogen weight fraction were increased from 5.7% to 15%. Thus, there are two alternative paths to the lightest aircraft. The lightest aircraft is marked as an intermediate aircraft. It is a benchmark aircraft for what modern automobile fuel cells can possibly achieve if tailored to eVTOL; either with a low pressure stack or a higher weight fraction storage. If both are accomplished, the resulting aircraft would converge to the R22. This aircraft is marked as the objective aircraft. Once again, the vertical band covers for the unknown heating behavior of the stack. So the aircraft could be between 1370 lb and 2000 lb. The intermediate aircraft lies just beyond a 300 Wh/kg battery aircraft (pack level, available and installed). The objective aircraft is far beyond.

Figure 68 shows the corresponding tank volumes. The intermediate aircraft requires $2.5\times$ the R22 tank volume (200 L). The objective aircraft about $3\times$ (300 L). This extra volume has to be found. Since new aircraft need not conform to an existing shape or form, it might be possible to find this extra volume. The intermediate aircraft would require 7.5 kg hydrogen at 700 bar 15°C. The intermediate band is between $7.5 - 11.2$ kg. The objective aircraft would require 10.4 kg hydrogen. The objective band is between $10.4 - 15$ kg.

Figure 69 shows the installed power levels (net electrical). The intermediate aircraft requires 175 hp (131 kWe) of installed power. The hover power is $1/1.25\times$ installed power and equals 105 kWe. The maximum continuous power is the stack design power which is $1/1.58\times$ installed power and equals 83 kWe. On the other end of the intermediate band is an aircraft of 275 hp (205 kWe). The hover power is 164 kWe and the maximum continuous power is 130 kWe. So the intermediate band is between $175 - 275$ hp ($131 - 205$ kWe). The objective band starts from the objective aircraft at 145 hp (108 kWe). The hover power is $1/1.25\times$ installed power and equals 87 kWe. The maximum continuous power is $1/1.58\times$ installed power and equals 67.5 kWe which is near the cruise power of the R22. On the other end of the objective band is an aircraft of 200 hp (149 kWe). The hover power is 119 kWe and the maximum continuous power is 94 kWe which is near the hover power of the R22. So the objective band is between $145 - 200$ hp ($108 - 149$ kWe).

Figure 70 shows the energy consumed per mission. The intermediate aircraft consumes about 80 kWh ($80 - 135$ kWh band) whereas the objective aircraft consumes 130 kWh ($130 - 180$ kWh band).

15 Summary and Conclusions

A method for design and analysis of a Proton Exchange Membrane fuel cell (PEMFC) system was developed. The software sizes a PEMFC system to meet certain specified requirements (design) and then predicts its performance for a set of conditions (analysis).

The design and analysis were based on simplified formulas and a reduced set of parameters to serve as a common-basis for technology assessment by fuel cell, hydrogen, and rotorcraft specialists. Examples of stack designs were provided for comparative assessment. Examples of rotorcraft designs were provided for illustration of how fuel cells affect sizing.

Examples of stack designs were provided for 80 kWe and 500 kWe net power. The 80 kWe example is a template for a modern automobile stack such as the U.S. Department of Defense Hydrogen Program (DOE HP) benchmark stack. Calibration will require subsystem weights and power data. The 500 kWe example is a template for a hypothetical eVTOL stack.

Examples of eVTOL designs were provided for 250 and 400 lb payload. The 250 lb example is similar to a hypothetical NASA electric quad-rotor. The 400 lb examples are similar to a hypothetical Maryland electric quad-rotor and the Robinson R22 Beta II helicopter. Comparisons were presented with piston engine and batteries. The objectives were to establish the key technology drivers and requirements for fuel cell hydrogen eVTOL. The R22-like example allowed technology targets to be anchored to a certified aircraft.

Cost analysis was left out of scope.

Based on the above study, the following key conclusions were drawn.

1. **Hydrogen supply:** There is sufficient hydrogen infrastructure in the U.S. for a pilot program on hydrogen eVTOL aircraft. There are 49 retail hydrogen stations in the U.S. as of July 2021, 48 in California, and one in Hawaii. There are 19 additional private stations, 5 of which are in California, 2 each in Massachusetts, Michigan, and Ohio, and one each in Arizona, Colorado, Connecticut, Delaware, Hawaii, New York, Virginia and Washington. About 12,000 kg of retail hydrogen is available for fuel cell ground vehicles per day in California. This is 0.05% of the 9 million metric tons of hydrogen produced in the U.S. in 2020.

2. **Fueling:** Fueling infrastructure is available for 350 bar tanks (buses and forklift) and 700 bar tanks (cars) of gaseous hydrogen. The fueling standards are defined in SAE J2601. The fueling rate is about $20-30$ g/s during which there is a rapid rise in temperature. The rate is limited by the maximum temperature of the automobile fuel cell tanks which is about 85°C. To maximize fill-in, the fuel delivery temperature is kept as low as -40°C. The tank temperature can reach $50-80$°C during fueling then settles into an equilibrium at $15^\circ - 20^\circ$C. Thus, 700 bar $15^\circ - 20^\circ$C storage is an appropriate sizing specification for the fuel tank if current infrastructure is to be used. The density of hydrogen under these conditions is 40.17 kg/m^3.

3. **PEM cell:** The performance of a modern air-breathing PEM fuel cell has reached near pure-oxygen levels. A modern cell can produce maximum power densities p_c of 0.8 W/cm^2 at 1 atm to 1.1 W/cm^2 at 2.5 atm. The corresponding cell current density and voltages are 0.5 A/cm^2 and 0.5 V and 2.0 A/cm^2 and 0.55 V respectively. The cell efficiencies are 0.34 and 0.37, and cell heating 0.53 W/cm^2 and 0.7 W/cm^2 respectively. The maximum power density occurs at a low voltage hence low efficiency and high heating. Hence a lighter stack consumes more fuel for the same power and produces more heat. The highest voltage and efficiency occur at or near zero power and is therefore not a practical design point. So a moderate voltage and efficiency is selected so that the power density is adequate.

State-of-the-art voltages and current densities are around 0.65 V and 0.75 A/cm^2 for a 1 atm stack (efficiency 0.44 and power density 0.5 W/cm^2) and 0.7 V and 1.0 A/cm^2 for a 2.5 atm stack (efficiency 0.48 and power density 0.7 W/cm^2). Leakage current can reduce efficiency significantly.

4. **Stack Weight:** The stack weight W is proportional to stack power generated P and inversely proportional to cell power density p_c. The constant of proportionality is a design factor DF which depends only on the structure and materials. So the specific power of a stack is $P/W = p_c/DF$ where electrochemistry impacts the numerator and structure impacts the denominator. A power density of $p_c \approx 1$ W/cm^2 gives $P/W = 10/DF$ kW/kg where DF has unit kg/m^2. The specific power of the stack can reach 2 kW/kg with a design factor of 5 kg/m^2. The design factors of commercial off-the-shelf stacks lie between $20 - 50$ kg/m^2.

5. **Specific Power:** The net specific power of the stack system P/W is the net electrical power divided by the stack system weight. The net electrical power in kWe is the gross power generated minus the balance of plant loss. The stack system weight is the weight of the stack plus all the ancillary subsystems. The steepest balance of plant loss is from the compressor in the air subsystem. The heaviest component is usually the radiator of the stack cooling subsystem. For example, a 2 kW/kg stack might effectively be $0.5 - 1$ kWe/kg stack system. Thus ancillary subsystems can produce more than 100-300% overhead.

6. **Stack Design:** A stack can be designed for minimum weight or high efficiency. The choice impacts the overhead. Minimum weight comes at the price of low efficiency hence high fuel consumption. High efficiency comes at the price of greater weight. Minimum weight also produces maximum heating which, depending on the cooling system, can more than offset any gain in stack weight. The decision is based on many factors such as: power level, mission time, altitude loss, and method of stack cooling, all of which depend on the application.

7. **Barrier to Specific Power:** The principal barrier to specific power in rotorcraft appears to be the weight of the stack cooling system. A straight-forward radiator-based automobile solution will not do. The burden may be relieved by leveraging the unique eVTOL mission. A short hover ($5-15$ min) opens the possibility of sizing the stack for maximum efficiency and continuous cooling only at cruise power while rating the hover power for duration. The secondary barrier to specific power is the compressor of the air intake system. Low altitude flight ($< 14,000$ ft) may open the possibility of eliminating this system or making it lighter by using a 1 atm stack and rating the ceiling appropriately. There is some precedence for such systems in fixed-wing experimental motor gliders and some published data on altitude loss. Then the air pre-cooling system can also be removed. Definitive conclusions require systematic parametric test data.

8. **Specific Energy:** A fuel cell is an engine and in itself has no specific energy. Specific energy is produced by the addition of the hydrogen subsystem. Likewise, the hydrogen subsystem in itself has no specific power. Therefore, comparison with batteries must include the stack system and the hydrogen subsystem. A 80 kWe stack system with 5 kg hydrogen in automobile-like long-term tanks of weight fraction 5.7% was predicted to deliver specific energy and specific power of $370 - 390$ Wh/kg and $0.56 - 0.48$ kW/kg respectively where the energy and power were net electrical and the weight included the total system. The specific power is equivalent to battery C-rates of $1.5 - 1.23$ with the lower value corresponding to the higher specific energy consistent with any electrochemical source.

9. **Barrier to Specific Energy:** The principal barrier to specific energy in rotorcraft is the weight fraction of hydrogen storage. A straight-forward low boil-off 5-8% automobile solution will not do. This barrier can be resolved by leveraging the short-term nature of storage in aviation, particularly in rotorcraft and eVTOL where only $1 - 3$ hr of storage is needed. Peer-reviewed efforts report 13% storage for tiny amounts of hydrogen (0.5 kg) in un-manned airplanes, 20-30% storage for moderate amounts

of hydrogen (30-35 kg) in the space shuttle, and massive storage fractions > 100% for massive amounts of hydrogen in launch vehicles. Once again, test data is needed for any definitive conclusion.

10. **Quad-rotor eVTOL of 250 lb Payload:** For a quad-rotor eVTOL of payload 250 lb, hover of 4 min, cruise at 70 kt with 20 min reserve, an automobile PEMFC system with 2.5 atm stacks sized to produce the aircraft maximum power as stack design power and 5.7% hydrogen storage, the design closes at a minimum gross weight of 4000 lb. But if sized to produce the same power as stack maximum power, of intermittent rating, leveraging the short hover time, with the aircraft cruise power equal to or lower than the stack design power, then an aircraft of 2200 lb is feasible. This solution surpasses a 150 Wh/kg battery aircraft (pack level, installed and available at 80% discharge at end of life) above a range of 25 nm and a 200 Wh/kg battery aircraft above 65 nm. At 15% hydrogen storage, a range of 125 nm is possible with an aircraft of 2750 lb which corresponds to a 400 Wh/kg battery solution. About 35 lb of hydrogen will be needed to fly this range which will require a large 450 L tank. The assessment changes significantly with low pressure 1 atm stacks. A range of 125 nm may now be possible with an aircraft of only 2000 lb. About 20 lb of hydrogen will be needed which will require a tank volume of only 260 L. This solution surpasses a 200 Wh/kg battery aircraft above 10 nm, a 300 Wh/kg above 35 nm, and a 400 Wh/kg above 65 nm.

11. **Quad-rotor eVTOL of 400 lb Payload:** For a quad-rotor eVTOL of payload 400 lb, hover of 4 min, cruise at 70 kt with 20 min reserve, an automobile PEMFC system with 2.5 atm stacks sized to produce the aircraft maximum power as stack design power and 5.7% hydrogen storage, the design closes at a minimum gross weight of 5500 lb. But if sized to produce the same power as stack maximum power, of intermittent rating, leveraging the short hover time, with the aircraft cruise power equal to or lower than the stack design power, then an aircraft of 3000 lb is feasible. This solution surpasses a 150 Wh/kg battery aircraft (pack level, installed and available at 80% discharge at end of life) above a range of 20 nm and a 200 Wh/kg battery aircraft above 55 nm. The assessment changes significantly with low pressure 1 atm stacks and 15% hydrogen storage. A range of 125 nm is now possible with an aircraft of only 3000 lb. About 44 lb of hydrogen will be needed which will require a tank volume of around 550 L. This solution surpasses a 200 Wh/kg battery aircraft above 10 nm and a 300 Wh/kg battery aircraft above 35 nm.

12. **Single main-rotor eVTOL of 400 lb Payload:** For a single main-rotor tail-rotor eVTOL of payload 400 lb, hover of 4 min, cruise at 80 kt (similar to a R22) with 20 min reserve, a PEMFC system with 1.0 atm stacks sized to produce the aircraft maximum power as stack maximum power at 1.09 kWe/kg and 15% hydrogen storage, would meet the R22 specifications. However, the usable tank volume would be 300 L which is $\approx 3\times$ the R22 usable volume. This solution is identified as the objective aircraft. A heavier stack system of 0.69 kWe/kg with the same 15% storage, or the same stack system but with a lower 5.7% storage, would achieve half the range (90 nm) with a 1800 lb aircraft (30% more than the R22). A tank size of 200 L would be needed. This solution is identified as an intermediate aircraft. The intermediate aircraft is similar to a 325 kWh/kg (1-1.5C) battery powered aircraft. The objective aircraft is far beyond batteries.

13. **H2 storage:** There appears little gain in volumetric hydrogen storage above a pressure of 700 bar. It is beneficial to lower the storage temperature below 15°C. The density of hydrogen at 700 bar and 0°C, -50°C, -100°C, and -150°C are 42, 48, 56 and 67 kg/m^3 respectively. The density of liquid hydrogen at 1 bar is 71 kg/m^3. Storage at -150°C, if it were possible, would be nearly as compact as liquid storage purely in terms of used volume.

16 Technology Drivers, Requirements and Recommendations

The report described conceptual stack designs and conceptual hydrogen eVTOL designs with trade-offs within those designs. The trade-offs included stack characteristics, hydrogen storage characteristics, and aircraft payload and range. The objectives were to the identify the key technology drivers of a hydrogen rotorcraft, establish the requirements, and offer recommendations on research to address fundamental barriers.

The key technology drivers were identified as follows.

1. High weight fraction ($> 15\%$) short term ($1 - 3$ hr) hydrogen storage.
2. High specific power stack system including ancillary subsystems (> 1.0 kWe/kg).
3. Ultra-light stack cooling.
4. Low temperature hydrogen storage ($< 15^\circ$ C).

Datta and Johnson 2014 gave a list of requirements for hydrogen eVTOL. The cell electrochemical requirements (voltage and power density) have now been met. The U.S. hydrogen infrastructure has also grown since then. In view of these advances, the requirements are re-derived systematically for an intermediate and an objective aircraft. These aircraft might be suitable candidates for a hydrogen eVTOL demonstrator.

Both the intermediate and objective aircraft are designed to carry the same payload as the Robinson R22 Beta II. The intermediate aircraft has half the range with 30% increase in gross take-off weight. The objective aircraft has the same range and gross weight as the R22. The intermediate aircraft needs only 50% more hydrogen than cars (7.5 kg) whereas the objective aircraft needs twice that of cars (11 kg). The intermediate aircraft needs twice the usable tank volume than the R22 (200 L) whereas the objective aircraft needs three times the volume (300 L).

The technology requirements are listed in Table 30. The present column is loosely based on Toyota Mirai. The stack system data (cell electrochemical properties, P/W, stack pressure and heat) are from the DOE HP for a stack of similar class. The power to weight P/W (net electrical power ÷ stack system weight including air, cooling, and electrical systems) was not readily available, so the predicted value from the 2.5 atm, maximum efficiency, liquid cooled stack in Table 29 is used, which has a specific power of 0.53 kWe/kg at design power. It is the same model as the liquid cooled stack in Table 28, which has a specific power of 0.63 kWe/kg, only designed for 5000-ft/20°C. The Δv_c/ft is the average loss in cell voltage due to altitude. This value is based on DLR airplane measurements.

The electrochemical requirements are assumed met for the purposes of this report. The stack pressure is not a requirement, only a possible path to the target specific power. The specific power corresponds to the bands for intermediate and objective aircraft. The target is the higher value of 1.1 kWe/kg with 0.7 kWe/kg continuous. The fall back is the lower value 0.7 kWe/kg continuous. The power levels correspond to the bands for intermediate and objective aircraft. The target aircraft is the lower value; intermediate target is 131 kWe max with a minimum continuous power of 83 kWe, the objective target is 108 kWe max with a minimum continuous power of 68 kWe. The hover power is $1/1.25\times$ maximum power. If these targets cannot be met, then the higher values should be pursued with a heavier 0.7 kWe/kg system. The objective aircraft is smaller, hence requires lower power.

The heat rejection criteria is the stack heat load Q divided by the difference in temperature ΔT between the stack coolant outlet temperature ($\approx$ stack temperature T_S) and the maximum ambient temperature of the radiator T_a for which the system is still able to reject the heat. The ratio $Q/\Delta T < 1.45$ kW/°C is a target for automobile stacks of 80-kWe at $T_a = 40$°C. The criteria for rotorcraft is unknown without actual

Variable	Unit	Present car	Intermediate aircraft	Objective aircraft
Stack system				
cell v_c, p_c	V, W/cm^2	0.70, 0.75 (2.5 atm)	- met -	- met -
		0.65, 0.50 (1.0 atm)	- met -	- met -
P/W stack sys	kWe/kg	0.83 max	1.10 max	1.10 max
	kWe/kg	0.56 con	0.70 con	0.70 con
stack pressure	atm	2.5	1.0	1.0
Δv_c/h	V/1000-ft	-0.0056	-accept-	-minimize-
Power	kWe	100 for 30 sec	$131 - 205$ for 30 sec	$108 - 149$ for 30 sec
	kWe		$105 - 164$ for 4 min	$87 - 120$ for 4 min
	kWe	73	$83 - 130$ con	$68 - 94$ con
Energy	kWe-hr	100	$80 - 135$	$130 - 180$
Heat Q/ΔT	kW/°C	1.45	3.1	2.5
H2 storage				
H2 wt%		5.7	5.7	15
H2 mass	kg	5.0	$7.5 - 11$	$11 - 15$
Tank vol	L	122	$200 - 320$	$300 - 400$
Tank vol%	kg/L	0.041	-accept-	-maximize-
H2 fill	bar, °C	700/15°C	-accept-	cooler °C or liquid

Table 30: **PEMFC requirements for a two-seat hydrogen eVTOL. $\Delta v_c/h$ is the cell voltage loss with altitude. P/W is net electrical power ÷ weight of stack system including air, cooling, and electrical systems (everything except fuel and fuel tank).**

measurements. The criteria shown is simply 1.45 scaled to maximum power (the lower values in the bands) and multiplied with the factor $\Delta T_{\text{rc}}/\Delta T_{\text{car}} = 1.625$. The car $\Delta T \approx 40$°C is based on stack and ambient temperatures of 40°C whereas the rotorcraft $\Delta T \approx 65$°C is based on a reduced ambient of $T_a = 15$°C for aviation.

The technology targets are compared to state-of-the-art rotorcraft engines in Figs 71 (all engines) and 72 (piston engine). The engine data is from Rosen 2008. The PEMFC concept engines sized earlier are also plotted. The maximum power is used on the horizontal axis. The intermediate and objective P/W target of 1.09 kWe/kg is marked with a straight line. The 1.5 kWe/kg target marked as ultimate is aspirational, informed only by the electric quad-rotor analysis in Fig 64, but otherwise arbitrary.

Combustion engines scale well with power (higher P/W at higher power). PEMFC has no engine exhaust and their low temperature operation do not lend well to radiator cooling. The scatter in the predicted PEMFC weights is due to differences in type of design (minimum weight versus maximum efficiency), stack pressure (2.5 atm versus 1 atm), and cooling system modeling (baseline versus liquid). No scaling trends can be established without weights data from the cooling system. A PEMFC engine should strictly include the electric motor. It is left out because motor weights scale with torque, not power, and the gear-box in a rotorcraft can be sized to reduce the total weight of gear-box and motor independently from the energy source. The Datta and Johnson 2014 targets (2.5 – 4.0 kW/kg for motors) provide ample margin. The combustion engines are plotted based on their dry weight.

The impact on hydrogen infrastructure in California is shown in Table 31. The calculation for cars is based on 10,000 fuel cell cars each requiring 1.2 kg/day of hydrogen on an average. With 5 kg tanks it amounts to

0.24 fill/day for each car. The calculation for eVTOL are based on 7.5 – 11 kg hydrogen and 80 – 135 kWh energy per flight for an intermediate fleet and 11 – 15 kg hydrogen and 130 – 180 kWh energy per flight for an objective fleet. The intermediate fleet is assumed to be 100 aircraft with 3 flights/day requiring at least 22.5 kg/day of hydrogen. The objective fleet is assumed to be 1000 aircraft with 5 flights/day requiring 55 kg/day of hydrogen. This is an extreme case; for context, only about 4600 R22 were built between 1979 and 2015. Fleet energy is total energy dispensed/day = Energy/fill × Fills/day × Fleet, given in units of megawatt-hr (MWh). About 2.25 metric tons of hydrogen are needed per day for the intermediate fleet, and 55 metric tons for the objective fleet. The energy required to produce this hydrogen is calculated based on Section 2 estimates of 49 kWh/kg for grey and 62 kWh/kg for green (generation and compression into 700 bar tanks). The intermediate fleet will require about 140 MWh of renewable electricity per day for green hydrogen. For context, 250 MW are presently deployed world wide for green hydrogen. So half an hour of generation is sufficient to operate the intermediate fleet. The higher capacity of the objective fleet is due to more aircraft and flights per day. If scaled back to the same number of aircraft and flights as the intermediate fleet, about 3.3 ton of hydrogen will be needed per day, and 205 MWh of renewable electricity per day for green hydrogen (not shown in Table 31).

Variable	Unit	Present cars	Intermediate fleet	Objective fleet
Fueling rate	min/kg	0.6	accept	accept
Fueling temp	°C	-40	accept	lower or liquid
Vehicle range	n-miles	312	90	180
H2/fill	kg	5	7.5 – 11	11 – 15
Fills/day		0.24	3	5
H2/day	Metric ton	12	2.25 – 3.3	55 – 75
Energy/fill	kWh	100	80 – 135	130 – 180
Fleet	vehicles	10,000	100	1000
Energy dispensed	MWh/day	240	24 – 41	650 – 900
Energy to produce H2				
Grey	MWh/day	588	110 – 162	2700 – 3650
Green	MWh/day	744	140 – 204	3410 – 4650

Table 31: **Hydrogen eVTOL impact on California infrastructure. Present fleet: 10,000 cars, 1.2 kg/day, 1.2/5 fill/day. Intermediate fleet: 100 A/C, 3 flights a day, 3 fill/day, 33 kg/day. Objective fleet: 1000 A/C, 5 flights a day, 5 fill/day, 55 kg/day.**

The assessment presented is only a crude approximation without a careful calibration of the tool with measurements. Dedicated research and engineering investments are needed to acquire these measurements in a rotorcraft environment. A demonstrator testbed is ultimately needed to support new aircraft development. These and other recommendations are listed below.

1. The design and analysis tool should be calibrated with weights and performance data from a 80-100 kW automobile fuel cell.

2. Short-term, high weight fraction hydrogen storage should be fabricated and tested. Boil-off and hydrogen leakage rates should be measured.

3. A hydrogen eVTOL demonstrator aircraft should be built and flown. The objective aircraft is the ideal target. If this target is not feasible, the intermediate aircraft can be the fall back option.

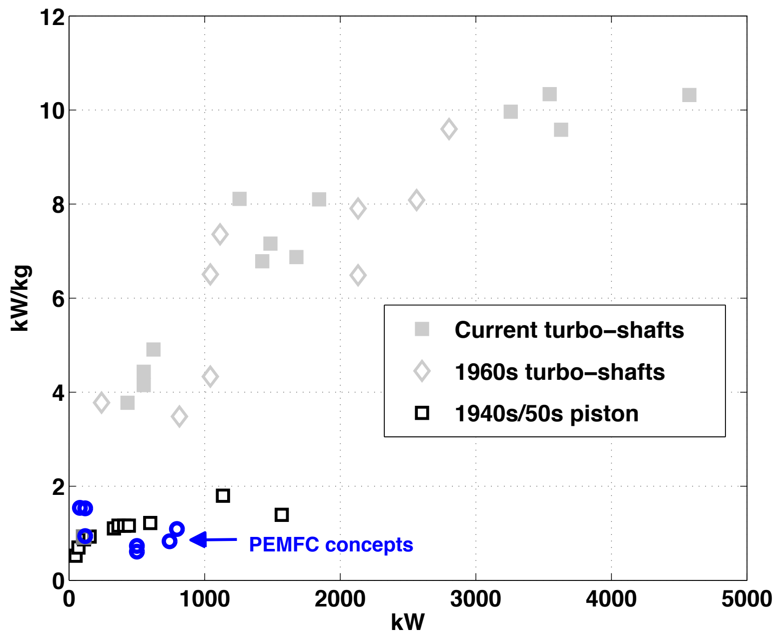

Figure 71: **Comparison of the PEMFC systems with state-of-the-art rotorcraft combustion engines. PEMFC power is maximum net electrical power in kWe. Weight includes stack and all ancillary subsystems including coolant. Engine weights are dry weights.**

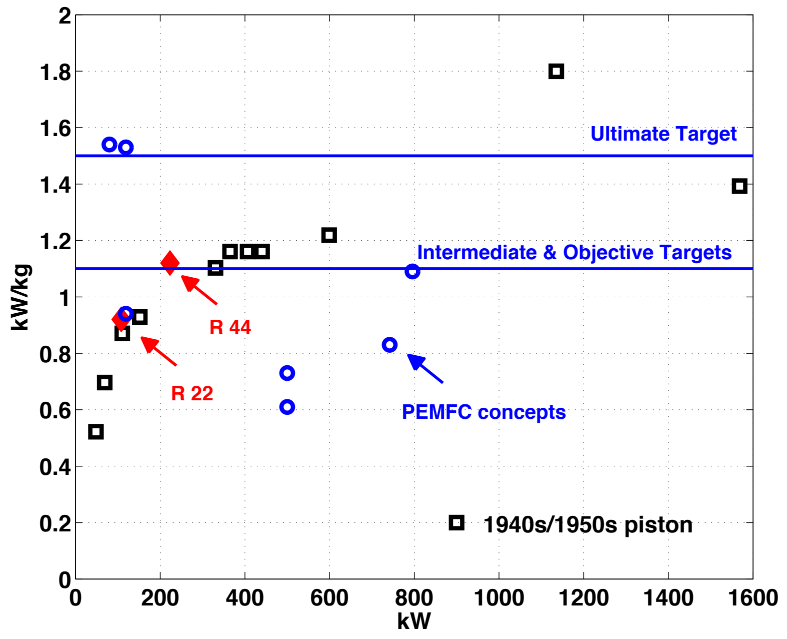

Figure 72: **Comparison of the PEMFC systems with state-of-the-art rotorcraft piston engines. PEMFC power is maximum net electrical power in kWe. Weight includes stack and all ancillary subsystems including coolant. Engine weights are dry weights.**

4. The demonstrator aircraft should fly interchangeable high and low pressure stacks, interchangeable low and high power stacks, measure wake and downwash effects on stack cooling, attempt 15% and higher weight fraction tanks, and carry a high C-rate battery in parallel for power sharing in maneuvers as well as contingency back up.

Rotorcraft aeromechanics would drive the architecture of a PEMFC hydrogen eVTOL. Aeromechanics sets the requirements and the specifications of the power plant. It determines how it should be unified within the high wake, high vibratory loading environment of a rotorcraft to mitigate its cooling barrier and maintain its structural integrity. A $2-3$ year lead time is often needed for new engine development for a new rotorcraft. Power and platform are integrally coupled in VTOL, and must be designed in an unified manner. Isolated efforts to drop-in or retro-fit existing solutions from elsewhere are unlikely to produce viable aircraft.

Acknowledgment

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18 Source Code and Input Decks

Files in *src* directory.

```
% DATA_gptH2.m
% Gas Phase Thermochemistry data
% NIST Standard Reference Database 69; NIST Chemistry WebBook
% New parameter fit October 2001
% Molecular weight: 2.01588
% Ref: Chase,  M. W., Jr., NIST-JANAF Thermochemical Tables,
%      Fourth Edition, J. Phys. Chem. Ref. Data, Monograph 9, 1998, 1-1951.

% T   temperature in Kelvin (K)
% g   standard Gibbs free energy kJ/mol
% s   standard entropy kJ/mol.K
% h   standard enthalpy kJ/mol
% cp  heat capacity J/mol.K


function [g, h, s, cp] = DATA_gptH2(T);
if T >= 298 && T < 1000
    A =  33.066178;
    B = -11.363417;
    C =  11.432816;
    D = -2.772874;
    E = -0.158558;
    F = -9.980797;
    G =  172.707974;
    H =  0.;
elseif T >= 1000 && T < 2500
    A =  18.563083;
    B =  12.257353;
    C = -2.859786;
    D =  0.268238;
    E =  1.977990;
    F = -1.147438;
    G =  156.288133;
    H =  0.;
elseif T >= 2500 && T <= 6000
    A =  43.413560;
    B = -4.293079;
    C =  1.272428;
    D = -0.096876;
    E = -20.533862;
    F = -38.515158;
    G = 162.081354;
    H =  0.;
else
    error('DATA_gptH2.m: no data for T < 298K or > 6000K');
end


h298 = 0;
t = T/1000;
t2 = t*t;
t3 = t2*t;
t4 = t3*t;
```

```
cp = A + B*t + C*t2 + D*t3 + E/t2;
h  = h298 + A*t + B*t2/2 + C*t3/3 + D*t4/4 - E/t + F - H;
s  = A*log(t) + B*t + C*t2/2 + D*t3/3 - E/(2*t2) + G;
s  = s*1e-3;
g  = h - T*s;
end
```

```
% DATA_gptO2.m
% Gas Phase Thermochemistry data
% NIST Standard Reference Database 69; NIST Chemistry WebBook
% New parameter fit January 2009
% Molecular weight: 31.9988

% T   temperature in Kelvin (K)
% g   standard Gibbs free energy kJ/mol
% s   standard entropy kJ/mol.K
% h   standard enthalpy kJ/mol
% cp  heat capacity J/mol.K


function [g, h, s, cp] = DATA_gptO2(T);


if T >= 100 && T < 700
    A =  31.32234;
    B = -20.23531;
    C =  57.86644;
    D = -36.50624;
    E = -0.007374;
    F = -8.903471;
    G =  246.7945;
    H =  0.;
elseif T >= 700 && T < 2000
    A =  30.03235;
    B =  8.772972;
    C = -3.988133;
    D =  0.788313;
    E = -0.741599;
    F = -11.32468;
    G =  236.1663;
    H =  0.;
elseif T >= 2000 && T <= 6000
    A =  20.91111;
    B =  10.72071;
    C = -2.020498;
    D =  0.146449;
    E =  9.245722;
    F =  5.337651;
    G =  237.6185;
    H =  0.;
else
    error('DATA_gptO2.m: no data for T < 100K or > 6000K');
end


h298 = 0;
t = T/1000;
t2 = t*t;
t3 = t2*t;
t4 = t3*t;
```

```
cp = A + B*t + C*t2 + D*t3 + E/t2;
h  = h298 + A*t + B*t2/2 + C*t3/3 + D*t4/4 - E/t + F - H;
s  = A*log(t) + B*t + C*t2/2 + D*t3/3 - E/(2*t2) + G;
s  = s*1e-3;
g  = h - T*s;
end
```

```
% DATA_gptH2Ol.m
% Liquid Phase Thermochemistry data
% NIST Standard Reference Database 69; NIST Chemistry WebBook
% New parameter fit January 2009
% Molecular weight: 18.0153
% Ref: Chase,  M. W., Jr., NIST-JANAF Thermochemical Tables,
%      Fourth Edition, J. Phys. Chem. Ref. Data, Monograph 9, 1998, 1-1951.

% T   temperature in Kelvin (K)
% g   standard Gibbs free energy kJ/mol
% s   standard entropy kJ/mol.K
% h   standard enthalpy kJ/mol
% cp  heat capacity J/mol.K

function [g, h, s, cp] = DATA_gptH2Ol(T);


if T >= 273 && T < 500
    % NIST range is 298 < T < 500
    A =  -203.6060;
    B =   1523.290;
    C =  -3196.413;
    D =   2474.455;
    E =   3.855326;
    F =  -256.5478;
    G =  -488.7163;
    H =  -285.8305;
else
    error('DATA_gptH2Ol.m: no data for T < 273K or > 500K');
end

h298 = -285.8304;
t = T/1000;
t2 = t*t;
t3 = t2*t;
t4 = t3*t;
cp = A + B*t + C*t2 + D*t3 + E/t2;
h  = h298 + A*t + B*t2/2 + C*t3/3 + D*t4/4 - E/t + F - H;
s  = A*log(t) + B*t + C*t2/2 + D*t3/3 - E/(2*t2) + G;
s  = s*1e-3;
g  = h - T*s;

end
```

```
% DATA_gptH2Ov.m
% Gas Phase Thermochemistry data
% NIST Standard Reference Database 69; NIST Chemistry WebBook
% Data last reviewed in March 1979
% Molecular weight: 18.0153
% Ref: Chase,  M. W., Jr., NIST-JANAF Thermochemical Tables,
%      Fourth Edition, J. Phys. Chem. Ref. Data, Monograph 9, 1998, 1-1951.

% T   temperature in Kelvin (K)
% g   standard Gibbs free energy kJ/mol
% s   standard entropy kJ/mol.K
% h   standard enthalpy kJ/mol
% cp  heat capacity J/mol.K

function [g, h, s, cp] = DATA_gptH2Ov(T);


if T >= 280 && T < 1700
    % NIST range is 500 < T < 1700
    A =  30.092;
    B =  6.832514;
    C =  6.793435;
    D = -2.534480;
    E =  0.082139;
    F = -250.8810;
    G =  223.3967;
    H = -241.8264;
elseif T >= 1700 && T <= 6000
    A =  41.96426;
    B =  8.622053;
    C = -1.499780;
    D =  0.098119;
    E = -11.15764;
    F = -272.1797;
    G =  219.7809;
    H = -241.8264;
else
    error('DATA_gptH2Ol.m: no data for T < 280K or > 6000K');
end

h298 = -241.8264;
t = T/1000;
t2 = t*t;
t3 = t2*t;
t4 = t3*t;
cp = A + B*t + C*t2 + D*t3 + E/t2;
h  = h298 + A*t + B*t2/2 + C*t3/3 + D*t4/4 - E/t + F - H;
s  = A*log(t) + B*t + C*t2/2 + D*t3/3 - E/(2*t2) + G;
s  = s*1e-3;
g  = h - T*s;

end
```

```
% Dry air Cp, Cv, Gamma
% Cp and Cv in J/kg-K

function [Cp, Cv, Gamma] = DATA_DryAirCp(C)

    K = C + 273.15;

% T(Kelvin)   Cp (kJ/kg-K)  Cv (kJ/kg-K)  Cp/Cv
m=[
175  1.0023  0.7152  1.401
200  1.0025  0.7154  1.401
225  1.0027  0.7156  1.401
250  1.0031  0.7160  1.401
275  1.0038  0.7167  1.401
300  1.0049  0.7178  1.400
325  1.0063  0.7192  1.400
350  1.0082  0.7211  1.398
375  1.0106  0.7235  1.397
400  1.0135  0.7264  1.395
450  1.0206  0.7335  1.391
500  1.0295  0.7424  1.387
550  1.0398  0.7527  1.381
600  1.0511  0.7640  1.376
650  1.0629  0.7758  1.370
700  1.0750  0.7879  1.364
750  1.0870  0.7999  1.359
800  1.0987  0.8116  1.354
850  1.1101  0.8230  1.349
900  1.1209  0.8338  1.344
950  1.1313  0.8442  1.340
1000  1.1411  0.8540  1.336
1050  1.1502  0.8631  1.333
1100  1.1589  0.8718  1.329
1150  1.1670  0.8799  1.326
1200  1.1746  0.8875  1.323
1250  1.1817  0.8946  1.321
];

Cp = interp1(m(:,1),m(:,2),K)*1e3;
Cv = interp1(m(:,1),m(:,3),K)*1e3;
Gamma = interp1(m(:,1),m(:,4),K);
```

```
% watervp.m
% T is temperature in dec C
% p is vapor pressure in Pa


% Arden Buck equations 1996
% Empirical correlations of saturation vapor pressure to temperature
% for moist air.
% See:
% Buck, A. L. (1981), "New equations for computing vapor pressure
%       and enhancement factor", J. Appl. Meteorol., 20: 1527-1532
% Buck (1996), Buck Research CR-1A User's Manual, Appendix 1.


function p = watervp(T)

if T >= 0
    % over liquid water
    p = 6.1121 * exp( (18.678 - T/234.5) * ( T / (257.14+T) ) );
else
    % over ice
    p = 6.1115 * exp( (23.036 - T/333.7) * ( T / (279.82+T) ) );
end

p = p * 100;
```

```
% create i-v data

function [ivdata, Eh, Er, ipmax] = createiv...
    (tempC, PstackA, xO2, deltai, Er, ...
     j0C, alphaC, j0A, alphaA, ASR, jL, jleak, C)

F   = 96485;            % Faraday's constant Coulombs/mole
R   = 8.3144621;        % Ideal gas const J/mol.K
T   = 273.15 + tempC;   % temperature in K
RT  = R*T;
nA  = 2;                % Anode: moles of e per mole of H2
nC  = 4;                % Cathode: moles of e per mole of O2

% Find Eh and Er
[Hg, Hh, Hs, Hcp]     =  DATA_gptH2(T);
[Og, Oh, Os, Ocp]     =  DATA_gptO2(T);
[Wlg, Wlh, Wls, Wlcp] =  DATA_gptH2Ol(T);
Delh_kJm = Wlh - (Hh + 0.5*Oh);
Delg_kJm = Wlg - (Hg + 0.5*Og);
Delg_kJm = Delg_kJm + RT*1e-3*log(1/(1*(PstackA*xO2)^0.5));
Eh = -Delh_kJm*1e3/(nA*F);

if Er==0
    % Er unknown, calculate
    Er = -Delg_kJm*1e3/(nA*F);
else
    % Er known, no action
end

% Create i-v
aA = - RT * log(j0A) / (alphaA * nA * F);
bA =   RT / (alphaA * nA * F);
aC = - RT * log(j0C) / (alphaC * nC * F);
bC =   RT / (alphaC * nC * F);

% jL-jleak-0.001 to prevent undefined concentration loss
irange=deltai:deltai:jL-jleak-0.001;
ic(1)=0; vc(1)=Er; pc(1)=0;
for icurrent = 2:length(irange)+1
    j = irange(icurrent-1);
    activationlossA = aA + bA*log(j+jleak);
    activationlossC = aC + bC*log(j+jleak);
    activationloss = activationlossA + activationlossC;
    ohmicloss = j*ASR;
    concentrationloss = C * log( jL/(jL-j-jleak) );
    ic(icurrent) = j;
    vc(icurrent) = Er - activationloss - ohmicloss - concentrationloss;
    pc(icurrent) = vc(icurrent)*j;
end

ivdata=[ic' vc' pc'];
ipmax=1; pmax=pc(1);
for i=2:length(ic);
```

```
    if pc(i) > pmax
        ipmax=i;
        pmax=pc(i);
    end
end
```

```
% h2density.m

% K      temperature in Kelvin
% MPa    pressure in Mega Pascal
% r      density in kg/m3
% Z      compressibility factor ( p = Z r RT )



function [r, Z] = h2density(K, MPa)


v=[
% i       a            b           c
1     0.05888460       1.325       1.0
2    -0.06136111       1.87        1.0
3    -0.002650473      2.5         2.0
4     0.002731125      2.8         2.0
5     0.001802374      2.938       2.42
6    -0.001150707      3.14        2.63
7     0.9588528e-4     3.37        3.0
8    -0.1109040e-6     3.75        4.0
9     0.1264403e-9     4.0         5.0
];


Z = 1;
for i=1:9
    a = v(i,2); b = v(i,3); c = v(i,4);
    Z = Z + a * (100/K)^b * MPa^c;
end

M = 2.01588;    % molar mass, g/mole
R = 8.314472;   % Universal Gas Constant, J / (mole.Kelvin)

r = MPa * 1e6 * M * 1e-3 / (Z * R * K); % kg/m3
```

```
% ISA up to 11 km
% short version: for PEMFC only need temp, pressure, and density

% t     : Temp in K
% p     : Pressure in N/m2
% d     : Density in kg/m3
% v     : viscosity in Pa-s
% s     : speed of sound in m/s

% iflag : 0  for standard day (i.e. SL temperature = 15.15 C
%         1  for non-standard day, temperature correction applied

% TC    : temperature at altitude h (for hot/cold day) in K.
%         ignored if iflag = 0.
%         if iflag = 1,
%         TC=temperature in C.

function [t,p,d,v,s] = ISAtmosphere(iflag,h,TC)

if h > 11000;
    disp('need h < 11 km')
    stop
end

PSL=1.01325*1e5;   % N/m2
DSL=1.225;         % kg/m3
TSL=288.15;        % K
g0=9.81;           % m/s2
a1=-0.0065;        % lapse rate in 1st gradient zone (0 < h < 11 km)
R=287.05;          % J/kg K
gamma=1.414;       % Cp/Cv for speed of sound calculation

% Calculate standard day

t=TSL+a1*h;
p=PSL * (t/TSL)^(-(g0/(a1*R)));
d=DSL * (t/TSL)^(-(g0/(a1*R)+1));


% Non-standard day correction
%     pressure remains as standard, correct density for temperature
if iflag ~= 0
    t=TC+273.15;
    d=p/(R*t);
end

s = sqrt(gamma*R*t);
   v0 = 18.27*1e-6;     % Pa.s
   T0 = 291.15;         % K
   C = 120;             % K
v = v0*((T0+C)/(t+C))*(t/T0)^1.5;
```

```
% ivpolarization.m

function [I,V,P,Pmax,iPmax,vPmax,Imax,Vmax] = ivpolarization...
    (ivdatainput, ipmax, iflag, Z, l, ...
     vcorr, h, h0, f, ...
     icorr, tfi, deltav)

% INPUTS:
% ivdata   matrix of cols i, v, p
% ipmax    row number of maximum p
% iflag
%          0         given   current   find   voltage,power
%          1         given   voltage   find   current,power
%          2         given   power,l   find   current,voltage
%                         l = 1  current range is lower side of max power
%                         l = 0  current range is higher side of max power
%                    for iflag=0,1 l is not used
%         > 2        returns current & voltage for max power
% Z                  current, voltage, or power per iflag
% vcorr              1-voltage correction, 0 otherwise
% h                  altitude, ft
% h0                 base altitude, ft, above which correction applies
% f                  vc degradation factor per 1000 ft
% icorr              1-current correction, 0 otherwise
% tfi                Tech Factor; for a given v, changes i
% deltav             for a given i, reduction in v

% OUTPUTS:
% current I, voltage V, power P
% max power Pmax, current & voltage at Pmax: iPmax, vPmax
% max current Imax, max voltage Vmax


ivdata = ivdatainput;


% first, tech factor on current
if icorr==1
    ivdata(:,1) = ivdata(:,1)*tfi;
    ivdata(:,3) = ivdata(:,3)*tfi;
end
% second, correction on voltage
if vcorr==1
    if h > h0
        vfac = 1+( (h-h0)/1000)*f;
    else
        vfac = 1;
    end
    ivdata(:,2) = ivdata(:,2)*vfac - deltav;
    ivdata(:,3) = ivdata(:,3)*vfac - ivdata(:,1)*deltav;
end
```

```
iPmax = ivdata(ipmax,1);
vPmax = ivdata(ipmax,2);
imax = length(ivdata);

Vmin = min(ivdata(:,2));
Vmax = max(ivdata(:,2));

Imin = min(ivdata(:,1));
Imax = max(ivdata(:,1));

Pmin = min(ivdata(:,3));
Pmax = max(ivdata(:,3));


% defaults
I = iPmax;
V = vPmax;
P = Pmax;

if iflag==0
    I = Z;
    if I > Imax
        disp('ic > imax ?')
        disp(I)
        disp(Imax)
        error('Error: ivpolarization.m')
    end
    if I < Imin
        disp('ic < imin ?')
        disp(I)
        disp(Imin)
        error('Error: ivpolarization.m')
    end
    V = interp1(ivdata(:,1),ivdata(:,2),I,'linear');
    P = I*V;
end


if iflag==1
    V = Z;
    if V > Vmax;
        disp('vc > vmax ?')
        disp(V)
        disp(Vmax)
        error('Error: ivpolarization.m')
    end
    if V < Vmin;
        disp('vc < vmin ?')
        disp(V)
        disp(Vmin)
        error('Error: ivpolarization.m')
    end
    I = interp1(ivdata(:,2),ivdata(:,1),V,'linear');
    P = I*V;
end


if iflag==2
    P = Z;
    if P > Pmax+0.0005;
        disp('pc > pmax ?')
        disp(P)
        disp(Pmax)
        error('Error: ivpolarization.m')
    end
    if P < Pmin;
        disp('pc < pmin ?')
        disp(P)
        disp(Pmin)
        error('Error: ivpolarization.m')
    end
    if l==1
        I = interp1(ivdata(1:ipmax,3),ivdata(1:ipmax,1),P,'linear');
        V = interp1(ivdata(1:ipmax,1),ivdata(1:ipmax,2),I,'linear');
    elseif l==0
        I = interp1(ivdata(ipmax:imax,3),ivdata(ipmax:imax,1),P,'linear');
        V = interp1(ivdata(ipmax:imax,1),ivdata(ipmax:imax,2),I,'linear');
    else
        disp('range variable l must be 0 or 1')
        error('Error: ivpolarization.m')
    end
end
```

```
% pemfc.m
% A Datta
% U Maryland 2020



POWERdsg_kW           = FCST.POWERdsg_kW;
Vstack_V              = FCST.Voltage;
Whydrogen             = FCST.Whydrogen;
Endurance_min         = FCST.Endurance_min;
iplotVIP              = FCST.iplotVIP;
iprntSCR              = FCST.iprntSCR;
fAirFlowRate          = FCST.fAirFlowRate;
fH2utilization        = FCST.fH2utilization;
TstackC               = FCST.TstackC;
Pstack                = FCST.Pstack;
xO2                   = FCST.xO2;
DPstack               = FCST.DPstack;
altitude              = FCST.altitudeDesign;
vcorr                 = FCST.vcorr;
vrate                 = FCST.vrate1000ft;
h0                    = FCST.vdropaboveft;
icorr                 = FCST.icorr;
tfi                   = FCST.TFi;
deltav                = FCST.deltav;
OneAtm                = FCST.OneAtm;
temperatureC          = FCST.temperatureC;
temperatureK          = FCST.temperatureC + 273.15;
CompEta               = FCST.CompEta;
TurbEta               = FCST.TurbEta;

tcell                 = FCST.tcell;
dcell                 = FCST.dcell;
porosity              = FCST.porosity;
dendplate             = FCST.dendplate;
dinsulplate           = FCST.dinsulplate;
dcollplate            = FCST.dcollplate;
dpem                  = FCST.dpem;
dgdl                  = FCST.dgdl;
dbipolarplate         = FCST.dbipolarplate;
dgasket               = FCST.dgasket;
Aendplatefrac         = FCST.Aendplatefrac;
tendplatecons         = FCST.tendplatecons;
tendplatefrac         = FCST.tendplatefrac;
Ainsulplatefrac       = FCST.Ainsulplatefrac;
tinsulplatecons       = FCST.tinsulplatecons;
tinsulplatefrac       = FCST.tinsulplatefrac;
Acollplatefrac        = FCST.Acollplatefrac;
tcollplatecons        = FCST.tcollplatecons;
tcollplatefrac        = FCST.tcollplatefrac;
Wfastener             = FCST.Wfastener;
tpem                  = FCST.tpem;
tgdl                  = FCST.tgdl;
```

```
tbipolarplate         = FCST.tbipolarplate;
pbipolarplate         = FCST.pbipolarplate;
Abipolarplatefrac     = FCST.Abipolarplatefrac;
tgasket               = FCST.tgasket;
Agasketfrac           = FCST.Agasketfrac;
Wstackother           = FCST.Wstackother;
TFwstack              = FCST.TFwstack;
dWwstack              = FCST.dWwstack;
dSstack               = FCST.dSsstack;
TFsstack              = FCST.TFsstack;


crh                   = FCST.crh;
arh                   = FCST.arh;
deltai                = FCST.deltai;
ASR0t                 = FCST.ASR0t;
ASRtk2                = FCST.ASRtk2;
ASRtk1                = FCST.ASRtk1;
jL0t                  = FCST.jL0t;
jLtk2                 = FCST.jLtk2;
jLtk1                 = FCST.jLtk1;
ASR0arh               = FCST.ASR0arh;
ASR0crh               = FCST.ASR0crh;
ASRrhk1a              = FCST.ASRrhk1a;
ASRrhk2a              = FCST.ASRrhk2a;
ASRrhk1a1c            = FCST.ASRrhk1a1c;
ASRrhk1a2c            = FCST.ASRrhk1a2c;
ASRrhk2a2c            = FCST.ASRrhk2a2c;
ASRrhk2a1c            = FCST.ASRrhk2a1c;
ASRrhk2c              = FCST.ASRrhk2c;
ASRrhk1c              = FCST.ASRrhk1c;
jL0arh                = FCST.jL0arh;
jL0crh                = FCST.jL0crh;
jLrhk1a               = FCST.jLrhk1a;
jLrhk2a               = FCST.jLrhk2a;
jLrhk1a1c             = FCST.jLrhk1a1c;
jLrhk1a2c             = FCST.jLrhk1a2c;
jLrhk2a2c             = FCST.jLrhk2a2c;
jLrhk2a1c             = FCST.jLrhk2a1c;
jLrhk2c               = FCST.jLrhk2c;
jLrhk1c               = FCST.jLrhk1c;

fovpower              = FCST.fovpower;
fovweight             = FCST.fovweight;
fovvolume             = FCST.fovvolume;


if iprntSCR==1
disp(' ')
disp(' ')
disp('           ===========================')
disp('              PEMFC SYSTEM DESIGN')
disp('           ===========================')
disp(' ')
disp(' ')
end



% Constants:
Runival = 0.0820578;      % universal gas constant in atm.L/mol.K
Runivkj = 8.3144621*1e-3; % universal gas constant in kJ/mol.K
Faraday = 96485.3365;     % Coulomb/mole
N       = 2;              % no of electrons per molecule of H2
SH2     = 1;              % H2 stoichiometry
MH2     = 2.01588*1e-3;   % molecular mass of H2 (kg/mole)
SAir    = 1/2;            % Air stoichiometry
MAir    = 28.965*1e-3;    % molecular mass of Air (kg/mole)
SO2     = 1/2;            % O2 stoichiometry
MO2     = 31.9988*1e-3;   % molecular mass of O2 (kg/mole)
MH2O    = 18.0153*1e-3;   % molecular mass of H2O (kg/mole)
SN2     = 1/2;            % N2 stoichiometry
MN2     = 28.0134*1e-3;   % molecular mass of N2 (kg/mole)
Rdryair = Runivkj/MAir;   % dry air gas constant, kJ/kg.K
SB      = 5.670367*1e-8;  % Stephan-Boltzmann W/m2/K4



if iprntSCR==1
disp(' ')
disp('                    ENVIRONMENT')
disp(' ')
end

% atmosphere; need temperature and pressure; only for compressor
iatm                  = FCST.iatm;
switch iatm
    case 0
        % ISA
        [temperatureK, pressure, density, viscosity, sound]=...
            ISAtmosphere(0,altitude,0);
        temperatureC = temperatureK - 273.15;
        FCST.temperatureC = temperatureC;
        FCST.pressure = pressure;
        FCST.density = density;
    case 1
        % ISA + temp correction
        [temperatureK, pressure, density, viscosity, sound]=...
            ISAtmosphere(1,altitude,temperatureC);
        FCST.pressure = pressure;
        FCST.density = density;
    otherwise
        % input temp and pressure directly
        pressure = FCST.pressure;
        density = pressure/(1e3 * Rdryair * temperatureK);
        FCST.density = density;
```

```
end

if iprntSCR==1
fprintf('%s %8.4f %s\n','Altitude        = ',altitude,'m');
fprintf('%s %8.4f %s\n','Pressure        = ',pressure/OneAtm,'Atm');
fprintf('%s %8.4f %s\n','Temperature     = ',temperatureK-273.15,'C');
fprintf('%s %8.4f %s\n','Density         = ',density,'kg/m3');
disp(' ')
end





%                       ============
%                          STACK
%                       ============


if iprntSCR==1
disp(' ')
disp('                         STACK')
disp(' ')
end


% w/compressor or blower Pstack = atmospheric pressure
if FCST.iCompressor==0 && FCST.iBlower==0
    fprintf('%s\n','WARNING:');
    fprintf('%s\n','No compressor or blower.');
    fprintf('%s\n','Re-setting stack pressure to atmospheric pressure.');
    Pstack = pressure;
end


% Ideal thermodynamic quantities. Need only for efficiencies.
% H-Hydrogen; O-Oxygen; Wv-H2O vapor; Wl-H2O liquid
% Thermodynamic values at stack temperature
Pstack_atm = Pstack / OneAtm;
TstackK = TstackC + 273.15;
[Hg, Hh, Hs, Hcp]     =  DATA_gptH2(TstackK);
[Og, Oh, Os, Ocp]     =  DATA_gptO2(TstackK);
[Wvg, Wvh, Wvs, Wvcp] =  DATA_gptH2Ov(TstackK);
[Wlg, Wlh, Wls, Wlcp] =  DATA_gptH2Ol(TstackK);
Delh_kJm = Wlh - (Hh + 0.5*Oh);
Delg_kJm = Wlg - (Hg + 0.5*Og);
% Correct for stack pressure
Delg_kJm = Delg_kJm + Runivkj*TstackK*log(1/(1*(Pstack_atm*xO2)^0.5));
Eh_V = -Delh_kJm*1e3/(N*Faraday);
Er_V = -Delg_kJm*1e3/(N*Faraday);
Delh_MJkg = Delh_kJm * 1e-3 / MH2;
Delg_MJkg = Delg_kJm * 1e-3 / MH2;
```

```
Delh_LHV_kJm = Wvh - (Hh + 0.5*Oh);
Delg_LHV_kJm = Wvg - (Hg + 0.5*Og);
Delg_LHV_kJm = Delg_LHV_kJm + Runivkj*TstackK*log(1/(1*(Pstack_atm*xO2)^0.5));
Eh_LHV_V = -Delh_LHV_kJm*1e3/(N*Faraday);
Er_LHV_V = -Delg_LHV_kJm*1e3/(N*Faraday);
Delh_LHV_MJkg = Delh_LHV_kJm * 1e-3 / MH2;
Delg_LHV_MJkg = Delg_LHV_kJm * 1e-3 / MH2;


if iprntSCR==1
fprintf('%s %8.4f %s\n','Stack temp      = ', TstackC,'C');
fprintf('%s %8.4f %s\n','Stack pres      = ', Pstack_atm,'atm');
fprintf('%s %8.4f %s\n','O2 mole frac    = ', xO2,' ');
fprintf('%s\n',' ');
fprintf('%s\n','Higher Heating Value (HHV)');
fprintf('%s %8.4f %s\n','   Enthalpy     = ', -Delh_kJm,'kJ/mole');
fprintf('%s %8.4f %s\n','                  ', -Delh_MJkg,'MJ/kg');
fprintf('%s %8.4f %s\n','   Gibbs        = ', -Delg_kJm,'kJ/mole');
fprintf('%s %8.4f %s\n','                  ', -Delg_MJkg,'MJ/kg');
fprintf('%s %8.4f %s\n','   Eh           = ', Eh_V,'V');
fprintf('%s %8.4f %s\n','   Er           = ', Er_V,'V');
fprintf('%s %8.4f %s\n','   eta ideal    = ', Er_V/Eh_V,'Er/Eh');
fprintf('%s\n',' ');
fprintf('%s\n','Lower Heating Value (LHV)');
fprintf('%s %8.4f %s\n','   Enthalpy     = ', -Delh_LHV_kJm,'kJ/mole');
fprintf('%s %8.4f %s\n','                  ', -Delh_LHV_MJkg,'MJ/kg');
fprintf('%s %8.4f %s\n','   Gibbs        = ', -Delg_LHV_kJm,'kJ/mole');
fprintf('%s %8.4f %s\n','                  ', -Delg_LHV_MJkg,'MJ/kg');
fprintf('%s %8.4f %s\n','   Eh           = ', Eh_LHV_V,'V');
fprintf('%s %8.4f %s\n','   Er           = ', Er_LHV_V,'V');
fprintf('%s %8.4f %s\n','   eta ideal    = ', Er_LHV_V/Eh_LHV_V,'Er/Eh');
fprintf('%s\n',' ');
end



% create i-v data
if FCST.ioptiv == 1
    % use i-v-p data from input
    FCST.ivdata = FCST.ivdatainp;
    FCST.ipmax  = FCST.ipmaxinp;
else
    switch FCST.ioptiv
        case 2
            % take parameters from input
            Er       = FCST.Er;
            j0C      = FCST.j0C;
            alphaC   = FCST.alphaC;
            j0A      = FCST.j0A;
            alphaA   = FCST.alphaA;
            ASR      = FCST.ASR;
            jL       = FCST.jL;
            jleak    = FCST.jleak;
            C        = FCST.C;
        case 3
            % find parameters from PMAT
            PMAT = FCST.PMAT;
            [m,n]=size(PMAT);
            Pinterp = Pstack_atm;
            if Pstack_atm > PMAT(1,n)
                disp('ERROR: Pstack > PMAT table pressure')
                disp('       Re-setting Pstack=1 atm')
                Pstack_atm = PMAT(1,n);
                Pinterp = Pstack_atm;
            end
            if Pstack_atm < PMAT(1,1)
                % start from here
                Pinterp = PMAT(1,1);
                % use  theoretical correction in createiv for below
            end
            j0C    = interp1(PMAT(1,1:n),PMAT(2,1:n),Pinterp,'linear');
            alphaC = interp1(PMAT(1,1:n),PMAT(3,1:n),Pinterp,'linear');
            j0A    = interp1(PMAT(1,1:n),PMAT(4,1:n),Pinterp,'linear');
            alphaA = interp1(PMAT(1,1:n),PMAT(5,1:n),Pinterp,'linear');
            ASR    = interp1(PMAT(1,1:n),PMAT(6,1:n),Pinterp,'linear');
            jL     = interp1(PMAT(1,1:n),PMAT(7,1:n),Pinterp,'linear');
            jleak  = interp1(PMAT(1,1:n),PMAT(8,1:n),Pinterp,'linear');
            C      = interp1(PMAT(1,1:n),PMAT(9,1:n),Pinterp,'linear');
            Er     = 0.; % find from thermodynamic constants
        otherwise
            % default
            Er       = FCST.Er;
            j0C      = FCST.j0C;
            alphaC   = FCST.alphaC;
            j0A      = FCST.j0A;
            alphaA   = FCST.alphaA;
            ASR      = FCST.ASR;
            jL       = FCST.jL;
            jleak    = FCST.jleak;
            C        = FCST.C;
    end
    % create i-v-p data from parameters
    % temperature correction to ASR
    t         = TstackC - ASR0t;
    dASRt     = ASRtk2*t^2 + ASRtk1*t;
    % temperature correction to jL
    t        =  TstackC - jL0t;
    djLt     =  jLtk2*t^2 + jLtk1*t;
    % relative humidity corrections to ASR and jL
    a = arh - ASR0arh; c = crh - ASR0crh;
    dASRrh = ASRrhk1a * a + ...
             ASRrhk2a * a^2 + ...
             ASRrhk1a1c * a * c + ...
             ASRrhk2a1c * a^2 * c + ...
             ASRrhk2a2c * a^2 * c^2 + ...
             ASRrhk1a2c * a * c^2 + ...
```

```
             ASRrhk2c * c^2 + ...
             ASRrhk1c * c;
    a = arh - jL0arh; c = crh - jL0crh;
    djLrh =  jLrhk1a * a + ...
             jLrhk2a * a^2 + ...
             jLrhk1a1c * a * c + ...
             jLrhk2a1c * a^2 * c + ...
             jLrhk2a2c * a^2 * c^2 + ...
             jLrhk1a2c * a * c^2 + ...
             jLrhk2c * c^2 + ...
             jLrhk1c * c;
    ASR =  ASR +  dASRt + dASRrh;
    jL  =  jL  +  djLt + djLrh;
    [ivdata, Eh, Er, ipmax] = createiv(TstackC, Pstack_atm, ...
        xO2, deltai, Er, ...
        j0C, alphaC, j0A, alphaA, ASR, jL, jleak, C);
    FCST.ivdata = ivdata;
    FCST.ipmax  = ipmax;
end



% Find cell voltage vc
altitude_ft = altitude/0.3048;
switch FCST.ioptp
    case 1
        % flag=3 returns I, V, P for max power.
        % I=I_Pmax; V=V_Pmax; P=Pmax.
       [I,V,P,Pmax,I_Pmax,V_Pmax,~,~]...
           = ivpolarization(FCST.ivdata,FCST.ipmax,3,0,1,...
           vcorr,altitude_ft,h0,vrate,icorr,tfi,deltav);
        Vcell_V = V;
    case 2
        Vcell_V = FCST.etac * Eh_V;
        [I,Vmax,P,Pmax,I_Pmax,V_Pmax,~,~]...
           = ivpolarization(FCST.ivdata,FCST.ipmax,0,0,1,...
           vcorr,altitude_ft,h0,vrate,icorr,tfi,deltav);
        if Vcell_V >= Vmax
            disp('etac too high; no cell voltage found.')
            fprintf('%s %6.4f \n','Max efficiency = ',Vmax/Eh_V)
            error('Error: pemfc.m')
        end
    case 3
        Vcell_V = FCST.vc;
        [I,Vmax,P,Pmax,I_Pmax,V_Pmax,~,~]...
           = ivpolarization(FCST.ivdata,FCST.ipmax,0,0,1,...
           vcorr,altitude_ft,h0,vrate,icorr,tfi,deltav);
        if Vcell_V >= Vmax
            disp('vc too high; no cell voltage found.')
            fprintf('%s %6.4f \n','Max vc = ',Vmax)
            error('Error: pemfc.m')
        end
    otherwise
```

```
        disp('Wrong FCST.ioptp')
        disp('Use 0, 1, or 2')
        error('Error: pemfc.m')
end
Ncell = floor(Vstack_V/Vcell_V);      % Ncell rounded to integer
Vcell_V = Vstack_V/Ncell;             % refine Vcell after rounding


% using vc, find ic from polarization curve
[Icell_Acm2,~,~,Pcell_max_Wcm2,Icell_Pmax,Vcell_Pmax,icmax,vcmax] = ...
    ivpolarization(FCST.ivdata,FCST.ipmax,1,Vcell_V,0,...
    vcorr,altitude_ft,h0,vrate,icorr,tfi,deltav);


Istackdsg_A = POWERdsg_kW*1e3/Vstack_V;               % design current
Aactive_cm2 = Istackdsg_A/Icell_Acm2;                 % Active area
POWERmax_kW = Aactive_cm2*Ncell*Pcell_max_Wcm2*1e-3;  % max power
IstackPmax  = Aactive_cm2*Icell_Pmax;
VstackPmax  = Ncell*Vcell_Pmax;
Istackmax_A = Aactive_cm2*icmax;                      % max current
Vstackmax_V = Ncell*vcmax;                            % max voltage
Aactive     = Aactive_cm2 * 1e-4; % Active area in m2
Pcell_Wcm2  = Icell_Acm2 * Vcell_V;
Qcell_Wcm2  = Icell_Acm2 * (Eh_LHV_V - Vcell_V);
fleakage    = Icell_Acm2 / (Icell_Acm2 + jleak);
fleakagemax = Icell_Pmax / (Icell_Pmax + jleak);
FCST.icdsg  = Icell_Acm2*1e4;
FCST.vcdsg  = Vcell_V;
FCST.pcdsg  = Pcell_Wcm2 * 1e4;
FCST.qcdsg  = Qcell_Wcm2 * 1e4;
FCST.etadsgHHV   = Vcell_V * fleakage / Eh_V;
FCST.etadsgLHV   = Vcell_V * fleakage /Eh_LHV_V;
FCST.etamaxHHV   = Vcell_Pmax * fleakage /Eh_V;
FCST.etamaxLHV   = Vcell_Pmax * fleakage /Eh_LHV_V;
FCST.etavdsgHHV  = Vcell_V * fleakage /Er_V;
FCST.etavdsgLHV  = Vcell_V * fleakage /Er_LHV_V;


if iprntSCR==1
    if vcorr == 1
        fprintf('%s %4.2f %s %5.0f %s\n','Cell vc reduction = ',...
             vrate*100,'% per 1000 ft above', h0,'ft');
    end
    disp(' ')
    fprintf('%s %8.4f %s\n','No of cells     = ',Ncell,'');
    fprintf('%s %8.4f %s\n','Active area     = ',Aactive_cm2,'cm2');
    fprintf('%s %8.4f %s\n','Cell voltage vc = ',Vcell_V,'V');
    fprintf('%s %8.4f %s\n','Cell current ic = ',Icell_Acm2,'A/cm2');
    fprintf('%s %8.4f %s\n','Cell power   pc = ',Pcell_Wcm2,'W/cm2');
    fprintf('%s %8.4f %s\n','Cell heat    qc = ',Qcell_Wcm2,'W/cm2');
    fprintf('%s %8.4f %s\n','Efficiency      = ',FCST.etadsgHHV,'vc/Eh (HHV)');
    fprintf('%s %8.4f %s\n','           v     ',FCST.etavdsgHHV,'vc/Er (HHV)');
    fprintf('%s %8.4f %s\n','                 ',FCST.etadsgLHV,'vc/Eh (LHV)');
    fprintf('%s %8.4f %s\n','            v      ',FCST.etavdsgLHV,'vc/Er (LHV)');
    fprintf('%s %8.4f %s\n','Stack current   = ',Istackdsg_A,'A');
    fprintf('%s %8.4f %s\n','Stack voltage   = ',Vstack_V,'V');
end


switch FCST.ioptsw
    case 0
        Sstack           = Ncell * Aactive * tcell;
        Lstack           = Ncell * tcell;
        Astack           = Aactive;
        Wstack.total     = Ncell * Aactive * tcell * dcell * porosity;
        FCST.Wstackk1    = Wstack.total/(Ncell * Aactive);
        Wstack.total     = TFwstack * Wstack.total + dWwstack;
        Sstack           = TFsstack * Sstack + dSstack;
        FCST.Wstackk0    = dWwstack;

    otherwise
    % cell  =  pem + gdl + gasket + bipolarplate
    %       =        mea          + bipolarplate
        Apem             = Aactive;
        Agdl             = Aactive;
        Agasket          = Aactive * Agasketfrac;
        Abipolarplate    = Aactive * Abipolarplatefrac;
        Amea             = Abipolarplate;
        tmea             = tpem + 2.0 * tgdl;
        Wpem             = tpem * Apem * dpem;
        Wgdl             = tgdl * Agdl * dgdl * 2.0;
        Wgasket          = tgasket * Agasket * dgasket * 2.0;
        Wmea             = Wpem + Wgdl + Wgasket;
        Wbipolarplate    = tbipolarplate * Abipolarplate * ...
                           dbipolarplate * pbipolarplate * 2.0;
        Wcell            = Wmea + Wbipolarplate;
        tacell           = tpem + 2.0 * (tgdl + tbipolarplate);
        tunit            = tacell * Ncell;
        Wunit            = Wcell * Ncell;

        % stack = endplate + insulating plate + collector plate + cells
        tendplate     = tendplatecons + tendplatefrac * tunit;
        Aendplate     = Aactive * Aendplatefrac;
        Wendplate     = tendplate * Aendplate * dendplate * 2.0;

        tinsulplate   = tinsulplatecons + tinsulplatefrac * tunit;
        Ainsulplate   = Aactive * Ainsulplatefrac;
        Winsulplate   = tinsulplate * Ainsulplate * dinsulplate * 2.0;

        tcollplate     = tcollplatecons + tcollplatefrac * tunit;
        Acollplate     = Aactive * Acollplatefrac;
        Wcollplate     = tcollplate * Acollplate * dcollplate * 2.0;

        tstack   = tendplate + tinsulplate + tcollplate + tunit;
        Astack   = Aendplate;
        Sstack   = TFsstack * tstack * Astack + dSstack;
        Lstack   = Sstack / Astack;
        W        = Wendplate + ...
                   Winsulplate + ...
                   Wcollplate + ...
                   Wunit + ...
                   Wfastener + ...
                   Wstackother;
        tcell    = tstack / Ncell;
        dcell    = W / (tstack * Aactive * porosity);
        W        = TFwstack * W + dWwstack;

        Wstack.total            = W;
        Wstack.endplate         = TFwstack * Wendplate;
        Wstack.insulplate       = TFwstack * Winsulplate;
        Wstack.collplate        = TFwstack * Wcollplate;
        Wstack.unit             = TFwstack * Wunit;
        Wstack.cell             = TFwstack * Wcell;
        Wstack.mea              = TFwstack * Wmea;
        Wstack.bipolarplate     = TFwstack * Wbipolarplate;
        Wstack.pem              = TFwstack * Wpem;
        Wstack.gdl              = TFwstack * Wgdl;
        Wstack.gasket           = TFwstack * Wgasket;
        Wstack.other            = TFwstack * Wstackother + dWwstack;

        fendplate               = Wstack.endplate/W;
        finsulplate             = Wstack.insulplate/W;
        fcollplate              = Wstack.collplate/W;
        funit                   = Wstack.unit/W;
        fmea                    = Wstack.mea*Ncell/W;
        fbipolarplate           = Wstack.bipolarplate*Ncell/W;
        fpem                    = Wstack.pem*Ncell/W;
        fgdl                    = Wstack.gdl*Ncell/W;
        fgasket                 = Wstack.gasket*Ncell/W;
        fother                  = Wstack.other/W;

        FCST.Wstackk1 = (W - dWwstack - Wstackother)/(Ncell*Aactive);
        FCST.Wstackk0 = dWwstack + Wstackother;

end
SP      = POWERdsg_kW / Wstack.total;
SPmax   = POWERmax_kW / Wstack.total;
DFstack = Wstack.total/(Ncell * Aactive);


if iprntSCR==1
    disp(' ')
    fprintf('%s %8.4f %s\n','P design        = ',POWERdsg_kW,'kW');
    fprintf('%s %8.4f %s\n','P max           = ',POWERmax_kW,'kW');
    fprintf('%s %8.4f %s\n','W stack         = ',Wstack.total,'kg');
    fprintf('%s %8.4f %s\n','k0              = ',FCST.Wstackk0,'kg');
    fprintf('%s %8.4f %s\n','k1              = ',FCST.Wstackk1,'kg/m2');
    if FCST.ioptsw ~= 0
    disp(' ')
    fprintf('%s %8.4f %s %5.2f %s\n','  endplate      = ',Wstack.endplate,'kg', fendplate*100, '%');
    fprintf('%s %8.4f %s %5.2f %s\n','  insulator     = ',Wstack.insulplate,'kg', finsulplate*100, '%');
    fprintf('%s %8.4f %s %5.2f %s\n','  collector     = ',Wstack.collplate,'kg', fcollplate*100, '%');
    fprintf('%s %8.4f %s\n','  cell unit     = ',Wstack.unit,'kg');
    fprintf('%s %8.4f %s\n','  one cell      = ',Wstack.cell,'kg');
    fprintf('%s %8.4f %s %5.2f %s\n','      bp plate  = ',Wstack.bipolarplate,'kg', fbipolarplate*100, '%');
    fprintf('%s %8.4f %s\n','      one mea   = ',Wstack.mea,'kg');
    fprintf('%s %8.4f %s %5.2f %s\n','            gas = ',Wstack.gdl,'kg', fgasket*100, '%');
    fprintf('%s %8.4f %s %5.2f %s\n','            gdl = ',Wstack.gasket,'kg', fgdl*100, '%');
    fprintf('%s %8.4f %s %5.2f %s\n','            pem = ',Wstack.pem,'kg', fpem*100, '%');
    fprintf('%s %8.4f %s %5.2f %s\n','  other         = ',Wstack.other,'kg', fother*100, '%');
    disp(' ')
    end
    fprintf('%s %8.4f %s\n','S stack         = ',Sstack,'m3');
    fprintf('%s %8.4f %s\n','SP              = ',SP,'kW/kg');
    fprintf('%s %8.4f %s\n','SP @ P max      = ',SPmax,'kW/kg');
    fprintf('%s %8.4f %s\n','Design Factor   = ',DFstack,'kg/m2');
    disp(' ')
end


FCST.Eh                        = Eh_V;
FCST.Er                        = Er_V;
FCST.Aactive_cm2               = Aactive_cm2;
FCST.Ncell                     = Ncell;
FCST.Istackdsg                 = Istackdsg_A;
FCST.Vstackdsg                 = Vstack_V;
FCST.Istackmax                 = Istackmax_A;
FCST.Vstackmax                 = Vstackmax_V;
FCST.IstackPmax                = IstackPmax;
FCST.VstackPmax                = VstackPmax;
FCST.Wstack                    = Wstack;
FCST.Sstack                    = Sstack;
FCST.Lstack                    = Lstack;
FCST.Astack                    = Astack;
FCST.SP                        = SP;
FCST.SPmax                     = SPmax;
FCST.DFstack                   = DFstack;


if iprntSCR==1
disp(' ')
disp('                    FLOW & HEAT')
disp(' ')
end



                       % Hydrogen input
                 % Anode:   H2 --> (2H+) + (2e-)

MassH2rate_kgs      = SH2*(MH2/(N*Faraday))*POWERdsg_kW*1e3/Vcell_V/fleakage;
MassH2rate_Pmax_kgs = SH2*(MH2/(N*Faraday))*POWERmax_kW*1e3/Vcell_Pmax/fleakagemax;
MassH2rate_kgs      = fH2utilization * MassH2rate_kgs;
```

```
MassH2rate_Pmax_kgs = fH2utilization * MassH2rate_Pmax_kgs;
switch FCST.ioptf
    case 2
        % Endurance given, find Whydrogen
        MassH2_kg       = MassH2rate_kgs * Endurance_min * 60;
        MassH2_Pmax_kg  = MassH2rate_kgs * Endurance_min * 60;

        FCST.Whydrogen      = MassH2_kg;
        FCST.Whydrogen_Pmax = MassH2_Pmax_kg;

    otherwise
        % Whydrogen given, find endurance
%         Endurance_min = Whydrogen * ...
%             N*Faraday*Vcell_V / (SH2*MH2*POWERdsg_kW*1e3*60);
%         Endurance_Pmax_min = Whydrogen * ...
%             N*Faraday*Vcell_Pmax / (SH2*MH2*POWERmax_kW*1e3*60);
        Endurance_min = Whydrogen / (MassH2rate_kgs * 60);
        Endurance_Pmax_min = Whydrogen / (MassH2rate_kgs * 60);

        FCST.Endurance_min = Endurance_min;
        FCST.Endurance_Pmax_min = Endurance_Pmax_min;

        MassH2_kg      = Whydrogen;
        MassH2_Pmax_kg = Whydrogen;
end
ENERGY_Pdsg_kWh = POWERdsg_kW*(Endurance_min/60);
ENERGY_Pmax_kWh = POWERmax_kW*(Endurance_Pmax_min/60);
SFCPdsg_kgkWh   = MassH2rate_kgs*3600/POWERdsg_kW;
SFCPmax_kgkWh   = MassH2rate_Pmax_kgs*3600/POWERmax_kW;

FCST.MassH2rate_kgs      = MassH2rate_kgs;
FCST.MassH2rate_Pmax_kgs = MassH2rate_Pmax_kgs;
FCST.SFCPdsg_kgkWh       = SFCPdsg_kgkWh;
FCST.SFCPmax_kgkWh       = SFCPmax_kgkWh;

if iprntSCR==1
fprintf('%s %8.4f %s\n','H2 amount          = ',MassH2_kg,'kg');
fprintf('%s %8.4f %s\n','H2 in              = ',MassH2rate_kgs,'kg/s');
fprintf('%s %8.4f %s\n','H2 in @ Pmax       = ',MassH2rate_Pmax_kgs,'kg/s');
fprintf('%s %8.4f %s\n','SFC                = ',SFCPdsg_kgkWh,'kg/kW-hr');
fprintf('%s %8.4f %s\n','SFC @ Pmax         = ',SFCPmax_kgkWh,'kg/kW-hr');
end


                        % Air input
          % Cathode:   (1/2) O2 + (2e-) + (2H+) --> H2O

MassAirrate_kgs =  SAir * (MAir/(N*xO2*Faraday)) * POWERdsg_kW*1e3 * ...
                   fAirFlowRate/Vcell_V/fleakage;
MassAirrate_Ls = (MassAirrate_kgs/MAir)*Runival*TstackK/Pstack_atm;
if iprntSCR==1
fprintf('%s %8.4f %s\n','Air in             = ',MassAirrate_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Air in vol         = ',MassAirrate_Ls,'L/sec');
```

```
end
FCST.MassAirrate_kgs = MassAirrate_kgs;
FCST.MassAirrate_Ls  = MassAirrate_Ls;


% max flows determine compressor/expander size.
% max air flow occurs at vpmax - cell voltage at max power
MassAirrate_max_kgs = SAir * (MAir/(N*xO2*Faraday)) * POWERmax_kW*1e3 * ...
                      fAirFlowRate/Vcell_Pmax/fleakagemax;
MassAirrate_max_Ls = (MassAirrate_max_kgs/MAir)*Runival*TstackK/Pstack_atm;
if iprntSCR==1
fprintf('%s %8.4f %s\n','Air in P max       = ',MassAirrate_max_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Air in P max vol   = ',MassAirrate_max_Ls,'L/sec');
end
FCST.MassAirrate_max_kgs = MassAirrate_max_kgs;
FCST.MassAirrate_max_Ls  = MassAirrate_max_Ls;


% O2 input - need for air and water output
MassO2rate_kgs =  SO2 * (MO2/(N*Faraday)) * POWERdsg_kW*1e3 / Vcell_V /fleakage;
MassO2rate_Pmax_kgs =  SO2 * (MO2/(N*Faraday)) * POWERmax_kW*1e3 / Vcell_Pmax / fleakage;


                            % Air output
                % Air input - O2 used up in reaction

MassAirrate_out_kgs = MassAirrate_kgs - MassO2rate_kgs;
MassAirrate_out_max_kgs = MassAirrate_max_kgs - MassO2rate_Pmax_kgs;
if iprntSCR==1
fprintf('%s %8.4f %s\n','Air out            = ',MassAirrate_out_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Air out P max      = ',MassAirrate_out_max_kgs,'kg/s');
end
FCST.MassAirrate_out_kgs     = MassAirrate_out_kgs;
FCST.MassAirrate_out_max_kgs = MassAirrate_out_max_kgs;


                           % Water output

% rate of water production
MassWaterrate_kgs = MassH2rate_kgs + MassO2rate_kgs;
MassWaterrate_Pmax_kgs = MassH2rate_Pmax_kgs + MassO2rate_Pmax_kgs;

% output humidity and vapor
% decide on humidifer based on design power, not max power.
% if not needed at design power, no humidifier.
% let stack dry up at max power.

% exit pr = stack pr - pr drop in stack
Pstack_exit = Pstack - DPstack;
% part of exit pr is vapor pressure. rest is dry air pressure.
Pvaporstack_out = watervp(TstackC);
Pdryairstack_out = Pstack_exit - Pvaporstack_out;
% humidity
```

```
Humidity_ratio = MH2O*Pvaporstack_out / (MAir*Pdryairstack_out);
% vapor needed to saturate output air
hm = Humidity_ratio * MassAirrate_out_kgs;
if hm < MassWaterrate_kgs
    % vapor < water means enough water to saturate output air
    % assume humidifier not needed
    MassVaporrate_out_kgs = hm;
    iHumid = 0;
else
    % vapor > water means less water than needed to saturate output air
    MassVaporrate_out_kgs = MassWaterrate_kgs;
    % assume humidifier needed
    % cathode will dry up without a humidifier
    iHumid = 1;
end
hmmax = Humidity_ratio * MassAirrate_out_max_kgs;
if hmmax < MassWaterrate_Pmax_kgs
    MassVaporrate_out_max_kgs = hmmax;
else
    MassVaporrate_out_max_kgs = MassWaterrate_kgs;
end

% rate of liq water leaving cell
MassLiquidrate_kgs = MassWaterrate_kgs - MassVaporrate_out_kgs;
MassLiquidrate_Pmax_kgs = ...
    MassWaterrate_Pmax_kgs - MassVaporrate_out_max_kgs;
MassTankrate_kgs = 0;
MassTankrate_Pmax_kgs = 0;
if MassLiquidrate_kgs > 0
    frac = FCST.waterstoragefrac;
    MassTankrate_kgs = frac * MassLiquidrate_kgs;
    MassTankrate_Pmax_kgs = frac * MassLiquidrate_Pmax_kgs;
end

% stop for non-physical results
if Pstack_exit < 0
    disp(' ')
    fprintf('%s\n','ERROR:');
    fprintf('%s\n','Pressure too low. Pstack cannot be < DPstack:');
    fprintf('%s %8.4f %s\n','P stack     = ',Pstack_atm,'atm');
    fprintf('%s %8.4f %s\n','DP stack    = ',DPstack/OneAtm,'atm');
    fprintf('%s\n','Use compressor.');
    stop
end
if Humidity_ratio < 0.01
    disp(' ')
    fprintf('%s\n','ERROR:');
    fprintf('%s\n','Output humidity zero. Temperature too low');
    fprintf('%s %8.4f %s\n','Sat humidity ratio = ',Humidity_ratio,' ');
    fprintf('%s\n','Use compressor.');
    stop
end
```

```
if iprntSCR==1
fprintf('%s %8.4f %s\n','Water produced     = ',MassWaterrate_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Sat humidity ratio = ',Humidity_ratio,' ');
fprintf('%s %8.4f %s\n','Vapor out satu     = ',hm,'kg/s');
fprintf('%s %8.4f %s\n','Vapor out true     = ',MassVaporrate_out_kgs,'kg/s');
if MassWaterrate_kgs > MassVaporrate_out_kgs
    fprintf('%s \n','    Water produced > Vapor to saturate air out.');
    fprintf('%s \n','    Humidifier not be needed.');
else
    fprintf('%s \n','    Water produced < Vapor to saturate air out.');
    fprintf('%s \n','    Humidifier needed.');
    fprintf('%s \n','    Include in weight budget under AIR SYSTEM.');
end
fprintf('%s %8.4f %s\n','Liq water          = ',MassLiquidrate_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Liq water stored   = ',MassTankrate_kgs,'kg/s');
end

FCST.Humidity_ratio         = Humidity_ratio;
FCST.MassWaterrate_kgs      = MassWaterrate_kgs;
FCST.MassVaporrate_kgs      = MassVaporrate_out_kgs;
FCST.MassLiquidrate_kgs     = MassLiquidrate_kgs;
FCST.MassTankrate_kgs       = MassTankrate_kgs;


               % Total exit air flow rate

Massrate_out_kgs = MassAirrate_out_kgs;
Massrate_out_max_kgs = MassAirrate_out_max_kgs;
if iprntSCR==1
fprintf('%s %8.4f %s\n','Flow out           = ',Massrate_out_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Flow out P max     = ',Massrate_out_max_kgs,'kg/s');
end


                       % Heating

if FCST.iHV==1;
    % use higher heating value (product water is liquid)
    Eforheat = Eh_V;
    if iprntSCR==1;
    fprintf('%s %8.4f %s\n','Eh of HHV (liq water)',Eh_V,'V');
    end
else
    % use lower heating value (product water is vapor)
    Eforheat = Eh_LHV_V;
    if iprntSCR==1;
    fprintf('%s %8.4f %s\n','Eh of LHV (vap water)',Eh_LHV_V,'V');
    end
end
HEATINGstack_kW = (Eforheat-Vcell_V)*Icell_Acm2*Aactive_cm2*Ncell*1e-3/fleakage;
HEATINGstack_max_kW = (Eforheat-Vcell_Pmax)*Icell_Pmax*Aactive_cm2*Ncell*1e-3/fleakagemax;
```

```
if iprntSCR==1
fprintf('%s %8.4f %s\n','Heat out           = ',HEATINGstack_kW,'kW');
fprintf('%s %8.4f %s\n','Heat out P max     = ',HEATINGstack_max_kW,'kW');
disp(' ')
end





%                       ===========================
%                              AIR SYSTEM
%                       ===========================

if iprntSCR==1
disp(' ')
disp('                    AIR SYSTEM')
disp(' ')
end


Wblower       = 0;
Pblower_kW    = 0;
Pblowermax_kW = 0;
Wcomp         = 0;
Pcomp_kW      = 0;
Pcompmax_kW   = 0;
Scomp         = 0;
Wturb         = 0;
Pturb_kW      = 0;
Pturbmax_kW   = 0;
Sturb         = 0;



% Blower
if FCST.iBlower==1
    if iprntSCR==1
    disp(' ')
    disp('Blower: ')
    disp(' ')
    end
    DeltaP = Pstack - pressure;
    if DeltaP > 100000
        fprintf('%s\n','WARNING:');
        fprintf('%s %8.4f %s\n','Delta P    = ',DeltaP*1e-3,'kPa');
        fprintf('%s\n','Blower/Pump impractical above 100 kPa');
        fprintf('%s\n','Use compressor.');
    end
end

if FCST.iBlower==1
```

```
    rhoAir = pressure / (1e3 * Rdryair * TstackK);
    volAirrate_max_m3s = MassAirrate_max_kgs / rhoAir;
    volAirrate_m3s = MassAirrate_kgs / rhoAir;
    CFM = volAirrate_m3s * 2118.88;
    CFMmax = volAirrate_max_m3s * 2118.88;
    Pblower_kW = DeltaP * volAirrate_m3s * 1e-3;
    Pblowermax_kW = DeltaP * volAirrate_max_m3s * 1e-3;
    x  = MassAirrate_max_kgs;
    y  = Pblowermax_kW * 1e3;
    dW = FCST.BlowW0;
    tf = FCST.BlowWtf;
    ex = FCST.BlowWfe;
    ey = FCST.BlowWpe;
    k  = FCST.BlowWk;
    Wblower = dW + tf * k * x^ex * y^ey;
    dS = FCST.BlowS0;
    tf = FCST.BlowStf;
    ex = FCST.BlowSfe;
    ey = FCST.BlowSpe;
    k  = FCST.BlowSk;
    Sblower = dS + tf * k * x^ex * y^ey;
    if iprntSCR==1
        fprintf('%s %8.4f %s\n','Delta P    = ',DeltaP/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','Flow       = ',MassAirrate_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','CFM        = ',CFM,'ft3/min');
        fprintf('%s %8.4f %s\n','Flow max   = ',MassAirrate_max_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','CFM max    = ',CFMmax,'ft3/min');
        fprintf('%s %8.4f %s\n','Weight     = ',Wblower,'kg');
        fprintf('%s %8.4f %s\n','Volume     = ',Sblower*1e3,'L');
    end
    FCST.Sblower = Sblower;
    FCST.Pblower_kW = Pblower_kW;
    FCST.Pblowermax_kW = Pblowermax_kW;
end


% Compressor-Turbine
iTurbine = 0;
iCompressor = 0;

if FCST.iCompressor==1

    if iprntSCR==1
    disp(' ')
    disp('Compressor: ')
    disp(' ')
    end

    T1compC = temperatureC;
    [Cp_air, Cv_air, Gamma_air] = DATA_DryAirCp(T1compC);
    P1comp = pressure;
    P2comp = Pstack;
    T1comp = T1compC + 273.15;
    fac1 = (Gamma_air-1)/Gamma_air;
    fac2 = (P2comp/P1comp)^fac1 - 1;
    DeltaT = fac2 * T1comp / CompEta;
    T2comp = T1comp + DeltaT;
    Pcomp_kW = Cp_air * MassAirrate_kgs * DeltaT * 1e-3;
    Pcompmax_kW = Cp_air * MassAirrate_max_kgs * DeltaT * 1e-3;

    if Pcompmax_kW < 0.1
        if iprntSCR==1
        fprintf('%s %8.4f %s\n','Power max  = ',Pcompmax_kW,'kW');
        fprintf('%s \n','Compressor not needed ');
        end
        Pcomp_kW = 0.;
        Pcompmax_kW = 0.;
        iTurbine = 0;
        iCompressor = 0;
    else
        iTurbine = 1;
        iCompressor = 1;
    end

    FCST.Pcomp_kW = Pcomp_kW;
    FCST.Pcompmax_kW = Pcompmax_kW;

end

if iCompressor==1

    % size
    x  = MassAirrate_max_kgs;
    y  = Pcompmax_kW * 1e3;
    dW = FCST.CompW0;
    tf = FCST.CompWtf;
    k  = FCST.CompWk;
    ex = FCST.CompWfe;
    ey = FCST.CompWpe;
    Wcomp = dW + tf * k * x^ex * y^ey;
    dS = FCST.CompS0;
    tf = FCST.CompStf;
    k  = FCST.CompSk;
    ex = FCST.CompSfe;
    ey = FCST.CompSpe;
    Scomp = dS + tf * k * x^ex * y^ey;
    if iprntSCR==1
        fprintf('%s %8.4f %s\n','Cp/Cv      = ',Gamma_air,' ');
        fprintf('%s %8.4f %s\n','Cp         = ',Cp_air,'J/kg.K ');
        fprintf('%s %8.4f %s\n','P1         = ',P1comp/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','P2         = ',P2comp/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','T1         = ',T1compC,'deg C');
        fprintf('%s %8.4f %s\n','T2         = ',T2comp-273.15,'deg C');
        fprintf('%s %8.4f %s\n','Delta T    = ',DeltaT,'deg C');
        fprintf('%s %8.4f %s\n','Flow       = ',MassAirrate_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','Flow max   = ',MassAirrate_max_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','Efficiency = ',CompEta,' ');
        fprintf('%s %8.4f %s\n','Power      = ',Pcomp_kW,'kW');
        fprintf('%s %8.4f %s\n','Power max  = ',Pcompmax_kW,'kW');
        fprintf('%s %8.4f %s\n','Weight     = ',Wcomp,'kg');
        fprintf('%s %8.4f %s\n','Volume     = ',Scomp*1e3,'L');
    end
    FCST.Wcomp = Wcomp;
    FCST.Scomp = Scomp;

    % low temperature cooling system
    % only when compressor is needed
    DeltaT = T2comp - TstackK;
    if DeltaT <= 0.
        % no cooling system needed
        HEATINGair    = 0;
        HEATINGairmax = 0;
    else
        % heat to be removed
        HEATINGair    = Cp_air * MassAirrate_kgs * DeltaT;
        HEATINGairmax = Cp_air * MassAirrate_max_kgs * DeltaT;
    end

    iLTR = FCST.iLTR;
    if iLTR==0
        HEATINGdsg = HEATINGair;
    else
        HEATINGdsg = HEATINGairmax;
    end
    hr             = FCST.ltRadiatorhr + 273.15;
    Tr             = FCST.ltRadiatorTr + 273.15;
    Ta             = FCST.ltRadiatorTa + 273.15;
    epr            = FCST.ltRadiatorepr;
    fac            = hr*(Tr - Ta) + epr*SB*(Tr^4 - Ta^4);
    Ar             = 1;
    if fac > 0
        Ar         = HEATINGdsg/fac;
    end
    RadiatorS0     = FCST.ltRadiatorS0;
    k              = FCST.ltRadiatorSk;
    e              = FCST.ltRadiatorSe;
    f              = FCST.ltRadiatorSf;
    Sradiator      = RadiatorS0 + k * HEATINGdsg^e * Ar^f;
    RadiatorW0     = FCST.ltRadiatorW0;
    k              = FCST.ltRadiatorWk;
    e              = FCST.ltRadiatorWe;
    f              = FCST.ltRadiatorWf;
    Wradiator      = RadiatorW0 + k * HEATINGdsg^e * Ar^f;
    CoolantW0      = FCST.ltCoolantW0;
    k              = FCST.ltCoolantWk;
    e              = FCST.ltCoolantWe;
    f              = FCST.ltCoolantWf;
    Cpcool         = FCST.ltCoolantCp;
    dTcool         = FCST.ltCoolantdT;
```

```
Cflowdsg       = HEATINGdsg / ( Cpcool*dTcool );
Wcoolant       = CoolantW0 + k * HEATINGdsg^e * Cflowdsg^f;
CoolerW0       = FCST.ltCoolerW0;
k              = FCST.ltCoolerWk;
e              = FCST.ltCoolerWe;
Wcooler        = CoolerW0 + k * HEATINGdsg^e;
ltFilterW0     = FCST.ltFilterW0;
ltRegulatorW0  = FCST.ltRegulatorW0;
ltPowerW0      = FCST.ltPowerW0;
ltPlumbW0      = FCST.ltPlumbW0;
w              = FCST.ltPlumbWw;
l              = FCST.ltPlumbWl;
WPlumb         = ltPlumbW0 + w*l;
W = Wradiator + Wcoolant + Wcooler + WPlumb + ...
    ltFilterW0 + ltRegulatorW0 + ltPowerW0;
% handle the fractions
f1 = FCST.ltFilterfW;
f2 = FCST.ltRegulatorfW;
f3 = FCST.ltPowerfW;
Wtotal = W / (1-f1-f2-f3);
% final
FCST.Wlt.total      = Wtotal;
FCST.Wlt.filter     = ltFilterW0 + f1 * Wtotal;
FCST.Wlt.regulator  = ltRegulatorW0 + f2 * Wtotal;
FCST.Wlt.power      = ltPowerW0 + f3 * Wtotal;
FCST.Wlt.radiator   = Wradiator;
FCST.Wlt.coolant    = Wcoolant;
FCST.Wlt.cooler     = Wcooler;
FCST.Wlt.plumbing   = WPlumb;
FCST.Sltradiator    = Sradiator;
FCST.Altradiator    = Ar;
FCST.HEATINGltmax_kW= HEATINGairmax * 1e-3;
FCST.HEATINGlt_kW   = HEATINGair * 1e-3;
Cflowmax            = HEATINGairmax / ( Cpcool*dTcool );
Cflow               = HEATINGair / ( Cpcool*dTcool );
FCST.Cflowlt        = Cflow;
FCST.Cflowmaxlt     = Cflowmax;
FCST.Cflowdsglt     = Cflowdsg;
% any power consumption to circulate coolant
dP                  = FCST.Plt0;
k                   = FCST.Pltk;
e                   = FCST.Plte;
f                   = FCST.Pltf;
Cflow               = HEATINGair / ( Cpcool*dTcool );
Cflowmax            = HEATINGairmax / ( Cpcool*dTcool );
P                   = dP + k * HEATINGair^e * Cflow^f;
Pmax                = dP + k * HEATINGairmax^e * Cflow^f;
Plt_kW              = P*1e-3;
Pltmax_kW           = Pmax*1e-3;
FCST.Plt_kW         = Plt_kW;
FCST.Pltmax_kW      = Pltmax_kW;
```

```
    if iprntSCR==1
    disp(' ')
    disp('Low temperature cooling system: ')
    fprintf('%s %8.4f %s\n','Air-Stack  = ',DeltaT,'deg C')
    fprintf('%s %8.4f %s\n','Heat max   = ',HEATINGdsg*1e-3,'kW')
    fprintf('%s %8.4f %s\n','kW max/C   = ',HEATINGdsg*1e-3/DeltaT,'kW/deg C')
    fprintf('%s %8.4f %s\n','Coolant q  = ',FCST.ltCoolantWf*Cflowdsg,'kg/s')
    fprintf('%s %8.4f %s\n','A Radiator = ',FCST.Altradiator,'m2')
    fprintf('%s %8.4f %s\n','S Radiator = ',FCST.Sltradiator*1e3,'L')
    fprintf('%s\n','Weights');
    fprintf('%s %8.4f %s\n','Radiator   = ',FCST.Wlt.radiator,'kg')
    fprintf('%s %8.4f %s\n','Coolant    = ',FCST.Wlt.coolant,'kg')
    fprintf('%s %8.4f %s\n','Filter     = ',FCST.Wlt.filter,'kg')
    fprintf('%s %8.4f %s\n','Regulator  = ',FCST.Wlt.regulator,'kg')
    fprintf('%s %8.4f %s\n','Plumbing   = ',FCST.Wlt.plumbing,'kg')
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Wlt.power,'kg')
    fprintf('%s %8.4f %s\n','TOTAL      = ',FCST.Wlt.total,'kg')
    disp(' ')
    fprintf('%s\n','Accessory power');
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Plt_kW,'kW')
    end


end



% Turbine
if FCST.iTurbine==1 && iTurbine==1

    if iprntSCR==1
    disp(' ')
    disp('Expander:')
    disp(' ')
    end

    T1turbC = TstackC;
    [Cp_air, Cv_air, Gamma_air] = DATA_DryAirCp(T1turbC);
    P1turb = Pstack - DPstack;
    P2turb = pressure;
    T1turb = T1turbC + 273.15;
    fac1 = (Gamma_air-1)/Gamma_air;
    fac2 = (P2turb/P1turb)^fac1 - 1;
    DeltaT = fac2 * T1turb * TurbEta;
    T2turb = T1turb + DeltaT;
    Pturb_kW = Cp_air * Massrate_out_kgs * DeltaT * 1e-3;
    Pturbmax_kW = Cp_air * Massrate_out_max_kgs * DeltaT * 1e-3;
    % size
    % size
    x  = Massrate_out_max_kgs;
    y  = Pturbmax_kW * 1e3;
    dW = FCST.TurbW0;
    tf = FCST.TurbWtf;
    k  = FCST.TurbWk;
    ex = FCST.TurbWfe;
    ey = FCST.TurbWpe;
    WTurb = dW + tf * k * x^ex * y^ey;
    dS = FCST.TurbS0;
    tf = FCST.TurbStf;
    k  = FCST.TurbSk;
    ex = FCST.TurbSfe;
    ey = FCST.TurbSpe;
    STurb = dS + tf * k * x^ex * y^ey;

    if iprntSCR==1
        fprintf('%s %8.4f %s\n','Cp/Cv      = ',Gamma_air,' ');
        fprintf('%s %8.4f %s\n','Cp         = ',Cp_air,'J/kg.K ');
        fprintf('%s %8.4f %s\n','P1         = ',P1turb/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','P2         = ',P2turb/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','T1         = ',T1turbC,'deg C');
        fprintf('%s %8.4f %s\n','T2         = ',T2turb-273.15,'deg C');
        fprintf('%s %8.4f %s\n','Delta T    = ',DeltaT,'deg C');
        fprintf('%s %8.4f %s\n','Flow       = ',Massrate_out_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','Flow max   = ',Massrate_out_max_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','Efficiency = ',TurbEta,' ');
        fprintf('%s %8.4f %s\n','Power      = ',Pturb_kW,'kW');
        fprintf('%s %8.4f %s\n','Power max  = ',Pturbmax_kW,'kW');
        fprintf('%s %8.4f %s\n','Weight     = ',Wturb,'kg');
        fprintf('%s %8.4f %s\n','Volume     = ',Sturb,'m3');
        disp(' ')
        fprintf('%s %8.4f %s\n','Net C-T    = ',Pcomp_kW + Pturb_kW,'kW');
        fprintf('%s %8.4f %s\n','Net max    = ',...
            Pcompmax_kW + Pturbmax_kW,'kW');
        disp(' ')
    end

    FCST.Wturb = Wturb;
    FCST.Sturb = Sturb;
    FCST.Pturb_kW = Pturb_kW;
    FCST.Pturbmax_kW = Pturbmax_kW;

end

Pair_kW         = Pblower_kW + Pcomp_kW + Pturb_kW;
Pairmax_kW      = Pblowermax_kW + Pcompmax_kW + Pturbmax_kW;

AirPlumbW0 = FCST.AirPlumbW0;
w = FCST.AirPlumbWw;
l = FCST.AirPlumbWl;
WPlumb = AirPlumbW0 + w*l;
% handle the fractions
AirFilterW0 = FCST.AirFilterW0;
AirRegulatorW0 = FCST.AirRegulatorW0;
AirPowerW0 = FCST.AirPowerW0;
AirHumidifierW0 = FCST.AirHumidifierW0;
W = Wblower + Wcomp + Wturb + WPlumb + ...
```

```
    AirFilterW0 + AirRegulatorW0 + AirPowerW0;
f1 = FCST.AirFilterfW;
f2 = FCST.AirRegulatorfW;
f3 = FCST.AirPowerfW;
f4 = 0;
if iHumid==1
    f4 = FCST.AirHumidifierfW;
    W = W + AirHumidifierW0;
end
Wtotal = W / (1-f1-f2-f3-f4);
% final
FCST.Wair.total      = Wtotal;
FCST.Wair.blower     = Wblower;
FCST.Wair.comp       = Wcomp;
FCST.Wair.turb       = Wturb;
FCST.Wair.filter     = AirFilterW0 + f1 * Wtotal;
FCST.Wair.regulator  = AirRegulatorW0 + f2 * Wtotal;
FCST.Wair.power      = AirPowerW0 + f3 * Wtotal;
if iHumid==1
    FCST.Wair.humidifier = AirHumidifierW0 + f4 * Wtotal;
end
FCST.Wair.plumbing   = WPlumb;
FCST.Pair_kW         = Pblower_kW + Pcomp_kW + Pturb_kW;
FCST.Pairmax_kW      = Pblowermax_kW + Pcompmax_kW + Pturbmax_kW;
if iprntSCR==1
    disp(' ')
    fprintf('%s\n','Accessory Weights');
    fprintf('%s %8.4f %s\n','Precooler  = ',FCST.Wair.precooler,'kg')
    fprintf('%s %8.4f %s\n','Humidifier = ',FCST.Wair.humidifier,'kg')
    fprintf('%s %8.4f %s\n','Filter     = ',FCST.Wair.filter,'kg')
    fprintf('%s %8.4f %s\n','Regulator  = ',FCST.Wair.regulator,'kg')
    fprintf('%s %8.4f %s\n','Plumbing   = ',FCST.Wair.plumbing,'kg')
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Wair.power,'kg')
    fprintf('%s %8.4f %s\n','TOTAL      = ',FCST.Wair.total,'kg')
end




%                      ==========================
%                              Storage/tanks
%                      ==========================

if iprntSCR==1
disp(' ')
disp(' ')
disp('                    H2 TANK')
disp(' ')
end

% hydrogen tank weight and volume
K     = FCST.h2tankTC + 273.15;
MPa   = FCST.h2tankBar*0.1;
```

```
[r,Z] = h2density(K,MPa);
itanktype = FCST.itanktype;
switch itanktype
    case 1
        h2tankwfg = FCST.h2tankwfg;
        h2tankwfs = FCST.h2tankwfs;
        Wh2tank   = MassH2_kg / h2tankwfg;
        Sh2tank   = MassH2_kg / h2tankwfs;
        Sh2       = MassH2_kg / r;
        h2tanksf  = Sh2/Sh2tank;
        h2tankSov = 1 - h2tanksf;
        FCST.h2tanksf = h2tanksf;
        FCST.h2tankSov = h2tankSov;
    case 2
        r = 70.99;
        h2tankSov = FCST.h2tankSov;
        h2tankwfg = FCST.h2tankwfg;
        h2tanksf  = 1 - h2tankSov;
        Sh2       = MassH2_kg / r;
        Sh2tank   = Sh2/h2tanksf;
        h2tankwfs = MassH2_kg / Sh2tank;
        Wh2tank   = MassH2_kg / h2tankwfg;
        FCST.h2tanksf = h2tanksf;
        FCST.h2tankwfs = h2tankwfs;
    otherwise
        h2tankSov = FCST.h2tankSov;
        h2tankwfg = FCST.h2tankwfg;
        h2tanksf  = 1 - h2tankSov;
        Sh2       = MassH2_kg / r;
        Sh2tank   = Sh2/h2tanksf;
        h2tankwfs = MassH2_kg / Sh2tank;
        Wh2tank   = MassH2_kg / h2tankwfg;
        FCST.h2tanksf = h2tanksf;
        FCST.h2tankwfs = h2tankwfs;
end



if iprntSCR==1
    switch itanktype
    case 1
        disp('STORAGE: input wt% gravimetric & effective density')
    case 2
        disp('STORAGE: liquid h2')
        disp('         input wt% gravimetric & tank volume overhead')
        fprintf('%s %8.4f %s\n','density    = ',r,'kg/m3');
    otherwise
        disp('STORAGE: input wt% gravimetric &')
        disp('         T, P, tank volume overhead')
        disp(' ')
        fprintf('%s %8.4f %s\n','H2 density = ',r,'kg/m3');
        fprintf('%s %8.4f %s\n','H2 compres = ',Z,'non-dim');
    end
    fprintf('%s %8.4f %s\n','W fuel     = ',MassH2_kg,'kg');
    fprintf('%s %8.4f %s\n','W tank     = ',Wh2tank,'kg including fuel');
    fprintf('%s %8.4f %s\n','S tank     = ',Sh2tank*1e3,'L');
    fprintf('%s %8.4f %s\n','vol frac   = ',h2tanksf,'m3/m3');
    fprintf('%s %8.4f %s\n','vol ov     = ',h2tankSov,'(Stank-SH2)/Stank');
    fprintf('%s %8.4f %s\n','wt% grav   = ',h2tankwfg,'kg/kg');
    fprintf('%s %8.4f %s\n','wt% volu   = ',h2tankwfs*1e-3,'kg/L');
    disp(' ')
end

FuelFilterW0 = FCST.FuelFilterW0;
FuelRegulatorW0 = FCST.FuelRegulatorW0;
FuelPowerW0 = FCST.FuelPowerW0;
FuelPlumbW0 = FCST.FuelPlumbW0;
w = FCST.FuelPlumbWw;
l = FCST.FuelPlumbWl;
WFuelPlumb = FuelPlumbW0 + w*l;
W = Wh2tank + WFuelPlumb + ...
    FuelFilterW0 + FuelRegulatorW0 + FuelPowerW0;
% handle the fractions
f1 = FCST.FuelFilterfW;
f2 = FCST.FuelRegulatorfW;
f3 = FCST.FuelPowerfW;
Wtotal = W / (1-f1-f2-f3);
% final
FCST.Sh2tank         = Sh2tank;
FCST.Wfuel.total     = Wtotal;
FCST.Wfuel.H2        = MassH2_kg;
FCST.Wfuel.tank      = Wh2tank - MassH2_kg;
FCST.Wfuel.filter    = FuelFilterW0 + f1 * Wtotal;
FCST.Wfuel.regulator = FuelRegulatorW0 + f2 * Wtotal;
FCST.Wfuel.power     = FuelPowerW0 + f3 * Wtotal;
FCST.Wfuel.plumbing  = WFuelPlumb;
% any power consumption to store / deliver fuel
dP = FCST.Pfuel0;
k  = FCST.Pfuelk;
e  = FCST.Pfuele;
P  = dP + k * MassH2rate_kgs^e;
Pfuel_kW = P*1e-3;
FCST.Pfuel_kW = Pfuel_kW;
P  = dP + k * MassH2rate_Pmax_kgs^e;
Pfuelmax_kW = P*1e-3;
FCST.Pfuelmax_kW = Pfuelmax_kW;

if iprntSCR==1
    disp(' ')
    fprintf('%s\n','Accessory weights');
    fprintf('%s %8.4f %s\n','Filter     = ',FCST.Wfuel.filter,'kg')
    fprintf('%s %8.4f %s\n','Regulator  = ',FCST.Wfuel.regulator,'kg')
    fprintf('%s %8.4f %s\n','Plumbing   = ',FCST.Wfuel.plumbing,'kg')
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Wfuel.power,'kg')
    fprintf('%s %8.4f %s\n','TOTAL      = ',FCST.Wfuel.total,'kg')
    disp(' ')
    fprintf('%s\n','Accessory power');
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Pfuel_kW,'kW')
end



%                       ==========================
%                              Stack Cooling
%                       ==========================

if iprntSCR==1
disp(' ')
disp(' ')
disp('                    STACK COOLING')
disp(' ')
end

FCST.HEATINGstack_max_kW  = HEATINGstack_max_kW;
FCST.HEATINGstack_kW      = HEATINGstack_kW;

% heat used to raise inlet air temp to stack temperature
HEATINGairin_kW = 0;
HEATINGairin_max_kW = 0;
if FCST.iCompressor==0 %&& iCompressor==1
    % then there is no compressor
    % air must be heated on entry; this heat can be taken from stack
    [Cp_air, Cv_air, Gamma_air] = DATA_DryAirCp(temperatureC);
    DeltaT = TstackK-temperatureK;
    HEATINGairin_kW = Cp_air * MassAirrate_kgs * DeltaT * 1e-3;
    HEATINGairin_max_kW = Cp_air * MassAirrate_max_kgs * DeltaT * 1e-3;
end
FCST.HEATINGairin_kW = HEATINGairin_kW;
FCST.HEATINGairin_max_kW = HEATINGairin_kW;


% heat carried out by air flow
[Cp_air, Cv_air, Gamma_air] = DATA_DryAirCp(temperatureC);
DeltaT = TstackK-temperatureK;
HEATINGairout_kW = Cp_air * MassAirrate_out_kgs * DeltaT * 1e-3;
HEATINGairout_max_kW = Cp_air * MassAirrate_out_max_kgs * DeltaT * 1e-3;
FCST.HEATINGairout_kW = HEATINGairout_kW;
FCST.HEATINGairout_max_kW = HEATINGairout_max_kW;


% heat dissipated
fd  = FCST.htStackfd;
hs  = FCST.htStackhs;
Ta  = FCST.htStackTa + 273.15;
eps = FCST.htStackeps;
fs  = FCST.htStackfs;
As  = fs * Aactive * Ncell;
HEATINGdiscon_kW = As * hs * (TstackK - Ta) * 1e-3;
HEATINGdisrad_kW = As * eps * SB * (TstackK^4 - Ta^4) * 1e-3;
```

```
HEATINGdis_kW  = fd*HEATINGstack_kW + ...
                 HEATINGdiscon_kW + HEATINGdisrad_kW;
FCST.HEATINGdiscon_kW = HEATINGdiscon_kW;
FCST.HEATINGdisrad_kW = HEATINGdisrad_kW;
FCST.HEATINGdis_kW    = HEATINGdis_kW;

HEATING        = (HEATINGstack_kW - HEATINGairin_kW - ...
                 HEATINGairout_kW - HEATINGdis_kW) * 1e3;
HEATINGmax     = (HEATINGstack_max_kW - HEATINGairin_max_kW - ...
                 HEATINGairout_max_kW - HEATINGdis_kW) * 1e3;

% high temperature cooling system
iHTR = FCST.iHTR;
if iHTR==0
    % size to design heat load
    HEATINGdsg = HEATING;
else
    % size to max heat load
    HEATINGdsg = HEATINGmax;
end
DeltaT         = TstackK-temperatureK;
hr             = FCST.htRadiatorhr + 273.15;
Tr             = FCST.htRadiatorTr + 273.15;
Ta             = FCST.htRadiatorTa + 273.15;
epr            = FCST.htRadiatorepr;
fac            = hr*(Tr - Ta) + epr*SB*(Tr^4 - Ta^4);
Ar             = 1;
if fac > 0
    Ar         = HEATINGdsg/fac;
end
RadiatorS0     = FCST.htRadiatorS0;
k              = FCST.htRadiatorSk;
e              = FCST.htRadiatorSe;
f              = FCST.htRadiatorSf;
Sradiator      = RadiatorS0 + k * HEATINGdsg^e * Ar^f;
RadiatorW0     = FCST.htRadiatorW0;
k              = FCST.htRadiatorWk;
e              = FCST.htRadiatorWe;
f              = FCST.htRadiatorWf;
Wradiator      = RadiatorW0 + k * HEATINGdsg^e * Ar^f;
CoolantW0      = FCST.htCoolantW0;
k              = FCST.htCoolantWk;
e              = FCST.htCoolantWe;
f              = FCST.htCoolantWf;
Cpcool         = FCST.htCoolantCp;
dTcool         = FCST.htCoolantdT;
Cflowdsg       = HEATINGdsg / ( Cpcool*dTcool );
Wcoolant       = CoolantW0 + k * HEATINGdsg^e * Cflowdsg^f;
htFilterW0     = FCST.htFilterW0;
htRegulatorW0  = FCST.htRegulatorW0;
htPowerW0      = FCST.htPowerW0;
htPlumbW0      = FCST.htPlumbW0;
w              = FCST.htPlumbWw;
```

```
l              = FCST.htPlumbWl;
WPlumb         = htPlumbW0 + w*l;
W = Wradiator + Wcoolant + WPlumb + ...
    htFilterW0 + htRegulatorW0 + htPowerW0;
% handle the fractions
f1 = FCST.htFilterfW;
f2 = FCST.htRegulatorfW;
f3 = FCST.htPowerfW;
Wtotal = W / (1-f1-f2-f3);
% final
FCST.Wht.total      = Wtotal;
FCST.Wht.filter     = htFilterW0 + f1 * Wtotal;
FCST.Wht.regulator  = htRegulatorW0 + f2 * Wtotal;
FCST.Wht.power      = htPowerW0 + f3 * Wtotal;
FCST.Wht.radiator   = Wradiator;
FCST.Wht.coolant    = Wcoolant;
FCST.Wair.plumbing  = WPlumb;
FCST.Shtradiator    = Sradiator;
FCST.Ahtradiator    = Ar;
FCST.HEATINGhtc_kW  = HEATINGdsg * 1e-3;
Cflow               = HEATING / ( Cpcool*dTcool );
Cflowmax            = HEATINGmax / ( Cpcool*dTcool );
FCST.Cflowht        = Cflow;
FCST.Cflowdsght     = Cflowdsg;
FCST.Cflowmaxht     = Cflowmax;
% any power consumption to circulate coolant
dP                  = FCST.Pht0;
k                   = FCST.Phtk;
e                   = FCST.Phte;
f                   = FCST.Phtf;
P                   = dP + k * HEATINGmax^e * Cflowmax^f;
Phtmax_kW           = P*1e-3;
FCST.Phtmax_kW      = Phtmax_kW;
P                   = dP + k * HEATING^e * Cflow^f;
Pht_kW              = P*1e-3;
FCST.Pht_kW         = Pht_kW;

if iprntSCR==1
    disp(' ')
    fprintf('%s %8.4f %s\n','Stack-Amb  = ',DeltaT,'deg C')
    if iHTR==0
    fprintf('%s\n','Heat load at dsg power used in sizing:');
    fprintf('%s %8.4f %s\n','Heat gen   = ',HEATINGstack_kW,'kW')
    fprintf('%s %8.4f %s\n','     sent to inlet  ',HEATINGairin_kW,'kW')
    fprintf('%s %8.4f %s\n','     out in exhaust ',HEATINGairout_kW,'kW')
    fprintf('%s %8.4f %s\n','     convect loss   ',HEATINGdiscon_kW,'kW')
    fprintf('%s %8.4f %s\n','     radiation loss ',HEATINGdisrad_kW,'kW')
    fprintf('%s %8.4f %s\n','Heat load  = ',HEATING*1e-3,'kW')
    fprintf('%s %8.4f %s\n','kW / C     = ',HEATING*1e-3/DeltaT,'kW/deg C')
    fprintf('%s %8.4f %s\n','Coolant q  = ',FCST.htCoolantWf*Cflowdsg,'kg/s')
    lpm = (FCST.htCoolantWf*Cflowdsg / FCST.htCoolantrho) * 60000;
    fprintf('%s %8.4f %s\n','            ',lpm,'L/min')
    else
    fprintf('%s\n','Heat load at max power used in sizing:');
    fprintf('%s %8.4f %s\n','Heat gen   = ',HEATINGstack_max_kW,'kW')
    fprintf('%s %8.4f %s\n','     sent to inlet  ',HEATINGairin_max_kW,'kW')
    fprintf('%s %8.4f %s\n','     out in exhaust ',HEATINGairout_max_kW,'kW')
    fprintf('%s %8.4f %s\n','     convect loss   ',HEATINGdiscon_kW,'kW')
    fprintf('%s %8.4f %s\n','     radiation loss ',HEATINGdisrad_kW,'kW')
    fprintf('%s %8.4f %s\n','Heat load  = ',HEATINGmax*1e-3,'kW')
    fprintf('%s %8.4f %s\n','kW max/C   = ',HEATINGmax*1e-3/DeltaT,'kW/deg C')
    fprintf('%s %8.4f %s\n','Coolant q  = ',FCST.htCoolantWf*Cflowmax,'kg/s')
    lpm = (FCST.htCoolantWf*Cflowmax / FCST.htCoolantrho) * 60000;
    fprintf('%s %8.4f %s\n','             ',lpm,'L/min')
    end
    disp(' ')
    fprintf('%s %8.4f %s\n','A Radiator = ',FCST.Ahtradiator,'m2')
    fprintf('%s %8.4f %s\n','S Radiator = ',FCST.Shtradiator*1e3,'L')
    fprintf('%s\n','Weights:');
    fprintf('%s %8.4f %s\n','Radiator   = ',FCST.Wht.radiator,'kg')
    fprintf('%s %8.4f %s\n','Coolant    = ',FCST.Wht.coolant,'kg')
    fprintf('%s %8.4f %s\n','Filter     = ',FCST.Wht.filter,'kg')
    fprintf('%s %8.4f %s\n','Regulator  = ',FCST.Wht.regulator,'kg')
    fprintf('%s %8.4f %s\n','Plumbing   = ',FCST.Wht.plumbing,'kg')
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Wht.power,'kg')
    fprintf('%s %8.4f %s\n','TOTAL      = ',FCST.Wht.total,'kg')
    disp(' ')
    fprintf('%s\n','Accessory power');
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Pht_kW,'kW')
end




%                       ==========================
%                             Water System
%                       ==========================

if iprntSCR==1
disp(' ')
disp(' ')
disp('                        WATER EXHAUST')
disp(' ')
end
frac              = FCST.waterstoragefrac;
hour              = FCST.waterstoragehour;
Sov               = FCST.watertankSov;
Wov               = FCST.watertankWov;
mrate             = MassTankrate_kgs;
mratemax          = MassTankrate_Pmax_kgs;
vrate             = mrate / FCST.rhowater;
Wwater            = mrate * hour * 3600;
Swater            = Wwater / FCST.rhowater;
Wwatertank        = Wwater * (1 + Wov);
Swatertank        = Swater * (1 + Sov);
waterPlumbW0      = FCST.waterPlumbW0;
```

```
w                 = FCST.waterPlumbWw;
l                 = FCST.waterPlumbWl;
WPlumb            = waterPlumbW0 + w*l;
waterFilterW0     = FCST.waterFilterW0;
waterRegulatorW0  = FCST.waterRegulatorW0;
waterPowerW0      = FCST.waterPowerW0;
W = WPlumb + Wwatertank + ...
    waterFilterW0 + waterRegulatorW0 + waterPowerW0;
% handle the fractions
f1 = FCST.htFilterfW;
f2 = FCST.waterRegulatorfW;
f3 = FCST.waterPowerfW;
Wtotal = W / (1-f1-f2-f3);
% final
FCST.Wwater.total      = Wtotal;
FCST.Wwater.tank       = Wwatertank;
FCST.Wwater.filter     = waterFilterW0 + f1 * Wtotal;
FCST.Wwater.regulator  = waterRegulatorW0 + f2 * Wtotal;
FCST.Wwater.power      = waterPowerW0 + f3 * Wtotal;
FCST.Wwater.plumbing   = WPlumb;
FCST.Swatertank        = Swatertank;
% any power consumption to pump water
dP                  = FCST.Pwater0;
k                   = FCST.Pwaterk;
e                   = FCST.Pwatere;
P                   = dP + k * mrate^e;
Pwater_kW           = P*1e-3;
FCST.Pwater_kW      = Pwater_kW;
P                   = dP + k * mratemax^e;
Pwatermax_kW        = P*1e-3;
FCST.Pwatermax_kW   = Pwatermax_kW;

if iprntSCR==1
    disp(' ')
    fprintf('%s %8.4f %s\n','Water rate = ',mrate,'kg/s')
    fprintf('%s %8.4f %s\n','Water flow = ',vrate*1000/60,'L/min')
    fprintf('%s %8.4f %s\n','S tank     = ',FCST.Swatertank*1e3,'L')
    disp(' ')
    fprintf('%s\n','Accessory Weights');
    fprintf('%s %8.4f %s\n','Tank       = ',FCST.Wwater.tank,'kg')
    fprintf('%s %8.4f %s\n','Filter     = ',FCST.Wwater.filter,'kg')
    fprintf('%s %8.4f %s\n','Regulator  = ',FCST.Wwater.regulator,'kg')
    fprintf('%s %8.4f %s\n','Plumbing   = ',FCST.Wwater.plumbing,'kg')
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Wwater.power,'kg')
    fprintf('%s %8.4f %s\n','TOTAL      = ',FCST.Wwater.total,'kg')
    disp(' ')
    fprintf('%s\n','Accessory power');
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Pwater_kW,'kW')
end


%                      ==========================
%                            Electrical System
%                      =========================

if iprntSCR==1
disp(' ')
disp(' ')
disp('                    ELECTRICAL')
disp(' ')
end
elecPlumbW0       = FCST.elecPlumbW0;
k                 = FCST.elecPlumbWwk;
e                 = FCST.elecPlumbWwe;
w                 = k * Istackmax_A^e;
l                 = FCST.elecPlumbWl;
WPlumb            = elecPlumbW0 + w*l;
elecControlW0     = FCST.elecControlW0;
k                 = FCST.elecControlWk;
e                 = FCST.elecControlWe;
Wcontroller       = elecControlW0 + k * Istackmax_A^e;
elecPowerW0       = FCST.elecPowerW0;
elecRegulatorW0   = FCST.elecRegulatorW0;
W = WPlumb + Wcontroller + elecPowerW0 + elecRegulatorW0;
% handle the fractions
f1 = FCST.elecPowerfW;
f2 = FCST.elecRegulatorfW;
Wtotal = W / (1-f1-f2);
% final
FCST.Welec.total       = Wtotal;
FCST.Welec.plumbing    = WPlumb;
FCST.Welec.controller  = Wcontroller;
FCST.Welec.power       = elecPowerW0 + f1 * Wtotal;
FCST.Welec.regulator   = elecRegulatorW0 + f2 * Wtotal;
% any power consumption
dP                 = FCST.Pelec0;
k                  = FCST.Peleck;
e                  = FCST.Pelece;
P                  = dP + k * 1e3 * POWERdsg_kW^e;
Pelec_kW           = P*1e-3;
FCST.Pelec_kW      = Pelec_kW;
P                  = dP + k * 1e3 * POWERmax_kW^e;
Pelecmax_kW        = P*1e-3;
FCST.Pelecmax_kW   = Pelecmax_kW;

if iprntSCR==1
    disp(' ')
    fprintf('%s\n','Weights');
    fprintf('%s %8.4f %s\n','Controller = ',FCST.Welec.controller,'kg')
    fprintf('%s %8.4f %s\n','Plumbing   = ',FCST.Welec.plumbing,'kg')
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Welec.regulator,'kg')
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Welec.power,'kg')
    fprintf('%s %8.4f %s\n','TOTAL      = ',FCST.Welec.total,'kg')
    disp(' ')
    fprintf('%s\n','Accessory power');
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Pelec_kW,'kW')
```

```
end



%                           ================
%                            Other overhead
%                           ===============

if iprntSCR==1
disp(' ')
disp(' ')
disp('                    OTHER')
disp(' ')
end

Pother_kW     = POWERdsg_kW * fovpower;
Pothermax_kW  = POWERmax_kW * fovpower;
Wtotal  = FCST.Wstack.total + ...
          FCST.Wfuel.total + ...
          FCST.Wair.total + ...
          FCST.Wlt.total + ...
          FCST.Wht.total + ...
          FCST.Wwater.total + ...
          FCST.Welec.total;
Stotal  = FCST.Sstack + ...
          FCST.Sblower + FCST.Scomp + FCST.Sturb + ...
          FCST.Sh2tank + ...
          FCST.Sltradiator + FCST.Shtradiator + ...
          FCST.Swatertank;
Wother  = Wtotal * fovweight;
Sother  = Stotal * fovvolume;

if iprntSCR==1
    fprintf('%s %8.4f %s\n','Other P    = ',Pother_kW,'kW');
    fprintf('%s %8.4f %s\n','Other W    = ',Wother,'kg');
    fprintf('%s %8.4f %s\n','Other S    = ',Sother*1e3,'Liter');
end

FCST.Wother = Wother;
FCST.Sother = Sother;
FCST.Pother_kW = Pother_kW;
FCST.Pothermax_kW = Pothermax_kW;




%                          ================
%                               TOTAL
%                          ================

% mass kg
Wtotal  = FCST.Wstack.total + ...
          FCST.Wfuel.total + ...
          FCST.Wair.total + ...
          FCST.Wlt.total + ...
          FCST.Wht.total + ...
          FCST.Wwater.total + ...
          FCST.Welec.total + ...
          FCST.Wother;



% volume m3
Stotal  = FCST.Sstack + ...
          FCST.Sblower + FCST.Scomp + FCST.Sturb + ...
          FCST.Sh2tank + ...
          FCST.Sltradiator + FCST.Shtradiator + ...
          FCST.Swatertank + ...
          FCST.Sother;
% in liter
StotalL = Stotal*1000;


% power kW
% balance of plant at design power
Pbop_kW = FCST.Pair_kW + ...
          FCST.Pfuel_kW + ...
          FCST.Plt_kW + FCST.Pht_kW + ...
          FCST.Pwater_kW + ...
          FCST.Pelec_kW + ...
          FCST.Pother_kW;
% balance of plant at max power
Pbopmax_kW = ...
          FCST.Pairmax_kW + ...
          FCST.Pfuelmax_kW + ...
          FCST.Pltmax_kW + FCST.Phtmax_kW + ...
          FCST.Pwatermax_kW + ...
          FCST.Pelecmax_kW + ...
          FCST.Pothermax_kW;


FCST.Wtotal      = Wtotal;
FCST.Stotal      = Stotal;
FCST.StotalL     = StotalL;
FCST.Pbop_kW     = Pbop_kW;
FCST.Pbopmax_kW  = Pbopmax_kW;


% Stack system (total - fuel system)
Wstacksys=FCST.Wstack.total + ...
          FCST.Wair.total + ...
          FCST.Wlt.total + ...
          FCST.Wht.total + ...
          FCST.Wwater.total + ...
          FCST.Welec.total + ...
          FCST.Wother;
```

```
Sstacksys=FCST.Sstack + ...
          FCST.Sblower + FCST.Scomp + FCST.Sturb + ...
          FCST.Sltradiator + FCST.Shtradiator + ...
          FCST.Swatertank + ...
          FCST.Sother;
% in liter
SstacksysL=Sstacksys*1000;

FCST.Wstacksys = Wstacksys;
FCST.Sstacksys = Sstacksys;




%                          ================
%                               SUMMARY
%                          ================

if iprntSCR==1
disp(' '); disp(' '); disp(' ');
disp('                    DESIGN SUMMARY')
disp(' ')
end

% power
POWERdsg_kWe    = POWERdsg_kW - Pbop_kW;
POWERfactor     = POWERdsg_kWe/POWERdsg_kW;
POWERmax_kWe    = POWERmax_kW - Pbopmax_kW;
POWERfactor_max = POWERmax_kWe/POWERmax_kW;

% net system efficiencies = stack x (P-Pbop)/P
EtadsgHHV   = FCST.etadsgHHV * POWERfactor;
EtadsgLHV   = FCST.etadsgLHV * POWERfactor;
EtamaxHHV   = FCST.etamaxHHV * POWERfactor;
EtamaxLHV   = FCST.etamaxLHV * POWERfactor;

% energy
ENERGY_Pdsg_kWh  = POWERdsg_kW*(Endurance_min/60);
ENERGY_Pmax_kWh  = POWERmax_kW*(Endurance_Pmax_min/60);
ENERGY_Pdsg_kWhe = POWERdsg_kWe*(Endurance_min/60);
ENERGY_Pmax_kWhe = POWERmax_kWe*(Endurance_Pmax_min/60);

% fuel gallon equivalents (Delh is -ve, hence the minus sign)
FGEHHV       = -(FCST.FGErho * FCST.FGEMJkg)/(r * Delh_MJkg);
FGELHV       = -(FCST.FGErho * FCST.FGEMJkg)/(r * Delh_LHV_MJkg);
FGEHHVtrue   = -(FCST.FGErho * FCST.FGEMJkg * FCST.FGEeta)/...
                (r * Delh_MJkg * EtadsgHHV);
FGELHVtrue   = -(FCST.FGErho * FCST.FGEMJkg * FCST.FGEeta)/...
                (r * Delh_LHV_MJkg * EtadsgLHV);

% Specific quantities
SFCPdsg_kgkWhe    = MassH2rate_kgs*3600/POWERdsg_kWe;
SFCPmax_kgkWhe    = MassH2rate_Pmax_kgs*3600/POWERmax_kWe;
```

```
SPower_Pdsg_kWkg  = POWERdsg_kW/Wtotal;
SPower_Pdsg_kWekg = POWERdsg_kWe/Wtotal;
SPower_Pmax_kWkg  = POWERmax_kW/Wtotal;
SPower_Pmax_kWekg = POWERmax_kWe/Wtotal;

SPower_Pdsg_Stack_kWkg      = POWERdsg_kW/FCST.Wstack.total;
SPower_Pdsg_Stack_kWekg     = POWERdsg_kWe/FCST.Wstack.total;
SPower_Pdsg_Stacksys_kWkg   = POWERdsg_kW/Wstacksys;
SPower_Pdsg_Stacksys_kWekg  = POWERdsg_kWe/Wstacksys;
SPower_Pmax_Stack_kWkg      = POWERmax_kW/FCST.Wstack.total;
SPower_Pmax_Stack_kWekg     = POWERmax_kWe/FCST.Wstack.total;
SPower_Pmax_Stacksys_kWkg   = POWERmax_kW/Wstacksys;
SPower_Pmax_Stacksys_kWekg  = POWERmax_kWe/Wstacksys;
DFstacksys                  = Wstacksys/(Ncell * Aactive);

EDensity_Pdsg_kWhL  = ENERGY_Pdsg_kWh/StotalL;
EDensity_Pdsg_kWheL = ENERGY_Pdsg_kWhe/StotalL;
EDensity_Pmax_kWhL  = ENERGY_Pmax_kWh/StotalL;
EDensity_Pmax_kWheL = ENERGY_Pmax_kWhe/StotalL;

SEnergy_Pdsg_kWhkg  = ENERGY_Pdsg_kWh/Wtotal;
SEnergy_Pdsg_kWhekg = ENERGY_Pdsg_kWhe/Wtotal;
SEnergy_Pmax_kWhkg  = ENERGY_Pmax_kWh/Wtotal;
SEnergy_Pmax_kWhekg = ENERGY_Pmax_kWhe/Wtotal;


if iprntSCR==1
disp(' ');
fprintf('%s \n\n','Power:');
fprintf('%s %8.4f %s\n','P gross    = ',POWERdsg_kW,'kW');
fprintf('%s %8.4f %s\n','P net      = ',POWERdsg_kWe,'kWe');
fprintf('%s %8.4f %s\n','% gross    = ',POWERfactor*100,'%');
fprintf('%s %8.4f %s\n','SFC        = ',SFCPdsg_kgkWh,'kg/kW-hr');
fprintf('%s %8.4f %s\n','Eta        = ',EtadsgHHV,'(HHV)');
fprintf('%s %8.4f %s\n','           = ',EtadsgLHV,'(LHV)');
disp(' ');
fprintf('%s \n','      P losses (balance of plant)');
fprintf('%s %8.4f %s\n','      Air          ',FCST.Pair_kW,'kW');
fprintf('%s %8.4f %s\n','      Fuel         ',FCST.Pfuel_kW,'kW');
fprintf('%s %8.4f %s\n','      LT cooling   ',FCST.Plt_kW,'kW');
fprintf('%s %8.4f %s\n','      HT cooling   ',FCST.Pht_kW,'kW');
fprintf('%s %8.4f %s\n','      Water        ',FCST.Pwater_kW,'kW');
fprintf('%s %8.4f %s\n','      Electrical   ',FCST.Pelec_kW,'kW');
fprintf('%s %8.4f %s\n','      Other        ',FCST.Pother_kW,'kW');
disp(' ')
fprintf('%s %8.4f %s\n','P gross max = ',POWERmax_kW,'kW');
fprintf('%s %8.4f %s\n','P net max   = ',POWERmax_kWe,'kWe');
fprintf('%s %8.4f %s\n','% gross     = ',POWERfactor_max*100,'%');
fprintf('%s %8.4f %s\n','SFC         = ',SFCPmax_kgkWh,'kg/kW-hr');
fprintf('%s %8.4f %s\n','Eta         = ',EtamaxHHV,'(HHV)');
fprintf('%s %8.4f %s\n','            = ',EtamaxLHV,'(LHV)');
```

```
disp(' '); disp(' '); disp(' ');
fprintf('%s \n\n','Energy:');
fprintf('%s %8.4f %s\n','Endurance  = ',Endurance_min,'min');
fprintf('%s %8.4f %s\n','@ Pmax     = ',Endurance_Pmax_min,'min');
fprintf('%s %8.4f %s\n','E          = ',ENERGY_Pdsg_kWh,'kWh');
fprintf('%s %8.4f %s\n','E net P    = ',ENERGY_Pdsg_kWhe,'kWhe');
fprintf('%s %8.4f %s\n','E Pmax     = ',ENERGY_Pmax_kWh,'kWh');
fprintf('%s %8.4f %s\n','E net Pmax = ',ENERGY_Pmax_kWhe,'kWhe');


disp(' '); disp(' '); disp(' ');
fprintf('%s \n\n','Fuel gallon equivalents @ Pdsg:');
fprintf('%s \n','Base fuel input:');
fprintf('%s %8.4f %s\n','Energy     = ',FCST.FGEMJkg,'MJ/kg');
fprintf('%s %8.4f %s\n','Density    = ',FCST.FGErho,'kg/m3');
fprintf('%s %8.4f %s\n','Engine eta = ',FCST.FGEeta,'(combustion)');
fprintf('%s \n','Fuel Cell compared to base fuel:');
fprintf('%s %8.4f %s\n','HHV        = ',FGEHHV,'U.S. gal');
fprintf('%s %8.4f %s\n','LHV        = ',FGELHV,'U.S. gal');
fprintf('%s %8.4f %s\n','HHV true   = ',FGEHHVtrue,'U.S. gal');
fprintf('%s %8.4f %s\n','LHV true   = ',FGELHVtrue,'U.S. gal');


disp(' '); disp(' '); disp(' ');
fprintf('%s \n\n','Weights:');
fprintf('%s %8.4f %s\n','W total     = ',Wtotal,'kg');
fprintf('%s %8.4f %s\n','W Stack Sys = ',Wstacksys,'kg');
fprintf('%s %8.4f %s\n','S Stack Sys = ',Sstacksys,'m3');
disp(' ');
fprintf('%s \n','      Break-down');
fprintf('%s %8.4f %s\n','      Stack        ',FCST.Wstack.total,'kg');
fprintf('%s %8.4f %s\n','      Air          ',FCST.Wair.total,'kg');
fprintf('%s %8.4f %s\n','      Fuel H2      ',FCST.Wfuel.H2,'kg');
fprintf('%s %8.4f %s\n','      Tank sys     ',FCST.Wfuel.total-FCST.Wfuel.H2,'kg');
fprintf('%s %8.4f %s\n','      HT cooling   ',FCST.Wht.total,'kg');
fprintf('%s %8.4f %s\n','      LT cooling   ',FCST.Wlt.total,'kg');
fprintf('%s %8.4f %s\n','      Water        ',FCST.Wwater.total,'kg');
fprintf('%s %8.4f %s\n','      Electrical   ',FCST.Welec.total,'kg');
fprintf('%s %8.4f %s\n','      Other        ',FCST.Wother,'kg');
disp(' ')


disp(' '); disp(' '); disp(' ');
fprintf('%s \n\n','Specific power and energy of total system:');
disp(' ')
fprintf('%s \n\n','Stack, Stack system:');
fprintf('%s %8.4f %s\n','SPstack    = ',SPower_Pdsg_Stack_kWkg,  'kW/kg');
fprintf('%s %8.4f %s\n','             ',SPower_Pdsg_Stack_kWekg, 'kWe/kg');
fprintf('%s %8.4f %s\n','    @ Pmax = ',SPower_Pmax_Stack_kWkg,  'kW/kg');
fprintf('%s %8.4f %s\n','             ',SPower_Pmax_Stack_kWekg, 'kWe/kg');
fprintf('%s %8.4f %s\n','SPstacksys = ',SPower_Pdsg_Stacksys_kWkg,'kW/kg');
fprintf('%s %8.4f %s\n','             ',SPower_Pdsg_Stacksys_kWekg,'kWe/kg');
fprintf('%s %8.4f %s\n','    @ Pmax = ',SPower_Pmax_Stacksys_kWkg,'kW/kg');
fprintf('%s %8.4f %s\n','             ',SPower_Pmax_Stacksys_kWekg,'kWe/kg');
fprintf('%s %8.4f %s\n','DF system  = ',DFstacksys,'kg/m2');
disp(' ')
fprintf('%s \n\n','Full system: Stack, stack system, fuel, fuel system:');
fprintf('%s %8.4f %s\n','SP         = ',SPower_Pdsg_kWkg,   'kW/kg');
fprintf('%s %8.4f %s\n','SE         = ',SEnergy_Pdsg_kWhkg, 'kWh/kg');
fprintf('%s %8.4f %s\n','ED         = ',EDensity_Pdsg_kWhL, 'kWh/L');
fprintf('%s %8.4f %s\n','SPe        = ',SPower_Pdsg_kWekg,  'kWe/kg');
fprintf('%s %8.4f %s\n','SEe        = ',SEnergy_Pdsg_kWhekg,'kWhe/kg');
fprintf('%s %8.4f %s\n','EDe        = ',EDensity_Pdsg_kWheL,'kWhe/L');
disp(' ')
fprintf('%s %8.4f %s\n','SP  @ Pmax = ',SPower_Pmax_kWkg,   'kW/kg');
fprintf('%s %8.4f %s\n','SE  @ Pmax = ',SEnergy_Pmax_kWhkg, 'kWh/kg');
fprintf('%s %8.4f %s\n','ED  @ Pmax = ',EDensity_Pmax_kWhL, 'kWh/L');
fprintf('%s %8.4f %s\n','SPe @ Pmax = ',SPower_Pmax_kWekg,  'kWe/kg');
fprintf('%s %8.4f %s\n','SEe @ Pmax = ',SEnergy_Pmax_kWhekg,'kWhe/kg');
fprintf('%s %8.4f %s\n','EDe @ Pmax = ',EDensity_Pmax_kWheL,'kWhe/L');
disp(' '); disp(' ');
end



FCST.SFCPdsg_kgkWhe            = SFCPdsg_kgkWhe;
FCST.SFCPmax_kgkWhe            = SFCPmax_kgkWhe;

FCST.POWERdsg_kWe              = POWERdsg_kWe;
FCST.POWERmax_kW               = POWERmax_kW;
FCST.POWERmax_kWe              = POWERmax_kWe;

FCST.EtadsgHHV                 = EtadsgHHV;
FCST.EtadsgLHV                 = EtadsgLHV;
FCST.EtamaxHHV                 = EtamaxHHV;
FCST.EtamaxLHV                 = EtamaxLHV;

FCST.FGEHHV                    = FGEHHV;
FCST.FGELHV                    = FGELHV;
FCST.FGEHHVtrue                = FGEHHVtrue;
FCST.FGELHVtrue                = FGELHVtrue;

FCST.ENERGY_Pdsg_kWh           = ENERGY_Pdsg_kWh;
FCST.ENERGY_Pdsg_kWhe          = ENERGY_Pdsg_kWhe;
FCST.ENERGY_Pmax_kWh           = ENERGY_Pmax_kWh;
FCST.ENERGY_Pmax_kWhe          = ENERGY_Pmax_kWhe;

FCST.SPower_Pdsg_kWkg           = SPower_Pdsg_kWkg;
FCST.SPower_Pdsg_kWekg          = SPower_Pdsg_kWekg;
FCST.SPower_Pmax_kWkg           = SPower_Pmax_kWkg;
FCST.SPower_Pmax_kWekg          = SPower_Pmax_kWekg;
FCST.SPower_Pdsg_Stack_kWkg     = SPower_Pdsg_Stack_kWkg;
FCST.SPower_Pdsg_Stack_kWekg    = SPower_Pdsg_Stack_kWekg;
FCST.SPower_Pdsg_Stacksys_kWkg  = SPower_Pdsg_Stacksys_kWkg;
FCST.SPower_Pdsg_Stacksys_kWekg = SPower_Pdsg_Stacksys_kWekg;
FCST.SPower_Pmax_Stacksys_kWkg  = SPower_Pmax_Stacksys_kWkg;
FCST.SPower_Pmax_Stacksys_kWekg = SPower_Pmax_Stacksys_kWekg;
FCST.DFstacksys                 = DFstacksys;

FCST.EDensity_Pdsg_kWhL        = EDensity_Pdsg_kWhL;
FCST.EDensity_Pdsg_kWheL       = EDensity_Pdsg_kWheL;
FCST.EDensity_Pmax_kWhL        = EDensity_Pmax_kWhL;
FCST.EDensity_Pmax_kWheL       = EDensity_Pmax_kWheL;

FCST.SEnergy_Pdsg_kWhkg        = SEnergy_Pdsg_kWhkg;
FCST.SEnergy_Pdsg_kWhekg       = SEnergy_Pdsg_kWhekg;
FCST.SEnergy_Pmax_kWhkg        = SEnergy_Pmax_kWhkg;
FCST.SEnergy_Pmax_kWhekg       = SEnergy_Pmax_kWhekg;


if iprntSCR==1
    disp(' ')
    disp(' ')
end
% ====================================================
% End - PEMFC system design




% Stack operating characteristics
% ====================================================

[~,~,~,~,~,~,imax,vmax]=ivpolarization(FCST.ivdata,FCST.ipmax,0,0,0,...
                        vcorr,altitude_ft,h0,vrate,icorr,tfi,deltav);
icrange=0:0.01:imax;
for ii=1:length(icrange)
    ic = icrange(ii);
    [~,vc,~,~,~,~,~,~]=ivpolarization(FCST.ivdata,FCST.ipmax,0,ic,0,...
                vcorr,altitude_ft,h0,vrate,icorr,tfi,deltav);
    Vrange(ii) = vc*Ncell;
    Irange(ii) = ic*Aactive_cm2;
    Prange(ii) = Irange(ii)*Vrange(ii);                        % Watt
    fleakage   = ic/(ic + jleak);
    etarange(ii) = vc * fleakage /Eh_V;
    H2flow(ii) = SH2*(MH2/(N*Faraday))*Prange(ii)/vc/fleakage; % kg/s
    SFCrange(ii) = H2flow(ii)*3600*1e3/Prange(ii);             % kg/kW-hr
end


if FCST.iplotVIP==1
        % Gross Power and Voltage vs Current
        figure;
        hold on; grid on; set(gca,'FontSize',16);
        plot(Irange,Vrange,'k-','LineWidth',3);
        plot(Irange,ones(1,length(Irange))*Vstack_V,'k--','LineWidth',3);
        plot(Irange,Prange*1e-3,'r-','LineWidth',3);
        plot(Irange,ones(1,length(Irange))*POWERdsg_kW,'r--','LineWidth',3);
        legend('Stack Voltage','Design value',...
               'Stack Power','Design value');
        xlabel('Stack current, A');
        ylabel('Stack voltage, V; Stack Power, kW');
        title('Stack voltage and power');

        % Efficiency vs Current
        figure;
        hold on; grid on; set(gca,'FontSize',16);
        plot(Irange,etarange,'k-','LineWidth',3);
        etadesign = FCST.etadsgHHV;
        plot(Irange,ones(1,length(Irange))*etadesign,'k--','LineWidth',3);
        legend('Efficiency range','Design efficiency');
        xlabel('Stack current, A');
        ylabel('Stack efficiency');
        title('Operating efficiency');

        % Fuel flow vs Current
        figure;
        hold on; grid on; set(gca,'FontSize',16);
        plot(Irange,H2flow*3600,'k-','LineWidth',3);
        plot(Irange,ones(1,length(Irange))*FCST.MassH2rate_kgs*3600,...
            'k--','LineWidth',3);
        legend('Hydrogen flow range','Design flow');
        xlabel('Stack current, A');
        ylabel('H2 flow, kg/hr');
        title('Operating H2 flow');

        % SFC vs Current
        figure;
        hold on; grid on; set(gca,'FontSize',16);
        plot(Irange,SFCrange,'k-','LineWidth',3);
        plot(Irange,ones(1,length(Irange))*FCST.SFCPdsg_kgkWh,'k--','LineWidth',3);
        legend('SFC','Design SFC');
        xlabel('Stack current, A');
        ylabel('SFC, kg / kW-hr');
        title('SFC');
end

clear Irange Vrange Prange SFCrange etarange ic vc


if iprntSCR==1
disp(' ')
disp(' ')
disp(' ')
end
```

```
% pemfcdesign.m
% Given Design power, size stack and system


FCST.iplotVIP = 0;

% Net power (KWe) needed from stack
kWe = FCST.POWERdsg_kW;

% Find jacobian
pemfc;
F0 = FCST.POWERdsg_kWe;
if kWe==0
    delta=0.01;
else
    delta = 0.05*kWe;
end
FCST.POWERdsg_kW = FCST.POWERdsg_kW + delta;
pemfc;
F = FCST.POWERdsg_kWe;
JAC = (F-F0)/delta;

% Iterate
driveriter=1;
while abs(kWe - FCST.POWERdsg_kWe) > 0.001
    FCST.POWERdsg_kW = FCST.POWERdsg_kW + (kWe - FCST.POWERdsg_kWe)/JAC;
    pemfc;
    driveriter = driveriter+1;
    fprintf('%s %3i\n','Iteration no = ',driveriter);
    if driveriter > 100; disp(' *** NO CONVERGENCE *** '); break; end
end

% Last call for plot
FCST.iplotVIP=1;
pemfc
```

```
% pemfcstatic.m
% A Datta
% U Maryland 2020


Istack                = FCOP.Istack;
altitude              = FCOP.altitude;
iatm                  = FCOP.iatm;
pressure              = FCOP.pressure;
temperatureC          = FCOP.temperatureC;
CompEta               = FCOP.CompEta;
TurbEta               = FCOP.TurbEta;
BlowEta               = FCOP.BlowEta;
vcorr                 = FCOP.vcorr;
vrate                 = FCOP.vrate1000ft;
h0                    = FCOP.vdropaboveft;
crh                   = FCOP.crh;
arh                   = FCOP.arh;


% Inputs, design, fixed
fAirFlowRate          = FCST.fAirFlowRate;
fH2utilization        = FCST.fH2utilization;
xO2                   = FCST.xO2;
DPstack               = FCST.DPstack;
Aactive_cm2           = FCST.Aactive_cm2;
Ncell                 = FCST.Ncell;
fovpower              = FCST.fovpower;
OneAtm                = FCST.OneAtm;
TstackC               = FCST.TstackC;
Pstack                = FCST.Pstack;
icorr                 = FCST.icorr;
tfi                   = FCST.TFi;
deltav                = FCST.deltav;
iplotVIP              = FCST.iplotVIP;
iprntSCR              = FCST.iprntSCR;
deltai                = FCST.deltai;
ASR0t                 = FCST.ASR0t;
ASRtk2                = FCST.ASRtk2;
ASRtk1                = FCST.ASRtk1;
jL0t                  = FCST.jL0t;
jLtk2                 = FCST.jLtk2;
jLtk1                 = FCST.jLtk1;
ASR0arh               = FCST.ASR0arh;
ASR0crh               = FCST.ASR0crh;
ASRrhk1a              = FCST.ASRrhk1a;
ASRrhk2a              = FCST.ASRrhk2a;
ASRrhk1a1c            = FCST.ASRrhk1a1c;
ASRrhk1a2c            = FCST.ASRrhk1a2c;
ASRrhk2a2c            = FCST.ASRrhk2a2c;
ASRrhk2a1c            = FCST.ASRrhk2a1c;
ASRrhk2c              = FCST.ASRrhk2c;
ASRrhk1c              = FCST.ASRrhk1c;
```

```
jL0arh              = FCST.jL0arh;
jL0crh              = FCST.jL0crh;
jLrhk1a             = FCST.jLrhk1a;
jLrhk2a             = FCST.jLrhk2a;
jLrhk1a1c           = FCST.jLrhk1a1c;
jLrhk1a2c           = FCST.jLrhk1a2c;
jLrhk2a2c           = FCST.jLrhk2a2c;
jLrhk2a1c           = FCST.jLrhk2a1c;
jLrhk2c             = FCST.jLrhk2c;
jLrhk1c             = FCST.jLrhk1c;



if iprntSCR==1
disp(' ')
disp(' ')
disp('          ===========================')
disp('             PEMFC SYSTEM OPERATION'  )
disp('                  STEADY-STATE     '  )
disp('          ===========================')
disp(' ')
disp(' ')
end



% Constants:
Runival = 0.0820578;      % universal gas constant in atm.L/mol.K
Runivkj = 8.3144621*1e-3; % universal gas constant in kJ/mol.K
Faraday = 96485.3365;     % Coulomb/mole
N       = 2;              % no of electrons per molecule of H2
SH2     = 1;              % H2 stoichiometry
MH2     = 2.01588*1e-3;   % molecular mass of H2 (kg/mole)
SAir    = 1/2;            % Air stoichiometry
MAir    = 28.965*1e-3;    % molecular mass of Air (kg/mole)
SO2     = 1/2;            % O2 stoichiometry
MO2     = 31.9988*1e-3;   % molecular mass of O2 (kg/mole)
MH2O    = 18.0153*1e-3;   % molecular mass of H2O (kg/mole)
SN2     = 1/2;            % N2 stoichiometry
MN2     = 28.0134*1e-3;   % molecular mass of N2 (kg/mole)
Rdryair = Runivkj/MAir;   % dry air gas constant, kJ/kg.K
SB      = 5.670367*1e-8;  % Stephan-Boltzmann W/m2/K4




switch iatm
    case 0
        % ISA
        [temperatureK, pressure, density, viscosity, sound]=...
            ISAtmosphere(0,altitude,0);
        temperatureC = temperatureK - 273.15;
        FCOP.pressure = pressure;
        FCOP.temperatureC = temperatureC;
        FCOP.density  = density;
    case 1
        % ISA + temp correction
        [temperatureK, pressure, density, viscosity, sound]=...
            ISAtmosphere(1,altitude,temperatureC);
        FCOP.pressure = pressure;
        FCOP.density  = density;
    otherwise
        % input temp and pressure directly
        pressure = FCOP.pressure;
        density = pressure/(1e3 * Rdryair * temperatureK);
        FCOP.density = density;
end




% Ideal thermodynamic quantities. Need only for efficiencies.
% H-Hydrogen; O-Oxygen; Wv-H2O vapor; Wl-H2O liquid
% Thermodynamic values at stack temperature
Pstack_atm = Pstack / OneAtm;
TstackK = TstackC + 273.15;
[Hg, Hh, Hs, Hcp]     =  DATA_gptH2(TstackK);
[Og, Oh, Os, Ocp]     =  DATA_gptO2(TstackK);
[Wvg, Wvh, Wvs, Wvcp] =  DATA_gptH2Ov(TstackK);
[Wlg, Wlh, Wls, Wlcp] =  DATA_gptH2Ol(TstackK);
Delh_kJm = Wlh - (Hh + 0.5*Oh);
Delg_kJm = Wlg - (Hg + 0.5*Og);
% Correct for stack pressure
Delg_kJm = Delg_kJm + Runivkj*TstackK*log(1/(1*(Pstack_atm*xO2)^0.5));
Eh_V = -Delh_kJm*1e3/(N*Faraday);
Er_V = -Delg_kJm*1e3/(N*Faraday);
Delh_MJkg = Delh_kJm * 1e-3 / MH2;
Delg_MJkg = Delg_kJm * 1e-3 / MH2;

Delh_LHV_kJm = Wvh - (Hh + 0.5*Oh);
Delg_LHV_kJm = Wvg - (Hg + 0.5*Og);
Delg_LHV_kJm = Delg_LHV_kJm + Runivkj*TstackK*log(1/(1*(Pstack_atm*xO2)^0.5));
Eh_LHV_V = -Delh_LHV_kJm*1e3/(N*Faraday);
Er_LHV_V = -Delg_LHV_kJm*1e3/(N*Faraday);
Delh_LHV_MJkg = Delh_LHV_kJm * 1e-3 / MH2;
Delg_LHV_MJkg = Delg_LHV_kJm * 1e-3 / MH2;




% estimate the leakage factor
% first get leakage current
% create i-v data
if FCST.ioptiv == 1
    % no action
else
```

```
    switch FCST.ioptiv
        case 2
            % take parameters from input
            jleak    = FCST.jleak;
        case 3
            % find parameters from PMAT
            PMAT = FCST.PMAT;
            [m,n]=size(PMAT);
            Pinterp = Pstack_atm;
            if Pstack_atm > PMAT(1,n)
                disp('ERROR: Pstack > PMAT table pressure')
                disp('       Re-setting Pstack=1 atm')
                Pstack_atm = PMAT(1,n);
                Pinterp = Pstack_atm;
            end
            if Pstack_atm < PMAT(1,1)
                % start from here
                Pinterp = PMAT(1,1);
                % use  theoretical correction in createiv for below
            end
            jleak  = interp1(PMAT(1,1:n),PMAT(8,1:n),Pinterp,'linear');
        otherwise
            % default
            jleak  = FCST.jleak;
    end
end
Icell_Acm2  = Istack / Aactive_cm2;
fleakage    = Icell_Acm2 / (Icell_Acm2 + jleak);



                       % Hydrogen input
                 % Anode:   H2 --> (2H+) + (2e-)

MassH2rate_kgs      = SH2*(MH2/(N*Faraday)) * Istack * Ncell/fleakage;
MassH2rate_kgs      = fH2utilization * MassH2rate_kgs;

if iprntSCR==1
fprintf('%s %8.4f %s\n','H2 in              = ',MassH2rate_kgs,'kg/s');
end



                         % Air input
          % Cathode:   (1/2) O2 + (2e-) + (2H+) --> H2O

MassAirrate_kgs =  SAir * (MAir/(N*xO2*Faraday)) * Istack * Ncell * ...
                   fAirFlowRate / fleakage;
MassAirrate_Ls = (MassAirrate_kgs/MAir)*Runival*TstackK/Pstack_atm;
if iprntSCR==1
fprintf('%s %8.4f %s\n','Air in             = ',MassAirrate_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Air in vol         = ',MassAirrate_Ls,'L/sec');
end
```

```
% O2 input - need for air and water output
MassO2rate_kgs =  SO2 * (MO2/(N*Faraday)) * Istack * Ncell / fleakage;


                         % Air output
                % Air input - O2 used up in reaction

MassAirrate_out_kgs = MassAirrate_kgs - MassO2rate_kgs;
if iprntSCR==1
fprintf('%s %8.4f %s\n','Air out            = ',MassAirrate_out_kgs,'kg/s');
end

                         % Water output

% rate of water production
MassWaterrate_kgs = MassH2rate_kgs + MassO2rate_kgs;

% exit pr out of stack = stack pr - pr drop in stack
Pstack_exit = Pstack - DPstack;
% part of exit pr is vapor pressure. rest is dry air pressure.
Pvaporstack_out = watervp(TstackC);
Pdryairstack_out = Pstack_exit - Pvaporstack_out;
% humidity
Humidity_ratio = MH2O*Pvaporstack_out / (MAir*Pdryairstack_out);
% vapor needed to saturate output air
hm = Humidity_ratio * MassAirrate_out_kgs;
if hm < MassWaterrate_kgs
    % vapor < water means enough water to saturate output air
    MassVaporrate_out_kgs = hm;
else
    % vapor > water means less water than needed to saturate output air
    MassVaporrate_out_kgs = MassWaterrate_kgs;
end

% rate of liq water leaving cell
MassLiquidrate_kgs = MassWaterrate_kgs - MassVaporrate_out_kgs;
MassTankrate_kgs = 0;
if MassLiquidrate_kgs > 0
    frac = FCST.waterstoragefrac;
    MassTankrate_kgs = frac * MassLiquidrate_kgs;
end

% stop for non-physical results
if Pstack_exit < 0
    disp(' ')
    fprintf('%s\n','ERROR:');
    fprintf('%s\n','Pressure too low. Pstack cannot be < DPstack:');
    fprintf('%s %8.4f %s\n','P stack     = ',Pstack_atm,'atm');
    fprintf('%s %8.4f %s\n','DP stack    = ',DPstack/OneAtm,'atm');
    fprintf('%s\n','Use compressor.');
    stop
end
if Humidity_ratio < 0.01
```

```
    disp(' ')
    fprintf('%s\n','ERROR:');
    fprintf('%s\n','Output humidity zero. Temperature too low');
    fprintf('%s %8.4f %s\n','Sat humidity ratio = ',Humidity_ratio,' ');
    fprintf('%s\n','Use compressor.');
    stop
end


if iprntSCR==1
fprintf('%s %8.4f %s\n','Water produced     = ',MassWaterrate_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Humidity           = ',Humidity_ratio,'ratio');
fprintf('%s %8.4f %s\n','Vapor out satu     = ',hm,'kg/s');
fprintf('%s %8.4f %s\n','Vapor out true     = ',MassVaporrate_out_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Water produced     = ',MassLiquidrate_kgs,'kg/s');
fprintf('%s %8.4f %s\n','Water to tank      = ',MassTankrate_kgs,'kg/s');
end

               % Total exit air flow rate

Massrate_out_kgs = MassAirrate_out_kgs;
if iprntSCR==1
fprintf('%s %8.4f %s\n','Flow out           = ',Massrate_out_kgs,'kg/s');
end




Pblower_kW    = 0;
Pcomp_kW      = 0;
Pturb_kW      = 0;


% Blower
if FCST.iBlower==1
    DeltaP = Pstack - pressure;
    rhoAir = pressure / (1e3 * Rdryair * TstackK);
    volAirrate_m3s = MassAirrate_kgs / rhoAir;
    CFM = volAirrate_m3s * 2118.88;
    Pblower_kW = DeltaP * volAirrate_m3s * 1e-3;
    if iprntSCR==1
        disp(' ')
        fprintf('%s \n','Blower:');
        fprintf('%s %8.4f %s\n','Delta P    = ',DeltaP/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','Flow       = ',MassAirrate_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','CFM        = ',CFM,'ft3/min');
        fprintf('%s %8.4f %s\n','CFM        = ',CFM,'ft3/min');
    end
    if Pblower_kW > FCST.Pblowermax_kW + 0.01
        Pblower_kW = FCST.Pblowermax_kW;
        DeltaP = FCST.Pblowermax_kW / (volAirrate_m3s * 1e-3);
        Pstack = pressure + DeltaP;
        Pstack_atm = Pstack/OneAtm;
```

```
if iprntSCR==1
    disp(' ')
    fprintf('%s \n','Blower power > max');
    fprintf('%s \n','Re-setting power to max');
    fprintf('%s %8.4f %s\n','Power      = ',FCST.Pblowermax_kW,'kW');
    fprintf('%s \n','Update stack P');
    fprintf('%s %8.4f %s\n','P new      = ',Pstack_atm,'atm');
    disp(' ')
end
switch FCST.ioptiv
    case 1
        %nothing can be done since i-v is direct table input
        %disp(' ');
        %fprintf('%s \n','ERROR: compressor power > max');
        %fprintf('%s %8.4f %s\n','P comp max      = ',FCST.Pcompmax_kW,'kW');
        %fprintf('%s \n','Size stack for operating T and altitude');
        %fprintf('%s %8.4f %s\n','Altitude        = ',altitude/0.3048,'ft');
        %fprintf('%s %8.4f %s\n','Temperature     = ',temperatureC,'C');
        %disp(' '); disp(' '); stop
    case 2
        % apply theoretical pressure correction in createiv
        Er       = FCST.Er;
        j0C      = FCST.j0C;
        alphaC   = FCST.alphaC;
        j0A      = FCST.j0A;
        alphaA   = FCST.alphaA;
        ASR      = FCST.ASR;
        jL       = FCST.jL;
        jleak    = FCST.jleak;
        C        = FCST.C;
    case 3
        % used measured pressure corrections
        % find parameters from PMAT
        PMAT = FCST.PMAT;
        [m,n]=size(PMAT);
        Pinterp = Pstack_atm;
        if Pstack_atm > PMAT(1,n)
            disp('WARNING: Pstack > PMAT table pressure')
            disp('         Resetting Pstack to max table')
            Pstack_atm = PMAT(1,n);
            Pinterp = Pstack_atm;
        end
        if Pstack_atm < PMAT(1,1)
            % start from here
            Pinterp = PMAT(1,1);
            % use  theoretical correction in createiv for below
        end
        j0C    = interp1(PMAT(1,1:n),PMAT(2,1:n),Pinterp,'linear');
        alphaC = interp1(PMAT(1,1:n),PMAT(3,1:n),Pinterp,'linear');
        j0A    = interp1(PMAT(1,1:n),PMAT(4,1:n),Pinterp,'linear');
        alphaA = interp1(PMAT(1,1:n),PMAT(5,1:n),Pinterp,'linear');
        ASR    = interp1(PMAT(1,1:n),PMAT(6,1:n),Pinterp,'linear');
        jL     = interp1(PMAT(1,1:n),PMAT(7,1:n),Pinterp,'linear');
```

```
                jleak  = interp1(PMAT(1,1:n),PMAT(8,1:n),Pinterp,'linear');
                C      = interp1(PMAT(1,1:n),PMAT(9,1:n),Pinterp,'linear');
                Er     = 0.; % find from thermodynamic constants
            otherwise
                % no action
        end
    end
end


% Compressor-Turbine
iCompressor = 0;
iTurbine = 0;

if FCST.iCompressor==1
    T1compC = temperatureC;
    [Cp_air, Cv_air, Gamma_air] = DATA_DryAirCp(T1compC);
    P1comp = pressure;
    P2comp = Pstack;
    T1comp = T1compC + 273.15;
    fac1 = (Gamma_air-1)/Gamma_air;
    fac2 = (P2comp/P1comp)^fac1 - 1;
    DeltaT = fac2 * T1comp / CompEta;
    T2comp = T1comp + DeltaT;
    Pcomp_kW = Cp_air * MassAirrate_kgs * DeltaT * 1e-3;

    if Pcomp_kW < 0.1
        if iprntSCR==1
            fprintf('%s \n','Compressor not needed ');
        end
        Pcomp_kW = 0.;
        iTurbine = 0;
        iCompressor = 0;
    else
        iTurbine = 1;
        iCompressor = 1;
    end

    if iprntSCR==1
        disp(' ')
        fprintf('%s \n','Compressor:');
        fprintf('%s %8.4f %s\n','Cp/Cv      = ',Gamma_air,' ');
        fprintf('%s %8.4f %s\n','Cp         = ',Cp_air,'J/kg.K ');
        fprintf('%s %8.4f %s\n','P1         = ',P1comp/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','P2         = ',P2comp/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','T1         = ',T1compC,'deg C');
        fprintf('%s %8.4f %s\n','T2         = ',T2comp-273.15,'deg C');
        fprintf('%s %8.4f %s\n','Delta T    = ',DeltaT,'deg C');
        fprintf('%s %8.4f %s\n','Flow       = ',MassAirrate_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','Efficiency = ',CompEta,' ');
        fprintf('%s %8.4f %s\n','Power      = ',Pcomp_kW,'kW');
    end
```

```
% Pcomp > Pcomp max from design ?
Pcompmax_kW = FCST.Pcompmax_kW;
if Pcomp_kW > Pcompmax_kW + 0.01
    % limit power to max power
    Pcomp_kW = Pcompmax_kW;
    % find new P2comp and T2comp
    DeltaT = Pcompmax_kW * 1e3 / (Cp_air * MassAirrate_kgs);
    T2comp = T1comp + DeltaT;
    fac1 = Gamma_air / (Gamma_air-1);
    fac2 = (CompEta * DeltaT / T1comp + 1)^fac1;
    P2comp = P1comp * fac2;
    Pstack = P2comp;
    Pstack_atm = Pstack/OneAtm;
    if iprntSCR==1
        disp(' ')
        fprintf('%s \n','Compressor power > max');
        fprintf('%s \n','Re-setting power to max');
        fprintf('%s %8.4f %s\n','Power      = ',FCST.Pcompmax_kW,'kW');
        fprintf('%s \n','Update stack P');
        fprintf('%s %8.4f %s\n','P new      = ',Pstack_atm,'atm');
        disp(' ')
    end
    switch FCST.ioptiv
        case 1
            %nothing can be done since i-v is direct table input
            %disp(' ');
            %fprintf('%s \n','ERROR: compressor power > max');
            %fprintf('%s %8.4f %s\n','P comp max      = ',FCST.Pcompmax_kW,'kW');
            %fprintf('%s \n','Size stack for operating T and altitude');
            %fprintf('%s %8.4f %s\n','Altitude        = ',altitude/0.3048,'ft');
            %fprintf('%s %8.4f %s\n','Temperature     = ',temperatureC,'C');
            %disp(' '); disp(' '); stop
        case 2
            % apply theoretical pressure correction in createiv
            Er       = FCST.Er;
            j0C      = FCST.j0C;
            alphaC   = FCST.alphaC;
            j0A      = FCST.j0A;
            alphaA   = FCST.alphaA;
            ASR      = FCST.ASR;
            jL       = FCST.jL;
            jleak    = FCST.jleak;
            C        = FCST.C;
        case 3
            % used measured pressure corrections
            % find parameters from PMAT
            PMAT = FCST.PMAT;
            [m,n]=size(PMAT);
            Pinterp = Pstack_atm;
            if Pstack_atm > PMAT(1,n)
                disp('WARNING: Pstack > PMAT table pressure')
                disp('         Resetting Pstack to max table')
                Pstack_atm = PMAT(1,n);
                Pinterp = Pstack_atm;
            end
            if Pstack_atm < PMAT(1,1)
                % start from here
                Pinterp = PMAT(1,1);
                % use  theoretical correction in createiv for below
            end
            j0C    = interp1(PMAT(1,1:n),PMAT(2,1:n),Pinterp,'linear');
            alphaC = interp1(PMAT(1,1:n),PMAT(3,1:n),Pinterp,'linear');
            j0A    = interp1(PMAT(1,1:n),PMAT(4,1:n),Pinterp,'linear');
            alphaA = interp1(PMAT(1,1:n),PMAT(5,1:n),Pinterp,'linear');
            ASR    = interp1(PMAT(1,1:n),PMAT(6,1:n),Pinterp,'linear');
            jL     = interp1(PMAT(1,1:n),PMAT(7,1:n),Pinterp,'linear');
            jleak  = interp1(PMAT(1,1:n),PMAT(8,1:n),Pinterp,'linear');
            C      = interp1(PMAT(1,1:n),PMAT(9,1:n),Pinterp,'linear');
            Er     = 0.; % find from thermodynamic constants
        otherwise
            % no action
    end
  end
end

    % create i-v-p data from parameters
    % temperature correction to ASR
    t         = TstackC - ASR0t;
    dASRt     = ASRtk2*t^2 + ASRtk1*t;
    % temperature correction to jL
    t        =  TstackC - jL0t;
    djLt     =  jLtk2*t^2 + jLtk1*t;
    % relative humidity corrections to ASR and jL
    a = arh - ASR0arh; c = crh - ASR0crh;
    dASRrh = ASRrhk1a * a + ...
             ASRrhk2a * a^2 + ...
             ASRrhk1a1c * a * c + ...
             ASRrhk2a1c * a^2 * c + ...
             ASRrhk2a2c * a^2 * c^2 + ...
             ASRrhk1a2c * a * c^2 + ...
             ASRrhk2c * c^2 + ...
             ASRrhk1c * c;
    a = arh - jL0arh; c = crh - jL0crh;
    djLrh =  jLrhk1a * a + ...
             jLrhk2a * a^2 + ...
             jLrhk1a1c * a * c + ...
             jLrhk2a1c * a^2 * c + ...
             jLrhk2a2c * a^2 * c^2 + ...
             jLrhk1a2c * a * c^2 + ...
             jLrhk2c * c^2 + ...
             jLrhk1c * c;
    ASR =  ASR +  dASRt + dASRrh;
    jL  =  jL  +  djLt + djLrh;
    [ivdata, Eh, Er, ipmax] = createiv(TstackC, Pstack_atm, ...
        xO2, deltai, Er, ...
        j0C, alphaC, j0A, alphaA, ASR, jL, jleak, C);
    FCOP.ivdata = ivdata;
    FCOP.ipmax  = ipmax;




% low temperature cooling system
Plt_kW = 0;
if FCST.iCompressor==1 && iCompressor==1
    % only when compressor is needed
    DeltaT = T2comp - TstackK;
    if DeltaT <= 0.
        % no cooling system needed
        HEATINGair    = 0;
    else
        % heat to be removed
        HEATINGair    = Cp_air * MassAirrate_kgs * DeltaT;
    end
    Cpcool  = FCST.ltCoolantCp;
    dTcool  = FCST.ltCoolantdT;
    Cflow   = HEATINGair / ( Cpcool*dTcool );
    % power consumption
    dP                  = FCST.Plt0;
    k                   = FCST.Pltk;
    e                   = FCST.Plte;
    f                   = FCST.Pltf;
    P                   = dP + k * HEATINGair^e * Cflow^f;
    Plt_kW              = P*1e-3;
    if iprntSCR==1
    disp(' ')
    disp('Low temperature cooling system: ')
    fprintf('%s %8.4f %s\n','Delta T    = ',DeltaT,'deg C')
    fprintf('%s %8.4f %s\n','Heat       = ',HEATINGair*1e-3,'kW')
    fprintf('%s %8.4f %s\n','kW/C       = ',HEATINGair*1e-3/DeltaT,'kW/deg C')
    fprintf('%s %8.4f %s\n','Coolant q  = ',f*Cflow,'kg/s')
    fprintf('%s %8.4f %s\n','Power      = ',Plt_kW,'kW')
    end
end



Icell_Acm2  = Istack / Aactive_cm2;
altitude_ft = altitude/0.3048;
% from ic find vc from polarization curve
[I,V,P,Pmax,iPmax,vPmax,Imax,Vmax] = ...
    ivpolarization(FCOP.ivdata,FCOP.ipmax,0,Icell_Acm2,1,...
    vcorr,altitude_ft,h0,vrate,icorr,tfi,deltav);
Vcell = V;
etac = Vcell * fleakage / Eh_V;
Vstack = Vcell * Ncell;
POWER_kW = Istack * Vstack * 1e-3;
```

```
% Turbine
if FCST.iTurbine==1 && iTurbine==1

    T1turbC = TstackC;
    [Cp_air, Cv_air, Gamma_air] = DATA_DryAirCp(T1turbC);
    P1turb = Pstack - DPstack;
    P2turb = pressure;
    T1turb = T1turbC + 273.15;
    fac1 = (Gamma_air-1)/Gamma_air;
    fac2 = (P2turb/P1turb)^fac1 - 1;
    DeltaT = fac2 * T1turb * TurbEta;
    T2turb = T1turb + DeltaT;
    Pturb_kW = Cp_air * Massrate_out_kgs * DeltaT * 1e-3;

    if iprntSCR==1
        disp(' ')
        fprintf('%s \n','Turbine:');
        fprintf('%s %8.4f %s\n','Cp/Cv      = ',Gamma_air,' ');
        fprintf('%s %8.4f %s\n','Cp         = ',Cp_air,'J/kg.K ');
        fprintf('%s %8.4f %s\n','P1         = ',P1turb/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','P2         = ',P2turb/OneAtm,'atm');
        fprintf('%s %8.4f %s\n','T1         = ',T1turbC,'deg C');
        fprintf('%s %8.4f %s\n','T2         = ',T2turb-273.15,'deg C');
        fprintf('%s %8.4f %s\n','Delta T    = ',DeltaT,'deg C');
        fprintf('%s %8.4f %s\n','Flow       = ',Massrate_out_kgs,'kg/s');
        fprintf('%s %8.4f %s\n','Efficiency = ',TurbEta,' ');
        fprintf('%s %8.4f %s\n','Power      = ',Pturb_kW,'kW');
        fprintf('%s %8.4f %s\n','Net C-T    = ',Pcomp_kW + Pturb_kW,'kW');
        disp(' ')
    end


end

Pair_kW = Pblower_kW + Pcomp_kW + Pturb_kW;



%                              Storage/tanks

% any power consumption to store / deliver fuel
dP = FCST.Pfuel0;
k  = FCST.Pfuelk;
e  = FCST.Pfuele;
P  = dP + k * MassH2rate_kgs^e;
Pfuel_kW = P*1e-3;
if iprntSCR==1
    disp(' ')
    fprintf('%s\n','H2 Tank: ');
    fprintf('%s %8.4f %s\n','Power      = ',Pfuel_kW,'kW')
```

```
end



%                              Stack Cooling


if iprntSCR==1
    disp(' ')
    fprintf('%s\n','High temperature cooling system: ');
end
if FCST.iHV==1;
    % use higher heating value (product water is liquid)
    Eforheat = Eh_V;
    if iprntSCR==1;
    fprintf('%s %8.4f %s\n','Eh of HHV (liq water)',Eh_V,'V');
    end
else
    % use lower heating value (product water is vapor)
    Eforheat = Eh_LHV_V;
    if iprntSCR==1;
    fprintf('%s %8.4f %s\n','Eh of LHV (vap water)',Eh_LHV_V,'V');
    end
end
HEATINGstack_kW = (Eforheat-Vcell)*Icell_Acm2*Aactive_cm2*Ncell*1e-3/fleakage;

% heat used to raise inlet air temp to stack temperature
HEATINGairin_kW = 0;
if FCST.iCompressor==0
    % then there is no compressor
    % air must be heated on entry; this heat can be taken from stack
    [Cp_air, Cv_air, Gamma_air] = DATA_DryAirCp(temperatureC);
    DeltaT = TstackK-temperatureK;
    HEATINGairin_kW = Cp_air * MassAirrate_max_kgs * DeltaT * 1e-3;
end
FCOP.HEATINGairin_kW = HEATINGairin_kW;

% heat carried out by air flow
[Cp_air, Cv_air, Gamma_air] = DATA_DryAirCp(temperatureC);
DeltaT = TstackK-temperatureK;
HEATINGairout_kW = Cp_air * MassAirrate_out_kgs * DeltaT * 1e-3;
FCOP.HEATINGairout_kW = HEATINGairout_kW;

% heat dissipated
fd  = FCST.htStackfd;
hs  = FCST.htStackhs;
Ta  = FCST.htStackTa + 273.15;
eps = FCST.htStackeps;
fs  = FCST.htStackfs;
As  = fs * Aactive * Ncell;
HEATINGdiscon_kW = As * hs * (TstackK - Ta) * 1e-3;
HEATINGdisrad_kW = As * eps * SB * (TstackK^4 - Ta^4) * 1e-3;
HEATINGdis_kW    = fd*HEATINGstack_max_kW + ...
```

```
                   HEATINGdiscon_kW + HEATINGdisrad_kW;
FCOP.HEATINGdiscon_kW = HEATINGdiscon_kW;
FCOP.HEATINGdisrad_kW = HEATINGdisrad_kW;
FCOP.HEATINGdis_kW    = HEATINGdis_kW;


% high temperature cooling system
DeltaT  = TstackK-temperatureK;
HEATING = (HEATINGstack_kW - HEATINGairin_kW - ...
           HEATINGairout_kW - HEATINGdis_kW) * 1e3;
Cpcool  = FCST.htCoolantCp;
dTcool  = FCST.htCoolantdT;
Cflow   = HEATING / ( Cpcool*dTcool );
% any power consumption to circulate coolant
dP                   = FCST.Pht0;
k                    = FCST.Phtk;
e                    = FCST.Phte;
f                    = FCST.Phtf;
P                    = dP + k * HEATING^e * Cflow^f;
Pht_kW               = P*1e-3;
if iprntSCR==1
    fprintf('%s %8.4f %s\n','Delta T    = ',DeltaT,'deg C')
    fprintf('%s %8.4f %s\n','Heat out   = ',HEATING*1e-3,'kW');
    fprintf('%s %8.4f %s\n','kW/C       = ',HEATING*1e-3/DeltaT,'kW/deg C')
    fprintf('%s %8.4f %s\n','Coolant q  = ',f*Cflow,'kg/s')
    fprintf('%s %8.4f %s\n','Power      = ',Pht_kW,'kW');
end




%                             Water System

% any power consumption to pump water
dP                   = FCST.Pwater0;
k                    = FCST.Pwaterk;
e                    = FCST.Pwatere;
P                    = dP + k * mrate^e;
Pwater_kW            = P*1e-3;
if iprntSCR==1
    disp(' ')
    fprintf('%s\n','Water : ');
    fprintf('%s %8.4f %s\n','Power      = ',Pwater_kW,'kW');
    disp(' ')
end



%                             Electrical System

% any power consumption
dP                   = FCST.Pelec0;
```

```
k                    = FCST.Peleck;
e                    = FCST.Pelece;
P                    = dP + k * 1e3 * POWER_kW^e;
Pelec_kW             = P*1e-3;
if iprntSCR==1
    disp(' ')
    fprintf('%s\n','Electrical : ');
    fprintf('%s %8.4f %s\n','Power      = ',Pelec_kW,'kW');
    disp(' ')
end


%                           Other overhead


Pother_kW     = POWER_kW * fovpower;
if iprntSCR==1
    disp(' ')
    fprintf('%s\n','Other : ');
    fprintf('%s %8.4f %s\n','Power      = ',Pother_kW,'kW');
    disp(' ')
end


%                              TOTAL

% power kW
% balance of plant at design power
Pbop_kW = Pair_kW + ...
          Pfuel_kW + ...
          Plt_kW + Pht_kW + ...
          Pwater_kW + ...
          Pelec_kW + ...
          Pother_kW;



%                              SUMMARY


POWER_kWe    = POWER_kW - Pbop_kW;
POWERfactor  = POWER_kWe/POWER_kW;
etas         = etac * POWERfactor;  % final system efficiency
SFC_kgkWhe   = MassH2rate_kgs*3600/POWER_kWe;
SFC_kgkWh    = MassH2rate_kgs*3600/POWER_kW;


if iprntSCR==1
   disp(' ')
   fprintf('%s \n','Summary:');
   fprintf('%s %8.4f %s\n','P          = ',POWER_kW,'kW');
   fprintf('%s %8.4f %s\n','SFC        = ',SFC_kgkWh,'kg/kW-hr');
   fprintf('%s %8.4f %s\n','P bop      = ',Pbop_kW,'kW');
   fprintf('%s %8.4f %s\n','      Air          ',Pair_kW,'kW');
   fprintf('%s %8.4f %s\n','      Fuel         ',Pfuel_kW,'kW');
   fprintf('%s %8.4f %s\n','      LT cooling   ',Plt_kW,'kW');
   fprintf('%s %8.4f %s\n','      HT cooling   ',Pht_kW,'kW');
   fprintf('%s %8.4f %s\n','      Water        ',Pwater_kW,'kW');
   fprintf('%s %8.4f %s\n','      Electrical   ',Pelec_kW,'kW');
   fprintf('%s %8.4f %s\n','      Other        ',Pother_kW,'kW');
   fprintf('%s %8.4f %s\n','P net      = ',POWER_kWe,'kWe');
   fprintf('%s %8.4f %s\n','SFC P net  = ',SFC_kgkWhe,'kg/kWe-hr');
   disp(' '); disp(' ')
end


FCOP.SFC_kgkWh                 = SFC_kgkWh;
FCOP.SFC_kgkWhe                = SFC_kgkWhe;
FCOP.POWER_kW                  = POWER_kW;
FCOP.Pair_kW                   = Pair_kW;
FCOP.Pfuel_kW                  = Pfuel_kW;
FCOP.Plt_kW                    = Plt_kW;
FCOP.Pht_kW                    = Pht_kW;
FCOP.Pwater_kW                 = Pwater_kW;
FCOP.Pelec_kW                  = Pelec_kW;
FCOP.Pother_kW                 = Pother_kW;
FCOP.Pblower_kW                = Pblower_kW;
FCOP.Pcomp_kW                  = Pcomp_kW;
FCOP.Pturb_kW                  = Pturb_kW;
FCOP.POWER_kWe                 = POWER_kWe;
FCOP.Pbop_kW                   = Pbop_kW;
FCOP.etac                      = etac;
FCOP.etas                      = etas;
FCOP.Vstack                    = Vstack;
FCOP.MassH2rate_kgs            = MassH2rate_kgs;
FCOP.MassAirrate_kgs           = MassAirrate_kgs;
FCOP.MassAirrate_Ls            = MassAirrate_Ls;
FCOP.MassO2rate_kgs            = MassO2rate_kgs;
FCOP.MassAirrate_out_kgs       = MassAirrate_out_kgs;
FCOP.MassWaterrate_kgs         = MassWaterrate_kgs;
FCOP.MassLiquidrate_kgs        = MassLiquidrate_kgs;
FCOP.MassTankrate_kgs          = MassTankrate_kgs;
FCOP.MassVaporrate_out_kgs     = MassVaporrate_out_kgs;
FCOP.Humidity                  = Humidity_ratio;
FCOP.HEATINGairin_kW           = HEATINGairin_kW;
FCOP.HEATINGstack_kW           = HEATINGstack_kW;
```

Drivers in the run directories.

```
% Driver for createiv

% Use this code to determine parameters
% Er j0C alphaC j0A alphaA ASR jL jLeak C
% and
% temperature and (cathode & anode) humidity corrections to ASR and jL
% to best fit an available i-v test data.

% Example:
% Paramaters were extracted from i-v data pressure 1, 1.25, 1.5, 2, 2.5 atm.
% These are given in the PMAT matrix below.

clear all
rundir = pwd;
srcdir = '../src';


% inputs ----------------------------------------------------------
TstackC                 =  80;
Pstack_atm              =  2.5;
xO2                     =  0.2095;
Er                      =  0.0;

% Parameters extracted from raw data by Yan et al 2006:
% row    1: pressures in atm
% rows 2-9: j0C alphaC j0A alphaA ASR jL jLeak C
% Er can be input separately by inspection of data (Er = v @ i=0), or
% set to zero. If set to zero, createiv will use its theoritical value.

% % 1. Use these to build model
% j0C       = 0.0001;
% alphaC    = 0.1800;
% j0A       = 0.1000;
% alphaA    = 0.5000;
% ASR       = 0.0400;
% jL        = 1.7500;
% jleak     = 0.1500;
% C         = 0.0300;

%2. Set up table look-up with model constants vs pressure
PMAT=[
    1        1.25     1.5      2.0        2.5
    0.0001   0.0001   0.0001   0.0001     0.0001
    0.1800   0.1900   0.2000   0.2100     0.2200
    0.1000   0.1000   0.1000   0.1000     0.1000
    0.5000   0.5000   0.5000   0.5000     0.5000
    0.0400   0.0400   0.0400   0.0400     0.0400
    1.7500   1.8500   1.9500   2.2500     2.4500
    0.1500   0.1500   0.2000   0.2500     0.3000
    0.0300   0.0350   0.0350   0.0350     0.0350
%Er=1.1713   1.1729   1.1743   1.1765     1.1782 <- not needed
    ];
[m,n]=size(PMAT);
```

```
if Pstack_atm > PMAT(1,n)
    disp('ERROR: Pstack > PMAT table pressure; Re-setting Pstack=1 atm')
    Pstack_atm = 1;
end
if Pstack_atm < PMAT(1,1)
    disp('ERROR: Pstack < PMAT table pressure; Re-setting Pstack=1 atm')
    Pstack_atm = 1;
end
j0C       = interp1(PMAT(1,1:n),PMAT(2,1:n),Pstack_atm,'linear');
alphaC    = interp1(PMAT(1,1:n),PMAT(3,1:n),Pstack_atm,'linear');
j0A       = interp1(PMAT(1,1:n),PMAT(4,1:n),Pstack_atm,'linear');
alphaA    = interp1(PMAT(1,1:n),PMAT(5,1:n),Pstack_atm,'linear');
ASR       = interp1(PMAT(1,1:n),PMAT(6,1:n),Pstack_atm,'linear');
jL        = interp1(PMAT(1,1:n),PMAT(7,1:n),Pstack_atm,'linear');
jleak     = interp1(PMAT(1,1:n),PMAT(8,1:n),Pstack_atm,'linear');
C         = interp1(PMAT(1,1:n),PMAT(9,1:n),Pstack_atm,'linear');


% for temperature and humidity corrections in ASR and jL
arh                       =  100;
crh                       =  100;
% temp corrections
ASR0t                     =  80;        % ref temp, no corr at this temp
ASRtk2                    =  0.0006;
ASRtk1                    =  0.002;
jL0t                      =  80;        % ref temp, no corr at this temp
jLtk2                     = -0.0015;
jLtk1                     = -0.003;
% humidity corrections
ASR0arh    =  100;        % ref anode rh, no corr at this arh
ASR0crh    =  100;        % ref cathode rh,  no corr at this crh
ASRrhk1a   = -0.005;
ASRrhk2a   =  0.;
ASRrhk1a1c =  0.;
ASRrhk1a2c =  2.0e-6;
ASRrhk2a2c =  0.;
ASRrhk2a1c =  0.;
ASRrhk2c   =  7.0e-5;
ASRrhk1c   =  0.0021;
jL0arh     =  100;
jL0crh     =  100;
jLrhk1a    =  0.;
jLrhk2a    =  0.;
jLrhk1a1c  =  0.;
jLrhk1a2c  =  0.;
jLrhk2a2c  =  0.;
jLrhk2a1c  =  0.;
jLrhk2c    = -0.0001;
jLrhk1c    = -0.0076;

% i-v data current resolution
deltai                    =  0.02;
% end-inputs -------------------------------------------------------
% temperature correction to ASR
t         =  TstackC - ASR0t;
dASRt     =  ASRtk2*t^2 + ASRtk1*t;

% temperature correction to jL
t        =  TstackC - jL0t;
djLt     =  jLtk2*t^2 + jLtk1*t;

% relative humidity corrections to ASR and jL

a = arh - ASR0arh; c = crh - ASR0crh;
dASRrh    =  ASRrhk1a * a + ...
             ASRrhk2a * a^2 + ...
             ASRrhk1a1c * a * c + ...
             ASRrhk2a1c * a^2 * c + ...
             ASRrhk2a2c * a^2 * c^2 + ...
             ASRrhk1a2c * a * c^2 + ...
             ASRrhk2c * c^2 + ...
             ASRrhk1c * c;


a = arh - jL0arh; c = crh - jL0crh;
djLrh    =   jLrhk1a * a + ...
             jLrhk2a * a^2 + ...
             jLrhk1a1c * a * c + ...
             jLrhk2a1c * a^2 * c + ...
             jLrhk2a2c * a^2 * c^2 + ...
             jLrhk1a2c * a * c^2 + ...
             jLrhk2c * c^2 + ...
             jLrhk1c * c;

ASR =  ASR +  dASRt + dASRrh;
jL  =  jL  +  djLt + djLrh;


cd(srcdir)
[ivdata, Eh, Er, ipmax] = createiv(TstackC, Pstack_atm, xO2, deltai, Er, ...
                          j0C, alphaC, j0A, alphaA, ASR, jL, jleak, C);

cd(rundir);

ic = ivdata(:,1);
vc = ivdata(:,2);
pc = ivdata(:,3);
fl = ic./(ic + jleak);
et = vc.*fl/Eh;
etv = vc.*fl/Er;
```

```
figure
hold on; grid on; set(gca,'FontSize',16);
plot(ic,vc,'k-','LineWidth',2);
xlabel('Current density, A/cm^2');
plot(ic,pc,'k--','LineWidth',2); hold on;
ylabel('Cell voltage, V; Power density, W/cm^2');
legend('Cell voltage, V','Cell power density, W/cm^2')

figure
hold on; grid on; set(gca,'FontSize',16);
plot(ic,et,'k-','LineWidth',3);
plot(ic,etv,'k--','LineWidth',3);
xlabel('Current density, A/cm^2');
ylabel('Cell efficiency');
legend('True','Voltage')


disp(' ')
disp('i - current density A/cm^2')
disp('v - voltage Volt')
disp('p - power density Watt/cm^2')
disp(' ')
disp('     i          v         p')
disp(' ')
disp(ivdata);
disp(' ');
fprintf('%s %i\n','ipmax = ', ipmax);
disp(' ')
```

```
% Driver for pemfc
% Size stack for an input gross power kW or vc or stack efficiency.

clear all

rundir = pwd;
srcdir = '../src';


fcvariables;
cd(srcdir)
pemfc;
cd(rundir);
```

```
% Driver for Design.
% Size stack for an input net power kWe or vc or stack efficiency.


clear all

rundir = pwd;
srcdir = '../src';


fcvariables;
cd(srcdir)
pemfcdesign;
cd(rundir);
```

```
% Driver for Analysis.
% For a fixed design, find operating space with altitude.

clear all

rundir = pwd;
srcdir = '../src';




fcvariables;
cd(srcdir)

% turn off plots
FCST.iplotVIP=0;
FCST.iprntSCR=1;


% design
pemfcdesign;



% Versus altitude

% First, Set operating current to design, max power, or max current
% (FCST.Istackdsg, FCST.IstackPmax, or FCST.Istackmax)
FCOP.Istack = FCST.Istackdsg;
% Now, vary altitude
Xrange = [0:500:14000]*0.3048;
FCOP.Istack = FCST.Istackdsg;
for iloop = 1:length(Xrange)
    FCOP.altitude        = Xrange(iloop);
    pemfcstatic;
    H1000ft(iloop)       = FCOP.altitude/304.8;
    pout(iloop)          = FCOP.POWER_kW;
    poute(iloop)         = FCOP.POWER_kWe;
    sfcout(iloop)        = FCOP.SFC_kgkWhe;
    H2SE(iloop)          = 1/FCOP.SFC_kgkWhe;
    H2in(iloop)          = FCOP.MassH2rate_kgs;
end
cd(rundir);
FCOP.Pevs1000ft          = [H1000ft; pout; poute; sfcout; H2in*3600];


% power vs altitude
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(H1000ft, pout,'k-','LineWidth',3);
xlabel('Altitude, x 1000 ft'); ylabel('Gross Power,kW');
figure;
hold on; grid on; set(gca,'FontSize',16);
```

```
plot(H1000ft, poute,'k-','LineWidth',3);
xlabel('Altitude, x 1000 ft'); ylabel('Net Power,kWe');

% SFC vs altitude
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(H1000ft, sfcout,'k-','LineWidth',3);
xlabel('Altitude, x 1000 ft'); ylabel('SFC,kg/kWhe');

% fuel flow vs altitude
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(H1000ft, H2in*3600,'k-','LineWidth',3);
xlabel('Altitude, x 1000 ft'); ylabel('H2 flow,kg/hr');
```

```
% Driver for Analysis.
% For a fixed design, find operating space.

clear all

rundir = pwd;
srcdir = '../src';

fcvariables;
cd(srcdir)

% turn off plots
FCST.iplotVIP=0;
FCST.iprntSCR=1;


% design
FCST.POWERdsg_kW = 500;
pemfcdesign;


% Versus Current (i.e. fuel flow)
FCOP.iatm         = 1;
FCOP.altitude     = 5000*0.3048;
FCOP.temperatureC = 20;
% use a fraction of FCST.IstackPmax at higher altitude
Xrange = [100:100:1.0*FCST.IstackPmax];

for iloop = 1:length(Xrange)
    FCOP.Istack          = Xrange(iloop);
    pemfcstatic;
    vout(iloop)          = FCOP.Vstack;
    pout(iloop)          = FCOP.POWER_kW;
    sfcout(iloop)        = FCOP.SFC_kgkWhe;
    H2SE(iloop)          = 1/FCOP.SFC_kgkWhe;
    poute(iloop)         = FCOP.POWER_kWe;
    H2in(iloop)          = FCOP.MassH2rate_kgs;
    Airin(iloop)         = FCOP.MassAirrate_kgs;
    Waterliqout(iloop)   = FCOP.MassWaterrate_kgs;
    Watervapout(iloop)   = FCOP.MassVaporrate_out_kgs;
    Dryairout(iloop)     = FCOP.MassAirrate_out_kgs - ...
                           FCOP.MassVaporrate_out_kgs;
end
cd(rundir);
FCOP.SFCvsPe = [poute; sfcout];                        % kg/kWe-hr vs kWe
FCOP.SFCvsPefrac = [poute/FCST.POWERdsg_kWe; sfcout]; % kg/kWe-hr vs frac
FCOP.H2flowvsPe = [poute; H2in*3600];                 % kg/hr vs kWe


% fuel flow vs net power
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(poute,H2in*3600,'k-','LineWidth',3);
```

```
        plot(poute,ones(1,length(poute))*FCST.MassH2rate_kgs*3600,...
            'k-','LineWidth',1);
xlabel('Net Power, kWe'); ylabel('H2 flow, kg/hr');

% SFC vs net power and netpower/design power
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(poute,sfcout,'k-','LineWidth',3);
        plot(poute,ones(1,length(poute))*FCST.SFCPdsg_kgkWhe,...
            'k-','LineWidth',1);
xlabel('Net Power, kWe'); ylabel('SFC, kg/kWe-hr');
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(poute/FCST.POWERdsg_kWe,sfcout,'k-','LineWidth',3);
        plot(poute/FCST.POWERdsg_kWe,ones(1,length(poute))*FCST.SFCPdsg_kgkWhe,...
            'k-','LineWidth',1);
xlabel('Fraction of Net Design Power'); ylabel('SFC, kg/kWe-hr');

figure;
hold on; grid on; set(gca,'FontSize',16);
plot(poute,H2SE,'k-','LineWidth',3);
        plot(poute,ones(1,length(poute))*(1/FCST.SFCPdsg_kgkWhe),...
            'k-','LineWidth',1);
xlabel('Net Power, kWe'); ylabel('Available Sp Energy, kWh/kg');
title('Available Specific Energy (1/SFC) vs Power');


% Voltage and current vs net power
figure
hold on; grid on; set(gca,'FontSize',16);
plot(poute,vout,'k-','LineWidth',3);
plot(poute,pout,'r--','LineWidth',3);
        plot(poute,ones(1,length(poute))*FCST.Vstackdsg,...
            'k-','LineWidth',1);
        plot(poute,ones(1,length(poute))*FCST.POWERdsg_kWe,...
            'r-','LineWidth',1);
legend('Voltage','Current');
xlabel('Net Power, kWe'); ylabel('Stack Voltage, V; Stack Current, A');
title('Voltage and Current vs Power');

% Voltage and power vs current
figure
hold on; grid on; set(gca,'FontSize',16);
plot(Xrange,vout,'k-','LineWidth',3);
plot(Xrange,pout,'r--','LineWidth',3);
plot(Xrange,poute,'r.-','LineWidth',3);
        plot(Xrange,ones(1,length(poute))*FCST.Vstackdsg,...
            'k-','LineWidth',1);
        plot(Xrange,ones(1,length(poute))*FCST.POWERdsg_kWe,...
            'r-','LineWidth',1);
legend('Voltage','Power','Power net');
xlabel('Stack current, A'); ylabel('Stack Voltage, V; Stack Power, kW');
title('Voltage and Power vs Current');
```

```
% Driver for Design.
% Design for fixed gross power kW (net power kWe + balance of plant).
% Vary power.


clear all

rundir = pwd;
srcdir = '../src';


fcvariables;
FCST.iplotVIP = 0;
vecPOWERdsg_kW = [50:50:600];

for iloopvecPOWERdsg_kW = 1:length(vecPOWERdsg_kW);

    FCST.POWERdsg_kW = vecPOWERdsg_kW( iloopvecPOWERdsg_kW );
    cd(srcdir)
    pemfc;
    cd(rundir);

    Pout( iloopvecPOWERdsg_kW )         = FCST.POWERdsg_kW;
    Poute( iloopvecPOWERdsg_kW )        = FCST.POWERdsg_kWe;
    Poutmax( iloopvecPOWERdsg_kW )      = FCST.POWERmax_kW;
    Poutmaxe( iloopvecPOWERdsg_kW )     = FCST.POWERmax_kWe;
    Wout( iloopvecPOWERdsg_kW )         = FCST.Wtotal;
    Wstacksysout( iloopvecPOWERdsg_kW ) = FCST.Wstacksys;
    Wstackout( iloopvecPOWERdsg_kW )    = FCST.Wstack.total;
    Wairout( iloopvecPOWERdsg_kW )      = FCST.Wair.total;
    Wfuelout( iloopvecPOWERdsg_kW )     = FCST.Wfuel.H2;
    Wtankout( iloopvecPOWERdsg_kW )     = FCST.Wfuel.tank;
    Wltcout( iloopvecPOWERdsg_kW )      = FCST.Wlt.total;
    Whtcout( iloopvecPOWERdsg_kW )      = FCST.Wht.total;
    Wwaterout( iloopvecPOWERdsg_kW )    = FCST.Wwater.total;
    Weleccout( iloopvecPOWERdsg_kW )    = FCST.Welec.total;
    Wotherout( iloopvecPOWERdsg_kW )    = FCST.Wother;
    Sout( iloopvecPOWERdsg_kW )         = FCST.Stotal;
    Eout( iloopvecPOWERdsg_kW )         = FCST.EtadsgHHV;
end
FCST.StackWvsP   = [Pout;   Wstackout];
FCST.StackSystemWvsPe  = [Poute;  Poutmaxe; Wstacksysout];



% Stack weight vs gross design power
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(Wstackout,Pout,'k-','LineWidth',3);
plot(Wstackout,Poutmax,'k--','LineWidth',3);
legend('Design','Maximum');
xlabel('Stack weight, kg'); ylabel('Gross Power, kW');
% Power-to-weight
```

```
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(Wstackout,Pout./Wstackout,'k-','LineWidth',3);
xlabel('Stack weight, kg'); ylabel('Power/Weight, kW/kg');



% Stack system weight vs net design power
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(Wstacksysout,Poute,'k-','LineWidth',3);
plot(Wstacksysout,Poutmaxe,'k--','LineWidth',3);
legend('Design','Maximum');
xlabel('Stack system weight, kg'); ylabel('Net Power, kWe');
% Power-to-weight
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(Wstacksysout,Poute./Wstacksysout,'k-','LineWidth',3);
xlabel('Stack system Weight, kg'); ylabel('Power/Weight, kWe/kg');



% System weight break-down by components
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(Wout,Pout,'k-','LineWidth',3);
plot(Wstackout,Pout,'k--','LineWidth',3);
plot(Wairout,Pout,'r--','LineWidth',2);
plot(Wltcout,Pout,'r--','LineWidth',2);
plot(Whtcout,Pout,'r-','LineWidth',2);
plot(Weleccout,Pout,'k-','LineWidth',1);
plot(Wwaterout,Pout,'k--','LineWidth',1);
ylabel('Gross Power, kW'); xlabel('Weight, kg');
legend('Total','Stack','Air','ltc','htc','elec','water');
```

```
% Ragone plot

clear all

rundir = pwd;
srcdir = '../src';


fcvariables;
cd(srcdir)

FCST.iplotVIP = 0;      % no performance plots
FCST.iprntSCR = 0;      % no screen outputs
FCST.ioptf    = 1;      % fuel option 1


Fuelkg = 1:0.25:20;
for iloop=1:length(Fuelkg)
    FCST.Whydrogen = Fuelkg(iloop);
    pemfc;

    % net @ design power
    out10(iloop)= FCST.SEnergy_Pdsg_kWhekg;
    out20(iloop)= FCST.SPower_Pdsg_kWekg;
    out30(iloop)= FCST.EDensity_Pdsg_kWheL;
    % net @ max power
    out100(iloop)= FCST.SEnergy_Pmax_kWhekg;
    out200(iloop)= FCST.SPower_Pmax_kWekg;
    out300(iloop)= FCST.EDensity_Pmax_kWheL;
end
cd(rundir);


% net @ design power
figure
color = 'b-';
x = out30*1e3; y = out20*1e3;
loglog(x,y,color,'LineWidth',3); hold on; grid on; set(gca,'FontSize',16);
xlabel('Energy density, Whe/L'); ylabel('Specific power, We/kg');
title('@ Design Power')
axis([50 1000 50 5000])

figure
x = out10*1e3; y = out20*1e3;
loglog(x,y,color,'LineWidth',3); hold on; grid on; set(gca,'FontSize',16);
xlabel('Specific energy, Whe/kg'); ylabel('Specific power, We/kg');
title('@ Design Power')
axis([50 1000 50 5000])
clear x;
x=[1 1000];
y=x./(4.0); loglog(x,y,'k--','LineWidth',2);      % 4 hr line
y=x./(2.0); loglog(x,y,'k--','LineWidth',2);      % 2 hr line
y=x./(1.0); loglog(x,y,'k--','LineWidth',2);      % 1 hr line
```

```
y=x./(0.5); loglog(x,y,'k--','LineWidth',2);      % 30 min
y=x./(0.25); loglog(x,y,'k--','LineWidth',2);     % 15 min
y=x./(0.125); loglog(x,y,'k--','LineWidth',2);    % 7.5 min
h=text(700,200,' 4 hours'); set(h,'Rotation',30,'FontSize',14);
h=text(700,400,' 2 hours'); set(h,'Rotation',30,'FontSize',14);
h=text(700,800,' 1 hour'); set(h,'Rotation',30,'FontSize',14);
h=text(700,1600,'30 mins'); set(h,'Rotation',30,'FontSize',14);
h=text(600,2600,'15 mins'); set(h,'Rotation',30,'FontSize',14);
h=text(300,2600,'7.5 mins'); set(h,'Rotation',30,'FontSize',14);


% net @ max power
color = 'r-';
figure
x = out300*1e3; y = out200*1e3;
loglog(x,y,color,'LineWidth',3); hold on; grid on; set(gca,'FontSize',16);
xlabel('Energy density, Whe/L'); ylabel('Specific power, We/kg');
title('@ Max Power')
axis([50 1000 50 2000])

figure
x = out100*1e3; y = out200*1e3;
loglog(x,y,color,'LineWidth',3); hold on; grid on; set(gca,'FontSize',16);
xlabel('Specific energy, Whe/kg'); ylabel('Specific power, We/kg');
title('@ Max Power')
axis([50 1000 50 5000])
clear x;
x=[1 1000];
y=x./(4.0); loglog(x,y,'k--','LineWidth',2);      % 4 hr line
y=x./(2.0); loglog(x,y,'k--','LineWidth',2);      % 2 hr line
y=x./(1.0); loglog(x,y,'k--','LineWidth',2);      % 1 hr line
y=x./(0.5); loglog(x,y,'k--','LineWidth',2);      % 30 min
y=x./(0.25); loglog(x,y,'k--','LineWidth',2);     % 15 min
y=x./(0.125); loglog(x,y,'k--','LineWidth',2);    % 7.5 min
h=text(700,200,' 4 hours'); set(h,'Rotation',30,'FontSize',14);
h=text(700,400,' 2 hours'); set(h,'Rotation',30,'FontSize',14);
h=text(700,800,' 1 hour'); set(h,'Rotation',30,'FontSize',14);
h=text(700,1600,'30 mins'); set(h,'Rotation',30,'FontSize',14);
h=text(600,2600,'15 mins'); set(h,'Rotation',30,'FontSize',14);
h=text(300,2600,'7.5 mins'); set(h,'Rotation',30,'FontSize',14);
```

```
% Driver for Design.
% Design for fixed net power kWe (gross power kW - balance of plant).
% Vary altitude.


clear all

rundir = pwd;
srcdir = '../src';


fcvariables;
FCST.iplotVIP = 0;
kWe = 80;
vecaltitudeDesign = [0:1000:14000]*0.3048;  % m

for iloopvec = 1:length(vecaltitudeDesign);

    FCST.altitudeDesign = vecaltitudeDesign( iloopvec );
    FCST.POWERdsg_kW    = kWe;

    cd(srcdir)
    pemfcdesign; close all;
    cd(rundir);

    H1000ft( iloopvec )    = FCST.altitudeDesign/304.8;
    Pout( iloopvec )       = FCST.POWERdsg_kW;
    Poute( iloopvec )      = FCST.POWERdsg_kWe;
    Wout( iloopvec )       = FCST.Wtotal;
    Wstackout( iloopvec )  = FCST.Wstack.total;
    Wairout( iloopvec )    = FCST.Wair.total;
    Wfuelout( iloopvec )   = FCST.Wfuel.H2;
    Wtankout( iloopvec )   = FCST.Wfuel.tank;
    Wltcout( iloopvec )    = FCST.Wlt.total;
    Whtcout( iloopvec )    = FCST.Wht.total;
    Wwaterout( iloopvec )  = FCST.Wwater.total;
    Weleccout( iloopvec )  = FCST.Welec.total;
    Wotherout( iloopvec )  = FCST.Wother;
    Sout( iloopvec )       = FCST.Stotal;
    Eout( iloopvec )       = FCST.EtadsgHHV;

end
W = Wout - Wfuelout - Wtankout;
FCST.StackSystemWvsH   = [vecaltitudeDesign;   W];


% altitude vs Stack weight
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(Wstackout,H1000ft,'k-','LineWidth',3);
ylabel('Altitude, x 1000 ft'); xlabel('Stack weight, kg');
```

```
% altitude vs Stack system weight
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(W,H1000ft,'k-','LineWidth',3);
ylabel('Altitude, x 1000 ft'); xlabel('Stack system weight, kg');


% weight break-down by components
figure;
hold on; grid on; set(gca,'FontSize',16);
plot(W,H1000ft,'k-','LineWidth',3);
plot(Wstackout,H1000ft,'k--','LineWidth',3);
plot(Wairout,H1000ft,'b--','LineWidth',2);
plot(Wltcout,H1000ft,'r--','LineWidth',2);
plot(Whtcout,H1000ft,'r-','LineWidth',2);
plot(Weleccout,H1000ft,'k-','LineWidth',1);
plot(Wwaterout,H1000ft,'k--','LineWidth',1);
ylabel('Altitude, x 1000 ft'); xlabel('Weight, kg');
legend('Total','Stack','Air','ltc','htc','elec','water');
```

The input decks *fcvariables.m*.

Only the inputs for *run Baseline 80kW*, *run Modern 80 kW*, and *run Modern 80 kW liqcool* are listed. The others need small changes that are described in the report.

```
% run Baseline 80kW
% SI units unless otherwise appended


% ======================================================
% I N P U T S - Design
% ======================================================


% design options for power
FCST.ioptp                      = 1;     % option, design power, kW
FCST.POWERdsg_kW                = 80;    % design stack power, kW
FCST.etac                       = 0.37;  % cell efficiency
FCST.vc                         = 0.60;  % cell voltage, V
FCST.Voltage                    = 250;


% atmospheric conditions affecting design
FCST.iatm                       = 0;             % environment option
FCST.OneAtm                     = 101325;        % Standard atm, Pa
FCST.temperatureC               = 15;            % temp of environ, C
FCST.altitudeDesign             = 0000*0.3048;   % altitude, m
FCST.pressure                   = 0.0*101325;    % pressure, Pa


% cell i-v variables
FCST.TstackC                    = 80;          % stack temp, C
FCST.Pstack                     = 2.5*101325;  % stack pres, Pa
FCST.xO2                        = 0.2095;      % mole fraction of O2 in air
FCST.ioptiv                     = 3;           % 1-data, 2-make, 3-PMAT
FCST.ivdatainp = [
         0    1.1782         0
    0.0100    0.9645    0.0096
    0.0200    0.9304    0.0186
    0.0300    0.9057    0.0272
    0.0400    0.8861    0.0354
    0.0500    0.8697    0.0435
    0.0600    0.8556    0.0513
    0.0700    0.8430    0.0590
    0.0800    0.8318    0.0665
    0.0900    0.8215    0.0739
    0.1000    0.8120    0.0812
    0.1100    0.8031    0.0883
    0.1200    0.7948    0.0954
    0.1300    0.7870    0.1023
    0.1400    0.7795    0.1091
    0.1500    0.7724    0.1159
    0.1600    0.7656    0.1225
    0.1700    0.7591    0.1290
    0.1800    0.7528    0.1355
    0.1900    0.7467    0.1419
    0.2000    0.7408    0.1482
    0.2100    0.7351    0.1544
```

0.2200	0.7295	0.1605
0.2300	0.7241	0.1665
0.2400	0.7187	0.1725
0.2500	0.7135	0.1784
0.2600	0.7084	0.1842
0.2700	0.7035	0.1899
0.2800	0.6985	0.1956
0.2900	0.6937	0.2012
0.3000	0.6890	0.2067
0.3100	0.6843	0.2121
0.3200	0.6796	0.2175
0.3300	0.6751	0.2228
0.3400	0.6706	0.2280
0.3500	0.6661	0.2331
0.3600	0.6617	0.2382
0.3700	0.6573	0.2432
0.3800	0.6529	0.2481
0.3900	0.6486	0.2530
0.4000	0.6444	0.2577
0.4100	0.6401	0.2624
0.4200	0.6359	0.2671
0.4300	0.6317	0.2716
0.4400	0.6275	0.2761
0.4500	0.6233	0.2805
0.4600	0.6192	0.2848
0.4700	0.6150	0.2891
0.4800	0.6109	0.2932
0.4900	0.6068	0.2973
0.5000	0.6027	0.3013
0.5100	0.5985	0.3053
0.5200	0.5944	0.3091
0.5300	0.5903	0.3129
0.5400	0.5862	0.3165
0.5500	0.5821	0.3201
0.5600	0.5779	0.3236
0.5700	0.5738	0.3271
0.5800	0.5696	0.3304
0.5900	0.5655	0.3336
0.6000	0.5613	0.3368
0.6100	0.5571	0.3398
0.6200	0.5529	0.3428
0.6300	0.5486	0.3456
0.6400	0.5443	0.3484
0.6500	0.5400	0.3510
0.6600	0.5357	0.3535
0.6700	0.5313	0.3559
0.6800	0.5268	0.3582
0.6900	0.5224	0.3604
0.7000	0.5178	0.3625
0.7100	0.5133	0.3644
0.7200	0.5086	0.3662
0.7300	0.5039	0.3679
0.7400	0.4992	0.3694

```
    0.7500    0.4943    0.3707
    0.7600    0.4894    0.3720
    0.7700    0.4844    0.3730
    0.7800    0.4793    0.3738
    0.7900    0.4741    0.3745
    0.8000    0.4687    0.3750
    0.8100    0.4633    0.3753
    0.8200    0.4577    0.3753
    0.8300    0.4520    0.3751
    0.8400    0.4460    0.3747
    0.8500    0.4399    0.3739
    0.8600    0.4336    0.3729
    0.8700    0.4270    0.3715
    0.8800    0.4202    0.3698
    0.8900    0.4131    0.3677
    0.9000    0.4056    0.3651
    0.9100    0.3978    0.3620
    0.9200    0.3894    0.3583
    0.9300    0.3805    0.3539
    0.9400    0.3710    0.3487
    0.9500    0.3606    0.3426
    0.9600    0.3493    0.3354
    0.9700    0.3368    0.3267
    0.9800    0.3225    0.3161
    0.9900    0.3061    0.3030
    1.0000    0.2864    0.2864
    1.0100    0.2615    0.2641
    1.0200    0.2272    0.2317
    1.0300    0.1699    0.1750
    ];
FCST.ipmaxinp                  =  83;
FCST.Er                        =  1.1782;
FCST.j0C                       =  0.0001;
FCST.alphaC                    =  0.155;
FCST.j0A                       =  0.1;
FCST.alphaA                    =  0.5;
FCST.ASR                       =  0.1075;
FCST.jL                        =  1.05;
FCST.jleak                     =  0.01;
FCST.C                         =  0.08;
FCST.PMAT = [
    1        2        3        4           % Presssure, atm
    0.0001   0.0001   0.0001   0.0001      % j0C
    0.1500   0.1550   0.1550   0.1600      % alphaC
    0.1000   0.1000   0.1000   0.1000      % j0A
    0.5000   0.5000   0.5000   0.5000      % alphaA
    0.1200   0.1150   0.1000   0.0600      % ASR, Ohm cm2
    0.7300   1.0000   1.1000   1.2300      % jL, A/cm2
    0.0100   0.0100   0.0100   0.0100      % jLeak, A/cm2
    0.0800   0.0800   0.0800   0.1200      % C, Volt
    ];
FCST.crh                       =  100;
FCST.arh                       =  100;
FCST.deltai                   =  0.02;
% temp corr
FCST.ASR0                     =  0.12;      % ref value
FCST.ASR0t                    =  80;        % ref temp
FCST.ASRtk2                   =  0.0006;
FCST.ASRtk1                   =  0.002;
FCST.jL0                      =  0.73;      % ref value
FCST.jL0t                     =  80;        % ref temp
FCST.jLtk2                    = -0.0015;
FCST.jLtk1                    = -0.003;
% humidity corrections in ASR and jL
FCST.ASR0arh                  =  100;        % ref anode rh
FCST.ASR0crh                  =  100;        % ref cathode rh
FCST.ASRrhk1a                 = -0.005;
FCST.ASRrhk2a                 =  0.;
FCST.ASRrhk1a1c               =  0.;
FCST.ASRrhk1a2c               =  2.0e-6;
FCST.ASRrhk2a2c               =  0.;
FCST.ASRrhk2a1c               =  0.;
FCST.ASRrhk2c                 =  7.0e-5;
FCST.ASRrhk1c                 =  0.0021;
FCST.jL0arh                   =  100;
FCST.jL0crh                   =  100;
FCST.jLrhk1a                  =  0.;
FCST.jLrhk2a                  =  0.;
FCST.jLrhk1a1c                =  0.;
FCST.jLrhk1a2c                =  0.;
FCST.jLrhk2a2c                =  0.;
FCST.jLrhk2a1c                =  0.;
FCST.jLrhk2c                  = -0.0001;
FCST.jLrhk1c                  = -0.0076;
% other corrections
FCST.icorr                    =  0;         % ic corr (0/1)
FCST.TFi                      =  1.0;       % changes ic to ic.Tfi
FCST.vcorr                    =  0;         % vc corr for alt (0/1)
FCST.deltav                   = -0.01;      % cell to stack correction
FCST.vrate1000ft              = -0.0056;    % percent drop / 1000 ft
FCST.vdropaboveft             =  1000;      % above 1000-ft


% stack weight variables
FCST.TFwstack                 = 1.0;       % Tech factor stack weight
FCST.dWwstack                 = 0.0;       % correction stack weight
FCST.TFsstack                 = 1.0;       % Tech factor stack volume
FCST.dSsstack                 = 0.0;       % correction stack volume
FCST.ioptsw                   = 1;         % 0: simple, 1: detailed
FCST.tcell                    = 0.001381;  % cell thickness, m
FCST.dcell                    = 2584;      % cell density, kg/m3
FCST.porosity                 = 1.0;
% Component breakdown
FCST.dendplate                = 1600;    % end plate, Al
FCST.dinsulplate              = 1420;    % polyoxy-methylene
FCST.dcollplate               = 8940;    % collector plate, Cu
FCST.dpem                      = 500;     % membrane, Nafion
FCST.dgdl                      = 440;     % gas diffusion layer
FCST.dbipolarplate             = 2160;    % graphite
FCST.dgasket                   = 1000;    % rubber
FCST.Aendplatefrac             = 1.1;     % Area = 1.1 Active area
FCST.tendplatecons             = 0.0;     % thickness constant
FCST.tendplatefrac             = 0.020;   % thickness = 0.05 tunit
FCST.Ainsulplatefrac           = 1.1;     % fraction active area
FCST.tinsulplatecons           = 0.0;     % thickness constrant
FCST.tinsulplatefrac           = 0.005;   % thickness = 0.02 tunit
FCST.Acollplatefrac            = 0.5;     % fraction active area
FCST.tcollplatecons            = 0.0;     % thickness constant
FCST.tcollplatefrac            = 0.005;   % thickness = 0.01 tunit
FCST.Wfastener                 = 0.0;     % bolt/clamp, steel, 7850 kg/m3
FCST.tpem                      = 0.00010; % membrane, Nafion
FCST.tgdl                      = 0.00020; % gas diffusion layer
FCST.tbipolarplate             = 0.00100; % graphite; avg seal + mea region
FCST.pbipolarplate             = 0.60000; % porosity due to channels
FCST.Abipolarplatefrac         = 1.10;    % fraction active area
FCST.tgasket                   = 0.00200; % rubber
FCST.Agasketfrac               = 0.03;    % fraction active area
FCST.Wstackother               = 0.00;    % any other weight




% H2 tank system
FCST.fH2utilization            = 1.0;
FCST.itanktype                 = 3;
FCST.ioptf                     = 1;     % fuel option; 1-h2, 2-endurance
FCST.Whydrogen                 = 5;
FCST.Endurance_min             = 60;
% 350 bar: 0.0515, 17.2; 700 bar: 0.057, 40;
% cryo-compressed Al: 0.09, 41; cryo-compressed steel: 0.055, 41
% liq: 0.075, 65;
FCST.h2tankwfg                 = 0.057; % kg/kg wt fraction gravimetric
FCST.h2tankwfs                 = 40.00; % kg/m3 wt fraction volumetric
FCST.h2tankBar                 = 700;   % tank storage pressure
FCST.h2tankTC                  = 15;    % tank storage temperature C
FCST.h2tankSov                 = 0.10;  % tank volume overhead
FCST.FuelFilterW0              = 0.;
FCST.FuelFilterfW              = 0.;
FCST.FuelRegulatorW0           = 0;     % kg
FCST.FuelRegulatorfW           = 0.01;  % fraction fuel sys wt
FCST.FuelPowerW0               = 0;     % kg wt of power supply
FCST.FuelPowerfW               = 0.01;  % fraction fuel sys wt
FCST.FuelPlumbW0               = 0;     % kg plumbing constant wt
FCST.FuelPlumbWw               = 0;     % kg/m plumb wt per length
FCST.FuelPlumbWl               = 0;     % m plumb length
FCST.Pfuel0                    = 0;     % prescribed power
FCST.Pfuelk                    = 0;     % prop to fuel flow rate
FCST.Pfuele                    = 0;     % expo to fuel flow rate
FCST.FGErho                    = 722.13;% density of gasoline
FCST.FGEMJkg                   = 44;    % heating value of gasoline, MJ/kg
FCST.FGEeta                    = 0.25;  % efficiency of FGE fuel




% Air system variables
FCST.fAirFlowRate              = 2.5;
FCST.DPstack                   = 30000;     % pres drop in stack, Pa
% Blower
FCST.iBlower                   = 0;
FCST.BlowEta                   = 0.75;
FCST.BlowW0                    = 0;
FCST.BlowWtf                   = 1.0;
FCST.BlowWk                    = 1/1000;
FCST.BlowWfe                   = 0;
FCST.BlowWpe                   = 1;
FCST.BlowS0                    = 0;
FCST.BlowStf                   = 1.0;
FCST.BlowSk                    = 1/3e6;
FCST.BlowSfe                   = 0;
FCST.BlowSpe                   = 1;
% Compressor
FCST.iCompressor               = 1;
FCST.CompEta                   = 0.75;
FCST.CompW0                    = 0.;
FCST.CompWtf                   = 1.0;
FCST.CompWk                    = 7e-4;   % 122 kg/kg/s or 7e-4 kg/W
FCST.CompWfe                   = 0;
FCST.CompWpe                   = 1;
FCST.CompS0                    = 0.;
FCST.CompStf                   = 1.0;
FCST.CompSk                    = 4.14e-7; % 0.0722 m3/kg/s or 4.14e-7 m3/W
FCST.CompSfe                   = 0;
FCST.CompSpe                   = 1;
% Turbo-expander (axial-flow turbine)
FCST.iTurbine                  = 1;
FCST.TurbEta                   = 0.75;
FCST.TurbW0                    = 0.;
FCST.TurbWtf                   = 1.0;
FCST.TurbWk                    = 0;
FCST.TurbWfe                   = 0;
FCST.TurbWpe                   = 0;
FCST.TurbS0                    = 0.;
FCST.TurbStf                   = 1.0;
FCST.TurbSk                    = 0;
FCST.TurbSfe                   = 0;
FCST.TurbSpe                   = 0;
% accessories
FCST.AirFilterW0               = 0;     % kg
FCST.AirFilterfW               = 0.01;  % fraction Air sys wt
FCST.AirRegulatorW0            = 0;     % kg
FCST.AirRegulatorfW            = 0.01;  % fraction Air sys wt
FCST.AirPowerW0                = 0;     % kg controller
FCST.AirPowerfW                = 0.05;  % fraction Air sys wt
FCST.AirPlumbW0                = 0;     % kg plumbing constant wt
FCST.AirPlumbWw                = 0;     % kg/m plumb wt per length
FCST.AirPlumbWl                = 0;     % m plumb length
FCST.AirHumidifierW0           = 0;     % kg
FCST.AirHumidifierfW           = 0.05;  % fraction Air sys wt



% Heat system variables
% low temp system (air pre-cooler, only if compressor used)
FCST.iLTR                     = 0;     % 0-Pdsg load, 1-Pmax load
FCST.ltRadiatorhr             = 1015;  % rad conv ht trans coeff W/m2/K
FCST.ltRadiatorTa             = 20;    % rad Ta in C
FCST.ltRadiatorTr             = 95;    % rad Tr in C
FCST.ltRadiatorepr            = 0.8;   % rad emissivity
FCST.ltRadiatorW0             = 0;     % kg
FCST.ltRadiatorWk             = 1e-4;  % kg/W (f=0) 1e-4 or kg/m2 (e=0) 3.5405
FCST.ltRadiatorWe             = 1;
FCST.ltRadiatorWf             = 0;
FCST.ltRadiatorS0             = 0;     % m3
FCST.ltRadiatorSk             = 1e-7;  % m3/W (f=0) 1e-7 or thick (e=0) 0.035
FCST.ltRadiatorSe             = 1;
FCST.ltRadiatorSf             = 0;
FCST.ltCoolantCp              = 4180;  % J/kg-K (water)
FCST.ltCoolantdT              = 10;    % coolant deltaT (max 15)
FCST.ltCoolantW0              = 0;     % kg
FCST.ltCoolantWk              = 1e-6;  % kg/W (f=0) 1e-6 or time sec (e=0) 30
FCST.ltCoolantWe              = 1;
FCST.ltCoolantWf              = 0;
FCST.ltCoolerW0               = 0;     % kg
FCST.ltCoolerWk               = 1e-6;  % kg/W
FCST.ltCoolerWe               = 1;     %
FCST.ltFilterW0               = 0;     % kg
FCST.ltFilterfW               = 0.01;  % fraction Heat sys wt
FCST.ltRegulatorW0            = 0;     % kg
FCST.ltRegulatorfW            = 0.01;  % fraction Heat sys wt
FCST.ltPowerW0                = 0;     % kg
FCST.ltPowerfW                = 0.01;  % fraction Heat sys wt
FCST.ltPlumbW0                = 0;     % kg plumbing constant wt
FCST.ltPlumbWw                = 0;     % kg/m plumb wt per length
FCST.ltPlumbWl                = 0;     % m plumb length
FCST.Plt0                     = 0;     % any power needed
FCST.Pltk                     = 0.01;  % 0.01-heat, kPa-flow (water)
FCST.Plte                     = 1;     % expo to heating
FCST.Pltf                     = 0;     % expo to flow


% Heat system variables
% high temp system (stack cooling)
FCST.iHV                      = 0;     % 0-LHV, 1-HHV
FCST.iHTR                      = 0;     % 0-Pdsg load, 1-Pmax load
FCST.htStackfd                 = 0;     % stack frac stack heat dissipated
FCST.htStackhs                 = 3;     % stack conv ht trans coeff W/m2/K
FCST.htStackTa                 = 40;    % stack Ta in C
FCST.htStackeps                = 0.8;   % stack emissivity
FCST.htStackfs                 = 0.6;   % stack frac surface/active area
FCST.htRadiatorhr              = 1015;  % rad conv ht trans coeff W/m2/K
FCST.htRadiatorTa              = 20;    % rad Ta in C
FCST.htRadiatorTr              = 95;    % rad Tr in C
FCST.htRadiatorepr             = 0.8;   % rad emissivity
FCST.htRadiatorW0              = 0;     % kg
FCST.htRadiatorWk              = 1e-5;  % kg/W (f=0) 1e-5 or kg/m2 (e=0) 3.5405
FCST.htRadiatorWe              = 1;
FCST.htRadiatorWf              = 0;
FCST.htRadiatorS0              = 0;     % m3
FCST.htRadiatorSk              = 1e-7;  % m3/W (f=0) 1e-7 or thick (e=0) 0.035
FCST.htRadiatorSe              = 1;
FCST.htRadiatorSf              = 0;
FCST.htCoolantCp               = 4180;  % J/kg-K (water)
FCST.htCoolantrho              = 997;   % kg/m3 (water)
FCST.htCoolantdT               = 10;    % coolant Tc in C
FCST.htCoolantW0               = 0;     % kg
FCST.htCoolantWk               = 1e-6;  % kg/W (f=0) 1e-6 or time sec (e=0) 5
FCST.htCoolantWe               = 1;
FCST.htCoolantWf               = 0;
FCST.htFilterW0                = 0;     % kg
FCST.htFilterfW                = 0.01;  % fraction Heat sys wt
FCST.htRegulatorW0             = 0;     % kg
FCST.htRegulatorfW             = 0.01;  % fraction Heat sys wt
FCST.htPowerW0                 = 0;     % kg
FCST.htPowerfW                 = 0.01;  % fraction Heat sys wt
FCST.htPlumbW0                 = 0;     % kg plumbing constant wt
FCST.htPlumbWw                 = 0;     % kg/m plumb wt per length
FCST.htPlumbWl                 = 0;     % m plumb length
FCST.Pht0                      = 0;     % any power needed
FCST.Phtk                      = 0.01;  % 0.01-heat, kPa-flow (water)
FCST.Phte                      = 1;     % expo to heat
FCST.Phtf                      = 0;     % expo to coolant flow


% Water system variables
FCST.rhowater                  = 997;   % kg/m3
FCST.waterstoragefrac          = 0.2;   % fraction of produce stored
FCST.waterstoragehour          = 0.5;   % accumulated over this time
FCST.watertankSov              = 0;     % Stank = (1+Sov).Swater
FCST.watertankWov              = 0;     % Wtank = (1+Wov).Wwater
FCST.waterFilterW0             = 0;     % kg
FCST.waterFilterfW             = 0;     % fraction Heat sys wt
FCST.waterRegulatorW0          = 0;     % kg
FCST.waterRegulatorfW          = 0;     % fraction water sys wt
FCST.waterPowerW0              = 0;     % kg
FCST.waterPowerfW              = 0;     % fraction water sys wt
FCST.waterPlumbW0              = 0;     % kg plumbing constant wt
FCST.waterPlumbWw              = 0;     % kg/m plumb wt per length
FCST.waterPlumbWl              = 0;     % m plumb length
FCST.Pwater0                   = 0;     % any power needed
FCST.Pwaterk                   = 1e6;   % prop to liq water flow
FCST.Pwatere                   = 1;     % expo to liq water flow



% electrical system variables
FCST.elecControlW0            = 0;     % kg
FCST.elecControlWk            = 0.01;  % prop to stack current
FCST.elecControlWe            = 1.0;   % expo to stack current
FCST.elecRegulatorW0          = 0;     % kg
FCST.elecRegulatorfW          = 0;     % fraction elec sys wt
FCST.elecPowerW0              = 0;     % kg
FCST.elecPowerfW              = 0.01;  % fraction elec sys wt
FCST.elecPlumbW0              = 0;     % kg plumbing constant wt
FCST.elecPlumbWwk             = 0.01;  % prop to stack current
FCST.elecPlumbWwe             = 1.0;   % expo to stack current
FCST.elecPlumbWl              = 0;     % m plumb length
FCST.Pelec0                   = 0;     % any power needed
FCST.Peleck                   = 0.02;  % prop to stack power
FCST.Pelece                   = 1.0;   % expo to stack power


% additional gross overhead fractions
FCST.fovpower                   = 0.0;
FCST.fovweight                  = 0.0;
FCST.fovvolume                  = 0.0;


% output options
FCST.iplotVIP                   = 1;
FCST.iprntSCR                   = 1;



% =====================================================
% O U T P U T S - Design
% =====================================================


% overall weight, power, energy, and specific quantities
% stack efficiencies
FCST.etadsgHHV                  = 0.;
FCST.etadsgLHV                  = 0.;
FCST.etamaxHHV                  = 0.;
FCST.etamaxLHV                  = 0.;
% net system efficiencies
FCST.EtadsgHHV                  = 0.;
FCST.EtadsgLHV                  = 0.;
FCST.EtamaxHHV                  = 0.;
FCST.EtamaxLHV                  = 0.;
```

```
% Fuel gallon equivalents
FCST.FGEHHV                    = 0.;  % chemical
FCST.FGELHV                    = 0.;  % chemical
FCST.FGEHHVtrue                = 0.;  % engine (chemical x efficiency)
FCST.FGELHVtrue                = 0.;  % engine (chemical x efficiency)

FCST.Wtotal                    = 0.;
FCST.Stotal                    = 0.;
FCST.Wstacksys                 = 0.;
FCST.Sstacksys                 = 0.;
FCST.DFstacksys                = 0.;

FCST.SFCPdsg_kgkWh             = 0.;
FCST.SFCPmax_kgkWh             = 0.;
FCST.SFCPdsg_kgkWhe            = 0.;
FCST.SFCPmax_kgkWhe            = 0.;

FCST.POWERdsg_kWe              = 0.;
FCST.POWERmax_kW               = 0.;
FCST.POWERmax_kWe              = 0.;
FCST.ENERGY_Pdsg_kWh           = 0.;
FCST.ENERGY_Pdsg_kWhe          = 0.;
FCST.ENERGY_Pmax_kWh           = 0.;
FCST.ENERGY_Pmax_kWhe          = 0.;

FCST.SPower_Pdsg_Stack_kWkg    = 0.;
FCST.SPower_Pdsg_Stack_kWekg   = 0.;
FCST.SPower_Pmax_Stack_kWkg    = 0.;
FCST.SPower_Pmax_Stack_kWekg   = 0.;
FCST.SPower_Pdsg_Stacksys_kWkg = 0.;
FCST.SPower_Pdsg_Stacksys_kWekg= 0.;
FCST.SPower_Pmax_Stacksys_kWkg = 0.;
FCST.SPower_Pmax_Stacksys_kWekg= 0.;
FCST.SPower_Pdsg_kWkg          = 0.;
FCST.SPower_Pdsg_kWekg         = 0.;
FCST.SPower_Pmax_kWkg          = 0.;
FCST.SPower_Pmax_kWekg         = 0.;

FCST.EDensity_Pdsg_kWhL        = 0.;
FCST.EDensity_Pdsg_kWheL       = 0.;
FCST.EDensity_Pmax_kWhL        = 0.;
FCST.EDensity_Pmax_kWheL       = 0.;

FCST.SEnergy_Pdsg_kWhkg        = 0.;
FCST.SEnergy_Pdsg_kWhekg       = 0.;
FCST.SEnergy_Pmax_kWhkg        = 0.;
FCST.SEnergy_Pmax_kWhekg       = 0.;


% stack variables
% performance
FCST.ivdata                    = 0.;
FCST.Eh                        = 0.;
FCST.Er                        = 0.;
FCST.Aactive_cm2               = 0.;
FCST.Ncell                     = 0.;
FCST.Istackdsg                 = 0.;
FCST.Vstackdsg                 = 0.;
FCST.Istackmax                 = 0.;
FCST.Vstackmax                 = 0.;
FCST.IstackPmax                = 0.;
FCST.VstackPmax                = 0.;
FCST.SP                        = 0.;
FCST.SPmax                     = 0.;
FCST.vcdsg                     = 0.;
FCST.icdsg                     = 0.;
FCST.pcdsg                     = 0.;
FCST.etavdsgHHV                = 0.;
FCST.etavdsgLHV                = 0.;

% size
FCST.Sstack                   = 0.;
FCST.Lstack                   = 0.;
FCST.Astack                   = 0.;
FCST.DFstack                  = 0.;
Wstack.total                  = 0.;
Wstack.endplate               = 0.;
Wstack.insulplate             = 0.;
Wstack.collplate              = 0.;
Wstack.unit                   = 0.;
Wstack.cell                   = 0.;
Wstack.mea                    = 0.;
Wstack.bipolarplate           = 0.;
Wstack.pem                    = 0.;
Wstack.gdl                    = 0.;
Wstack.gasket                 = 0.;
Wstack.other                  = 0.;
FCST.Wstack                   = Wstack;
FCST.Wstackk0                 = 0.;
FCST.Wstackk1                 = 0.;



% Air system variables
FCST.Sblower                  = 0.;
FCST.Scomp                    = 0.;
FCST.Sturb                    = 0.;
Wair.total                    = 0.;
Wair.blower                   = 0.;
Wair.comp                     = 0.;
Wair.turb                     = 0.;
Wair.precooler                = 0.;
Wair.humidifier               = 0.;
Wair.filter                   = 0.;
Wair.plumbing                 = 0.;
Wair.power                    = 0.;
Wair.regulator               = 0.;
FCST.Wair                    = Wair;
FCST.Pair_kW                 = 0.;
FCST.Pairmax_kW              = 0.;
FCST.Pblower_kW              = 0.;
FCST.Pblowermax_kW           = 0.;
FCST.Pcomp_kW                = 0.;
FCST.Pcompmax_kW             = 0.;
FCST.Pturb_kW                = 0.;
FCST.Pturbmax_kW             = 0.;
FCST.MassAirrate_kgs         = 0.;
FCST.MassAirrate_max_kgs     = 0.;
FCST.MassAirrate_Ls          = 0.;
FCST.MassAirrate_max_Ls      = 0.;
FCST.MassAirrate_out_kgs     = 0.;
FCST.MassAirrate_out_max_kgs = 0.;
FCST.Humidity_ratio          = 0.;
FCST.MassWaterrate_kgs       = 0.;
FCST.MassVaporrate_kgs       = 0.;
FCST.MassLiquidrate_kgs      = 0.;



% Fuel system variables
FCST.Sh2tank                 = 0.;
Wfuel.total                  = 0.;
Wfuel.H2                     = 0.;
Wfuel.tank                   = 0.;
Wfuel.filter                 = 0.;
Wfuel.plumbing               = 0.;
Wfuel.power                  = 0.;
Wfuel.regulator              = 0.;
FCST.Wfuel                   = Wfuel;
FCST.Pfuel_kW                = 0.;
FCST.Pfuelmax_kW             = 0.;
FCST.MassH2rate_kgs          = 0.;
FCST.MassH2rate_Pmax_kgs     = 0.;
FCST.SFCPdsg_kgkWh           = 0.;
FCST.SFCPmax_kgkWh           = 0.;


% low temp cooling system variables (air pre-cooler)
Wlt.total                  = 0.;
Wlt.filter                 = 0.;
Wlt.plumbing               = 0.;
Wlt.regulator              = 0.;
Wlt.coolant                = 0.;
Wlt.radiator               = 0.;
Wlt.power                  = 0.;
FCST.Wlt                   = Wlt;
FCST.Plt_kW                = 0.;
FCST.Pltmax_kW             = 0.;
FCST.Sltradiator           = 0.;
FCST.Altradiator            = 0.;
FCST.HEATINGlt_kW           = 0.;
FCST.HEATINGlt_max_kW       = 0.;
FCST.Cflowmaxlt             = 0.;
FCST.Cflowdsglt             = 0.;
FCST.Cflowlt                = 0.;


% high temp cooling system variables (stack cooler)
Wht.total                   = 0.;
Wht.filter                  = 0.;
Wht.plumbing                = 0.;
Wht.regulator               = 0.;
Wht.coolant                 = 0.;
Wht.radiator                = 0.;
Wht.power                   = 0.;
FCST.Wht                    = Wht;
FCST.Pht_kW                 = 0.;
FCST.Phtmax_kW              = 0.;
FCST.Shtradiator            = 0.;
FCST.Ahtradiator            = 0.;
FCST.HEATINGstack_kW        = 0.;
FCST.HEATINGairin_kW        = 0.;
FCST.HEATINGairout_kW       = 0.;
FCST.HEATINGstack_max_kW    = 0.;
FCST.HEATINGairin_max_kW    = 0.;
FCST.HEATINGairout_max_kW   = 0.;
FCST.HEATINGdiscon_kW       = 0.;
FCST.HEATINGdisrad_kW       = 0.;
FCST.HEATINGhtc_max_kW      = 0.;
FCST.HEATINGhtc_kW          = 0.;
FCST.Cflowmaxht             = 0.;
FCST.Cflowdsght             = 0.;
FCST.Cflowht                = 0.;


% Water system
FCST.Swatertank               = 0.;
Wwater.total                  = 0.;
Wwater.tank                   = 0.;
Wwater.plumbing               = 0.;
Wwater.power                  = 0.;
Wwater.regulator              = 0.;
FCST.Wwater                   = Wwater;
FCST.Pwater_kW                = 0.;
FCST.Pwatermax_kW             = 0.;
FCST.MassTankrate_kgs         = 0.;


% Electrical system
Welec.total                   = 0.;
Welec.controller              = 0.;
Welec.power                   = 0.;
Welec.plumbing                 = 0.;
FCST.Welec                     = Welec;
FCST.Pelec_kW                 = 0.;
FCST.Pelecmax_kW              = 0.;


% Other
FCST.Wother                    = 0.;
FCST.Pother_kW                 = 0.;
FCST.Sother                    = 0.;




% =====================================================
% I N P U T S - Operations
% =====================================================

FCOP.Istack                 = 0;
FCOP.iatm                   = 0;
FCOP.altitude               = 0000*0.3048;
FCOP.temperatureC           = 0;
FCOP.BlowEta                = 0.75;
FCOP.CompEta                = 0.75;
FCOP.TurbEta                = 0.75;
FCOP.vcorr                  = 0;          % vc correction
FCOP.vrate1000ft            = -0.005;     % percent drop / 1000 ft
FCOP.vdropaboveft           = 1000;       % above 1000-ft
FCOP.arh                    = FCST.arh;
FCOP.crh                    = FCST.arh;


% =====================================================
% O U T P U T S - Operations
% =====================================================

FCOP.ivdata                 = 0;
FCOP.pressure               = 0;
FCOP.Vstack                 = 0;
FCOP.SFC_kgkWh              = 0;
FCOP.POWER_kW               = 0;
FCOP.Pair_kW                = 0;
FCOP.Pfuel_kW               = 0;
FCOP.Plt_kW                 = 0;
FCOP.Pht_kW                 = 0;
FCOP.Pwater_kW              = 0;
FCOP.Pelec_kW               = 0;
FCOP.Pother_kW              = 0;
FCOP.POWER_kWe              = 0;
FCOP.Pbop_kW                = 0;
FCOP.Pblower_kW             = 0;
FCOP.Pcomp_kW               = 0;
FCOP.Pturb_kW               = 0;
FCOP.etac                  = 0;
FCOP.etas                  = 0;
FCOP.Vstack                = 0;
FCOP.MassH2rate_kgs        = 0;
FCOP.MassAirrate_kgs       = 0;
FCOP.MassAirrate_Ls        = 0;
FCOP.MassO2rate_kgs        = 0;
FCOP.MassAirrate_out_kgs   = 0;
FCOP.MassWaterrate_kgs     = 0;
FCOP.MassLiquidrate_kgs    = 0;
FCOP.MassTankrate_kgs      = 0;
FCOP.MassVaporrate_out_kgs = 0;
FCOP.Humidity              = 0;
FCOP.HEATINGairin_kW       = 0;
FCOP.HEATINGstack_kW       = 0;
FCOP.SFCvsPe               = 0; % kg/kWe-hr vs kWe
FCOP.H2flowvsPe            = 0; % kg/hr vs kg/hr
FCOP.Pevs1000ft            = 0;  % [kW, h in 1000-ft]
FCOP.SFCvs1000ft           = 0;  % [kg/kWhe in 1000-ft]
FCOP.H2invs1000ft          = 0;  % [kg/s, h in 1000-ft]
```

```
% run Modern 80kW
% SI units unless otherwise appended


% ======================================================
% I N P U T S - Design
% ======================================================


% design options for power
FCST.ioptp                      = 3;     % option, design power, kW
FCST.POWERdsg_kW                = 80;    % design stack power, kW
FCST.etac                       = 0.37;  % cell efficiency
FCST.vc                         = 0.70;  % cell voltage, V
FCST.Voltage                    = 250;


% atmospheric conditions affecting design
FCST.iatm                       = 0;             % environment option
FCST.OneAtm                     = 101325;        % Standard atm, Pa
FCST.temperatureC               = 20;            % temp of environ, C
FCST.altitudeDesign             = 0000*0.3048;   % altitude, m
FCST.pressure                   = 0.0*101325;    % pressure, Pa


% cell i-v variables
FCST.TstackC                    = 80;          % stack temp, C
FCST.Pstack                     = 2.5*101325;  % stack pres, Pa
FCST.xO2                        = 0.2095;      % mole fraction of O2 in air
FCST.ioptiv                     = 3;           % 1-data, 2-make, 3-PMAT
FCST.ivdatainp = [
         0    1.1782         0
    0.0200    0.8580    0.0172
    0.0400    0.8529    0.0341
    0.0600    0.8481    0.0509
    0.0800    0.8434    0.0675
    0.1000    0.8390    0.0839
    0.1200    0.8347    0.1002
    0.1400    0.8305    0.1163
    0.1600    0.8264    0.1322
    0.1800    0.8225    0.1481
    0.2000    0.8187    0.1637
    0.2200    0.8150    0.1793
    0.2400    0.8114    0.1947
    0.2600    0.8078    0.2100
    0.2800    0.8044    0.2252
    0.3000    0.8010    0.2403
    0.3200    0.7977    0.2553
    0.3400    0.7945    0.2701
    0.3600    0.7913    0.2849
    0.3800    0.7881    0.2995
    0.4000    0.7850    0.3140
    0.4200    0.7820    0.3284
```

0.4400	0.7790	0.3428
0.4600	0.7761	0.3570
0.4800	0.7732	0.3711
0.5000	0.7703	0.3852
0.5200	0.7675	0.3991
0.5400	0.7647	0.4129
0.5600	0.7619	0.4267
0.5800	0.7592	0.4403
0.6000	0.7565	0.4539
0.6200	0.7538	0.4673
0.6400	0.7511	0.4807
0.6600	0.7485	0.4940
0.6800	0.7459	0.5072
0.7000	0.7433	0.5203
0.7200	0.7407	0.5333
0.7400	0.7381	0.5462
0.7600	0.7356	0.5591
0.7800	0.7331	0.5718
0.8000	0.7306	0.5845
0.8200	0.7281	0.5970
0.8400	0.7256	0.6095
0.8600	0.7231	0.6219
0.8800	0.7207	0.6342
0.9000	0.7182	0.6464
0.9200	0.7158	0.6585
0.9400	0.7134	0.6706
0.9600	0.7109	0.6825
0.9800	0.7085	0.6943
1.0000	0.7061	0.7061
1.0200	0.7037	0.7178
1.0400	0.7013	0.7293
1.0600	0.6989	0.7408
1.0800	0.6965	0.7522
1.1000	0.6941	0.7635
1.1200	0.6917	0.7747
1.1400	0.6893	0.7858
1.1600	0.6869	0.7968
1.1800	0.6845	0.8077
1.2000	0.6821	0.8185
1.2200	0.6797	0.8292
1.2400	0.6773	0.8399
1.2600	0.6749	0.8504
1.2800	0.6725	0.8607
1.3000	0.6700	0.8710
1.3200	0.6676	0.8812
1.3400	0.6651	0.8913
1.3600	0.6627	0.9012
1.3800	0.6602	0.9111
1.4000	0.6577	0.9208
1.4200	0.6552	0.9304
1.4400	0.6527	0.9399
1.4600	0.6501	0.9492
1.4800	0.6476	0.9584

```
    1.5000    0.6450    0.9675
    1.5200    0.6424    0.9764
    1.5400    0.6397    0.9852
    1.5600    0.6371    0.9938
    1.5800    0.6344    1.0023
    1.6000    0.6316    1.0106
    1.6200    0.6288    1.0187
    1.6400    0.6260    1.0267
    1.6600    0.6232    1.0344
    1.6800    0.6202    1.0420
    1.7000    0.6173    1.0493
    1.7200    0.6142    1.0565
    1.7400    0.6111    1.0633
    1.7600    0.6079    1.0700
    1.7800    0.6047    1.0763
    1.8000    0.6013    1.0823
    1.8200    0.5978    1.0880
    1.8400    0.5942    1.0934
    1.8600    0.5905    1.0983
    1.8800    0.5866    1.1028
    1.9000    0.5825    1.1067
    1.9200    0.5782    1.1101
    1.9400    0.5736    1.1128
    1.9600    0.5687    1.1147
    1.9800    0.5635    1.1157
    2.0000    0.5577    1.1154
    2.0200    0.5514    1.1137
    2.0400    0.5441    1.1101
    2.0600    0.5358    1.1037
    2.0800    0.5256    1.0933
    2.1000    0.5125    1.0763
    2.1200    0.4933    1.0458
    2.1400    0.4535    0.9705
    ];
FCST.ipmaxinp                  =  100;
FCST.Er                        =  1.1782;
FCST.j0C                       =  0.0001;
FCST.alphaC                    =  0.22;
FCST.j0A                       =  0.1;
FCST.alphaA                    =  0.5;
FCST.ASR                       =  0.04;
FCST.jL                        =  2.45;
FCST.jleak                     =  0.30;
FCST.C                         =  0.35;
FCST.PMAT = [
    1         1.25      1.5       2.0        2.5
    0.0001    0.0001    0.0001    0.0001     0.0001
    0.1800    0.1900    0.2000    0.2100     0.2200
    0.1000    0.1000    0.1000    0.1000     0.1000
    0.5000    0.5000    0.5000    0.5000     0.5000
    0.0400    0.0400    0.0400    0.0400     0.0400
    1.7500    1.8500    1.9500    2.2500     2.4500
    0.1500    0.1500    0.2000    0.2500     0.3000
    0.0300   0.0350   0.0350   0.0350     0.0350
    ];
FCST.crh                        =  100;
FCST.arh                        =  100;
FCST.deltai                     =  0.02;
% temp corr
FCST.ASR0                       =  0.12;      % ref value
FCST.ASR0t                      =  80;        % ref temp
FCST.ASRtk2                     =  0.0006;
FCST.ASRtk1                     =  0.002;
FCST.jL0                        =  0.73;      % ref value
FCST.jL0t                       =  80;        % ref temp
FCST.jLtk2                      = -0.0015;
FCST.jLtk1                      = -0.003;
% humidity corrections in ASR and jL
FCST.ASR0arh                    =  100;        % ref anode rh
FCST.ASR0crh                    =  100;        % ref cathode rh
FCST.ASRrhk1a                   = -0.005;
FCST.ASRrhk2a                   =  0.;
FCST.ASRrhk1a1c                 =  0.;
FCST.ASRrhk1a2c                 =  2.0e-6;
FCST.ASRrhk2a2c                 =  0.;
FCST.ASRrhk2a1c                 =  0.;
FCST.ASRrhk2c                   =  7.0e-5;
FCST.ASRrhk1c                   =  0.0021;
FCST.jL0arh                     =  100;
FCST.jL0crh                     =  100;
FCST.jLrhk1a                    =  0.;
FCST.jLrhk2a                    =  0.;
FCST.jLrhk1a1c                  =  0.;
FCST.jLrhk1a2c                  =  0.;
FCST.jLrhk2a2c                  =  0.;
FCST.jLrhk2a1c                  =  0.;
FCST.jLrhk2c                    = -0.0001;
FCST.jLrhk1c                    = -0.0076;
% other corrections
FCST.icorr                      =  0;         % ic corr (0/1)
FCST.TFi                        =  1.0;       % changes ic to ic.Tfi
FCST.vcorr                      =  0;         % vc corr for alt (0/1)
FCST.deltav                     = -0.01;      % cell to stack correction
FCST.vrate1000ft                = -0.0056;    % percent drop / 1000 ft
FCST.vdropaboveft               =  1000;      % above 1000-ft


% stack weight variables
FCST.TFwstack                   = 1.0;       % Tech factor stack weight
FCST.dWwstack                   = 0.0;       % correction stack weight
FCST.TFsstack                   = 1.0;       % Tech factor stack volume
FCST.dSsstack                   = 0.0;       % correction stack volume
FCST.ioptsw                     = 1;         % 0: simple, 1: detailed
FCST.tcell                      = 0.001381;  % cell thickness, m
FCST.dcell                      = 2584;      % cell density, kg/m3
FCST.porosity                   = 1.0;
```

```
% Component breakdown
FCST.dendplate                = 1600;    % end plate, Al
FCST.dinsulplate              = 1420;    % polyoxy-methylene
FCST.dcollplate               = 8940;    % collector plate, Cu
FCST.dpem                     = 500;     % membrane, Nafion
FCST.dgdl                     = 440;     % gas diffusion layer
FCST.dbipolarplate            = 2160;    % graphite
FCST.dgasket                  = 1000;    % rubber
FCST.Aendplatefrac            = 1.1;     % Area = 1.1 Active area
FCST.tendplatecons            = 0.0;     % thickness constant
FCST.tendplatefrac            = 0.020;   % thickness = 0.05 tunit
FCST.Ainsulplatefrac          = 1.1;     % fraction active area
FCST.tinsulplatecons          = 0.0;     % thickness constrant
FCST.tinsulplatefrac          = 0.005;   % thickness = 0.02 tunit
FCST.Acollplatefrac           = 0.5;     % fraction active area
FCST.tcollplatecons           = 0.0;     % thickness constant
FCST.tcollplatefrac           = 0.005;   % thickness = 0.01 tunit
FCST.Wfastener                = 0.0;     % bolt/clamp, steel, 7850 kg/m3
FCST.tpem                     = 0.00010; % membrane, Nafion
FCST.tgdl                     = 0.00020; % gas diffusion layer
FCST.tbipolarplate            = 0.00100; % graphite; avg seal + mea region
FCST.pbipolarplate            = 0.60000; % porosity due to channels
FCST.Abipolarplatefrac        = 1.10;    % fraction active area
FCST.tgasket                  = 0.00200; % rubber
FCST.Agasketfrac              = 0.03;    % fraction active area
FCST.Wstackother              = 0.00;    % any other weight




% H2 tank system
FCST.fH2utilization           = 1.0;
FCST.itanktype                = 3;
FCST.ioptf                    = 1;     % fuel option; 1-h2, 2-endurance
FCST.Whydrogen                = 5;
FCST.Endurance_min            = 60;
% 350 bar: 0.0515, 17.2; 700 bar: 0.057, 40;
% cryo-compressed Al: 0.09, 41; cryo-compressed steel: 0.055, 41
% liq: 0.075, 65;
FCST.h2tankwfg                = 0.057; % kg/kg wt fraction gravimetric
FCST.h2tankwfs                = 40.00; % kg/m3 wt fraction volumetric
FCST.h2tankBar                = 700;   % tank storage pressure
FCST.h2tankTC                 = 15;    % tank storage temperature C
FCST.h2tankSov                = 0.10;  % tank volume overhead
FCST.FuelFilterW0             = 0.;
FCST.FuelFilterfW             = 0.;
FCST.FuelRegulatorW0          = 0;     % kg
FCST.FuelRegulatorfW          = 0.01;  % fraction fuel sys wt
FCST.FuelPowerW0              = 0;     % kg wt of power supply
FCST.FuelPowerfW              = 0.01;  % fraction fuel sys wt
FCST.FuelPlumbW0              = 0;     % kg plumbing constant wt
FCST.FuelPlumbWw              = 0;     % kg/m plumb wt per length
FCST.FuelPlumbWl              = 0;     % m plumb length
FCST.Pfuel0                    = 0;     % prescribed power
FCST.Pfuelk                    = 0;     % prop to fuel flow rate
FCST.Pfuele                    = 0;     % expo to fuel flow rate
FCST.FGErho                    = 722.13;% density of gasoline
FCST.FGEMJkg                   = 44;    % heating value of gasoline, MJ/kg
FCST.FGEeta                    = 0.25;  % efficiency of FGE fuel




% Air system variables
FCST.fAirFlowRate              = 2.5;
FCST.DPstack                   = 30000;     % pres drop in stack, Pa
% Blower
FCST.iBlower                   = 0;
FCST.BlowEta                   = 0.75;
FCST.BlowW0                    = 0;
FCST.BlowWtf                   = 1.0;
FCST.BlowWk                    = 1/1000;
FCST.BlowWfe                   = 0;
FCST.BlowWpe                   = 1;
FCST.BlowS0                    = 0;
FCST.BlowStf                   = 1.0;
FCST.BlowSk                    = 1/3e6;
FCST.BlowSfe                   = 0;
FCST.BlowSpe                   = 1;
% Compressor
FCST.iCompressor               = 1;
FCST.CompEta                   = 0.75;
FCST.CompW0                    = 0.;
FCST.CompWtf                   = 1.0;
FCST.CompWk                    = 7e-4;   % 122 kg/kg/s or 7e-4 kg/W
FCST.CompWfe                   = 0;
FCST.CompWpe                   = 1;
FCST.CompS0                    = 0.;
FCST.CompStf                   = 1.0;
FCST.CompSk                    = 4.14e-7; % 0.0722 m3/kg/s or 4.14e-7 m3/W
FCST.CompSfe                   = 0;
FCST.CompSpe                   = 1;
% Turbo-expander (axial-flow turbine)
FCST.iTurbine                  = 1;
FCST.TurbEta                   = 0.75;
FCST.TurbW0                    = 0.;
FCST.TurbWtf                   = 1.0;
FCST.TurbWk                    = 0;
FCST.TurbWfe                   = 0;
FCST.TurbWpe                   = 0;
FCST.TurbS0                    = 0.;
FCST.TurbStf                   = 1.0;
FCST.TurbSk                    = 0;
FCST.TurbSfe                   = 0;
FCST.TurbSpe                   = 0;
```

```
% accessories
FCST.AirFilterW0                = 0;     % kg
FCST.AirFilterfW                = 0.01;  % fraction Air sys wt
FCST.AirRegulatorW0             = 0;     % kg
FCST.AirRegulatorfW             = 0.01;  % fraction Air sys wt
FCST.AirPowerW0                 = 0;     % kg controller
FCST.AirPowerfW                 = 0.05;  % fraction Air sys wt
FCST.AirPlumbW0                 = 0;     % kg plumbing constant wt
FCST.AirPlumbWw                 = 0;     % kg/m plumb wt per length
FCST.AirPlumbWl                 = 0;     % m plumb length
FCST.AirHumidifierW0            = 0;     % kg
FCST.AirHumidifierfW            = 0.05;  % fraction Air sys wt



% Heat system variables
% low temp system (air pre-cooler, only if compressor used)
FCST.iLTR                      = 0;     % 0-Pdsg load, 1-Pmax load
FCST.ltRadiatorhr              = 1015;  % rad conv ht trans coeff W/m2/K
FCST.ltRadiatorTa              = 20;    % rad Ta in C
FCST.ltRadiatorTr              = 95;    % rad Tr in C
FCST.ltRadiatorepr             = 0.8;   % rad emissivity
FCST.ltRadiatorW0              = 0;     % kg
FCST.ltRadiatorWk              = 1e-4;  % kg/W (f=0) 1e-4 or kg/m2 (e=0) 3.5405
FCST.ltRadiatorWe              = 1;
FCST.ltRadiatorWf              = 0;
FCST.ltRadiatorS0              = 0;     % m3
FCST.ltRadiatorSk              = 1e-7;  % m3/W (f=0) 1e-7 or thick (e=0) 0.035
FCST.ltRadiatorSe              = 1;
FCST.ltRadiatorSf              = 0;
FCST.ltCoolantCp               = 4180;  % J/kg-K (water)
FCST.ltCoolantdT               = 10;    % coolant deltaT (max 15)
FCST.ltCoolantW0               = 0;     % kg
FCST.ltCoolantWk               = 1e-6;  % kg/W (f=0) 1e-6 or time sec (e=0) 30
FCST.ltCoolantWe               = 1;
FCST.ltCoolantWf               = 0;
FCST.ltCoolerW0                = 0;     % kg
FCST.ltCoolerWk                = 1e-6;  % kg/W
FCST.ltCoolerWe                = 1;     %
FCST.ltFilterW0                = 0;     % kg
FCST.ltFilterfW                = 0.01;  % fraction Heat sys wt
FCST.ltRegulatorW0             = 0;     % kg
FCST.ltRegulatorfW             = 0.01;  % fraction Heat sys wt
FCST.ltPowerW0                 = 0;     % kg
FCST.ltPowerfW                 = 0.01;  % fraction Heat sys wt
FCST.ltPlumbW0                 = 0;     % kg plumbing constant wt
FCST.ltPlumbWw                 = 0;     % kg/m plumb wt per length
FCST.ltPlumbWl                 = 0;     % m plumb length
FCST.Plt0                      = 0;     % any power needed
FCST.Pltk                      = 0.01;  % 0.01-heat, kPa-flow (water)
FCST.Plte                      = 1;     % expo to heating
FCST.Pltf                      = 0;     % expo to flow
```

```
% Heat system variables
% high temp system (stack cooling)
FCST.iHV                     = 0;     % 0-LHV, 1-HHV
FCST.iHTR                    = 0;     % 0-Pdsg load, 1-Pmax load
FCST.htStackfd               = 0;     % stack frac stack heat dissipated
FCST.htStackhs               = 3;     % stack conv ht trans coeff W/m2/K
FCST.htStackTa               = 40;    % stack Ta in C
FCST.htStackeps              = 0.8;   % stack emissivity
FCST.htStackfs               = 0.6;   % stack frac surface/active area
FCST.htRadiatorhr            = 1015;  % rad conv ht trans coeff W/m2/K
FCST.htRadiatorTa            = 20;    % rad Ta in C
FCST.htRadiatorTr            = 95;    % rad Tr in C
FCST.htRadiatorepr           = 0.8;   % rad emissivity
FCST.htRadiatorW0            = 0;     % kg
FCST.htRadiatorWk            = 1e-5;  % kg/W (f=0) 1e-5 or kg/m2 (e=0) 3.5405
FCST.htRadiatorWe            = 1;
FCST.htRadiatorWf            = 0;
FCST.htRadiatorS0            = 0;     % m3
FCST.htRadiatorSk            = 1e-7;  % m3/W (f=0) 1e-7 or thick (e=0) 0.035
FCST.htRadiatorSe            = 1;
FCST.htRadiatorSf            = 0;
FCST.htCoolantCp             = 4180;  % J/kg-K (water)
FCST.htCoolantrho            = 997;   % kg/m3 (water)
FCST.htCoolantdT             = 10;    % coolant Tc in C
FCST.htCoolantW0             = 0;     % kg
FCST.htCoolantWk             = 1e-6;  % kg/W (f=0) 1e-6 or time sec (e=0) 5
FCST.htCoolantWe             = 1;
FCST.htCoolantWf             = 0;
FCST.htFilterW0              = 0;     % kg
FCST.htFilterfW              = 0.01;  % fraction Heat sys wt
FCST.htRegulatorW0           = 0;     % kg
FCST.htRegulatorfW           = 0.01;  % fraction Heat sys wt
FCST.htPowerW0               = 0;     % kg
FCST.htPowerfW               = 0.01;  % fraction Heat sys wt
FCST.htPlumbW0               = 0;     % kg plumbing constant wt
FCST.htPlumbWw               = 0;     % kg/m plumb wt per length
FCST.htPlumbWl               = 0;     % m plumb length
FCST.Pht0                    = 0;     % any power needed
FCST.Phtk                    = 0.01;  % 0.01-heat, kPa-flow (water)
FCST.Phte                    = 1;     % expo to heat
FCST.Phtf                    = 0;     % expo to coolant flow


% Water system variables
FCST.rhowater                = 997;   % kg/m3
FCST.waterstoragefrac        = 0.2;   % fraction of produce stored
FCST.waterstoragehour        = 0.5;   % accumulated over this time
FCST.watertankSov            = 0;     % Stank = (1+Sov).Swater
FCST.watertankWov            = 0;     % Wtank = (1+Wov).Wwater
FCST.waterFilterW0           = 0;     % kg
FCST.waterFilterfW           = 0;     % fraction Heat sys wt
FCST.waterRegulatorW0        = 0;     % kg
FCST.waterRegulatorfW         = 0;     % fraction water sys wt
FCST.waterPowerW0             = 0;     % kg
FCST.waterPowerfW             = 0;     % fraction water sys wt
FCST.waterPlumbW0             = 0;     % kg plumbing constant wt
FCST.waterPlumbWw             = 0;     % kg/m plumb wt per length
FCST.waterPlumbWl             = 0;     % m plumb length
FCST.Pwater0                  = 0;     % any power needed
FCST.Pwaterk                  = 1e6;   % prop to liq water flow
FCST.Pwatere                  = 1;     % expo to liq water flow



% electrical system variables
FCST.elecControlW0           = 0;     % kg
FCST.elecControlWk           = 0.01;  % prop to stack current
FCST.elecControlWe           = 1.0;   % expo to stack current
FCST.elecRegulatorW0         = 0;     % kg
FCST.elecRegulatorfW         = 0;     % fraction elec sys wt
FCST.elecPowerW0             = 0;     % kg
FCST.elecPowerfW             = 0.01;  % fraction elec sys wt
FCST.elecPlumbW0             = 0;     % kg plumbing constant wt
FCST.elecPlumbWwk            = 0.01;  % prop to stack current
FCST.elecPlumbWwe            = 1.0;   % expo to stack current
FCST.elecPlumbWl             = 0;     % m plumb length
FCST.Pelec0                  = 0;     % any power needed
FCST.Peleck                  = 0.02;  % prop to stack power
FCST.Pelece                  = 1.0;   % expo to stack power


% additional gross overhead fractions
FCST.fovpower                 = 0.0;
FCST.fovweight                = 0.0;
FCST.fovvolume                = 0.0;


% output options
FCST.iplotVIP                 = 1;
FCST.iprntSCR                 = 1;



% =====================================================
% O U T P U T S - Design
% =====================================================


% overall weight, power, energy, and specific quantities
% stack efficiencies
FCST.etadsgHHV                = 0.;
FCST.etadsgLHV                = 0.;
FCST.etamaxHHV                = 0.;
FCST.etamaxLHV                = 0.;
% net system efficiencies
```

```
FCST.EtadsgHHV                 = 0.;
FCST.EtadsgLHV                 = 0.;
FCST.EtamaxHHV                 = 0.;
FCST.EtamaxLHV                 = 0.;
% Fuel gallon equivalents
FCST.FGEHHV                    = 0.;  % chemical
FCST.FGELHV                    = 0.;  % chemical
FCST.FGEHHVtrue                = 0.;  % engine (chemical x efficiency)
FCST.FGELHVtrue                = 0.;  % engine (chemical x efficiency)

FCST.Wtotal                    = 0.;
FCST.Stotal                    = 0.;
FCST.Wstacksys                 = 0.;
FCST.Sstacksys                 = 0.;
FCST.DFstacksys                = 0.;

FCST.SFCPdsg_kgkWh             = 0.;
FCST.SFCPmax_kgkWh             = 0.;
FCST.SFCPdsg_kgkWhe            = 0.;
FCST.SFCPmax_kgkWhe            = 0.;

FCST.POWERdsg_kWe              = 0.;
FCST.POWERmax_kW               = 0.;
FCST.POWERmax_kWe              = 0.;
FCST.ENERGY_Pdsg_kWh           = 0.;
FCST.ENERGY_Pdsg_kWhe          = 0.;
FCST.ENERGY_Pmax_kWh           = 0.;
FCST.ENERGY_Pmax_kWhe          = 0.;

FCST.SPower_Pdsg_Stack_kWkg    = 0.;
FCST.SPower_Pdsg_Stack_kWekg   = 0.;
FCST.SPower_Pmax_Stack_kWkg    = 0.;
FCST.SPower_Pmax_Stack_kWekg   = 0.;
FCST.SPower_Pdsg_Stacksys_kWkg = 0.;
FCST.SPower_Pdsg_Stacksys_kWekg= 0.;
FCST.SPower_Pmax_Stacksys_kWkg = 0.;
FCST.SPower_Pmax_Stacksys_kWekg= 0.;
FCST.SPower_Pdsg_kWkg          = 0.;
FCST.SPower_Pdsg_kWekg         = 0.;
FCST.SPower_Pmax_kWkg          = 0.;
FCST.SPower_Pmax_kWekg         = 0.;

FCST.EDensity_Pdsg_kWhL        = 0.;
FCST.EDensity_Pdsg_kWheL       = 0.;
FCST.EDensity_Pmax_kWhL        = 0.;
FCST.EDensity_Pmax_kWheL       = 0.;

FCST.SEnergy_Pdsg_kWhkg        = 0.;
FCST.SEnergy_Pdsg_kWhekg       = 0.;
FCST.SEnergy_Pmax_kWhkg        = 0.;
FCST.SEnergy_Pmax_kWhekg       = 0.;

% Design factor of system
```

```
FCST.DFsystem                = 0.; % cell pc / (P/W) in kW/kg
FCST.DFsysteme               = 0.; % cell pc / (P/W) in kWe/kg

% stack variables
% performance
FCST.ivdata                  = 0.;
FCST.Eh                      = 0.;
FCST.Er                      = 0.;
FCST.Aactive_cm2             = 0.;
FCST.Ncell                   = 0.;
FCST.Istackdsg               = 0.;
FCST.Vstackdsg               = 0.;
FCST.Istackmax               = 0.;
FCST.Vstackmax               = 0.;
FCST.IstackPmax              = 0.;
FCST.VstackPmax              = 0.;
FCST.SP                      = 0.;
FCST.vcdsg                   = 0.;
FCST.icdsg                   = 0.;
FCST.pcdsg                   = 0.;
FCST.etadsg                  = 0.;
FCST.etavdsgHHV              = 0.;
FCST.etavdsgLHV              = 0.;

% size
FCST.Sstack                 = 0.;
FCST.Lstack                 = 0.;
FCST.Astack                 = 0.;
FCST.DFstack                = 0.;
Wstack.total                = 0.;
Wstack.endplate             = 0.;
Wstack.insulplate           = 0.;
Wstack.collplate            = 0.;
Wstack.unit                 = 0.;
Wstack.cell                 = 0.;
Wstack.mea                  = 0.;
Wstack.bipolarplate         = 0.;
Wstack.pem                  = 0.;
Wstack.gdl                  = 0.;
Wstack.gasket               = 0.;
Wstack.other                = 0.;
FCST.Wstack                 = Wstack;
FCST.Wstackk0               = 0.;
FCST.Wstackk1               = 0.;


% Air system variables
FCST.Sblower                = 0.;
FCST.Scomp                  = 0.;
FCST.Sturb                  = 0.;
Wair.total                  = 0.;
Wair.blower                 = 0.;
Wair.comp                   = 0.;
Wair.turb                    = 0.;
Wair.precooler               = 0.;
Wair.humidifier              = 0.;
Wair.filter                  = 0.;
Wair.plumbing                = 0.;
Wair.power                   = 0.;
Wair.regulator               = 0.;
FCST.Wair                    = Wair;
FCST.Pair_kW                 = 0.;
FCST.Pairmax_kW              = 0.;
FCST.Pblower_kW              = 0.;
FCST.Pblowermax_kW           = 0.;
FCST.Pcomp_kW                = 0.;
FCST.Pcompmax_kW             = 0.;
FCST.Pturb_kW                = 0.;
FCST.Pturbmax_kW             = 0.;
FCST.MassAirrate_kgs         = 0.;
FCST.MassAirrate_max_kgs     = 0.;
FCST.MassAirrate_Ls          = 0.;
FCST.MassAirrate_max_Ls      = 0.;
FCST.MassAirrate_out_kgs     = 0.;
FCST.MassAirrate_out_max_kgs = 0.;
FCST.Humidity_ratio          = 0.;
FCST.MassWaterrate_kgs       = 0.;
FCST.MassVaporrate_kgs       = 0.;
FCST.MassLiquidrate_kgs      = 0.;



% Fuel system variables
FCST.Sh2tank                 = 0.;
Wfuel.total                  = 0.;
Wfuel.H2                     = 0.;
Wfuel.tank                   = 0.;
Wfuel.filter                 = 0.;
Wfuel.plumbing               = 0.;
Wfuel.power                  = 0.;
Wfuel.regulator              = 0.;
FCST.Wfuel                   = Wfuel;
FCST.Pfuel_kW                = 0.;
FCST.Pfuelmax_kW             = 0.;
FCST.MassH2rate_kgs          = 0.;
FCST.MassH2rate_Pmax_kgs     = 0.;
FCST.SFCPdsg_kgkWh           = 0.;
FCST.SFCPmax_kgkWh           = 0.;


% low temp cooling system variables (air pre-cooler)
Wlt.total                  = 0.;
Wlt.filter                 = 0.;
Wlt.plumbing               = 0.;
Wlt.regulator              = 0.;
Wlt.coolant                = 0.;
Wlt.radiator                = 0.;
Wlt.power                   = 0.;
FCST.Wlt                    = Wlt;
FCST.Plt_kW                 = 0.;
FCST.Pltmax_kW              = 0.;
FCST.Sltradiator            = 0.;
FCST.Altradiator            = 0.;
FCST.HEATINGlt_kW           = 0.;
FCST.HEATINGlt_max_kW       = 0.;
FCST.Cflowmaxlt             = 0.;
FCST.Cflowdsglt             = 0.;
FCST.Cflowlt                = 0.;



% high temp cooling system variables (stack cooler)
Wht.total                   = 0.;
Wht.filter                  = 0.;
Wht.plumbing                = 0.;
Wht.regulator               = 0.;
Wht.coolant                 = 0.;
Wht.radiator                = 0.;
Wht.power                   = 0.;
FCST.Wht                    = Wht;
FCST.Pht_kW                 = 0.;
FCST.Phtmax_kW              = 0.;
FCST.Shtradiator            = 0.;
FCST.Ahtradiator            = 0.;
FCST.HEATINGstack_kW        = 0.;
FCST.HEATINGairin_kW        = 0.;
FCST.HEATINGairout_kW       = 0.;
FCST.HEATINGstack_max_kW    = 0.;
FCST.HEATINGairin_max_kW    = 0.;
FCST.HEATINGairout_max_kW   = 0.;
FCST.HEATINGdiscon_kW       = 0.;
FCST.HEATINGdisrad_kW       = 0.;
FCST.HEATINGhtc_max_kW      = 0.;
FCST.HEATINGhtc_kW          = 0.;
FCST.Cflowmaxht             = 0.;
FCST.Cflowdsght             = 0.;
FCST.Cflowht                = 0.;


% Water system
FCST.Swatertank               = 0.;
Wwater.total                  = 0.;
Wwater.tank                   = 0.;
Wwater.plumbing               = 0.;
Wwater.power                  = 0.;
Wwater.regulator              = 0.;
FCST.Wwater                   = Wwater;
FCST.Pwater_kW                = 0.;
FCST.Pwatermax_kW             = 0.;
FCST.MassTankrate_kgs          = 0.;


% Electrical system
Welec.total                    = 0.;
Welec.controller               = 0.;
Welec.power                    = 0.;
Welec.plumbing                 = 0.;
FCST.Welec                     = Welec;
Welec.Pelec_kW                 = 0.;
Welec.Pelecmax_kW              = 0.;


% Other
FCST.Wother                    = 0.;
FCST.Pother_kW                 = 0.;
FCST.Sother                    = 0.;




% =====================================================
% I N P U T S - Operations
% =====================================================

FCOP.Istack                 = 0;
FCOP.iatm                   = 0;
FCOP.altitude               = 0000*0.3048;
FCOP.temperatureC           = 20;
FCOP.BlowEta                = 0.75;
FCOP.CompEta                = 0.75;
FCOP.TurbEta                = 0.75;
FCOP.vcorr                  = 0;         % vc correction
FCOP.vrate1000ft            = -0.005;    % percent drop / 1000 ft
FCOP.vdropaboveft           = 1000;      % above 1000-ft
FCOP.arh                    = FCST.arh;
FCOP.crh                    = FCST.arh;


% =====================================================
% O U T P U T S - Operations
% =====================================================

FCOP.ivdata                 = 0;
FCOP.pressure               = 0;
FCOP.Vstack                 = 0;
FCOP.SFC_kgkWh              = 0;
FCOP.POWER_kW               = 0;
FCOP.Pair_kW                = 0;
FCOP.Pfuel_kW               = 0;
FCOP.Plt_kW                 = 0;
FCOP.Pht_kW                 = 0;
FCOP.Pwater_kW              = 0;
FCOP.Pelec_kW              = 0;
FCOP.Pother_kW             = 0;
FCOP.POWER_kWe             = 0;
FCOP.Pbop_kW               = 0;
FCOP.Pblower_kW            = 0;
FCOP.Pcomp_kW              = 0;
FCOP.Pturb_kW              = 0;
FCOP.etac                  = 0;
FCOP.etas                  = 0;
FCOP.Vstack                = 0;
FCOP.MassH2rate_kgs        = 0;
FCOP.MassAirrate_kgs       = 0;
FCOP.MassAirrate_Ls        = 0;
FCOP.MassO2rate_kgs        = 0;
FCOP.MassAirrate_out_kgs   = 0;
FCOP.MassWaterrate_kgs     = 0;
FCOP.MassLiquidrate_kgs    = 0;
FCOP.MassTankrate_kgs      = 0;
FCOP.MassVaporrate_out_kgs = 0;
FCOP.Humidity              = 0;
FCOP.HEATINGairin_kW       = 0;
FCOP.HEATINGstack_kW       = 0;
FCOP.SFCvsPe               = 0; % kg/kWe-hr vs kWe
FCOP.H2flowvsPe            = 0; % kg/hr vs kg/hr
FCOP.Pevs1000ft            = 0;  % [kW, h in 1000-ft]
FCOP.SFCvs1000ft           = 0;  % [kg/kWhe in 1000-ft]
FCOP.H2invs1000ft          = 0;  % [kg/s, h in 1000-ft]
```

```
% run Modern 80kW liqcool
% SI units unless otherwise appended


% ======================================================
% I N P U T S - Design
% ======================================================


% design options for power
FCST.ioptp                      = 3;     % option, design power, kW
FCST.POWERdsg_kW                = 80;    % design stack power, kW
FCST.etac                       = 0.37;  % cell efficiency
FCST.vc                         = 0.70;  % cell voltage, V
FCST.Voltage                    = 250;


% atmospheric conditions affecting design
FCST.iatm                       = 0;             % environment option
FCST.OneAtm                     = 101325;        % Standard atm, Pa
FCST.temperatureC               = 15;            % temp of environ, C
FCST.altitudeDesign             = 0000*0.3048;   % altitude, m
FCST.pressure                   = 0.0*101325;    % pressure, Pa


% cell i-v variables
FCST.TstackC                    = 80;          % stack temp, C
FCST.Pstack                     = 2.5*101325;  % stack pres, Pa
FCST.xO2                        = 0.2095;      % mole fraction of O2 in air
FCST.ioptiv                     = 3;           % 1-data, 2-make, 3-PMAT
FCST.ivdatainp = [
         0    1.1782         0
    0.0200    0.8580    0.0172
    0.0400    0.8529    0.0341
    0.0600    0.8481    0.0509
    0.0800    0.8434    0.0675
    0.1000    0.8390    0.0839
    0.1200    0.8347    0.1002
    0.1400    0.8305    0.1163
    0.1600    0.8264    0.1322
    0.1800    0.8225    0.1481
    0.2000    0.8187    0.1637
    0.2200    0.8150    0.1793
    0.2400    0.8114    0.1947
    0.2600    0.8078    0.2100
    0.2800    0.8044    0.2252
    0.3000    0.8010    0.2403
    0.3200    0.7977    0.2553
    0.3400    0.7945    0.2701
    0.3600    0.7913    0.2849
    0.3800    0.7881    0.2995
    0.4000    0.7850    0.3140
    0.4200    0.7820    0.3284
```

0.4400	0.7790	0.3428
0.4600	0.7761	0.3570
0.4800	0.7732	0.3711
0.5000	0.7703	0.3852
0.5200	0.7675	0.3991
0.5400	0.7647	0.4129
0.5600	0.7619	0.4267
0.5800	0.7592	0.4403
0.6000	0.7565	0.4539
0.6200	0.7538	0.4673
0.6400	0.7511	0.4807
0.6600	0.7485	0.4940
0.6800	0.7459	0.5072
0.7000	0.7433	0.5203
0.7200	0.7407	0.5333
0.7400	0.7381	0.5462
0.7600	0.7356	0.5591
0.7800	0.7331	0.5718
0.8000	0.7306	0.5845
0.8200	0.7281	0.5970
0.8400	0.7256	0.6095
0.8600	0.7231	0.6219
0.8800	0.7207	0.6342
0.9000	0.7182	0.6464
0.9200	0.7158	0.6585
0.9400	0.7134	0.6706
0.9600	0.7109	0.6825
0.9800	0.7085	0.6943
1.0000	0.7061	0.7061
1.0200	0.7037	0.7178
1.0400	0.7013	0.7293
1.0600	0.6989	0.7408
1.0800	0.6965	0.7522
1.1000	0.6941	0.7635
1.1200	0.6917	0.7747
1.1400	0.6893	0.7858
1.1600	0.6869	0.7968
1.1800	0.6845	0.8077
1.2000	0.6821	0.8185
1.2200	0.6797	0.8292
1.2400	0.6773	0.8399
1.2600	0.6749	0.8504
1.2800	0.6725	0.8607
1.3000	0.6700	0.8710
1.3200	0.6676	0.8812
1.3400	0.6651	0.8913
1.3600	0.6627	0.9012
1.3800	0.6602	0.9111
1.4000	0.6577	0.9208
1.4200	0.6552	0.9304
1.4400	0.6527	0.9399
1.4600	0.6501	0.9492
1.4800	0.6476	0.9584

```
    1.5000    0.6450    0.9675
    1.5200    0.6424    0.9764
    1.5400    0.6397    0.9852
    1.5600    0.6371    0.9938
    1.5800    0.6344    1.0023
    1.6000    0.6316    1.0106
    1.6200    0.6288    1.0187
    1.6400    0.6260    1.0267
    1.6600    0.6232    1.0344
    1.6800    0.6202    1.0420
    1.7000    0.6173    1.0493
    1.7200    0.6142    1.0565
    1.7400    0.6111    1.0633
    1.7600    0.6079    1.0700
    1.7800    0.6047    1.0763
    1.8000    0.6013    1.0823
    1.8200    0.5978    1.0880
    1.8400    0.5942    1.0934
    1.8600    0.5905    1.0983
    1.8800    0.5866    1.1028
    1.9000    0.5825    1.1067
    1.9200    0.5782    1.1101
    1.9400    0.5736    1.1128
    1.9600    0.5687    1.1147
    1.9800    0.5635    1.1157
    2.0000    0.5577    1.1154
    2.0200    0.5514    1.1137
    2.0400    0.5441    1.1101
    2.0600    0.5358    1.1037
    2.0800    0.5256    1.0933
    2.1000    0.5125    1.0763
    2.1200    0.4933    1.0458
    2.1400    0.4535    0.9705
    ];
FCST.ipmaxinp                  =  100;
FCST.Er                        =  1.1782;
FCST.j0C                       =  0.0001;
FCST.alphaC                    =  0.22;
FCST.j0A                       =  0.1;
FCST.alphaA                    =  0.5;
FCST.ASR                       =  0.04;
FCST.jL                        =  2.45;
FCST.jleak                     =  0.30;
FCST.C                         =  0.35;
FCST.PMAT = [
    1        1.25     1.5      2.0        2.5
    0.0001   0.0001   0.0001   0.0001     0.0001
    0.1800   0.1900   0.2000   0.2100     0.2200
    0.1000   0.1000   0.1000   0.1000     0.1000
    0.5000   0.5000   0.5000   0.5000     0.5000
    0.0400   0.0400   0.0400   0.0400     0.0400
    1.7500   1.8500   1.9500   2.2500     2.4500
    0.1500   0.1500   0.2000   0.2500     0.3000
    0.0300   0.0350   0.0350   0.0350     0.0350
    ];
FCST.crh                      =  100;
FCST.arh                      =  100;
FCST.deltai                   =  0.02;
% temp corr
FCST.ASR0                     =  0.12;      % ref value
FCST.ASR0t                    =  80;        % ref temp
FCST.ASRtk2                   =  0.0006;
FCST.ASRtk1                   =  0.002;
FCST.jL0                      =  0.73;      % ref value
FCST.jL0t                     =  80;        % ref temp
FCST.jLtk2                    = -0.0015;
FCST.jLtk1                    = -0.003;
% humidity corrections in ASR and jL
FCST.ASR0arh                  =  100;        % ref anode rh
FCST.ASR0crh                  =  100;        % ref cathode rh
FCST.ASRrhk1a                 = -0.005;
FCST.ASRrhk2a                 =  0.;
FCST.ASRrhk1a1c               =  0.;
FCST.ASRrhk1a2c               =  2.0e-6;
FCST.ASRrhk2a2c               =  0.;
FCST.ASRrhk2a1c               =  0.;
FCST.ASRrhk2c                 =  7.0e-5;
FCST.ASRrhk1c                 =  0.0021;
FCST.jL0arh                   =  100;
FCST.jL0crh                   =  100;
FCST.jLrhk1a                  =  0.;
FCST.jLrhk2a                  =  0.;
FCST.jLrhk1a1c                =  0.;
FCST.jLrhk1a2c                =  0.;
FCST.jLrhk2a2c                =  0.;
FCST.jLrhk2a1c                =  0.;
FCST.jLrhk2c                  = -0.0001;
FCST.jLrhk1c                  = -0.0076;
% other corrections
FCST.icorr                    =  0;         % ic corr (0/1)
FCST.TFi                      =  1.0;       % changes ic to ic.Tfi
FCST.vcorr                    =  0;         % vc corr for alt (0/1)
FCST.deltav                   = -0.01;      % cell to stack correction
FCST.vrate1000ft              = -0.0056;    % percent drop / 1000 ft
FCST.vdropaboveft             =  1000;      % above 1000-ft


% stack weight variables
FCST.TFwstack                 = 1.0;       % Tech factor stack weight
FCST.dWwstack                 = 0.0;       % correction stack weight
FCST.TFsstack                 = 1.0;       % Tech factor stack volume
FCST.dSsstack                 = 0.0;       % correction stack volume
FCST.ioptsw                   = 1;         % 0: simple, 1: detailed
FCST.tcell                    = 0.001381;  % cell thickness, m
FCST.dcell                    = 2584;      % cell density, kg/m3
FCST.porosity                 = 1.0;
```

```
% Component breakdown
FCST.dendplate                = 1600;    % end plate, Al
FCST.dinsulplate              = 1420;    % polyoxy-methylene
FCST.dcollplate               = 8940;    % collector plate, Cu
FCST.dpem                     = 500;     % membrane, Nafion
FCST.dgdl                     = 440;     % gas diffusion layer
FCST.dbipolarplate            = 2160;    % graphite
FCST.dgasket                  = 1000;    % rubber
FCST.Aendplatefrac            = 1.1;     % Area = 1.1 Active area
FCST.tendplatecons            = 0.0;     % thickness constant
FCST.tendplatefrac            = 0.020;   % thickness = 0.05 tunit
FCST.Ainsulplatefrac          = 1.1;     % fraction active area
FCST.tinsulplatecons          = 0.0;     % thickness constrant
FCST.tinsulplatefrac          = 0.005;   % thickness = 0.02 tunit
FCST.Acollplatefrac           = 0.5;     % fraction active area
FCST.tcollplatecons           = 0.0;     % thickness constant
FCST.tcollplatefrac           = 0.005;   % thickness = 0.01 tunit
FCST.Wfastener                = 0.0;     % bolt/clamp, steel, 7850 kg/m3
FCST.tpem                     = 0.00010; % membrane, Nafion
FCST.tgdl                     = 0.00020; % gas diffusion layer
FCST.tbipolarplate            = 0.00100; % graphite; avg seal + mea region
FCST.pbipolarplate            = 0.60000; % porosity due to channels
FCST.Abipolarplatefrac        = 1.10;    % fraction active area
FCST.tgasket                  = 0.00200; % rubber
FCST.Agasketfrac              = 0.03;    % fraction active area
FCST.Wstackother              = 0.00;    % any other weight



% H2 tank system
FCST.fH2utilization           = 1.0;
FCST.itanktype                = 3;
FCST.ioptf                    = 1;     % fuel option; 1-h2, 2-endurance
FCST.Whydrogen                = 5;
FCST.Endurance_min            = 60;
% 350 bar: 0.0515, 17.2; 700 bar: 0.057, 40;
% cryo-compressed Al: 0.09, 41; cryo-compressed steel: 0.055, 41
% liq: 0.075, 65;
FCST.h2tankwfg                = 0.057; % kg/kg wt fraction gravimetric
FCST.h2tankwfs                = 40.00; % kg/m3 wt fraction volumetric
FCST.h2tankBar                = 700;   % tank storage pressure
FCST.h2tankTC                 = 15;    % tank storage temperature C
FCST.h2tankSov                = 0.10;  % tank volume overhead
FCST.FuelFilterW0             = 0.;
FCST.FuelFilterfW             = 0.;
FCST.FuelRegulatorW0          = 0;     % kg
FCST.FuelRegulatorfW          = 0.01;  % fraction fuel sys wt
FCST.FuelPowerW0              = 0;     % kg wt of power supply
FCST.FuelPowerfW              = 0.01;  % fraction fuel sys wt
FCST.FuelPlumbW0              = 0;     % kg plumbing constant wt
FCST.FuelPlumbWw              = 0;     % kg/m plumb wt per length
FCST.FuelPlumbWl              = 0;     % m plumb length
FCST.Pfuel0                     = 0;     % prescribed power
FCST.Pfuelk                     = 0;     % prop to fuel flow rate
FCST.Pfuele                     = 0;     % expo to fuel flow rate
FCST.FGErho                     = 722.13;% density of gasoline
FCST.FGEMJkg                    = 44;    % heating value of gasoline, MJ/kg
FCST.FGEeta                     = 0.25;  % efficiency of FGE fuel





% Air system variables
FCST.fAirFlowRate               = 2.5;
FCST.DPstack                    = 30000;     % pres drop in stack, Pa
% Blower
FCST.iBlower                    = 0;
FCST.BlowEta                    = 0.75;
FCST.BlowW0                     = 0;
FCST.BlowWtf                    = 1.0;
FCST.BlowWk                     = 1/1000;
FCST.BlowWfe                    = 0;
FCST.BlowWpe                    = 1;
FCST.BlowS0                     = 0;
FCST.BlowStf                    = 1.0;
FCST.BlowSk                     = 1/3e6;
FCST.BlowSfe                    = 0;
FCST.BlowSpe                    = 1;
% Compressor
FCST.iCompressor                = 1;
FCST.CompEta                    = 0.75;
FCST.CompW0                     = 0.;
FCST.CompWtf                    = 1.0;
FCST.CompWk                     = 7e-4;   % 122 kg/kg/s or 7e-4 kg/W
FCST.CompWfe                    = 0;
FCST.CompWpe                    = 1;
FCST.CompS0                     = 0.;
FCST.CompStf                    = 1.0;
FCST.CompSk                     = 4.14e-7; % 0.0722 m3/kg/s or 4.14e-7 m3/W
FCST.CompSfe                    = 0;
FCST.CompSpe                    = 1;
% Turbo-expander (axial-flow turbine)
FCST.iTurbine                   = 1;
FCST.TurbEta                    = 0.75;
FCST.TurbW0                     = 0.;
FCST.TurbWtf                    = 1.0;
FCST.TurbWk                     = 0;
FCST.TurbWfe                    = 0;
FCST.TurbWpe                    = 0;
FCST.TurbS0                     = 0.;
FCST.TurbStf                    = 1.0;
FCST.TurbSk                     = 0;
FCST.TurbSfe                    = 0;
FCST.TurbSpe                    = 0;
```

```
% accessories
FCST.AirFilterW0                = 0;     % kg
FCST.AirFilterfW                = 0.01;  % fraction Air sys wt
FCST.AirRegulatorW0             = 0;     % kg
FCST.AirRegulatorfW             = 0.01;  % fraction Air sys wt
FCST.AirPowerW0                 = 0;     % kg controller
FCST.AirPowerfW                 = 0.05;  % fraction Air sys wt
FCST.AirPlumbW0                 = 0;     % kg plumbing constant wt
FCST.AirPlumbWw                 = 0;     % kg/m plumb wt per length
FCST.AirPlumbWl                 = 0;     % m plumb length
FCST.AirHumidifierW0            = 0;     % kg
FCST.AirHumidifierfW            = 0.05;  % fraction Air sys wt



% Heat system variables
% low temp system (air pre-cooler, only if compressor used)
FCST.iLTR                      = 1;     % 0-Pdsg load, 1-Pmax load
FCST.ltRadiatorhr              = 1015;  % rad conv ht trans coeff W/m2/K
FCST.ltRadiatorTa              = 20;    % rad Ta in C
FCST.ltRadiatorTr              = 95;    % rad Tr in C
FCST.ltRadiatorepr             = 0.8;   % rad emissivity
FCST.ltRadiatorW0              = 0;     % kg
FCST.ltRadiatorWk              = 3.5405;% kg/W (f=0) 1e-4 or kg/m2 (e=0) 3.5405
FCST.ltRadiatorWe              = 0;
FCST.ltRadiatorWf              = 1;
FCST.ltRadiatorS0              = 0;     % m3
FCST.ltRadiatorSk              = 0.035; % m3/W (f=0) 1e-7 or thick (e=0) 0.035
FCST.ltRadiatorSe              = 0;
FCST.ltRadiatorSf              = 1;
FCST.ltCoolantCp               = 4180;  % J/kg-K (water)
FCST.ltCoolantdT               = 10;    % coolant deltaT (max 15)
FCST.ltCoolantW0               = 0;     % kg
FCST.ltCoolantWk               = 20;    % kg/W (f=0) 1e-6 or time sec (e=0) 5
FCST.ltCoolantWe               = 0;
FCST.ltCoolantWf               = 1;
FCST.ltCoolerW0                = 0;     % kg
FCST.ltCoolerWk                = 1e-6;  % kg/W
FCST.ltCoolerWe                = 1;     %
FCST.ltFilterW0                = 0;     % kg
FCST.ltFilterfW                = 0.01;  % fraction Heat sys wt
FCST.ltRegulatorW0             = 0;     % kg
FCST.ltRegulatorfW             = 0.01;  % fraction Heat sys wt
FCST.ltPowerW0                 = 0;     % kg
FCST.ltPowerfW                 = 0.01;  % fraction Heat sys wt
FCST.ltPlumbW0                 = 0;     % kg plumbing constant wt
FCST.ltPlumbWw                 = 0;     % kg/m plumb wt per length
FCST.ltPlumbWl                 = 0;     % m plumb length
FCST.Plt0                      = 0;     % any power needed
FCST.Pltk                      = 300;   % 0.01-heat, kPa-flow (water)
FCST.Plte                      = 0;     % expo to heating
FCST.Pltf                      = 1;     % expo to flow
```

```
% Heat system variables
% high temp system (stack cooling)
FCST.iHV                      = 0;     % 0-LHV, 1-HHV
FCST.iHTR                     = 0;     % 0-Pdsg load, 1-Pmax load
FCST.htStackfd                = 0;     % stack frac stack heat dissipated
FCST.htStackhs                = 3;     % stack conv ht trans coeff W/m2/K
FCST.htStackTa                = 40;    % stack Ta in C
FCST.htStackeps               = 0.8;   % stack emissivity
FCST.htStackfs                = 0.6;   % stack frac surface/active area
FCST.htRadiatorhr             = 1015;  % rad conv ht trans coeff W/m2/K
FCST.htRadiatorTa             = 40;    % rad Ta in C
FCST.htRadiatorTr             = 95;    % rad Tr in C
FCST.htRadiatorepr            = 0.8;   % rad emissivity
FCST.htRadiatorW0             = 0;     % kg
FCST.htRadiatorWk             = 3.5405;% kg/W (f=0) 1e-5 or kg/m2 (e=0) 3.5405
FCST.htRadiatorWe             = 0;
FCST.htRadiatorWf             = 1;
FCST.htRadiatorS0             = 0;     % m3
FCST.htRadiatorSk             = 0.035; % m3/W (f=0) 1e-7 or thick (e=0) 0.035
FCST.htRadiatorSe             = 0;
FCST.htRadiatorSf             = 1;
FCST.htCoolantCp              = 4180;  % J/kg-K (water)
FCST.htCoolantrho             = 997;   % kg/m3 (water)
FCST.htCoolantdT              = 10;    % coolant Tc in C
FCST.htCoolantW0              = 0;     % kg
FCST.htCoolantWk              =20;     % kg/W (f=0) 1e-6 or time sec (e=0) 5
FCST.htCoolantWe              = 0;
FCST.htCoolantWf              = 1;
FCST.htFilterW0               = 0;     % kg
FCST.htFilterfW               = 0.01;  % fraction Heat sys wt
FCST.htRegulatorW0            = 0;     % kg
FCST.htRegulatorfW            = 0.01;  % fraction Heat sys wt
FCST.htPowerW0                = 0;     % kg
FCST.htPowerfW                = 0.01;  % fraction Heat sys wt
FCST.htPlumbW0                = 0;     % kg plumbing constant wt
FCST.htPlumbWw                = 0;     % kg/m plumb wt per length
FCST.htPlumbWl                = 0;     % m plumb length
FCST.Pht0                     = 0;     % any power needed
FCST.Phtk                     = 300;   % 0.01-heat, kPa-flow (water)
FCST.Phte                     = 0;     % expo to heat
FCST.Phtf                     = 1;     % expo to coolant flow


% Water system variables
FCST.rhowater                 = 997;   % kg/m3
FCST.waterstoragefrac         = 0.2;   % fraction of produce stored
FCST.waterstoragehour         = 0.5;   % accumulated over this time
FCST.watertankSov             = 0;     % Stank = (1+Sov).Swater
FCST.watertankWov             = 0;     % Wtank = (1+Wov).Wwater
FCST.waterFilterW0            = 0;     % kg
FCST.waterFilterfW            = 0;     % fraction Heat sys wt
FCST.waterRegulatorW0         = 0;     % kg
FCST.waterRegulatorfW         = 0;     % fraction water sys wt
FCST.waterPowerW0             = 0;     % kg
FCST.waterPowerfW             = 0;     % fraction water sys wt
FCST.waterPlumbW0             = 0;     % kg plumbing constant wt
FCST.waterPlumbWw             = 0;     % kg/m plumb wt per length
FCST.waterPlumbWl             = 0;     % m plumb length
FCST.Pwater0                  = 0;     % any power needed
FCST.Pwaterk                  = 1e6;   % prop to liq water flow
FCST.Pwatere                  = 1;     % expo to liq water flow



% electrical system variables
FCST.elecControlW0           = 0;     % kg
FCST.elecControlWk           = 0.01;  % prop to stack current
FCST.elecControlWe           = 1.0;   % expo to stack current
FCST.elecRegulatorW0         = 0;     % kg
FCST.elecRegulatorfW         = 0;     % fraction elec sys wt
FCST.elecPowerW0             = 0;     % kg
FCST.elecPowerfW             = 0.01;  % fraction elec sys wt
FCST.elecPlumbW0             = 0;     % kg plumbing constant wt
FCST.elecPlumbWwk            = 0.01;  % prop to stack current
FCST.elecPlumbWwe            = 1.0;   % expo to stack current
FCST.elecPlumbWl             = 0;     % m plumb length
FCST.Pelec0                  = 0;     % any power needed
FCST.Peleck                  = 0.02;  % prop to stack power
FCST.Pelece                  = 1.0;   % expo to stack power


% additional gross overhead fractions
FCST.fovpower                  = 0.0;
FCST.fovweight                 = 0.0;
FCST.fovvolume                 = 0.0;


% output options
FCST.iplotVIP                  = 1;
FCST.iprntSCR                  = 1;



% =====================================================
% O U T P U T S - Design
% =====================================================


% overall weight, power, energy, and specific quantities
% stack efficiencies
FCST.etadsgHHV                 = 0.;
FCST.etadsgLHV                 = 0.;
FCST.etamaxHHV                 = 0.;
FCST.etamaxLHV                 = 0.;
% net system efficiencies
```

```
FCST.EtadsgHHV                 = 0.;
FCST.EtadsgLHV                 = 0.;
FCST.EtamaxHHV                 = 0.;
FCST.EtamaxLHV                 = 0.;
% Fuel gallon equivalents
FCST.FGEHHV                    = 0.;  % chemical
FCST.FGELHV                    = 0.;  % chemical
FCST.FGEHHVtrue                = 0.;  % engine (chemical x efficiency)
FCST.FGELHVtrue                = 0.;  % engine (chemical x efficiency)

FCST.Wtotal                    = 0.;
FCST.Stotal                    = 0.;
FCST.Wstacksys                 = 0.;
FCST.Sstacksys                 = 0.;
FCST.DFstacksys                = 0.;

FCST.SFCPdsg_kgkWh             = 0.;
FCST.SFCPmax_kgkWh             = 0.;
FCST.SFCPdsg_kgkWhe            = 0.;
FCST.SFCPmax_kgkWhe            = 0.;

FCST.POWERdsg_kWe              = 0.;
FCST.POWERmax_kW               = 0.;
FCST.POWERmax_kWe              = 0.;
FCST.ENERGY_Pdsg_kWh           = 0.;
FCST.ENERGY_Pdsg_kWhe          = 0.;
FCST.ENERGY_Pmax_kWh           = 0.;
FCST.ENERGY_Pmax_kWhe          = 0.;

FCST.SPower_Pdsg_Stack_kWkg    = 0.;
FCST.SPower_Pdsg_Stack_kWekg   = 0.;
FCST.SPower_Pmax_Stack_kWkg    = 0.;
FCST.SPower_Pmax_Stack_kWekg   = 0.;
FCST.SPower_Pdsg_Stacksys_kWkg = 0.;
FCST.SPower_Pdsg_Stacksys_kWekg= 0.;
FCST.SPower_Pmax_Stacksys_kWkg = 0.;
FCST.SPower_Pmax_Stacksys_kWekg= 0.;
FCST.SPower_Pdsg_kWkg          = 0.;
FCST.SPower_Pdsg_kWekg         = 0.;
FCST.SPower_Pmax_kWkg          = 0.;
FCST.SPower_Pmax_kWekg         = 0.;

FCST.EDensity_Pdsg_kWhL        = 0.;
FCST.EDensity_Pdsg_kWheL       = 0.;
FCST.EDensity_Pmax_kWhL        = 0.;
FCST.EDensity_Pmax_kWheL       = 0.;

FCST.SEnergy_Pdsg_kWhkg        = 0.;
FCST.SEnergy_Pdsg_kWhekg       = 0.;
FCST.SEnergy_Pmax_kWhkg        = 0.;
FCST.SEnergy_Pmax_kWhekg       = 0.;
```

```
% stack variables
% performance
FCST.ivdata                    = 0.;
FCST.Eh                        = 0.;
FCST.Er                        = 0.;
FCST.Aactive_cm2               = 0.;
FCST.Ncell                     = 0.;
FCST.Istackdsg                 = 0.;
FCST.Vstackdsg                 = 0.;
FCST.Istackmax                 = 0.;
FCST.Vstackmax                 = 0.;
FCST.IstackPmax                = 0.;
FCST.VstackPmax                = 0.;
FCST.SP                        = 0.;
FCST.vcdsg                     = 0.;
FCST.icdsg                     = 0.;
FCST.pcdsg                     = 0.;
FCST.etadsg                    = 0.;
FCST.etavdsgHHV                = 0.;
FCST.etavdsgLHV                = 0.;


% size
FCST.Sstack                   = 0.;
FCST.Lstack                   = 0.;
FCST.Astack                   = 0.;
FCST.DFstack                  = 0.;
Wstack.total                  = 0.;
Wstack.endplate               = 0.;
Wstack.insulplate             = 0.;
Wstack.collplate              = 0.;
Wstack.unit                   = 0.;
Wstack.cell                   = 0.;
Wstack.mea                    = 0.;
Wstack.bipolarplate           = 0.;
Wstack.pem                    = 0.;
Wstack.gdl                    = 0.;
Wstack.gasket                 = 0.;
Wstack.other                  = 0.;
FCST.Wstack                   = Wstack;
FCST.Wstackk0                 = 0.;
FCST.Wstackk1                 = 0.;


% Air system variables
FCST.Sblower                  = 0.;
FCST.Scomp                    = 0.;
FCST.Sturb                    = 0.;
Wair.total                    = 0.;
Wair.blower                   = 0.;
Wair.comp                     = 0.;
Wair.turb                     = 0.;
Wair.precooler                = 0.;
Wair.humidifier              = 0.;
Wair.filter                  = 0.;
Wair.plumbing                = 0.;
Wair.power                   = 0.;
Wair.regulator               = 0.;
FCST.Wair                    = Wair;
FCST.Pair_kW                 = 0.;
FCST.Pairmax_kW              = 0.;
FCST.Pblower_kW              = 0.;
FCST.Pblowermax_kW           = 0.;
FCST.Pcomp_kW                = 0.;
FCST.Pcompmax_kW             = 0.;
FCST.Pturb_kW                = 0.;
FCST.Pturbmax_kW             = 0.;
FCST.MassAirrate_kgs         = 0.;
FCST.MassAirrate_max_kgs     = 0.;
FCST.MassAirrate_Ls          = 0.;
FCST.MassAirrate_max_Ls      = 0.;
FCST.MassAirrate_out_kgs     = 0.;
FCST.MassAirrate_out_max_kgs = 0.;
FCST.Humidity_ratio          = 0.;
FCST.MassWaterrate_kgs       = 0.;
FCST.MassVaporrate_kgs       = 0.;
FCST.MassLiquidrate_kgs      = 0.;



% Fuel system variables
FCST.Sh2tank                 = 0.;
Wfuel.total                  = 0.;
Wfuel.H2                     = 0.;
Wfuel.tank                   = 0.;
Wfuel.filter                 = 0.;
Wfuel.plumbing               = 0.;
Wfuel.power                  = 0.;
Wfuel.regulator              = 0.;
FCST.Wfuel                   = Wfuel;
FCST.Pfuel_kW                = 0.;
FCST.Pfuelmax_kW             = 0.;
FCST.MassH2rate_kgs          = 0.;
FCST.MassH2rate_Pmax_kgs     = 0.;
FCST.SFCPdsg_kgkWh           = 0.;
FCST.SFCPmax_kgkWh           = 0.;


% low temp cooling system variables (air pre-cooler)
Wlt.total                  = 0.;
Wlt.filter                 = 0.;
Wlt.plumbing               = 0.;
Wlt.regulator              = 0.;
Wlt.coolant                = 0.;
Wlt.radiator               = 0.;
Wlt.power                  = 0.;
```

```
FCST.Wlt                    = Wlt;
FCST.Plt_kW                 = 0.;
FCST.Pltmax_kW              = 0.;
FCST.Sltradiator            = 0.;
FCST.Altradiator            = 0.;
FCST.HEATINGlt_kW           = 0.;
FCST.HEATINGlt_max_kW       = 0.;
FCST.Cflowmaxlt             = 0.;
FCST.Cflowdsglt             = 0.;
FCST.Cflowlt                = 0.;



% high temp cooling system variables (stack cooler)
Wht.total                   = 0.;
Wht.filter                  = 0.;
Wht.plumbing                = 0.;
Wht.regulator               = 0.;
Wht.coolant                 = 0.;
Wht.radiator                = 0.;
Wht.power                   = 0.;
FCST.Wht                    = Wht;
FCST.Pht_kW                 = 0.;
FCST.Phtmax_kW              = 0.;
FCST.Shtradiator            = 0.;
FCST.Ahtradiator            = 0.;
FCST.HEATINGstack_kW        = 0.;
FCST.HEATINGairin_kW        = 0.;
FCST.HEATINGairout_kW       = 0.;
FCST.HEATINGstack_max_kW    = 0.;
FCST.HEATINGairin_max_kW    = 0.;
FCST.HEATINGairout_max_kW   = 0.;
FCST.HEATINGdiscon_kW       = 0.;
FCST.HEATINGdisrad_kW       = 0.;
FCST.HEATINGhtc_max_kW      = 0.;
FCST.HEATINGhtc_kW          = 0.;
FCST.Cflowmaxht             = 0.;
FCST.Cflowdsght             = 0.;
FCST.Cflowht                = 0.;


% Water system
FCST.Swatertank               = 0.;
Wwater.total                  = 0.;
Wwater.tank                   = 0.;
Wwater.plumbing               = 0.;
Wwater.power                  = 0.;
Wwater.regulator              = 0.;
FCST.Wwater                   = Wwater;
FCST.Pwater_kW                = 0.;
FCST.Pwatermax_kW             = 0.;
FCST.MassTankrate_kgs         = 0.;
```

```
% Electrical system
Welec.total                  = 0.;
Welec.controller             = 0.;
Welec.power                  = 0.;
Welec.plumbing               = 0.;
FCST.Welec                   = Welec;
Welec.Pelec_kW               = 0.;
Welec.Pelecmax_kW            = 0.;


% Other
FCST.Wother                  = 0.;
FCST.Pother_kW               = 0.;
FCST.Sother                  = 0.;




% =====================================================
% I N P U T S - Operations
% =====================================================

FCOP.Istack               = 0;
FCOP.iatm                 = 0;
FCOP.altitude             = 0000*0.3048;
FCOP.temperatureC         = 0;
FCOP.BlowEta              = 0.75;
FCOP.CompEta              = 0.75;
FCOP.TurbEta              = 0.75;
FCOP.vcorr                = 0;         % vc correction
FCOP.vrate1000ft          = -0.005;    % percent drop / 1000 ft
FCOP.vdropaboveft         = 1000;      % above 1000-ft
FCOP.arh                  = FCST.arh;
FCOP.crh                  = FCST.arh;


% =====================================================
% O U T P U T S - Operations
% =====================================================

FCOP.ivdata               = 0;
FCOP.pressure             = 0;
FCOP.Vstack               = 0;
FCOP.SFC_kgkWh            = 0;
FCOP.POWER_kW             = 0;
FCOP.Pair_kW              = 0;
FCOP.Pfuel_kW             = 0;
FCOP.Plt_kW               = 0;
FCOP.Pht_kW               = 0;
FCOP.Pwater_kW            = 0;
FCOP.Pelec_kW             = 0;
FCOP.Pother_kW            = 0;
FCOP.POWER_kWe             = 0;
FCOP.Pbop_kW               = 0;
FCOP.Pblower_kW            = 0;
FCOP.Pcomp_kW              = 0;
FCOP.Pturb_kW              = 0;
FCOP.etac                  = 0;
FCOP.etas                  = 0;
FCOP.Vstack                = 0;
FCOP.MassH2rate_kgs        = 0;
FCOP.MassAirrate_kgs       = 0;
FCOP.MassAirrate_Ls        = 0;
FCOP.MassO2rate_kgs        = 0;
FCOP.MassAirrate_out_kgs   = 0;
FCOP.MassWaterrate_kgs     = 0;
FCOP.MassLiquidrate_kgs    = 0;
FCOP.MassTankrate_kgs      = 0;
FCOP.MassVaporrate_out_kgs = 0;
FCOP.Humidity              = 0;
FCOP.HEATINGairin_kW       = 0;
FCOP.HEATINGstack_kW       = 0;
FCOP.SFCvsPe               = 0;  % kg/kWe-hr vs kWe
FCOP.H2flowvsPe            = 0;  % kg/hr vs kg/hr
FCOP.Pevs1000ft            = 0;  % [kW, h in 1000-ft]
FCOP.SFCvs1000ft           = 0;  % [kg/kWhe in 1000-ft]
FCOP.H2invs1000ft          = 0;  % [kg/s, h in 1000-ft]
```