N63-22708

NASA TECHNICAL NOTE

NASA TN D-1812

NASA TN D-1812

LIBRARY
National Aeronautics
Washington 25, D. C.

A STUDY OF SOME TRANSITION MATRIX ASSUMPTIONS IN CIRCUMLUNAR NAVIGATION THEORY

by Ruben L. Jones and Alton P. Mayo
Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1963

TECHNICAL NOTE D-1812

A STUDY OF SOME TRANSITION MATRIX ASSUMPTIONS IN CIRCUMLUNAR NAVIGATION THEORY

By Ruben L. Jones and Alton P. Mayo

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-1812

A STUDY OF SOME TRANSITION MATRIX ASSUMPTIONS IN CIRCUMLUNAR NAVIGATION THEORY

By Ruben L. Jones and Alton P. Mayo

SUMMARY

A study was made of the transition matrices utilized in midcourse navigation systems for circumlunar missions to compute future deviations in displacement and velocity resulting from present deviations from the reference trajectory. The method selected for computing the matrices involved the calculations of seven circumlunar trajectories of which the first was the reference or nominal trajectory and the remaining six were obtained by increasing the individual trajectory parameters by small increments. The differences between the parameters for the reference trajectory and the corresponding parameters for each of the remaining six trajectories were determined and divided by the increment added to a particular variable to obtain the elements of the columns of the transition matrices for that variable. The computations were performed on the IBM 7090 electronic data processing system.

The individual effects on the elements of the transition matrix of excluding various masses of the solar system from the trajectory computations and of increasing the disturbance in the insertion parameters and on the nominal partial derivatives of changing (shifting) the reference trajectory were studied. The results indicate that the effect of the planet masses is negligible. Further, it appears that a modified two-body approach to the small perturbation problem is applicable. The partial derivatives in the transition matrices may be accurately approximated as a function of the product of the disturbance or shift and of the corresponding time by a second-degree equation.

The analysis indicated that the small perturbation theory is applicable within an accuracy of four to five significant figures for perturbations or disturbances up to 2 statute miles. Furthermore, the partial derivatives were found to be, in general, nonlinear with respect to initial disturbances for shifts or changes in the reference trajectory.

INTRODUCTION

Circumlunar vehicles will be designed to fly a preplanned or nominal trajectory. Therefore, the onboard navigation system must determine the deviations in velocity and displacement at any time from the preplanned position and velocity, determine the future displacement due to the present displacement and velocity discrepancies, and determine the increment of velocity which must be added to

reach the desired transfer point into a lunar orbit and the desired conditions of transfer. Present navigation systems rely on transition matrices for projecting discrepancies at a present time to find those at a future time. (Refs. 1, 2, and 3 are examples.)

There are several proposed methods for computing the transition matrices. One method requires integration of the partial derivatives of the differential equations of motion at selected points along the reference trajectory to obtain the deviations in displacement and velocity with respect to the respective parameters of position and velocity (or the elements of the transition matrices). Another method determines the change at the time in question in the components of displacement and velocity per unit change in the individual components of the insertion position and velocity.

For the purposes of this study, the second method was applied, and various-sized "units" were used as initial displacements. This approach has two advantages: (1) it serves to show the range of displacement from the reference trajectory over which the transition matrices may be considered constant and (2) the results may be extrapolated to find the transition matrices which would have resulted from application of the first method in which the initial displacements are infinitesimal.

It is the purpose of this paper: (1) to show the variations of the transition matrices caused by varying the deviations in the insertion parameters; (2) to compare the transition matrices computed for one nominal trajectory with those computed for a closely related new trajectory (shifted trajectory); (3) to determine whether Earth oblateness and two-, three-, or four-body considerations are required for calculating transition matrices; and (4) to determine whether the transition matrices may be approximated by a function of the product of the disturbance or shift and the corresponding time of interest by a simple algebraic equation.

The fundamental assumption underlying the use of transition matrices requires the deviations in position and velocity at the time of insertion to be linearly related to the deviations at a later time. If the deviations are large, the assumption of linearity is violated.

Within the sphere of influence of the Moon the elements of the transition matrices will depart from linearity because of the large attraction of this body. However, a method utilizing a set of terminal coordinates which linearizes the matrices for a wide range of perturbation variables is described in reference 1. It is the purpose of this paper to analyze the linearity up to the Moon's sphere of influence (approximately 38,916 statute miles from the Moon's center).

SYMBOLS

In cases where distances are expressed in miles, the statute mile is intended. The following factors are included for use in converting English units to metric units: 1 statute mile = 1.6093440 kilometers, 1 foot = 0.3048 meter.

a_1, a_2, a_3, a_4, a_5 constants

R radial distance from Earth's center, statute miles

s product of t and Δx

t time elapsed from insertion, hr

t_o time at insertion, hr

V total velocity, miles/sec

w percent of nominal partial derivative

X, Y, Z inertial coordinate axes

x, y, z orthogonal components of Earth centered, inertial, rectangular coordinate system, statute miles (see fig. 1)

$\Delta x, \Delta y, \Delta z$ disturbance of reference-trajectory position in x-, y,- and z-direction, respectively, statute miles

$\delta x, \delta y, \delta z$ errors in displacement resulting from initial errors in position and velocity, statute miles

ϕ transition matrix

Subscripts:

o nominal- or reference-trajectory insertion point

t hours after insertion

Derivatives with respect to time are denoted by dots over the variable.

Primes denote changes of the reference-trajectory insertion position and instantaneous changes in the reference-trajectory insertion velocity.

A bar over a variable denotes a vector.

THEORY AND COMPUTATIONAL TECHNIQUE

To determine the elements of a column of the transition matrices, two trajectories were generated - a nominal or reference trajectory and one in which the initial value of the appropriate trajectory variable was changed by some given increment. Then the positions and velocities of the nominal trajectory at selected times after insertion were subtracted from those of the second trajectory at the corresponding times. The differences in position and velocity were normalized by dividing by the magnitude of the initial displacement or disturbance

in velocity. If the disturbance is small, the ratio of effect to cause (disturbance) approaches the partial derivative expressed as follows:

$$\frac{\partial \text{ Trajectory variable at time } t}{\partial \text{ Disturbance at insertion}}$$

For example, if the coordinate system is defined as being Earth centered, inertial, right handed, and rectangular with the X-axis pointed to Aries and positive in that direction, a disturbance in y_o of Δy_o at insertion causes an effect on x after t hours have elapsed which can be represented by

$$\Delta x_t = x_{t,\text{disturbed}} - x_{t,\text{nominal}}$$

Then, the partial derivative of x_t with respect to y_o is given by

$$\frac{\partial x_t}{\partial y_o} = \frac{x_{t,\text{disturbed}} - x_{t,\text{nominal}}}{\Delta y_o} \tag{1}$$

A disturbance Δx_o produces an effect in x_t, y_t, z_t, $\dot{x}_t$, $\dot{y}_t$, and $\dot{z}_t$; likewise, a disturbance in x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$, respectively, produces a corresponding effect in x_t, as well as in each of the remaining parameters. Thus, if the partial derivatives are linear,

$$\delta x_t = \frac{\partial x_t}{\partial x_o}\delta x_o + \frac{\partial x_t}{\partial y_o}\delta y_o + \frac{\partial x_t}{\partial z_o}\delta z_o + \frac{\partial x_t}{\partial \dot{x}_o}\delta \dot{x}_o + \frac{\partial x_t}{\partial \dot{y}_o}\delta \dot{y}_o + \frac{\partial x_t}{\partial \dot{z}_o}\delta \dot{z}_o \tag{2}$$

represents the change in x, t hours from insertion, as a result of initial errors in the parameters x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$. In like manner, the deviations δy_t, δz_t, $\delta \dot{x}_t$, $\delta \dot{y}_t$, and $\delta \dot{z}_t$ are obtained. In matrix notation

$$\begin{Bmatrix} \delta x_t \\ \delta y_t \\ \delta z_t \\ \delta \dot{x}_t \\ \delta \dot{y}_t \\ \delta \dot{z}_t \end{Bmatrix}_t = \phi(t_o,t) \begin{Bmatrix} \delta x_o \\ \delta y_o \\ \delta z_o \\ \delta \dot{x}_o \\ \delta \dot{y}_o \\ \delta \dot{z}_o \end{Bmatrix}_{t_o} \tag{3}$$

where $\phi(t_o,t)$, the transition matrix, is the square matrix of 36 partial derivatives obtained by perturbing each of the 6 parameters x_o, y_o, z_o, $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$ in turn and evaluating the elements, as described, for the interval contained between the times t_o and t, and

$$\begin{Bmatrix} \delta x_o \\ \delta y_o \\ \delta z_o \\ \delta \dot{x}_o \\ \delta \dot{y}_o \\ \delta \dot{z}_o \end{Bmatrix}_{t_o} \quad \text{and} \quad \begin{Bmatrix} \delta x_t \\ \delta y_t \\ \delta z_t \\ \delta \dot{x}_t \\ \delta \dot{y}_t \\ \delta \dot{z}_t \end{Bmatrix}_t$$

are, respectively, the column matrices of initial deviations and of the deviations resulting after t hours have elapsed. Thus, a square (6 by 6) matrix, called a transition matrix, is generated for each time point of the reference trajectory.

Present proposals for the midcourse guidance of the Apollo vehicle assume that the errors in position and velocity will have been all but eliminated prior to entering the Moon's sphere of influence. Up to the Moon's sphere of influence the transition matrices are considered to be linear with respect to the disturbances over a sufficiently wide range of variables and, as a consequence, are assumed to be used exclusively over this region. In the vicinity of the Moon the transition matrices depart from linearity with large disturbances because of the large attraction of that body. As stated previously, in the method described in reference 1 a set of terminal coordinates is so defined that the transition matrices in the vicinity of the Moon are linear over a wide range of variables. However, in any case the vehicle will be in its terminal phase of guidance in this region and any necessary corrections will normally be small.

In the appendix, the computational technique is described. Trajectory 1 is the nominal or reference trajectory and the remaining six are the disturbed trajectories obtained by altering the initial conditions of the nominal trajectory.

The procedure followed in this paper utilizes a trajectory-generation program based upon Encke's perturbation techniques. Six bodies, that is, the oblate Earth, Moon, Sun, Mars, Jupiter, and Venus, are included in the trajectory program. (See ref. 4.) The trajectory is computed in an Earth fixed, rectangular, inertial coordinate system.

To generate the nominal set of transition matrices, initial deviations were selected with $\Delta x_o = \Delta y_o = \Delta z_o = 1$ statute mile and $\Delta \dot{x}_o = \Delta \dot{y}_o = \Delta \dot{z}_o = 10$ ft/sec and the calculations were performed. In table I, the seven resulting trajectories are shown. In table II, the nominal transition matrices are shown. In tables III, IV, V, VI, VII, and VIII, the transition matrices are shown for the initial deviations in position of $\Delta x = \Delta y = \Delta z = 2, 6, 10, 25, 50$, and 150 statute miles and in velocity of $\Delta \dot{x} = \Delta \dot{y} = \Delta \dot{z} = 20, 40, 50, 75, 150$, and 300 ft/sec, respectively. The results in the tables are shown for every 8 hours from insertion.

In table I, the x, y, and z coordinates of the vehicle and its radial distance r from the Earth's center are in statute miles. The $\dot{x}$, $\dot{y}$, and $\dot{z}$ components of vehicle velocity are in statute miles/sec for each of the seven trajectories every 8 hours beginning with $t = 0$. In tables II to VIII, the partial derivatives of the trajectory parameters with respect to x_o are tabulated in the x column; the partial derivatives of the trajectory parameters with respect to y_o are in the y column, and so forth.

The results shown are for a fairly low energy lunar trajectory (nominal reference trajectory) such as would be expected for lunar orbit or landing. The vehicle was assumed to be inserted into the translunar orbit at approximately 10 p.m., March 18, 1968, and to enter the Moon's sphere of influence 56 hours later (ref. 5). (See fig. 1.) To simplify the analysis, only that portion of the trajectory up to the Moon's sphere of influence, approximately 38,916 statute

miles from the Moon's center, was considered. The orientation of the Moon and Sun will be described in a subsequent section.

RESULTS AND DISCUSSION

Thirty-six partial derivatives were evaluated for each time, as in equation (1), for disturbances of the trajectory and for changes of the reference trajectory and were analyzed. However, discussion of the partial derivatives $\partial x_t/\partial x_o$ and $\partial \dot{x}_t/\partial \dot{x}_o$ is considered sufficient to establish the characteristic nature of the transition matrix. The remaining partial derivatives in the transition matrix vary, in general, as $\partial x_t/\partial x_o$ although the individual rates of change of the various partial derivatives vary from one to the other. (Hereinafter the term partial derivative denotes $\partial x_t/\partial x_o$ or $\partial \dot{x}_t/\partial \dot{x}_o$.)

The results discussed in this study are plotted in figures 2 to 12. Tables II to X are furnished so that the results may be studied in greater detail if desired. Tables IX and X contain the $\partial x_t/\partial x_o$ and $\partial \dot{x}_t/\partial \dot{x}_o$ for both the disturbances and the trajectory shifts in terms of percents of the corresponding nominal partial derivatives.

The Nominal Partial Derivatives for the Reference Trajectory

In figure 2 the $\partial x_t/\partial x_o$ is seen to be practically linear with both time and radial distance. The slopes vary from approximately 2.5 to about 2.1 for the time curve and from approximately 0.44 to about 1.0 for the radial-distance curve. However, the curves of figure 3 start at small values, increase rapidly to large values and then slowly decrease until the attraction of the Moon in the x-direction begins to approach that of the Earth, at which point the partial derivative begins to increase again. (See table I and the appendix.) The rapid increase of the partial derivative initially is attributed to the large attraction of the Earth in the positive x-direction which caused the velocity to change from a negative quantity to a positive quantity at approximately 2 hours after insertion.

Limits of Transition Matrix Assumptions

The effect on the displacement partial derivative of increasing the disturbance is shown in figure 4. As Δx_o increases, the partial derivatives depart from their nominal values, the rate of departure increasing with time. (For example, the partial derivative based on $\Delta x_o = 50$ statute miles is 99.2029 percent of the nominal value after the lapse of 8 hours; this partial derivative decreases to 98.3750 percent of the nominal value after a lapse of 16 hours.)

The displacement partial derivatives may be considered constant within certain limits, depending on the degree of accuracy required. For instance, if $\partial x_t/\partial x_o$ based on a 1-mile initial displacement were used to calculate the x-deviation due to a 2-mile actual initial displacement, the error 8 hours after insertion would be in the sixth significant figure. Forty hours after insertion the error would be in the fifth significant figure.

In figure 5 is shown the effect upon the velocity partial derivative of increasing the disturbance in velocity. The fact that the velocity partial derivatives are much more critical than the displacement partial derivatives is evident.

If the approximate magnitudes of the partial derivatives $\partial x_t/\partial x_o$ and $\partial \dot{x}_t/\partial \dot{x}_o$ at zero are desired, they may be obtained by extrapolating the curves of figures 4 and 5 to the ordinate axis.

Comparison of Transition Matrices of Neighboring Trajectories

In order to determine the effects of using the nominal transition matrices to compute deviations from a neighboring trajectory, the trajectory studied in this paper was displaced by increasing the x, y, and z coordinates of the insertion point by increments of $\Delta x_o' = \Delta y_o' = \Delta z_o' = 5$, 10, 25, and 75 statute miles. Under such circumstances, the insertion point will for each of these displacements be shifted in the same direction for a total displacement of $\Delta x_o'\sqrt{3}$ miles. Transition matrices were then computed for the new trajectory as previously described with increments of 1 mile and 10 ft/sec in the initial parameters of the trajectory.

In figure 6 the displacement partial derivatives, expressed as percents of the nominal partial derivatives, are plotted as a function of the shift in the reference trajectory in terms of $\Delta x_o'$. Nearer insertion, the percentages decreased in magnitude. Further increase in time served to increase the magnitude of the partial derivatives. (See fig. 6.)

In figure 7 the velocity partial derivatives, expressed as percents of the nominal partial derivatives, are plotted as a function of the same shifts in the trajectory as for figure 6. These curves are seen to increase smoothly at all times. However, the shifts have a greater effect on the velocity partial derivatives than on the displacement partial derivatives.

Figures 8 and 9 show, respectively, the displacement and velocity partial derivatives, expressed as percents of the nominal partial derivatives, plotted as a function of the instantaneous changes in velocity. The changes in velocity were derived in the same manner as were the shifts in figures 6 and 7. That is, the magnitude of the vector increase in velocity is equal to $\Delta \dot{x}_o'\sqrt{3}$ where $\Delta \dot{x}_o' = \Delta \dot{y}_o' = \Delta \dot{z}_o' = 10$, 20, and 40 ft/sec and is always in the same direction. The abscissa of figures 8 and 9 is in terms of $\Delta \dot{x}_o'$.

The curves of figures 8 and 9 appear to be essentially straight lines. However, a close examination of table X(b) will show some curvature.

A comparison of figures 6 and 7 with figures 8 and 9 indicates that changes in the vehicle velocity affect the partial derivatives much more radically than do shifts of the reference trajectory. In figures 6 to 9 the nominal partial derivatives are seen to vary nonlinearly with the trajectory shift and change in

initial trajectory velocity. However, the ratios of the partial derivatives based on the new reference trajectory to the corresponding nominal partial derivatives are believed to be essentially the same as those plotted in figures 4 and 5. Therefore, the results in figures 4 and 5 are considered to be general.

The Effects of Small Perturbations on the Vehicle Trajectory

Figures 10 and 11 show the effect of the various perturbing forces of the solar system on displacement and velocity, respectively, of a circumlunar trajectory. The effects of the perturbing forces are shown as absolute magnitudes of the vector differences in displacement and velocity. Of the factors affecting the vehicle, shown in figure 10, the Earth's oblateness is most important. One interesting result shown in figure 10 is that, if the mass of the Sun is omitted, a displacement error of less than 100 miles results after 56 hours of flight.

At the time of insertion the Sun lies approximately on the positive X-axis and the Moon is in a plane approximately 38.253° from the negative Y-axis measured clockwise from it. The vehicle is located in the second quadrant 21.3116° counterclockwise from the positive Y-axis. After 56 hours the vehicle has advanced to within approximately 38,916 miles of the Moon; the Moon is still located in the third quadrant of the coordinate system and is approximately 2.0649° clockwise from the negative Y-axis. The vehicle is in the fourth quadrant and 7.3132° counterclockwise from the negative Y-axis. Therefore, the individual effects of the Sun and Moon should oppose each other slightly.

The most striking characteristic of figure 11 is the fact that the data which resulted from omitting the Earth's oblateness effect (curve 1) and from omitting the oblateness effect along with other masses (curve 4) indicate that $|\overline{\Delta V}|$ increases rapidly (to approximately 0.0043 mile/sec and 0.004 mile/sec, respectively) 2 hours after insertion. This is explainable. Near the Earth the attraction of the equatorial bulge is greater than the single or aggregate attraction of Moon, Sun, and planet masses. If the effect of the equatorial bulge is neglected, the Earth may be considered a sphere of equal mass; the equatorial bulge, therefore, exerts a larger or smaller force than the oblate ellipsoid in accordance with the proximity to the equatorial plane or the polar axis, respectively. Thus, the velocity will be affected by a larger acceleration, for this trajectory, due to the changed potential field plus the effect due to the change in the field that results from a change in position. Finally, the results show that the effect of the planets on the lunar trajectory is negligible for both velocity and displacement.

Effects of One-, Two-, Three-, and Four-Body Considerations on the Transition Matrix

The effects discussed in the preceding section were found to be small for the Sun and planets. Furthermore, the Earth's oblateness was found to be most critical initially. If the combined effects of the Sun's and Moon's masses and the Earth's oblateness can be omitted from the transition matrix computations

without affecting the accuracy greatly, the problem of computing them can be reduced to a series of two-body problems.

In figures 12(a) and 12(b), the displacement and velocity partial derivatives resulting from omitting various perturbing forces, expressed as percents of the nominal partial derivatives, are plotted against time. For all cases investigated the contribution to the partial derivatives by the masses of the planets was found to be negligible. The Sun's mass was found to contribute less than 0.1 percent to the displacement partial derivative and approximately 0.1 percent to the velocity partial derivative after 56 hours. Thus, for this trajectory, it appears that the mass of the Sun as well as the masses of the planets Mars, Venus, and Jupiter might be omitted from the computational scheme (for computing the partial derivatives of a translunar trajectory) without greatly affecting the results.

Up to 14 hours after insertion the effect on the position partial derivative of omitting the Earth's oblateness was practically negligible. Thereafter the effect became greater and the position partial derivative increased smoothly to approximately 100.6 percent of the nominal partial derivative after 56 hours. The velocity partial derivative increased smoothly to about 103.8 percent of the nominal value. (See fig. 12.)

If the Moon's mass is omitted from the computations, the effect on the x component of position and velocity is practically negligible up to 16 hours after insertion. After 16 hours the position partial derivative based on no Moon mass begins to decrease gradually until it is approximately 95.9 percent of nominal partial derivative after 56 hours. The velocity partial derivative decreases rapidly to approximately 82.8 percent of the nominal partial derivative after 56 hours from insertion.

Although the effect upon the partial derivatives of omitting individually the Sun's mass, the Moon's mass, and the Earth's oblateness can be significant, the combined effect of omitting all three will be quite tolerable for this trajectory up to approximately the Moon's sphere of influence (about 38,916 statute miles from the Moon's center) because of the opposing effects of the Moon's mass and the Earth's oblateness. Up to 40 hours after insertion, the maximum discrepancy introduced into the position partial derivatives by omitting the total effect was approximately 0.1 percent of the nominal value. The velocity partial derivatives deviate from the nominal value by a maximum of about 0.2 percent over a 27-hour interval. Thereafter the velocity partial derivatives decrease very rapidly.

The results discussed in this section and in the preceding section will vary with changes in the insertion parameters and orientations of the Sun and Moon. However, the orientation of the Sun and Moon is not expected to change the results greatly.

An Algebraic Approximation to the Partial Derivatives

The partial derivatives represented by the curves of figures 4 to 9 are not, in reality, strictly linear with respect to disturbance in, or shifts of, the

reference trajectory. Neither are the partial derivatives linear with respect to time. However, the curves in each figure do vary smoothly and systematically with time and shift of reference trajectory or disturbance.

From the curves in figures 4 to 9 and tables IX and X, the following general equation is assumed to represent the partial derivatives:

$$a_1 s^2 + 2a_2 ws + a_3 w^2 + 2a_4 s + 2a_5 w + 1 = 0 \tag{4}$$

where

a_1, a_2, a_3, a_4, and a_5 constants

s product of disturbance or shift and time from insertion

w percent of nominal partial derivative

By the method of least squares, empirical values for a_1, a_2, a_3, a_4, and a_5 can be computed from at least five percentages of each partial derivative. Constants are computed for the disturbances with time maintained constant, and for shifts with the deviations maintained constant.

For test cases in which six different values of w were processed, in general, an accuracy up to five significant figures was obtained for interpolated disturbances and times, whereas in some cases agreement to six significant figures was obtained for the percentages employed in the least squares fit. The scatter of errors in the predicted percents was found to be generally symmetric about zero. Thus, the approximation is considered good.

Only 85 percent of the 72 partial derivatives (36 for disturbances and 36 for shifts) were analyzed. However, the approximating polynominal was found to be, in general, applicable with an accuracy of four to five significant figures for disturbances up to 50 miles and shifts up to 10 or 15 miles. An increase in data points should improve the approximation.

Thus, if it became necessary to change partial derivatives during a translunar flight, it could be done with a simple analytical expression and the appropriate constants. The constant of the analytical expressions for the various partial derivatives of the particular reference trajectory could be tabulated and stored in a computer along with the corresponding precomputed nominal partial derivatives.

CONCLUDING REMARKS

A study was made of the transition matrices utilized in midcourse navigation systems for circumlunar missions to compute future deviations in displacement and velocity resulting from present deviations from the reference trajectory. The results have shown that:

1. The elements of the transition matrices are effectively linear within certain limits, depending upon the degree of accuracy required. For initial disturbances of 2 statute miles, accuracy of four significant figures is obtainable for a period of 40 hours after insertion, and accuracy of five significant figures is obtainable in the first 8 hours.

2. The partial derivatives were found to be, in general, nonlinear with respect to initial disturbances for shifts or changes in the reference trajectory.

3. Since the curves produced by varying the disturbances and by shifting the reference trajectory are smooth and change systematically with both time and displacement, it is possible to represent these data as functions of the product of the disturbance or shift and the time from insertion by analytical expressions. Constants of the analytical expressions for the various elements of the transition matrices of a particular reference trajectory may be tabulated and stored in a computer along with the corresponding precomputed nominal partial derivatives and reference trajectory. Thus, if it became necessary to change references and/or partial derivatives during a translunar flight, it could be done with a simple analytical expression and the appropriate constants.

4. The effect of the masses of Mars, Venus, Jupiter, and the Sun on the partial derivatives is negligible and may for all practical purposes be eliminated, for this trajectory, from the computational scheme.

5. The combined effects of the Sun's and Moon's mass and of the Earth's oblateness could be omitted from the partial derivative computation scheme up to 40 hours after insertion with a reduction in accuracy of no more than 0.05 percent. Hence, the problem of computing transition matrices may be reduced to a series of two-body problems for the aforestated accuracy.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., June 26, 1963.

APPENDIX

COMPUTATIONAL PROCEDURE

Over each given interval seven trajectories are generated by the Encke method (ref. 4) with the following initial conditions:

Trajectory	Time	Components					
		Displacement			Velocity		
1	t_o	x_o	y_o	z_o	$\dot{x}_o$	$\dot{y}_o$	$\dot{z}_o$
2	t_o	$x_o + \Delta x_o$	y_o	z_o	$\dot{x}_o$	$\dot{y}_o$	$\dot{z}_o$
3	t_o	x_o	$y_o + \Delta y_o$	z_o	$\dot{x}_o$	$\dot{y}_o$	$\dot{z}_o$
4	t_o	x_o	y_o	$z_o + \Delta z_o$	$\dot{x}_o$	$\dot{y}_o$	$\dot{z}_o$
5	t_o	x_o	y_o	z_o	$\dot{x}_o + \Delta\dot{x}_o$	$\dot{y}_o$	$\dot{z}_o$
6	t_o	x_o	y_o	z_o	$\dot{x}_o$	$\dot{y}_o + \Delta\dot{y}_o$	$\dot{z}_o$
7	t_o	x_o	y_o	z_o	$\dot{x}_o$	$\dot{y}_o$	$\dot{z}_o + \Delta\dot{z}_o$

The following six partial derivatives can be generated by using trajectories 1 and 2 with Δx_o:

$$\frac{\partial x}{\partial x_o} = \frac{x_2 - x_1}{\Delta x_o} \qquad \frac{\partial \dot{x}}{\partial x_o} = \frac{\dot{x}_2 - \dot{x}_1}{\Delta x_o}$$

$$\frac{\partial y}{\partial x_o} = \frac{y_2 - y_1}{\Delta x_o} \qquad \frac{\partial \dot{y}}{\partial x_o} = \frac{\dot{y}_2 - \dot{y}_1}{\Delta x_o}$$

$$\frac{\partial z}{\partial x_o} = \frac{z_2 - z_1}{\Delta x_o} \qquad \frac{\partial \dot{z}}{\partial x_o} = \frac{\dot{z}_2 - \dot{z}_1}{\Delta x_o}$$

Numerical subscripts refer to trajectory numbers.

The other partial derivatives are generated by a similar process with trajectories and disturbances used in the computations as indicated in the following table:

Trajectories	Disturbance	Six partial derivatives generated with respect to -
1 and 3	Δy_o	y_o
1 and 4	Δz_o	z_o
1 and 5	$\Delta \dot{x}_o$	$\dot{x}_o$
1 and 6	$\Delta \dot{y}_o$	$\dot{y}_o$
1 and 7	$\Delta \dot{z}_o$	$\dot{z}_o$

For example, six partial derivatives with respect to y_o are generated by using trajectories 1 and 3 with Δy_o.

The computations were performed on the IBM 7090 electronic data processing system.

REFERENCES

1. Noton, A. R. M., Cutting, E., and Barnes, F. L.: Analysis of Radio-Command Mid-Course Guidance. Tech. Rep. No. 32-28 (Contract No. NASw-6), Jet Propulsion Lab., C.I.T., Sept. 8, 1960.

2. McLean, John D., Schmidt, Stanley F., and McGee, Leonard A.: Optimal Filtering and Linear Prediction Applied to a Midcourse Navigation System for the Circumlunar Mission. NASA TN D-1208, 1962.

3. Battin, Richard H.: A Statistical Optimizing Navigation Procedure for Space Flight. R-341 (Contracts NAS-9-103 and NAS-9-153), Instrumentation Lab., M.I.T., Sept. 1961.

4. Pines, Samuel, and Wolf, Henry: Interplanetary Trajectory by the Encke Method Programmed for the IBM 704. Rep. No. RAC-656-450 (Contract NASw-109 (NASA)), Republic Aviation Corp., Dec. 15, 1959.

5. Gapcynski, John P., and Woolston, Donald S.: Characteristics of Three Precision Circumlunar Trajectories for the Year 1968. NASA TN D-1028, 1962.

TABLE I.- [illegible] USED TO COMPUTE THE NOMINAL TRAJECTORY

Trajectory parameter	Nominal trajectory	Trajectories derived from nominal trajectory with deviations of -					
		Δx_0 = 1 mile	Δy_0 = 1 mile	Δz_0 = 1 mile	$\Delta \dot{x}_0$ = 10 ft/sec	$\Delta \dot{y}_0$ = 10 ft/sec	$\Delta \dot{z}_0$ = 10 ft/sec
				t = 0			
x	-0.13078878×10^4	-0.13088879×10^4	-0.13078878×10^4	-0.13078878×10^4	-0.13078878×10^4	-0.13078878×10^4	-0.13078878×10^4
y	0.33525649×10^4	0.33525649×10^4	0.33535649×10^4	0.33525649×10^4	0.33525649×10^4	0.33525649×10^4	0.33525649×10^4
z	0.17863838×10^4	0.17863838×10^4	0.17863838×10^4	0.17873839×10^4	0.17863838×10^4	0.17863838×10^4	0.17863838×10^4
r	0.40176368×10^4	0.40175145×10^4	0.40184742×10^4	0.40180845×10^4	0.40176368×10^4	0.40176368×10^4	0.40176368×10^4
$\dot{x}$	-0.65367520×10	-0.65367520×10	-0.65367520×10	-0.65367381×10	-0.65367520×10	-0.65367520×10	-0.65367520×10
$\dot{y}$	-0.16348018×10	-0.16348018×10	-0.16348018×10	-0.16348018×10	-0.16348018×10	-0.16348018×10	-0.16348018×10
$\dot{z}$	-0.17603715×10	-0.17603715×10	-0.17603715×10	-0.17603715×10	-0.17603715×10	-0.17603715×10	-0.17603715×10
				t = 8			
x	-0.11391691×10^5	-0.11355229×10^5	-0.11443077×10^5	-0.11417016×10^5	-0.11320740×10^5	-0.11328144×10^5	-0.11384340×10^5
y	-0.54653896×10^5	-0.54641651×10^5	-0.54699908×10^5	-0.54668774×10^5	-0.54556791×10^5	-0.54621349×10^5	-0.54439924×10^5
z	-0.30910217×10^5	-0.30900682×10^5	-0.30938496×10^5	-0.30933968×10^5	-0.30849169×10^5	-0.30897304×10^5	-0.30891148×10^5
r	0.63814299×10^5	0.63792659×10^5	0.63868288×10^5	0.63843197×10^5	0.63676475×10^5	0.63788356×10^5	0.63788404×10^5
$\dot{x}$	0.15989187	0.16097590	0.15814321	0.15899860	0.16467863	0.16036466	0.16037787
$\dot{y}$	-0.13437574×10	-0.13428798×10	-0.13460608×10	-0.13447820×10	-0.13381924×10	-0.13437341×10	-0.13437341×10
$\dot{z}$	-0.73590138	-0.73533536	-0.73706479	-0.73689441	-0.73256057	-0.73527498	-0.73527341
				t = 16			
x	-0.58483604×10^4	-0.57837664×10^4	-0.59469311×10^4	-0.58984079×10^4	-0.57790885×10^4	-0.58317035×10^4	-0.58511168×10^4
y	-0.87347006×10^5	-0.87299082×10^5	-0.87462841×10^5	-0.87393916×10^5	-0.87051660×10^5	-0.87263740×10^5	$-0.87298[illegible] \times 10^5$
z	-0.48720164×10^5	-0.48691586×10^5	-0.48779560×10^5	-0.48775026×10^5	-0.48545107×10^5	-0.48681698×10^5	-0.48677897×10^5
r	0.10018765×10^6	0.10012718×10^6	0.10035228×10^6	0.10029712×10^6	0.99828876×10^5	0.10009433×10^6	0.10011461×10^6
$\dot{x}$	0.20923231	0.21012284	0.20770065	0.20844440	0.21338441	0.20970349	0.20971619
$\dot{y}$	-0.97958363	-0.97835930	-0.98269775	-0.98109358	-0.97188489	-0.97773348	-0.97781620
$\dot{z}$	-0.53131486	-0.53057400	-0.53296786	-0.53247461	-0.52683372	-0.53018796	-0.53004804
				t = 24			
x	0.28598773×10^3	0.37429852×10^3	0.14603246×10^3	0.21462245×10^3	0.46713873×10^3	0.31097627×10^3	$0.3[illegible] \times 10^3$
y	-0.11267971×10^6	-0.11259783×10^6	-0.11289923×10^6	-0.11278039×10^6	-0.11214476×10^6	-0.11254752×10^6	-0.11138111×10^6
z	-0.62421531×10^5	-0.62369685×10^5	-0.62533810×10^5	-0.62511855×10^5	-0.62104618×10^5	-0.62350223×10^5	-0.62343456×10^5
r	0.12881477×10^6	0.12871826×10^6	0.12906097×10^6	0.12894644×10^6	0.12819423×10^6	0.12866026×10^6	$0.12869[illegible] \times 10^6$
$\dot{x}$	0.21404207	0.21480484	0.21269537	0.21334772	0.21767351	0.21449140	0.21448705
$\dot{y}$	-0.79339820	-0.79309639	-0.79911780	-0.79725961	-0.78617746	-0.79316872	[illegible]
$\dot{z}$	-0.42885256	-0.42798329	-0.43085846	-0.43015184	-0.42352054	-0.42760489	[illegible]
				t = 32			
x	0.65921165×10^4	0.65009162×10^4	0.62157370×10^4	0.63019659×10^4	0.68712827×10^4	0.64344878×10^4	$0.64494[illegible] \times 10^4$
y	-0.13574889×10^6	-0.13562152×10^6	-0.13408321×10^6	-0.13390753×10^6	$-0.13[illegible] \times 10^6$	$-0.13554[illegible] \times 10^6$	-0.13397454×10^6
z	-0.73794189×10^5	-0.73675709×10^5	-0.73928621×10^5	-0.73883768×10^5	$-0.73473[illegible] \times 10^5$	-0.73644749×10^5	$-0.73[illegible] \times 10^5$
r	0.15287022×10^6	0.15275551×10^6	0.15323963×10^6	0.15306780×10^6	0.15193997×10^6	0.15253552×10^6	$0.15[illegible] \times 10^6$
$\dot{x}$	0.20910167	0.20976562	0.20791455	0.20848935	0.21228930	0.20951178	0.20950[illegible]
$\dot{y}$	-0.67478889	-0.67310511	-0.67903336	-0.67694989	-0.66415040	-0.67[illegible]2804	[illegible]
$\dot{z}$	-0.36207672	-0.36109836	-0.36438130	-0.36350209	-0.35602[illegible]	-0.36064461	[illegible]
				t = 40			
x	0.12293221×10^5	0.12419995×10^5	0.12084564×10^5	0.12180474×10^5	0.12868834×10^5	0.12346829×10^5	$0.12[illegible] \times 10^5$
y	-0.15189167×10^6	-0.15167293×10^6	-0.15231536×10^6	-0.15207657×10^6	-0.15070482×10^6	-0.15196724×10^6	$-0.15[illegible] \times 10^6$
z	-0.83446482×10^5	-0.83338347×10^5	-0.83692278×10^5	-0.83618853×10^5	-0.82781296×10^5	-0.83292798×10^5	$-0.83[illegible] \times 10^5$
r	0.17370483×10^6	0.17350566×10^6	0.17421319×10^6	0.17397574×10^6	0.17242399×10^6	0.17338616×10^6	$0.17[illegible] \times 10^6$
$\dot{x}$	0.20018366	0.20077235	0.19912144	0.19963603	0.20301936	0.20055679	0.20054[illegible]
$\dot{y}$	-0.58641376	-0.58453063	-0.59215096	-0.58885308	-0.57451146	-0.58354523	-0.58[illegible]
$\dot{z}$	-0.31325181	-0.31217131	-0.31583148	-0.31479772	-0.30652500	-0.31164847	-0.31[illegible]
				t = 48			
x	0.17991782×10^5	0.18034861×10^5	0.17853453×10^5	0.17769693×10^5	0.18336160×10^5	0.17955983×10^5	0.17941714×10^5
y	-0.16771732×10^6	-0.16748141×10^6	-0.16834456×10^6	-0.16801646×10^6	-0.16620923×10^6	-0.16734587×10^6	$-0.16[illegible] \times 10^6$
z	-0.91903709×10^5	-0.91761982×10^5	-0.92225701×10^5	-0.92121331×10^5	-0.91034207×10^5	-0.91700468×10^5	$-0.91[illegible] \times 10^5$
r	0.19208155×10^6	0.19182164×10^6	0.19274434×10^6	0.19243605×10^6	0.19041093×10^6	0.19166647×10^6	$0.19[illegible] \times 10^6$
$\dot{x}$	0.18796626	0.18852006	0.18694604	0.18744000	0.19065537	0.18834008	0.18832118
$\dot{y}$	-0.51820227	-0.51611799	-0.52345689	-0.52092227	-0.50500400	-0.51502720	-0.51[illegible]
$\dot{z}$	-0.27555440	-0.27450[illegible]	-0.27838648	-0.27720106	-0.26813796	-0.27370292	-0.27[illegible]
				t = 56			
x	0.23065699×10^5	0.23023374×10^5	0.22793367×10^5	0.22925106×10^5	0.23790859×10^5	0.23140325×10^5	0.23145095×10^5
y	-0.18189202×10^6	-0.18155247×10^6	-0.18261823×10^6	-0.18223406×10^6	-0.17994271×10^6	-0.18138441×10^6	$-0.18149[illegible] \times 10^6$
z	-0.99399105×10^5	-0.99222595×10^5	-0.99808489×10^5	-0.99667566×10^5	-0.98307046×10^5	-0.99143463×10^5	-0.99186594×10^5
r	0.20852403×10^6	0.20819643×10^6	0.20935781×10^6	0.20896994×10^6	0.20642106×10^6	0.20800290×10^6	0.20812182×10^6
$\dot{x}$	0.16958784	0.17024413	0.16828100	0.16891309	0.17288618	0.17013273	0.17007391
$\dot{y}$	-0.46610854	-0.46376041	-0.47104053	-0.46916126	-0.45136111	-0.46059300	-0.46[illegible]
$\dot{z}$	-0.24649519	-0.24518813	-0.24966432	-0.24829246	-0.23833701	-0.24455205	-0.24499118

TABLE II.- NOMINAL TRANSITION MATRICES

Trajectory parameter						Trajectory parameter
x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	
			t = 0			
0.09999999×10	0	0	0	0	0	x
0	0.09999999×10	0	0	0	0	y
0	0	0.09999999×10	0	0	0	z
0	0	0	0.99999905	0	0	$\dot{x}$
0	0	0	0	0.99999118	0	$\dot{y}$
0	0	0	0	0	0.99997544	$\dot{z}$
			t = 8			
0.36421753×10^{2}	$-0.51426147 \times 10^{2}$	$-0.25865722 \times 10^{2}$	0.74400949×10^{5}	0.18514805×10^{4}	0.48078164×10^{4}	x
0.12245605×10^{2}	$-0.42011718 \times 10^{2}$	$-0.14877441 \times 10^{2}$	0.51271687×10^{5}	0.16670413×10^{5}	0.94893046×10^{4}	y
0.95358886×10	$-0.18279053 \times 10^{2}$	$-0.23750732 \times 10^{2}$	0.32235585×10^{5}	0.68184961×10^{4}	0.10068609×10^{5}	z
$0.10840315 \times 10^{-2}$	$-0.17486587 \times 10^{-2}$	$-0.89327060 \times 10^{-3}$	0.25274100×10	0.22058791	0.25644743	$\dot{x}$
$0.87766349 \times 10^{-3}$	$-0.23033470 \times 10^{-2}$	$-0.10244846 \times 10^{-2}$	0.29383843×10	0.71983409	0.52977920	$\dot{y}$
$0.56602061 \times 10^{-3}$	$-0.11634156 \times 10^{-2}$	$-0.99303573 \times 10^{-3}$	0.17639479×10	0.38300908	0.35796976	$\dot{z}$
			t = 16			
0.64593994×10^{2}	$-0.98570678 \times 10^{2}$	$-0.50047485 \times 10^{2}$	0.14217560×10^{6}	0.87948867×10^{4}	0.12275806×10^{5}	x
0.42924804×10^{2}	$-0.12083495 \times 10^{3}$	$-0.51910156 \times 10^{2}$	0.15330304×10^{6}	0.41324765×10^{5}	0.28112905×10^{5}	y
0.28578125×10^{2}	$-0.59395996 \times 10^{2}$	$-0.54862304 \times 10^{2}$	0.92430163×10^{5}	0.20309695×10^{5}	0.21788508×10^{5}	z
$0.89053623 \times 10^{-3}$	$-0.15316550 \times 10^{-2}$	$-0.78790262 \times 10^{-3}$	0.21923140×10	0.24874780	0.25548953	$\dot{x}$
$0.12243316 \times 10^{-2}$	$-0.31141192 \times 10^{-2}$	$-0.15099496 \times 10^{-2}$	0.40649331×10	0.97740662	0.74808741	$\dot{y}$
$0.74225664 \times 10^{-3}$	$-0.16535968 \times 10^{-2}$	$-0.11603534 \times 10^{-2}$	0.23710047×10	0.54209626	0.45205700	$\dot{z}$
			t = 24			
0.88310783×10^{2}	$-0.13995527 \times 10^{3}$	$-0.71365283 \times 10^{2}$	0.20124745×10^{6}	0.15833925×10^{5}	0.19355577×10^{5}	x
0.81874999×10^{2}	$-0.21951953 \times 10^{3}$	$-0.10063574 \times 10^{3}$	0.28272285×10^{6}	0.72436547×10^{5}	0.52060593×10^{5}	y
0.51846679×10^{2}	$-0.11227880 \times 10^{3}$	$-0.90323730 \times 10^{2}$	0.16733011×10^{6}	0.37650679×10^{5}	0.35946796×10^{5}	z
$0.76276809 \times 10^{-3}$	$-0.13467036 \times 10^{-2}$	$-0.69435500 \times 10^{-3}$	0.19173988×10	0.23724899	0.23494962	$\dot{x}$
$0.14718100 \times 10^{-2}$	$-0.37196055 \times 10^{-2}$	$-0.18614158 \times 10^{-2}$	0.48949481×10	0.11771664×10	0.90945232	$\dot{y}$
$0.86926669 \times 10^{-3}$	$-0.20059086 \times 10^{-2}$	$-0.12992844 \times 10^{-2}$	0.28153080×10	0.65802848	0.52927451	$\dot{z}$
			t = 32			
0.10879968×10^{3}	$-0.17637945 \times 10^{3}$	$-0.90150573 \times 10^{2}$	0.25299975×10^{6}	0.22372066×10^{5}	0.25779187×10^{5}	x
0.12736523×10^{3}	$-0.33432617 \times 10^{3}$	$-0.15864062 \times 10^{3}$	0.43419131×10^{6}	0.10889175×10^{6}	0.80283843×10^{5}	y
0.78479492×10^{2}	$-0.17443164 \times 10^{3}$	$-0.12957910 \times 10^{3}$	0.25400409×10^{6}	0.58048031×10^{5}	0.52191562×10^{5}	z
$0.66395104 \times 10^{-3}$	$-0.11871103 \times 10^{-2}$	$-0.61231293 \times 10^{-3}$	0.16831052×10	0.21626356	0.21115145	$\dot{x}$
$0.16837865 \times 10^{-2}$	$-0.42444691 \times 10^{-2}$	$-0.21609962 \times 10^{-2}$	0.56118120×10	0.13516548×10	0.10480989×10	$\dot{y}$
$0.97835809 \times 10^{-3}$	$-0.23045801 \times 10^{-2}$	$-0.14253706 \times 10^{-2}$	0.31969068×10	0.75647056	0.59790069	$\dot{z}$
			t = 40			
0.12677417×10^{3}	$-0.20865698 \times 10^{3}$	$-0.10679687 \times 10^{3}$	0.29864355×10^{6}	0.28304783×10^{5}	0.31549095×10^{5}	x
0.17873828×10^{3}	$-0.46369336 \times 10^{3}$	$-0.22490625 \times 10^{3}$	0.60553968×10^{6}	0.15018094×10^{6}	0.11233406×10^{6}	y
0.10813476×10^{3}	$-0.24479590 \times 10^{3}$	$-0.17237109 \times 10^{3}$	0.35121796×10^{6}	0.81144937×10^{5}	0.70345172×10^{5}	z
$0.58868714 \times 10^{-3}$	$-0.10622200 \times 10^{-2}$	$-0.54762140 \times 10^{-3}$	0.14972497×10	0.19701397	0.19083086	$\dot{x}$
$0.18831342 \times 10^{-2}$	$-0.47371984 \times 10^{-2}$	$-0.24393126 \times 10^{-2}$	0.62844156×10	0.15145855×10	0.11768045×10	$\dot{y}$
$0.10804981 \times 10^{-2}$	$-0.25796741 \times 10^{-2}$	$-0.15459098 \times 10^{-2}$	0.35517570×10	0.84656686	0.66240299	$\dot{z}$
			t = 48			
0.14307934×10^{3}	$-0.23832837 \times 10^{3}$	$-0.12208984 \times 10^{3}$	0.34023193×10^{6}	0.33898089×10^{5}	0.36924292×10^{5}	x
0.23589453×10^{3}	$-0.60724609 \times 10^{3}$	$-0.29915430 \times 10^{3}$	0.79626834×10^{6}	0.19612002×10^{6}	0.14806274×10^{6}	y
0.14072753×10^{3}	$-0.32299121 \times 10^{3}$	$-0.21862109 \times 10^{3}$	0.45856903×10^{6}	0.10678335×10^{6}	0.90337499×10^{5}	z
$0.55380538 \times 10^{-3}$	$-0.10212138 \times 10^{-2}$	$-0.52625500 \times 10^{-3}$	0.14198550×10	0.19736803	0.18740246	$\dot{x}$
$0.20893216 \times 10^{-2}$	$-0.52376240 \times 10^{-2}$	$-0.27200058 \times 10^{-2}$	0.69686833×10	0.16764382×10	0.13050538×10	$\dot{y}$
$0.11841170 \times 10^{-2}$	$-0.28520785 \times 10^{-2}$	$-0.16666614 \times 10^{-2}$	0.39056886×10	0.93391925	0.72603786	$\dot{z}$
			t = 56			
0.15987427×10^{3}	$-0.27033227 \times 10^{3}$	$-0.13859375 \times 10^{3}$	0.38392986×10^{6}	0.40458386×10^{5}	0.42976828×10^{5}	x
0.29955468×10^{3}	$-0.76620898 \times 10^{3}$	$-0.38203711 \times 10^{3}$	0.10081180×10^{7}	0.24690084×10^{6}	0.18766274×10^{6}	y
0.17651074×10^{3}	$-0.40938281 \times 10^{3}$	$-0.26845996 \times 10^{3}$	0.57660745×10^{6}	0.13497979×10^{6}	0.11220618×10^{6}	z
$0.65628439 \times 10^{-3}$	$-0.13068356 \times 10^{-2}$	$-0.67474879 \times 10^{-3}$	0.17415237×10	0.28769839	0.25664708	$\dot{x}$
$0.23483261 \times 10^{-2}$	$-0.58319978 \times 10^{-2}$	$-0.30527227 \times 10^{-2}$	0.77866408×10	0.18562000×10	0.14506064×10	$\dot{y}$
$0.13070609 \times 10^{-2}$	$-0.31591300 \times 10^{-2}$	$-0.17974693 \times 10^{-2}$	0.43075283×10	0.10259835×10	0.79412100	$\dot{z}$

TABLE III.- TRANSITION MATRICES FOR $\Delta x_0 = \Delta y_0 = \Delta z_0 = 2$ STATUTE MILES AND $\Delta\dot{x}_0 = \Delta\dot{y}_0 = \Delta\dot{z}_0 = 10$ FT/SEC

Trajectory parameter						Trajectory parameter
x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	
			t = 0			
0.09999999 × 10	0	0	0	0	0	x
0	0.09999999 × 10	0	0	0	0	y
0	0	0.09999999 × 10	0	0	0	z
0	0	0	0.10000148 × 10	0	0	$\dot{x}$
0	0	0	0	0.10000069 × 10	0	$\dot{y}$
0	0	0	0	0	0.100000.9 × 10	$\dot{z}$
			t = 8			
0.36417114 × 10^2	-0.51420532 × 10^2	-0.25868774 × 10^2	0.74394535 × 10^5	0.18478388 × 10^4	0.48009843 × 10^4	x
0.12250976 × 10^2	-0.41976807 × 10^2	-0.14868896 × 10^2	0.51370816 × 10^5	0.16666160 × 10^5	0.94846641 × 10^4	y
0.95386962 × 10	-0.18297324 × 10^2	-0.23748046 × 10^2	0.32291531 × 10^5	0.68166914 × 10^4	0.10063517 × 10^5	z
0.10836003 × 10^{-2}	-0.17468543 × 10^{-2}	-0.89346245 × 10^{-3}	0.25257818 × 10	0.12050972	0.13601386	$\dot{x}$
0.87793171 × 10^{-3}	-0.23012087 × 10^{-2}	-0.10259109 × 10^{-2}	0.29448162 × 10	0.71964525	0.52958250	$\dot{y}$
0.56615844 × 10^{-3}	-0.11619963 × 10^{-2}	-0.99294260 × 10^{-3}	0.17674825 × 10	0.38298154	0.30772979	$\dot{z}$
			t = 16			
0.64575226 × 10^2	-0.98579650 × 10^2	-0.50058502 × 10^2	0.14207844 × 10^6	0.87902622 × 10^4	0.17267819 × 10^5	x
0.42938965 × 10^2	-0.12071045 × 10^3	-0.51875976 × 10^2	0.15366708 × 10^6	0.41317289 × 10^5	0.28103882 × 10^5	y
0.28584961 × 10^2	-0.59316650 × 10^2	-0.54854980 × 10^2	0.92629066 × 10^5	0.70308535 × 10^4	0.71776777 × 10^5	z
0.88990573 × 10^{-3}	-0.15325528 × 10^{-2}	-0.76836491 × 10^{-3}	0.21876272 × 10	0.64872960	0.25529890	$\dot{x}$
0.12248084 × 10^{-2}	-0.31102449 × 10^{-2}	-0.15088990 × 10^{-2}	0.40769552 × 10	0.97742826	0.74795169	$\dot{y}$
0.74247271 × 10^{-3}	-0.16511306 × 10^{-2}	-0.11601374 × 10^{-2}	0.23774642 × 10	0.54290247	0.45182490	$\dot{z}$
			t = 24			
0.88270595 × 10^2	-0.14000428 × 10^3	-0.71389792 × 10^2	0.20096637 × 10^6	0.15828118 × 10^5	0.19339930 × 10^5	x
0.81908203 × 10^2	-0.21925830 × 10^3	-0.10056543 × 10^3	0.28352053 × 10^6	0.72438679 × 10^5	0.52048476 × 10^5	y
0.51862305 × 10^2	-0.11211279 × 10^3	-0.90308105 × 10^2	0.16776207 × 10^6	0.37654804 × 10^5	0.35928878 × 10^5	z
0.76192059 × 10^{-3}	-0.13485783 × 10^{-2}	-0.69494639 × 10^{-3}	0.19092979 × 10	0.23713982	0.23467916	$\dot{x}$
0.14724806 × 10^{-2}	-0.37138090 × 10^{-2}	-0.18597767 × 10^{-2}	0.49131129 × 10	0.11774772 × 10	0.90945035	$\dot{y}$
0.86957030 × 10^{-3}	-0.20023286 × 10^{-2}	-0.12988709 × 10^{-2}	0.28249934 × 10	0.65828910	0.52905634	$\dot{z}$
			t = 32			
0.10873239 × 10^3	-0.17649734 × 10^3	-0.90196747 × 10^2	0.25242993 × 10^6	0.20362366 × 10^5	0.25759618 × 10^5	x
0.12742187 × 10^3	-0.33386914 × 10^3	-0.15891269 × 10^3	0.43560773 × 10^6	0.10890906 × 10^6	0.80275077 × 10^5	y
0.78505371 × 10^2	-0.17414746 × 10^3	-0.12994883 × 10^3	0.25476464 × 10^6	0.58062468 × 10^5	0.52170163 × 10^5	z
0.66291941 × 10^{-3}	-0.11901008 × 10^{-2}	-0.61323401 × 10^{-3}	0.16709892 × 10	0.21608703	0.21080527	$\dot{x}$
0.16846992 × 10^{-2}	-0.42366199 × 10^{-2}	-0.21488097 × 10^{-2}	0.56368081 × 10	0.13523079 × 10	0.10482582 × 10	$\dot{y}$
0.97879209 × 10^{-3}	-0.22998247 × 10^{-2}	-0.14247354 × 10^{-2}	0.32102074 × 10	0.75693476	0.59781611	$\dot{z}$
			t = 40			
0.12667224 × 10^3	-0.20888049 × 10^3	-0.10687353 × 10^3	0.29765484 × 10^6	0.28287606 × 10^5	0.31509715 × 10^5	x
0.17882617 × 10^3	-0.46297754 × 10^3	-0.22470801 × 10^3	0.60778317 × 10^6	0.15021600 × 10^6	0.11233457 × 10^6	y
0.10817529 × 10^3	-0.24435645 × 10^3	-0.17251835 × 10^3	0.35241885 × 10^6	0.81175616 × 10^5	0.70323773 × 10^5	z
0.58736838 × 10^{-3}	-0.10667005 × 10^{-2}	-0.54892804 × 10^{-3}	0.14798633 × 10	0.19668353	0.19040649	$\dot{x}$
0.18842854 × 10^{-2}	-0.47270879 × 10^{-2}	-0.24364591 × 10^{-2}	0.63169078 × 10	0.15156712 × 10	0.11771723 × 10	$\dot{y}$
0.10810588 × 10^{-2}	-0.25736373 × 10^{-2}	-0.15449729 × 10^{-2}	0.35691124 × 10	0.84726807	0.66242856	$\dot{z}$
			t = 48			
0.14293188 × 10^3	-0.23871569 × 10^3	-0.12221301 × 10^3	0.33862008 × 10^6	0.33865669 × 10^5	0.36869701 × 10^5	x
0.23601855 × 10^3	-0.60620605 × 10^3	-0.29886719 × 10^3	0.79955803 × 10^6	0.19619273 × 10^6	0.14807460 × 10^6	y
0.14078662 × 10^3	-0.32235644 × 10^3	-0.21853759 × 10^3	0.46033427 × 10^6	0.10683827 × 10^6	0.90317648 × 10^5	z
0.55186823 × 10^{-3}	-0.10285964 × 10^{-2}	-0.52835699 × 10^{-3}	0.13925812 × 10	0.19658124	0.18669435	$\dot{x}$
0.20906851 × 10^{-2}	-0.52251779 × 10^{-2}	-0.27164891 × 10^{-2}	0.70088485 × 10	0.16780373 × 10	0.13056696 × 10	$\dot{y}$
0.11848174 × 10^{-2}	-0.28446596 × 10^{-2}	-0.16853296 × 10^{-2}	0.39275710 × 10	0.93490469	0.72621685	$\dot{z}$
			t = 56			
0.15965100 × 10^3	-0.27105676 × 10^3	-0.13881213 × 10^3	0.38114864 × 10^6	0.40386327 × 10^5	0.42890074 × 10^5	x
0.29971484 × 10^3	-0.76477441 × 10^3	-0.38163672 × 10^3	0.10126586 × 10^7	0.24703439 × 10^6	0.18769575 × 10^6	y
0.17659131 × 10^3	-0.40850781 × 10^3	-0.26832666 × 10^3	0.57907781 × 10^6	0.13507003 × 10^6	0.11219458 × 10^6	z
0.65210834 × 10^{-3}	-0.13263179 × 10^{-2}	-0.68010390 × 10^{-3}	0.16778706 × 10	0.28528543	0.25477946	$\dot{x}$
0.23495555 × 10^{-2}	-0.58167949 × 10^{-2}	-0.30485801 × 10^{-2}	0.78327334 × 10	0.18587059 × 10	0.14516046 × 10	$\dot{y}$
0.13078982 × 10^{-2}	-0.31496789 × 10^{-2}	-0.17991990 × 10^{-2}	0.43351389 × 10	0.10275437 × 10	0.79462700	$\dot{z}$

TABLE IV.- TRANSITION MATRICES FOR $\Delta x_o = \Delta y_o = \Delta z_o = 6$ STATUTE MILES AND $\Delta \dot{x}_o = \Delta \dot{y}_o = \Delta \dot{z}_o = 40$ FT/SEC

Trajectory parameter						Trajectory parameter
x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	
			t = 0			
0.09999999×10	0	0	0	0	0	x
0	0.09999999×10	0	0	0	0	y
0	0	0.09999999×10	0	0	0	z
0	0	0	0.10000148×10	0	0	$\dot{x}$
0	0	0	0	0.10000049×10	0	$\dot{y}$
0	0	0	0	0	0.10000069×10	$\dot{z}$
			t = 8			
0.36393879×10^{2}	$-0.51396748 \times 10^{2}$	$-0.25886209 \times 10^{2}$	0.74378309×10^{5}	0.18390249×10^{4}	0.47872881×10^{4}	x
0.12274658×10^{2}	$-0.41837808 \times 10^{2}$	$-0.14836913 \times 10^{2}$	0.51567978×10^{5}	0.16654687×10^{5}	0.94756406×10^{4}	y
0.95505777×10	$-0.18170003 \times 10^{2}$	$-0.23740356 \times 10^{2}$	0.32402551×10^{5}	0.68118896×10^{4}	0.10053269×10^{5}	z
$0.10819562 \times 10^{-2}$	$-0.17493454 \times 10^{-2}$	$-0.89433354 \times 10^{-3}$	0.25223621×10	0.22032016	0.25579507	$\dot{x}$
$0.87903440 \times 10^{-3}$	$-0.22926653 \times 10^{-2}$	$-0.10218893 \times 10^{-2}$	0.29576309×10	0.71917319	0.52916394	$\dot{y}$
$0.56665266 \times 10^{-3}$	$-0.11563909 \times 10^{-2}$	$-0.99273026 \times 10^{-3}$	0.17745036×10	0.38285172	0.35723215	$\dot{z}$
			t = 16			
0.64490987×10^{2}	$-0.98615285 \times 10^{2}$	$-0.50111267 \times 10^{2}$	0.14187483×10^{6}	0.87775488×10^{4}	0.12236071×10^{5}	x
0.43003254×10^{2}	$-0.12022640 \times 10^{3}$	$-0.51757975 \times 10^{2}$	0.15439411×10^{6}	0.41296535×10^{5}	0.28084417×10^{5}	y
0.28615071×10^{2}	$-0.59007161 \times 10^{2}$	$-0.54835205 \times 10^{2}$	0.93026677×10^{5}	0.20303120×10^{5}	0.21752349×10^{5}	z
$0.88737874 \times 10^{-3}$	$-0.15362560 \times 10^{-2}$	$-0.78989037 \times 10^{-3}$	0.21779535×10	0.24860642	0.25481339	$\dot{x}$
$0.12264848 \times 10^{-2}$	$-0.30947787 \times 10^{-2}$	$-0.15049402 \times 10^{-2}$	0.41010778×10	0.97732598	0.74765861	$\dot{y}$
$0.74321280 \times 10^{-3}$	$-0.16413020 \times 10^{-2}$	$-0.11594482 \times 10^{-2}$	0.23904157×10	0.54232049	0.45135283	$\dot{z}$
			t = 24			
0.88101269×10^{2}	$-0.14019706 \times 10^{3}$	$-0.71501999 \times 10^{2}$	0.20038593×10^{6}	0.15810715×10^{5}	0.19295347×10^{5}	x
0.82030598×10^{2}	$-0.21822298 \times 10^{3}$	$-0.10030338 \times 10^{3}$	0.28511858×10^{6}	0.72416823×10^{5}	0.52024370×10^{5}	y
0.51918294×10^{2}	$-0.11145817 \times 10^{3}$	$-0.90258218 \times 10^{2}$	0.16862523×10^{6}	0.37657447×10^{5}	0.35892656×10^{5}	z
$0.75852808 \times 10^{-3}$	$-0.13558932 \times 10^{-2}$	$-0.69749262 \times 10^{-3}$	0.18926528×10	0.23693624	0.23417218	$\dot{x}$
$0.14749616 \times 10^{-2}$	$-0.36909133 \times 10^{-2}$	$-0.18537119 \times 10^{-2}$	0.49497690×10	0.11779257×10	0.90941298	$\dot{y}$
$0.87069161 \times 10^{-3}$	$-0.19881651 \times 10^{-2}$	$-0.12974602 \times 10^{-2}$	0.28445262×10	0.65872823	0.52869835	$\dot{z}$
			t = 32			
0.10845292×10^{3}	$-0.17696109 \times 10^{3}$	$-0.90398305 \times 10^{2}$	0.25125590×10^{6}	0.22336400×10^{5}	0.25693658×10^{5}	x
0.12763020×10^{3}	$-0.33205827 \times 10^{3}$	$-0.15804394 \times 10^{3}$	0.43846120×10^{6}	0.10890902×10^{6}	0.80253937×10^{5}	y
0.78600422×10^{2}	$-0.17301887 \times 10^{3}$	$-0.12944531 \times 10^{3}$	0.25629552×10^{6}	0.58082835×10^{5}	0.52125562×10^{5}	z
$0.65860587 \times 10^{-3}$	$-0.12017103 \times 10^{-2}$	$-0.61691304 \times 10^{-3}$	0.16461247×10	0.21565995	0.21014880	$\dot{x}$
$0.16881153 \times 10^{-2}$	$-0.42056316 \times 10^{-2}$	$-0.21504425 \times 10^{-2}$	0.56875486×10	0.13534359×10	0.10485483×10	$\dot{y}$
$0.98038216 \times 10^{-3}$	$-0.22810393 \times 10^{-2}$	$-0.14224139 \times 10^{-2}$	0.32371846×10	0.75775351	0.59762384	$\dot{z}$
			t = 40			
0.12625415×10^{3}	$-0.20975614 \times 10^{3}$	$-0.10720149 \times 10^{3}$	0.29561806×10^{6}	0.28245438×10^{5}	0.31431146×10^{5}	x
0.17914583×10^{3}	$-0.46015071 \times 10^{3}$	$-0.22396321 \times 10^{3}$	0.61231861×10^{6}	0.15026678×10^{6}	0.11232684×10^{6}	y
0.10832259×10^{3}	$-0.24261588 \times 10^{3}$	$-0.17213297 \times 10^{3}$	0.35484526×10^{6}	0.81225890×10^{5}	0.70276465×10^{5}	z
$0.58193194 \times 10^{-3}$	$-0.10841396 \times 10^{-2}$	$-0.55416767 \times 10^{-3}$	0.14441425×10	0.19592256	0.18953021	$\dot{x}$
$0.18887222 \times 10^{-2}$	$-0.46873514 \times 10^{-2}$	$-0.24256073 \times 10^{-2}$	0.63832186×10	0.15176293×10	0.11778745×10	$\dot{y}$
$0.10831896 \times 10^{-2}$	$-0.25498649 \times 10^{-2}$	$-0.15415164 \times 10^{-2}$	0.36044817×10	0.84854610	0.66245954	$\dot{z}$
			t = 48			
0.14232959×10^{3}	$-0.24023747 \times 10^{3}$	$-0.12273083 \times 10^{3}$	0.33531071×10^{6}	0.33791484×10^{5}	0.36759003×10^{5}	x
0.23647754×10^{3}	$-0.60210612 \times 10^{3}$	$-0.29777311 \times 10^{3}$	0.80623769×10^{6}	0.19631390×10^{6}	0.14809394×10^{6}	y
0.14100391×10^{3}	$-0.31985514 \times 10^{3}$	$-0.21823177 \times 10^{3}$	0.46391206×10^{6}	0.10693340×10^{6}	0.90275367×10^{5}	z
$0.54420375 \times 10^{-3}$	$-0.10580026 \times 10^{-2}$	$-0.53683233 \times 10^{-3}$	0.13373155×10	0.19502342	0.18526413	$\dot{x}$
$0.20959426 \times 10^{-2}$	$-0.51763070 \times 10^{-2}$	$-0.27031253 \times 10^{-2}$	0.70913297×10	0.16810133×10	0.13068467×10	$\dot{y}$
$0.11075120 \times 10^{-2}$	$-0.28154471 \times 10^{-2}$	$-0.16603202 \times 10^{-2}$	0.39723221×10	0.93676150	0.72654386	$\dot{z}$
			t = 56			
0.15873555×10^{3}	$-0.27395235 \times 10^{3}$	$-0.13971138 \times 10^{3}$	0.37553851×10^{6}	0.40233251×10^{5}	0.42715663×10^{5}	x
0.30032454×10^{3}	$-0.75911621 \times 10^{3}$	$-0.38012271 \times 10^{3}$	0.10219099×10^{7}	0.24726075×10^{6}	0.18775737×10^{6}	y
0.17689323×10^{3}	$-0.40506038 \times 10^{3}$	$-0.26783349 \times 10^{3}$	0.58409290×10^{6}	0.13523090×10^{6}	0.11216918×10^{6}	z
$0.63560034 \times 10^{-3}$	-0.10985990	$-0.70205672 \times 10^{-3}$	0.15567812×10	0.28041501	0.25111306	$\dot{x}$
$0.23542220 \times 10^{-2}$	$-0.42172583 \times 10^{-3}$	$-0.30326241 \times 10^{-2}$	0.79271334×10	0.18634187×10	0.14535233×10	$\dot{y}$
$0.13110470 \times 10^{-2}$	$0.34315207 \times 10^{-2}$	$-0.17861991 \times 10^{-2}$	0.43905243×10	0.10304873×10	0.79558908	$\dot{z}$

TABLE V.- TRANSITION MATRICES FOR $\Delta x_o = \Delta y_o = \Delta z_o = 10$ STATUTE MILES AND $\Delta \dot{x}_o = \Delta \dot{y}_o = \Delta \dot{z}_o = 50$ FT/SEC

Trajectory parameter						Trajectory parameter
x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	
			t = 0			
0.09999999×10	0	0	0	0	0	x
0	0.09999999×10	0	0	0	0	y
0	0	0.09999999×10	0	0	0	z
0	0	0	0.10000053×10	0	0	$\dot{x}$
0	0	0	0	0.10000006×10	0	$\dot{y}$
0	0	0	0	0	0.10000022×10	$\dot{z}$
			t = 8			
0.36370470×10^{2}	$-0.51373120 \times 10^{2}$	$-0.25903516 \times 10^{2}$	0.74368826×10^{5}	0.18346582×10^{4}	0.47801789×10^{4}	x
0.12298193×10^{2}	$-0.41699609 \times 10^{2}$	$-0.14805175 \times 10^{2}$	0.51665882×10^{5}	0.16649170×10^{5}	0.94706906×10^{4}	y
0.95623534×10	$-0.18083300 \times 10^{2}$	$-0.23732764 \times 10^{2}$	0.32457614×10^{5}	0.68094984×10^{4}	0.10047958×10^{5}	z
$0.10803084 \times 10^{-2}$	$-0.17498090 \times 10^{-2}$	$-0.89520066 \times 10^{-3}$	0.25205794×10	0.22022894	0.25551313	$\dot{x}$
$0.88012964 \times 10^{-3}$	$-0.22841974 \times 10^{-2}$	$-0.10198458 \times 10^{-2}$	0.29640302×10	0.71893559	0.52894678	$\dot{y}$
$0.56714042 \times 10^{-3}$	$-0.11508457 \times 10^{-2}$	$-0.99250823 \times 10^{-3}$	0.17780038×10	0.38278563	0.35697605	$\dot{z}$
			t = 16			
0.64406518×10^{2}	$-0.98649506 \times 10^{2}$	$-0.50163946 \times 10^{2}$	0.14176861×10^{6}	0.87711550×10^{4}	0.12222426×10^{5}	x
0.43065820×10^{2}	$-0.11974707 \times 10^{3}$	$-0.51640137 \times 10^{2}$	0.15475762×10^{6}	0.41286815×10^{5}	0.28073512×10^{5}	y
0.28644042×10^{2}	$-0.58700634 \times 10^{2}$	$-0.54814941 \times 10^{2}$	0.93225206×10^{5}	0.20300620×10^{5}	0.21739523×10^{5}	z
$0.88482051 \times 10^{-3}$	$-0.15398695 \times 10^{-2}$	$-0.79150143 \times 10^{-3}$	0.21729859×10	0.24854717	0.25458846	$\dot{x}$
$0.12281819 \times 10^{-2}$	$-0.30795089 \times 10^{-2}$	$-0.15009969 \times 10^{-2}$	0.41131849×10	0.97727287	0.74750598	$\dot{y}$
$0.74395984 \times 10^{-3}$	$-0.16316018 \times 10^{-2}$	$-0.11587582 \times 10^{-2}$	0.23969095×10	0.54237793	0.45111287	$\dot{z}$
			t = 24			
0.87930868×10^{2}	$-0.14038485 \times 10^{3}$	$-0.71613441 \times 10^{2}$	0.20008656×10^{6}	0.15801981×10^{5}	0.19274520×10^{5}	x
0.82153222×10^{2}	$-0.21720049 \times 10^{3}$	$-0.10004307 \times 10^{3}$	0.28592055×10^{6}	0.72409424×10^{5}	0.52010990×10^{5}	y
0.51974072×10^{2}	$-0.11081138 \times 10^{3}$	$-0.90208545 \times 10^{2}$	0.16905791×10^{6}	0.37658929×10^{5}	0.35873681×10^{5}	z
$0.75513794 \times 10^{-3}$	$-0.13630095 \times 10^{-2}$	$-0.70000737 \times 10^{-3}$	0.18841117×10	0.23683219	0.23390084	$\dot{x}$
$0.14774464 \times 10^{-2}$	$-0.36684132 \times 10^{-2}$	$-0.18476888 \times 10^{-2}$	0.49682714×10	0.11781436×10	0.90937364	$\dot{y}$
$0.87181590 \times 10^{-3}$	$-0.19742440 \times 10^{-2}$	$-0.12960552 \times 10^{-2}$	0.28543780×10	0.65894430	0.52850697	$\dot{z}$
			t = 32			
0.10817168×10^{3}	$-0.17741124 \times 10^{3}$	$-0.90597790 \times 10^{2}$	0.25065146×10^{6}	0.22323398×10^{5}	0.25664325×10^{5}	x
0.12783789×10^{3}	$-0.33027676 \times 10^{3}$	$-0.15757734 \times 10^{3}$	0.43989835×10^{6}	0.10891237×10^{6}	0.80241562×10^{5}	y
0.78695117×10^{2}	$-0.17190791 \times 10^{3}$	$-0.12934267 \times 10^{3}$	0.25706597×10^{6}	0.58093200×10^{5}	0.52102359×10^{5}	z
$0.65429918 \times 10^{-3}$	$-0.12129787 \times 10^{-2}$	$-0.62054023 \times 10^{-3}$	0.16333465×10	0.21544904	0.20983025	$\dot{x}$
$0.16915440 \times 10^{-2}$	$-0.41752934 \times 10^{-2}$	$-0.21421558 \times 10^{-2}$	0.57132967×10	0.13540034×10	0.10486748×10	$\dot{y}$
$0.98197721 \times 10^{-3}$	$-0.22626384 \times 10^{-2}$	$-0.14201093 \times 10^{-2}$	0.32508619×10	0.75816450	0.59751831	$\dot{z}$
			t = 40			
0.12583332×10^{3}	$-0.21060654 \times 10^{3}$	$-0.10752625 \times 10^{3}$	0.29457009×10^{6}	0.28224333×10^{5}	0.31390850×10^{5}	x
0.17946738×10^{3}	$-0.45737636 \times 10^{3}$	$-0.22322343 \times 10^{3}$	0.61461262×10^{6}	0.15029210×10^{6}	0.11232106×10^{6}	y
0.10847118×10^{3}	$-0.24090722 \times 10^{3}$	$-0.17194853 \times 10^{3}$	0.35607093×10^{6}	0.81251052×10^{5}	0.70251637×10^{5}	z
$0.57645980 \times 10^{-3}$	$-0.11011118 \times 10^{-2}$	$-0.55934563 \times 10^{-3}$	0.14257779×10	0.19553423	0.18908577	$\dot{x}$
$0.18931798 \times 10^{-2}$	$-0.46486280 \times 10^{-2}$	$-0.24148888 \times 10^{-2}$	0.64170702×10	0.15186122×10	0.11781987×10	$\dot{y}$
$0.10853331 \times 10^{-2}$	$-0.25266770 \times 10^{-2}$	$-0.15380963 \times 10^{-2}$	0.36225122×10	0.84918802	0.66246200	$\dot{z}$
			t = 48			
0.14172409×10^{3}	$-0.24172261 \times 10^{3}$	$-0.12324367 \times 10^{3}$	0.33361169×10^{6}	0.33754436×10^{5}	0.36702625×10^{5}	x
0.23694062×10^{3}	$-0.59809043 \times 10^{3}$	$-0.29668964 \times 10^{3}$	0.80962921×10^{6}	0.19637474×10^{6}	0.14810111×10^{6}	y
0.14122236×10^{3}	$-0.31740449 \times 10^{3}$	$-0.21792842 \times 10^{3}$	0.46572611×10^{6}	0.10698115×10^{6}	0.90252627×10^{5}	z
$0.53652935 \times 10^{-3}$	$-0.10872430 \times 10^{-2}$	$-0.54526366 \times 10^{-3}$	0.13092574×10	0.19423369	0.18455057	$\dot{x}$
$0.21012533 \times 10^{-2}$	$-0.51288187 \times 10^{-2}$	$-0.26899479 \times 10^{-2}$	0.71337091×10	0.16825034×10	0.13074110×10	$\dot{y}$
$0.11902194 \times 10^{-2}$	$-0.27869913 \times 10^{-2}$	$-0.16553319 \times 10^{-2}$	0.39952280×10	0.93769030	0.72669286	$\dot{z}$
			t = 56			
0.15782160×10^{3}	$-0.27685779 \times 10^{3}$	$-0.14061256 \times 10^{3}$	0.37270249×10^{6}	0.40156540×10^{5}	0.42627183×10^{5}	x
0.30093984×10^{3}	$-0.75356112 \times 10^{3}$	$-0.37862305 \times 10^{3}$	0.10266261×10^{7}	0.24737501×10^{6}	0.18778526×10^{6}	y
0.17719589×10^{3}	$-0.40167069 \times 10^{3}$	$-0.26733906 \times 10^{3}$	0.58664089×10^{6}	0.13531196×10^{6}	0.11215431×10^{6}	z
$0.61960294 \times 10^{-3}$	$-0.66566026 \times 10^{-1}$	$-0.65796164 \times 10^{-1}$	0.14987625 $\times 10$	0.27799836	0.24929500	$\dot{x}$
$0.23590352 \times 10^{-2}$	$-0.24920973 \times 10^{-2}$	$0.18369927 \times 10^{-3}$	0.79757937×10	0.18657496×10	0.14544407×10	$\dot{y}$
$0.13141842 \times 10^{-2}$	$0.85389428 \times 10^{-3}$	$0.21490502 \times 10^{-2}$	0.44185603×10	0.10319427×10	0.79604369	$\dot{z}$

TABLE VI.- TRANSITION MATRICES FOR $\Delta x_o = \Delta y_o = \Delta z_o = 25$ STATUTE MILES AND $\Delta \dot{x}_o = \Delta \dot{y}_o = \Delta \dot{z}_o = 75$ FT/SEC

Trajectory parameter						Trajectory parameter
x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	
		t = 0				
0.09999999×10	0	0	0	0	0	x
0	0.09999999×10	0	0	0	0	y
0	0	0.09999999×10	0	0	0	z
0	0	0	0.99999698	0	0	$\dot{x}$
0	0	0	0	0.99999698	0	$\dot{y}$
0	0	0	0	0	0.99999803	$\dot{z}$
		t = 8				
0.36282612×10^{2}	$-0.51281767 \times 10^{2}$	$-0.25967641 \times 10^{2}$	0.74346063×10^{5}	0.18240321×10^{4}	0.47621837×10^{4}	x
0.12385800×10^{2}	$-0.41188104 \times 10^{2}$	$-0.14686503 \times 10^{2}$	0.51913641×10^{5}	0.16635403×10^{5}	0.94586597×10^{4}	y
0.96063475×10	$-0.17763349 \times 10^{2}$	$-0.23703495 \times 10^{2}$	0.32596988×10^{5}	0.68037064×10^{4}	0.10034698×10^{5}	z
$0.10740953 \times 10^{-2}$	$-0.17512946 \times 10^{-2}$	$-0.89841693 \times 10^{-3}$	0.25161196×10	0.22000910	0.25490758	$\dot{x}$
$0.88423491 \times 10^{-3}$	$-0.22529840 \times 10^{-2}$	$-0.10122061 \times 10^{-2}$	0.29802259×10	0.71835339	0.52838871	$\dot{y}$
$0.56896686 \times 10^{-3}$	$-0.11304408 \times 10^{-2}$	$-0.99163680 \times 10^{-3}$	0.17868695×10	0.38262802	0.35633274	$\dot{z}$
		t = 16				
0.64087890×10^{2}	$-0.98759530 \times 10^{2}$	$-0.50358471 \times 10^{2}$	0.14150152×10^{6}	0.87559400×10^{4}	0.12187446×10^{5}	x
0.43301913×10^{2}	$-0.11798426 \times 10^{3}$	$-0.51201133 \times 10^{2}$	0.15567819×10^{6}	0.41261964×10^{5}	0.28047250×10^{5}	y
0.28753007×10^{2}	$-0.57575253 \times 10^{2}$	$-0.54738300 \times 10^{2}$	0.93728012×10^{5}	0.20294347×10^{5}	0.21707847×10^{5}	z
$0.87514035 \times 10^{-3}$	$-0.15525216 \times 10^{-2}$	$-0.79744070 \times 10^{-3}$	0.21603543×10	0.24839591	0.25399740	$\dot{x}$
$0.12345161 \times 10^{-2}$	$-0.30237802 \times 10^{-2}$	$-0.14863238 \times 10^{-2}$	0.41439505×10	0.97715934	0.74709451	$\dot{y}$
$0.74673831 \times 10^{-3}$	$-0.15962171 \times 10^{-2}$	$-0.11561467 \times 10^{-2}$	0.24134082×10	0.54253346	0.45049421	$\dot{z}$
		t = 24				
0.87287554×10^{2}	$-0.14103305 \times 10^{3}$	$-0.72025061 \times 10^{2}$	0.19932577×10^{6}	0.15781299×10^{5}	0.19221698×10^{5}	x
0.82613710×10^{2}	$-0.21346781 \times 10^{3}$	$-0.99073867 \times 10^{2}$	0.28795903×10^{6}	0.72390864×10^{5}	0.51976514×10^{5}	y
0.52182968×10^{2}	$-0.10845295 \times 10^{3}$	$-0.90020780 \times 10^{2}$	0.17015745×10^{6}	0.37662729×10^{5}	0.35825763×10^{5}	z
$0.74225947 \times 10^{-3}$	$-0.13879713 \times 10^{-2}$	$-0.70923454 \times 10^{-3}$	0.18622411×10	0.23657151	0.23323346	$\dot{x}$
$0.14868092 \times 10^{-2}$	$-0.35870638 \times 10^{-2}$	$-0.18254616 \times 10^{-2}$	0.50154714×10	0.11787269×10	0.90926378	$\dot{y}$
$0.87604120 \times 10^{-3}$	$-0.19239078 \times 10^{-2}$	$-0.12908452 \times 10^{-2}$	0.28795017×10	0.65950581	0.52802783	$\dot{z}$
		t = 32				
0.10710889×10^{3}	$-0.17897797 \times 10^{3}$	$-0.91330236 \times 10^{2}$	0.24910785×10^{6}	0.22292084×10^{5}	0.25589936×10^{5}	x
0.12861922×10^{3}	$-0.32381234 \times 10^{3}$	$-0.15585148 \times 10^{3}$	0.44356085×10^{6}	0.10892269×10^{6}	0.80210076×10^{5}	y
0.79051093×10^{2}	$-0.16787699 \times 10^{3}$	$-0.12895934 \times 10^{3}$	0.25902841×10^{6}	0.58119877×10^{5}	0.52043476×10^{5}	z
$0.63794299 \times 10^{-3}$	$-0.12523164 \times 10^{-2}$	$-0.63376396 \times 10^{-3}$	0.16004552×10	0.21494105	0.20899063	$\dot{x}$
$0.17045545 \times 10^{-2}$	$-0.40666872 \times 10^{-2}$	$-0.21117382 \times 10^{-2}$	0.57793222×10	0.13554584×10	0.10489657×10	$\dot{y}$
$0.98802759 \times 10^{-3}$	$-0.21967039 \times 10^{-2}$	$-0.14116278 \times 10^{-2}$	0.32859153×10	0.75921383	0.59724099	$\dot{z}$
		t = 40				
0.12423952×10^{3}	$-0.21357592 \times 10^{3}$	$-0.10871486 \times 10^{3}$	0.29188279×10^{6}	0.28172608×10^{5}	0.31288891×10^{5}	x
0.18068039×10^{3}	$-0.44737515 \times 10^{3}$	$-0.22049765 \times 10^{3}$	0.62047742×10^{6}	0.15035927×10^{6}	0.11230450×10^{6}	y
0.10903125×10^{3}	$-0.23474351 \times 10^{3}$	$-0.17126500 \times 10^{3}$	0.35920356×10^{6}	0.81316402×10^{5}	0.70188663×10^{5}	z
$0.55566534 \times 10^{-3}$	$-0.11610196 \times 10^{-2}$	$-0.57827464 \times 10^{-3}$	0.13784125×10	0.19458073	0.18797491	$\dot{x}$
$0.19102185 \times 10^{-2}$	$-0.45114546 \times 10^{-2}$	$-0.23758029 \times 10^{-2}$	0.65043841×10	0.15211158×10	0.11789954×10	$\dot{y}$
$0.10935099 \times 10^{-2}$	$-0.24443581 \times 10^{-2}$	$-0.15255631 \times 10^{-2}$	0.36689651×10	0.85081825	0.66245782	$\dot{z}$
		t = 48				
0.13943155×10^{3}	$-0.24699587 \times 10^{3}$	$-0.12512680 \times 10^{3}$	0.32925216×10^{6}	0.33663146×10^{5}	0.36560563×10^{5}	x
0.23869937×10^{3}	$-0.58370858 \times 10^{3}$	$-0.29271258 \times 10^{3}$	0.81833664×10^{6}	0.19653123×10^{6}	0.14811569×10^{6}	y
0.14205047×10^{3}	$-0.30861168 \times 10^{3}$	$-0.21680609 \times 10^{3}$	0.47037823×10^{6}	0.10710315×10^{6}	0.90194158×10^{5}	z
$0.50771347 \times 10^{-3}$	$-0.11974235 \times 10^{-2}$	$-0.57683370 \times 10^{-3}$	0.12377150×10	0.19227611	0.18277376	$\dot{x}$
$0.91218484 \times 10^{-2}$	$-0.49606442 \times 10^{-2}$	$-0.26421436 \times 10^{-2}$	0.72438469×10	0.16862927×10	0.13088044×10	$\dot{y}$
$0.12006248 \times 10^{-2}$	$-0.26856398 \times 10^{-2}$	$-0.16367608 \times 10^{-2}$	0.40545402×10	0.94004817	0.72705702	$\dot{z}$
		t = 56				
0.15440519×10^{3}	$-0.28796186 \times 10^{3}$	$-0.14400963 \times 10^{3}$	0.36551674×10^{6}	0.39966764×10^{5}	0.42405482×10^{5}	x
0.30329984×10^{3}	$-0.73324969 \times 10^{3}$	$-0.37310757 \times 10^{3}$	0.10387933×10^{7}	0.24766624×10^{6}	0.18785071×10^{6}	y
0.17834964×10^{3}	$-0.38924538 \times 10^{3}$	$-0.26545621 \times 10^{3}$	0.59319337×10^{6}	0.13551732×10^{6}	0.11211496×10^{6}	z
$0.56343182 \times 10^{-3}$	$-0.27896740 \times 10^{-1}$	$-0.26845206 \times 10^{-1}$	0.13590841×10	0.27202492	0.24478341	$\dot{x}$
$0.23785850 \times 10^{-2}$	$-0.41168427 \times 10^{-2}$	$-0.16735315 \times 10^{-2}$	0.81034488×10	0.18715674×10	0.14566860×10	$\dot{y}$
$0.13261139 \times 10^{-2}$	$-0.13189248 \times 10^{-2}$	$-0.16482144 \times 10^{-3}$	0.44908399×10	0.10355721×10	0.79713892	$\dot{z}$

TABLE VII.- TRANSITION MATRICES FOR $\Delta x_o = \Delta y_o = \Delta z_o = 50$ STATUTE MILES AND $\Delta \dot{x}_o = \Delta \dot{y}_o = \Delta \dot{z}_o = 150$ FT/SEC

Trajectory parameter						Trajectory parameter
x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	
			t = 0			
0.09999999×10	0	0	0	0	0	x
0	0.99999817	0	0	0	0	y
0	0	0.99999786	0	0	0	z
0	0	0	0.10000011×10	0	0	$\dot{x}$
0	0	0	0	0.10000037×10	0	$\dot{y}$
0	0	0	0	0	0.10000005×10	$\dot{z}$
			t = 8			
0.36131425×10^2	-0.51122189×10^2	-0.26071957×10^2	0.74272515×10^5	0.17919816×10^4	0.47088339×10^4	x
0.12529794×10^2	-0.40359970×10^2	-0.14489219×10^2	0.52671506×10^5	0.16594514×10^5	0.94226515×10^4	y
0.96781249×10	-0.17248403×10^2	-0.23652422×10^2	0.33022979×10^5	0.67863039×10^4	0.99953991×10^4	z
$0.10635275 \times 10^{-2}$	$-0.17529363 \times 10^{-2}$	$-0.90364269 \times 10^{-3}$	0.25020556×10	0.21935239	0.25312459	$\dot{x}$
$0.89092224 \times 10^{-3}$	$-0.22028308 \times 10^{-2}$	$-0.99956005 \times 10^{-3}$	0.30799304×10	0.71661196	0.52673907	$\dot{y}$
$0.57190611 \times 10^{-3}$	$-0.10977823 \times 10^{-2}$	$-0.99007606 \times 10^{-3}$	0.18140486×10	0.38215175	0.35441954	$\dot{z}$
			t = 16			
0.63544289×10^2	-0.98886442×10^2	-0.50673691×10^2	0.14065294×10^6	0.87098128×10^4	0.12084172×10^5	x
0.43688007×10^2	-0.11516751×10^3	-0.50477597×10^2	0.15851694×10^6	0.41188915×10^5	0.27968668×10^5	y
0.28929579×10^2	-0.55783300×10^2	-0.54605820×10^2	0.95276637×10^5	0.20276059×10^5	0.21613487×10^5	z
$0.85866164 \times 10^{-3}$	$-0.15709019 \times 10^{-2}$	$-0.80700170 \times 10^{-3}$	0.21200617×10	0.24795905	0.25225343	$\dot{x}$
$0.12449138 \times 10^{-2}$	$-0.29359678 \times 10^{-2}$	$-0.14623670 \times 10^{-2}$	0.42396848×10	0.97681916	0.74588913	$\dot{y}$
$0.75125120 \times 10^{-3}$	$-0.15405395 \times 10^{-2}$	$-0.11517674 \times 10^{-2}$	0.24646781×10	0.54300027	0.44866467	$\dot{z}$
			t = 24			
0.86190275×10^2	-0.14194221×10^3	-0.72687194×10^2	0.19689001×10^6	0.15718663×10^5	0.19065229×10^5	x
0.83370507×10^2	-0.20757716×10^3	-0.97490663×10^2	0.29430272×10^6	0.72337168×10^5	0.51875346×10^5	y
0.52523154×10^2	-0.10473770×10^3	-0.89705263×10^2	0.17357441×10^6	0.37675069×10^5	0.35684137×10^5	z
$0.72025231 \times 10^{-3}$	$-0.14241736 \times 10^{-2}$	$-0.72389546 \times 10^{-3}$	0.17914113×10	0.23580052	0.23124342	$\dot{x}$
$0.15023773 \times 10^{-2}$	$-0.34612309 \times 10^{-2}$	$-0.17895956 \times 10^{-2}$	0.51645977×10	0.11805040×10	0.90896609	$\dot{y}$
$0.88301919 \times 10^{-3}$	$-0.18460243 \times 10^{-2}$	$-0.12823441 \times 10^{-2}$	0.29587504×10	0.66120511	0.52661109	$\dot{z}$
			t = 32			
0.10529246×10^3	-0.18121639×10^3	-0.92499897×10^2	0.24411532×10^6	0.22197353×10^5	0.25369856×10^5	x
0.12991359×10^3	-0.31373776×10^3	-0.15305519×10^3	0.45508175×10^6	0.10895723×10^6	0.80117399×10^5	y
0.79636385×10^2	-0.16159718×10^3	-0.12832799×10^3	0.26519140×10^6	0.58201928×10^5	0.51869605×10^5	z
$0.60985289 \times 10^{-3}$	$-0.13092489 \times 10^{-2}$	$-0.65460353 \times 10^{-3}$	0.14923350×10	0.21340479	0.20651291	$\dot{x}$
$0.17265087 \times 10^{-2}$	$-0.39018423 \times 10^{-2}$	$-0.20632464 \times 10^{-2}$	0.59914400×10	0.13599027×10	0.10499050×10	$\dot{y}$
$0.99818772 \times 10^{-3}$	$-0.20964468 \times 10^{-2}$	$-0.13980273 \times 10^{-2}$	0.33983008×10	0.76240683	0.59644960	$\dot{z}$
			t = 40			
0.12150923×10^3	-0.21787272×10^3	-0.11060401×10^3	0.28311032×10^6	0.28015869×10^5	0.30986312×10^5	x
0.18271094×10^3	-0.43198605×10^3	-0.21611742×10^3	0.63914434×10^6	0.15056682×10^6	0.11226042×10^6	y
0.10996267×10^3	-0.22524941×10^3	-0.17015435×10^3	0.36915261×10^6	0.81515294×10^5	0.70003278×10^5	z
$0.51987074 \times 10^{-3}$	$-0.12510826 \times 10^{-2}$	$-0.60826357 \times 10^{-3}$	0.12213904×10	0.19167900	0.18462899	$\dot{x}$
$0.19394774 \times 10^{-2}$	$-0.43071354 \times 10^{-2}$	$-0.23142963 \times 10^{-2}$	0.67903812×10	0.15287803×10	0.11814662×10	$\dot{y}$
$0.11074737 \times 10^{-2}$	$-0.23211861 \times 10^{-2}$	$-0.15055817 \times 10^{-2}$	0.38205983×10	0.85579304	0.66249294	$\dot{z}$
			t = 48			
0.13550772×10^3	-0.25505523×10^3	-0.12814460×10^3	0.31497127×10^6	0.33386512×10^5	0.36138093×10^5	x
0.24167836×10^3	-0.56173659×10^3	-0.28637222×10^3	0.84642703×10^6	0.19701412×10^6	0.14816801×10^6	y
0.14344365×10^3	-0.29513752×10^3	-0.21498222×10^3	0.48533804×10^6	0.10747580×10^6	0.90024927×10^5	z
$0.45914992 \times 10^{-3}$	$-0.14333755 \times 10^{-1}$	$-0.62993321 \times 10^{-3}$	0.10051869×10	0.18637518	0.17746645	$\dot{x}$
$0.21582384 \times 10^{-2}$	$-0.31366914 \times 10^{-2}$	$-0.25671741 \times 10^{-2}$	0.76134567×10	0.16978731×10	0.13130946×10	$\dot{y}$
$0.12187356 \times 10^{-2}$	$-0.12444961 \times 10^{-2}$	$-0.16053895 \times 10^{-2}$	0.42518802×10	0.94723199	0.72819823	$\dot{z}$
			t = 56			
0.14867640×10^3	-0.30466086×10^3	-0.14973271×10^3	0.34252128×10^6	0.39399525×10^5	0.41748841×10^5	x
0.30737683×10^3	-0.69845210×10^3	-0.36410730×10^3	0.10787060×10^7	0.24855517×10^6	0.18805806×10^6	y
0.18031402×10^3	-0.36816753×10^3	-0.26206679×10^3	0.61453126×10^6	0.13614115×10^6	0.11200299×10^6	z
$0.47870628 \times 10^{-3}$	$-0.15128520 \times 10^{-1}$	$-0.13995260 \times 10^{-4}$	0.95691162	0.25469789	0.23173057	$\dot{x}$
$0.24162599 \times 10^{-2}$	$-0.39468864 \times 10^{-2}$	$-0.21776102 \times 10^{-2}$	0.85496023×10	0.18887428×10	0.14633341×10	$\dot{y}$
$0.13470973 \times 10^{-2}$	$-0.16441561 \times 10^{-2}$	$-0.84359317 \times 10^{-3}$	0.47351832×10	0.10462688×10	0.80028595	$\dot{z}$

TABLE VIII.- TRANSITION MATRICES FOR $\Delta x_o = \Delta y_o = \Delta z_o = 150$ STATUTE MILES AND $\Delta \dot{x}_o = \Delta \dot{y}_o = \Delta \dot{z}_o = 300$ FT/SEC

Trajectory parameter						Trajectory parameter
x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	
t = 0						
0.09999999×10	0	0	0	0	0	x
0	0.99999939	0	0	0	0	y
0	0	0.99999928	0	0	0	z
0	0	0	0.10000022×10	0	0	$\dot{x}$
0	0	0	0	0.10000022×10	0	$\dot{y}$
0	0	0	0	0	0.10000003×10	$\dot{z}$
t = 8						
0.35471401×10^{2}	$-0.50409794 \times 10^{2}$	$-0.26457945 \times 10^{2}$	0.74086279×10^{5}	0.17272492×10^{4}	0.46013945×10^{4}	x
0.13076790×10^{2}	$-0.37318297 \times 10^{2}$	$-0.13707884 \times 10^{2}$	0.54244316×10^{5}	0.16511833×10^{5}	0.93492695×10^{4}	y
0.99449071×10	$-0.15391689 \times 10^{2}$	$-0.23421080 \times 10^{2}$	0.33904745×10^{5}	0.67507816×10^{4}	0.99167750×10^{4}	z
$0.10187337 \times 10^{-2}$	$-0.17511209 \times 10^{-2}$	$-0.92295523 \times 10^{-3}$	0.24701975×10	0.21801654	0.24953382	$\dot{x}$
$0.91556916 \times 10^{-3}$	$-0.20226207 \times 10^{-2}$	$-0.95016807 \times 10^{-3}$	0.31341942×10	0.71306567	0.52335513	$\dot{y}$
$0.58228721 \times 10^{-3}$	$-0.98181897 \times 10^{-3}$	$-0.98257685 \times 10^{-3}$	0.18708996×10	0.38115581	0.35057388	$\dot{z}$
t = 16						
0.61222373×10^{2}	$-0.98843139 \times 10^{2}$	$-0.51827879 \times 10^{2}$	0.13869341×10^{6}	0.86165181×10^{4}	0.11876309×10^{5}	x
0.45135338×10^{2}	$-0.10517729 \times 10^{3}$	$-0.47684374 \times 10^{2}$	0.16453901×10^{6}	0.41039985×10^{5}	0.27806934×10^{5}	y
0.29569902×10^{2}	$-0.49499703 \times 10^{2}$	$-0.54022151 \times 10^{2}$	0.98550856×10^{5}	0.20237490×10^{5}	0.21423788×10^{5}	z
$0.78866366 \times 10^{-3}$	$-0.16187795 \times 10^{-2}$	$-0.84134248 \times 10^{-3}$	0.20267324×10	0.24706798	0.24875288	$\dot{x}$
$0.12841994 \times 10^{-2}$	$-0.26358927 \times 10^{-2}$	$-0.13722488 \times 10^{-2}$	0.44476945×10	0.97608850	0.74335045	$\dot{y}$
$0.76768178 \times 10^{-3}$	$-0.13512781 \times 10^{-2}$	$-0.11338851 \times 10^{-2}$	0.25756843×10	0.54390401	0.44495919	$\dot{z}$
t = 24						
0.81497502×10^{2}	$-0.14398105 \times 10^{3}$	$-0.75066884 \times 10^{2}$	0.19117640×10^{6}	0.15592056×10^{5}	0.18750786×10^{5}	x
0.86254042×10^{2}	$-0.18733273 \times 10^{3}$	$-0.91520221 \times 10^{2}$	0.30810223×10^{6}	0.72226567×10^{5}	0.51664627×10^{5}	y
0.53779127×10^{2}	$-0.92056162 \times 10^{2}$	$-0.88408401 \times 10^{2}$	0.18097847×10^{6}	0.37697498×10^{5}	0.35398317×10^{5}	z
$0.62522013 \times 10^{-3}$	$-0.15212845 \times 10^{-2}$	$-0.77497952 \times 10^{-3}$	0.16207946×10	0.23422293	0.22726162	$\dot{x}$
$0.15640652 \times 10^{-2}$	$-0.30503027 \times 10^{-2}$	$-0.16590379 \times 10^{-2}$	0.55017809×10	0.11841113×10	0.90823425	$\dot{y}$
$0.91003323 \times 10^{-3}$	$-0.15919089 \times 10^{-2}$	$-0.12502276 \times 10^{-2}$	0.31372089×10	0.66463495	0.52372662	$\dot{z}$
t = 32						
0.97459199×10^{2}	$-0.18688428 \times 10^{3}$	$-0.96628079 \times 10^{2}$	0.23209827×10^{6}	0.22004606×10^{5}	0.24927794×10^{5}	x
0.13498629×10^{3}	$-0.28015245 \times 10^{3}$	$-0.14274273 \times 10^{3}$	0.48086695×10^{6}	0.10902758×10^{6}	0.79920138×10^{5}	y
0.81872623×10^{2}	$-0.14072680 \times 10^{3}$	$-0.12586807 \times 10^{3}$	0.27892420×10^{6}	0.58366446×10^{5}	0.51517932×10^{5}	z
$0.48587140 \times 10^{-3}$	$-0.14701158 \times 10^{-2}$	$-0.72596241 \times 10^{-3}$	0.12202971×10	0.21023124	0.20154334	$\dot{x}$
$0.18181319 \times 10^{-2}$	$-0.33879016 \times 10^{-2}$	$-0.18924139 \times 10^{-2}$	0.64934452×10	0.13690611×10	0.10516834×10	$\dot{y}$
$0.10399600 \times 10^{-2}$	$-0.17827087 \times 10^{-2}$	$-0.13488214 \times 10^{-2}$	0.36629526×10	0.76894137	0.59483296	$\dot{z}$
t = 40						
0.10960112×10^{3}	$-0.23027757 \times 10^{3}$	$-0.11723469 \times 10^{3}$	0.26136934×10^{6}	0.27694901×10^{5}	0.30378480×10^{5}	x
0.19094809×10^{3}	$-0.38186476 \times 10^{3}$	$-0.20032212 \times 10^{3}$	0.68231401×10^{6}	0.15099566×10^{6}	0.11215819×10^{6}	y
0.11366741×10^{3}	$-0.19431570 \times 10^{3}$	$-0.16595434 \times 10^{3}$	0.39204154×10^{6}	0.81920282×10^{5}	0.69628333×10^{5}	z
$0.35909029 \times 10^{-3}$	$-0.58482333 \times 10^{-2}$	$-0.71596754 \times 10^{-3}$	0.81019402	0.18566590	0.17790889	$\dot{x}$
$0.20689417 \times 10^{-2}$	$-0.28132498 \times 10^{-2}$	$-0.21043215 \times 10^{-2}$	0.75046971×10	0.15447286×10	0.11863954×10	$\dot{y}$
$0.11684471 \times 10^{-2}$	$-0.13263732 \times 10^{-2}$	$-0.14309719 \times 10^{-2}$	0.41966245×10	0.86607746	0.66256664	$\dot{z}$
t = 48						
0.11825771×10^{3}	$-0.27713403 \times 10^{3}$	$-0.13901747 \times 10^{3}$	0.27864809×10^{6}	0.32819668×10^{5}	0.35290693×10^{5}	x
0.25427772×10^{3}	$-0.48705836 \times 10^{3}$	$-0.26361333 \times 10^{3}$	0.91394711×10^{6}	0.19801636×10^{6}	0.14826030×10^{6}	y
0.14923523×10^{3}	$-0.24973074 \times 10^{3}$	$-0.20759903 \times 10^{3}$	0.52104837×10^{6}	0.10824060×10^{6}	0.89682449×10^{5}	z
$0.24477089 \times 10^{-3}$	$-0.16069633 \times 10^{-2}$	$-0.51327803 \times 10^{-2}$	0.38880778	0.17443918	0.16695824	$\dot{x}$
$0.23326051 \times 10^{-2}$	$-0.36725461 \times 10^{-2}$	$-0.17490567 \times 10^{-2}$	0.86015064×10	0.17220190×10	0.13217684×10	$\dot{y}$
$0.13033541 \times 10^{-2}$	$-0.19369680 \times 10^{-2}$	$-0.10064129 \times 10^{-2}$	0.47724430×10	0.96211188	0.73049524	$\dot{z}$
t = 56						
0.12397543×10^{3}	$-0.32147390 \times 10^{3}$	$-0.16339421 \times 10^{3}$	0.28394923×10^{6}	0.38268544×10^{5}	0.40451481×10^{5}	x
0.32564419×10^{3}	$-0.59415745 \times 10^{3}$	$-0.32894397 \times 10^{3}$	0.11794450×10^{7}	0.25039176×10^{6}	0.18845956×10^{6}	y
0.18890714×10^{3}	$-0.30633312 \times 10^{3}$	$-0.24718691 \times 10^{3}$	0.66777667×10^{6}	0.13741779×10^{6}	0.11177076×10^{6}	z
$0.16111887 \times 10^{-3}$	$-0.14605300 \times 10^{-2}$	$-0.82192985 \times 10^{-3}$	$-0.86886227 \times 10^{-2}$	0.22245477	0.20749206	$\dot{x}$
$0.26327994 \times 10^{-2}$	$-0.37654679 \times 10^{-2}$	$-0.22509822 \times 10^{-2}$	0.98779433×10	0.19228441×10	0.14761472×10	$\dot{y}$
$0.14562739 \times 10^{-2}$	$-0.19942962 \times 10^{-2}$	$-0.13230559 \times 10^{-2}$	0.54381294×10	0.10673357×10	0.80596763	$\dot{z}$

TABLE IX.- NOMINAL PARTIAL DERIVATIVES AND PARTIAL DERIVATIVES

RESULTING FROM INCREASES IN DISTURBANCE

Partial derivative	Disturbance		Elapsed time from insertion, t, hr						
	Variable	Increment	8	16	24	32	40	48	56
(a) Nominal partial derivatives									
$\partial x_t/\partial x_o$	Δx_o, mile	1	36.421753	64.593933	88.310780	108.79968	126.77417	143.07934	159.87402
$\partial \dot{x}_t/\partial \dot{x}_o$	$\Delta \dot{x}_o$, ft/sec	10	2.5274100	2.1923149	1.9173978	1.6831032	1.4972470	1.4198540	1.7415227
(b) Percent of nominal partial derivative									
$\partial x_t/\partial x_o$	Δx_o, miles	2	99.998857	99.971040	99.954496	99.938150	99.919600	99.896940	99.860503
		6	99.923469	99.840630	99.762760	99.681290	99.589810	99.475990	99.287896
		10	99.859197	99.709857	99.569800	99.422790	99.257850	99.052798	98.716227
		25	99.646543	99.216578	98.841340	98.445960	98.006573	97.450510	96.579288
		50	99.202873	98.375010	97.598800	96.776440	95.846990	94.708096	92.995970
		150	97.390700	94.780380	92.284880	89.576730	86.453830	82.651842	77.545700
$\partial \dot{x}_t/\partial \dot{x}_o$	$\Delta \dot{x}_o$, ft/sec	20	99.935578	99.786176	99.577558	99.280260	98.838910	98.079180	96.345030
		40	99.800274	99.344920	98.709450	97.802960	96.453590	94.186832	89.391960
		50	99.729740	99.118330	98.263996	97.043750	95.226590	92.210710	86.060460
		75	99.553280	98.542150	97.123360	95.089550	92.063090	87.171990	78.039990
		150	98.996820	96.704250	93.429980	88.665690	81.575710	70.795090	54.946840
		300	97.736320	92.447140	84.530950	72.502810	54.112223	27.383645	-.04989095

TABLE X.- PARTIAL DERIVATIVES RESULTING FROM SHIFTS IN REFERENCE TRAJECTORY AND INSTANTANEOUS CHANGES IN INSERTION VELOCITY

Partial derivative	Change in trajectory		Percent of nominal value for elapsed hours from insertion of -						
	Variable	Increment	8	16	24	32	40	48	56
(a) Shifts in reference trajectory									
$\partial x_t/\partial x_o$	$\Delta x_o'$, miles	5	99.661156	99.839932	99.926988	100.35763	100.71014	101.18812	102.11485
		10	99.319629	99.671740	100.12657	100.68008	101.36249	102.30525	104.22084
		25	98.307789	99.140326	100.20078	101.47205	103.04351	105.31974	110.52737
		75	94.940458	97.039424	99.598594	102.59187	106.44042	112.60483	115.92266
$\partial \dot{x}_t/\partial \dot{x}_o$	$\Delta \dot{x}_o'$, miles	5	100.10176	100.60314	101.35150	102.45181	104.18660	107.43349	116.31476
		10	100.19911	101.17991	102.63745	104.77485	108.15493	114.78247	134.59661
		25	100.46501	102.77271	106.13096	111.02117	119.01275	137.05800	187.64829
		75	101.10774	106.86097	114.80748	126.47029	149.66146	177.29956	88.872490
(b) Instantaneous changes in insertion velocity									
$\partial x_t/\partial x_o$	$\Delta x_o'$, ft/sec	10	99.945368	99.792122	99.613661	99.404006	99.151586	98.820584	98.245703
		20	99.919224	99.609377	99.249848	98.825180	98.313008	97.641512	96.513154
		40	99.836106	99.204485	98.464467	97.589782	96.526643	95.145148	92.944437
$\partial \dot{x}_t/\partial \dot{x}_o$	$\Delta \dot{x}_o'$, ft/sec	10	99.909800	99.543140	99.003340	98.215010	97.024380	94.941020	90.215798
		20	99.816140	99.070760	97.976220	96.370360	93.929870	89.785830	81.059592
		40	99.607720	98.071460	95.797580	92.441320	87.350270	79.067120	64.012240

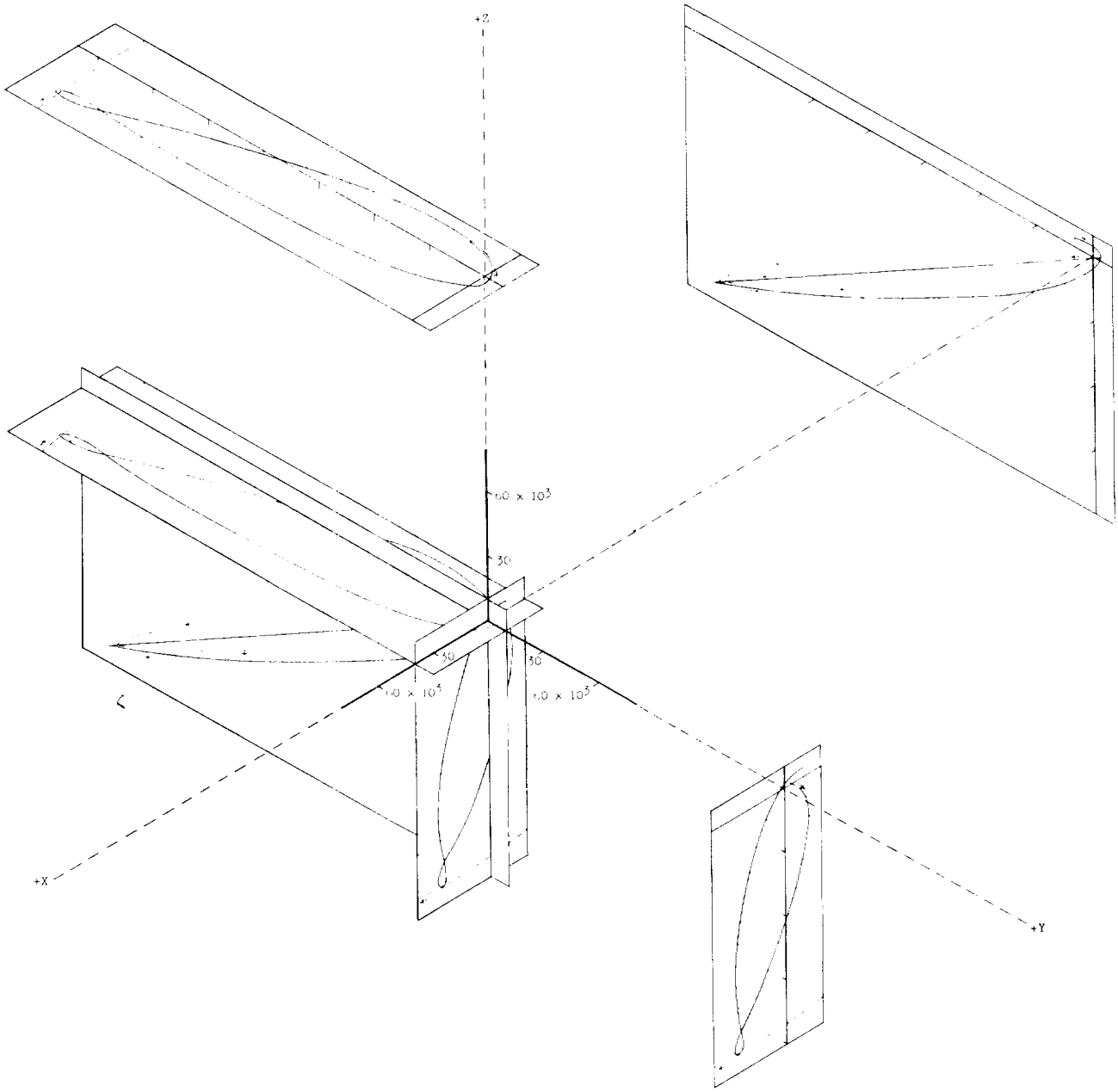


Figure 1.- Circumlunar trajectory projected to XY-, YZ-, and ZX-planes. All dimensions are in statute miles.

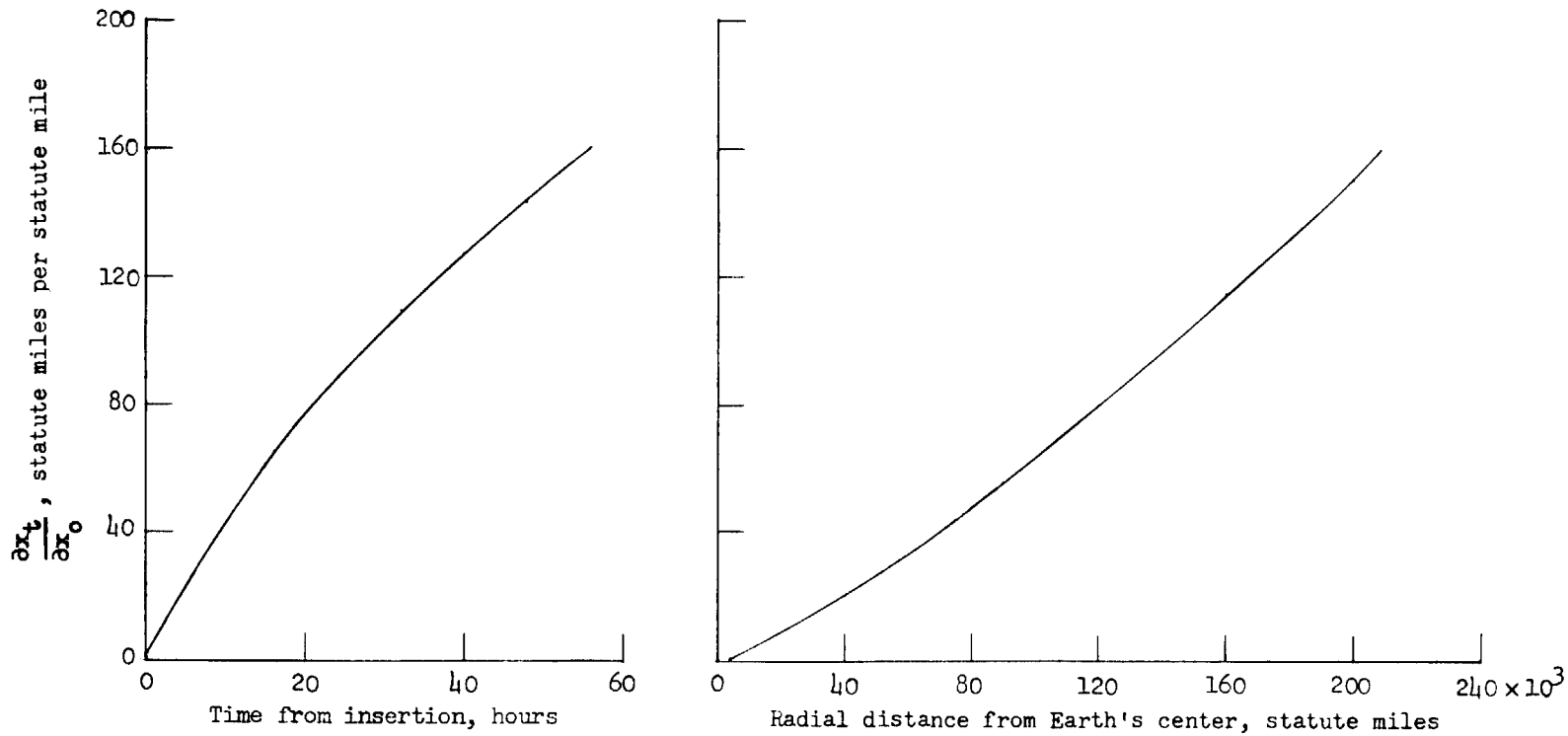


Figure 2.- Partial derivative of x component of nominal trajectory with respect to x_o as a function of time from insertion and of radial distance from center of Earth.

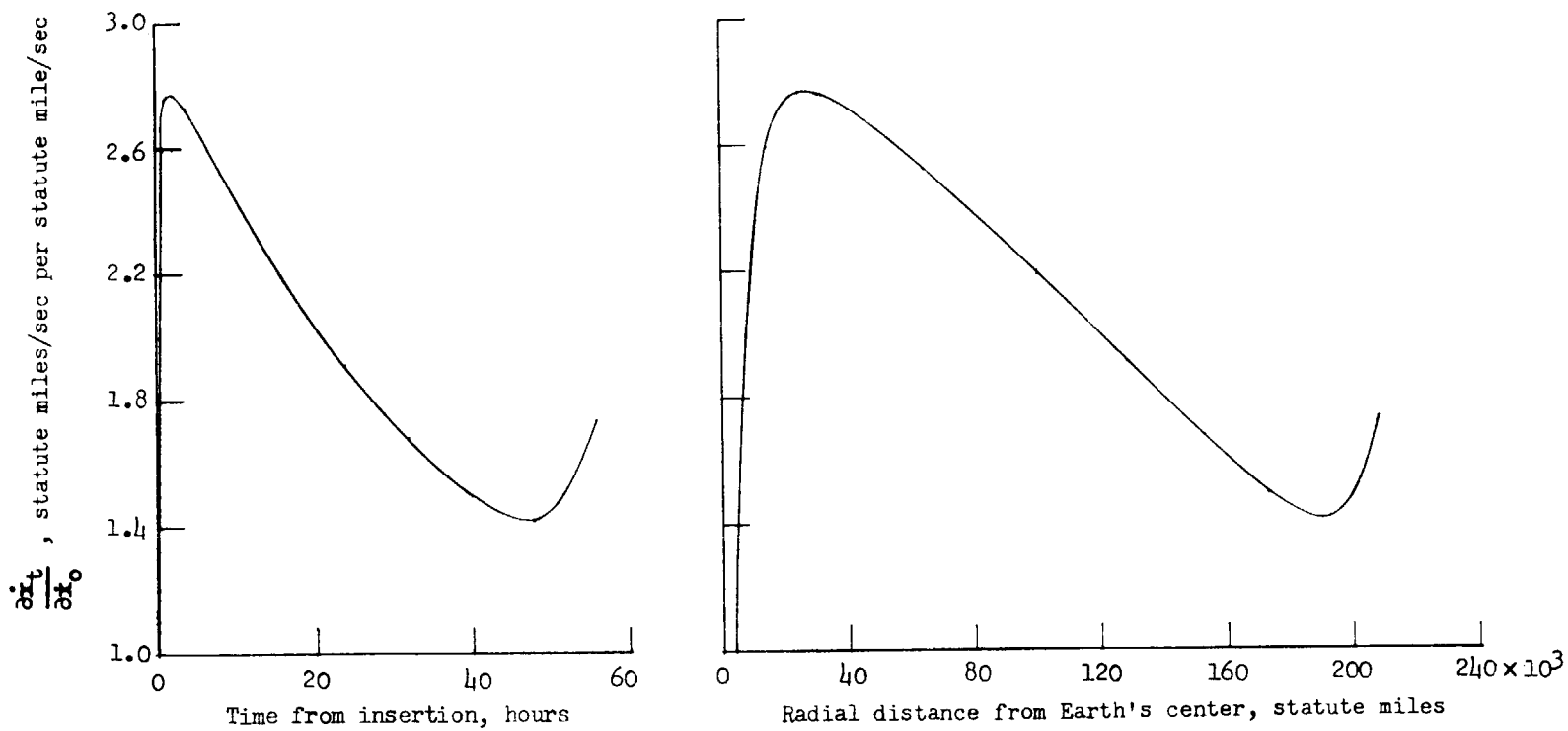


Figure 3.- Partial derivative of $\dot{x}_t$ with respect to $\dot{x}_o$ as a function of time and of distance from Earth's center.

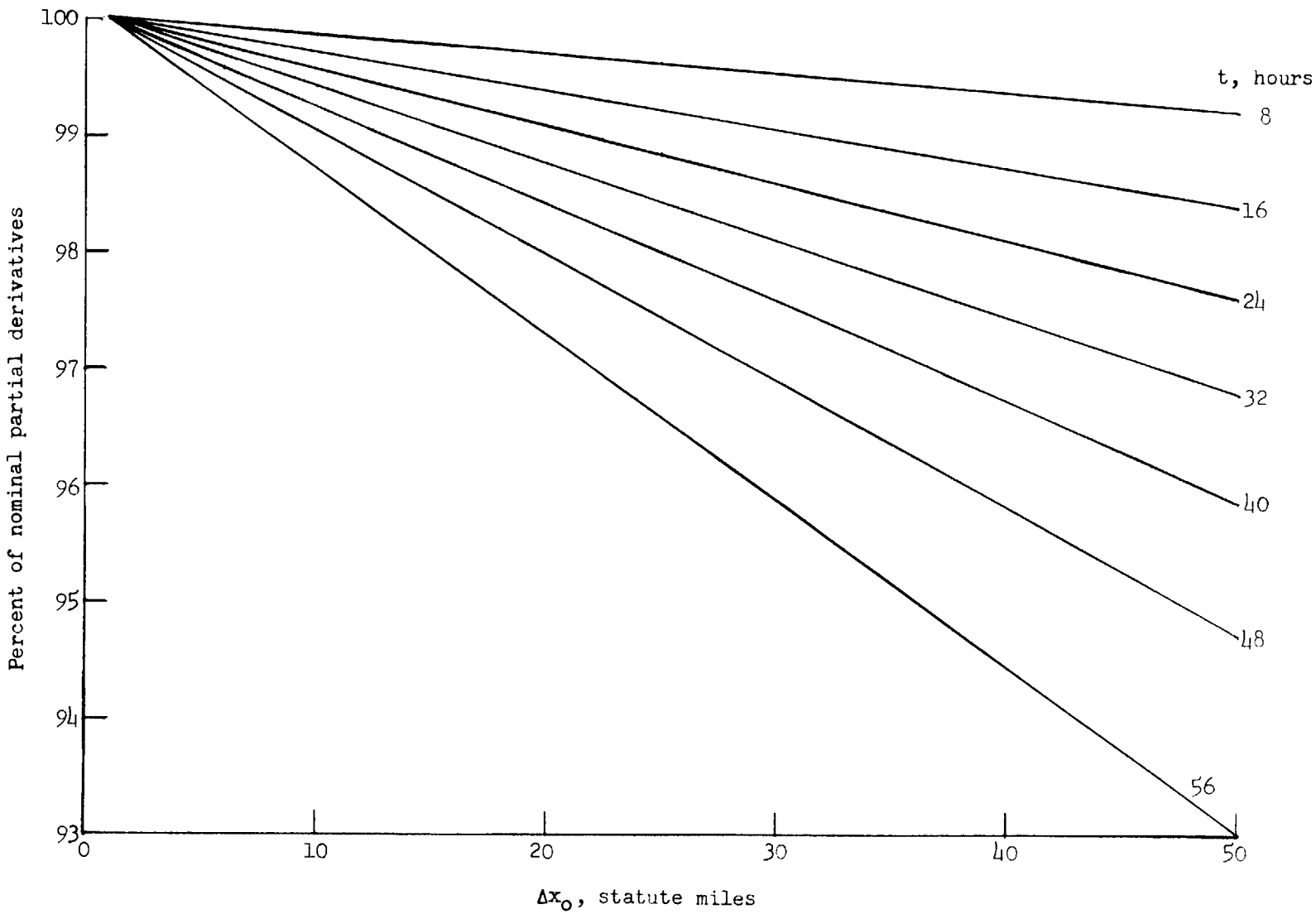


Figure 4.- Displacement partial derivative $\frac{\partial x_t}{\partial x_o}$ as a function of increasing disturbance Δx_o.

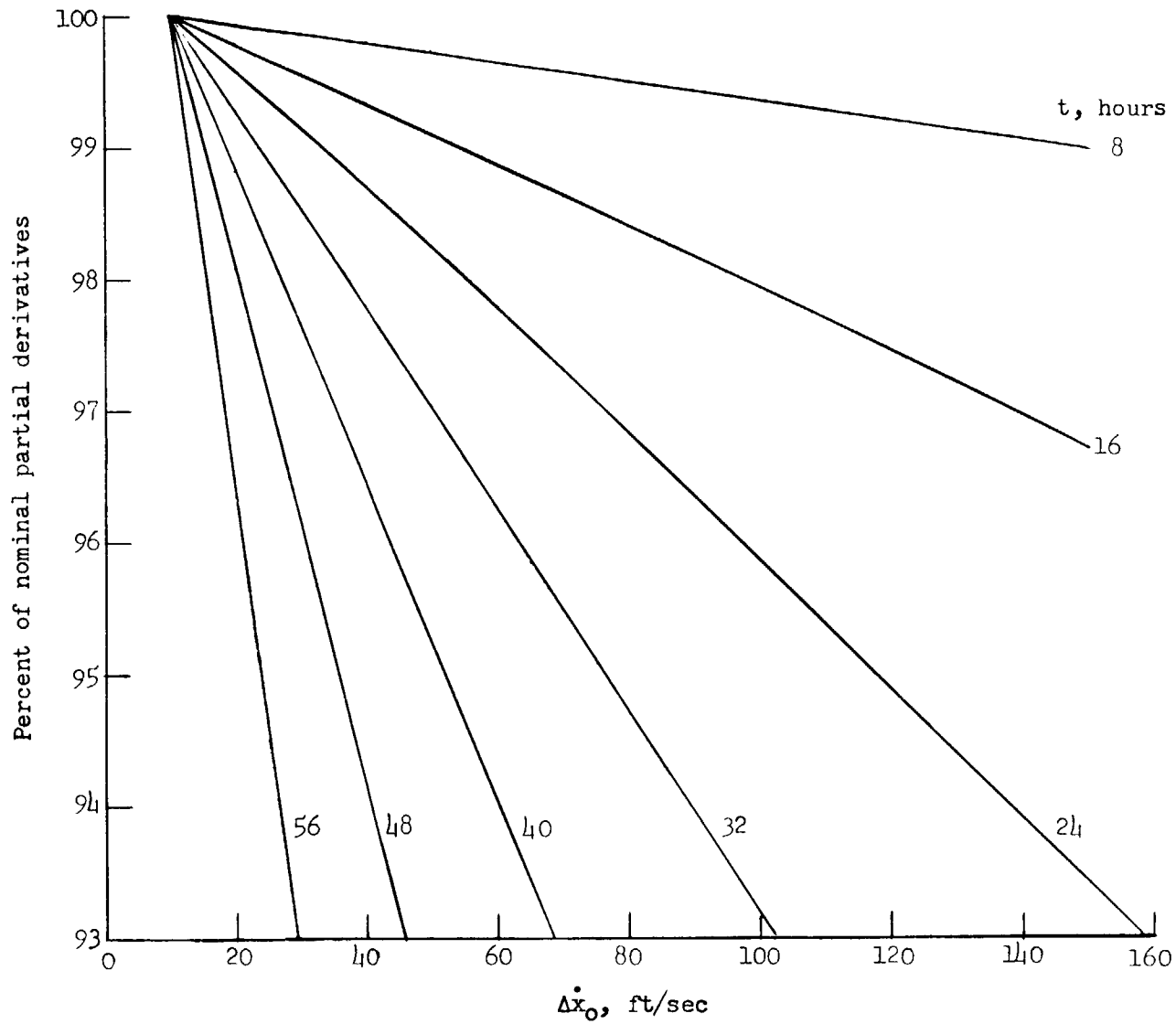


Figure 5.- Velocity partial derivative $\frac{\partial \dot{x}_t}{\partial \dot{x}_o}$ as a function of increasing disturbance $\Delta\dot{x}_o$.

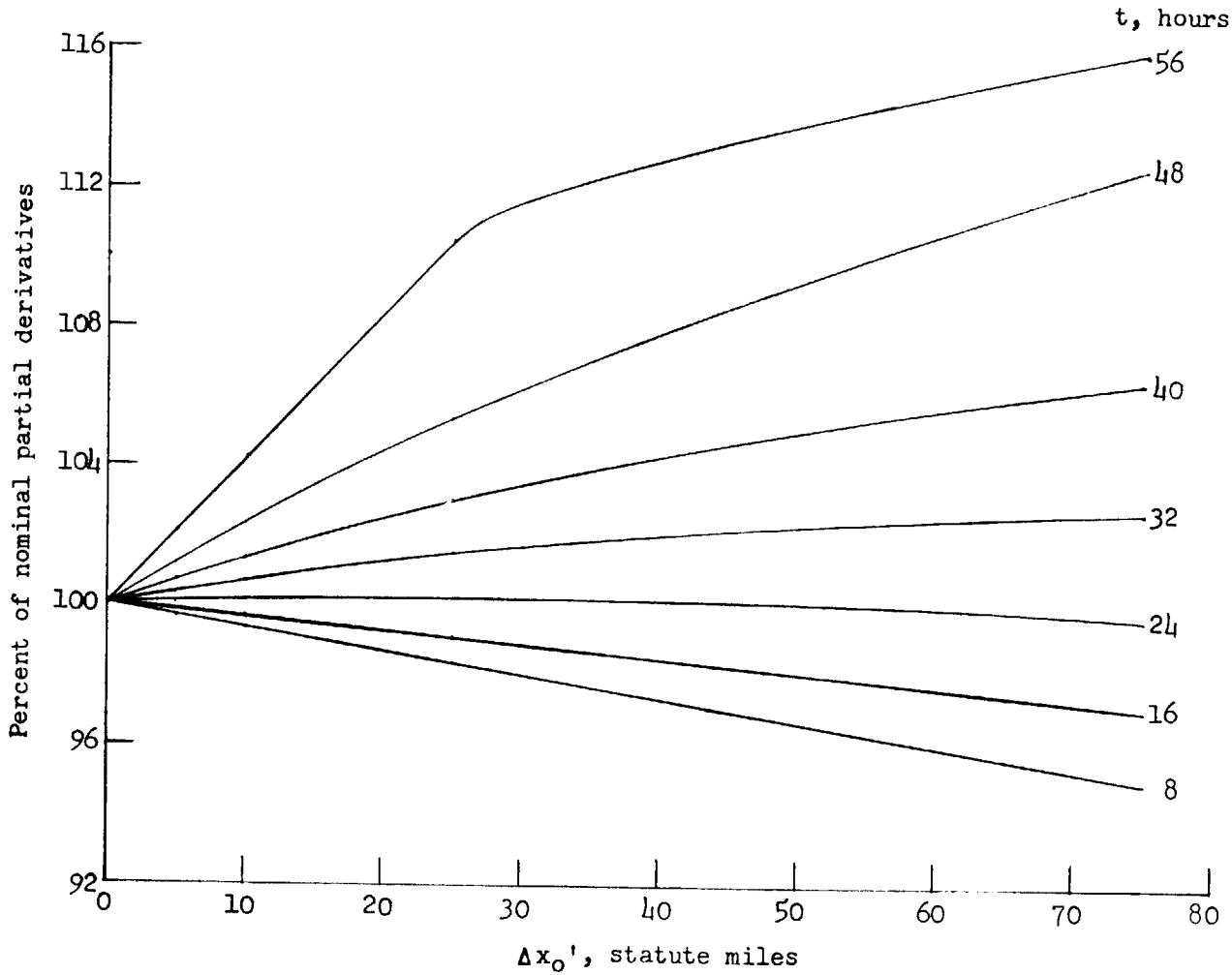


Figure 6.- Displacement partial derivative $\frac{\partial x_t'}{\partial x_o'}$ as a function of shifts in reference trajectory $\Delta x_o'$.

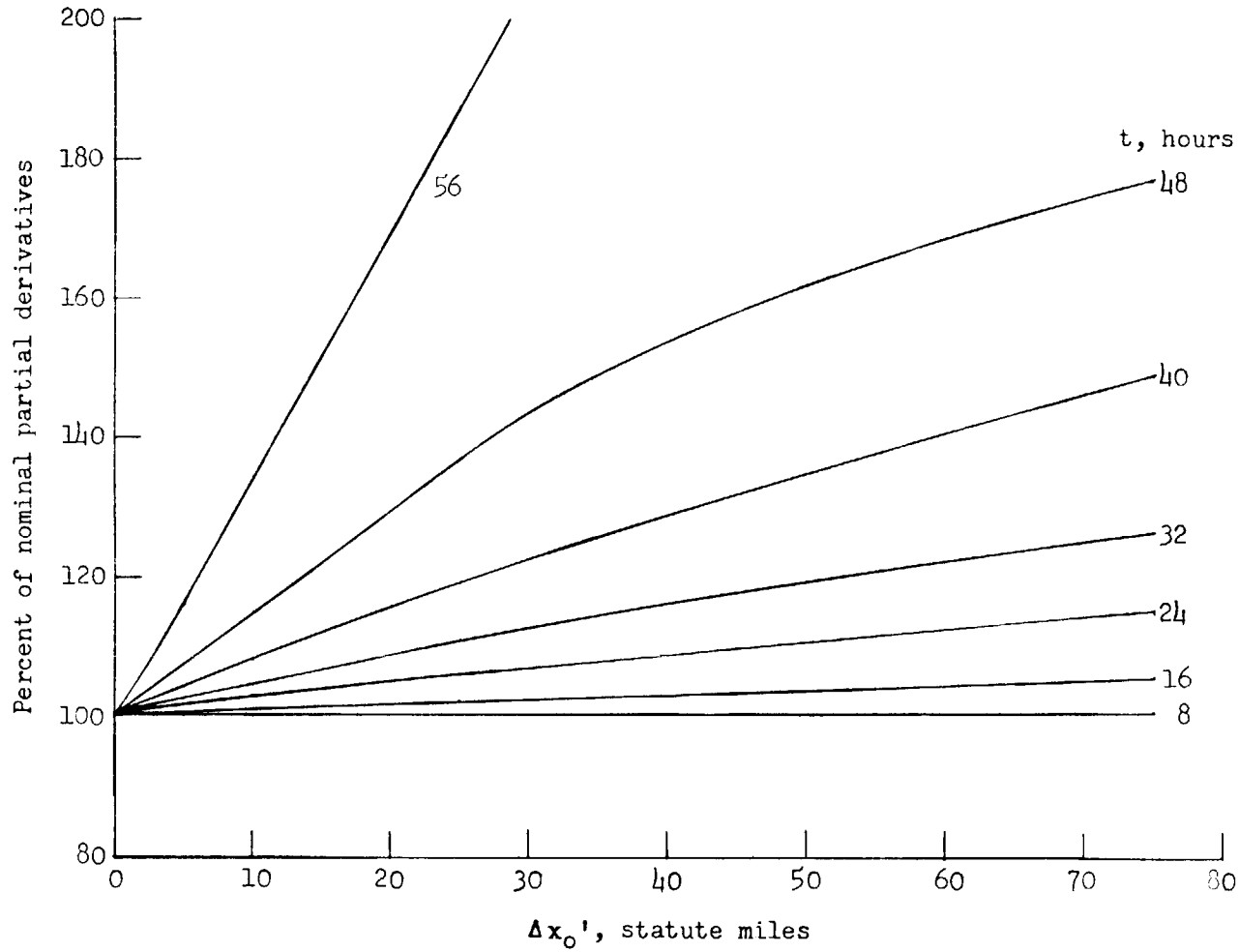


Figure 7.- Velocity partial derivative $\frac{\partial \dot{x}_t'}{\partial \dot{x}_o'}$ as a function of shifts in reference trajectory $\Delta x_o'$.

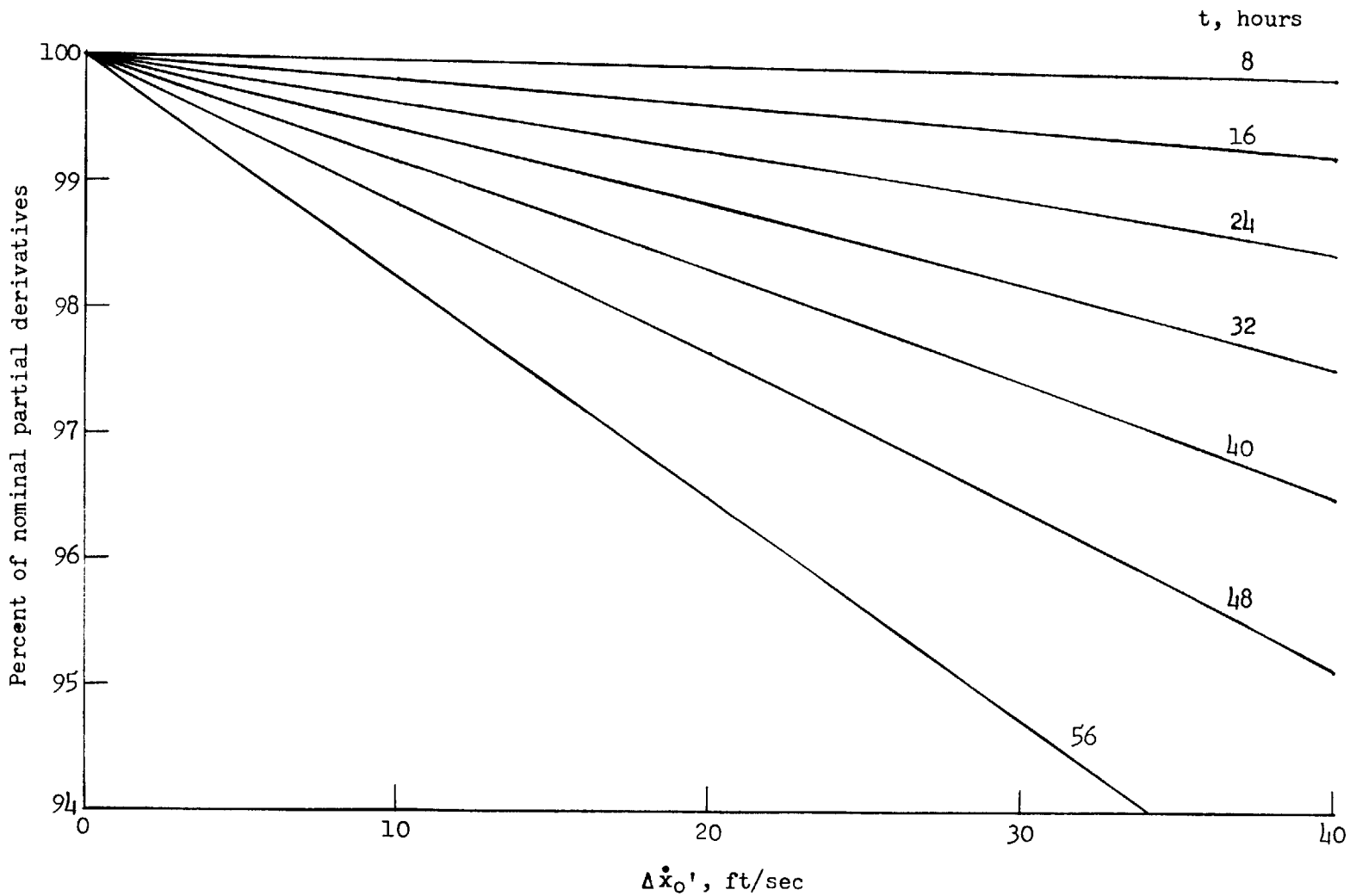


Figure 8.- Displacement partial derivative $\frac{\partial x_t'}{\partial x_o'}$ as a function of instantaneous change in reference-trajectory velocity $\Delta \dot{x}_o'$.

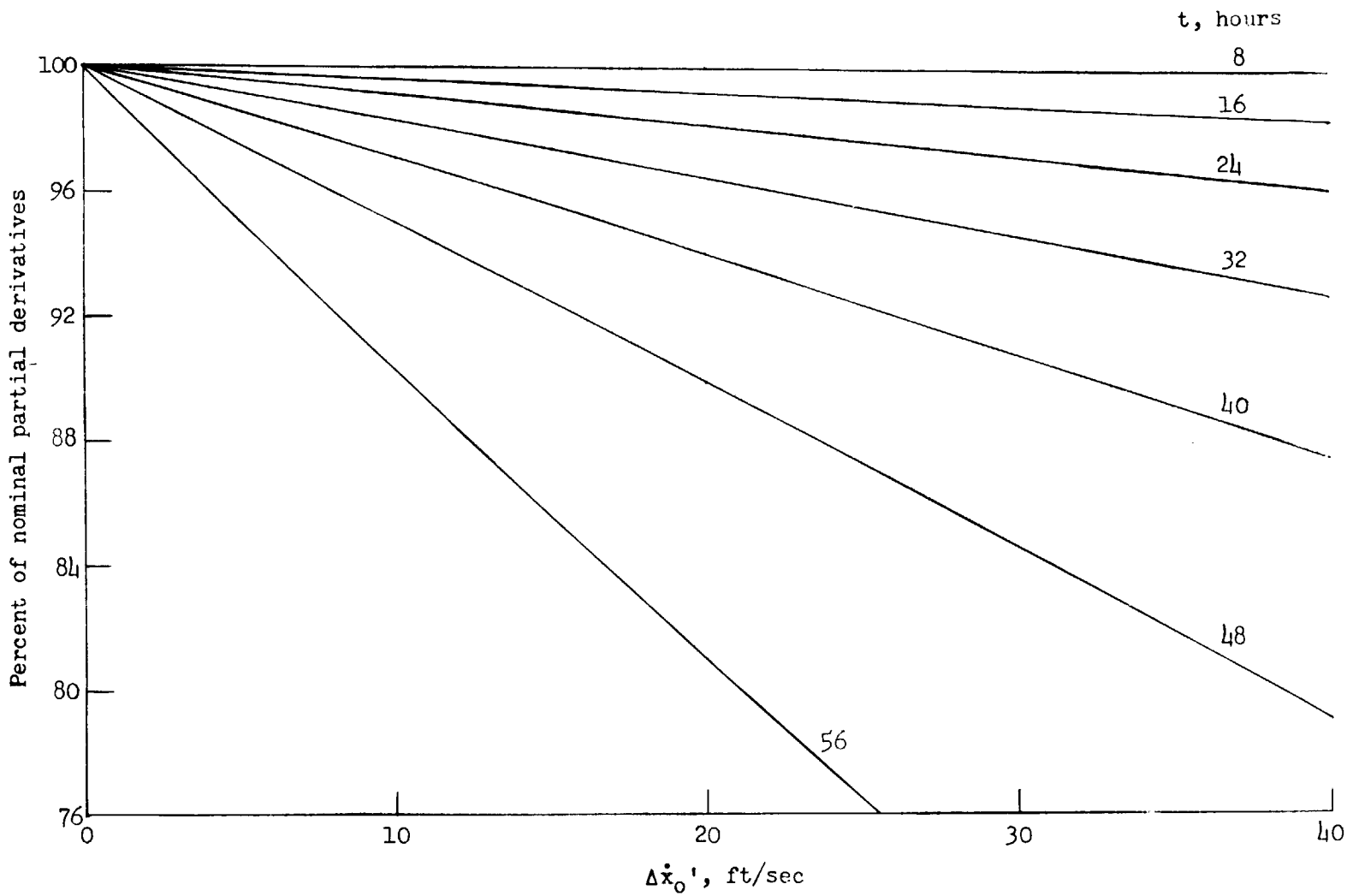


Figure 9.- Velocity partial derivative $\frac{\partial\dot{x}_t'}{\partial\dot{x}_o'}$ as a function of instantaneous change in reference-trajectory velocity $\Delta\dot{x}_o'$.

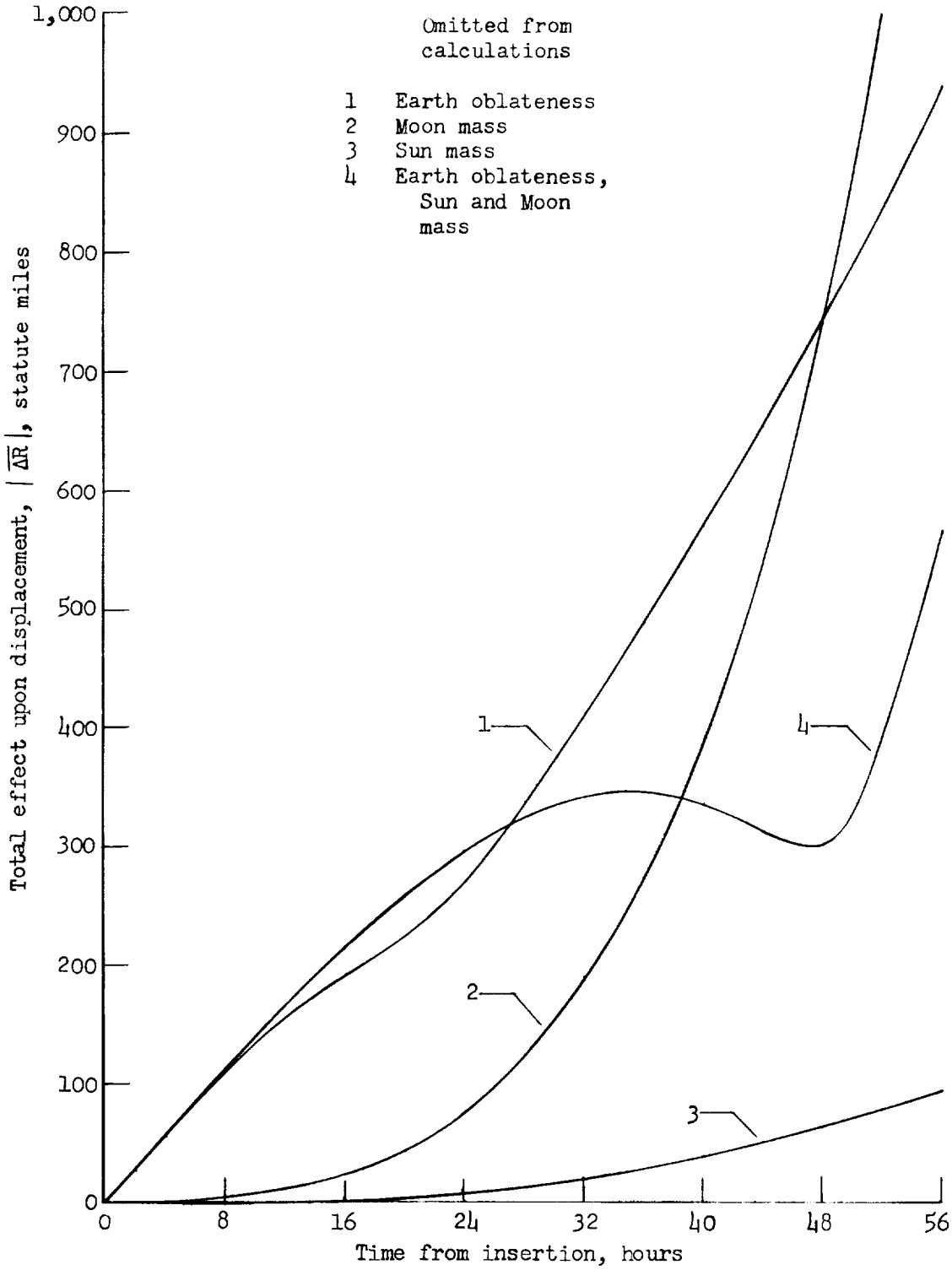


Figure 10.- Effect on trajectory displacement of omitting mass.

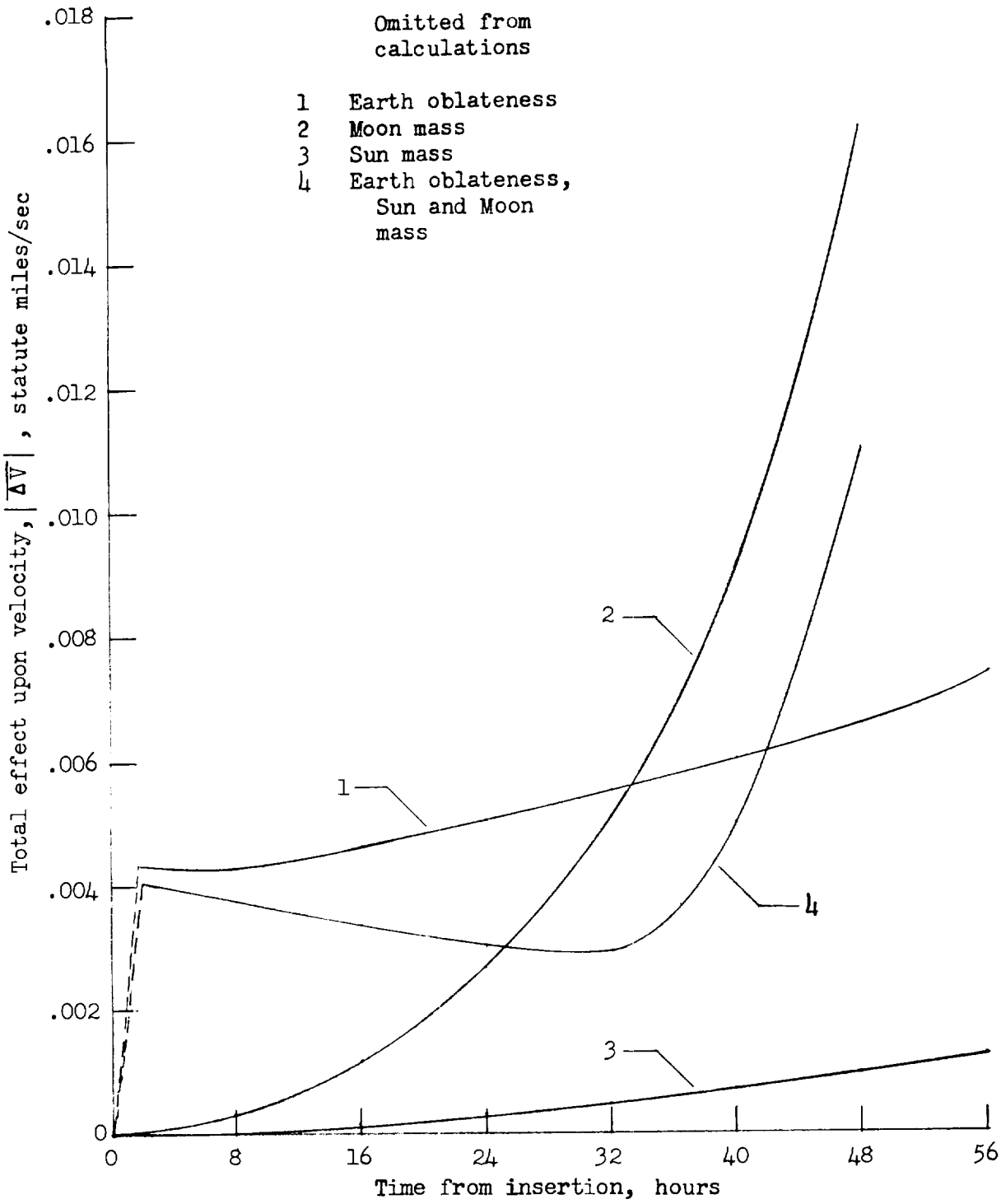
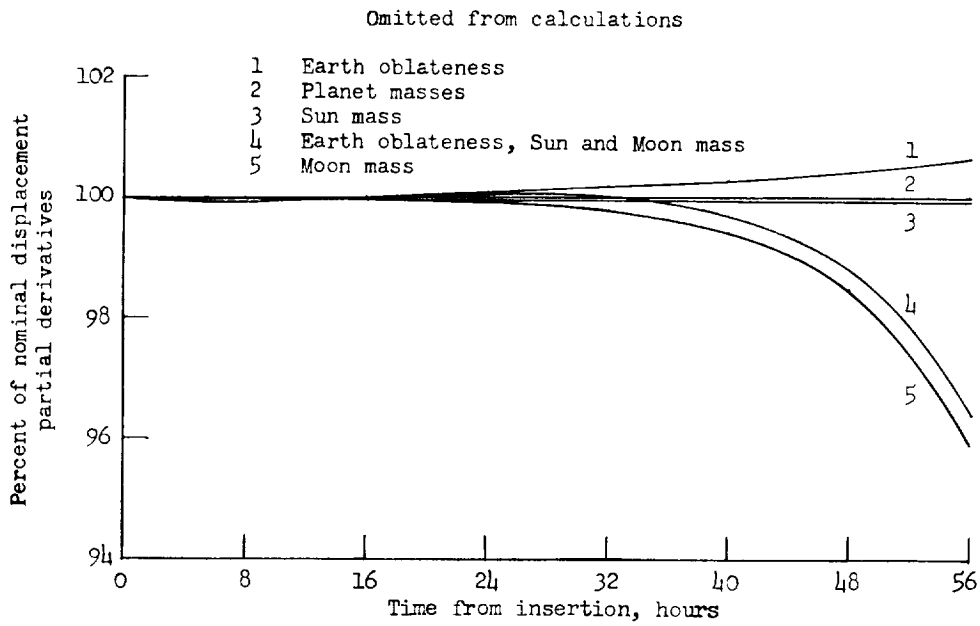
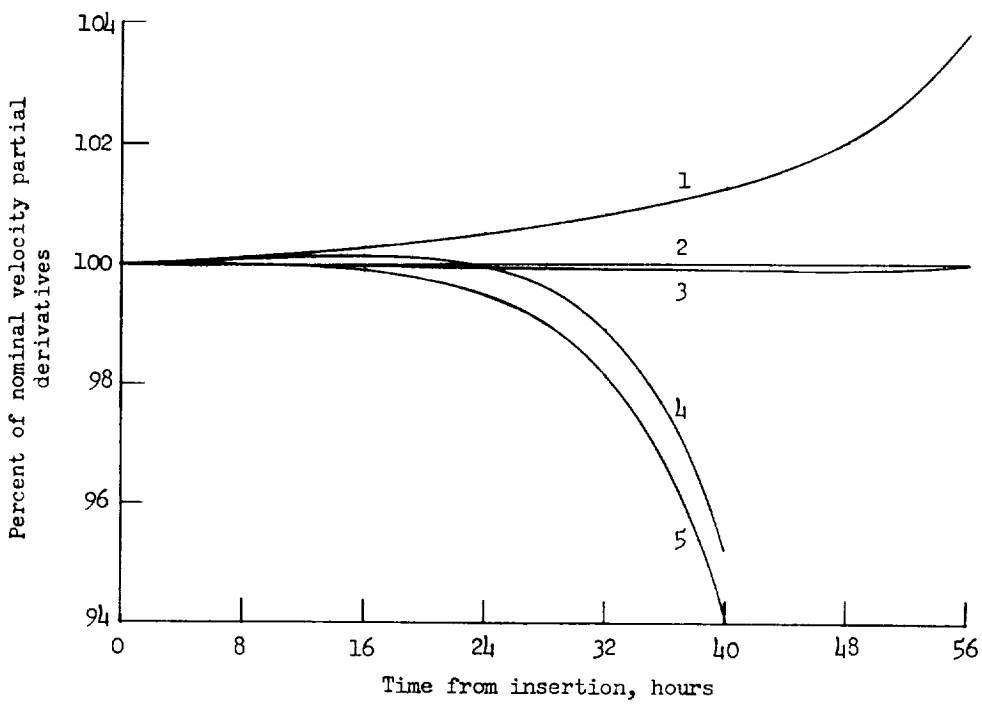


Figure 11.- Effect on trajectory velocity of omitting mass.



(a) Effect on $\frac{\partial x_t}{\partial x_o}$.



(b) Effect on $\frac{\partial \dot{x}_t}{\partial \dot{x}_o}$.

Figure 12.- Effect on position partial derivative $\frac{\partial x_t}{\partial x_o}$ and velocity partial derivative $\frac{\partial \dot{x}_t}{\partial \dot{x}_o}$ of omitting mass.

NASA-Langley, 1963 L-3461

NASA TN D-1812
National Aeronautics and Space Administration.
A STUDY OF SOME TRANSITION MATRIX ASSUMPTIONS IN CIRCUMLUNAR NAVIGATION THEORY.
Ruben L. Jones and Alton P. Mayo. October 1963.
36p. OTS price, $1.00.
(NASA TECHNICAL NOTE D-1812)

A study was made of the transition matrices utilized in midcourse navigation systems for circumlunar missions to compute future deviations from the reference trajectory. The columns of the matrices were obtained by computing the effect on the individual components of the reference-trajectory position and velocity per unit change in the respective components of the insertion position and velocity. The individual effects of excluding various masses from the trajectory computation scheme, the effect of increasing the disturbance, and the effect on the nominal partial derivatives of changing the reference trajectory were studied.

I. Jones, Ruben L.
II. Mayo, Alton P.
III. NASA TN D-1812

NASA

NASA TN D-1812
National Aeronautics and Space Administration.
A STUDY OF SOME TRANSITION MATRIX ASSUMPTIONS IN CIRCUMLUNAR NAVIGATION THEORY.
Ruben L. Jones and Alton P. Mayo. October 1963.
36p. OTS price, $1.00.
(NASA TECHNICAL NOTE D-1812)

A study was made of the transition matrices utilized in midcourse navigation systems for circumlunar missions to compute future deviations from the reference trajectory. The columns of the matrices were obtained by computing the effect on the individual components of the reference-trajectory position and velocity per unit change in the respective components of the insertion position and velocity. The individual effects of excluding various masses from the trajectory computation scheme, the effect of increasing the disturbance, and the effect on the nominal partial derivatives of changing the reference trajectory were studied.

I. Jones, Ruben L.
II. Mayo, Alton P.
III. NASA TN D-1812

NASA

NASA TN D-1812
National Aeronautics and Space Administration.
A STUDY OF SOME TRANSITION MATRIX ASSUMPTIONS IN CIRCUMLUNAR NAVIGATION THEORY.
Ruben L. Jones and Alton P. Mayo. October 1963.
36p. OTS price, $1.00.
(NASA TECHNICAL NOTE D-1812)

A study was made of the transition matrices utilized in midcourse navigation systems for circumlunar missions to compute future deviations from the reference trajectory. The columns of the matrices were obtained by computing the effect on the individual components of the reference-trajectory position and velocity per unit change in the respective components of the insertion position and velocity. The individual effects of excluding various masses from the trajectory computation scheme, the effect of increasing the disturbance, and the effect on the nominal partial derivatives of changing the reference trajectory were studied.

I. Jones, Ruben L.
II. Mayo, Alton P.
III. NASA TN D-1812

NASA

NASA TN D-1812
National Aeronautics and Space Administration.
A STUDY OF SOME TRANSITION MATRIX ASSUMPTIONS IN CIRCUMLUNAR NAVIGATION THEORY.
Ruben L. Jones and Alton P. Mayo. October 1963.
36p. OTS price, $1.00.
(NASA TECHNICAL NOTE D-1812)

A study was made of the transition matrices utilized in midcourse navigation systems for circumlunar missions to compute future deviations from the reference trajectory. The columns of the matrices were obtained by computing the effect on the individual components of the reference-trajectory position and velocity per unit change in the respective components of the insertion position and velocity. The individual effects of excluding various masses from the trajectory computation scheme, the effect of increasing the disturbance, and the effect on the nominal partial derivatives of changing the reference trajectory were studied.

I. Jones, Ruben L.
II. Mayo, Alton P.
III. NASA TN D-1812

NASA