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THE INFLUENCE OF PRECESSION OF EARTH RENDEZVOUS ORBITS ON LUNAR MISSION REQUIREMENTS

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SUMMARY

A study has been made to determine the influence of precession of an earth rendezvous orbit due to the earth's oblateness on lunar mission requirements. The nature of the regression along the earth-moon plane of the nodal line formed by the intersection of the earth rendezvous orbital and earth-moon planes is investigated. This investigation applies for the years 1964 to 1972. With this regression taken into account, two methods for correcting for more than anticipated delay times in earth orbit are considered. The two methods considered are a plane change and an increase in injection velocity. Results of the two methods indicate that the plane-change technique requires considerably less additional velocity. Based on the results of the analysis, launch windows corresponding to lunar positions or declinations throughout the month are presented.

INTRODUCTION

The concept of orbital rendezvous has been receiving, and will continue to receive, serious consideration for application in lunar and planetary space exploration missions requiring large payloads. By either assembling spacecraft systems in orbit about the earth, or fueling already assembled vehicles in orbit, missions well beyond the payload capabilities of launch vehicles currently under development can be accomplished.

For a lunar mission, the assembled vehicle must be injected from the earth rendezvous orbit into a trajectory which will carry it to a desired position relative to the moon. Since approximately 70 percent of the velocity required for the lunar trajectory has already been attained in the earth rendezvous orbit, it is desirable, and most efficient, to apply the additional injection velocity in the plane of the earth rendezvous orbit. If this is the case, the plane of the lunar trajectory will be the same as that of the earth rendezvous orbit at the time of injection. The problem for design of lunar trajectories is to insure that the vehicle and moon both arrive sufficiently close to the node line of the trajectory plane and the moon-earth orbital plane at the same time.

The fact that the earth rendezvous orbit plane is subject to a precession about the polar axis of the earth because of the second harmonic of the gravitational potential influences the lunar trajectory design. This precession, which is a regression of the node of the rendezvous orbital plane along the earth equatorial plane, also results in a regression of the node of the earth rendezvous orbital plane along the moon-earth orbital plane. The regression must be taken into account in determining at what times of the month the position of the moon will be such as to permit use of a lunar trajectory with injection in the plane of the earth rendezvous orbit.

In designing the initial space orientation of the plane of the earth rendezvous orbit, allowance will be made for the precession of the orbit during the time period required for the completion of the rendezvous operation, for check-out of the vehicle system, and for preparation for injection into the lunar trajectory. For nominal conditions of rendezvous orbit establishment and anticipated delay time, a predetermined time can thus be established for injection from the rendezvous orbit into the lunar trajectory. This predetermined time will be chosen such that the additional injection velocity is applied in the plane of the earth rendezvous orbit and for the earth-moon transit time specified, such that the vehicle will arrive at the desired position relative to the moon.

The question arises as to what penalties exist in the event of off-nominal situations, either in establishment of the earth rendezvous orbit plane or, particularly, in the event of greater than anticipated delay time in the rendezvous orbit. It is clear that the precession of the earth rendezvous orbit plane during the additional delay time will require modifications in the lunar trajectory injection velocity, application of this velocity out of the plane of the rendezvous orbit, or other considerations. The purpose of the present analysis is to define the situation which exists in the event of prolonged delay time in the rendezvous orbit, to investigate procedures for obtaining acceptable lunar trajectories for the off-nominal situations, and to estimate the additional velocity requirements.

SYMBOLS

Illustrations of some of the angular parameters defined are given in figures 1 and 3. In this paper distances are measured in international statute miles (1 international statute mile = 1.60944 kilometers).

a	semimajor axis of orbit, miles
e	eccentricity
i	inclination of earth rendezvous orbit to earth's equatorial plane, deg
J	coefficient of second harmonic of earth's gravitational potential, 0.001624
R	equatorial radius of earth, 3963.34 miles
r	radial distance of vehicle from center of earth, miles

r_p	radial distance of vehicle from center of earth at perigee, miles
T	delay time in earth rendezvous orbit from nominal injection time, days or hours
V	injection velocity, ft/sec
V_p	parabolic velocity at injection altitude, 35303.3 ft/sec at 315 miles above the earth's surface
V_θ	normal component of vehicle velocity at some point along trajectory, ft/sec
ΔV	velocity increment needed to correct for off-nominal situation, ft/sec
δ_M	inclination of moon's orbit to earth's equatorial plane, deg
θ	true anomaly, deg
ρ	angular travel of vehicle between point of plane change and position of moon, deg
σ	angle through which normal component of velocity is rotated when plane change is made, deg
ϕ	angle between earth rendezvous orbital plane and earth moon plane, deg
$\Delta\phi$	regressional rate of earth rendezvous orbital plane about polar axis of earth, radian/revolution of satellite
$\dot{\omega}$	angular velocity of moon, approximately 13.2° per day
Ω_E	angle measured in equatorial plane from moon's ascending node to node of earth rendezvous orbital plane and equatorial plane, deg
Ω_M	angle measured in earth-moon plane from moon's ascending node to node of earth rendezvous orbital plane and earth-moon plane, deg
Ω_T	sum of regression of node along earth-moon plane and forward motion of moon for a given delay time in orbit, deg

Subscripts:

0	nominal value
1	value at point of plane change

GENERAL CONSIDERATIONS

Precession of Node of Earth Rendezvous Orbital Plane and Earth Equatorial Plane

The nodal line defined by the intersection of the earth orbital plane of a near satellite and the earth's equatorial plane will experience a regression along the equator due to the oblateness of the earth (or second harmonic of the earth's gravitational potential). The regression of the nodes is at a constant mean rate, the magnitude of which is determined by the orbital elements of the satellite orbit. This regressional rate for $i > 0$ is given in reference 1 as:

$$\Delta\phi = \frac{2\pi J R^2 \cos i}{a^2\left(1 - e^2\right)^2} \tag{1}$$

The value of the inclination i chosen for this analysis is 30°. This choice of inclination is consistent with the findings of reference 2 which states that, for once a day or more frequent rendezvous from the earth, it is desirable to have the orbit inclination somewhat greater than the launch latitude. Since the launch latitude will be at about 28.5° N., corresponding to Cape Canaveral, a 30° inclination would be appropriate. A somewhat arbitrary choice of a circular orbit at 315 statute miles was chosen for the rendezvous orbit of this investigation. Three values of δ_M, the inclination of the earth-moon plane to the earth's equatorial plane (also the magnitude of the maximum lunar declination), are treated in this analysis. These values are 23.0°, 25.0°, and 28.5° and correspond to the conditions between the years 1964 and 1972.

From equation (1) it is found that, for the assumed altitude and inclination, the node of the earth satellite orbital plane and the equatorial plane will regress at a mean rate of 6.6° per day along the equator in the direction indicated in figure 1(a).

Precession of Node of Earth Rendezvous Orbital Plane and Earth-Moon Plane

The node defined by the intersection of the earth rendezvous orbital plane and the earth-moon plane also experiences a regression along the earth-moon plane. The rate at which this node regresses, however, is not constant with time since the earth-moon plane is inclined at an angle δ_M to the equatorial plane.

The location of the nodal points of the earth-moon and earth's equatorial planes with the earth rendezvous orbital plane is given at any time by the angles Ω_M and Ω_E measured in the earth-moon and equatorial planes, respectively. These angles are defined in figure 1(b) and are positive in the direction of regression.

The following expression which relates these nodal positions can be obtained from figure 1(b) by applying the laws of spherical trigonometry.

$$\Omega_M = \cot^{-1}\left(\frac{\cos \delta_M \cos \Omega_E - \sin \delta_M \cot i}{\sin \Omega_E}\right) \tag{2}$$

Also defined in figure 1(b) are the inclination of the earth rendezvous orbital plane i, the inclination of the earth-moon plane δ_M and the angle between the earth rendezvous orbital and earth-moon planes ϕ. The angle between earth rendezvous orbital and earth-moon planes varies with Ω_E since the moon's orbital plane is inclined to the equatorial plane. The value of the angle ϕ at any time is a function of the position of the nodes and can be obtained from figure 1(b) with the laws of spherical trigonometry.

$$\phi = \sin^{-1}\left(\frac{\sin i \sin \Omega_E}{\sin \Omega_M}\right) \tag{3}$$

The variations of Ω_M and ϕ with Ω_E as obtained from equations (2) and (3) are shown in figure 2 for values of δ_M of 23.0°, 25.0°, and 28.5°. The curves for Ω_M in figure 2 show in every case that the node of the earth-moon plane with the earth rendezvous orbital plane moves more rapidly than that of the equatorial plane with the earth rendezvous orbital plane in a small region on either side of the moon's ascending node $\left(\Omega_E = \Omega_M = 0\right)$ but moves more slowly in a larger region on either side of the moon's decending node $\left(\Omega_E = \Omega_M = 180^\circ\right)$. For instance, from figure 2(c), it can be seen that a 13° (about 2 days of regression) change in Ω_E from a nominal value of Ω_E of zero results in approximately an 80° change in Ω_M. (Comparisons of figs. 2(a) to 2(c) for Ω_M indicate that this initial rate decreases with decreasing δ_M.) However, if the nominal values of Ω_E and Ω_M are 180°, the same 13° change in Ω_E results in only 7° change in Ω_M.

The variation of ϕ for 360° of regression of the earth rendezvous orbital plane is also given in figure 2. In every case it is seen that the value of ϕ has initially a minimum value given by $i - \delta_M$ when the nodal points are at the moon's ascending node; this angle increases to a maximum value of $i + \delta_M$ when the nodal points are at the moon's descending node. Further regression of the nodes back to the moon's ascending node causes ϕ to return again to a value of $i - \delta_M$.

TECHNIQUES FOR MODIFICATION OF OFF-NOMINAL SITUATIONS

Since the earth rendezvous orbital plane regresses about the earth's polar axis, any time spent in the earth rendezvous orbit beyond the nominal time will require that the vehicle be injected into the lunar trajectory in a plane having a different orientation from that planned (assuming that injection into the lunar trajectory is made in the plane of earth rendezvous orbit). Consequently, the moon, in general, will not be at the nodal point of the earth rendezvous orbital and earth-moon planes at the time the vehicle arrives there. In order for the moon and vehicle to arrive at a common point simultaneously, a change will have to be made in the trajectory. This change could be in the form of a plane change at injection accomplished by turning the velocity vector through an appropriate angle while injecting into the lunar phase. (See ref. 3.) The present analysis, however, corrects the situation by two methods:

(1) Modification of lunar trajectory plane: Since the nominal nodal point lies out of the plane of the regressed trajectory, a plane change can be initiated somewhere along the flight path so that the vehicle and moon will arrive at a common point simultaneously. This method is illustrated in figure 3 where the earth rendezvous orbital plane at the time of the plane change, the earth-moon plane, and the plane containing the modified trajectory are given. At the point marked, the vehicle is changed from the regressed orbital plane to the indicated plane and remains in this plane until it reaches the moon.

(2) Increase in injection velocity: An increase in the injection velocity over the nominal value will decrease the flight time and can be chosen to cause both vehicle and moon to reach the regressed node simultaneously. No plane change is used in this technique.

Modification of Lunar Trajectory Plane

In order to simplify the analysis, it is assumed in this method that the vehicle is injected into the lunar trajectory with a parabolic velocity and 0° injection angle. The assumption of parabolic velocity is well justified in that a small variation in injection velocity will only serve to increase or decrease the total energy of the trajectory without appreciably affecting the additional velocity needed to correct for the off-nominal situation. The condition of 0° injection angle simply means that the delays in the earth rendezvous orbit must be such that the vehicle is in a position in the orbit to inject with 0° injection angle and reach the desired position relative to the moon. The lunar trajectory remains in the earth rendezvous orbit plane until the vehicle reaches an optimum point along the trajectory for which a plane change can be initiated with the least velocity increment. At this point (generally around 60,000 miles from the center of the earth) the lunar trajectory is transferred to a plane (shown in fig. 3) which allows the vehicle and moon to arrive at a common point simultaneously. The plane change is accomplished by applying a velocity increment ΔV necessary to turn the normal component of velocity through the desired angle σ. In order that the trajectory remain parabolic after the plane change and have the same flight time as nominally planned, it is necessary that the total angular travel from

injection to the vicinity of the moon $(\theta_1 + \rho)$ be equal to the true anomaly of the vehicle at a distance equal to the moon's distance from injection.

The two-body expressions for the position of the vehicle along a parabolic trajectory is

$$\frac{r}{r_p} = \sec^2 \frac{\theta}{2} \tag{4}$$

The true anomaly θ of a vehicle at the moon's mean distance (r = 238,857 miles) from this expression is approximately 164.7° (injection at 315 statute miles being assumed). The velocity increment needed to accomplish the plane change is developed in the appendix and is

$$\frac{\Delta V}{V_p} = 2 \cos^2 \frac{\theta}{2} \sin \frac{\sigma}{2} \tag{5}$$

where

$$\sin \sigma = \frac{\sin \Omega_T \sin\phi}{\sin \rho} \tag{6}$$

$$\rho = 164.7 - \theta_1 \tag{7}$$

and

$$\Omega_T = \Omega_M - (\Omega_M)_0 + \dot{\omega}T \tag{8}$$

The optimum point, that is, the point where the plane change is assumed to take place for this analysis for a given delay time and nominal condition, was determined as the value of θ used in equation (5) that gives the minimum value of $\Delta V/V_p$. This minimum $\Delta V/V_p$ is the value used in figures 4 and 5.

The results of expressions (5) to (8) are shown in figures 4 for values of δ_M of 23.0°, 25.0°, and 28.5°. Shown in these figures is the variation of $\Delta V/V_p$ needed to make the plane change with nominal angles ϕ_0 between earth rendezvous orbital and earth-moon planes for delay times from 2 to 36 hours. (The curve marked "increase in injection velocity" is to be discussed later in the analysis.) The two branches of these curves for the longer delay periods given by the solid and dashed lines are due to the fact that there are two positions, 180° apart, of the moon in its orbit which correspond to the same value of ϕ_0. (See fig. 2.) The solid lines are representative of lunar positions for which the moon's declinations are negative and the dashed lines are for positive lunar declinations. These branches for the short delay times are not distinguishable on the curves given here. A comparison of the velocities associated with the two branches indicates that, for a given value of ϕ_0 and delay time, a smaller velocity increment is required to change the plane for positive lunar declinations. For instance, from figure 4(c), $\phi_0 = 10^\circ$ and a delay time of 1 day requires a $\Delta V/V_p$ of 0.019 (ΔV = 671 fps) when the moon is at a negative declination and a $\Delta V/V_p$ of 0.0011 (ΔV = 388 fps) when the moon is at a positive declination.

The results of figure 4 for the plane-change method indicate that, in general, the ratio $\Delta V/V_p$ increases with increasing values of ϕ_0 and that a minimum occurs generally between 0° and 10° in ϕ_0. It is interesting to note that the minimum value of $\Delta V/V_p$ does not occur when the nominal angle ϕ_0 is a minimum but at some other value. For instance, figure 4(c) shows that a minimum value of $\Delta V/V_p$ occurs for a value of ϕ_0 of about 5° rather than for the minimum value of 1.5°. This effect occurs because the velocity increment is determined by the nominal regression rate along the earth-moon plane as well as the nominal angle ϕ_0. The value of ϕ_0 for which the velocity increment is a minimum is therefore determined by a trade-off between these two quantities. It can be seen from figure 2(c) that at $\phi = 1.5^\circ$ the regression rate along the earth-moon plane (that is, the change in Ω_M with Ω_E) is very high but decreases with increasing ϕ. In fact, when ϕ has increased to only 5° this rate is reduced considerably.

For purposes of comparison the results of the method of making the plane change totally at injection are given in table I for 2, 4, 6, 12, 24, and 36 hours and two nominal lunar positions $(\Omega_M)_0$ of 0° and 90°.

Increase in Injection Velocity

It is also possible to reach a desired position relative to the moon after an off-nominal delay without a plane change. This could be done by simply increasing the injection velocity by an amount above the nominal so that the flight time to the moon would be decreased. This additional injection velocity made in the earth rendezvous orbital plane would allow the vehicle to intercept the moon at the location of the regressed node. If it is assumed that in the vicinity of the moon such maneuvers as a rendezvous orbit or a soft landing will be performed, a velocity increment in the form of a retrovelocity will be needed to slow the vehicle relative to the moon. The velocity relative to the moon, however, is strongly dependent on the energy of the earth-moon trajectory. In fact, approximately a 4-foot-per-second increment is needed at the moon to cancel every 1-foot-per-second increment applied at injection in excess of the nominal value. This fact is illustrated in figure 6. Figure 6 gives the velocities relative to the moon at various distances from the lunar surface as a function of injection velocity ratio, as determined by a direct application of Jacobi's integral of the three-body problem. The 4-to-1 ratio of velocities quoted is a consequence of the fact that the slopes of these curves are fairly constant for all injection velocities and nearly 4 when the injection velocity ratio is expressed as a velocity. Since the velocity increment required in the plane-change method does not serve to increase the velocity of the vehicle relative to the moon as does the velocity increment associated with the increase in injection velocity method, the retrovelocity needed to cancel out the effect at the moon of the increased injection velocity must be included in the comparison of the two methods.

The results of this method for a 12-hour delay in orbit is given in figure 4. (Other delay times have been treated but are not given here since the 12-hour delay sufficiently represents this method.) The variation of velocity increment with nominal angle between earth rendezvous orbital and earth-moon planes is given by the curves marked. These curves were constructed for a 60-hour nominal flight

to the moon or an injection velocity of about 0.995 parabolic velocity. The variation in flight time with injection velocity was taken from reference 4. A comparison in the two methods for a 12-hour delay indicates that the plane-change technique applied at the optimum point along the trajectory is considerably less expensive. For this reason the remainder of the analysis is based on the plane-change method.

LAUNCH WINDOWS

Previously, results of delay times in earth rendezvous orbit of only a few hours have been discussed. If delay times of several days are considered, other interesting results are brought to view. Figure 5 (based on the plane-change method) gives the delay time in days available for a given velocity and each nominal position of the moon. These curves were obtained from a cross plot of results of the type of figure 4 by using the relationship between ϕ and Ω_M given by figure 2.

A striking feature of these curves is that for each $(\Omega_M)_0$ and $\Delta V/V_p$ there are two possible delay times. It can be seen from figure 5(c), for instance, for a nominal lunar position $(\Omega_M)_0$ of 0° and a velocity ratio of 0.02, the injection into the lunar trajectory from earth rendezvous may be made after a delay past the nominal of 1.3 days or after an even longer delay of 4.9 days. These two times are available because of the manner in which the node regresses along the earth-moon plane. It must be remembered that the earth rendezvous orbit plane intersects the earth-moon plane at two nodal points 180° apart and that, for a delay in earth rendezvous orbit, both of these points regress in the opposite direction of the moon's rotation about the earth. Thus, if injection into the lunar trajectory to reach one of the nodes is not initiated after the first delay period mentioned, it is possible to remain in earth rendezvous orbit an additional amount of time, inject with the same velocity increment, and reach the moon in the vicinity of the second nodal point.

It is further seen from figure 5 that certain additional delays in earth rendezvous orbit will allow the vehicle to be injected from the earth rendezvous orbit into the lunar trajectory to reach the proper vicinity of the moon with no additional velocity increment. These conditions or delay times correspond to the curve marked $\Delta V/V_p = 0$ in figure 5. Since the regression of the nodes is periodic, it is found that there are several such curves corresponding to $\Delta V/V_p = 0$ for increasingly longer delay times in earth rendezvous orbit. The table below gives such delay times for the case of $\delta_M = 28.5^\circ$ for nominal lunar declinations of zero (moon at ascending node), maximum negative, zero (moon descending node), and maximum positive:

Delay times in days for $\frac{\Delta V}{V_p} = 0$ for $(\Omega_M)_0$ of -			
0°	90°	180°	270°
5.9	10.5	10.6	3.5
16.5	21.1	21.2	14.1
27.1	31.7	27.0	19.4
37.7	42.3	31.0	25.3
48.3	50.6	41.5	35.9

From figure 5(c) for $\delta_M = 28.5^\circ$, it can be seen that a very desirable launch window from earth rendezvous orbit exists for a nominal lunar position of approximately 290° (moon near maximum positive declination and $\phi_0 = 5^\circ$). For this nominal position a maximum delay time is available for a minimum cost in velocity increment. The nominal lunar position in this region is also desirable because of the relatively short delay times between opportunities to reach the vicinity of the moon when it is at either of the two nodes mentioned previously for the same velocity increment. (The branches of the velocity curves are separated vertically by smaller amounts of time.) This lunar position has similar properties for each of the δ_M values of figures 5(a) and 5(b).

From figure 5 it can also be seen that the least favorable launch windows occur when the moon is nominally in the vicinity of its descending node $\left((\Omega_M)_0 = 180^\circ\right)$. Only relatively short delay times in earth rendezvous orbit are allowed for reasonable velocity increments for these lunar positions. In addition to this, long delay times are necessary between opportunities to reach the vicinity of the moon when it is at either of the two nodes. (The branches of the velocity curves are separated vertically by larger amounts of time.)

CONCLUDING REMARKS

Of the two methods of correcting for the off-nominal delay times in earth rendezvous orbit, it is concluded that the plane-change method applied totally at the optimum point is the more favorable.

It is found that for the period considered (that is, from 1964 to 1972) the plane-change technique allows several hours delay in earth rendezvous orbit for reasonable velocity increments if the nominal lunar position is chosen properly. For instance, nominal lunar positions near maximum positive declination allow maximum delay times for a minimum velocity increment. It is shown that, for the same nominal angle between earth rendezvous orbital and earth-moon planes and delay time in orbit, smaller velocity increments are consistently required to make the plane change for the positive nominal lunar declinations.

An important consequence of the relative motion between the node line (formed by the intersection of the earth rendezvous orbital and earth-moon planes) and moon along the earth-moon plane is that numerous delays in earth rendezvous orbit occur which require little or no additional velocity over that of the nominal injection velocity to reach the vicinity of the moon. These delay times exist (although they have different values) for each nominal lunar position.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., August 27, 1962.

APPENDIX

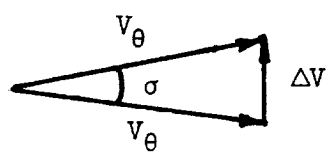
DEVELOPMENT OF EQUATIONS USED IN ANALYSIS

In order to express simply the velocity increments associated with delay time in orbit, it is assumed that the vehicle is injected into a parabolic lunar trajectory. The two-body expression for the position of the vehicle along the trajectory is:

$$r = r_p \sec^2 \frac{\theta}{2} \tag{A1}$$

where r_p is the distance to the perigee point from the center of the earth and θ is the true anomaly of the vehicle at any time after injection. If injection is made at approximately 315 statute miles or an r_p of approximately 4,275 miles, the value of θ at the moon's distance (r = 238,857 miles) is about 164.7°.

A parabolic trajectory for the entire flight may be obtained by making the necessary plane change in the form of a rotation of the normal component of velocity V_θ through the angle σ as illustrated in the following sketch:



Sketch 1

The velocity increment ΔV needed to do this is given as

$$\frac{\Delta V}{V_p} = \frac{2V_\theta}{V_p} \sin \frac{\sigma}{2} \tag{A2}$$

This expression can be expressed in terms of the angles σ and θ by use of the condition of constant angular momentum. That is,

$$V_p r_p = V_\theta r \tag{A3}$$

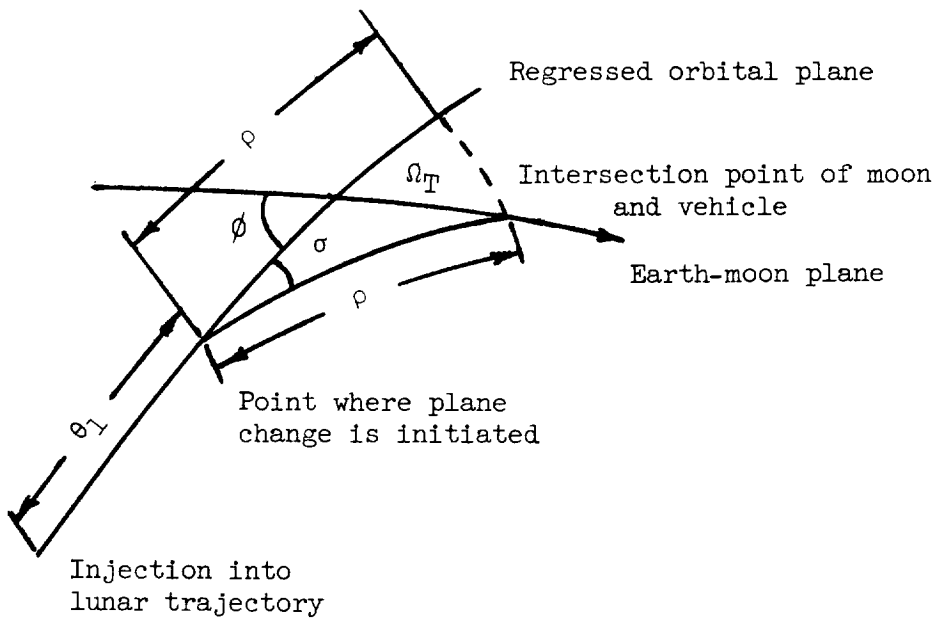
or

$$\frac{V_\theta}{V_p} = \frac{r_p}{r} = \cos^2 \frac{\theta}{2} \tag{A4}$$

The expression for the velocity increment is then:

$$\frac{\Delta V}{V_p} = 2 \cos^2 \frac{\theta}{2} \sin \frac{\sigma}{2} \tag{A5}$$

The angle σ can be expressed in terms of the known variables of the problem by considering the following sketch:



Sketch 2

Application of spherical trigonometry yields the expression for σ:

$$\sigma = \sin^{-1}\left(\frac{\sin \Omega_T \sin \phi}{\sin \rho}\right) \tag{A6}$$

The angle Ω_T is the sum of the regression of the node along the earth-moon plane $\Omega_M - (\Omega_M)_0$ and the forward motion of the moon $\dot{\omega}T$ during this delay time T, that is,

$$\Omega_T = \Omega_M - (\Omega_M)_0 + \dot{\omega}T \tag{A7}$$

where $\dot{\omega}$ is approximately 13.2° per day.

In order that the trajectory remain parabolic after the plane change and have the same flight time as originally planned, it is necessary that the injection be made at such point that the sum of the angles ρ and θ_1 be equal to θ at the

moon's distance, that is,

$$\rho + \theta_1 = 164.7^{\circ} \tag{A8}$$

The angle θ_1 is the value of θ at the time the plane change is initiated.

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TABLE I.- VELOCITY INCREMENTS ASSOCIATED WITH

A TOTAL PLANE CHANGE AT INJECTION

(a) $\delta_M = 23.0^\circ$

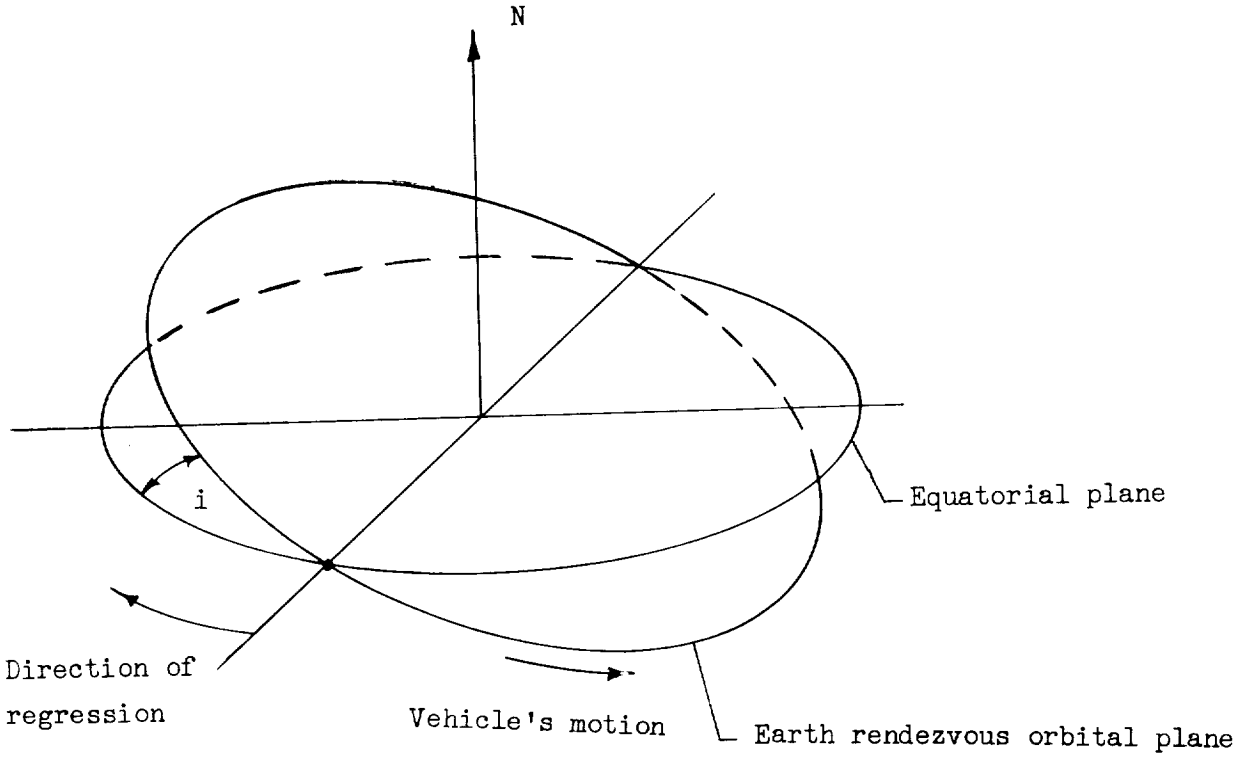
Delay time, hr	$\Delta V/V_p$ for -	
	$(\Omega_M)_0 = 0^\circ$; $\phi_0 = 7^\circ$	$(\Omega_M)_0 = 90^\circ$; $\phi_0 = 20^\circ$
2	0.002	0.005
4	.005	.010
6	.010	.020
12	.030	.064
24	.087	.235
36	.155	.490

(b) $\delta_M = 25.0^\circ$

Delay time, hr	$\Delta V/V_p$ for -	
	$(\Omega_M)_0 = 0^\circ$; $\phi_0 = 5^\circ$	$(\Omega_M)_0 = 90^\circ$; $\phi_0 = 17.5$
2	0.002	0.002
4	.006	.007
6	.013	.014
12	.038	.045
24	.115	.175
36	.205	.380

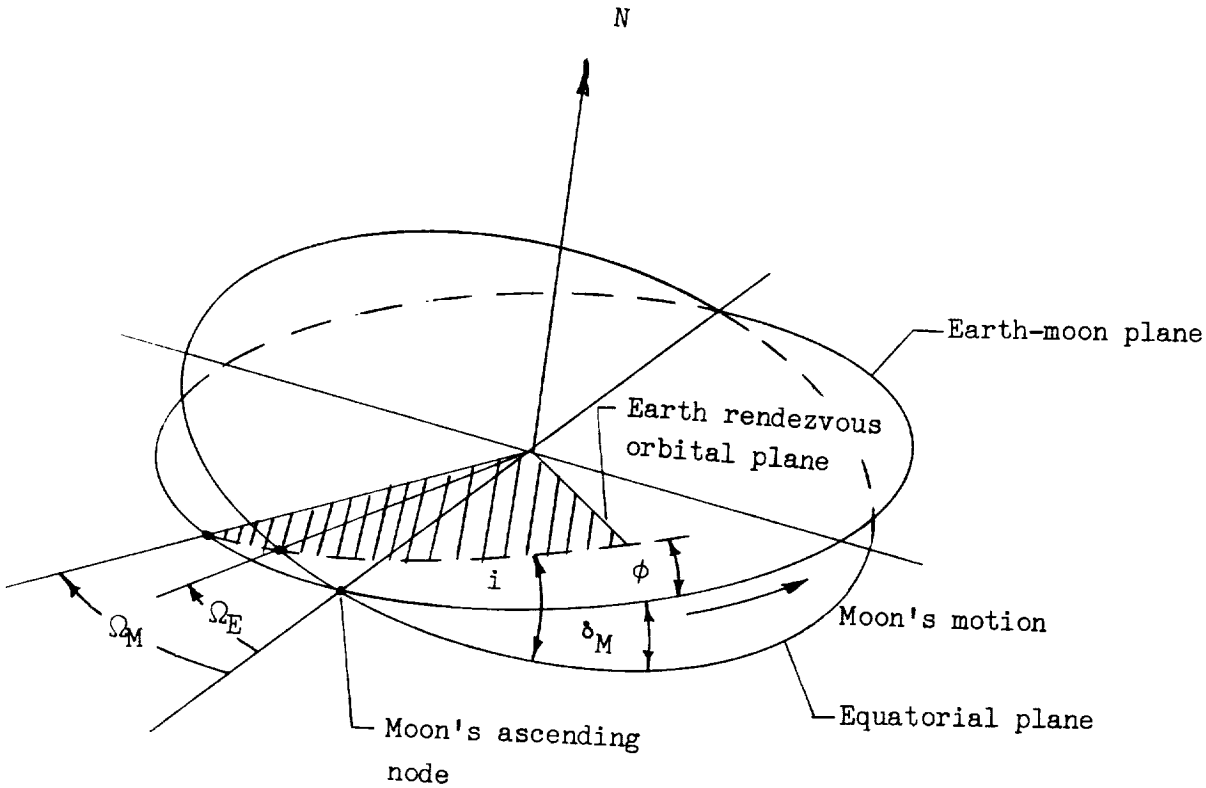
(c) $\delta_M = 28.5^\circ$

Delay time, hr	$\Delta V/V_p$ for -	
	$(\Omega_M)_0 = 0^\circ$; $\phi_0 = 1.5^\circ$	$(\Omega_M)_0 = 90^\circ$; $\phi_0 = 10^\circ$
2	0.001	0.001
4	.002	.002
6	.005	.004
12	.017	.011
24	.060	.060
36	.114	.180



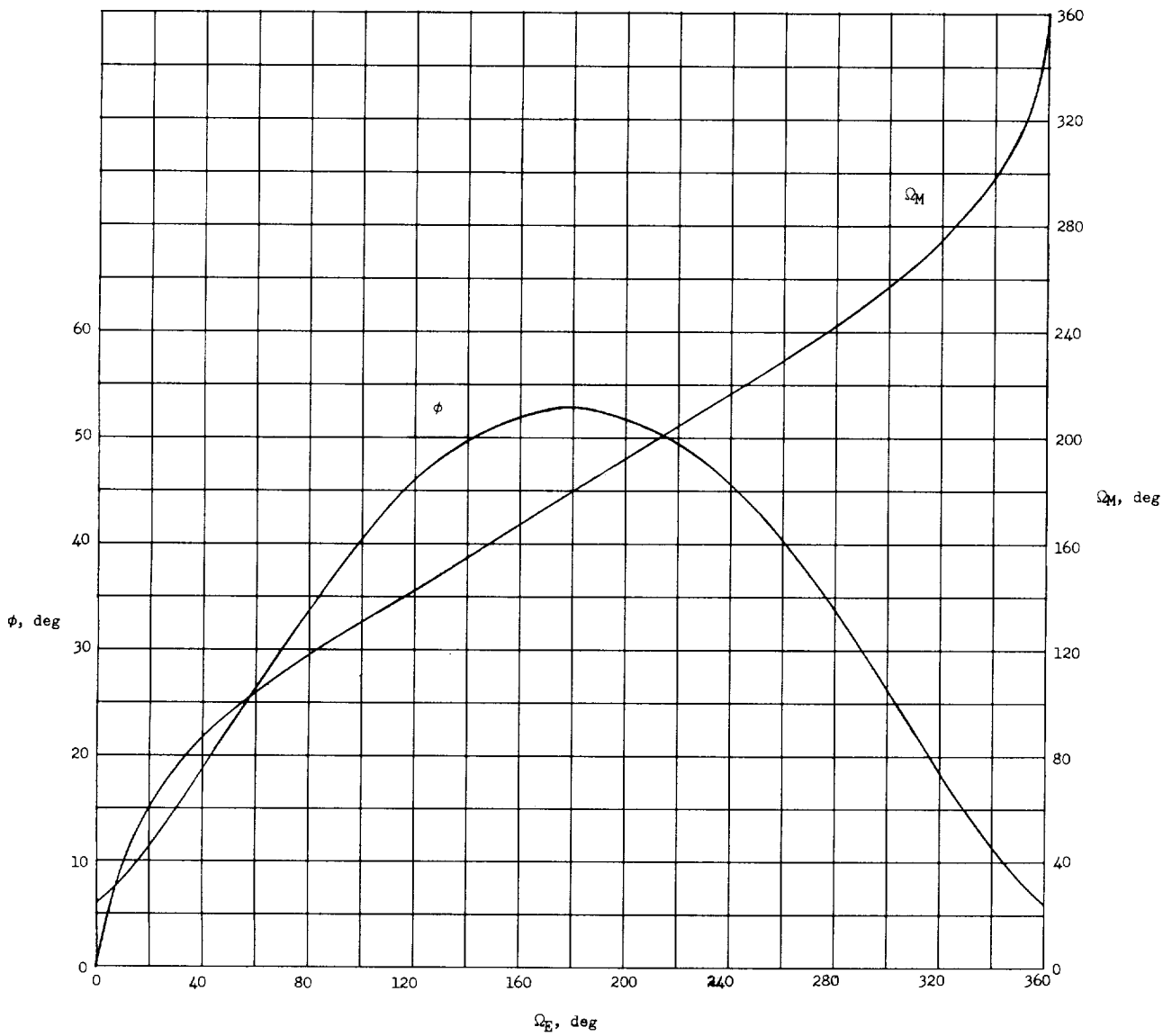
(a) Direction of regression.

Figure 1.- Illustration of pertinent angles and planes.



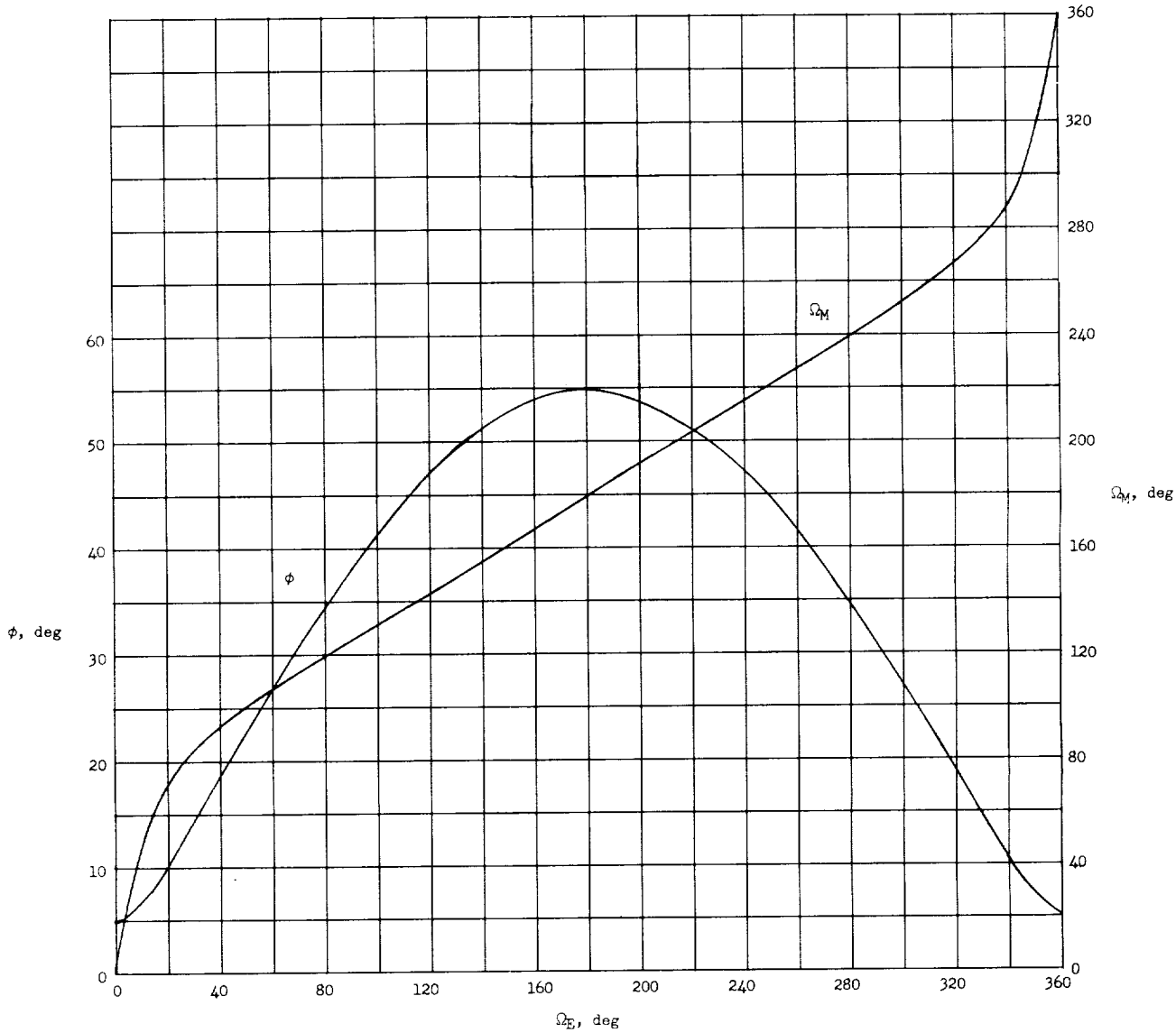
(b) Regression and other pertinent angles.

Figure 1.- Concluded.



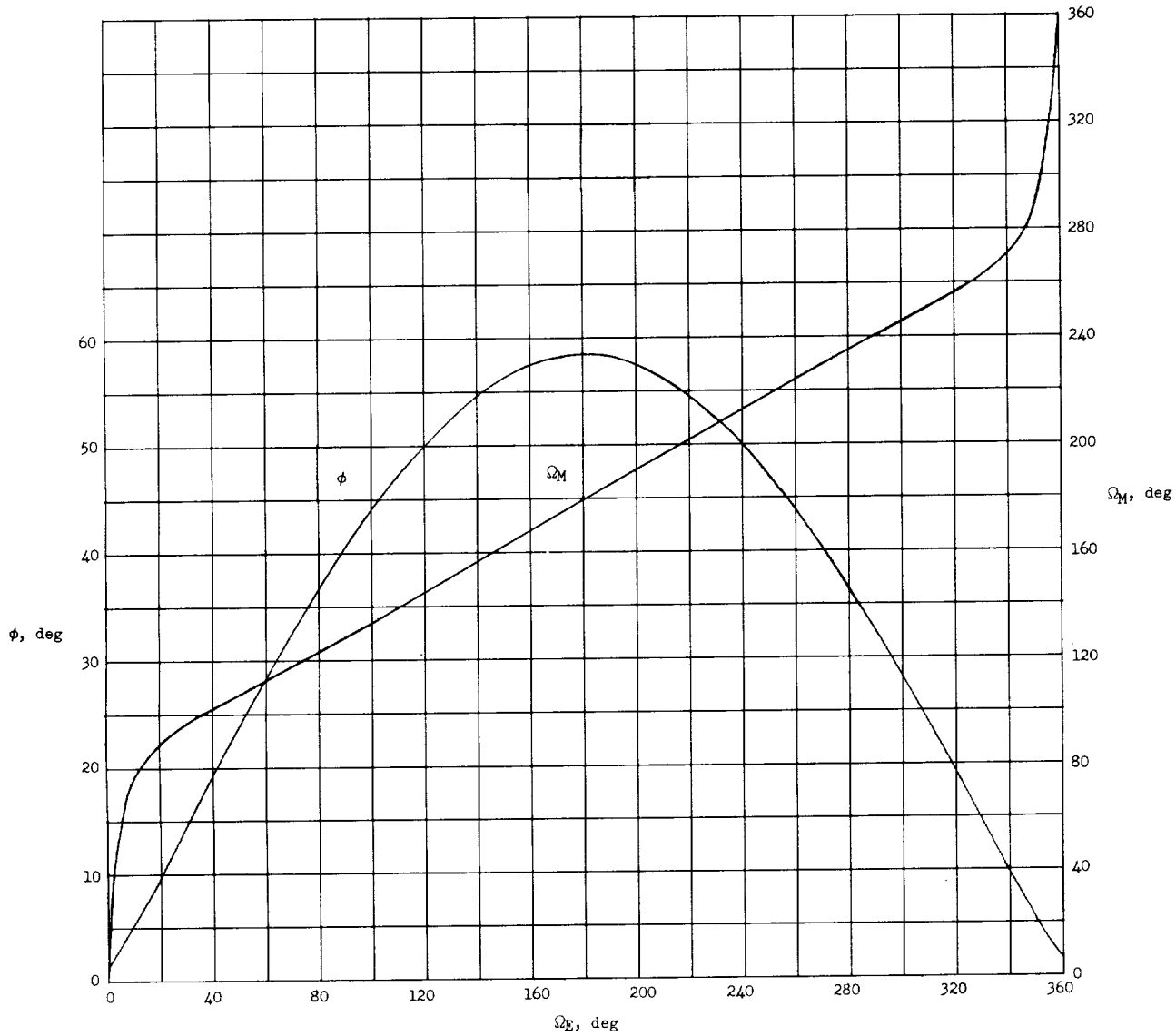
(a) $\delta_M = 23.0^\circ$.

Figure 2.- Variation of nodal position along earth-moon plane and angle between earth-moon and earth rendezvous orbital planes with nodal position along equatorial plane.



(b) $\delta_M = 25.0^\circ$.

Figure 2.- Continued.



(c) $\delta_M = 28.5^\circ$.

Figure 2.- Concluded.

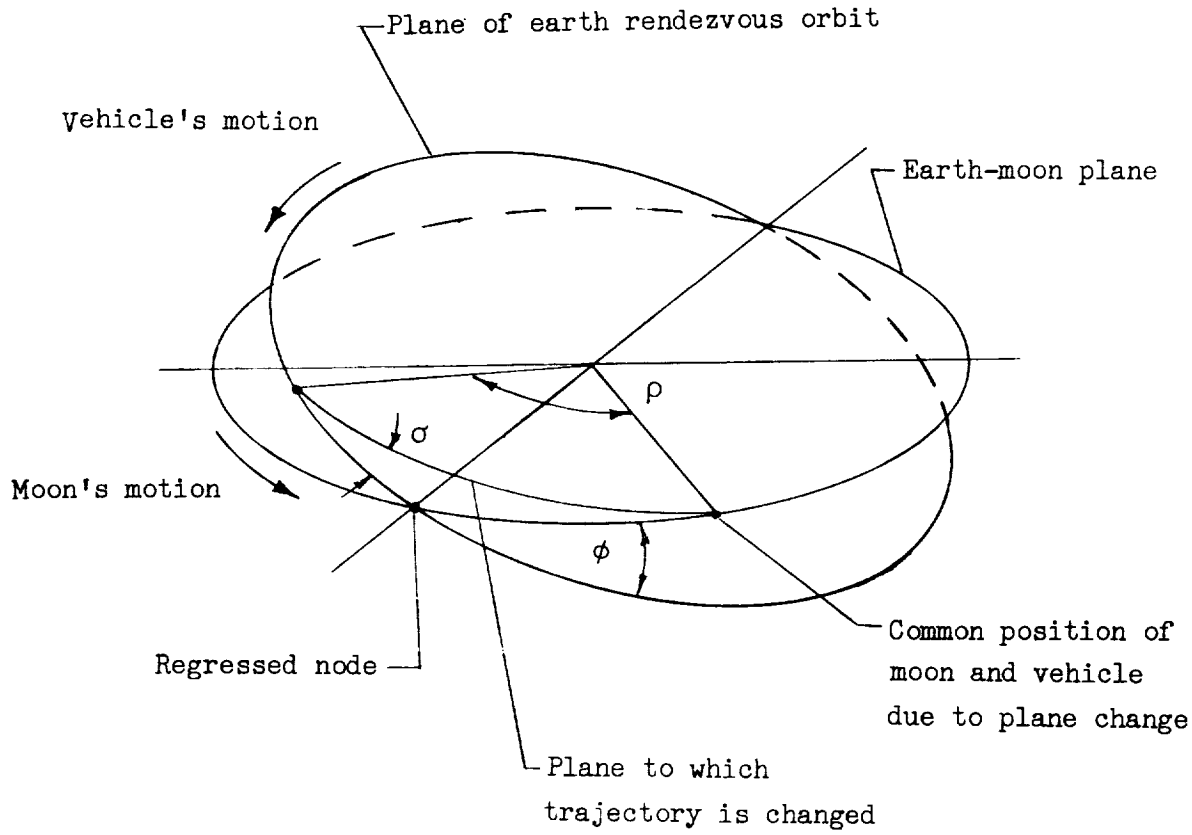
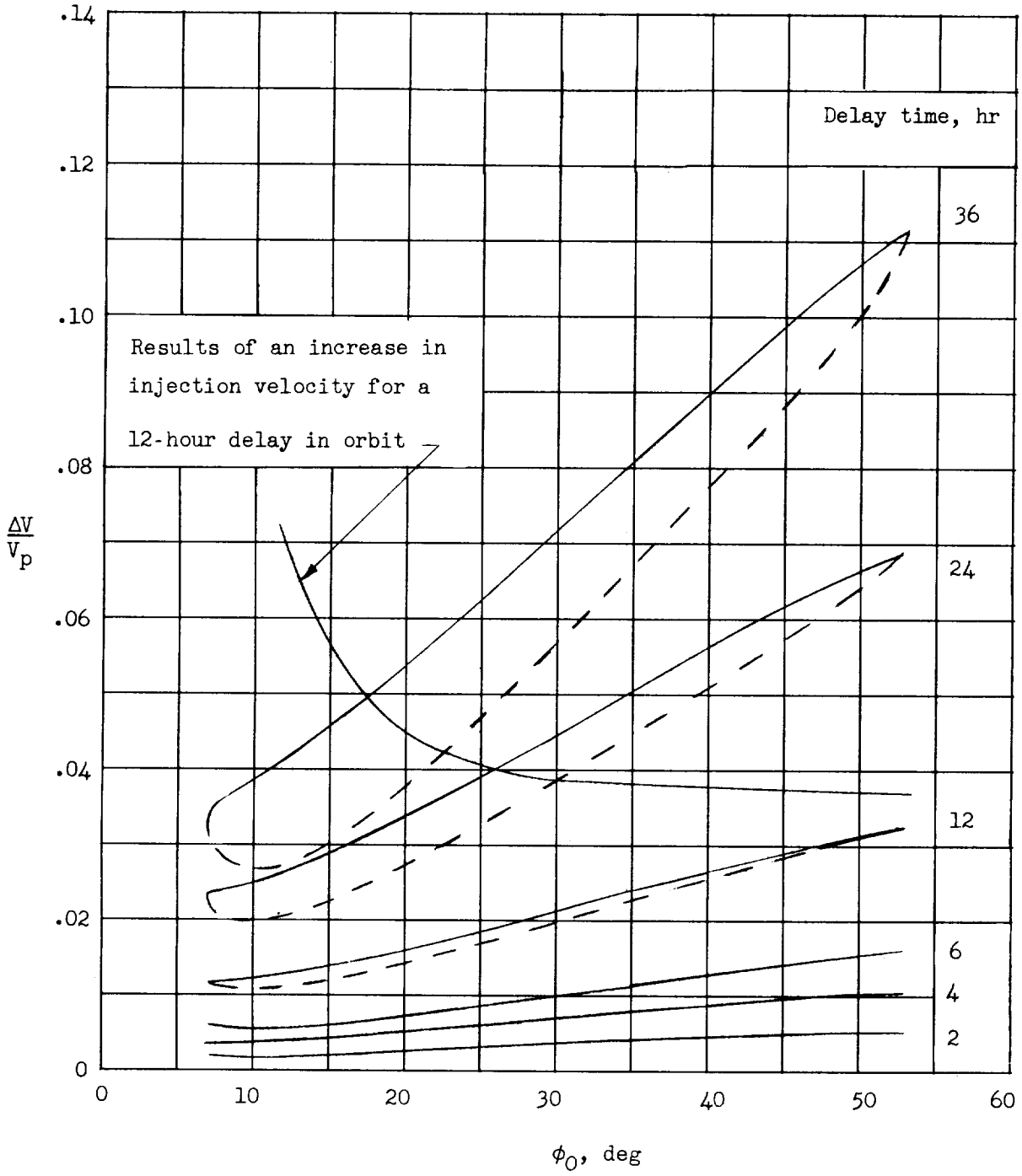
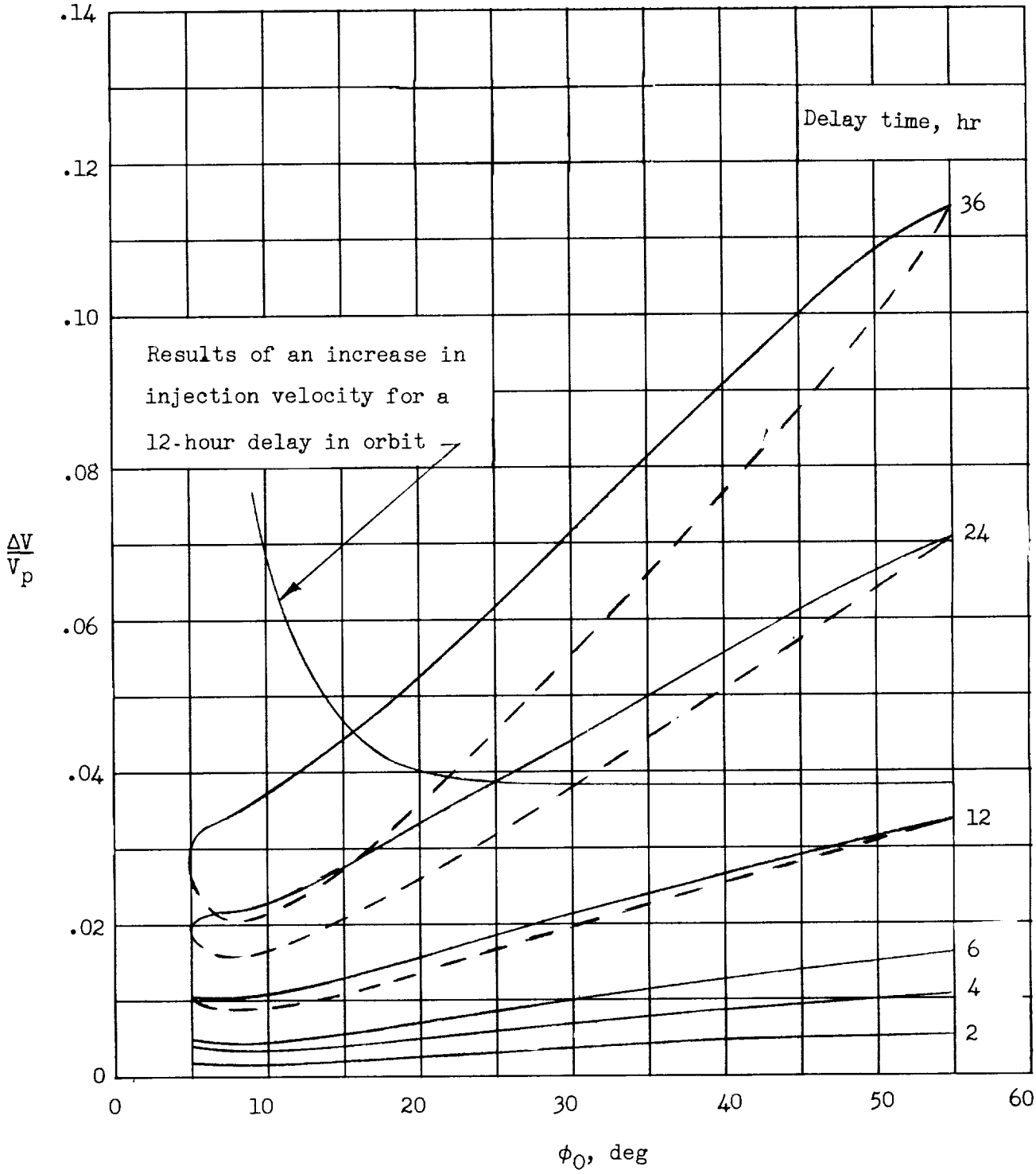


Figure 3.- Illustration of plane-change technique.



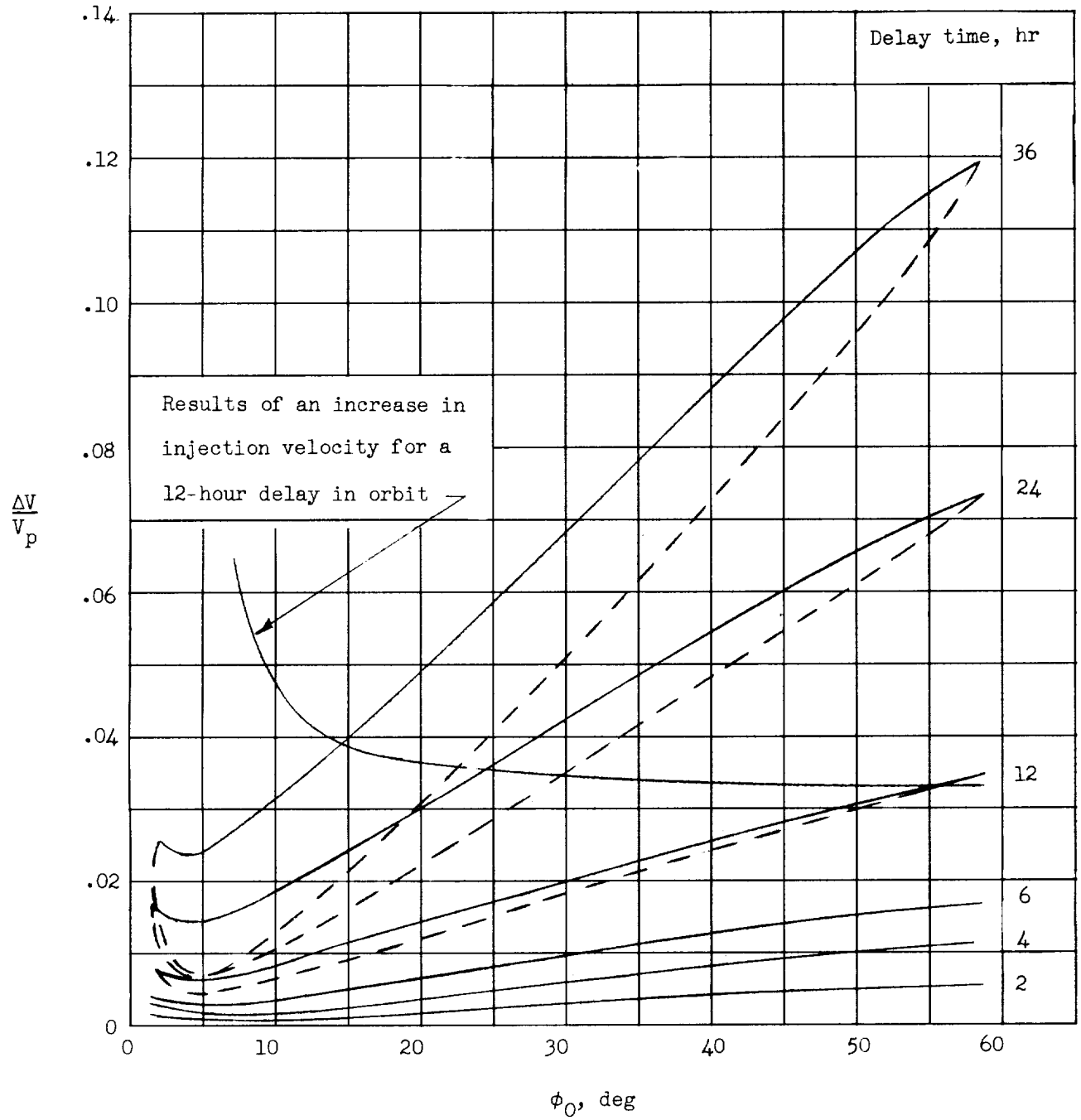
(a) $\delta_M = 23.0^\circ$.

Figure 4.- Variation of velocity increments for off-nominal situations with nominal angles between earth rendezvous orbital and earth-moon planes. Solid and dashed curves indicate negative and positive lunar declinations, respectively.



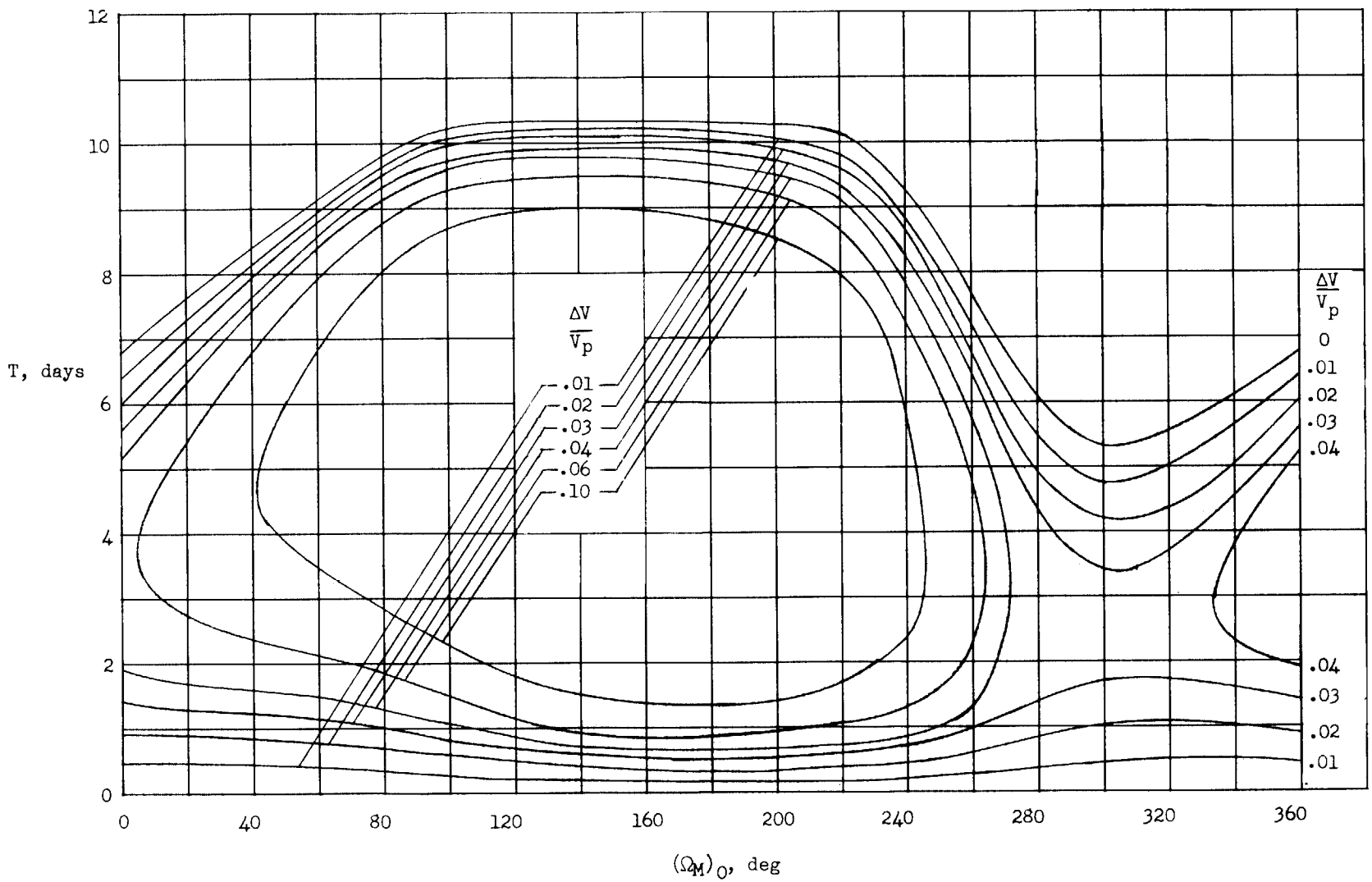
(b) $\delta_M = 25.0^\circ$.

Figure 4.- Continued.



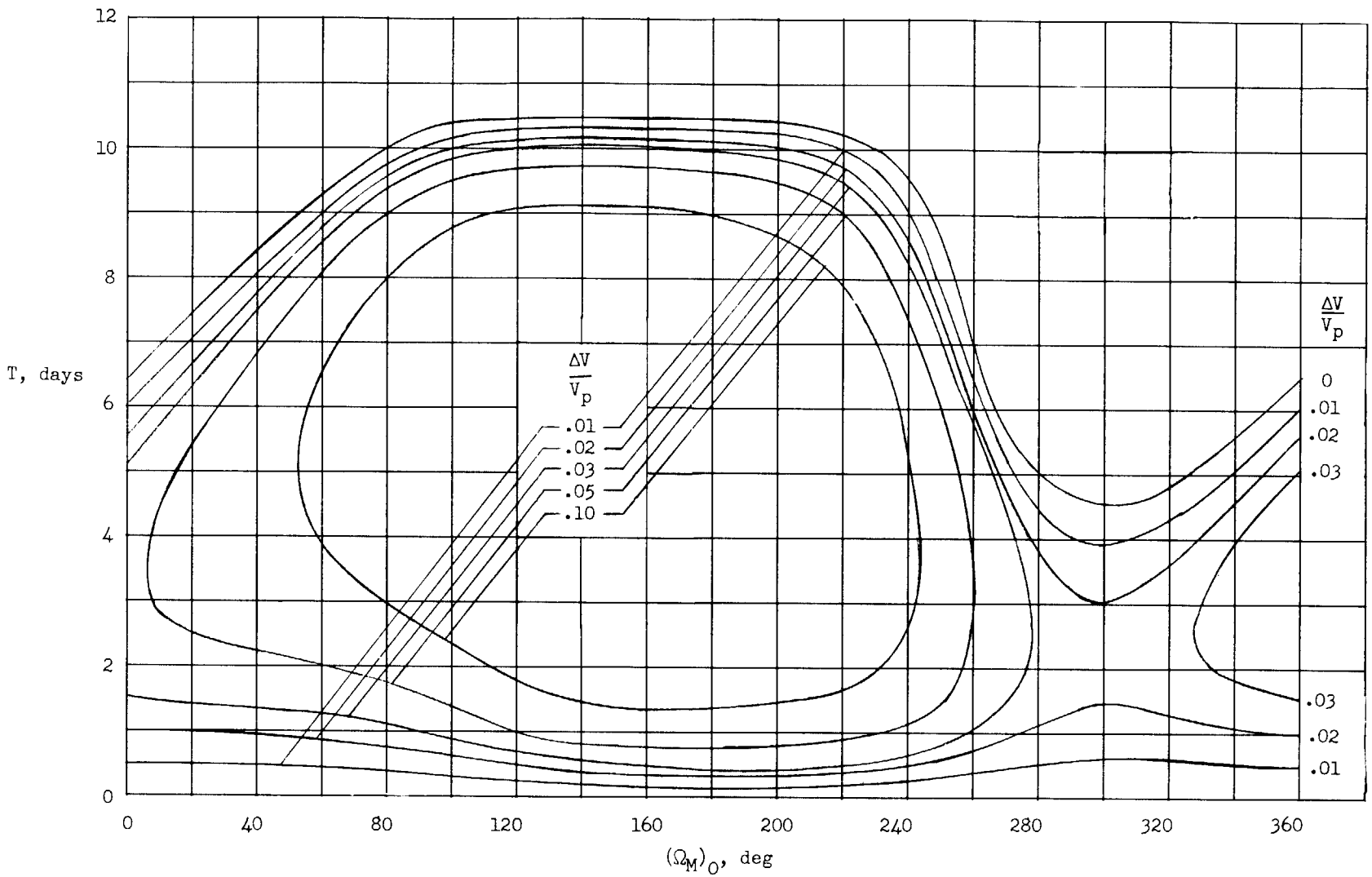
(c) $\delta_M = 28.5^\circ$.

Figure 4.- Concluded.



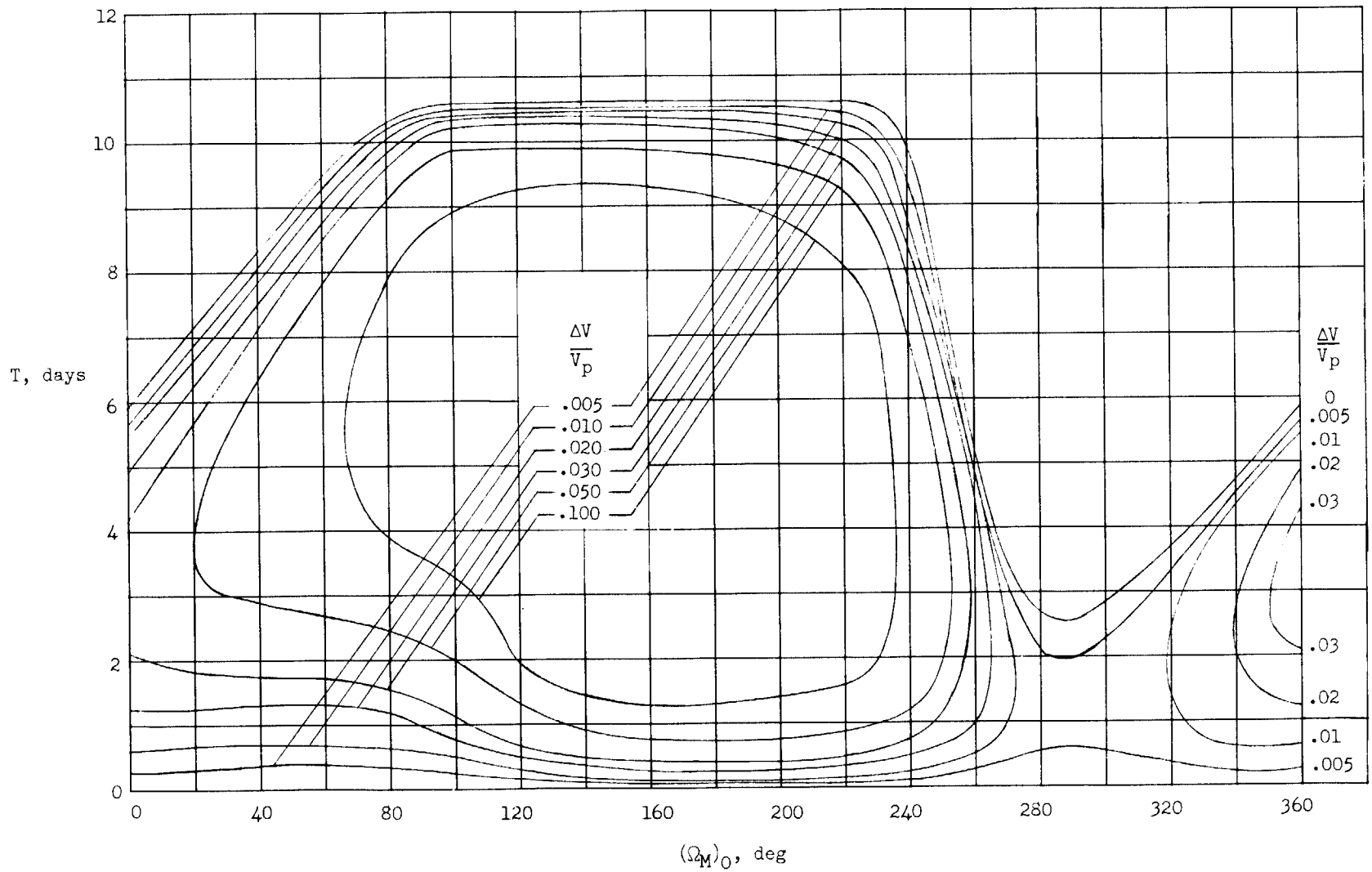
(a) $\delta_M = 23.0^\circ$.

Figure 5.- Delay times in earth orbit past the nominal for nominal lunar positions and various velocity increments.



(b) $\delta_M = 25.0^\circ$.

Figure 5.- Continued.



(c) $\delta_M = 28.5^{\circ}$.

Figure 5.- Concluded.

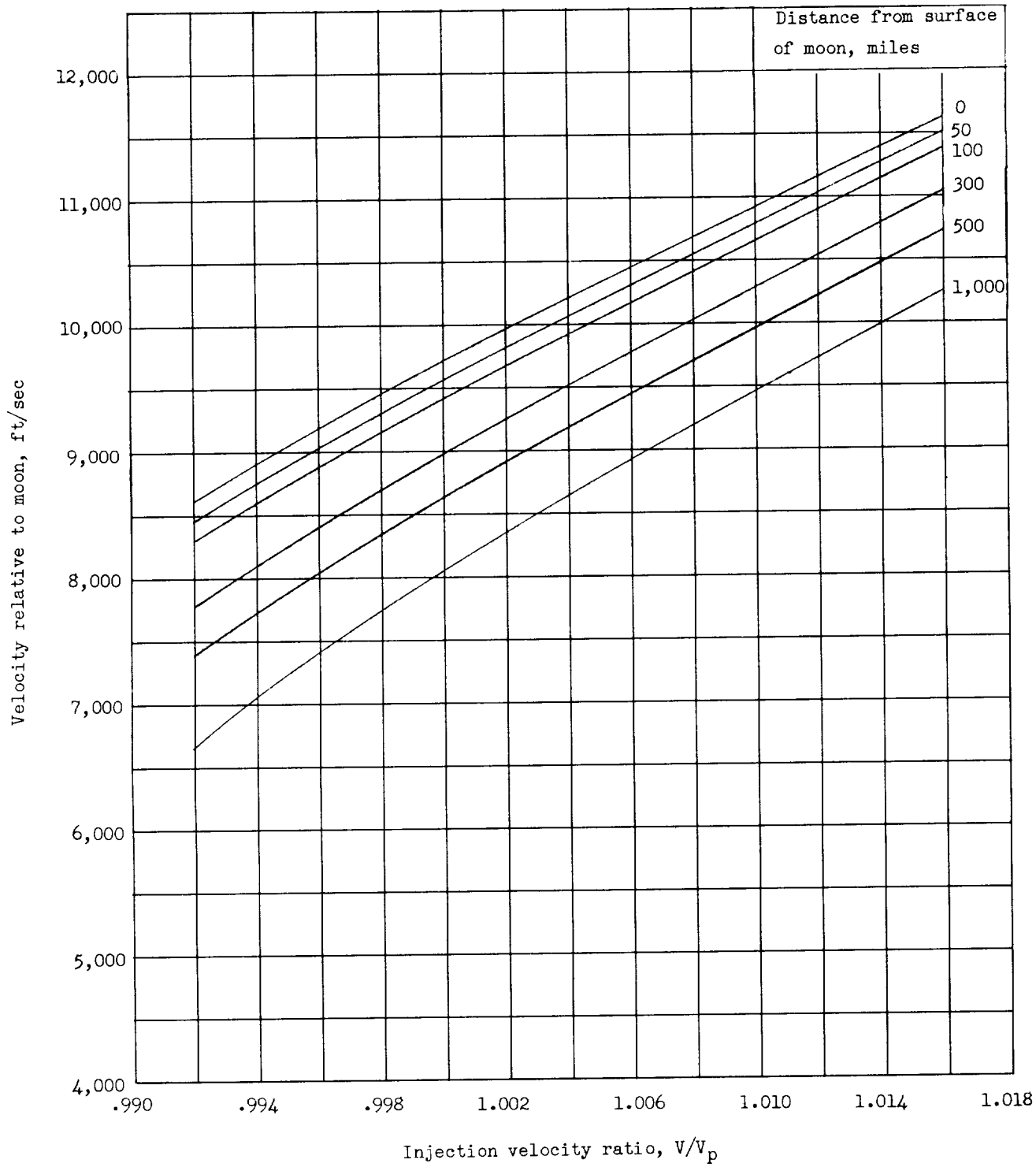


Figure 6.- Velocity relative to moon for various injection velocity ratios.

NASA-Langley, 1962 L-3121

NASA TN D-1512
National Aeronautics and Space Administation.
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(NASA TECHNICAL NOTE D-1512)

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