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ENERGY AVERAGES IN LOOSELY COUPLED VIBRATING SYSTEMS, WITH APPLICATIONS TO NOISE REDUCTION

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ABSTRACT

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Vibrations of complex dynamical systems are treated in terms of energy quantities, neglecting details of structure and excitation. Relations between the average energies in loosely and conservatively coupled systems are obtained in terms of internal losses and coupling losses. It is shown that the products of modal density and coupling loss factor are the same for each pair of subsystems.

The general theory is applied to study the noise reduction of a rectangular box, and experimental results are compared with theoretical predictions with generally good agreement. The transmission behavior of a box at a given frequency is shown to depend on the relation of this frequency to the lowest mechanical and acoustical resonance frequencies and the critical frequency of the walls. Theory and experiment show that near the lowest mechanical resonances the pressure in the box may considerably exceed that in the incident sound field.

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I. INTRODUCTION

The advent of powerful flight vehicles has increased the need for protection against wide band random noise and vibration. It has become more important to understand the response of structures to such excitation, not only from the point of the structures themselves but also from the point of the acoustic shielding that enclosures can provide.

There is little hope of obtaining detailed solutions for complicated dynamical systems, consisting of an agglomeration, for example, of beams, plates, shells, and fluid-filled spaces by classical methods [1,2,3]. The very complication of such systems, however, makes these particularly amenable to statistical treatment [4,5,6]. Of course, in applying statistical methods we sacrifice detailed information for the sake of mathematical tractability.

The statistical method discussed in this paper deals with mean energies (averaged over time, space, and frequency) rather than with detailed velocities and pressures. The distinction between acoustical and mechanical systems largely disappears. The method utilizes only gross parameters of the systems such as total masses, average modal densities, and loss factors. The results so obtained pertain best to a typical system—that is,

to an average of an ensemble of similar systems under diffuse broadband random excitation. Although some extensions, such as the consideration of variances, are available [1], such refinements will not be considered here.

The basic statistical method is outlined and extended to multiple subsystems in Section II. Sections III and IV describe a study of the noise reduction of a rectangular box; Section III presents experimental results, and Section IV compares these with values predicted by means of the statistical and classical methods. This noise reduction study is presented not only to illustrate the utility of the statistical method, but also as a subject of interest per se which, although familiar, has not been fully treated previously [2,3,4].

II. VIBRATIONAL AVERAGES IN LOOSELY COUPLED SYSTEMS

(a) Loosely Coupled Systems

A complex classical dynamical system may generally be represented by an equivalent circuit consisting of resonators and certain coupling elements. Each of the resonators corresponds to one mode (a spatial function describing a free vibration of the system).

The spatial structure of a complex system is usually clear if its subsystems are loosely coupled to each other. It is then also usually clear how the coupling might be reduced in order to arrive at uncoupled subsystems under either "free" or "blocked" conditions[5]. Moreover, the modes of the system are then closely related to the modes of the subsystems, taken as uncoupled.

What we mean, more precisely, by a subsystem being loosely coupled to others is that the vibrational energy per cycle exchanged by it with other subsystems is small compared to the energy stored in it, at its resonances.

The present analysis will be limited to non-dissipative (or conservative) coupling. Many practical systems seem to be coupled in such a manner. For example, no appreciable power seems to be dissipated in the coupling between structures that

are joined in permanent contact, or in the coupling between a structure and a fluid in which the structure is immersed.

(b) Modal Energy Balance

Let the velocity $\vec{v}$ of a complex system be given by

$$\vec{v}(\vec{r},t) = \sum_i \vec{R}_i(\vec{r})T_i(t) \tag{1}$$

where the $\vec{R}_i$ are functions of position only ($\vec{r}$ being a position vector) and where the T_i are functions only of time t. If the velocity is statistically stationary, one may obtain the mean vibrational energy E by averaging $\rho|\vec{v}|^2$ over time and over the volume V occupied by the system. If the functions $\vec{R}_i$ are orthogonal or if the functions T_i are uncorrelated, the mean energy of the whole system is given by[8]

$$E = V\sum_i \langle\rho|\vec{R}_i|^2\rangle\langle T_i^2\rangle \ . \tag{2}$$

The following analysis is simplified considerably by considering only mean energies and related mean square quantities.

If we are treating a complex system consisting of loosely coupled subsystems, we may choose for the $\vec{R}_i$ functions the modes of the individual subsystems (considered as uncoupled). These modes are orthogonal, and Eq. (2) will then constitute a good approximation.

The component terms of the sum of Eq. (2) represent the time average modal energies

$$\theta_i = V\langle\rho|\vec{R}_i|^2\rangle\langle T_i^2\rangle \ . \tag{3}$$

(The letter θ is used because these energies are analogous to temperature in thermodynamics[4].) Assume that the i'th mode is on the average supplied with power Π_i from a source, that it dissipates power $\theta_i\phi_i^i$ internally, that it transmits power $\theta_i\phi_i^j$ to mode j, and that it accepts power $\theta_j\phi_j^i$ from mode j, where the ϕ's are positive factors[9]. If there is no power lost or gained otherwise (e.g., in coupling) the modal energy balance may be expressed as

$$\theta_i \sum_j \phi_i^j - \sum_j{}' \theta_j\phi_j^i = \Pi_i \ , \tag{4}$$

where the prime on the summation symbol denotes that the term where j equals i is to be omitted.

For the special case where all generator powers and all the factors ϕ_i^j except ϕ_1^2 and ϕ_2^1 vanish, Eq. (4) reduces to

$$\theta_1\phi_1^2 = \theta_2\phi_2^1 \ . \tag{5}$$

According to statistical mechanics, the average modal energies of two coupled systems are equal in absence of sources and

internal losses; hence, $\phi_1^2 = \phi_2^1$. We may generalize this result and write

$$\phi_i^j = \phi_j^i \quad . \tag{6}$$

That is, the matrix of coupling factors is symmetric.

(c) Modal averages

Let us determine multimodal averages of the various power terms. Such averages have to be taken over a frequency band Δf which contains at least several modes (which means generally that their resonances fall into the band). Average spectral densities are obtained from modal averages by multiplying them with modal densities.

The mean spectral density P_s of power fed (in the frequency band considered) directly into subsystem s is given by

$$P_s = \sum_{i(s,\Delta f)} \Pi_i/\Delta f = n_s \langle \Pi_i \rangle_{i(s,\Delta f)} \tag{7}$$

The summation index $i(s,\Delta f)$ includes only those modes which belong to subsystem s and which are contained in the band Δf. The symbol n_s denotes the modal density of the subsystem.

The mean spectral density of energy stored in subsystems s is similarly given by

$$S_s = \sum_{i(s,\Delta f)} \theta_i/\Delta f = n_s\langle\theta_i\rangle_{i(s,\Delta f)} , \tag{8}$$

the mean spectral density of power fed into subsystem r by subsystem $s \neq r$ by

$$P_s^r = \sum_{i(s,\Delta f)} \sum_{j(r,\Delta f)} \theta_i\phi_i^j/\Delta f =$$

$$= \Delta f \; n_s n_r \left\langle \theta_i \langle\phi_i^j\rangle_{j(r,\Delta f)} \right\rangle_{i(s,\Delta f)} \tag{9}$$

and the mean spectral density of power dissipated internally in subsystems s by

$$P_s^s = \sum_{i(s,\Delta f)} \theta_i\phi_i^i/\Delta f = n_s\langle\theta_i\phi_i^i\rangle_{i(s,\Delta f)} . \tag{10}$$

(d) Coupling Loss Factors

Coupling and internal losses may also be expressed in terms of the loss factors η_s^r and η_s^s, given by

$$\eta_s^r = P_s^r/\omega S_s \tag{11}$$

and

$$\eta_s^s = P_s^s/\omega S_s \tag{12}$$

where ω denotes the (radian) center frequency of the band Δf considered.

So far the only limitation imposed on the excitation Π_i is that it be stationary. Although the modal energies θ_i generally may be expected to depend on the spectral distribution of the excitation, the coefficients ϕ_i^j may be expected to be relatively independent of it if the physical loss and coupling mechanisms are much less dependent on frequency than the modal responses. Usually the details of the excitation are unknown or their inclusion in calculations is impractical, so that it is of interest to consider an "average" excitation which produces equipartition of energy between the modes of each subsystem in each narrow frequency band. Such equipartition seems reasonable if the directly supplied and internally dissipated energy terms are small compared to the energy storage terms.

If such equipartition holds or if the θ_i and $\langle\phi_i^j\rangle_{j(r,\Delta f)}$ are uncorrelated in a subsystem s, we may rewrite the average of the products in Eq. (9) as a product of averages,

$$\omega\eta_s^r S_s = \Delta f\, n_s n_r \langle\theta_i\rangle_{i(s,\Delta f)} \left\langle \langle\phi_i^j\rangle_{j(r,\Delta f)} \right\rangle_{i(s,\Delta f)}. \quad (13)$$

By combining this result with Eq. (8) we find

$$\omega\eta_s^r = \Delta f\ n_r \left\langle \langle\phi_i^j\rangle_{j(r,\Delta f)} \right\rangle_{i(s,\Delta f)} \ . \tag{14}$$

By interchanging the indexes s and r and making use of the symmetry of the ϕ_i^j we further obtain

$$\omega\eta_r^s = \Delta f\ n_s \left\langle \langle\phi_j^i\rangle_{j(s,\Delta f)} \right\rangle_{i(r,\Delta f)} \ . \tag{15}$$

Since the average values indicated in Eqs. (14) and (15) are identical we conclude that

$$n_s\eta_s^r = n_r\eta_r^s \ . \tag{16}$$

Equation (16)indicates that for each pair of subsystems the two products of modal density and coupling loss factor are equal. This is the central result of this discussion and, with Eqs.(11) and (12), provides a convenient relation between average vibrational energies and powers in multimodal systems. If only one mode of, say, subsystem s is involved in Δf, we only replace n_s by $1/\Delta f$. If the frequency band of interest is so narrow that it contains at the most only one mode of each subsystem, then there is no point in taking modal averages.

We may define a coupling or transfer resistance R_s^r between subsystem s and subsystem r by

$$R_s^r = \eta_s^r \omega M_s \quad , \tag{17}$$

where M_s is the total mass of subsystem s. R_s^r is a transfer resistance, not a driving point resistance, although for the special case of a structure s radiating into a large space r, this coupling resistance may be approximated by the radiation resistance of the structure for radiation into free space (which is a driving point resistance) [5,6]. In order to avoid possible confusion, we shall here avoid use of resistances in favor of the appropriate loss factors.

(e) Spectral Energy Balance

It is useful to write the energy balance for the subsystems in terms of mean spectral densities, as

$$S_s \omega \sum_r \eta_s^r - \omega \sum_r{}' S_r \eta_r^s = P_s \quad . \tag{18}$$

This system of linear equations is easily solved for the unknown S. If only $P_1 \neq 0$, for example, we find

$$\frac{S_r}{S_1} = \frac{\Delta_{1r}}{\Delta_{11}} \quad , \tag{19}$$

where the Δ denote co-factors of the matrix elements indicated by the subscripts.

For the special case where there are only the three subsystems, r,w,s, where direct excitation acts only on subsystem r, and where there is no direct coupling between the first and the third, Eq. (19) yields

$$\frac{S_w}{S_r} = \frac{\eta_r^w}{\eta_w^r+\eta_w^w+\eta_w^s\eta_s^s/(\eta_s^w+\eta_s^s)} \quad , \tag{20}$$

and

$$\frac{S_s}{S_r} = \frac{\eta_w^s}{\eta_s^s+\eta_s^w} \cdot \frac{\eta_r^w}{\eta_w^r+\eta_w^w+\eta_w^s\eta_s^s/(\eta_s^w+\eta_s^s)} \quad . \tag{21}$$

These equations express the sharing of vibrational energy between the subsystems of a system that is connected and excited in the described manner.

III. EXPERIMENTAL STUDY OF NOISE REDUCTION OF RECTANGULAR BOXES

Figure 1 is a schematic diagram of the experimental setup. The box to be tested was suspended in a large room from a string attached to one box corner. The loudspeaker provided to excite the room volume was placed sufficiently far from the box and the room was sufficiently irregular and reverberant to generate a diffuse sound field.

Two test boxes were used successively; the smaller had outside dimensions 283 x 181 x 130 mm, the larger 458 x 407 x 306 mm. Both were made from 1.6 mm thick 2024 aluminum panels bolted at their edges to 6.4 mm square aluminum bar stock, with a bolt spacing of about 50 mm. Each box was provided with a circular hole near one of its corners to permit the insertion of a small microphone into the box. All edges and holes were sealed by application of hot wax.

The sound pressure incident on the box was measured by means of a 1/2-inch diameter condenser microphone (Brüel and Kjaer 4133) placed about 1 m above the box. The same microphone, when inserted into the box, was used to measure the sound pressure inside the box. A light accelerometer (Gulton A317T, weight 3.7 grams) was glued (successively at several locations) to the box panels. A root-mean-square meter with a long integration time (Ballantine 320) was used to measure average values of the various signals.

The damping of vibrations of the box was determined by exciting the box with an electrodynamic shaker (Goodman Industries V-47), removing the excitation, and measuring the reverberation time (with a recently developed reverberation time instrument).

Third-octave band filters (General Radio 1554-A, Brüel and Kjaer 2110) were used in the various circuits as indicated in the figure.

Values of the loss factor η_b associated with all energy losses of the box will be needed later when the response to sound excitation is calculated. These values were obtained from measurements of reverberation (vibration decay) times T at various frequencies f by use of the relation $\eta_b = 2.2/fT$. Results of these measurements for the smaller and the larger box, respectively, are shown in Figs. 2 and 3. Values of about 3×10^{-3} were found; this is much greater than the loss factor of aluminum, which is about 10^{-4}. Evidently, energy must also have been dissipated by radiation, by the joints, or by the enclosed air.

The response of box walls to a sound field in the room was determined. Corresponding ratios of mean square transverse velocities of the walls of the larger box to the mean particle velocity in the incident sound field are shown in Fig. 4. The reported

wall vibrations were measured where the vibrations were biggest (usually near the panel centers). The local reduction of wall vibration due to loading by the accelerometer at higher frequencies has been taken into account in the calculations leading to this figure by means of appropriate calculated corrections.

There is some question concerning the relation between spatial mean square panel vibrations and the vibrations measured near panel centers. The sound field excites most strongly those modes which are symmetrical about the panel center lines. These modes have maxima at the panel centers, so that one might expect the measured vibration levels to be excessive by about 6 db (or more for low modes of clamped panels). However, experimental measurements showed the vibration levels to be fairly independent of location for frequencies a decade or more above the fundamental panel resonances.

The noise reduction of the two test boxes for outside noise was determined. We define the noise reduction of a system (as distinct from wall transmission loss) as the ratio of mean square sound pressure at a location before and after the system has been introduced. The pressure in the room before the introduction of the box may be well approximated by the incident pressure measured sufficiently far away from the box. The pressure in the box was obtained as a rough mean square taken over the enclosed space. Figure 5 shows the noise reduction measured for the smaller box. Figure 6 shows that for the larger box.

IV. COMPARISON WITH THEORY

It is convenient to carry out this comparison in distinct frequency ranges according to the number of modes expected in a considered frequency band. There will always be many active modes of the room in which the box is suspended. However, the presence of mechanical box modes or of modes of the enclosed space will depend on the position of the experimental bands with respect to the lowest mechanical resonance (associated with the lowest of the fundamental resonances of the box panels) and the lowest acoustic resonance (where the acoustic wavelength is twice the length of the longest box edge).

Whenever several modes of a subsystem are expected to be active in a considered frequency band, we shall assume that their modal energies are equal, in the average. For the box walls, this assumption is supported by experimental evidence of mechanical coupling between the walls. However, the following analysis will be facilitated at various stages if we take the box to consist of panels that are attached to a rigid frame and hence mechanically isolated. This procedure is in line with the statistical method which allows two systems to be interchanged if only their statistical properties can be shown or assumed to be in reasonable agreement. It will further be assumed throughout the following discussion that no energy is stored in the box frame.

In the two boxes under study the lowest mechanical resonances occurred below the lowest acoustical resonances. The fundamental panel resonances were found between the corresponding frequencies one would calculate for hinged and for clamped boundaries [10]. The lowest mechanical resonance of the smaller box was observed to occur at 250 cps (compared to theoretical values of 167 and 322 cps for the two boundary conditions); the lowest mechanical resonance of the larger box was found at 62 cps (compared to computed values of 45 and 80 cps). The lowest acoustical resonances of the smaller and larger boxes occurred, respectively, at 600 and 370 cps.

A. Low Frequencies

At frequencies below all mechanical resonances the box panels and the enclosed air act as pure stiffnesses. The sound pressure p_s in the enclosed space follows the net volume change due to the wall motion according to the law of adiabatic compression. We find

$$p_s = (ic^2\rho/\omega V)\ \Sigma\ A_{wj}\langle v_{wj}\rangle \tag{22}$$

where c is the velocity of sound in air, ρ the density of air, ω the angular frequency, V the enclosed volume; A_{wj} is the area and $\langle v_{wj}\rangle$ the spatial mean outward velocity of the j'th wall. The summation is taken over all six walls of the box.

The acoustic pressure on the box surface is nearly equal to the incident pressure p_r in the outside room. For a rigid box the deviation of surface pressure from p_r is expected to be less than 1 db, even for frequencies as high as the first acoustic resonance [11]. Box deformations cause the surface pressure to exceed p_r by an amount of the order of $(a/\lambda)^2 << 1$, where a is the greatest panel edge length and λ is the wavelength of sound in air. When we neglect both deviations, the noise reduction of the box becomes simply

$$NR = \left|\frac{p_r}{p_s}\right|^2 = \left[1 + (V/\rho c^2)\,\Sigma\, C_j\right]^2 , \qquad (23)$$

where the C_j are the compliances (ratios of displaced volume to pressure) of the walls [6].

By computing the compliances C_j of the experimental panels as if they were clamped plates we obtain NR = 22 db for the smaller box and NR = 7 db for the larger one. These values are indicated in Figs. 5 and 6 and may be seen to agree rather well with the experimental values. (Corresponding calculations assuming the panels to be simply supported lead to values that fall considerably below the observed ones.)

B. Intermediate Frequencies

This intermediate frequency region extends from the lowest mechanical resonance to somewhat below the lowest acoustic resonance. The air within the box still acts like a pure stiffness closely coupled to the walls and the pressure there obeys Eq. (22), but the walls exhibit resonant behavior. The enclosed volume need still not be treated as a separate subsystem; it loads the walls reactively and increases some of their resonance frequencies slightly, but this increase may be neglected because it barely changes the modal density of wall resonances n_{wj}. Nevertheless the various wall modes must be considered separately (not just in modal averages), because they are widely spaced, at least near the lower end of the intermediate frequency region, and because at their resonances they can produce considerable increases in the sound pressure within the box.

(a) Relation between Mean and Mean-square Wall Velocities

For frequencies below the acoustic resonance we may obtain a good approximation to the spatial mean velocity $\langle v_{wj} \rangle$ of a box wall by equating the power radiated from it to the power radiated from a piston of equal area and mean velocity[6]:

$$\langle v_{wj}^2 \rangle \, \eta_{wj}^r \, \omega M_{wj} = \langle v_{wj} \rangle^2 \, A_{wj}^2 \, \rho\omega^2/4\pi c \; . \qquad (24)$$

Here M_{wj} denotes the mass of a wall and η^r_{wj} denotes the loss factor associated with the flow of energy from a wall to the outside room. In accordance with Eq. (11),

$$\eta^r_{wj} = P^r_{wj}/\omega S_{wj} \ , \tag{25}$$

where P^r_{wj} is the spectral density of power flowing from the wall to the room surrounding the box (without considering the power P^{wj}_r flowing in the opposite direction) and S_{wj} is the energy spectral density of the wall. The spectral densities of the velocities in Eq. (24) must be related in the same manner as the velocities, namely,

$$S^v_{wj}\eta^r_{wj}\omega M_{wj} = S^{\langle v\rangle}_{wj}A^2_{wj}\rho\omega^2/4\pi c \ . \tag{26}$$

(b) Noise Reduction at Single Wall Resonances

Minimum noise reduction may be expected to occur when two opposite box panels vibrate at or near their common resonance frequency and move symmetrically in phase with each other. If the resonances of the other panels lie outside the frequency band considered, these other panels can be considered as rigid.

By making use of the aforementioned symmetry, i.e., by considering a volume V/2 with a single non-rigid wall, designated by the subscript j, and by applying Eq. (22) and Eq. (26) we find

$$S_s^p = \frac{16\pi\rho c^5}{\omega^3 V^2} \eta_{wj}^r M_{wj} S_{wj}^v \tag{27}$$

Since no net energy is transferred to the enclosed space, the power received by the wall is either dissipated mechanically or reradiated to the room outside the box. Thus,

$$S_r \eta_r^{wj} = S_{wj}(\eta_{wj}^{wj} + \eta_{wj}^r) \ . \tag{28}$$

The velocity spectral densities S_r^v and S_{wj}^v may be obtained by dividing the respective energy spectral densities by the masses. Combining Eqs. (16), (27) and (28), and the relation

$$S_r^p = \rho^2 c^2 S_r^v \tag{29}$$

between pressure and velocity spectral densities in the room, we obtain

$$S_s^p = \frac{16\pi c^3 M_r n_{wj} (\eta_{wj}^r)^2}{\rho\omega^3 V^2 n_r (\eta_{wj}^{wj} + \eta_{wj}^r)} \ S_r^p \ . \tag{30}$$

For a large test room the power lost by the wall to the room may be approximated well by the power that would be radiated into a half space. The corresponding coupling loss factor η_{wj}^r, calculated as if the box walls were baffled hinged plates, is found to be [6,12]

$$\eta^r_{wj} = \frac{32\rho\omega}{c\pi M_{wj} k_x^2 k_y^2} \tag{31}$$

for a mode well below the critical frequency (the lowest coincidence frequency) $f_{cj} = \sqrt{3}\ c_o^2/\pi h_j c_{wj}$ of the wall. The product of plate wave numbers in the x- and y-direction $k_x k_y$ may be approximated by $(k_x^2 + k_y^2)/2 = \sqrt{3}\ \omega/h_j c_{wj}$. Equation (31) then becomes

$$\eta^r_{wj} = \frac{16\rho h_j c_{wj}^2}{3\pi^2 f c A_{wj} \rho_{wj}}\ , \tag{32}$$

where c_{wj} and ρ_{wj} are respectively the plate speed of sound and the density of the wall material, and h_j the wall thickness.

By inserting the ratio of modal density to mass

$$n_r/M_r = 4\pi f^2/\rho c^3 \tag{33}$$

as well as Eq. (32) and $n_{wj} = 1/\Delta f$ into Eq. (30), we obtain the noise reduction

$$NR = \frac{S_r^p}{S_s^p} = \frac{\pi\Delta f}{2f}\ (\eta^{wj}_{wj} + \eta^r_{wj}) \left[\frac{3\pi^3 f^4 \rho_{wj} A_{wj} V}{8\rho h_j c^2 c_{wj}^2} \right]^2 . \tag{34}$$

By introducing into Eq. (34) the area of the smallest wall one obtains a lower bound to the noise reduction which applies also at resonances of the larger panels occurring below the fundamental of the smallest ones. By inserting the appropriate

numerical values into Eq. (34) and approximating the total wall loss factor $\eta_{wj}^{wj} + \eta_{wj}^{r}$ by the measured η_b values (Figs. 2 and 3) one obtains the lines labeled "lower bound" in Figs. 5 and 6. It may be seen that the experimental data indeed do exceed these lower limits, with one exception.

(c) Noise Reduction in Multimodal Bands

In reasonably wide frequency bands not too close to the fundamental wall resonances we may expect to encounter several active wall modes. Assuming that the wall motions are incoherent we find from Eq. (22) and Eq. (26) that the pressure spectral density in the enclosed space is given by

$$S_s^p = \frac{4\pi\rho c^5}{\omega^3 V^2} \sum \eta_{wj}^{r} S_{wj} \quad . \tag{35}$$

The wall spectral energy balance may be expressed as

$$S_r \sum \eta_r^{wj} = S_w \sum (\eta_{wj}^{wj} + \eta_{wj}^{r}) n_{wj}/n_w \quad , \tag{36}$$

where $S_w = \Sigma\, S_{wj}$, $n_w = \Sigma\, n_{wj}$, and

$$n_{wj} = \sqrt{3}\, A_{wj}/c_{wj} h_j \tag{37}$$

is the modal density of the j'th wall (approximated by its asymptotic formula). Average equipartition of wall modal energy is expressed by

$$S_{wj} = n_{wj}\, S_w/n_w \ . \tag{38}$$

After introducing Eqs. (36) and (38) into (35) we obtain

$$S_s^p = \frac{4\pi c^3 M_r S_r^p}{\rho\omega^3 V^2 n_r} \left[\sum \eta_{wj}^r n_{wj} \right]^2 \Big/ \sum (\eta_{wj}^{wj} + \eta_{wj}^r)\, n_{wj} \ . \tag{39}$$

The coupling loss factors appearing in Eq. (39) may be approximated by use of an available expression for the average power radiated into half-space by a baffled hinged panel vibrating in modal equilibrium [6,12]. For frequencies well below the critical frequency we find

$$\eta_{wj}^r = \frac{\rho c_{wj} L_{wj}}{\pi^3 c \rho_{wj} A_{wj}} \left[\frac{c_{wj} h_j}{3\pi\sqrt{3}\, f} \right]^{1/2} , \tag{40}$$

where L_{wj} is the perimeter of the j'th panel. When we introduce this result and Eqs. (33) and (37) into Eq. (39) we obtain the following expression for the noise reduction of the box:

$$NR = 6 \left[\sum (\eta_{wj}^{wj} + \eta_{wj}^r) n_{wj}/n_w \right] \left[\sum \frac{A_{wj}}{c_{wj} h_j} \right] \left[\frac{2\pi^5 f^3 V}{\rho c^2 \sum (c_{wj}/h_j)^{1/2} L_{wj}/\rho_{wj}} \right]^2 . \tag{41}$$

We may approximate the first bracketed term by η_b, the measured total loss factor of the box. (The approximation is exact if the panels also share modal energy equally throughout the decay experiment in which η_b is measured.) If we now specialize to the case where all walls are of the same material and of equal thickness, we obtain

$$NR = \frac{96\Sigma A_{wj}}{(\Sigma L_{wj})^2} \eta_b \left[\frac{\pi^5 f^3 \rho_w V}{2\rho c^2 c_w}\right]^2 . \tag{42}$$

For boxes that are roughly of cubic shape, the term $96\ \Sigma A_{wj}/(\Sigma L_{wj})^2 \approx 1$. For the experimental boxes considered here we find from Eq. (42) values corresponding to the dashed lines appearing in Figs. 5 and 6 directly above those labeled "lower bound." As evident from the figures, the lines based on Eq. (42) predict the average of the observed noise reductions reasonably well.

C. High Frequencies

At frequencies well above the fundamental mechanical and acoustic resonances we may rely on the presence of several mechanical as well as several acoustic modes in any of the considered frequency bands. The energy balance for the panels then may be expressed as

$$S_r \Sigma \eta_r^{wj} + S_s \Sigma \eta_s^{wj} = \Sigma S_{wj}(\eta_{wj}^{wj} + \eta_{wj}^{r} + \eta_{wj}^{s}) , \tag{43}$$

and that for the enclosed space as

$$\Sigma S_{wj} \eta_{wj}^{s} = S_s(\eta_s^s + \Sigma \eta_s^{wj}) . \tag{44}$$

The summations here are again taken over all the box walls.

Similarly, the energy balance pertaining to the decay experiment is given by

$$S_w \eta_b = \Sigma S_{wj}(\eta_{wj}^{wj} + \eta_{wj}^{r} + \eta_{wj}^{s}) - S_s \Sigma \eta_s^{wj} \tag{45}$$

If we assume that Eq. (44) and equipartition of wall modal energy, Eq. (38), hold also for the decay experiment, we find from Eqs. (16) and (45) that

$$\eta_b = \Sigma(\eta_{wj}^{wj} + \eta_{wj}^{r} + \eta_{wj}^{s})n_{wj}/n_w - \frac{n_w^{-1}(\Sigma\eta_{wj}^{s}n_{wj})^2}{n_s\eta_s^s+\Sigma\eta_{wj}^{s}n_{wj}} . \tag{46}$$

By use of this relation and Eqs. (16), (43), and (44), we may conclude that the energy levels of the box walls and of the enclosed space are given by

$$S_w = \frac{S_r}{n_r\eta_b} \Sigma \eta_{wj}^{r} n_{wj} \tag{47}$$

and

$$S_s = S_r \frac{n_s(\Sigma\eta^r_{wj}n_{wj})(\Sigma\eta^s_{wj}n_{wj})}{n_r n_w \eta_b (n_s\eta^s_s + \Sigma\eta^s_{wj}n_{wj})} \quad , \tag{48}$$

in correspondence to Eqs. (20) and (21), respectively.

By applying Eq. (33) to Eq. (47) we find that the velocity levels obey

$$\frac{S^v_w}{S^v_r} = \frac{M_r}{n_r M_w \eta_b} \Sigma\eta^r_{wj}n_{wj} \quad . \tag{49}$$

Then we find from Eqs. (29) and (48) that the noise reduction is given by

$$NR = \frac{n_w\eta_b}{r\Sigma\eta^r_{wj}n_{wj}} \left[1 + \frac{n_s\eta^s_s}{\Sigma\eta^s_{wj}n_{wj}} \right] \tag{50}$$

where

$$r = \frac{n_s M_r}{n_r M_s} = 1 + A_w c/8fV + Lc^2/32\pi f^2 V \tag{51}$$

and L denotes the sum of all box edge lengths. The ratio r represents a correction factor which expresses the deviation of the modal density of the enclosed space from the corresponding asymptotic formula. This ratio is plotted in Figs. 2 and 3 for the two test boxes.

The wall coupling losses appearing in Eqs. (49) and (50) may be approximated again in terms of the power radiated into free space by a baffled hinged plate vibrating in modal equilibrium [12]. For a plate wider than the acoustic wavelength and radiating into a half space below its critical frequency f_{cj} we find

$$\eta^r_{wj} = \frac{\rho c^2 L_{wj}\, g_2(f/f_{cj})}{\omega \rho_{wj} h_{wj} f_{cj} A_{wj}} , \tag{52}$$

where the function g_2 is that of Maidanik[12,13]. However, in this frequency regime the plates radiate mostly from their edges[12] so that the box walls radiate essentially outward into a 3/4-space and inward to a 1/4-space [6]. Modifying Eq. (52) accordingly, we obtain by use of Eq. (37) that

$$\Sigma \eta^r_{wj} n_{wj}/n_w = \frac{2\rho c^2}{3\omega} \frac{\Sigma L_{wj} g_2(f/f_{cj})/\rho_{wj} h_j^2 f_{cj} c_{wj}}{\Sigma A_{wj}/c_{wj} h_j} \tag{53}$$

and three times this value for $\Sigma \eta^s_{wj} n_{wj}/n_w$.

Computed values of the coupling losses for the two boxes under study are shown in Figs. 2 and 3. They lie generally well below the measured values of the total loss factor η_b, except when the frequency approaches the critical frequency.

The internal losses of the enclosed space are dominated by viscous and thermal losses at the boundary, so that the corresponding loss factor obeys

$$\eta_s^s = s_m c A_w / 4\omega V \ , \tag{54}$$

where s_m is a dimensionless quantity which, for air at room temperature, is given by[14] $s_m = 18 \times 10^{-5} f^{1/2}$, where f denotes frequency in cps. (Other losses, such as that due to internal absorption in the air are negligible.) The loss factors η_s^s associated with the two experimental boxes are indicated in Figs. 2 and 3.

Computations based on Eq. (49) result in vibration level values that, for the larger box, are in close agreement with measured values (see Fig. 4) at frequencies from one octave above the lowest acoustic resonance up to the critical frequency. Similarly, the noise reduction calculated from Eq. (50), as shown in Figs. 5 and 6, may be seen to be in good agreement with experimentally determined values up to about one octave below the critical frequency.

D. Classical Transmission Theory

For the sake of additional comparison, calculations based on classical wall transmission theory are presented, too. The

transmission coefficient τ of a wall is classically[1] defined as the ratio of transmitted to incident sound power, where the wall is taken as of infinite extent. If the mean sound intensity incident on the test box is denoted by

$$I_r = \langle p_r^2 \rangle / 4\rho c \ , \tag{55}$$

and the mean intensity in the enclosed space is denoted by I_s, then the power balance for the enclosed space may be written as

$$I_r \ \Sigma A_{wj} \tau_j = I_s \ \Sigma A_{wj} \tau_j + 4 I_s \omega \eta_s^s \ V/c \tag{56}$$

where the last term denotes the power absorbed in the enclosed space. The noise reduction of a box, specialized for walls of equal transmission coefficient, may then be found to be given by

$$NR = \frac{I_r}{I_s} = 1 + \frac{4\omega\eta_s^s V}{c A_w \tau} \tag{57}$$

We would expect this expression to be valid at high frequencies, where the box panels extend over many flexural wavelengths and the interior space encompasses many acoustic wavelengths. Figures 5 and 6 show NR values for the test boxes calculated from Eq. (57) and using a value of τ according to the "field incidence" law[1].

V. DISCUSSION AND CONCLUSION

The results obtained from vibration and sound measurements on rectangular aluminum boxes were found to agree reasonably well with theoretical predictions based on various models applicable in different frequency regions. It is also apparent from the analysis that the noise reduction of a box, as defined here, is the same for an external as for an internal noise source, provided that both are diffuse constant volume noise sources.

At very low frequencies, the noise reduction of a rectangular box is constant but much less than that which would be obtained with a spherical shell of the same material, weight, and enclosed volume. The noise reduction of a spherical shell of radius R may be shown to be given by $NR = [1 + 2hc_w^2\rho_w/3c^2\rho R]^2$. For spherical enclosures of the same material, weights, and volumes as the rectangular test boxes we calculate noise reductions of 77 and 69 db (compared to 22 and 7 db, respectively, for the rectangular boxes). Of course, the low noise reduction of a flat plate below its first mechanical resonance has long been known to acoustic practitioners.

At somewhat higher frequencies, the lowest mechanical resonances are reached. These are associated with irregular behavior of the noise reduction curve, which may even take on negative values. The behavior in this frequency region may be analyzed by taking the structural resonances into account, but assuming the enclosed volume to act as a stiffness. The frequency range in which this behavior occurs depends on the panel properties and box dimensions; for the smaller box studied it encompassed 1-1/2 octaves, for the larger 2-1/2 octaves. Negative noise reduction values as low as -18 db in some third octave band were observed.

At higher frequencies, modal details disappear and the noise reduction rises at a rate of about 3 db/octave in good agreement with theory. Noise reduction maxima (of about 17 db) were observed at about half an octave below the critical frequency of the panels.

About the critical frequency, the noise reduction of the two test boxes decreases to about 11 db, but the predicted values fall short of the measured ones. This is obviously due to an overestimate of the panel radiated power by the theoretical expression used to calculate the panel coupling loss factors in this frequency region. Still lower values are predicted about the critical frequency by the classical

wall transmission theory; but at lower frequencies this theory, which takes into account only forced waves, predicts somewhat too high values of noise reduction.

An extension of the analysis presented here to enclosures with nonuniform walls or with reinforcing ribs could be readily accomplished. Inclusion of these factors would complicate the computation of the various loss factors and modal densities only slightly.

A method for the analysis of an average system due to average excitation in terms of masses, modal densities, and loss factors, has been presented and illustrated. After internal and external loss factors have been calculated or estimated by means of convenient substitute systems, only elementary operations remain to be performed. The method treats sound and structural vibration equally and should be successful with complex systems which could not be analyzed otherwise without difficulty.

VI. ACKNOWLEDGMENT

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8. Here and throughout this paper the brackets <...> denote average values.

9. In Ref. 4, β_i is used for ϕ_i^i and g_{ij} for ϕ_i^j.

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13. Near the critical frequency the function g_2 has been limited in such a manner that the panel radiation efficiency did not exceed unity. The expression corresponding to Eq. (52) which resulted is $\eta^r_{wj}=\rho c/\omega\rho_{wj}h_j$ and was also used for calculations beyond the critical frequency.

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Fig. 6 Noise reduction of larger box

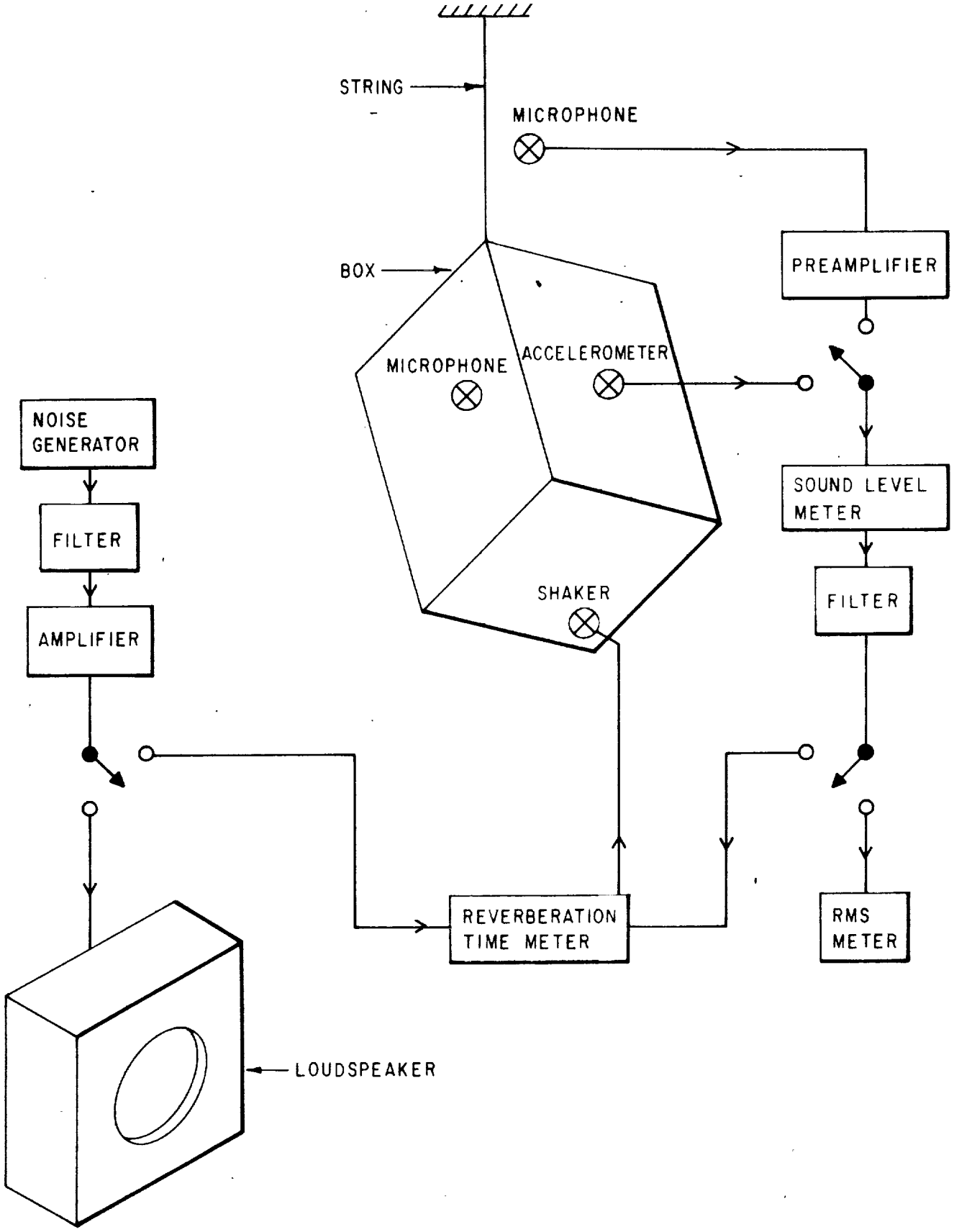

FIG. 1 EXPERIMENTAL SET-UP

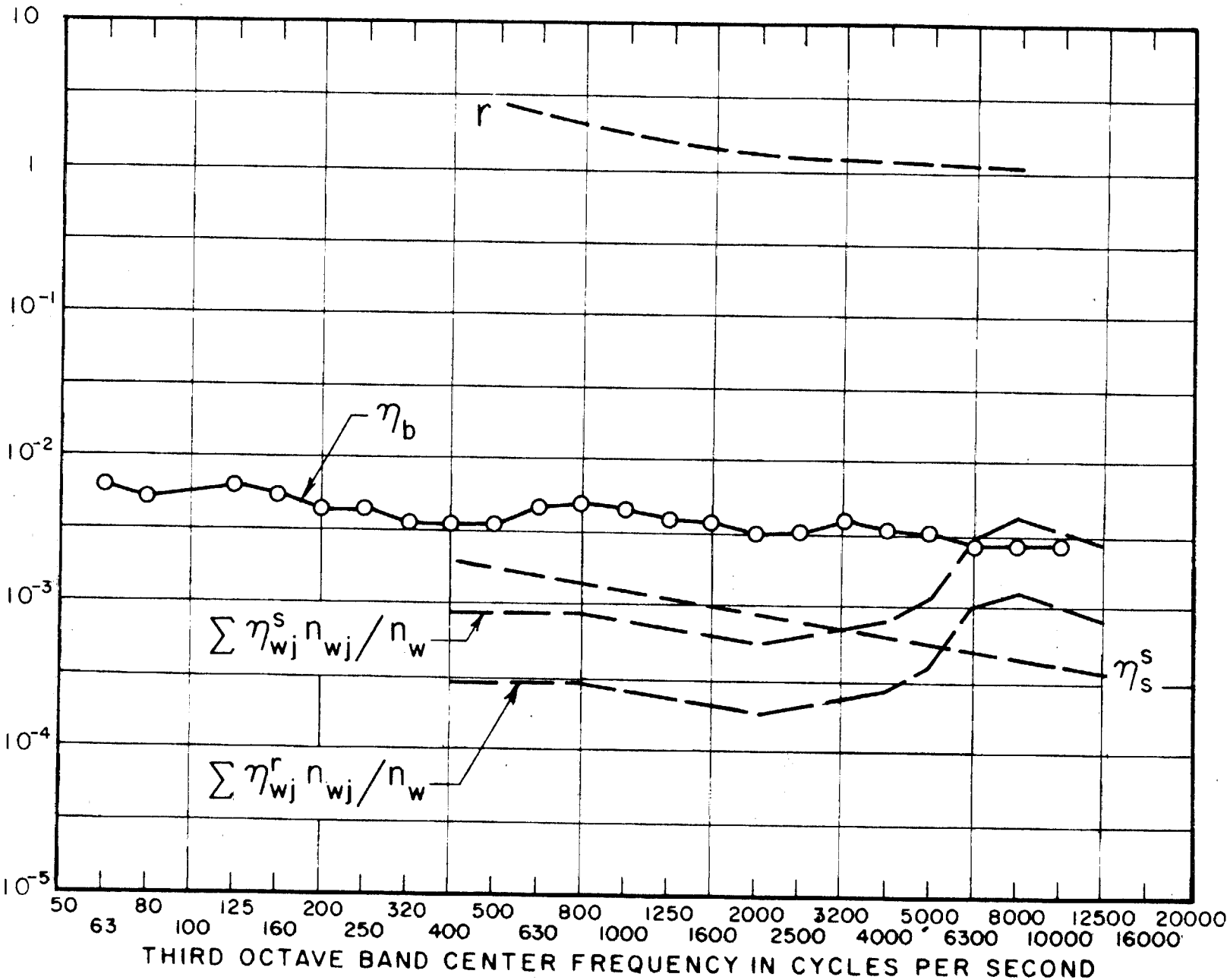

FIG.3 LOSS FACTORS η AND MODAL DENSITY CORRECTION r OF LARGER BOX

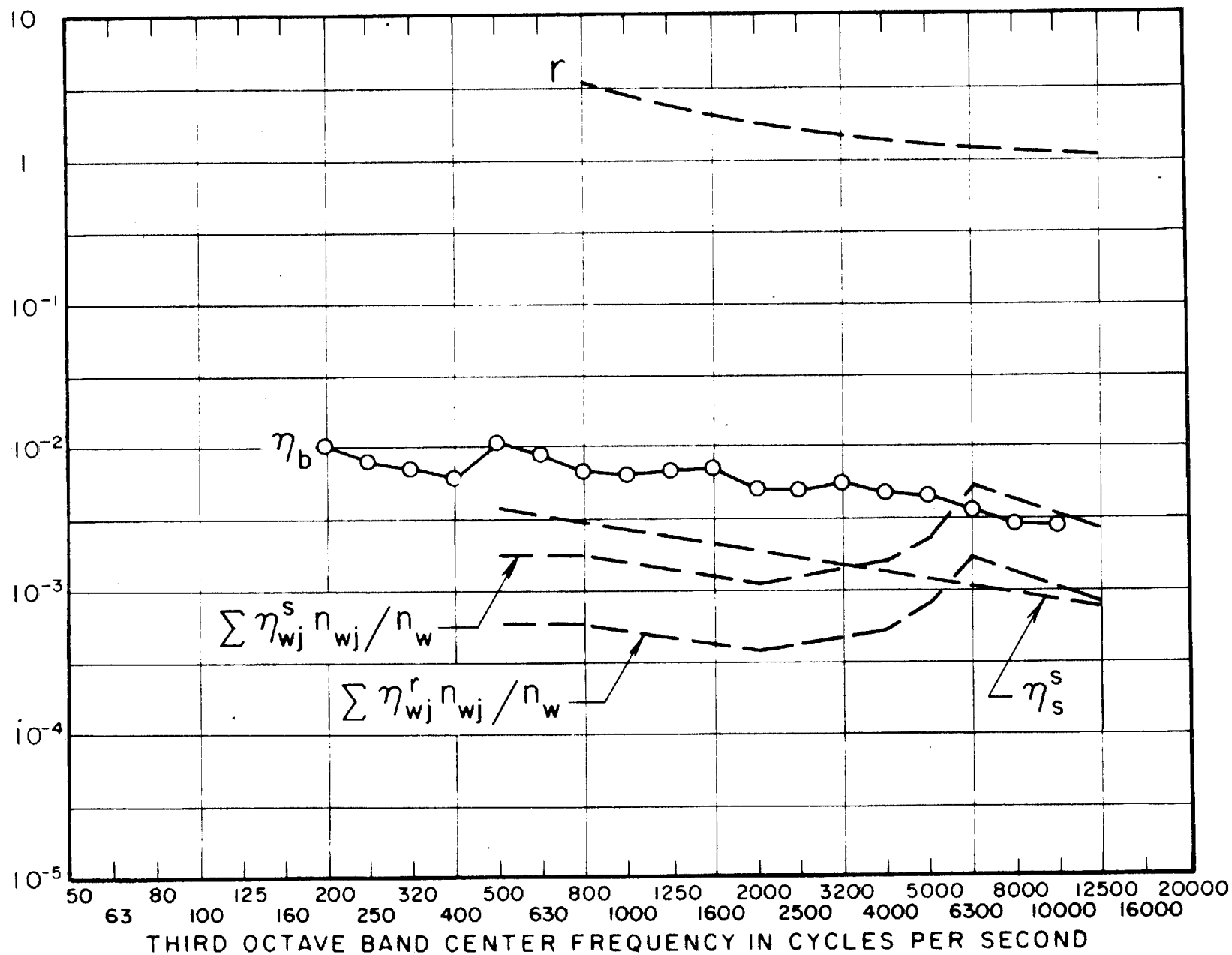

FIG.2 LOSS FACTORS η AND MODAL DENSITY CORRECTION r OF SMALLER BOX

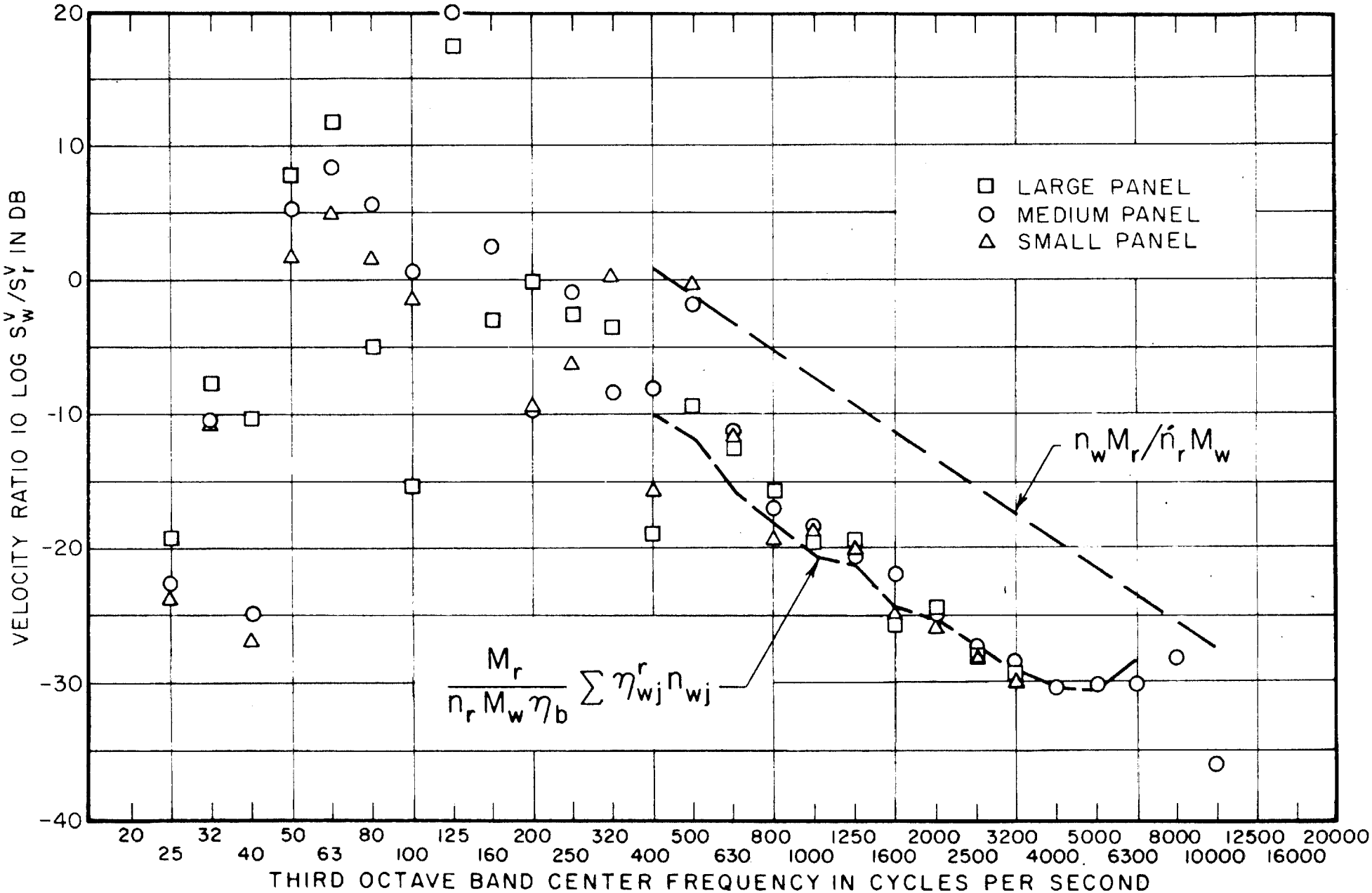

FIG.4 VIBRATORY RESPONSE OF LARGER BOX

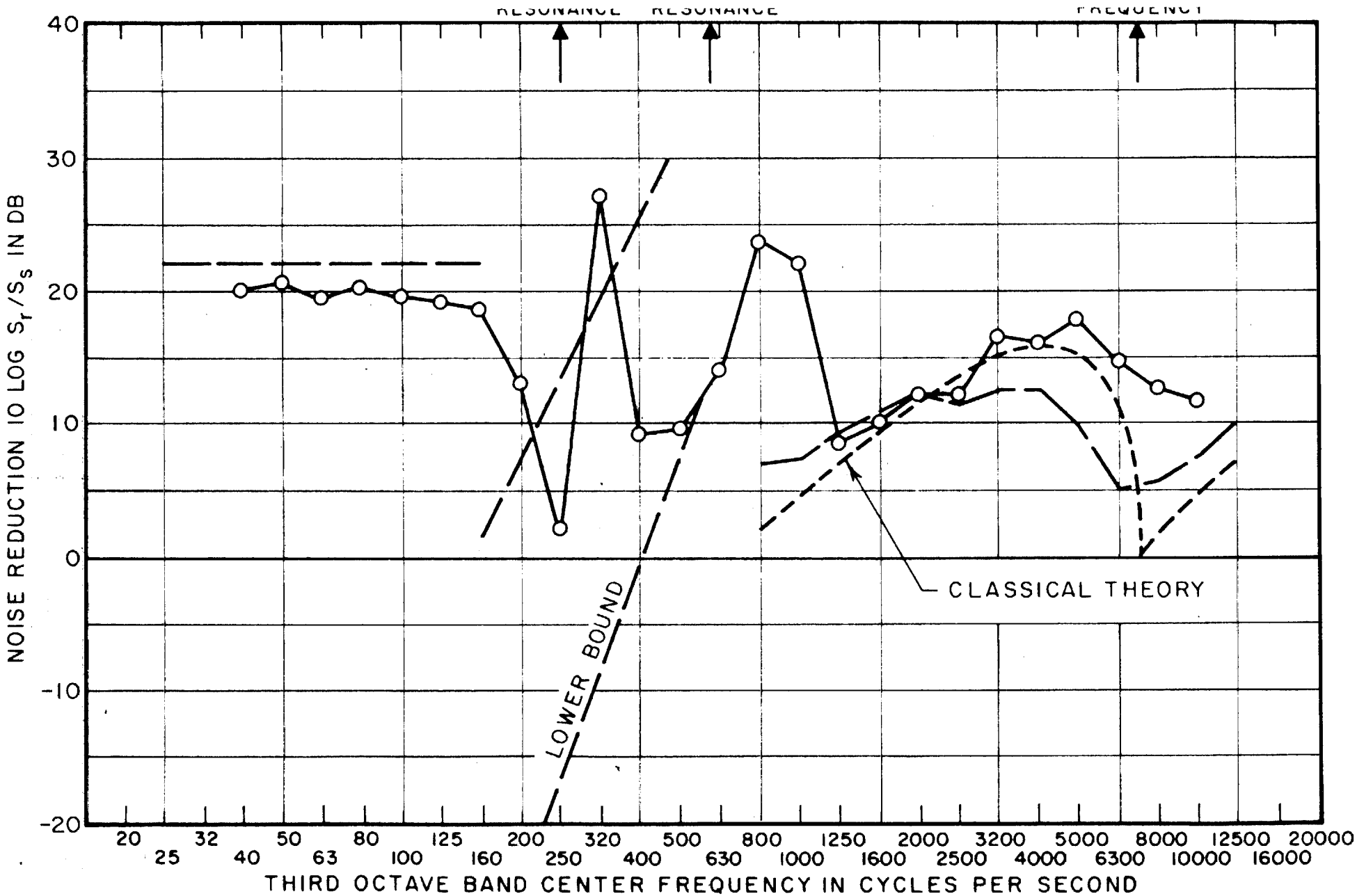

FIG. 5 NOISE REDUCTION OF SMALLER BOX

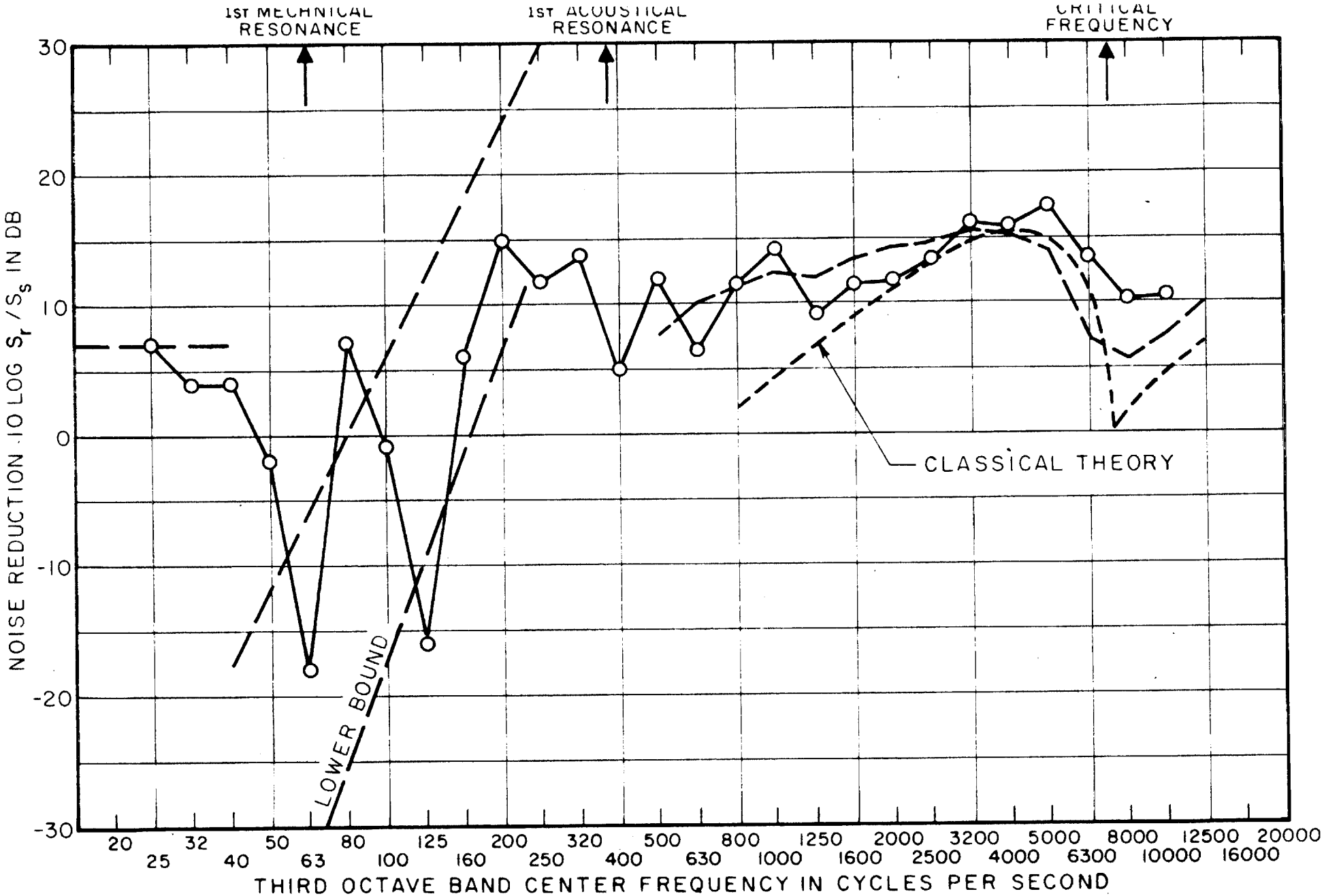

FIG.6 NOISE REDUCTION OF LARGER BOX