A Thermodynamical Model of Conductivity Dispersion in Plasmas and Semiconductors

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ABSTRACT

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In this paper, we first introduce a relation between the kinetic energy and the relaxation time, given by the Arrhenius equation, into the electrical conductivity equation of plasmas and semiconductors. Second, we investigate mathematically the change of the conductivity in the frequency and the time domains, and the nature of the distribution of relaxation times, of this new model of conductivity dispersion.

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1. Introduction

The electrical conductivity of plasmas and semiconductors due to different scattering processes has been much discussed in the literature[1]. The starting points of these discussions are: the conductivity dispersion has (i) a Maxwellian energy distribution and (ii) a power functional dependence of the relaxation time (τ) on the energy parameter ϵ. $\epsilon = mv^2/2kT$, where m is the electron mass, and v is the electron velocity with respect to the lattices or impurity centers in the case of semiconductors, and to the ions or molecules in the case of ionized plasma. Writing explicitly,

$$\tau = A_p \epsilon^{-p}, \qquad (1)$$

where p is the power and the coefficient A_p is independent of ϵ but may depend on the temperature.

The complex conductivity, σ^*, with Maxwellian energy distribution in the so-called relaxation time approximation, can be expressed as,

$$\sigma^*(\omega) = \frac{4e^2 n}{3\sqrt{\pi}\, m} \int_0^\infty \frac{\tau \epsilon^{3/2} e^{-\epsilon}}{1 + i\omega\tau} d\epsilon, \qquad (2)$$

where n is the density of the electrons, and ω is the frequency of the applied electric field.

Substituting the relation (1) into (2), the frequency dependence of the conductivity can be evaluated[2]. The mathematical tables for the evaluation are given by Dingle *et al.*[3].

Now, in some physical processes, instead of the relation given by Equation (1), we have,

$$\tau = \tau_o e^{\varepsilon} . \qquad (3)$$

This relation is known as Arrhenius equation. It has been well-established in the thermodynamical activation processes, and in the chemical reaction rate processes. If we introduce this relation into Equation (2), we have a model of the electrical conductivity of plasmas and semiconductors for which the nature of the dispersion is not known[4].

In the present paper, first, we will discuss the mathematical calculation of the conductivity of this thermodynamical model. Second, the nature of the distribution of relaxation times will be discussed. Finally, the transient time characteristics of this model will be presented.

2. Conductivity Integrals

Denoting by k/τ_o the coefficient of the integral in Equation (2), we have, separating σ^* into the real and the imaginary parts,

$$\sigma^*(\omega) = k\left[T_1(\omega) - i\,T_2(\omega)\right] , \qquad (4)$$

where the T functions are,

$$T_1(\omega) = \int_0^\infty \frac{\varepsilon^{3/2}}{1+\omega^2 \tau_o^2 e^{2\varepsilon}} \, d\varepsilon, \tag{5a}$$

$$T_2(\omega) = \int_0^\infty \frac{\omega \tau_o e^{\varepsilon} \varepsilon^{3/2}}{1+\omega^2 \tau_o^2 e^{2\varepsilon}} \, d\varepsilon \tag{5b}$$

The T_1 integral is the familiar Fermi-Dirac integral, therefore, its analytical nature is known and its numerical table exists[5]. To our knowledge, T_2 integral cannot be related to special functions. Therefore, numerical computation is carried out on T_2 and the general result is given in Figure 1, with numerical values given in Table 1. $T_1(\omega)$ is also recalculated and included in the figures and in the table for completeness.

Asymptotically, when $\omega\tau_o$ is very large, we have, by neglecting 1 in the denominator of the integral,

$$T_1(\omega) \sim 2^{-5/2} \Gamma(5/2) (\omega \tau_o)^{-2}, \tag{6a}$$

$$T_2(\omega) \sim \Gamma(5/2) (\omega \tau_o)^{-1}. \tag{6b}$$

When $\omega \tau_o$ is very small, asymptotic behavior can be obtained from a steepest descent method of Sommerfeld and Bethe[6], and well illustrated by Seitz[7]. The result is,

$$T_1(\omega) \sim 2/5 (\log 1/\omega\tau_o)^{1/2} \left[(\log 1/\omega \tau_o)^3 + \pi^2/8 \right], \tag{7a}$$

$$T_2(\omega) \sim (\log 1/\omega \tau_o)^{-1/2} \left[A (\log 1/\omega \tau_o)^2 + B/8 \right]. \tag{7b}$$

The evaluation of T_1 is almost identical to the example of Seitz[7]. The coefficients A and B of T_2 are the following definite integrals:

$$A = \int_{-\infty}^{\infty} \frac{\eta(1 - e^{-2\eta})\, e^{-\eta}}{(1 + e^{-2\eta})^2}\, d\eta, \tag{8a}$$

$$B = \int_{-\infty}^{\infty} \frac{\eta^3 (1 - e^{-2\eta})\, e^{-\eta}}{(1 + e^{-2\eta})^2}\, d\eta, \tag{8b}$$

with $e^{-\eta} = \omega\tau_o e^{\varepsilon}$. From the integration table [8], we have

$$\begin{aligned} A &= 2\sum_{n=o}^{\infty} (-1)^n (n+1) \left[\frac{1}{(2n+1)^2} - \frac{1}{(2n+3)^2}\right] \\ &= 2\sum_{n=o}^{\infty} \frac{(-1)^n}{2n+1} \\ &= \pi/2, \end{aligned} \tag{8a}$$

$$\begin{aligned} B &= 2\Gamma(4) \sum_{n=o}^{\infty} (-1)^n (n+1) \left[\frac{1}{(2n+1)^4} - \frac{1}{(2n+3)^4}\right] \\ &= 12 \sum_{n=o}^{\infty} \frac{(-1)^n}{(2n+1)^3} \\ &= 3/8\, \pi^3 . \end{aligned} \tag{8b}$$

Therefore, both T_1 and T_2 diverge as the frequency approaches zero. In more familiar dispersions, typified by Lorentzian dispersions (corresponding to the case $p = o$ in Equation 1), the corresponding T_1 is finite at $\omega = 0$ and decreases monotonically to $T_1 = 0$ as $\omega \to \infty$. The corresponding T_2 vanishes both at $\omega = 0$ and $\omega \to \infty$, with a maximum at a certain frequency. We find that T_1 and T_2 of the present model behave rather differently, but this is by no means a unique, or an impossible property; similar behavior is found, for example, in the hopping process model of the low frequency conductivity in semiconductors[9] and in the case of large deviation of the phonon distribution from equilibrium in metals[6].

3. Distribution Function of Relaxation Times

The basis of the distribution function $G(\tau)$, of relaxation times, τ, is [10], for the T^* function of (4), the following integral equation:

$$T^*(\omega) = T_1(\omega) - i\, T_2(\omega)$$

$$= \int_0^\infty \frac{G(\tau)}{1 + i\,\omega\tau}\, d\tau . \qquad (9)$$

Making a comparison between (9) and (2), we can interpret the present thermodynamical model in the formalism of the distribution function of relaxation times. We have, with (3)

$$G(\tau) = \frac{\tau_o}{\tau} \left(\log \frac{\tau}{\tau_o}\right)^{3/2} \quad \text{for } \tau \geq \tau_o$$

$$= o \text{ for } \tau < \tau_o . \qquad (10)$$

A graph of G (τ) is given in Figure 2. We note some special properties of G (τ):

(i) G (τ) has a cut-off relaxation time below which, there is null distribution;

(ii) G (τ) is asymmetrical with respect to log τ;

(iii) G (τ) approaches zero slowly as τ approaches infinity.

The slowness of approaching zero is responsible for the divergence of conductivity at the zero frequency and the non-normalizability of G (τ) -- i.e., normalization in the sense,

$$\int_0^\infty G(\tau)\, d\tau < \infty.$$

As can be proved, when the range of relaxation time is from 0 to ∞, (instead of the finite range), in order that the corresponding dispersion function be finite, G (τ) should approach zero faster than τ when τ approaches zero, and faster than τ^{-1} when τ approaches infinity.

The above remark is well illustrated by the distribution function of Davidson and Cole [11], and its modified form[12].

4. Decay Function

For experimental application, sometimes it is convenient to determine the relaxation characteristics from the transient time of the decay of the conductivity instead of from its frequency dispersion. Mathematically, this decay function, can be obtained from the Laplace transform of G (τ):[13]

$$A(\tau) = \int_0^1 (\log 1/s)^{3/2}\, e^{-t/\tau_0 s}\, ds \qquad (12)$$

Substituting s by e^y and expanding the exponential term,

$$A(t) = \sum_{n=0}^{\infty} \int_0^{\infty} \frac{(-1)^n}{n!} (t/\tau_0)^n \, y^{3/2} \, e^{-(n+1)y} \, dy$$

$$= \frac{3\sqrt{\pi}}{4} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (n+1)^{-5/2} (t/\tau_0)^n . \tag{13}$$

In the Lorentzian dispersion $A(t)$, with the same initial value, say $A_L(t)$, would be

$$A_L(t) = \frac{3\sqrt{\pi}}{4} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (t/\tau_0)^n .$$

Therefore, $A(t)$ has a smaller rate of decay than $A_L(t)$.

$A(t)$ is calculated numerically and the result is given in Table 1 as well as in Figure 3. In Figure 3, the decay function for a Lorentz dispersion $A_L(t)$ is also drawn for comparison.

5. Conclusions

A conductivity dispersion model with a thermodynamical relation between the relaxation time and energy analytical is investigated mathematically, and some mathematical problems of this dispersion function are discussed. With the availability of these results, it would be possible to test the model for some experimental results of conductivity dispersion in plasmas and semiconductors.

I would like to thank Dr. H. Oser of the National Bureau of Standards, who suggested the steepest descent method in finding asymptotic expressions. Mr. E. Monasterski, GSFC, has helped me in carrying out all the numerical computations.

TABLE I

x	$E_1(x)$	$E_2(x)$	$A(x)$
1.0 - 4	1.049 2	4.439 - 1	1.000 0
1.0 - 3	5.179 1	2.908 1	9.998 - 1
1.0 - 2	1.952 1	1.623 1	9.982 - 1
3.0 - 2	1.036 1	1.114 1	9.947 - 1
5.0 - 2	7.271 0	9.046 0	9.912 - 1
7.0 - 2	5.606 0	7.770 0	9.877 - 1
1.0 - 1	4.139 0	6.515 0	9.826 - 1
2.0 - 1	2.072 0	4.389 0	9.659 - 1
3.0 - 1	1.279 0	3.354 0	9.497 - 1
4.0 - 1	8.725 - 1	2.720 0	9.341 - 1
5.0 - 1	6.325 - 1	2.287 0	9.190 - 1
6.0 - 1	4.786 - 1	1.971 0	9.044 - 1
7.0 - 1	3.739 - 1	1.730 0	8.903 - 1
8.0 - 1	2.995 - 1	1.540 0	8.767 - 1
9.0 - 1	2.450 - 1	1.387 0	8.635 - 1
1.0 0	2.039 - 1	1.261 0	8.507 - 1
1.2 0	1.470 - 1	1.066 0	8.264 - 1
1.4 0	1.107 - 1	9.220 - 1	8.033 - 1
1.6 0	8.621 - 2	8.119 - 1	7.820 - 1
1.8 0	6.896 - 2	7.250 - 1	7.617 - 1
2.0 0	5.636 - 2	6.546 - 1	7.425 - 1
2.5 0	3.659 - 2	5.265 - 1	6.991 - 1
3.0 0	2.562 - 2	4.400 - 1	6.611 - 1
4.0 0	1.453 - 3	3.310 - 1	5.977 - 1
5.5 0	7.723 - 3	2.412 - 1	5.252 - 1
7.0 0	4.779 - 3	1.856 - 1	4.708 - 1
8.5 0	3.244 - 3	1.502 - 1	4.282 - 1
1.0 1	2.346 - 3	1.248 - 1	3.939 - 1
1.2 1	1.630 - 3	1.107 - 1	3.571 - 1
1.4 1	1.198 - 3	9.492 - 2	3.277 - 1
1.6 1	9.173 - 4	8.306 - 2	3.035 - 1
1.8 1	7.249 - 4	7.384 - 2	2.832 - 1
2.0 1	5.872 - 4	6.445 - 2	2.658 - 1
3.0 1	2.611 - 4	4.431 - 2	2.064 - 1
4.0 1	1.469 - 4	3.323 - 2	1.711 - 1
5.0 1	9.399 - 5	2.659 - 2	1.473 - 1
6.0 1	6.527 - 5	2.215 - 2	1.300 - 1
7.0 1	4.796 - 5	1.899 - 2	1.168 - 1
8.0 1	3.672 - 5	1.662 - 2	1.064 - 1
9.0 1	2.901 - 5	1.477 - 2	9.763 - 2
1.0 2	2.350 - 5	1.329 - 2	9.072 - 2
1.5 2	1.644 - 5	8.862 - 3	6.751 - 2
2.0 2	5.875 - 5	6.646 - 3	5.450 - 2
2.5 2	3.760 - 5	5.317 - 3	4.605 - 2
3.0 2	2.611 - 6	4.431 - 3	4.007 - 2
3.5 2	1.918 - 6	3.798 - 3	3.559 - 2
4.0 2	1.469 - 6	3.323 - 3	3.210 - 2
5.0 2	9.400 - 6	2.659 - 3	2.698 - 2
7.5 2	4.178 - 7	1.772 - 3	1.959 - 2
1.0 3	2.350 - 7	1.329 - 3	1.557 - 2

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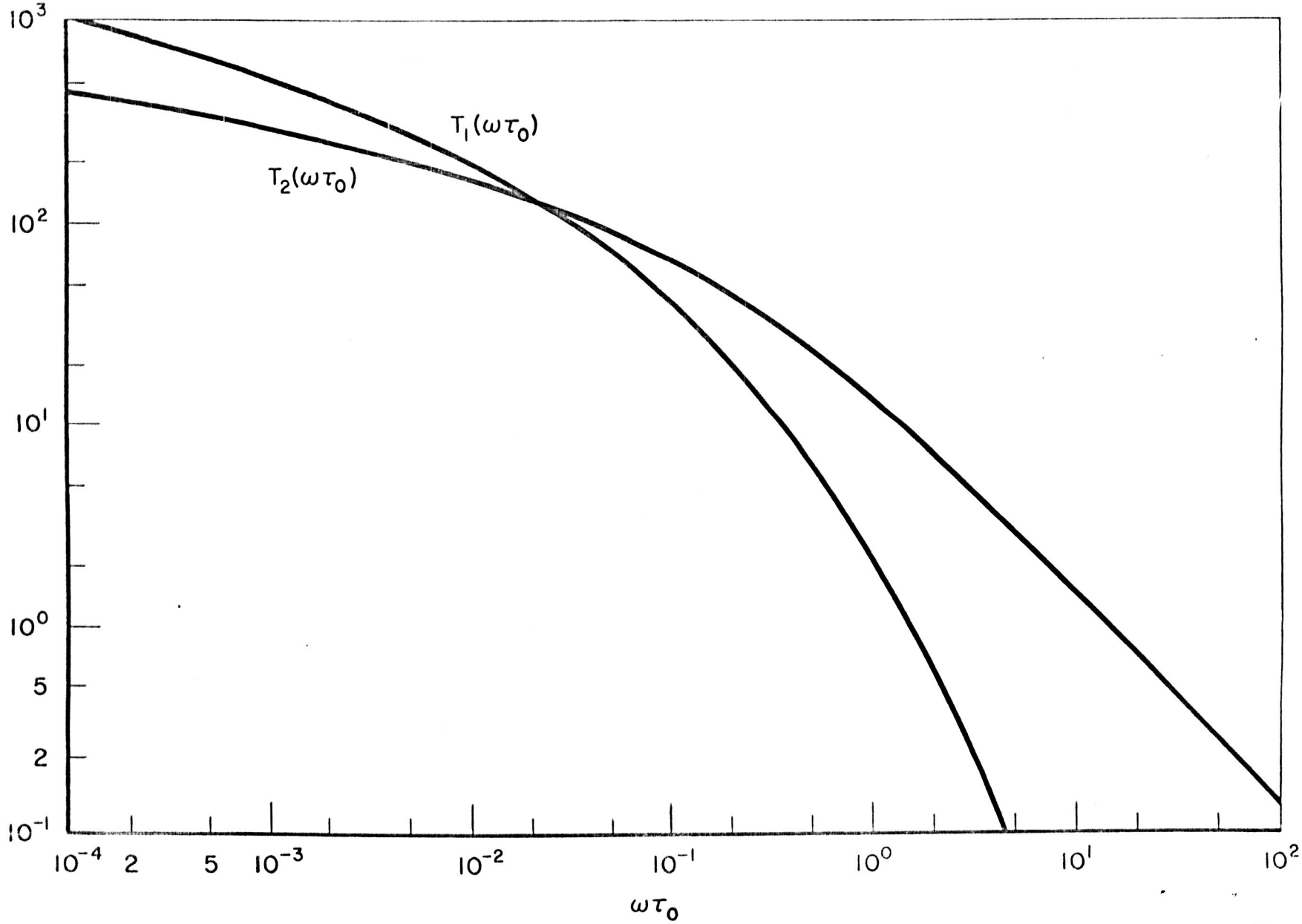
10^3
10^2
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5
2
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$T_1(\omega\tau_0)$
$T_2(\omega\tau_0)$
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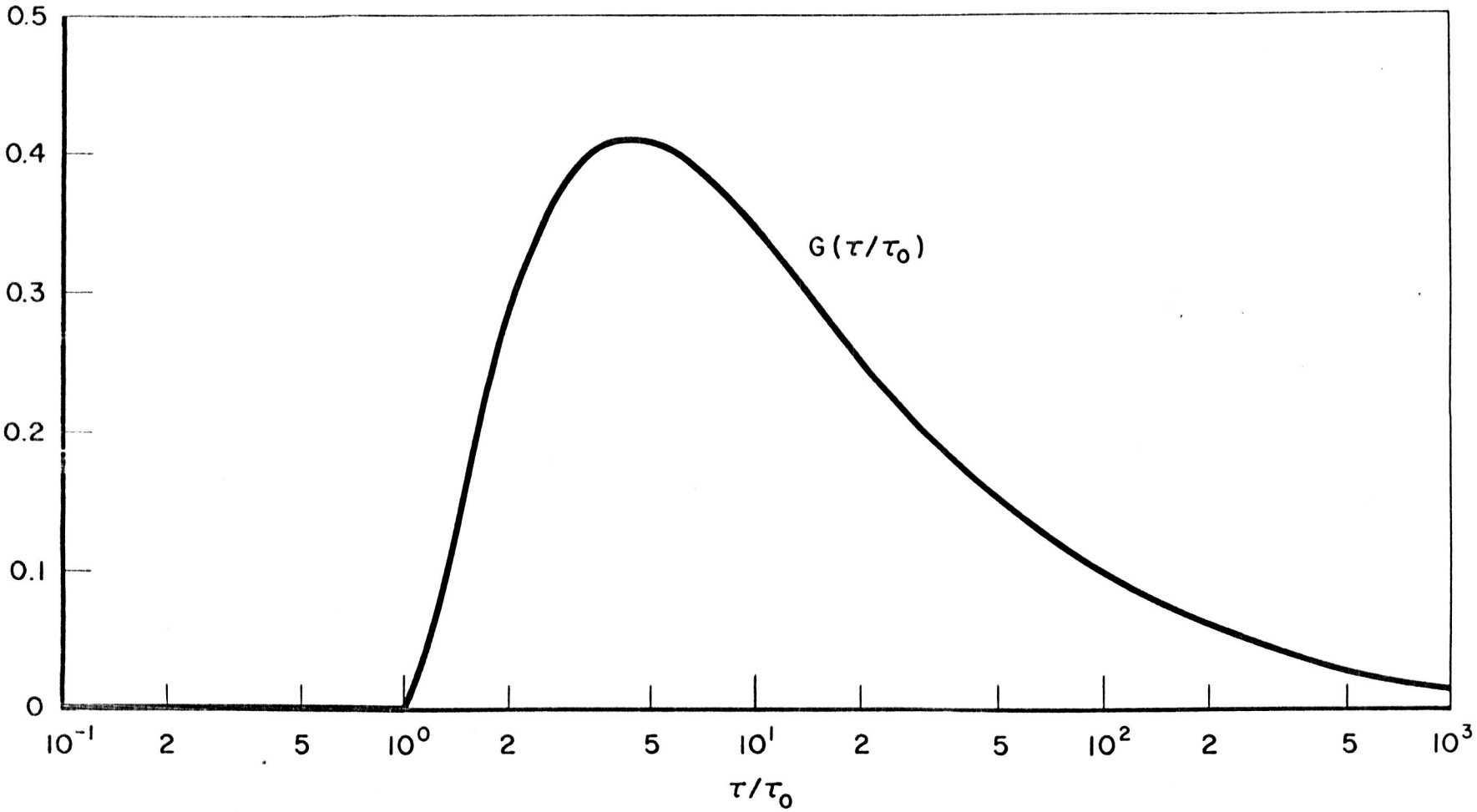
0.5
0.4
0.3
0.2
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$G(\tau/\tau_0)$
10^{-1}
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τ/τ_0