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PARAMETRIC PERFORMANCE ANALYSIS OF LUNAR MISSIONS.

PART I: BRAKE TO LUNAR ORBIT

By

Charles M. Akridge and Sam H. Harlin

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PART I

BRAKE TO LUNAR ORBIT

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ABSTRACT

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The effect of thrust-to-weight ratios, earth-lunar transfer time, and pericenter altitude on trajectory parameters has been investigated for brake to lunar orbit. A single stage with constant thrust directed against the velocity vector was used in all instances. Specific impulses of 300 and 420 and transfer times of 50, 60 and 72 hours were used for comparison.

The results of the study show the variations of trajectory parameters for earth thrust-to-weight ratios from 0.1 to 1.0. It was found that the velocity loss due to gravity is small, and for earth thrust-to-weight ratios greater than 0.4 losses can be neglected.

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FLIGHT OPERATIONS SECTION
ADVANCED FLIGHT SYSTEMS BRANCH
PROPULSION AND VEHICLE ENGINEERING DIVISION

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DEFINITION OF SYMBOLS

Symbol	Definition
F	Thrust, kp
g	Gravitational acceleration, m/sec^2
g_o	Earth gravitational acceleration, 9.80665 m/sec^2
h	Altitude, km
Δh	Altitude change, $h_o - h_f$, km
I_{sp}	Specific impulse, sec
m	Mass, $\frac{kp - sec^2}{m}$
r	Radius, km
t	Time, sec
T	Transfer time, hrs
Δt	Incremental time, sec
V	Velocity, m/sec
V^*	Comparative velocity, m/sec
ΔV	Characteristic velocity, m/sec
W_o	Gross weight, kp
F/W_o	Initial thrust-to-weight ratio (based on weight at sea level)
W_8	Propellant weight, kp

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
α	Thrust vector orientation angle measured from the velocity to the thrust vector (positive down), deg
ζ	Stage propellant mass fraction, W_8 / W_o
γ	Flight path angle from vertical, deg
μ	Gravitational constant for the moon, 4906 km^3/sec^2
ψ	Central angle, deg
Subscripts	
$\mathbb{C}$	Moon
p	Pericenter
e	Earth
B	Burnout
o	Initial
id	Ideal
f	Final
ex	Exhaust

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The effect of thrust-to-weight ratios, earth-lunar transfer time, and pericenter altitude on trajectory parameters has been investigated for brake to lunar orbit. A single stage with constant thrust directed against the velocity vector was used in all instances. Specific impulses of 300 and 420 and transfer times of 50, 60 and 72 hours were used for comparison.

The results of the study show the variations of trajectory parameters for earth thrust-to-weight ratios from 0.1 to 1.0. It was found that the velocity loss due to gravity is small, and for earth thrust-to-weight ratios greater than 0.4 losses can be neglected.

SECTION I. INTRODUCTION

Orbital operations are of great interest in lunar exploration because of requirements imposed by the landing site, energy, and tracking. Preliminary analysis of lunar missions requires a rapid method of sufficient accuracy for determining trajectory parameters.

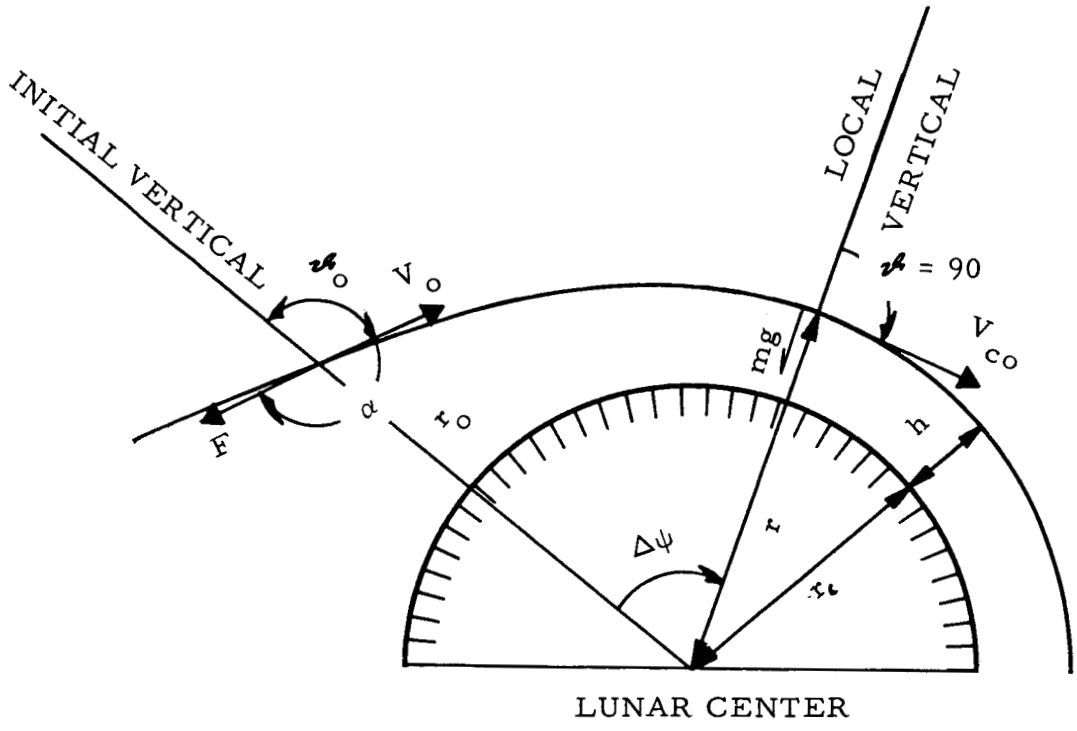
The purpose of this report is to present trajectory parameters for the powered lunar braking maneuver for various transfer times and lunar orbit altitudes. The parameters presented are: velocity, flight path angle, altitude, central angle, burning time, propellant mass fraction, time to pericenter at initiation of burning, and velocity losses.

The approach used was to determine the arrival velocity for a given transfer time and initiate burning so that circular orbit conditions are attained at burnout. The equations of motion were integrated on a RECOMP II computer, using Runge-Kutta numerical integration.

SECTION II. ANALYSIS

In lunar mission programs, it is assumed that one mode of lunar flight and landing will be by way of transfer from a circular orbit around the Moon. In general, a spacecraft will approach the vicinity of the Moon with hyperbolic flight velocity relative to the Moon.

The velocity requirements for injection from a lunar transfer were calculated using the equations of motion for a vehicle flying into orbit with α = 180 degrees. Relations of velocity and pericenter altitude from sphere-of-influence calculations were used for the earth-lunar transfer.



Referring to the sketch above, computations were made for a point mass moving in a plane using the following equations of motion:

$$\dot{V} = \frac{F \cos \alpha}{m} - \frac{g_o r_o^2}{r^2} \cos \vartheta \tag{1}$$

$$V\dot{\vartheta} = \frac{F \sin \alpha}{m} + \left(\frac{g_o r_o^2}{r^2} - \frac{V^2}{r} \right) \sin \vartheta \tag{2}$$

$$\dot{r} = V \cos \vartheta \tag{3}$$

$$\dot{\psi} = \frac{V \sin \vartheta}{r} \tag{4}$$

where

$$m = m_o + \int \dot{m}\, dt \tag{5}$$

and

$$\dot{m} = - \frac{F}{g_e I_{sp}} \tag{6}$$

The velocity and flight path angle may be obtained by integrating the equation of motion

$$V = \int \dot{V}\, dt \tag{7}$$

$$\vartheta = \int \dot{\vartheta}\, dt \tag{8}$$

The range and pericenter altitude can then be calculated by the relations

$$x = \int \frac{r_\ell}{r} v \sin \vartheta\, dt \tag{9}$$

$$h = h_o + \int \dot{r}\, dt \tag{10}$$

and the central angle is

$$\psi = \int \frac{\dot{x}}{r_\ell}\, dt \tag{11}$$

The initial weight of the vehicle is

$$W_o = W_c + W_8 \tag{12}$$

The arrival velocities for altitude versus transfer time were calculated using the sphere-of-influence method and are shown in FIG 1. This method is based on the sphere-of-influence concept, reducing the n-body problem to two 2-body problems. These velocities, though not exact, are sufficient for preliminary calculations.

The velocity expended by a vehicle is the characteristic velocity, or

$$\Delta V = V_{ex} \ln \frac{1}{1 - \zeta} \tag{13}$$

Then the velocity losses are the difference between the characteristic velocity and the change in comparative velocity, or

$$\Delta V_{loss} = \Delta V - \Delta V^* \tag{14}$$

where the comparative velocity is

$$V^* = \sqrt{V^2 + 2\mu_e \left(\frac{1}{r_o} - \frac{1}{r_f} \right)} \tag{15}$$

The change in comparative velocity during descent from $r = r_o$ to $r = r_f$ is

$$\Delta V^* = -\sqrt{V_f^{\,2} + 2\mu_e \left(\frac{1}{r_o} - \frac{1}{r_f} \right)} + V_o \tag{16}$$

and the velocity loss due to gravity is

$$\Delta V_{loss} = V_{ex} \ln \left(\frac{1}{1 - \zeta} \right) - \left[-\sqrt{V_f^{\,2} + 2\mu_e \left(\frac{1}{r_o} - \frac{1}{r_f} \right)} + V_o \right] \tag{17}$$

SECTION III. ASSUMPTIONS

A summary of the basic assumptions used in this analysis follows:

1. Deceleration of a single stage from earth-moon transfer to circular lunar orbit, using constant thrust directed against the velocity vector.

2. Initial pericenter altitudes:

$h_p = 50$ km

$h_p = 100$ km

$h_p = 200$ km

$h_p = 300$ km

3. For comparison, specific impulses of 300 and 420 were used.

4. The thrust-to-weight ratio for a chemical stage was varied parametrically from 0.1 to 1.0.

5. Mean spherical moon:

$\mu_{☾} = 4906 \text{ km}^3/\text{sec}^2$

$r = 1738.3$ km

SECTION IV. DISCUSSION OF RESULTS

The results of this analysis are shown in FIG 2 through 9, separated according to pericenter altitude. All variables are plotted versus earth thrust-to-weight ratios for 50, 60 and 72-hour earth-lunar transfer times and specific impulses of 300 and 420. These specific impulses are representative of present day storable and cryogenic stages.

The time of initiation of burning prior to Keplerian pericenter is shown in FIG 2. The characteristic velocity required to brake to lunar orbit is shown in FIG 3 and the corresponding stage propellant mass fraction is shown in FIG 4. The change in other trajectory variables is shown in FIG 5 through 8.

FIG 9 shows the velocity losses due to gravity. Beyond an earth thrust-to-weight ratio of 0.4, these gravity losses are small and considered to be zero. It should be noted that these losses are the difference between characteristic and change in comparative velocities during burning, i.e., true gravity losses.

SECTION V. CONCLUSIONS

From the results of this study, it can be concluded that velocity loss due to gravity is small throughout the range of earth thrust-to-weight ratios considered and that losses can be neglected for values of thrust-to-weight ratios greater than 0.4. It can also be concluded that variation of pericenter altitudes does not appreciably change the stage propellant mass fraction required. For most cases it is found that angular range is 30° or less.

SECTION VI. GRAPHIC PRESENTATION

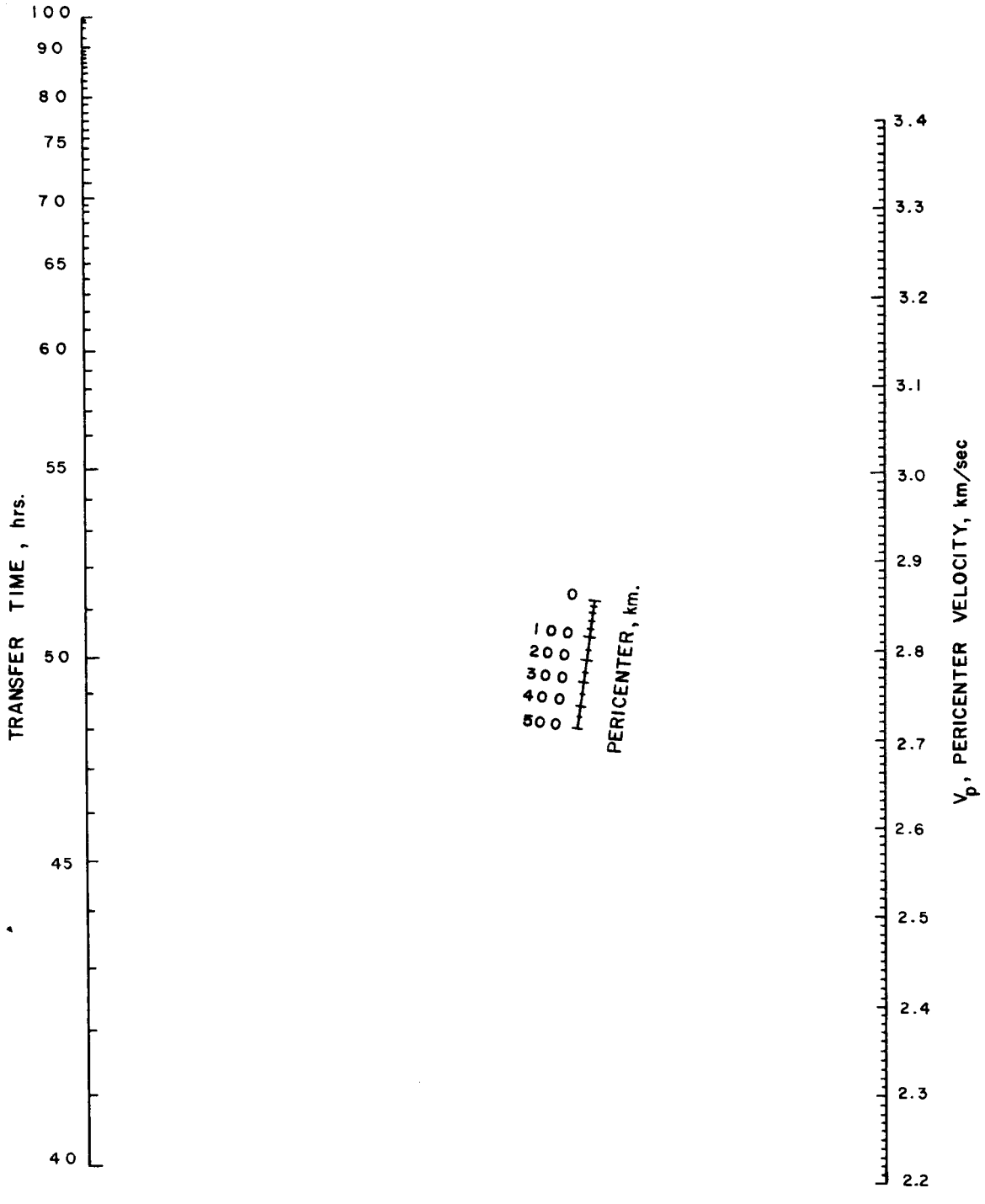


FIGURE 1. PERICENTER VELOCITY FOR EARTH-LUNAR TRANSFER.

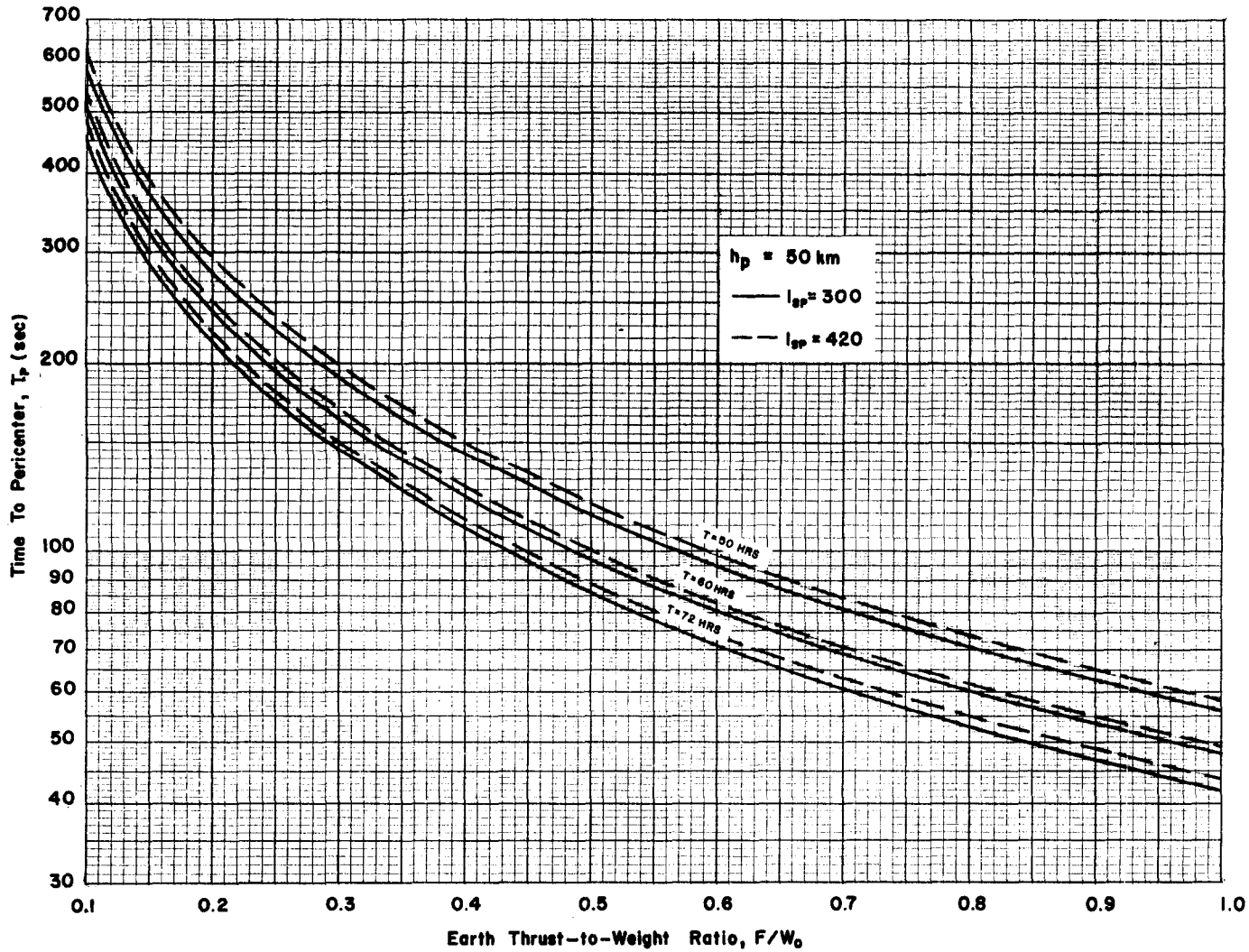


FIGURE 2a. TIME TO PERICENTER FOR BRAKE TO LUNAR ORBIT - h_p = 50 km.

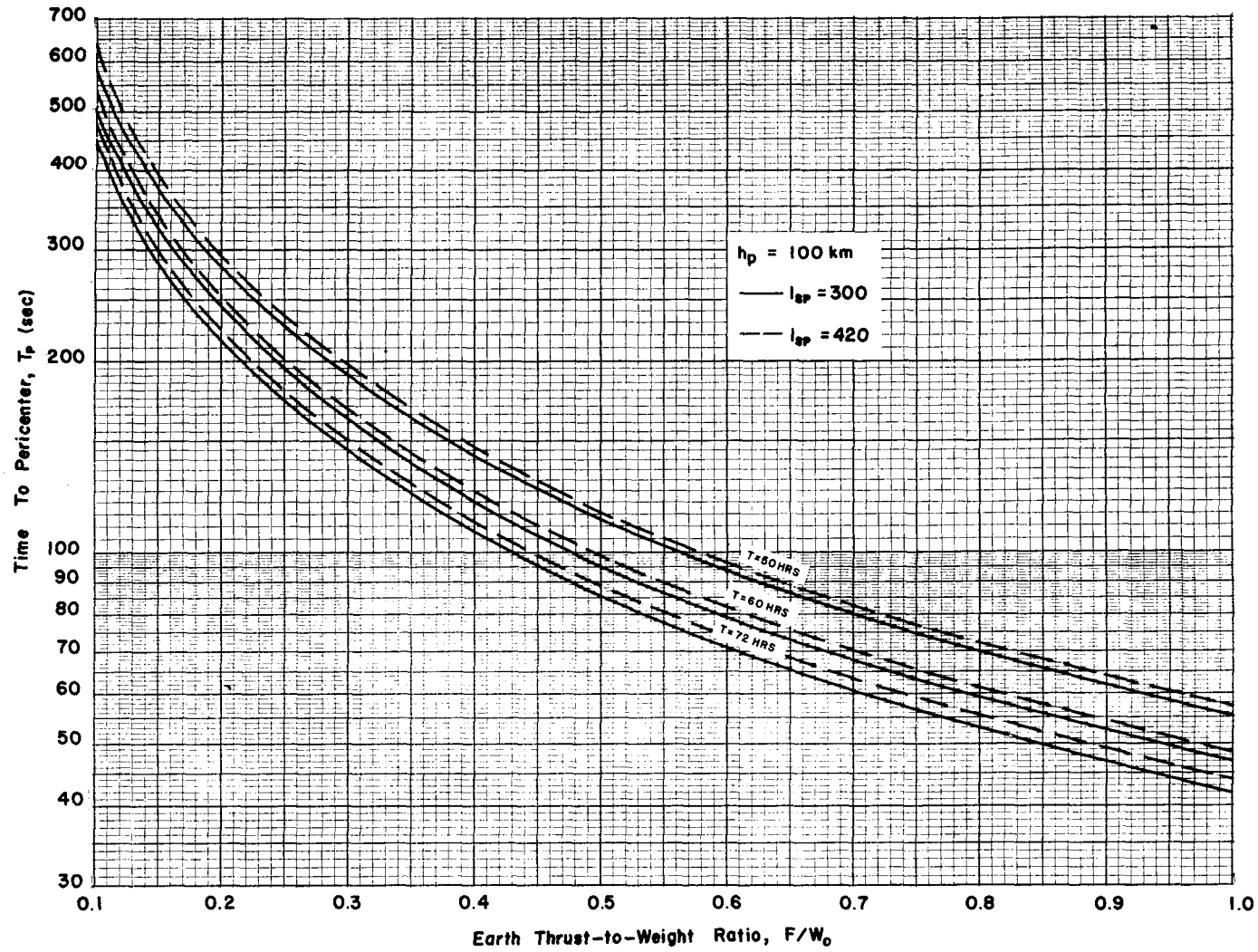


FIGURE 2b. TIME TO PERICENTER FOR BRAKE TO LUNAR ORBIT - h_p = 100 km.

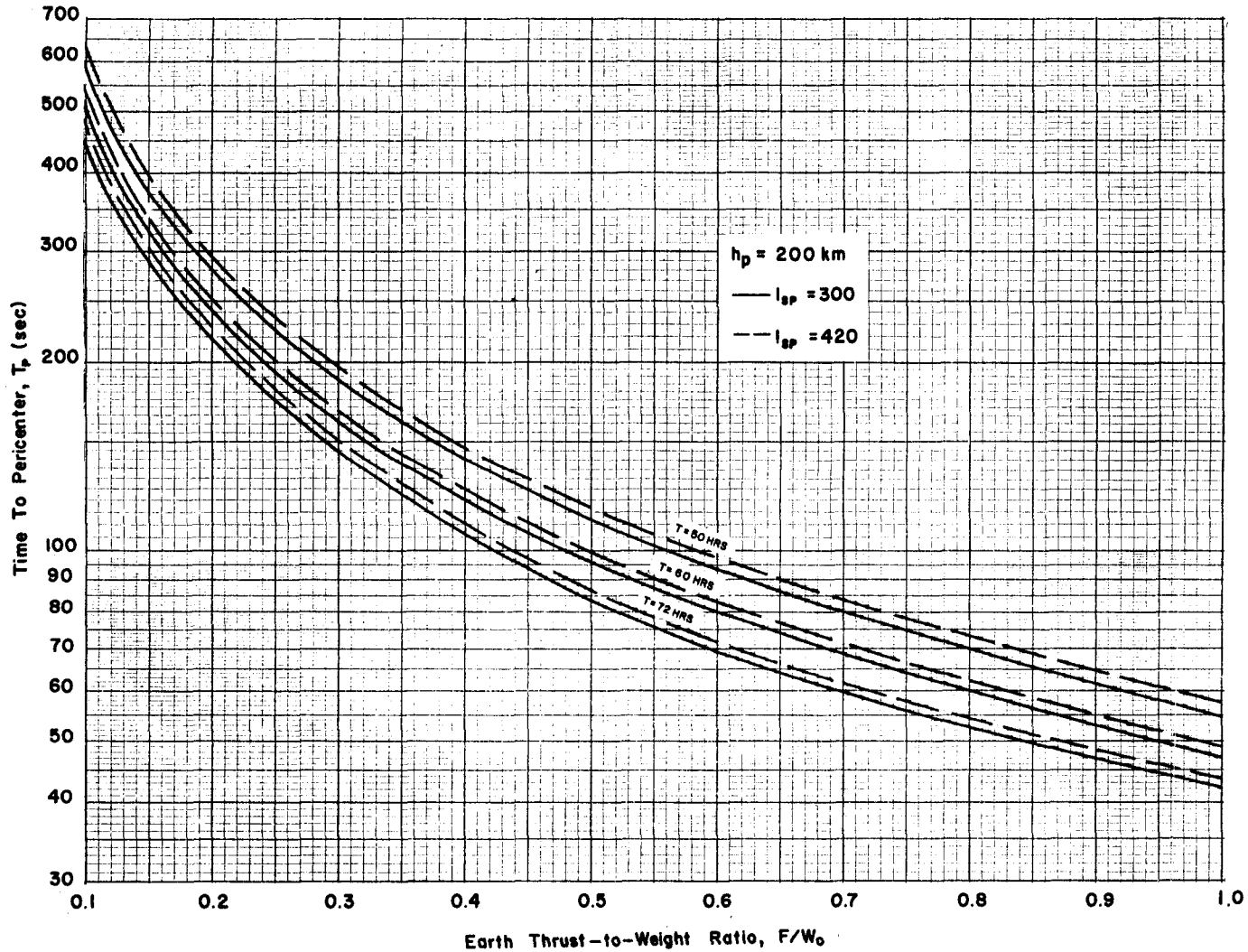


FIGURE 2c. TIME TO PERICENTER FOR BRAKE TO LUNAR ORBIT - h_p = 200 km.

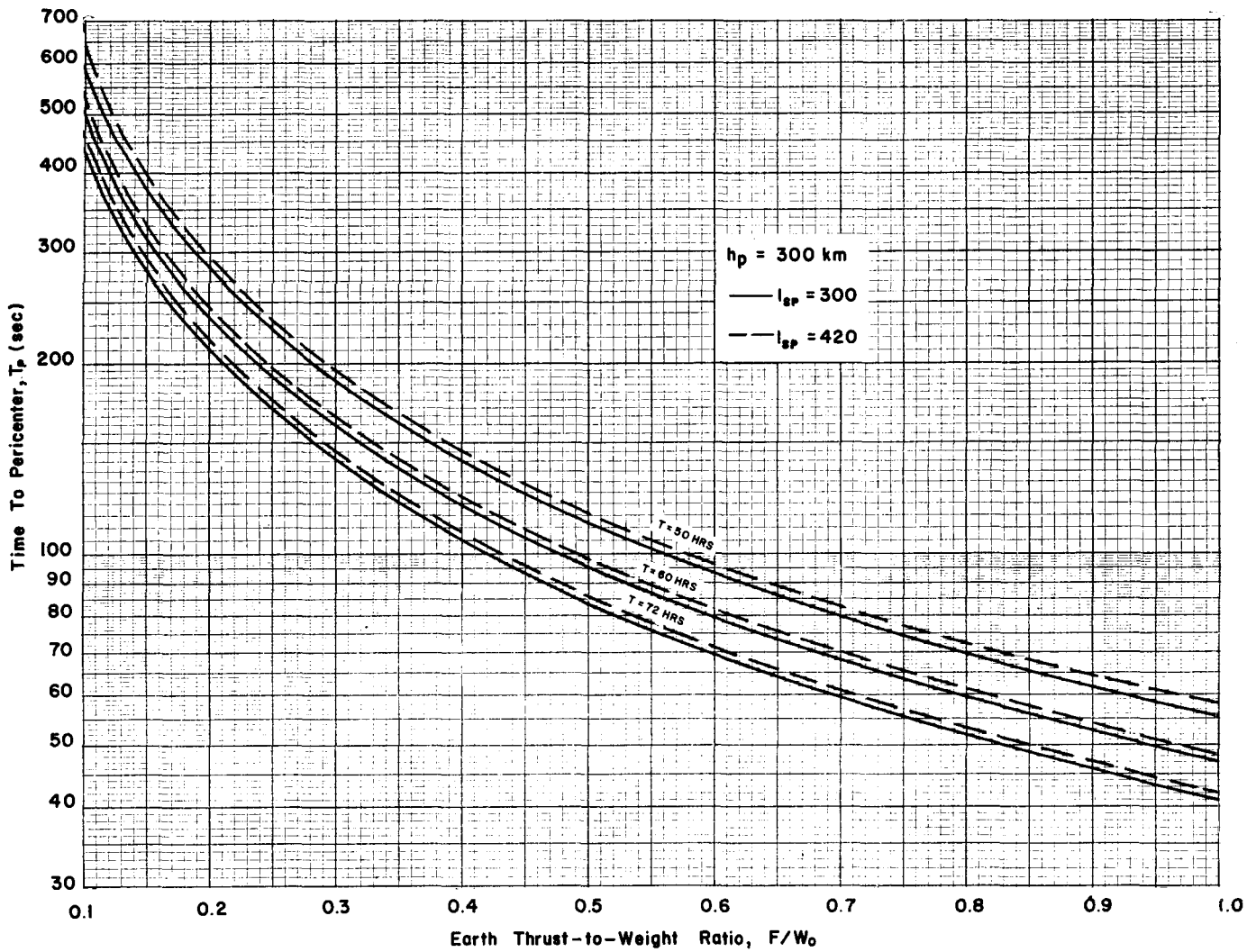


FIGURE 2d. TIME TO PERICENTER FOR BRAKE TO LUNAR ORBIT - h_p = 300 km.

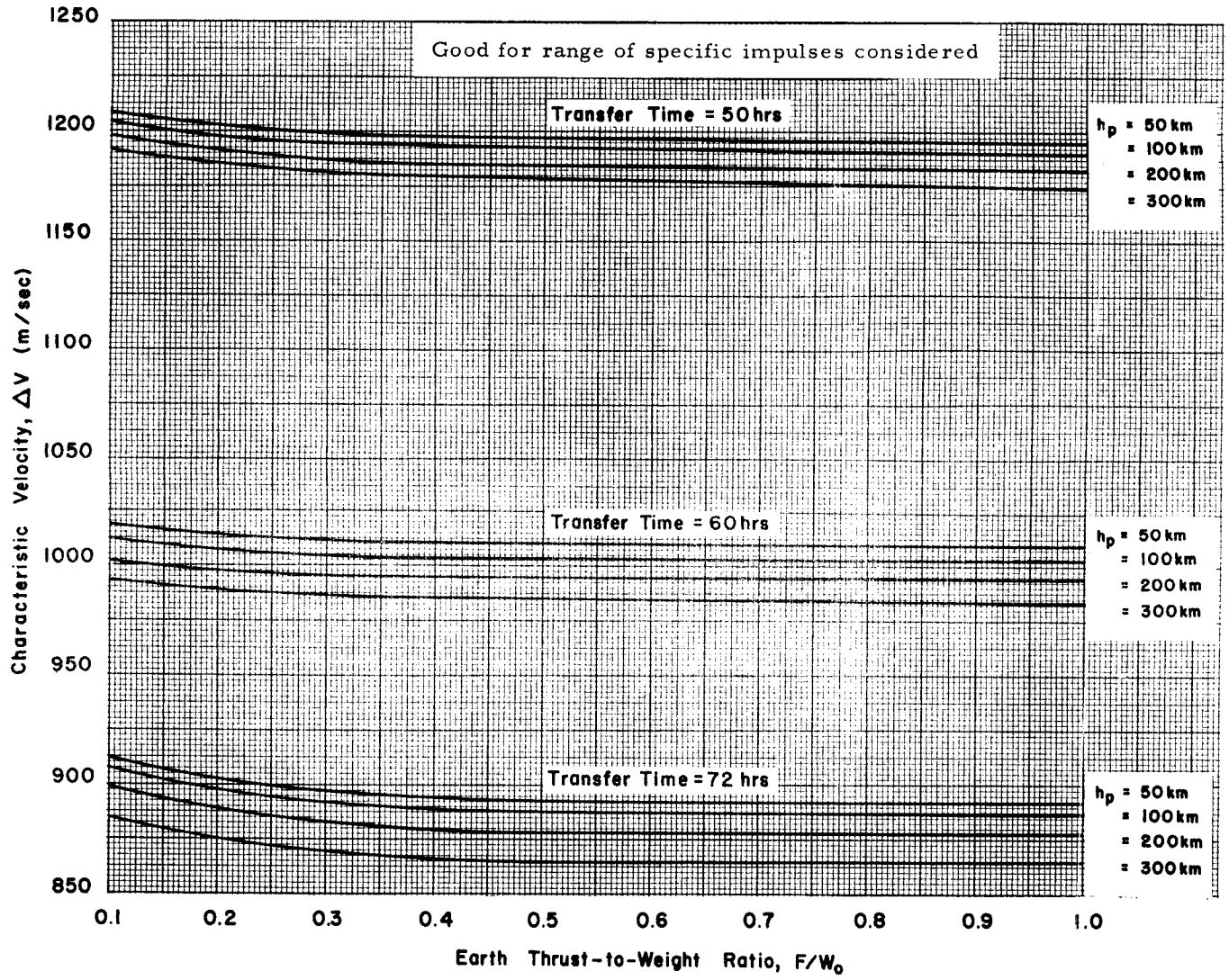


FIGURE 3. CHARACTERISTIC VELOCITY FOR BRAKE TO LUNAR ORBIT.

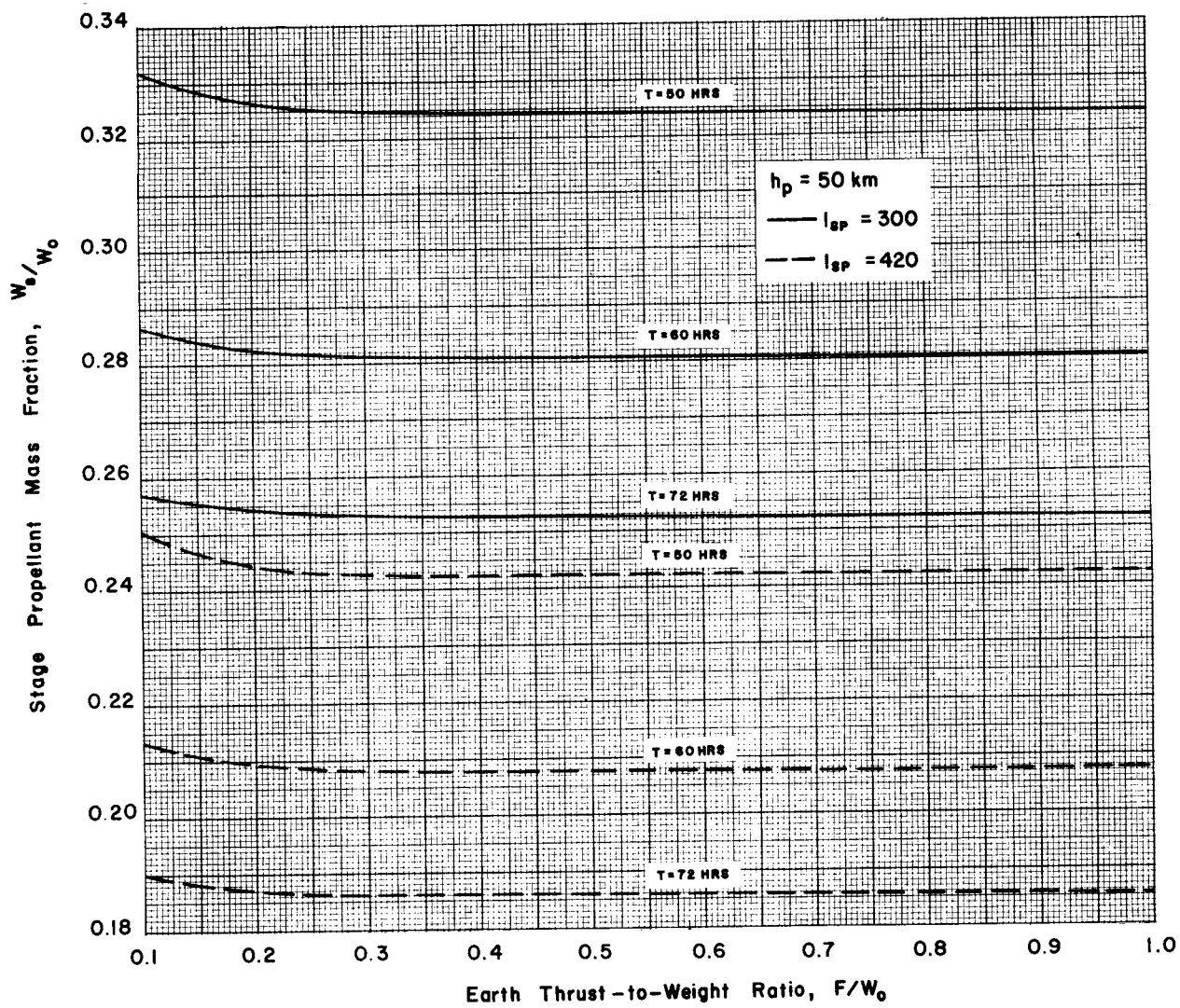


FIGURE 4a. STAGE PROPELLANT MASS FRACTION FOR BRAKE TO LUNAR ORBIT - h_p = 50 km.

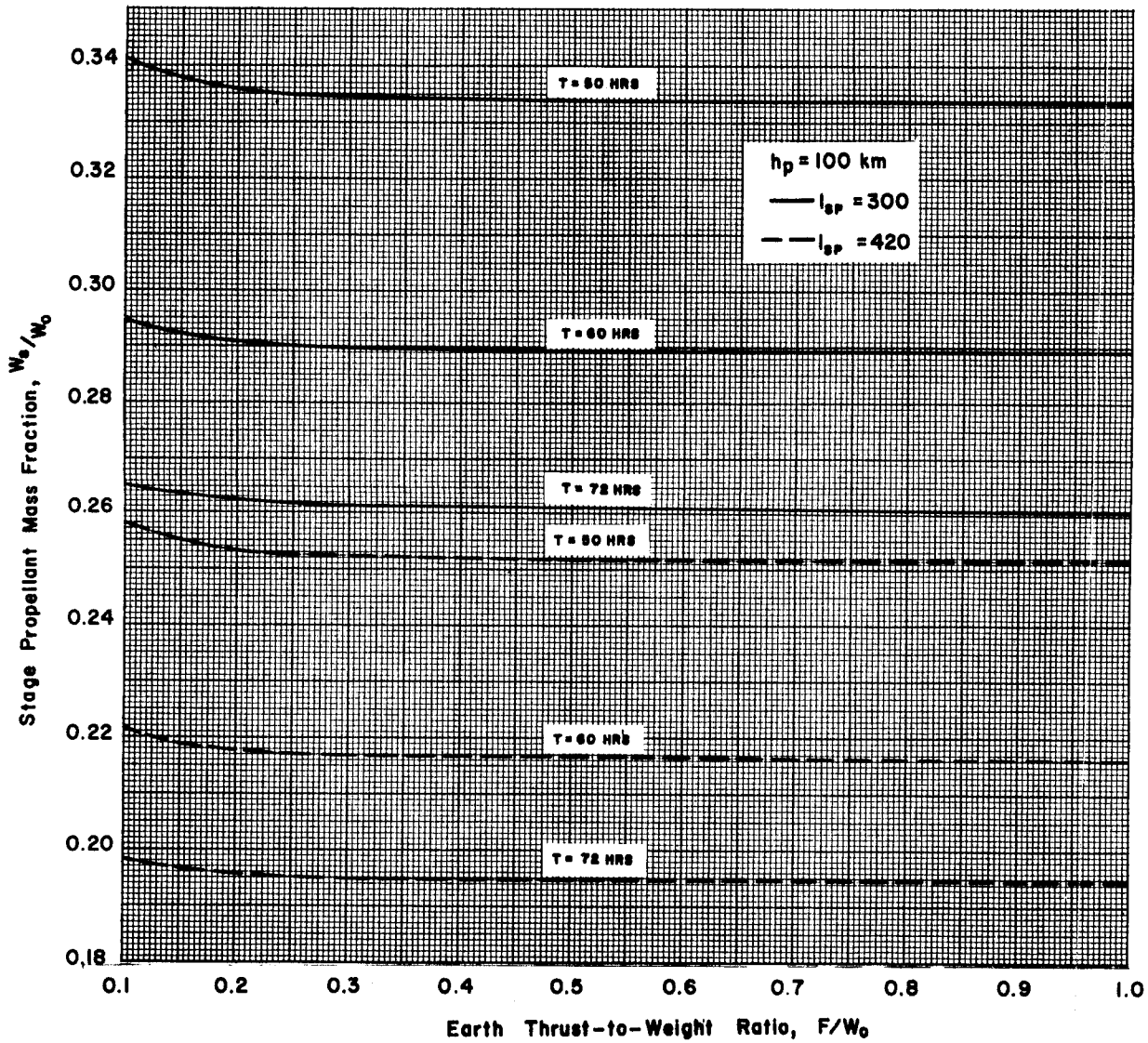


FIGURE 4b. STAGE PROPELLANT MASS FRACTION FOR BRAKE TO LUNAR ORBIT - h_p = 100 km.

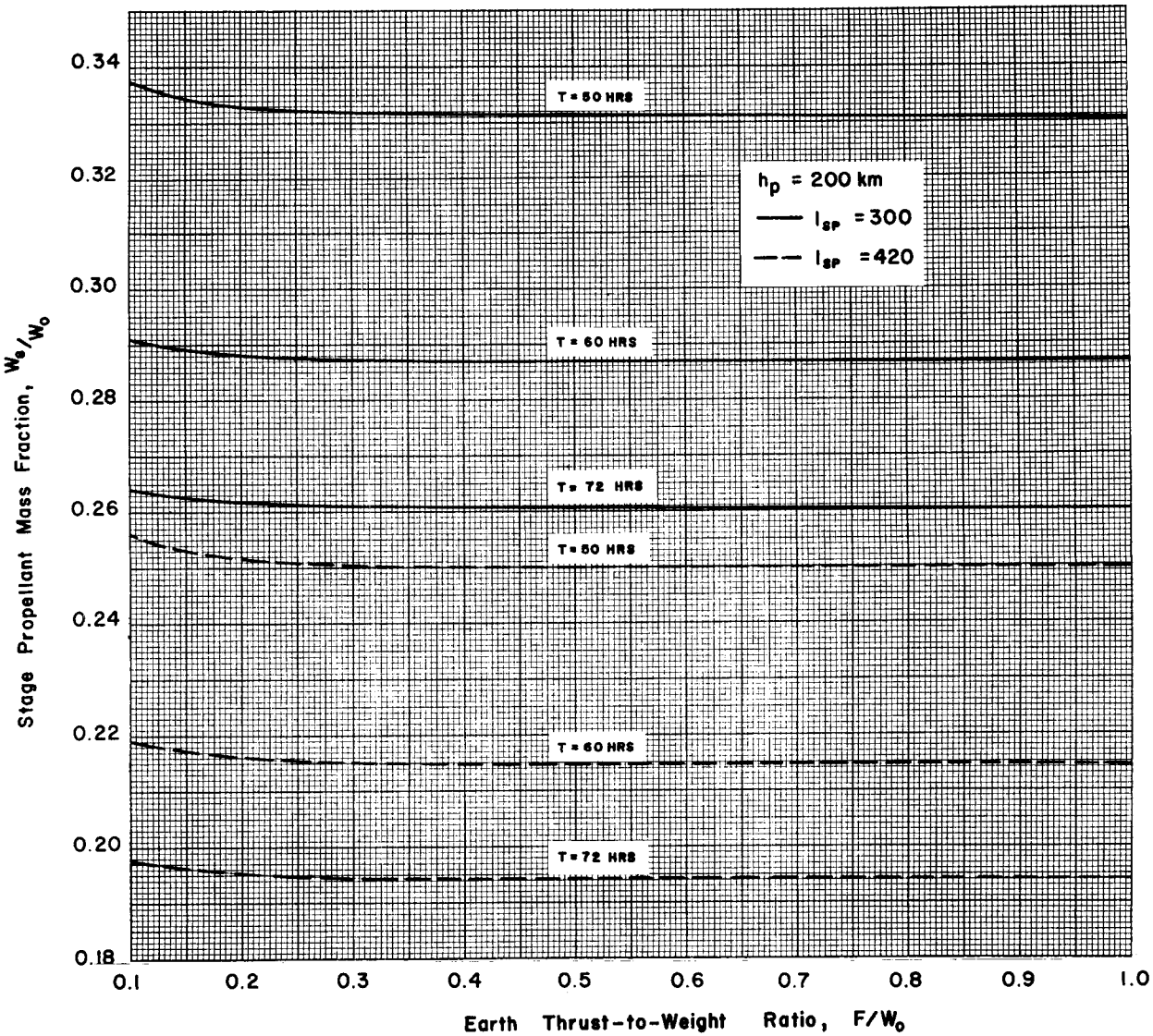


FIGURE 4c. STAGE PROPELLANT MASS FRACTION FOR BRAKE TO LUNAR ORBIT - h_p = 200 km.

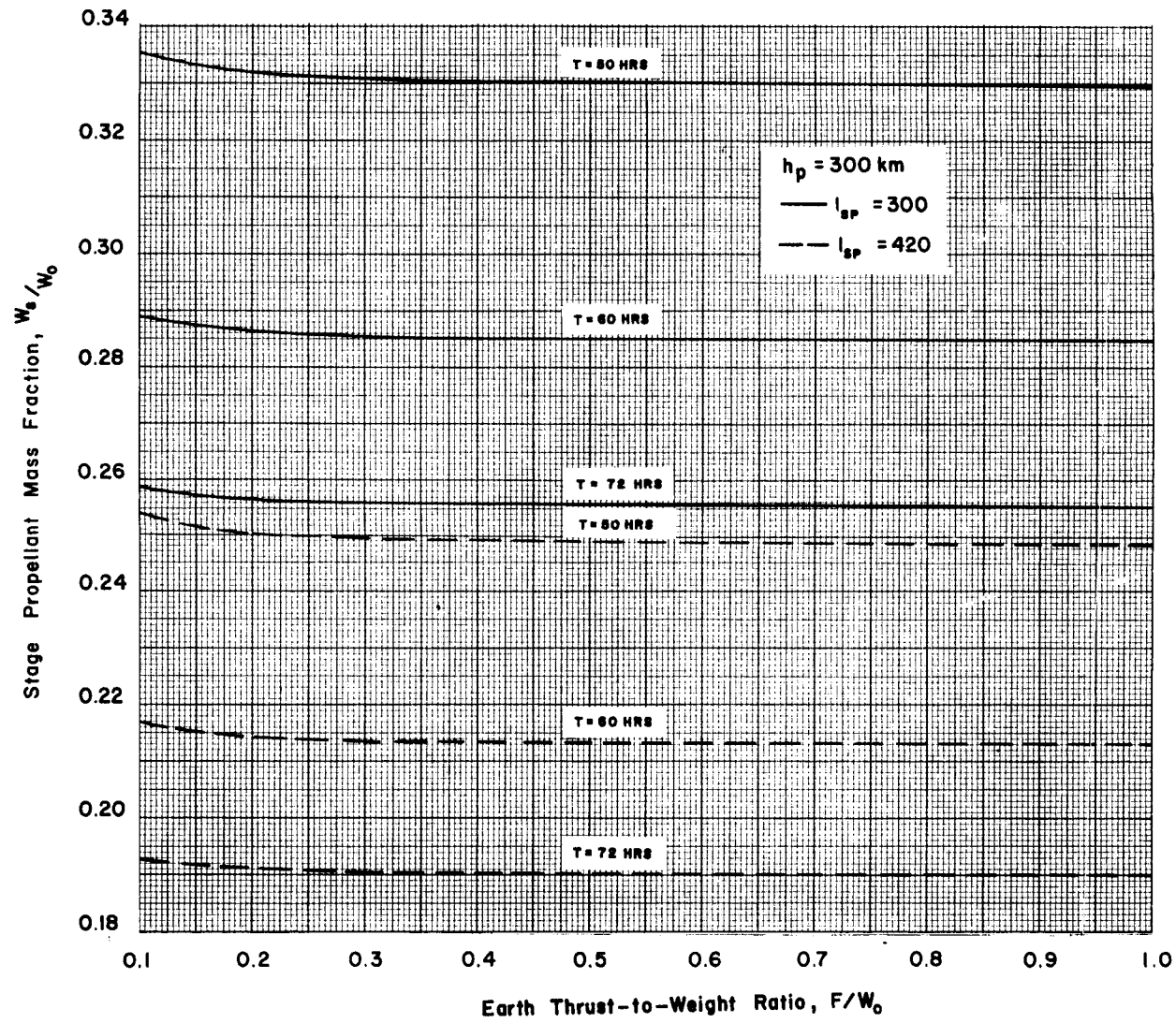


FIGURE 4d. STAGE PROPELLANT MASS FRACTION FOR BRAKE TO LUNAR ORBIT - h_p = 300 km.

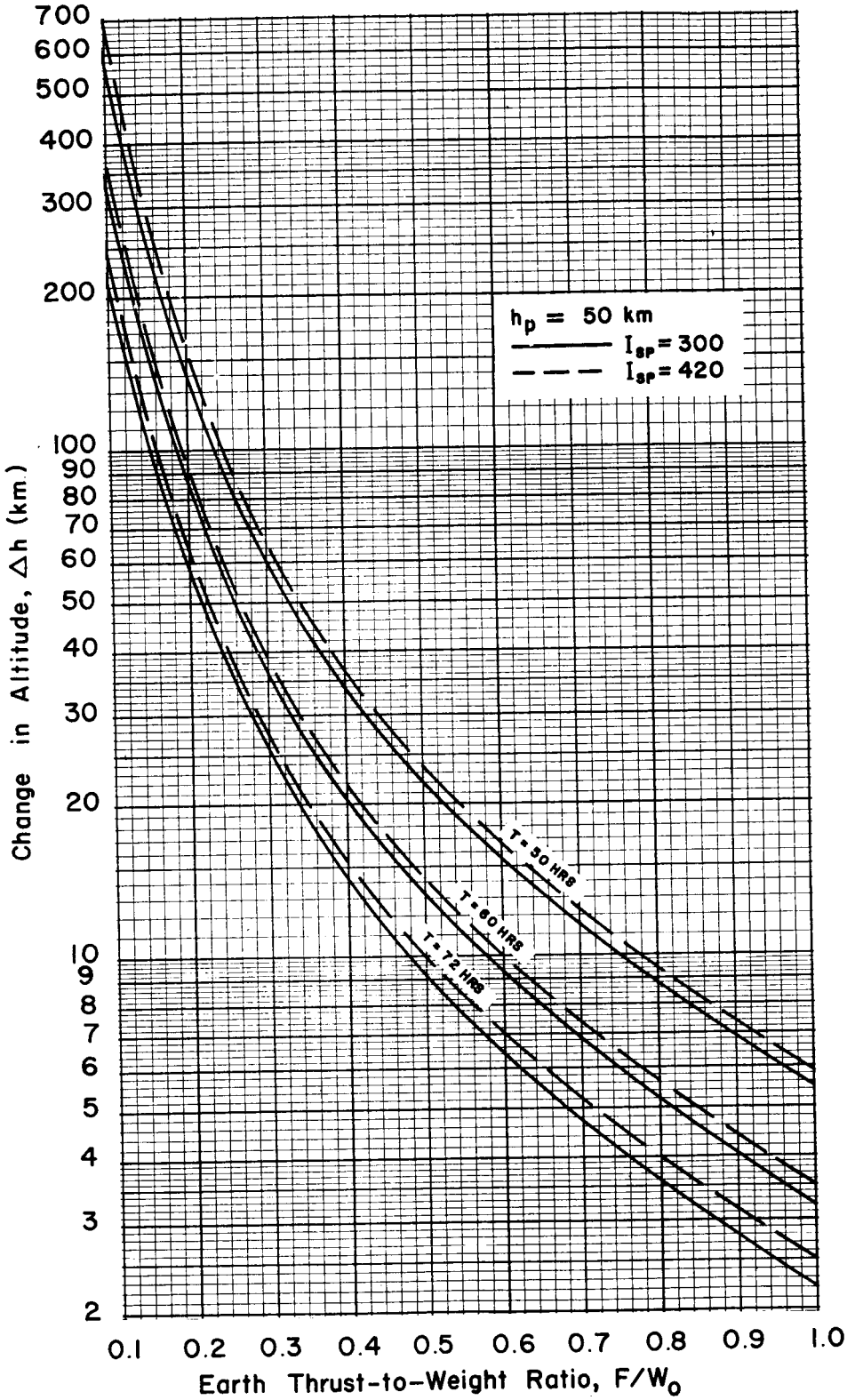


FIGURE 5a. CHANGE IN ALTITUDE FOR BRAKE TO LUNAR ORBIT - h_p = 50 km.

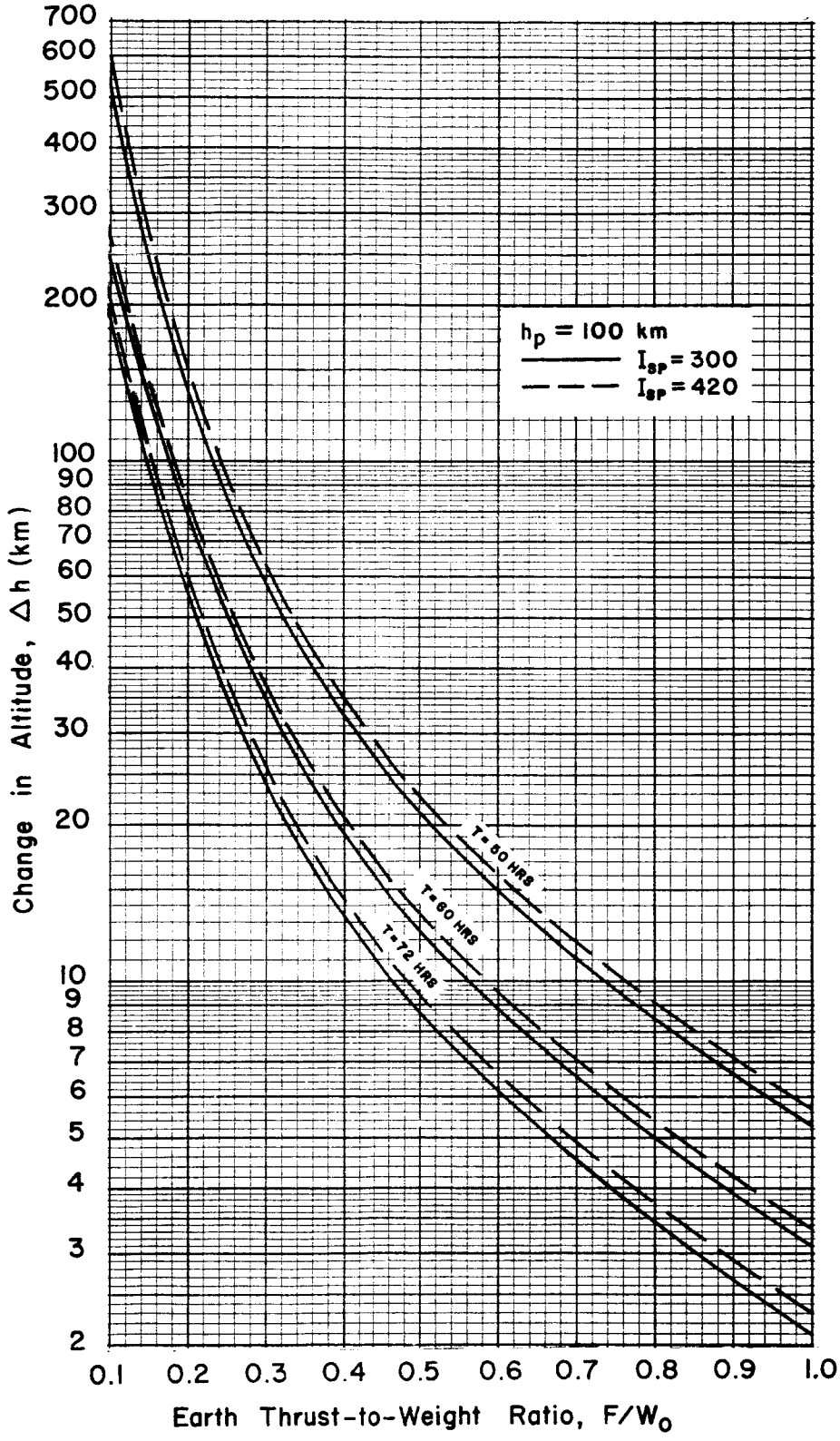


FIGURE 5b. CHANGE IN ALTITUDE FOR BRAKE TO LUNAR ORBIT - h_p = 100 km.

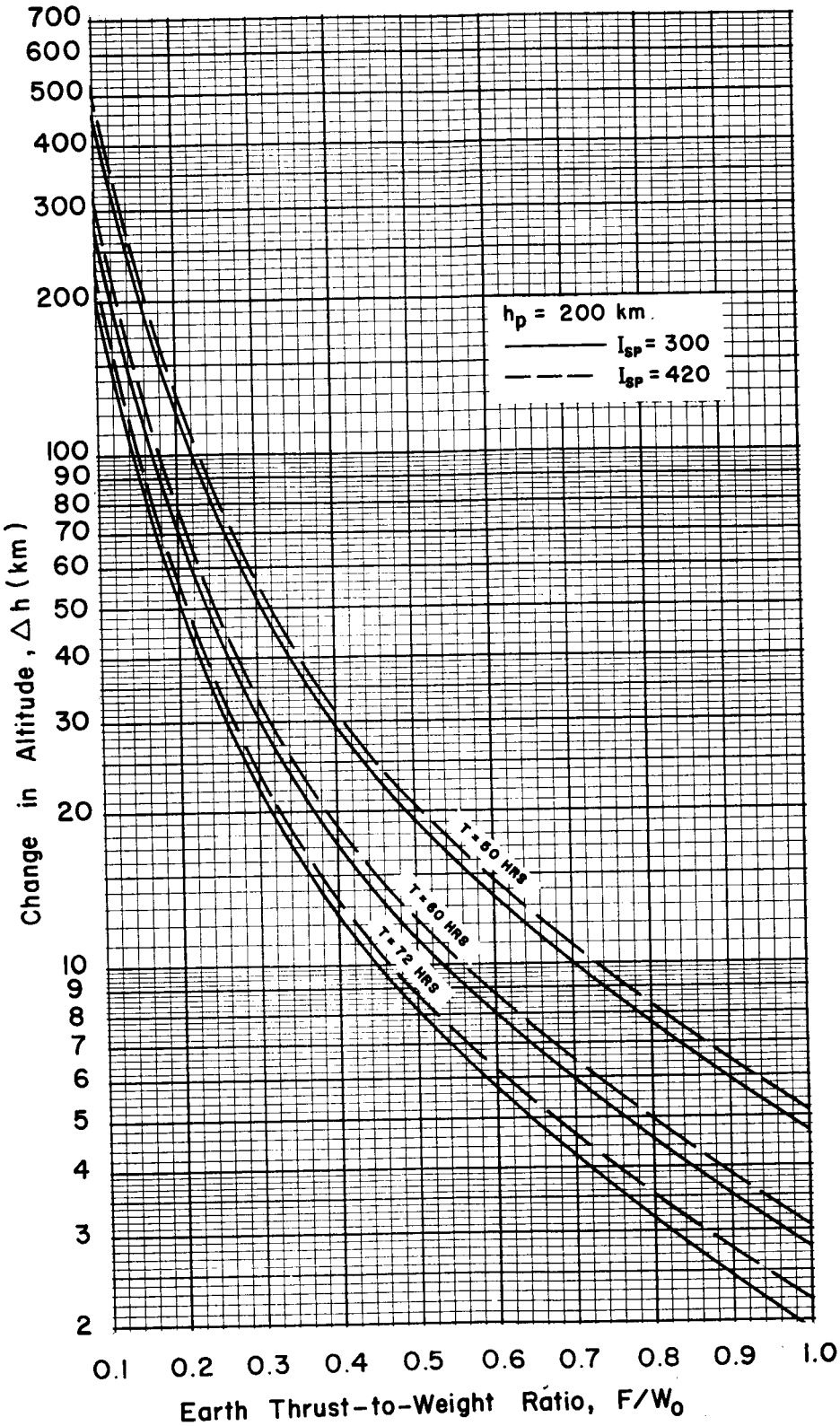


FIGURE 5c. CHANGE IN ALTITUDE FOR BRAKE TO LUNAR ORBIT - h_p = 200 km.

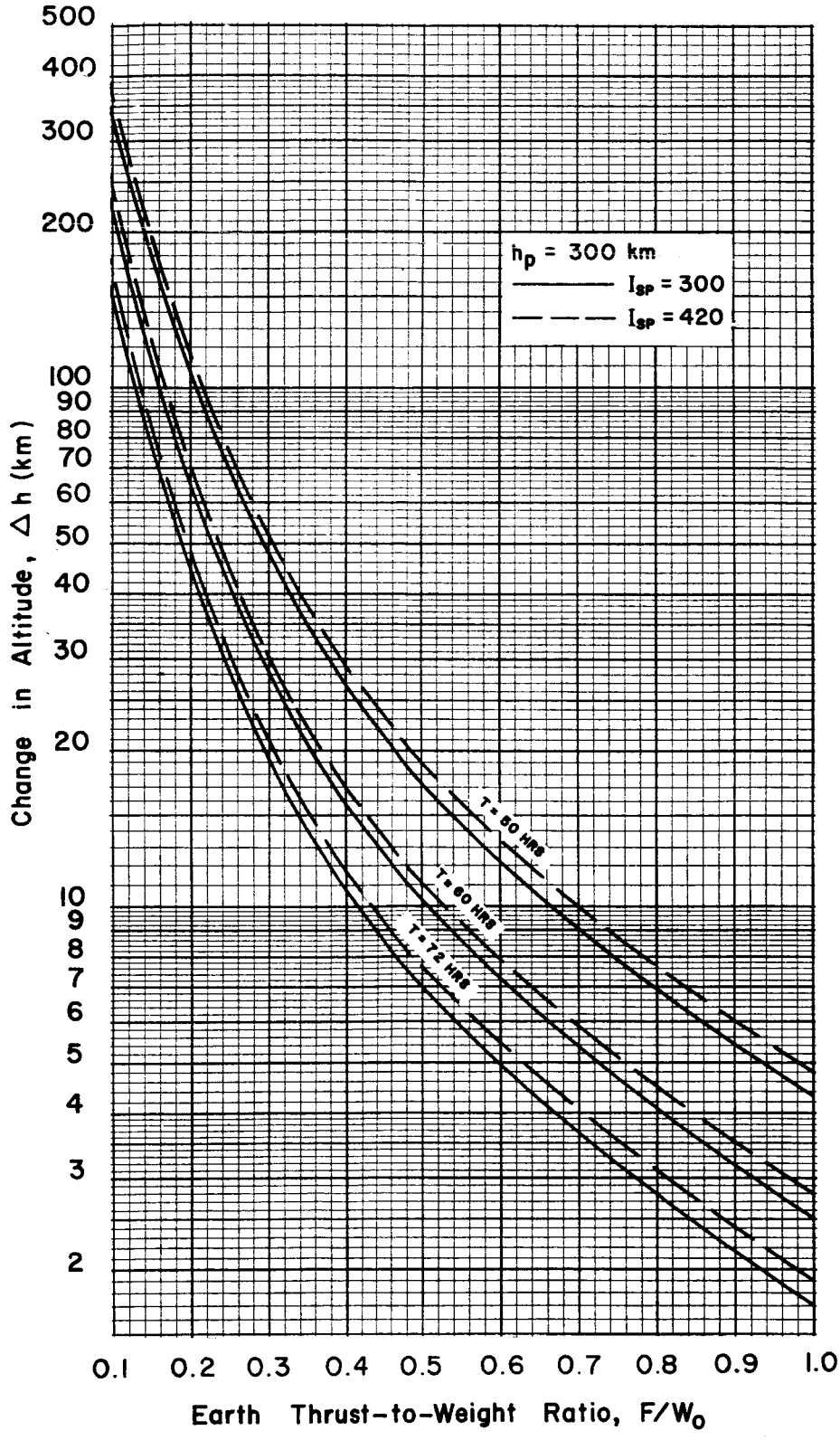


FIGURE 5d. CHANGE IN ALTITUDE FOR BRAKE TO LUNAR ORBIT - h_p = 300 km.

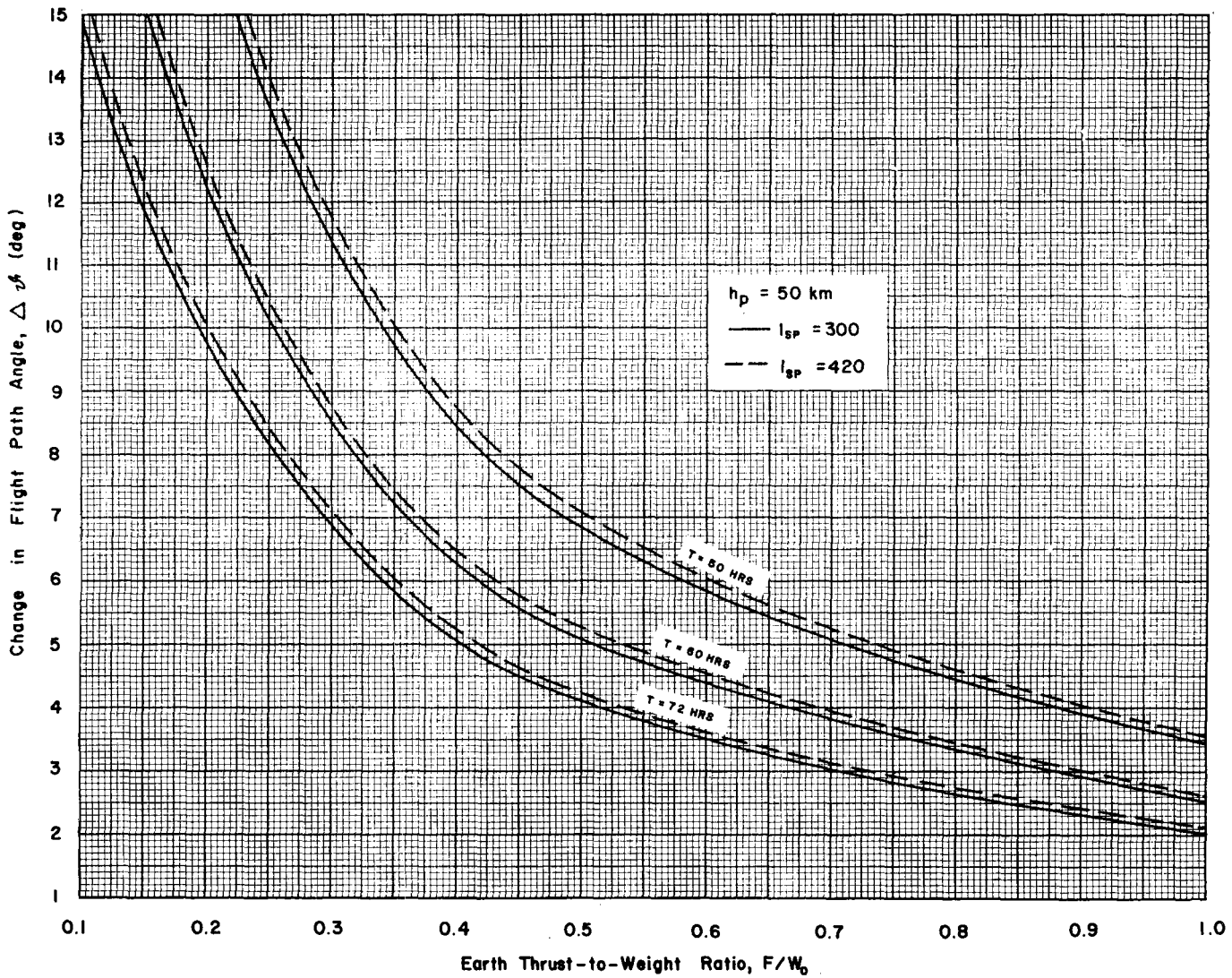


FIGURE 6a. CHANGE IN FLIGHT PATH ANGLE FOR BRAKE TO LUNAR ORBIT - h_p = 50 km.

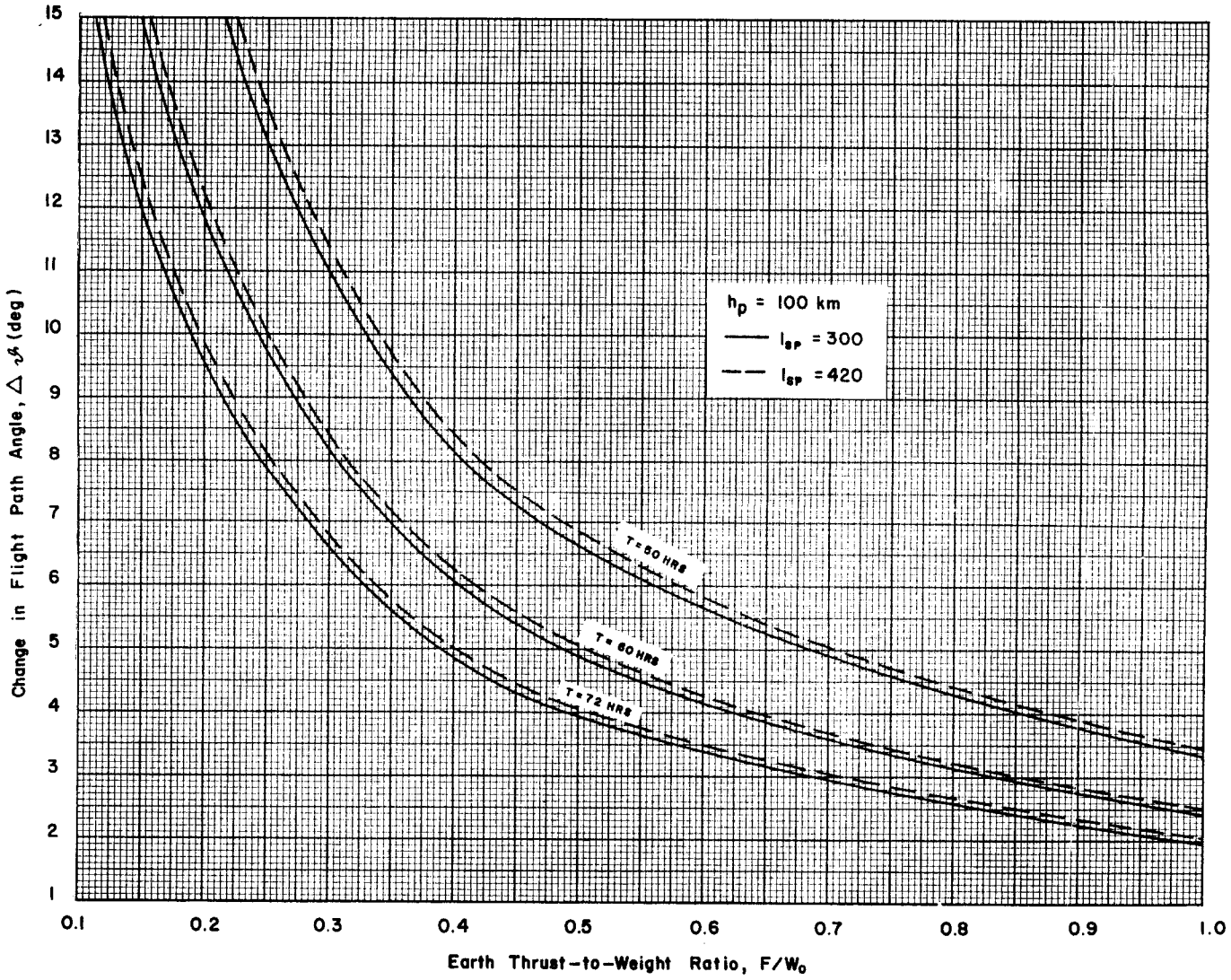


FIGURE 6b. CHANGE IN FLIGHT PATH ANGLE FOR BRAKE TO LUNAR ORBIT - h_p = 100 km.

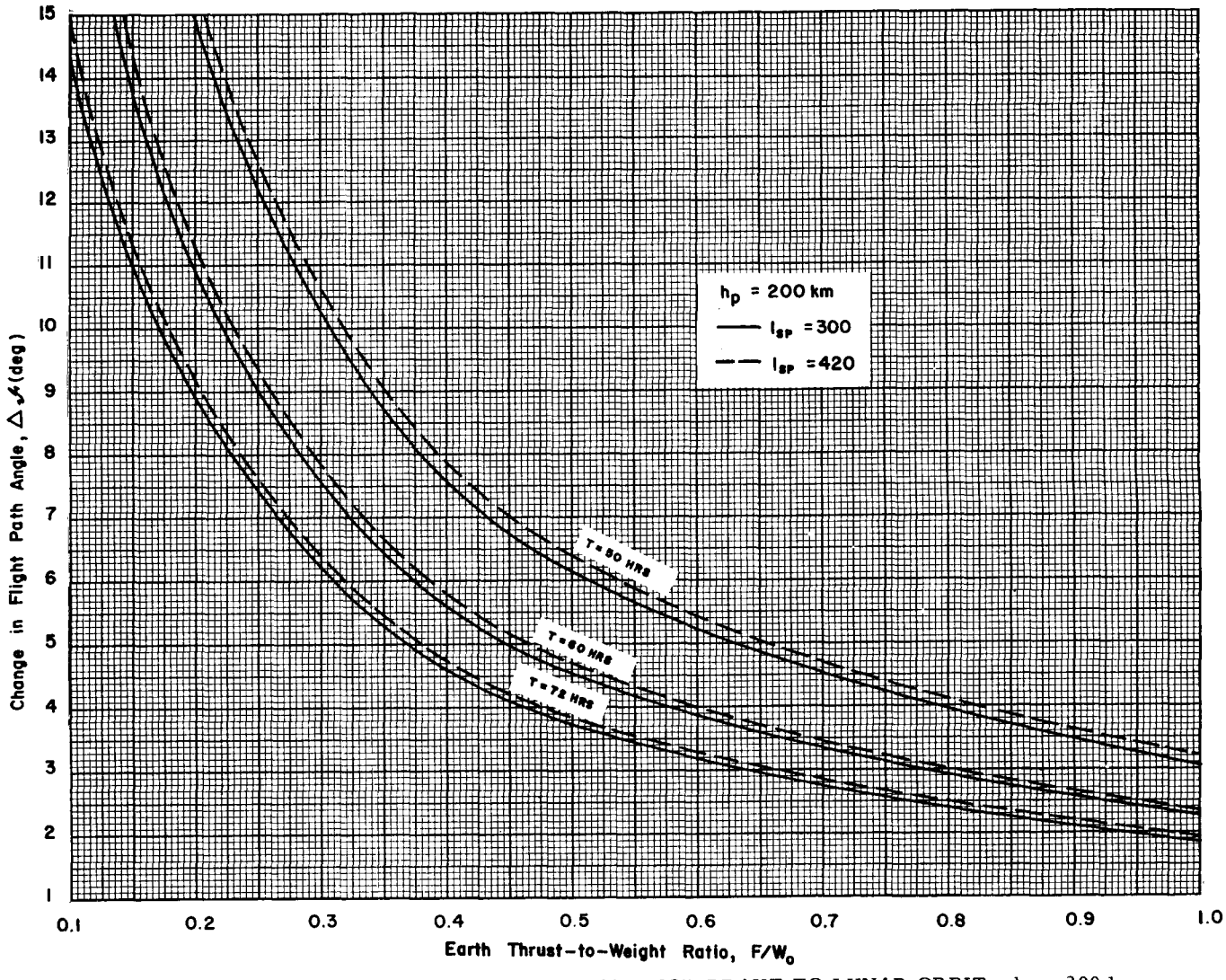


FIGURE 6c. CHANGE IN FLIGHT PATH ANGLE FOR BRAKE TO LUNAR ORBIT - h_p = 200 km.

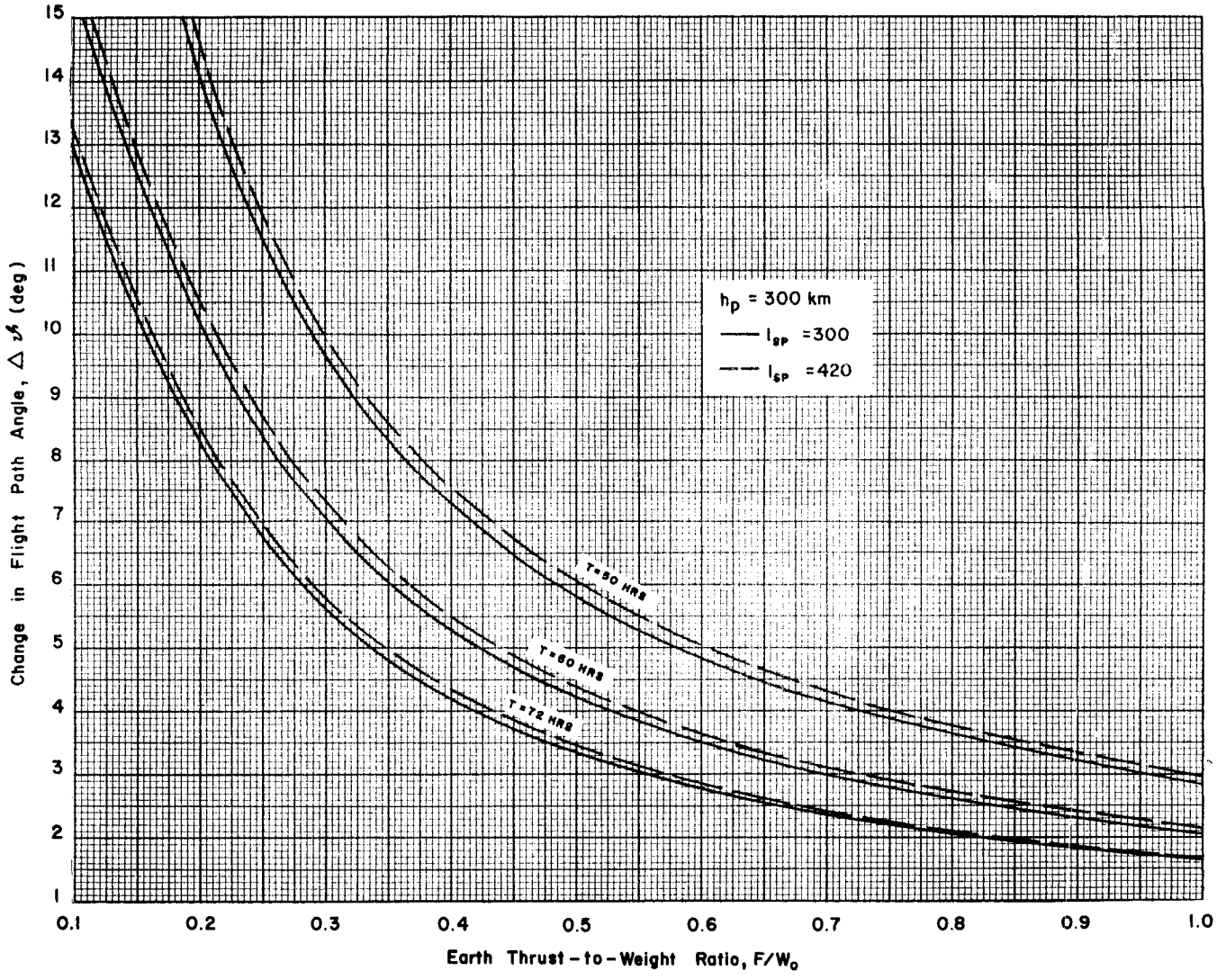


FIGURE 6d. CHANGE IN FLIGHT PATH ANGLE FOR BRAKE TO LUNAR ORBIT - h_p = 300 km.

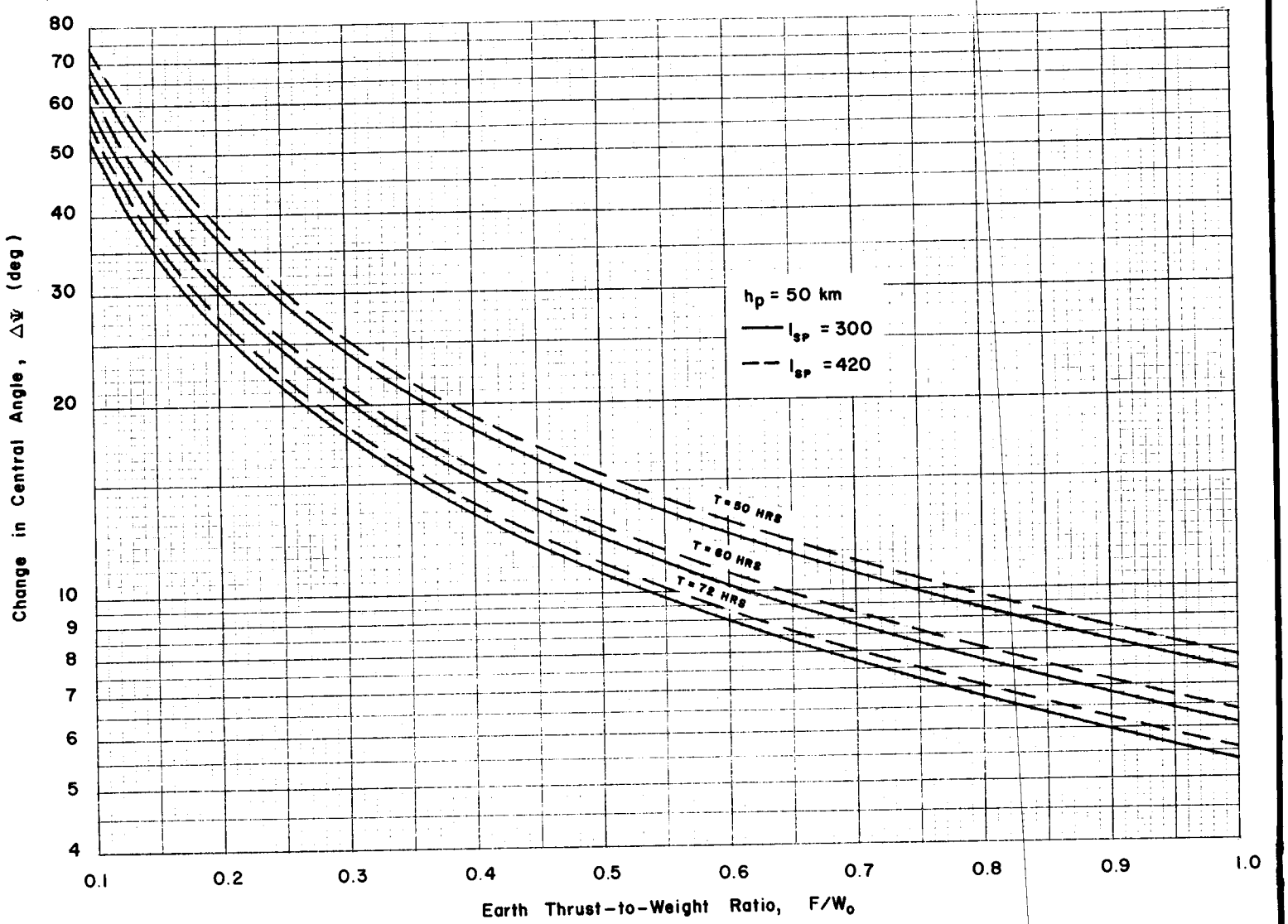


FIGURE 7a. CHANGE IN CENTRAL ANGLE FOR BRAKE TO LUNAR ORBIT - h_p = 50 km.

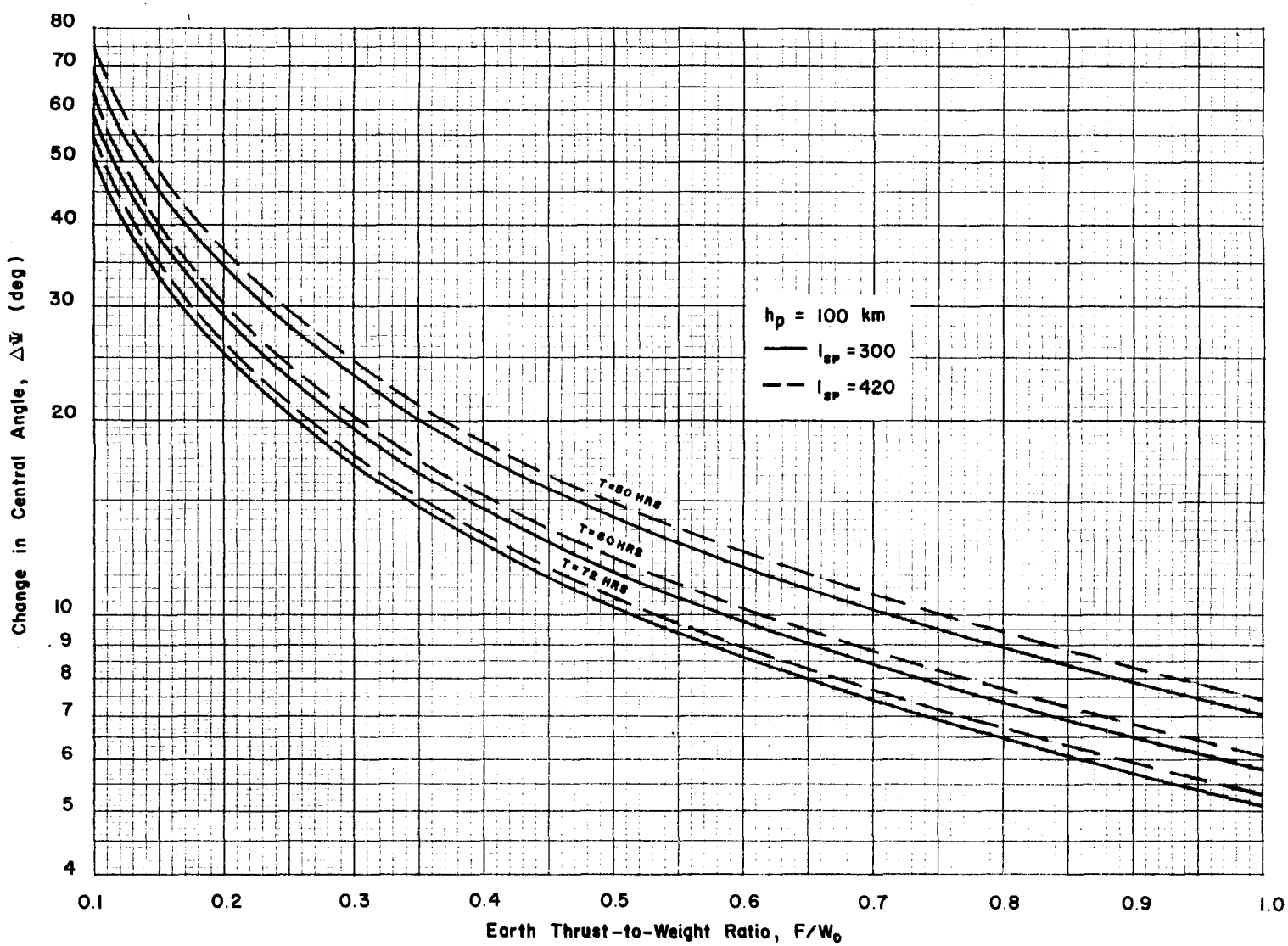


FIGURE 7b. CHANGE IN CENTRAL ANGLE FOR BRAKE TO LUNAR ORBIT - h_p = 100 km.

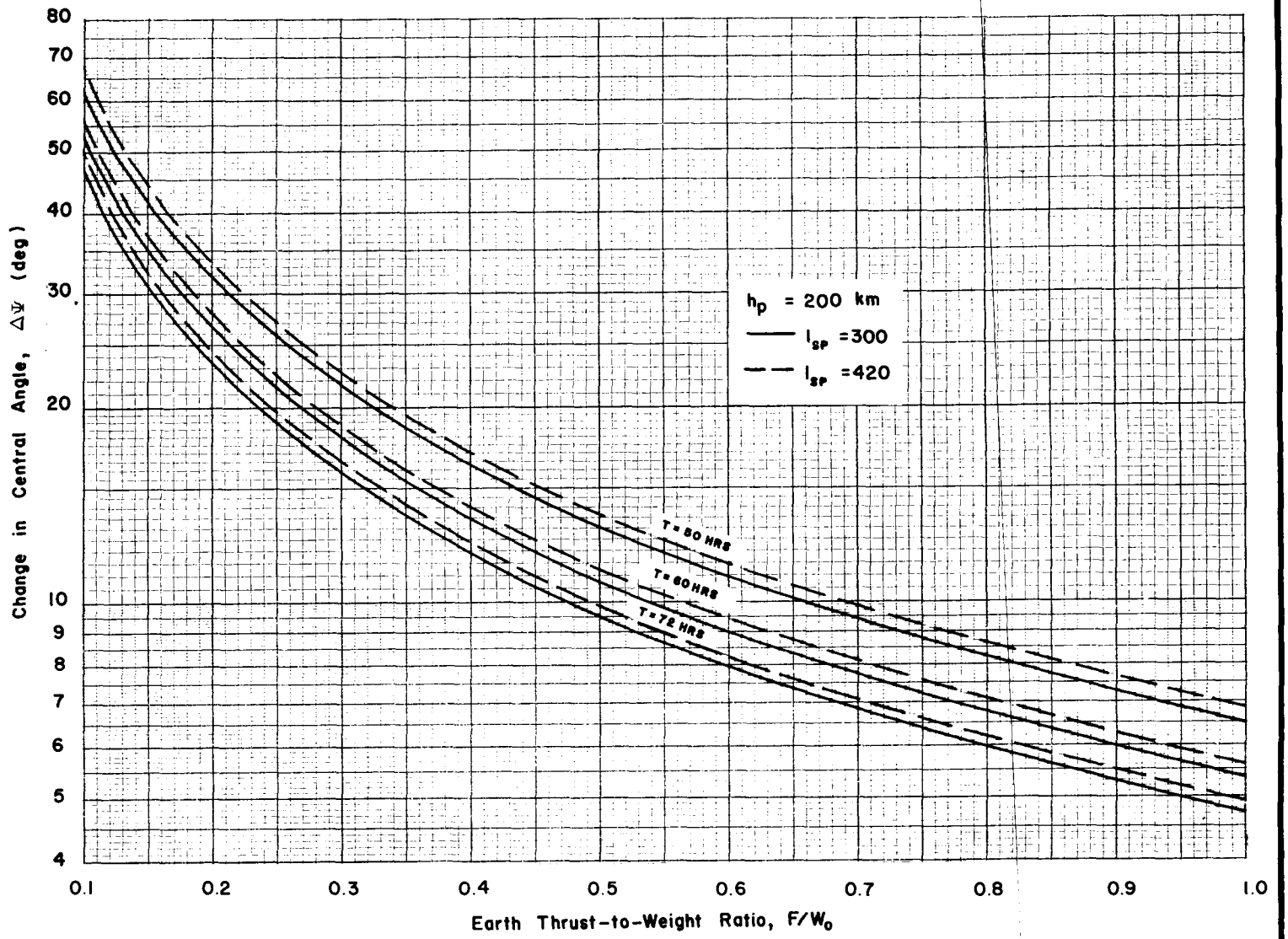


FIGURE 7c. CHANGE IN CENTRAL ANGLE FOR BRAKE TO LUNAR ORBIT - h_p = 200 km.

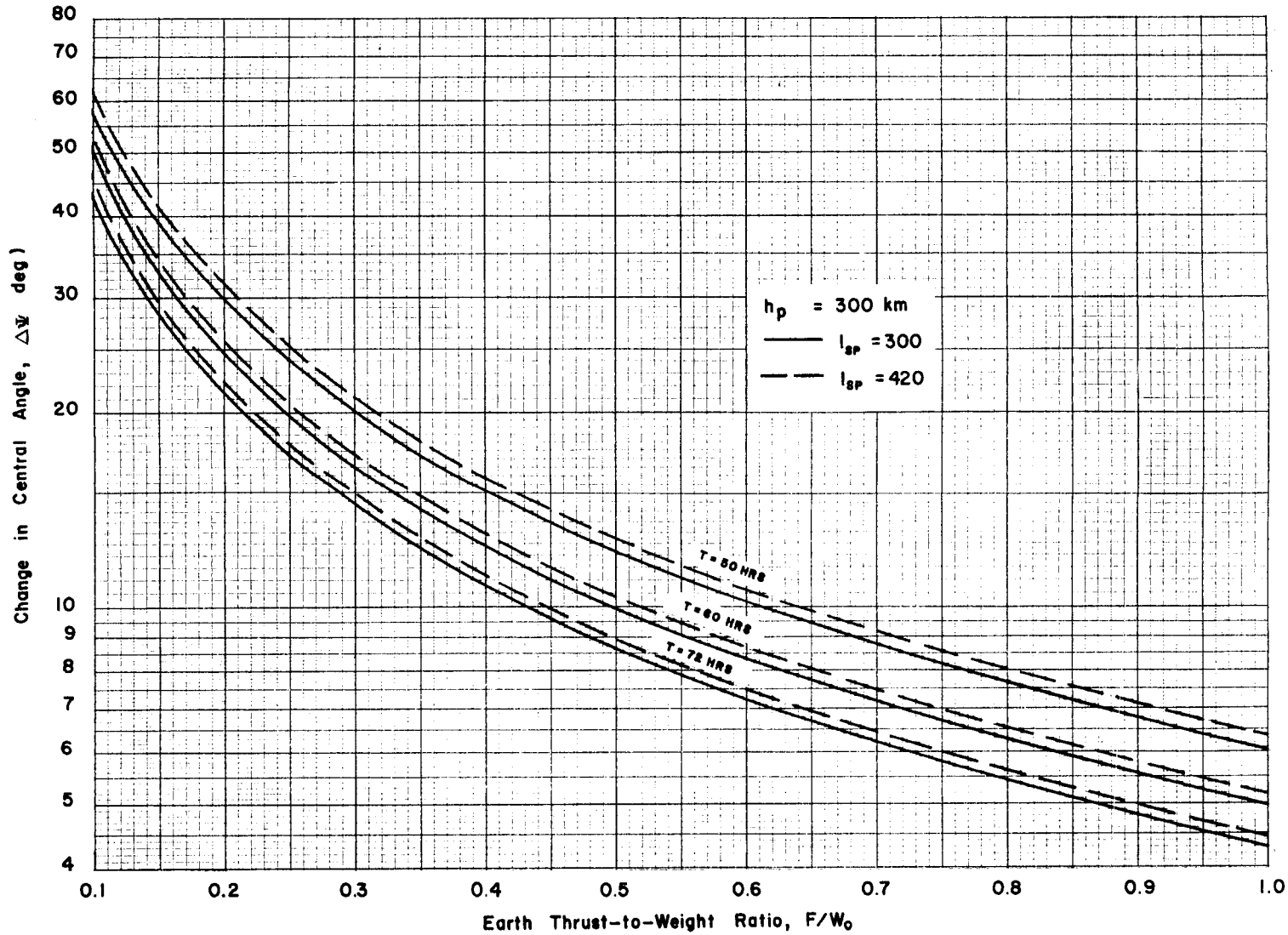


FIGURE 7d. CHANGE IN CENTRAL ANGLE FOR BRAKE TO LUNAR ORBIT - h_p = 300 km.

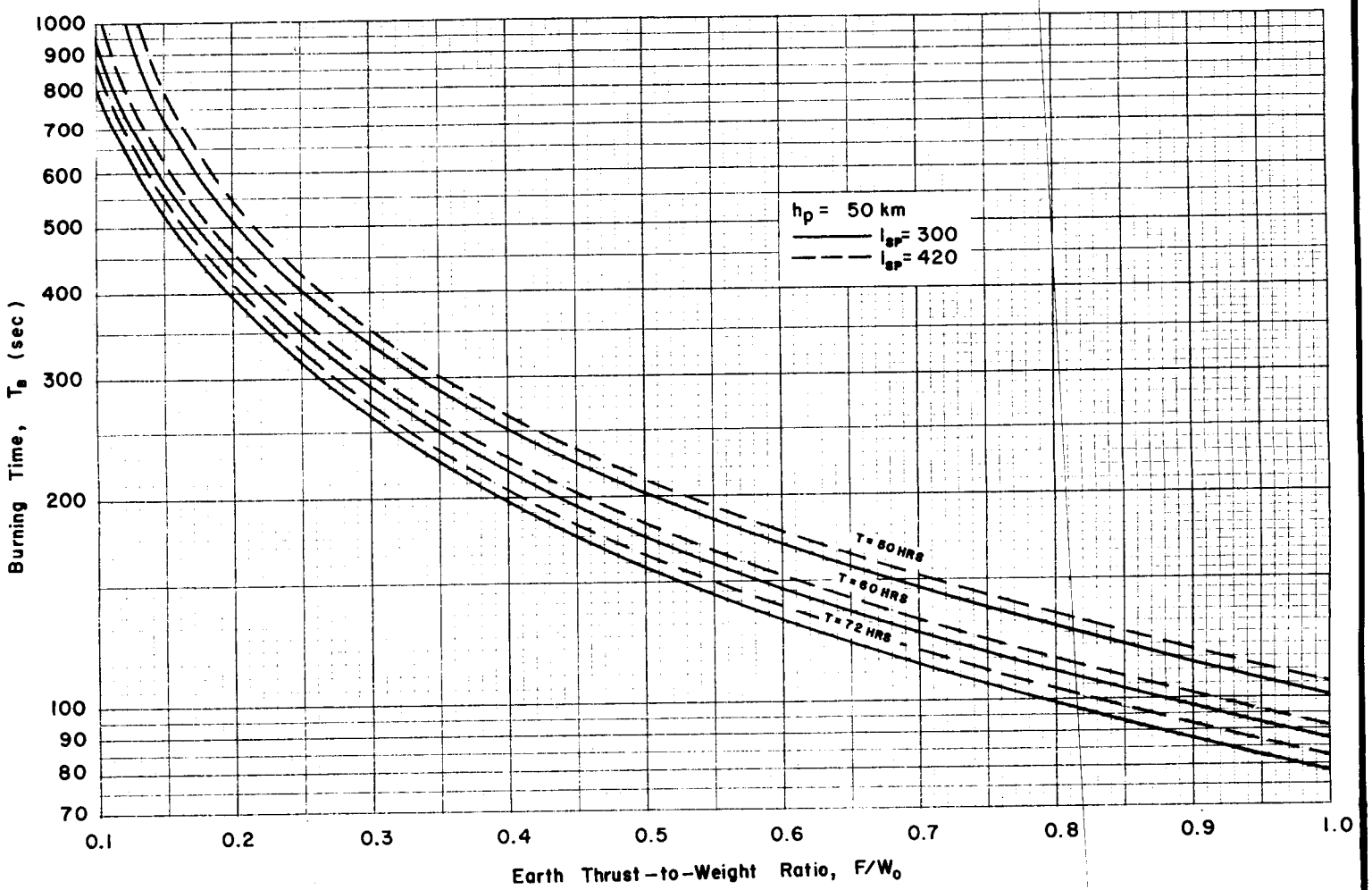


FIGURE 8a. BURNING TIME FOR BRAKE TO LUNAR ORBIT - h_p = 50 km.

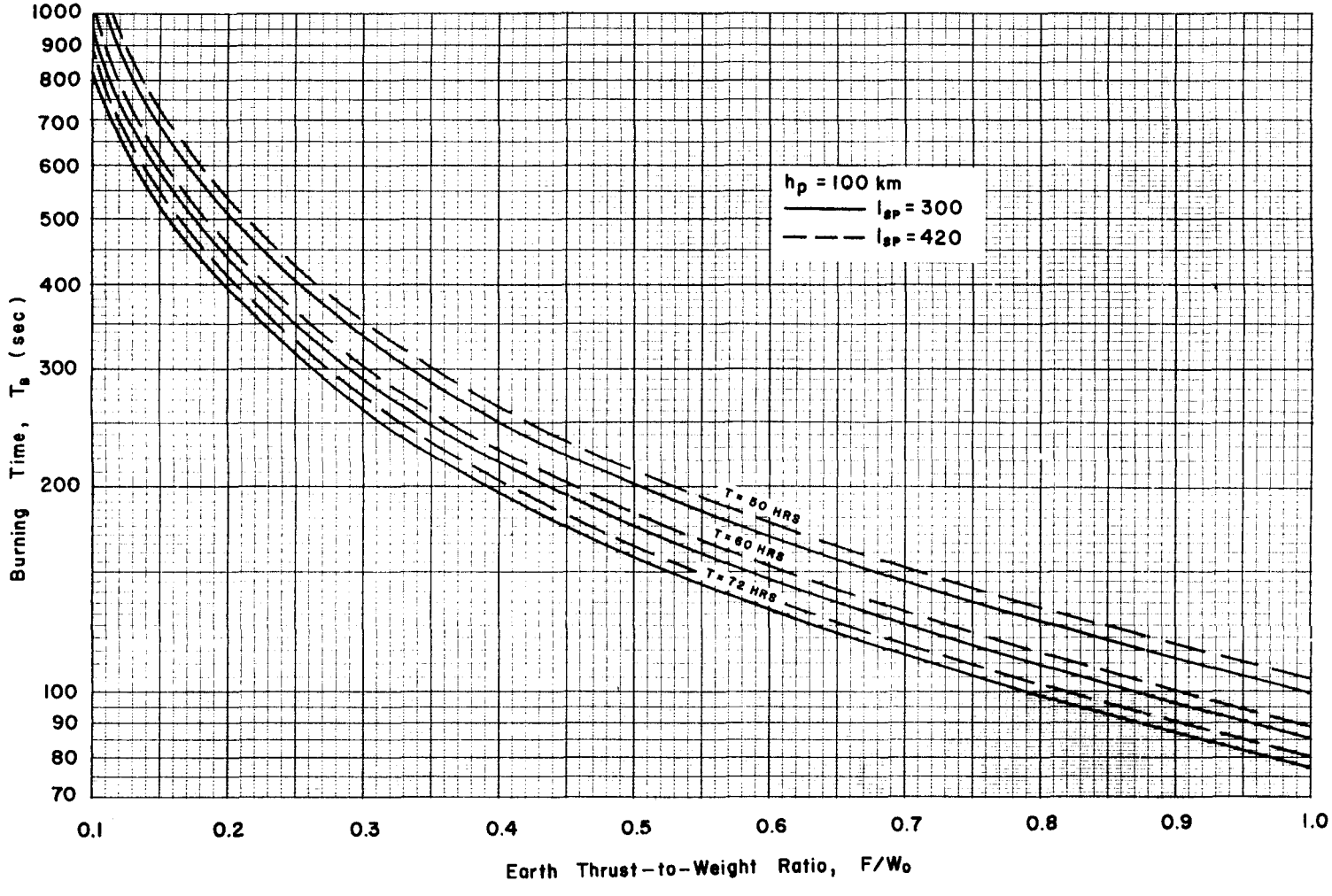


FIGURE 8b. BURNING TIME FOR BRAKE TO LUNAR ORBIT - h_p = 100 km.

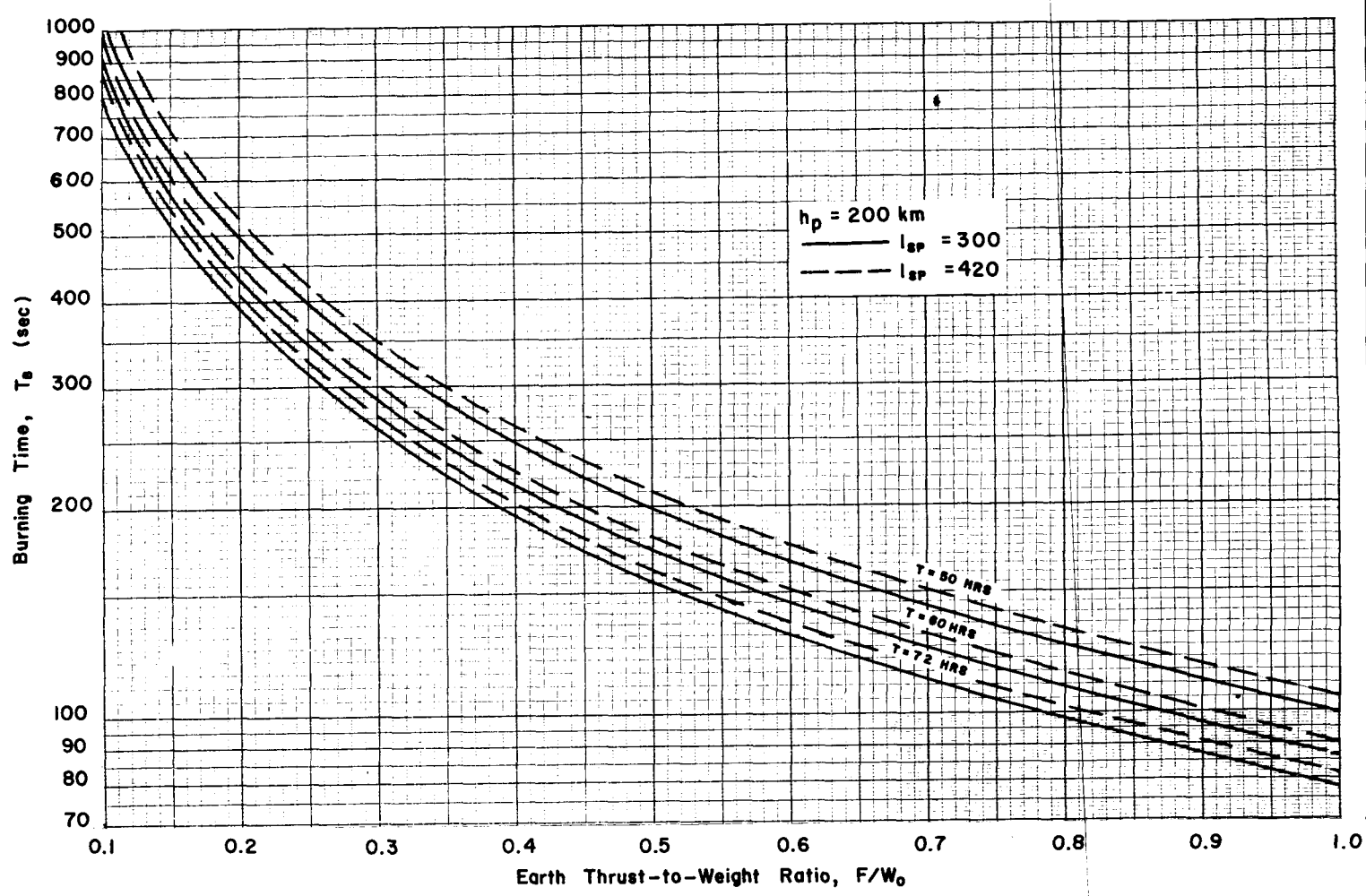


FIGURE 8c. BURNING TIME FOR BRAKE TO LUNAR ORBIT - h_p = 200 km.

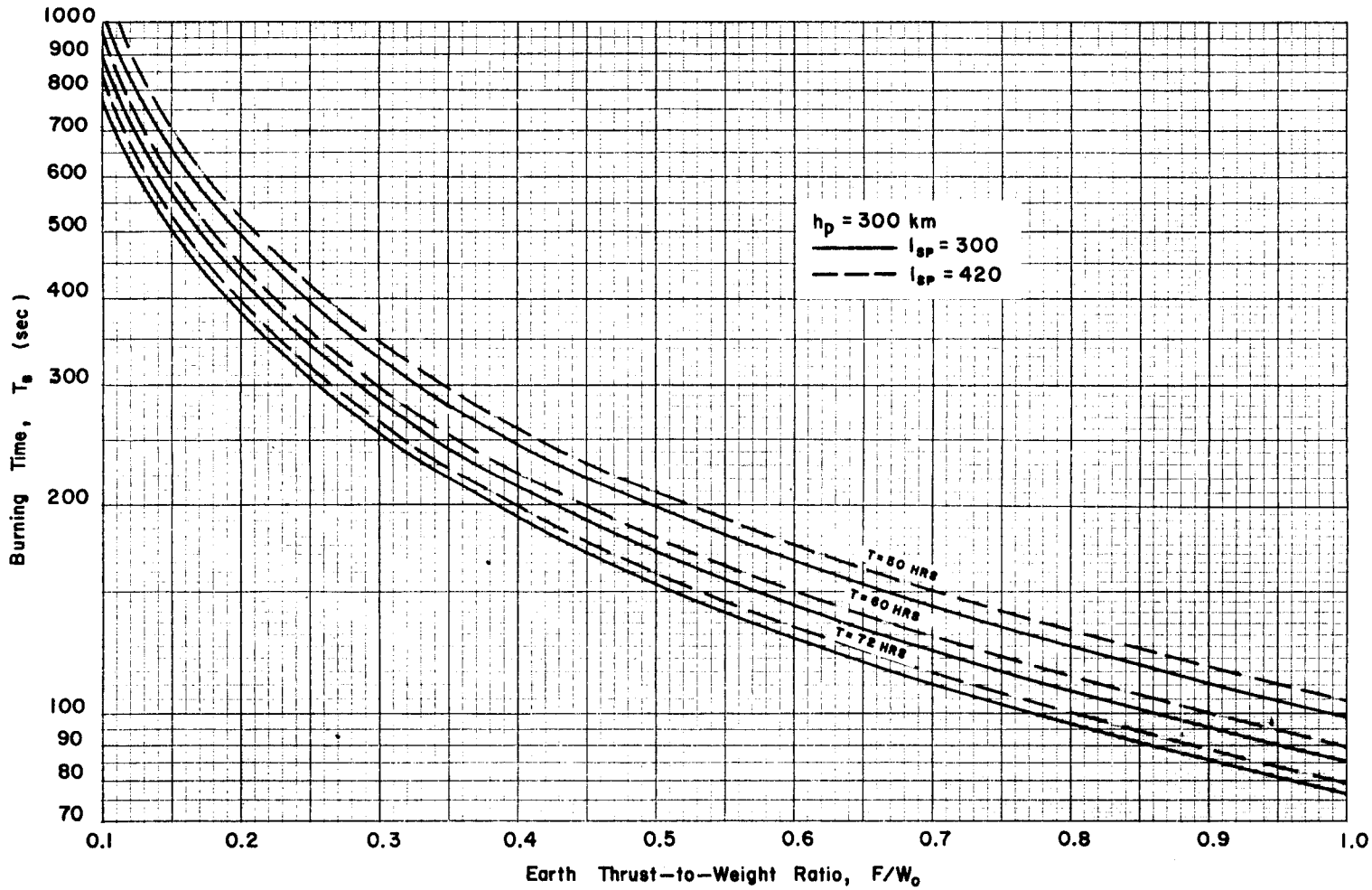


FIGURE 8d. BURNING TIME FOR BRAKE TO LUNAR ORBIT - h_p = 300 km.

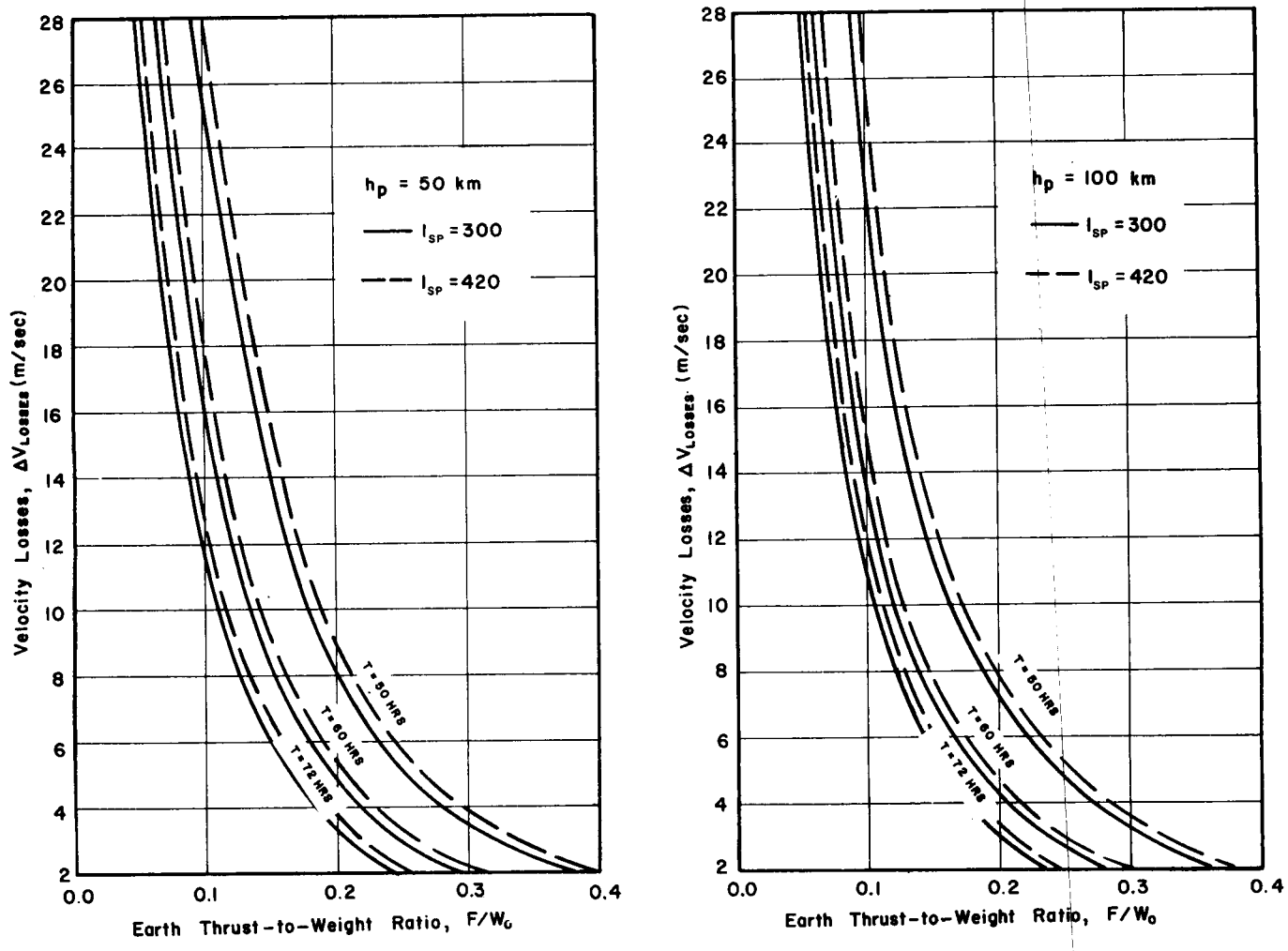


FIGURE 9a. VELOCITY LOSSES FOR BRAKE TO LUNAR ORBIT.

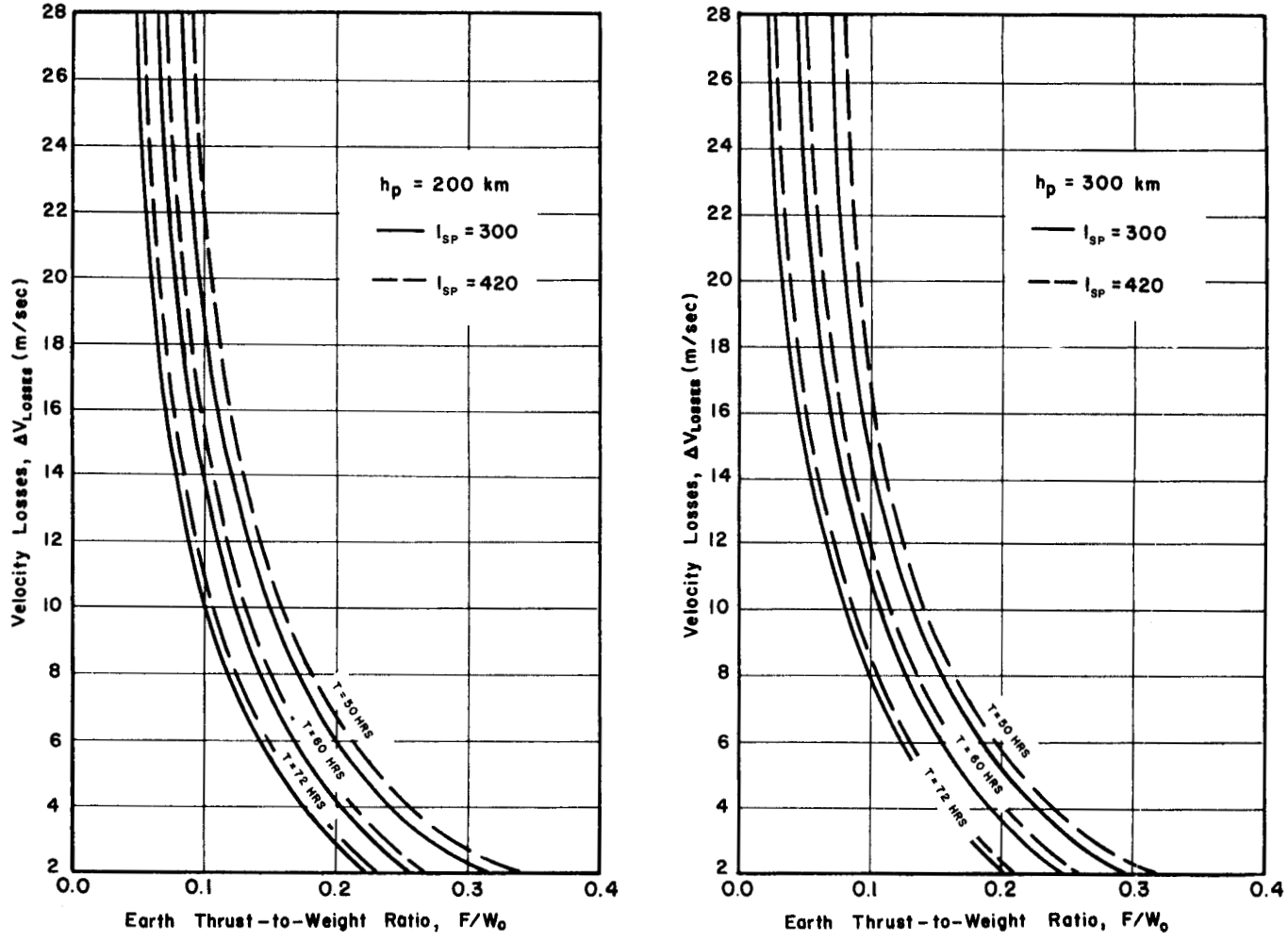


FIGURE 9b. VELOCITY LOSSES FOR BRAKE TO LUNAR ORBIT.

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PARAMETRIC PERFORMANCE ANALYSIS OF LUNAR MISSIONS

PART I

BRAKE TO LUNAR ORBIT

By Charles M. Akridge and Sam H. Harlin

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