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PRECISE ASPECTS OF TERRESTRIAL AND CELESTIAL REFERENCE FRAMES

by

George Veis

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George Veis

Smithsonian Institution
Astrophysical Observatory

Cambridge 38, Massachusetts

PRECISE ASPECTS OF TERRESTRIAL AND CELESTIAL REFERENCE FRAMES[1]

George Veis[2]

Abstract

Three distinct reference systems are involved in an analysis of observations of artificial satellites for geodetic purposes. A terrestrial system for earth-fixed points, a celestial system for observations, and an orbital system for the orbital elements. These reference systems have to be carefully defined, and their precise relation established for the fullest exploitation of satellites for geodetic purposes. In practice, both the terrestrial system and the celestial system are replaced by the geodetic systems and the astrometric systems. The transformation from the celestial system to the terrestrial is done through the use of a sidereal system, or of a modified sidereal system introduced by the author. It can be proved that the optimum system for orbital work is the same with the modified sidereal system. The existing geodetic triangulation nets are expected to be internally consistent to an accuracy of 10^{-5} and the accuracy with which directions in space can be determined by photographing the object against star background is expected to be better than 1" of arc.

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[2]Smithsonian Astrophysical Observatory, Cambridge, Mass.; and National Technical University, Athens, Greece.

Introduction

The astrometric determination of directions in space by photographing objects against the star background is a very powerful method. Since both terrestrial and celestial reference systems are involved, we need to define and relate them.

Theoretically, the earth-fixed system is defined by the mean terrestrial pole of a certain date and the mean meridian of Greenwich, and the celestial-fixed system by the mean equator and the ecliptic of a certain date. In reality, however, both systems are defined by a set of coordinates given to physical points. Specifically the terrestrial system is replaced by a number of geodetic systems, and the celestial by a system defined by the mean places and the proper motions given to stars belonging to a catalogue.

The two systems can be related when one knows the position of the instantaneous pole and the sidereal time at Greenwich as determined by observation, and the precessional and nutational constants. This is done through the use of the sidereal system or of a modified sidereal system introduced by the author.

In addition, a system is needed to which the orbital parameters of the artificial satellites can be referred. For this purpose, rectangular Cartesian coordinates rather than a polar or a cylindrical system will be used throughout this paper, as these coordinates have several advantages in analytical manipulation and can easily be transformed to any other Cartesian system by a simple matrix multiplication.

The terrestrial systems

The ideal terrestrial rectangular coordinate system (x^1, x^2, x^3) is defined as follows: The origin is at the earth's center of gravity and is oriented so that the third axis is directed toward the mean north pole as defined by the International Latitude Service. The 1-3 plane is parallel to the mean meridian of Greenwich, also called the meridian of the Mean Observatory, as defined by the Bureau Internationale de l'Heure, but any other standard meridian can be used. This terrestrial coordinate system is fixed with respect to the earth's surface; and the coordinates of any point on the earth do not change, assuming no movement of the crust.

Since the center of gravity of the earth is not known, this ideal reference system is replaced by a number of (rectangular) geodetic systems (u^1, u^2, u^3) defined theoretically by the geodetic coordinates φ_o, λ_o, and H_o of the origin of each geodetic datum. The origin of each geodetic system will not coincide in general with the earth's center of gravity, which is the origin of the ideal terrestrial system. The coordinates of the origin of a geodetic system (x_o^1, x_o^2, x_o^3) expressed in the terrestrial system will be a function of the absolute deflections of the vertical ξ_o, η_o, ζ_o, expressed in linear units, as the origin of the datum (see fig. 1). We can easily show that their relation will be given by:

$$\begin{pmatrix} x_o^1 \\ x_o^2 \\ x_o^3 \end{pmatrix} = \begin{pmatrix} \sin\varphi_o \cos\lambda_o & \sin\lambda_o & -\cos\varphi_o \cos\lambda_o \\ \sin\varphi_o \sin\lambda_o & -\cos\lambda_o & -\cos\varphi_o \sin\lambda_o \\ -\cos\varphi_o & 0 & -\cos\varphi_o \end{pmatrix} \begin{pmatrix} \xi_o \\ \eta_o \\ \zeta_o \end{pmatrix} \qquad (1)$$

We can also prove (Veis, 1960) that every geodetic system must be parallel to the terrestrial one and therefore to every other geodetic system. However, this parallelism will not hold unless the geodetic systems are properly oriented by astronomic methods. Let us assume an orientation error expressed by three components: dA (the error in azimuth of the system),

$d\xi$ (the error in tilt of the system on the meridian plane), and $d\eta$ (the error in tilt in the prime vertical plane) at the origin of the datum. Then the general transformation from the geodetic coordinates to the ideal geocentric terrestrial coordinates will be given by the following expression:

$$\underset{\sim}{x} = \underset{\sim}{u} + \underset{\sim}{x}_o + \underset{\sim}{K}\underset{\sim}{e} , \qquad (2)$$

where

$$\underset{\sim}{e} = \begin{pmatrix} dA \\ d\xi \\ d\eta \end{pmatrix} ;$$

$$\underset{\sim}{K} = \begin{pmatrix} \sin\varphi_o \Delta u^2 - \cos\varphi_o \sin\lambda_o \Delta u^3 & \cos\lambda_o \Delta u^3 & -\cos\varphi_o \Delta u^2 - \sin\varphi_o \sin\lambda_o \Delta u^3 \\ -\sin\varphi_o \Delta u^1 + \cos\varphi_o \cos\lambda_o \Delta u^3 & \sin\lambda_o \Delta u^3 & \cos\varphi_o \Delta u^1 + \sin\varphi_o \cos\lambda_o \Delta u^3 \\ \cos\varphi_o \sin\lambda_o \Delta u^1 - \cos\varphi_o \cos\lambda_o \Delta u^2 & \cos\lambda_o \Delta u^1 + \sin\lambda_o \Delta u^2 & \sin\varphi_o \sin\lambda_o \Delta u^1 - \sin\varphi_o \cos\lambda_o \Delta u^2 \end{pmatrix} ;$$

and

$$\Delta \underset{\sim}{u} = \underset{\sim}{u} - \underset{\sim}{u}_o .$$

In expression (2), the terms of squares of the angular errors have been omitted. Expression (2) can also be written in the form

$$\underset{\sim}{x} = \underset{\sim}{u} + \underset{\sim}{x}_o + \underset{\sim}{M} \Delta \underset{\sim}{u} , \qquad (3)$$

which is the general expression of the rectangular coordinate transformation. These errors in orientation will probably be no larger than 1" of arc, at least for major geodetic datums. However, both the error in the azimuth and the error in the tilt have a real meaning (Veis, 1960). In practice, a geodetic system is defined not in theoretical terms but by the specific set of coordinates given to actual triangulation points on the surface of the earth. Therefore, we might also consider the introduction of a different scale factor for every geodetic system, since nonconnected triangulations may be at a different scale.

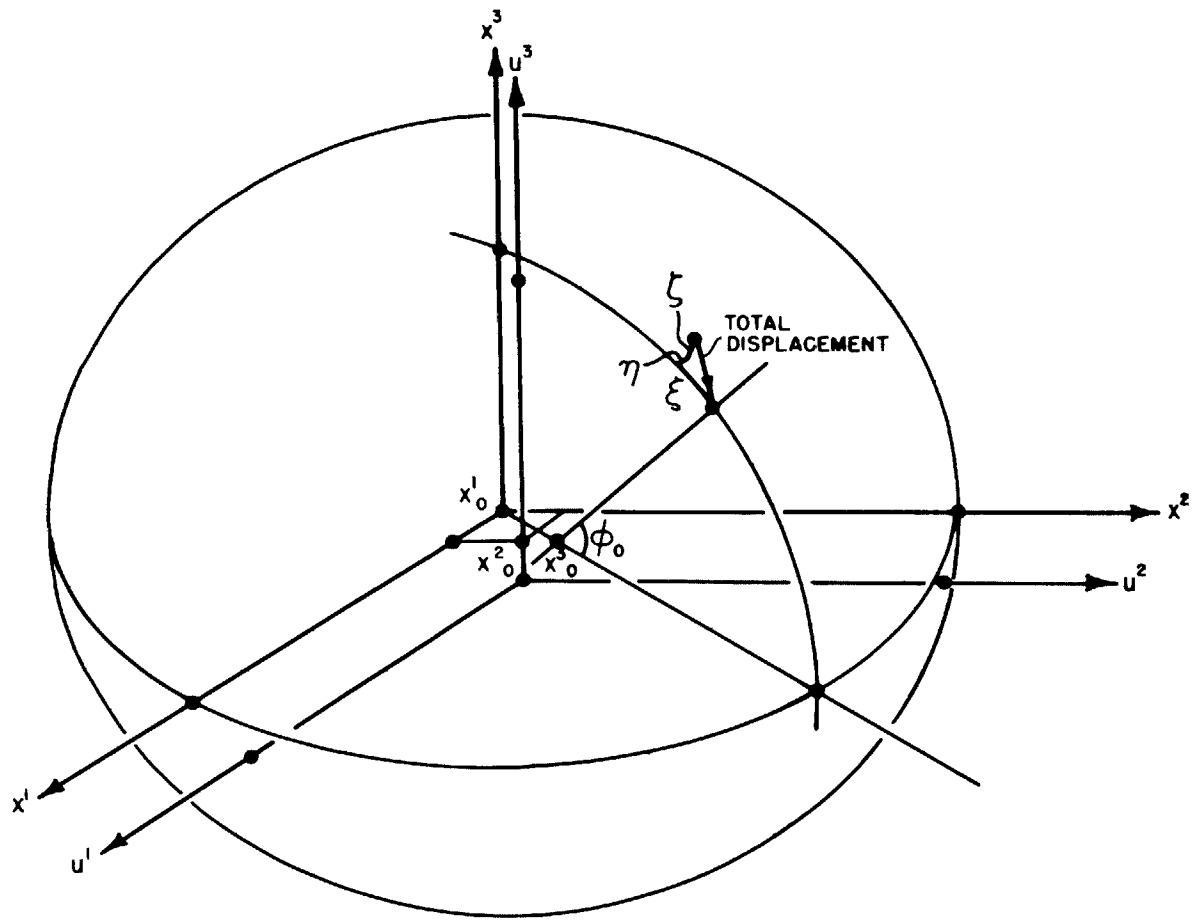

Fig. 1. Terrestrial and geodetic systems.

Modern geodetic nets have an internal consistency of about 10^{-5}. This represents the average accuracy of rather large triangulation nets. We can therefore assume that a geodetic system is accurate to one part in 10^5.

The celestial systems

A nonrotating system is convenient for points that do not rotate with the earth. The earth's axis of rotation is not fixed with respect to its surface. The actual motion of the true pole, defined by the instantaneous axis of the rotation of the earth, is determined by the International Latitude Service, which gives the true pole's position in terms of the angular coordinates (x,y) of the instantaneous pole with respect to a mean pole Q. This same mean pole is used to define the terrestrial system. On the other hand, the actual rotation of the earth around this instantaneous axis is measured by what is called sidereal time θ defined as the Greenwich hour angle of the (vernal) equinox. The use of the true or the mean equinox distinguishes true sidereal time θ from mean sidereal time $\bar{\theta}$. The difference between the true and the sidereal time is called the equation of the equinoxes.

Apart from the motion of the equinox due to precession and nutation, with the known irregular variations in the speed of the rotation of the earth, it is quite important that the sidereal time be considered an angle in space rather than elapsed time.

Both the mean and the true sidereal time are given as functions of solar time, more precisely of the universal time (UT). In practice, however, the sidereal time is directly observed, and UT is defined from the actually observed sidereal time according to an analytical expression. The universal time thus obtained and corrected for the displacement of the pole is called UT-1, which is the first step to a more uniform time (Stoyko, 1955). A further step is to subtract the effect of the seasonal variation in the rotation of the earth. The UT-1 time thus corrected is called UT-2, the system used by most of the time services throughout the world. It should be emphasized that UT-2 is not really a uniform time, since the seasonal variation is only an extrapolated expression obtained by analyzing the observations of some years (Stoyko, 1955). We will use the decimal system to divide the day (rather than using hours, minutes and seconds), and measure whole days starting from the Julian Date 2400000.5. This system is now in use at the Smithsonian Astrophysical Observatory. An epoch referred to this system is called Modified Julian Date (MJD). We have

$$\text{MJD} = \text{JD} - 2400000.5.$$

Depending on whether we refer the epoch to UT-1 or UT-2, we have MJD-1 or MJD-2.

The following expression gives $\bar{\theta}$ as a function of time:

$$\bar{\theta} = 100^\circ.075542 + 360^\circ.985647348(\text{MJD}-33282.0) + 0^\circ.2900 \times 10^{-12}(\text{MJD}-33282.0)^2 \ . \tag{4}$$

The expression that gives θ is

$$\begin{aligned}\theta = {} & 100^\circ.075542 + 360^\circ.985647348\ T + 0^\circ.2900 \times 10^{-12}\ T^2 \\ & - 4^\circ.392 \times 10^{-3} \sin(12^\circ.1128 - 0^\circ.052954\ T) \\ & + 0^\circ.053 \times 10^{-3} \sin 2(12^\circ.1128 - 0^\circ.052954\ T) \\ & - 0^\circ.325 \times 10^{-3} \sin 2(280^\circ.0812 - 0^\circ.985647\ T) \\ & - 0^\circ.050 \times 10^{-3} \sin 2(64^\circ.3824 + 13.176398\ T)\ ,\end{aligned} \tag{5}$$

where

$$T = \text{MJD} - 33282.0 \ .$$

Expression (5) is accurate to better than 0".2 (5 meters on the equator). If higher accuracy is needed, more terms for the nutation in R.A. have to be taken into account (H. M. Nautical Almanac Office, 1961).

According to what has been said, the independent variable T has to be expressed in UT-1. To refer this to UT-2, we must take into account the seasonal variation effect. It is important to note that the expression is precisely correct if we use UT-1, since it is from this expression that UT-1 is defined.

Ideally, the independent variable in orbital work should be the ephemeris time (H. M. Nautical Almanac Office, 1961). Unfortunately, the necessary reductions to this time system are made available only after a delay of several years. It can be replaced very efficiently with the system of Atomic Time A-1, which is not only quite adequate for orbital work of the present and the near future, but also has the great advantage of being immediately available, through time signals, and special correction bulletins (Markowitz, 1964). The introduction of the Atomic Time system implies also the use of the Atomic Julian Date (AJD), which is the Modified Julian Date expressed in Atomic Time.

There is no analytical expression relating A-1 to UT-2 or UT-1, but their differences are given in special bulletins published by the U. S. Naval Observatory. Since UT-2 is not really a uniform time, the difference between UT-2 and A-1 is not a linear function but oscillates around an average value. On the average, the rate of A-1 with respect to UT-2 is about 1.3 msec per day. (See figure 2.)

Since the A-1 system is a time system defined by a clock reading, that is, practically speaking, free of error, and UT-1 is the result of an observation, the sidereal time at a certain epoch expressed in A-1 can be determined to the same accuracy as that with which the observations provide the smoothed sidereal time. According to Markowitz (1964), this is about 5 msec (s.e.). To obtain the sidereal time for a certain epoch expressed in A-1, we apply the correction $\Delta U = (UT\text{-}1) - (A\text{-}1)$ to provide UT-1. With this as argument equations (4) and (5) will give the sidereal time. The sidereal time can also be given directly from the Atomic Julian Date (AJD):

$$\bar{\theta} = 100^\circ.075542 + 360^\circ.985647348(AJD-33282.0) + 0^\circ.2900 \times 10^{-12}(AJD-33282.0)^2 + \Delta\theta , \tag{6}$$

where

$$\Delta\theta = 1.002738\ \Delta U = 0^\circ.004177\ \Delta U_{(sec)} .$$

The true sidereal time θ will be obtained by adding to the above expression the nutation in R.A. as given by the periodic terms of equation (5).

Expressing the sidereal time as $\theta = \theta_o + \Delta\theta$, where θ_o is the uniform part of θ, we can introduce a fictitious meridian rotating at a uniform speed. Since its position is determined by θ_o expressed in Atomic Time, it will be called Atomic Meridian according to the Ephemeris Meridian. It will be to the west of the actual meridian of Greenwich (or any other standard meridian) by $\Delta\theta$ and since $\Delta\theta$ is always negative it will always be to the east of the actual meridian. A secular part of $\Delta\theta$ is $\Delta\theta_o$, and a periodic part $\delta\theta$, so that

$$\Delta\theta = \Delta\theta_o + \delta\theta .$$

The coordinate system (z^1, z^2, z^3) defined by the actual axis of rotation of the earth (true pole) as the 3-axis and the true vernal equinox as the 1-axis and centered at the center of gravity of the earth (fig. 3) will be called the <u>true sidereal system</u> or, simply, <u>the sidereal system</u>. It is related to the terrestrial system by the expression

$$\underline{x} = S\underline{z} \tag{7}$$

where

$$\underset{\sim}{S} = \begin{pmatrix} \cos\theta & \sin\theta & x \\ -\sin\theta & \cos\theta & -y \\ -x\cos\theta - y\sin\theta & -x\sin\theta + y\cos\theta & 1 \end{pmatrix},$$

where θ is the true sidereal time and (x,y) the coordinates of the instantaneous pole expressed in radians.

If we express θ as $\theta_o + \Delta\theta$, we can separate matrix S into two matrices, $\underset{\sim}{S} = \underset{\sim}{S}_\Delta \underset{\sim}{S}_o$, the matrix $\underset{\sim}{S}_o$ containing the major part of the rotation and $\underset{\sim}{S}_\Delta$ the small angles. We have

$$\underset{\sim}{S}_o = \begin{pmatrix} \cos\theta_o & \sin\theta_o & 0 \\ -\sin\theta_o & \cos\theta_o & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and

$$\underset{\sim}{S}_\Delta = \begin{pmatrix} 1 & \Delta\theta & x \\ -\Delta\theta & 1 & -y \\ -x & y & 1 \end{pmatrix}.$$

It is important for consistency that the sidereal time be measured from the same meridian, preferably that of the Mean Observatory. If another meridian is used as reference, the necessary reduction for the conventional longitude has to be applied.

However, this system of reference is not fixed with respect to the star background, since the axes move due to the lunisolar gravitational attraction on the earth's equatorial bulge. This motion is expressed by two components, the precession and the nutation. By subtracting the effect of the nutation, we obtain a system of reference that has only a smooth long-period motion with respect to an inertial system. We will call this the <u>mean sidereal system</u> $(\bar{z}^1, \bar{z}^2, \bar{z}^3)$; and it will always refer to the specific date (or epoch) of the observation.

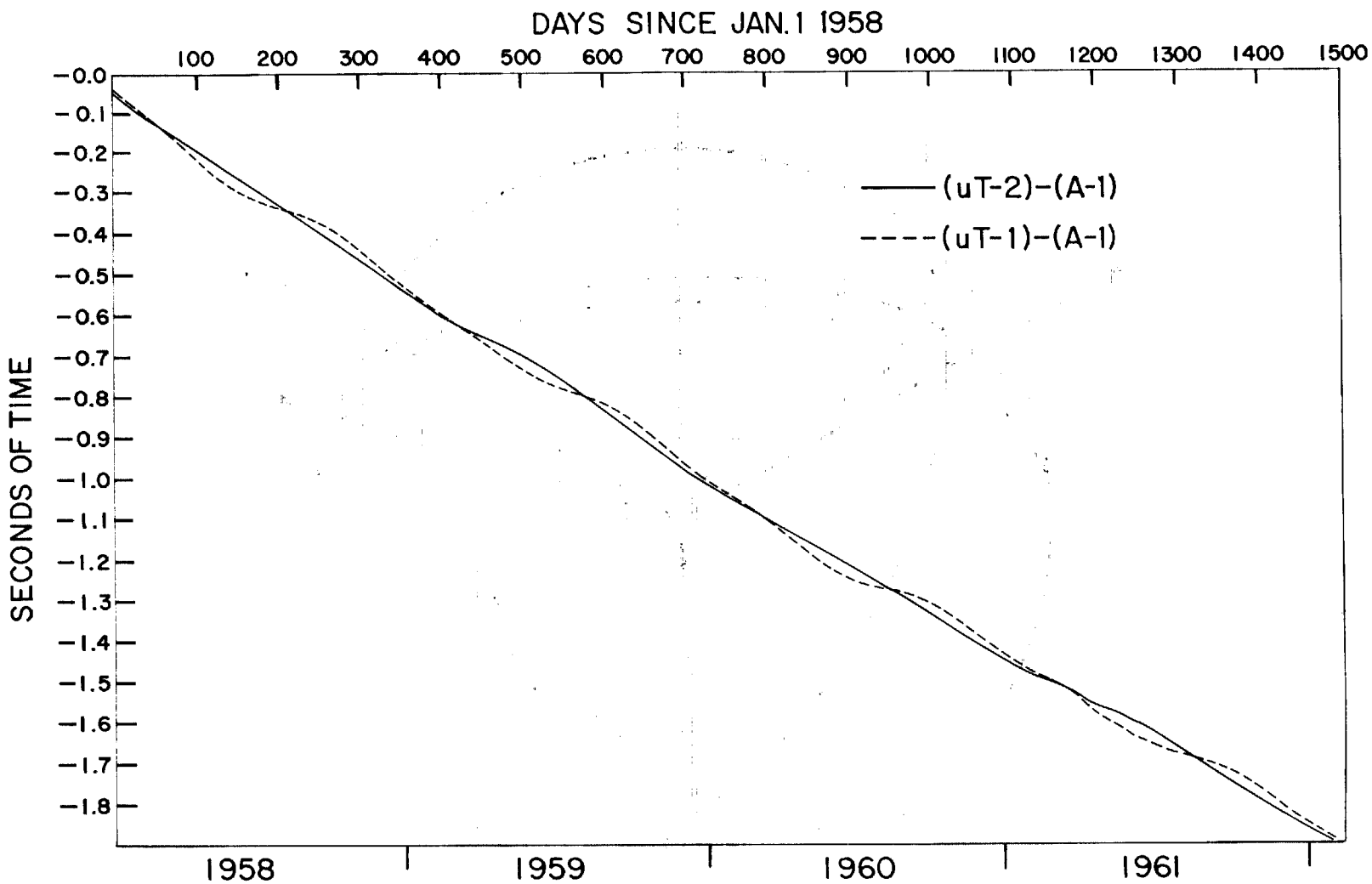

Fig. 2.--Relation between UT-1, UT-2, and A-1.

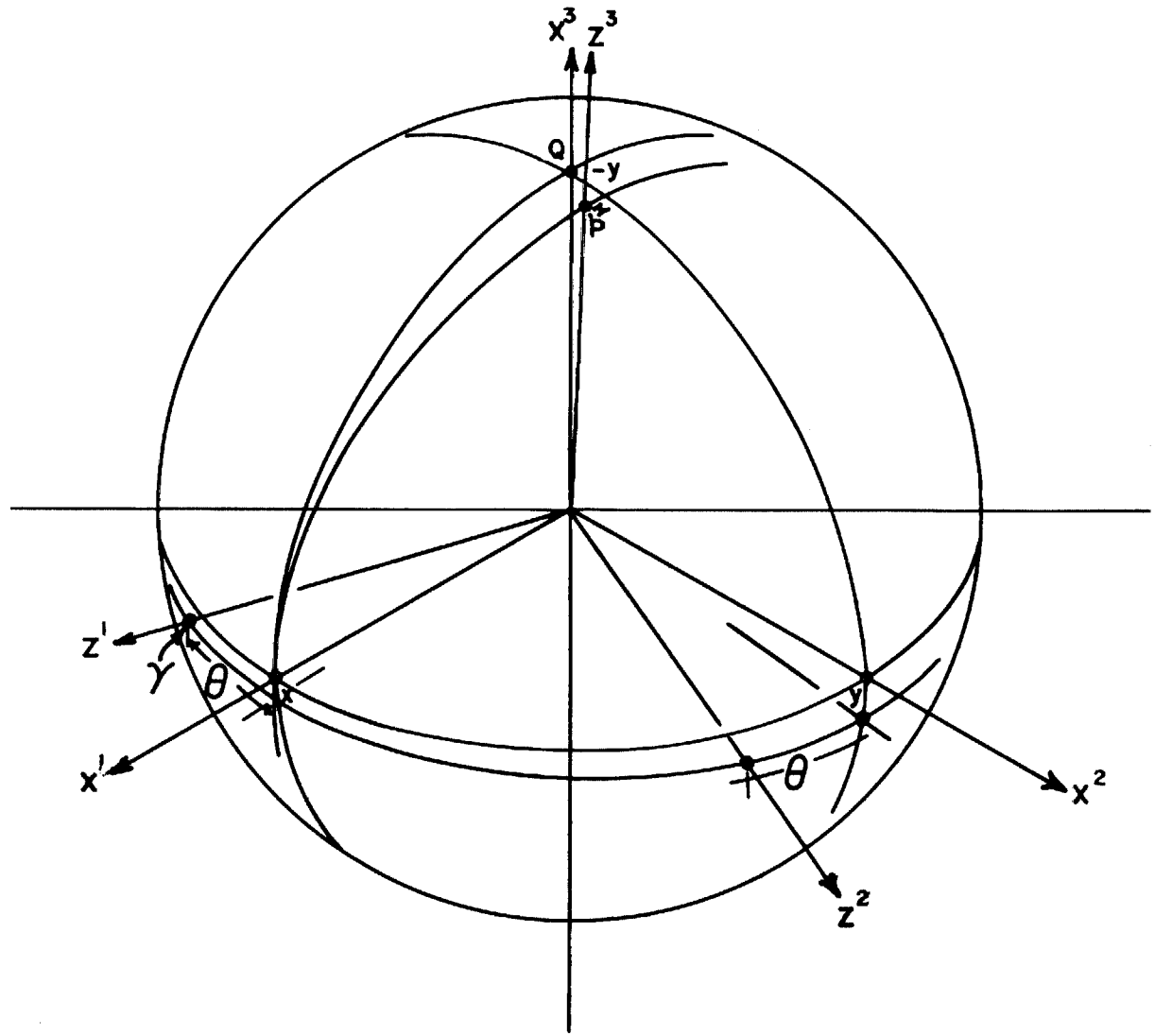

Fig. 3. Terrestrial and sidereal systems

The transformation from this mean system to the true one (fig. 4) is a function of time. It is given by

$$\underset{\sim}{z} = \underset{\sim}{N}\bar{\underset{\sim}{z}} ,$$

where $\underset{\sim}{N}$ is the following matrix (the second-order terms have been omitted):

$$\underset{\sim}{N} = \begin{pmatrix} 1 & -\Delta\mu & -\Delta\upsilon \\ \Delta\mu & 1 & -\Delta\epsilon \\ \Delta\upsilon & \Delta\epsilon & 1 \end{pmatrix} ;$$

and $\Delta\mu$, $\Delta\upsilon$, $\Delta\epsilon$ (nutation in right ascension, in declination, and in obliquity) are given to an accuracy better than 0".2 (or 10^{-6}) by

$$\begin{aligned}
\Delta\mu &= -76.7 \times 10^{-6}\sin(12^\circ.1128-0^\circ.0529539T) + 0.9 \times 10^{-6}\sin 2(12^\circ.1128-0.0529539T) \\
&\quad -5.7\times10^{-6}\sin 2(280^\circ.0812+0^\circ.9856473T) - 0.9\times10^{-6}\sin 2(64^\circ.3824+13^\circ.176396T), \\
\Delta\upsilon &= -33.3 \times 10^{-6}\sin(12^\circ.1128-0^\circ.0529539T) + 0.4 \times 10^{-6}\sin 2(12^\circ.1128-0^\circ.0529539T) \\
&\quad -2.5 \times 10^{-6}\sin 2(280^\circ.0812+0^\circ.9856473T) - 0.4 \times 10^{-6}\sin 2(64^\circ.3824+13^\circ.176396T) \\
\Delta\epsilon &= 44.7 \times 10^{-6}\cos(12^\circ.1128-0^\circ.0529539T) - 0.4 \times 10^{-6}\cos 2(12^\circ.1128-0^\circ.0529539T) \\
&\quad +2.7 \times 10^{-6}\cos 2(280^\circ.0812+0^\circ.9856473T) + 0.4 \times 10^{-6}\cos 2(64^\circ.3824+13^\circ.176396T),
\end{aligned} \tag{9}$$

where

$$T = MJD - 33282.0 .$$

By referring this mean (instantaneous) system to a mean system of a standard epoch, we obtain a fixed reference system, which we will call the <u>celestial system</u> (w^1, w^2, w^3) and for which we will use 1950.0 as the standard epoch. The transformation from this <u>celestial system</u> to the mean instantaneous one will be given by the relation

$$\bar{\underset{\sim}{z}} = \underset{\sim}{P}\underset{\sim}{w} , \tag{10}$$

where $\underline{P}$ is the following matrix:

$$\underline{P} = \begin{pmatrix} -\sin\varkappa\sin\omega+\cos\varkappa\cos\omega\cos\upsilon & -\cos\varkappa\sin\omega-\sin\varkappa\cos\omega\cos\upsilon & -\cos\omega\sin\upsilon \\ \sin\varkappa\cos\omega+\cos\varkappa\sin\omega\cos\upsilon & \cos\varkappa\cos\omega-\sin\varkappa\sin\omega\cos\upsilon & -\sin\omega\sin\upsilon \\ \cos\varkappa\sin\upsilon & -\sin\varkappa\sin\upsilon & \cos\upsilon \end{pmatrix}$$

and the values of $\varkappa, \omega, \upsilon$ (fig. 5)* are given by

$$\begin{aligned} \varkappa &= 23''.0495\ t + 0''.30 \times 10^{-4}\ t^2 , \\ \omega &= 23''.0495\ t + 1''.09 \times 10^{-4}\ t^2 , \\ \upsilon &= 20''.0426\ t - 0''.43 \times 10^{-4}\ t^2 , \end{aligned} \tag{11}$$

where t is the epoch counted in tropical years since 1950.0, or with sufficient accuracy:

$$\begin{aligned} \varkappa &= 0''.063107(\mathrm{MJD} - 33282.0) \\ \omega &= 0''.063107(\mathrm{MJD} - 33282.0) \\ \upsilon &= 0''.054875(\mathrm{MJD} - 33282.0) . \end{aligned}$$

Accordingly, a position referred to in the celestial system will be transformed in the terrestrial system by

$$\underline{x} = \underline{SNPw} \tag{12}$$

As the terrestrial system had to be replaced by a number of geodetic systems, the celestial system is also replaced in practice by a number of systems that are in reality defined by the coordinates given to a number of stars. Since these systems are used primarily for astrometric work, we will call them <u>astrometric systems</u>. Every astrometric system is associated with a star catalogue that gives the mean places and sometimes the proper motions of the stars. However, those positions and proper motions do not always refer to the same equator and equinox. Furthermore, systematic and periodic errors exist in each catalogue. The catalogues most often used as reference systems in precise satellite observations are the following:

*The symbols ζ, z, θ and r, s, j are often used for the angles $\varkappa, \omega, \upsilon$.

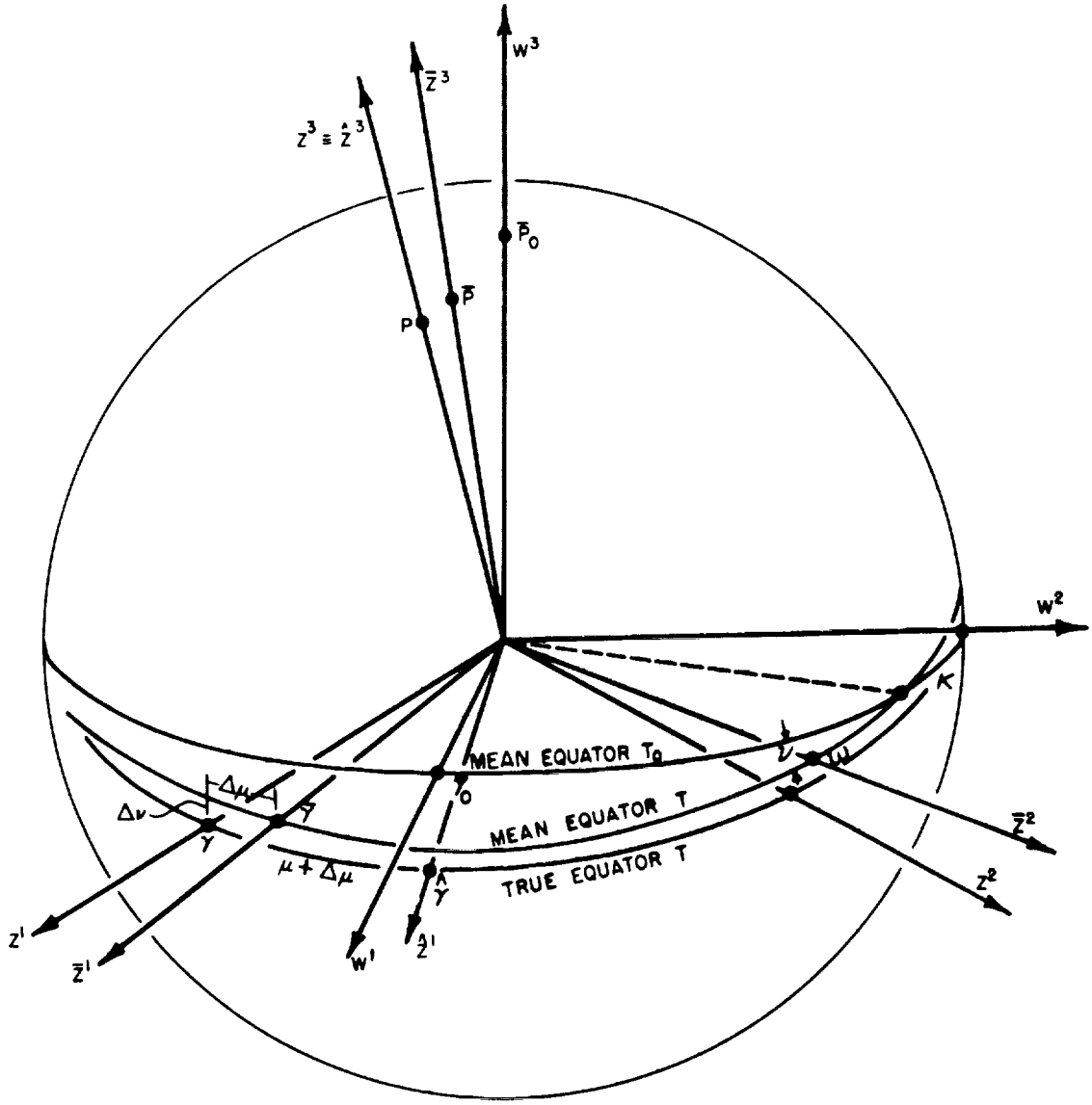

Fig. 4. Sidereal and celestial systems.

1. General Catalog (GC), which contains 33,342 stars covering the entire sky;

2. Zweiter Katalog der Astronomischen Gesellschaft (AGK2), which contains 183,520 stars from -2° to +90°;

3. Transactions of the Astronomical Observatory of Yale University (Yale Zone Catalogs), which contain 1,031 stars from +85° to +90°, 16,544 stars from +50° to +60°, and 128,093 stars from +30° to -30°, for a total of 145,668 stars;

4. Cape Photographic Catalogs (Cape Annals) containing 24,961 stars from -30° to -40°, and 23,925 stars from -52° to -64°, for a total of 48,886 stars;

5. Cape Zone Catalog, 1900 containing 20,843 stars from -40° to -52°.

The FK3 system (Dritter Fundamentalkatalog des Berliner Astronomischen Jahrbuchs), replaced by the FK4 from January 1, 1962, is the most consistent and uniform, and since it is accepted as the fundamental system for positional astronomy, it can be used to replace the celestial system. Unfortunately, it contains a very limited number of stars.

Although the last four catalogs contain a great number of stars, they are not in the same system, and do not cover the entire sky; even more serious, AGK2 does not contain proper motions. Using those four catalogs, one finds that the expected errors in position for the present range from 0".2 to 0".8 (standard error) with proper motions applied, and from 0".7 to 3".0 (standard error) without proper motions (Scott, 1964).

A catalog can be reduced to the system of another catalog by applying the appropriate systematic and periodic corrections (Newcomb, 1960). By combining existing catalogs, the Smithsonian Astrophysical Observatory has prepared a star catalog covering the entire sky and containing about 250,000 stars with their proper motions all reduced to the FK3 system. This Smithsonian Star Catalog will overcome the previous difficulty of having to use catalogs of different systems.

Orbital systems

A reference system for orbit determination is preferably an inertial one, such as the celestial system. Although the celestial system can refer to the mean equator and equinox of any date, it is preferable in orbital work to use the mid-epoch of the interval for which the orbit is computed. However, the earth and, therefore, its gravitational potential are not fixed in this inertial system because of the precession and the nutation of the earth's axis. Thus a periodic perturbation arises from this motion of the potential that must be taken into account in an orbit theory, although only the effect arising from the J_2 term of the earth's potential (the equatorial bulge) need be considered (Kozai, 1960).

It is also possible to refer an orbit to the sidereal system (true or mean), in which the main part of the earth's gravitational potential, i.e., all the zonal harmonics, are kept fixed, while the tesseral and sectorial harmonics rotate with the earth. However, because the sidereal system is not inertial, perturbations will arise.

These two systems can be combined to form a new one that reduces the perturbations. This new system will be defined by the 3-axis coinciding with the instantaneous true equator. The 1-axis is defined as being directed toward the position that the equator occupied at a certain date (e.g., 1950.0), assuming, however, that this equinox has followed the motion of the equatorial plane. In other words, the position of the 1-axis is obtained by a rotation of the sidereal system (true equinox) by the amount of precession and nutation in R.A. counted from a certain epoch (1950.0), but in the opposite direction (fig. 4). Since 1959 the author has used this system as a reference frame for satellite orbits computed at the Smithsonian Astrophysical Observatory. Kozai (1960) has proved that this system is indeed the least influenced by precession and nutation.

This system $(\hat{z}^1, \hat{z}^2, \hat{z}^3)$ will be called modified sidereal system. It is related to the true sidereal system by the expression

$$\hat{\underline{z}} = \underline{R}\underline{z} , \tag{13}$$

where

$$\underline{R} = \begin{pmatrix} \cos(\mu+\Delta\mu) & \sin(\mu+\Delta\mu) & 0 \\ -\sin(\mu+\Delta\mu) & \cos(\mu+\Delta\mu) & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

where $\mu = \kappa + \omega$, the precession in R.A. since 1950.0.

The angle $\hat{\theta}$ between the 1-axis of this modified system and the meridian of the Mean Observatory defines a new kind of sidereal time, since the intersection of the 1-axis with the true equator can be assumed to be modified equinox. To emphasize the fact that $\hat{\theta}$ is an angle we will call it the <u>sidereal angle</u>. It is obvious that

$$\hat{\theta} = \theta - (\mu + \Delta\mu) = \bar{\theta} - \Delta\mu ,$$

which means that the modified sidereal time θ is a simple linear expression of UT-1, or

$$\hat{\theta} = 100^{\circ}.075542 + 360^{\circ}.985612288 \text{ (MJD - 33282.0)} . \tag{14}$$

However, if UT-2 or A-1 is used as argument, the correction for the variation in the rotational speed of the earth must be added. Therefore,

$$\begin{aligned} \hat{\theta} &= 100^{\circ}.075542 + 360^{\circ}.985612288 \text{ (AJD - 33282.0)} + \Delta\theta \\ &= 0^{rev}.277987616 + 1^{rev}.00273781191 \text{ (AJD - 33282.0)} + \Delta\theta \end{aligned} \tag{15}$$

where $\Delta\theta$ is the same as in equation (6).

To obtain the modified coordinates from the celestial ones, we have

$$\hat{\underset{\sim}{z}} = \underset{\sim}{R}\underset{\sim}{N}\underset{\sim}{P}\underset{\sim}{w} . \tag{16}$$

We can write this expression in the form

$$\hat{\underset{\sim}{z}} = \underset{\sim}{M}\underset{\sim}{Q}\underset{\sim}{w} \tag{17}$$

by separating the $\underset{\sim}{R}$ matrix in two parts, one containing the precession, and the other the nutation, and then combining each with the corresponding matrix $\underset{\sim}{P}$ and $\underset{\sim}{N}$. We obtain

$$\underset{\sim}{Q} = \begin{pmatrix} \sin^2\varkappa+\cos^2\varkappa\cos\nu & \sin\varkappa\cos\varkappa-\sin\varkappa\cos\varkappa\cos\nu & -\cos\varkappa\sin\nu \\ \sin\varkappa\cos\varkappa-\cos^2\varkappa\cos\nu & \cos^2\varkappa+\sin^2\varkappa\cos\nu & \sin\varkappa\sin\nu \\ \cos\varkappa\sin\nu & -\sin\varkappa\sin\nu & \cos\nu \end{pmatrix} ,$$

$$\underset{\sim}{M} = \begin{pmatrix} 1 & 0 & -\cos\mu\Delta\nu-\sin\mu\Delta\epsilon \\ 0 & 1 & \sin\mu\Delta\nu-\cos\mu\Delta\epsilon \\ \cos\mu\Delta\nu+\sin\mu\Delta\epsilon & -\sin\mu\Delta\nu+\cos\mu\Delta\epsilon & 1 \end{pmatrix} .$$

The matrix $\underset{\sim}{Q}$ contains the effect of the precession alone and is independent of the nutation. The matrix $\underset{\sim}{M}$ contains the effect of the nutation but depends also on the precession to a minor degree. It becomes, however, a unit matrix for $\Delta\nu = \Delta\epsilon = 0$.

It is very simple to find the relation between the terrestrial system and the modified system. It must be the same as equation (7) except that the true sidereal time θ is replaced with the modified sidereal time $\hat{\theta}$, or

$$\underset{\sim}{x} = \underset{\sim}{T}\hat{\underset{\sim}{z}} , \tag{18}$$

where

$$\underset{\sim}{T} = \begin{pmatrix} \cos\hat{\theta} & \sin\hat{\theta} & x \\ -\sin\hat{\theta} & \cos\hat{\theta} & -y \\ -x\cos\hat{\theta} - y\sin\hat{\theta} & -x\sin\theta + y\cos\theta & 1 \end{pmatrix} .$$

Hence the terrestrial system is related to the celestial by the expression

$$\underset{\sim}{x} = \underset{\sim}{T}\underset{\sim}{M}\underset{\sim}{Q}\underset{\sim}{w} , \tag{19}$$

corresponding to equation (12).

The $\underset{\sim}{T}$ matrix can also be separated: $\underset{\sim}{T} = \underset{\sim}{T}_\Delta\underset{\sim}{T}_o$ as the $\underset{\sim}{S}$ matrix of equation (7). We have $\underset{\sim}{T}_\Delta \equiv \underset{\sim}{S}_\Delta$, and $\underset{\sim}{T}_o$ will be the $\underset{\sim}{S}_o$ with $\hat{\theta}_o$ instead of θ_o.

As can be seen, the modified sidereal reference system adequately replaces the instantaneous true sidereal system. Aside from the advantage of introducing the minimum perturbation that arises from precession and nutation, in satellite orbital theories, the sidereal time in this system has a very simple expression.

In essence, this new reference system eliminates the use of the ecliptic. If the main purpose is to establish the relation between the celestial and the terrestrial systems, the sidereal system is used only as an intermediate step. In this case, the equator is a necessary reference plane and therefore its motion due to the precession and the nutation cannot be ignored. However, the use of a moving equinox as 1-axis can be selected arbitrarily on the true equator, preferably using a fixed point. The mean equinox of a standard epoch (e.g., 1950.0) is a very convenient point to use as 1-axis since it is also the 1-axis of the celestial system. Since this mean equinox does not lie on the true equator of the same date, we should apply the necessary reduction for the nutation. This is done by the matrix $\underset{\sim}{M}$ in which case actually $\mu \equiv 0$. Figure 5 shows in more detail the relation between the different systems for each of the axes.

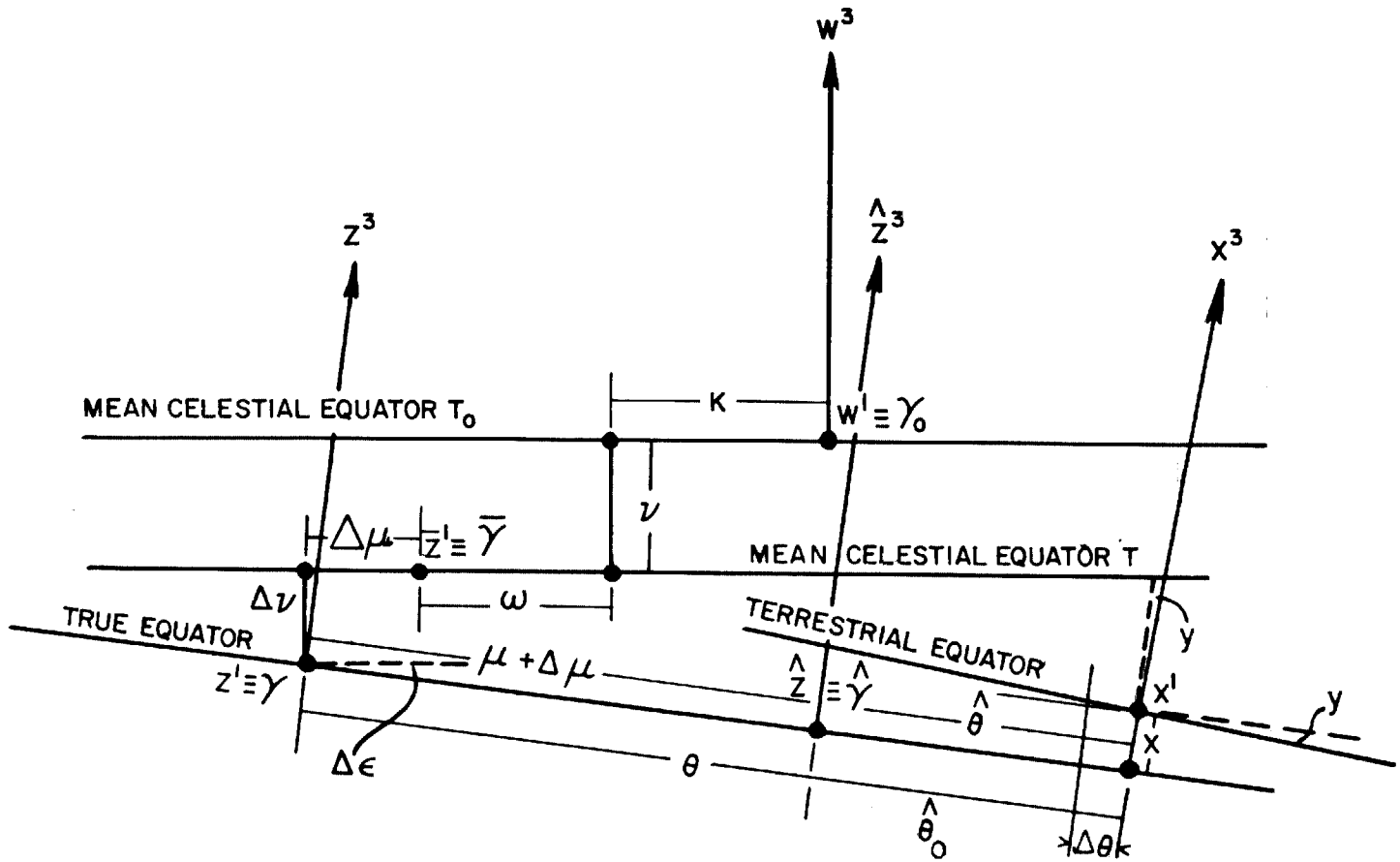

Fig. 5a. The 1-axis of the different reference systems.

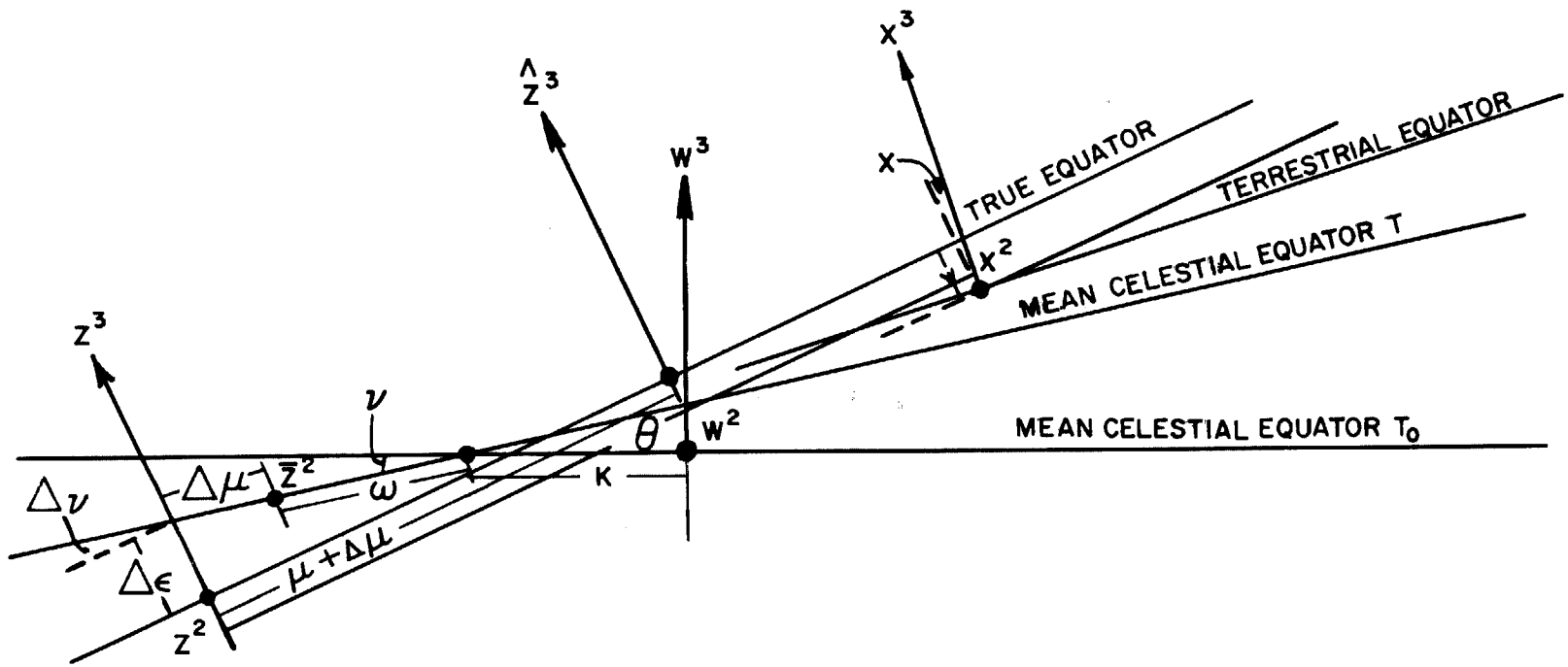

Fig. 5b. The 2-axis of the different reference systems.

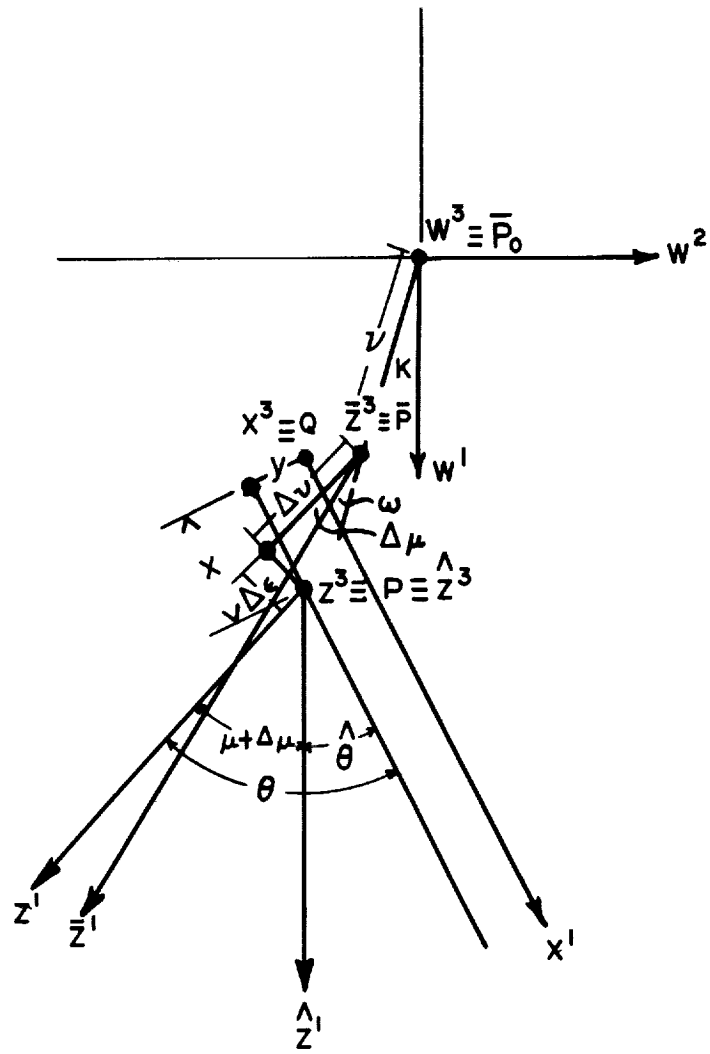

Fig. 5c. The 3-axis of the different reference systems.

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NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory. First issued to ensure the immediate dissemination of data for satellite tracking, the Reports have continued to provide a rapid distribution of catalogues of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals.

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