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Free Vibrations of Discrete Structures with Non-Diagonal Symmetric Mass Matrix

Sandra M. Barondess
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CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

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ABSTRACT

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The linear eigenvalue problem resulting from the free vibrations of an n-deg of freedom system is in the form

$$[K]\{x\} = \omega^2 [M_a]\{x\}$$

where $[K]$ is the stiffness matrix, $[M_a]$ the mass matrix, and ω the natural frequency. By suitable transformation this problem is reduced to the standard form

$$A\{x\} = \lambda\{x\}$$

and by successive rotations (Jacobi's method) $[A]$ is diagonalized to yield the eigenvalues (squares of the natural frequencies) and the associated eigenvectors (the natural modes of vibrations).

A digital computer program was developed to give the eigenvalues and the eigenvectors of the original problem.

AUTHOR

I. INTRODUCTION

This work was initiated to compute the natural frequencies and the associated modes of the Dynamic Model of the Advanced Antenna System for the JPL/NASA Deep Space Instrumentation Facility. The stiffness and mass matrices used for "debugging" the program were obtained from a computer program which was based on the preliminary studies of Blaw-Knox for the Advanced Antenna System. This latter program is given in Appendix A.

II. FORMULATION OF THE PROBLEM

The dynamic equilibrium equations (d'Alembert equations) of a discrete structure are of the form

$$[K]\{X\} = -[M_a]\{\ddot{X}\} \tag{1}$$

where $[K]$ is the stiffness matrix, $[M_a]$ the mass matrix, and $\{X\}$ the displacement vector. If the structure undergoes free vibration, it can be written

$$\{X\} = \{x\} \sin \omega t \tag{2}$$

Substituting $\{X\}$ from Eq. (2) into Eq. (1) yields the linear eigenvalue problem

$$[K]\{x\} = \omega^2 [M_a]\{x\} \tag{3}$$

To solve the above eigenvalue problem numerically, an iteration method can be applied. In this work, iteration with successive rotations is chosen because (a) both $[K]$ and $[M_a]$ are real symmetric matrices, (b) $[K]$ is positive definite, and (c) all eigenvalues and eigenvectors are computed with the same degree of accuracy, in contrast to Stodola's iteration in which the eigenvalues are computed one at a time with descending degree of accuracy.

A. Diagonalization with Successive Rotations[1]

The eigenvalue problem associated with a real positive definite symmetric matrix $[A]$ is:

$$[A]\{x\} = \lambda\{x\} \tag{4}$$

Let $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_n$ be the eigenvalues and $\{X_1\}, \{X_2\}, \{X_3\}, \ldots\ldots, \{X_n\}$ be the associated eigenvectors. Defining $[D]$ and $[M]$ as

$$[D] = \begin{bmatrix} \lambda_1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \lambda_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & & \cdot & & & & & & & \cdot \\ \cdot & & & \cdot & & & & & & \cdot \\ \cdot & & & & \cdot & & & & & \cdot \\ \cdot & & & & & \cdot & & & & \cdot \\ \cdot & & & & & & \cdot & & & \cdot \\ \cdot & & & & & & & \cdot & & \cdot \\ \cdot & & & & & & & & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_n \end{bmatrix}, \quad [M] = \begin{bmatrix} \uparrow & \uparrow & \uparrow & & \uparrow \\ \{X_1\} & \{X_2\} & \{X_3\} & \cdots\cdots & \{X_n\} \\ \downarrow & \downarrow & \downarrow & & \downarrow \end{bmatrix}, \tag{5, 6}$$

it can be written

$$[A][M] = [M][D] \tag{7}$$

If the eigenvectors are normalized

$$\{X_i\}^T \{X_j\} = \delta_{ij} \tag{8}$$

holds, and it follows that

$$[M]^T = [M]^{-1} \tag{9}$$

Using Eq. (9), Eq. (7) can be written as

$$[D] = [M]^T [A] [M] \tag{10}$$

or

$$[A] = [M][D][M]^T \tag{11}$$

Equation (10) implies that $[A]$ can be diagonalized by pre- and post-multiplying it by $[t_i]^T$ and $[t_i]$, respectively, where $[t_i]$ satisfies Eq. (9), as follows:

$$\begin{aligned} &[t_m]^T [t_{m-1}]^T \cdots [t_2]^T [t_1]^T \\ &[A] [t_1] [t_2] \cdots [t_{m-1}]^T [t_m] \end{aligned} \tag{12}$$

When $[A]$ is completely diagonalized, then

$$[M]^T = [t_m]^T [t_{m-1}]^T \cdots [t_2]^T [t_1]^T \tag{13}$$

$$[M] = [t_1] [t_2] \cdots [t_{m-1}] [t_m] \tag{14}$$

It can be shown that $a_{p,q}$ (where $p \neq q$) can be made zero by selecting

$$[t_i] = \begin{array}{c} \left[\begin{array}{cccccccccc} 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & & & & & & & & 0 \\ \cdot & & c & & & & -s & & & \cdot \\ \cdot & & & 1 & & & & & & \cdot \\ \cdot & & & & \cdot & & & & & \cdot \\ \cdot & & & & & \cdot & & & & \cdot \\ \cdot & & s & & & & c & & & \cdot \\ \cdot & & & & & & & 1 & & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{array}\right] \end{array} \begin{array}{l} \\ \\ -p \\ \\ \\ \\ -q \\ \\ \\ \end{array} \tag{15}$$

where

$$s = \sin \theta \tag{16}$$

$$c = \cos \theta \tag{17}$$

[1]See Ref. 1.

$$\tan 2\theta = \frac{2a_{pq}}{a_{pp} - a_{qq}} \tag{18}$$

An iterative scheme using $[t_i]$ in the form of Eq. (13) can be established to nullify all the off-diagonal elements of $[A]$. This iteration will converge if $[A]$ is a positive definite matrix.

B. Reduction of Eq. (3) to Eq. (4)

In order to solve the eigenvalue problem resulting from the Dynamic Analysis of a discrete structure by successive rotations, Eq. (3) must be reduced to the standard form of Eq. (4). This can be done as follows:

Define

$$1/\omega_i^2 = \lambda_i \tag{19}$$

and

$$[D] = \begin{bmatrix} \lambda_1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \lambda_2 & & & & & \cdot \\ \cdot & & \cdot & & & & \cdot \\ \cdot & & & \cdot & & & \cdot \\ \cdot & & & & \cdot & & \cdot \\ \cdot & & & & & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \lambda_n \end{bmatrix} \tag{20}$$

Then Eq. (3) can be written

$$[M_a]\,[M] = [K]\,[M]\,[D] \tag{21}$$

where $[M]$ is defined in Eq. (6). Since $[K]$ is positive definite it can be written

$$[K] = [m]\,[d]\,[m]^T \tag{22}$$

which makes possible the definition

$$[K]^{1/2} = [m]\,[d]^{1/2}\,[m]^T \tag{23}$$

$$[K]^{-1/2} = [m]\,[d]^{-1/2}\,[m]^T \tag{24}$$

Then Eq. (21) can be rewritten

$$[M_a]\,[K]^{-1/2}\,[K]^{1/2}\,[M] = [K]^{1/2}\,[K]^{1/2}\,[M]\,[D] \tag{25}$$

or

$$[K]^{-1/2}\,[M_a]\,[K]^{-1/2}\,[K]^{1/2}\,[M] = [K]^{1/2}\,[M]\,[D] \tag{26}$$

Defining

$$[\overline{M}_a] = [K]^{-1/2}\,[M_a]\,[K]^{-1/2} \tag{27}$$

and

$$[\overline{M}] = [K]^{1/2}\,[M] \tag{28}$$

then Eq. (3) reduces to the standard form

$$[\overline{M}_a]\,[\overline{M}] = [\overline{M}]\,[D] \tag{29}$$

As shown above, the procedure requires the determination of the eigenvalues and associated eigenvectors of $[K]$.

III. FORMULATION OF NUMERICAL SOLUTION

A Fortran program for the IBM 7090 was developed to reduce Eq. (3) to Eq. (29). This program calls a SHARE subroutine to perform the diagonalization by successive rotations for obtaining $[m]$ and $[d]$, $[\overline{M}]$ and $[D]$ associated with $[K]$ and $[\overline{M}_a]$, respectively. Since all operations are limited to the rapid access memory of a 32K machine, the stiffness and mass matrices can be as large as square matrices of order 50. The flow chart for the program is given in Fig. 1. Appendix B lists programs and instructions.

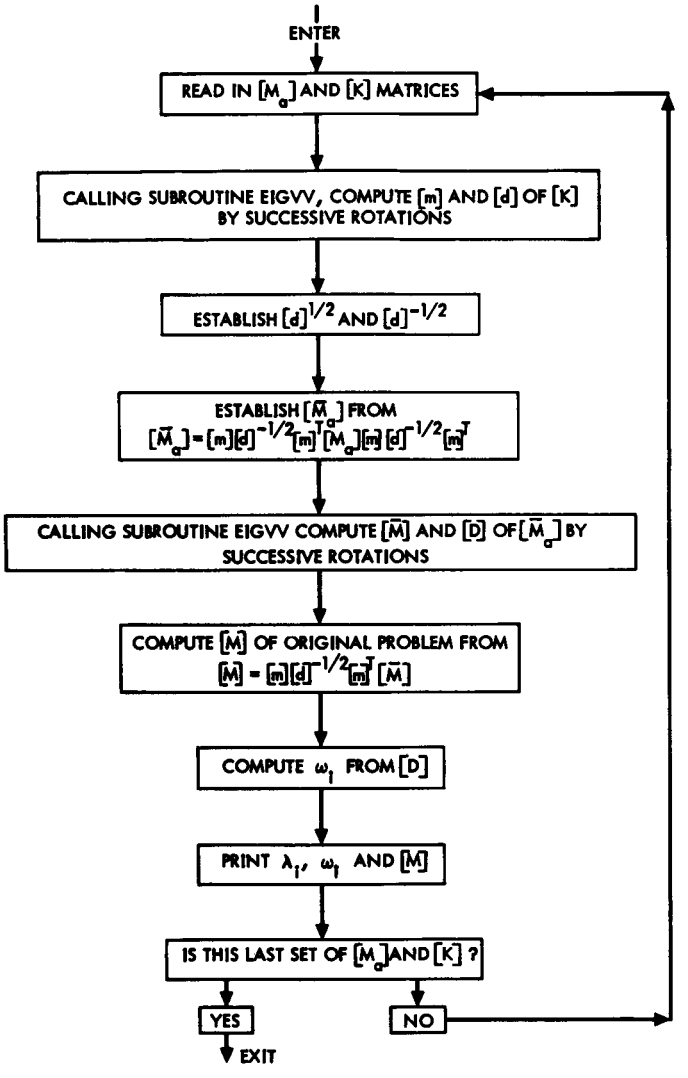

Fig. 1. Flow chart of the eigenvalue problem

NOMENCLATURE

$[A]$	A real symmetric matrix
$a_{p,q}$	Element of $[A]$ on p^{th} row and q^{th} column
c	Element of rotation matrix $[t_i]$ which is on p^{th} and q^{th} diagonal locations
$[D]$	Diagonal matrix established by the eigenvalues of $[A]$
$[d]$	Diagonal matrix established by the eigenvalues of $[K]$
$[d]^{1/2}$, $[d]^{-1/2}$	Matrices satisfying $[d]^{1/2}[d]^{1/2} = [d]$; $[d]^{1/2}[d]^{-1/2} = [d]^{-1/2}[d]^{1/2} = [I]$
$[I]$	Identity matrix
$[K]$	Stiffness matrix which is real symmetric and positive definite
$[K]^{1/2}$, $[K]^{-1/2}$	Matrices satisfying $[K]^{1/2}[K]^{1/2} = [K]$; $[K]^{1/2}[K]^{-1/2} = [K]^{-1/2}[K]^{1/2} = [I]$
$[M]$	Modal matrix established by the normalized eigenvectors of $[A]$
$[M_a]$	Mass matrix which is real and symmetric
$[\overline{M}]$	Modal matrix established by the normalized eigenvectors $[\overline{M}_a]$
$[\overline{M}_a]$	Modified symmetric mass matrix
$[m]$	Modal matrix established by the normalized eigenvectors of $[K]$
s	Absolute value of non-zero off-diagonal entries of rotation matrix $[t_i]$
t	Continuous time variable
$[t_i]$	Rotation matrix used to nullify $a_{p,q}$ element of $[A]$
$\{X\}$	Displacement vector of the discrete structure
$\{\ddot{X}\}$	Acceleration vector of the discrete structure
$\{x\}$	Amplitudes of displacements (and accelerations) of harmonic motion
$\{X_i\}$	i^{th} eigenvector
δ_{ij}	Kronecker delta ($\delta_{ij} = 0$ if $i \neq j$, $\delta_{ij} = 1$ if $i = j$)
λ_i	i^{th} eigenvalue
ω	Natural frequency

REFERENCES

1. Crandall, Stephen H., *Engineering Analysis*, McGraw-Hill Book Co., Inc., 1956.
2. Heikkinen, R. R., E. G. Abu-Saba, *Engineering Analysis Report*, ER-C-20-1, Blaw-Knox Company, Advanced Products Division, July 25, 1962.
3. Kane, T. R., "Dynamics of Nonholomonic Systems," *Journal of Applied Mechanics*, Vol. 28, No. 4, December 1961, pp. 574-578.
4. Heikkinen, R. R., E. G. Abu-Saba, *Interim Report*, No. C-20-E, Blaw-Knox Company, Advanced Products Division, June 6, 1962. (This reference includes Blaw-Knox corrections transmitted by telephone.)

APPENDIX A

Fortran Program for Obtaining the Matrices

In its actual form, the Advanced Antenna System involves a discrete structure of multi-thousand deg of freedom. To solve the associated eigenvalue problem of this structure is too difficult a job to handle with present-day computers. Moreover, the design of the Servo System of the Advanced Antenna requires comparatively few eigenvalues and associated eigenvectors. Therefore the structure is idealized to a 33-deg of freedom structure (Ref. 2). The governing equations of this dynamic model are obtained by Blaw-Knox by means of a special method (Ref. 3). These equations are given in Ref. (4). The following IBM 7090 Fortran program merely generates the stiffness and mass matrices of the Blaw-Knox equations. The resulting matrices are sketched in Figs. A-1 and A-2. The symbols used in Ref. (4) and the corresponding symbols used in the Fortran program are listed in Fig. A-3. The input for generating both matrices is given in Ref. (2).

The input of the program should be compatible with the Fortran statements:

```
READ INPUT TAPE 5, 10, A11K, A22K, A33K,
A44K, A55K, A66K, C11K, C22K, C33K, C12K,
1C44K, C55K, C66K, C45K, C34K, C35K, E22K,
E33K, E23K, E55K, E66K, E56K, E25K, E26K,
2E35K, E36K, G22K, G33K, G23K, G25K, G26K,
G35K, G36K, G55K, G66K, G56K, D22K, D33K,
3D44K, D55K, D34K, Q66K, R55K, CMK, B11K,
B22K, B33K, B55K, B66K, ZK, A4J, A5J, A6J, 4B4J,
B5J, B6J, C4J, C5J, C6J, D4J, D5J, D6J, E4J, E5J,
E6J, G4J, G5J, G6J, AM, BM, CM, DM, 5EM, GM,
P, AL, BL, PL, QL, RL, (SKA(I), I = 1, 5),
(ELA(I), I = 1, 5)

READ INPUT TAPE 5, 14, T

10 FORMAT (6E12.4)
14 FORMAT (F2.0)
```

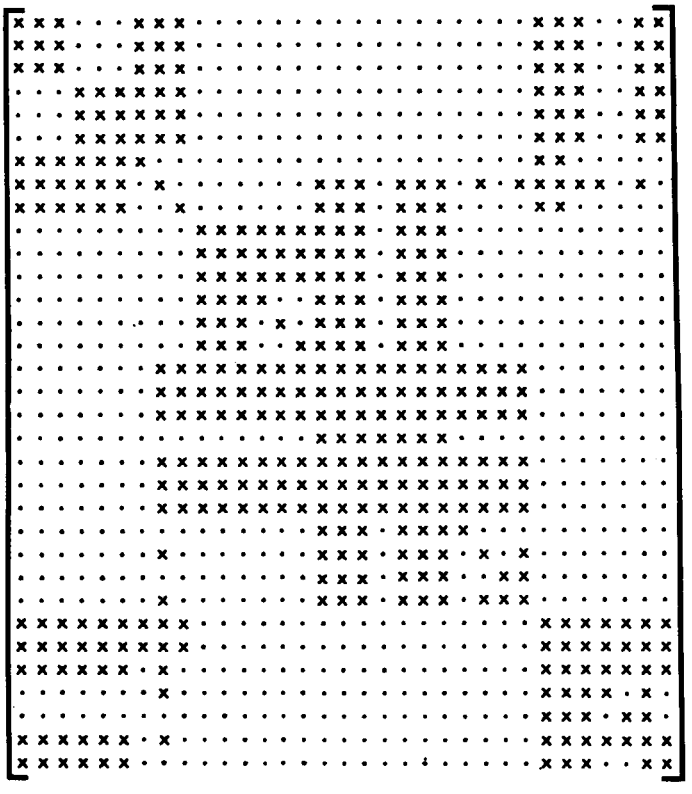

Fig. A-1. Mass matrix of Blaw-Knox dynamic model given in Ref. 2

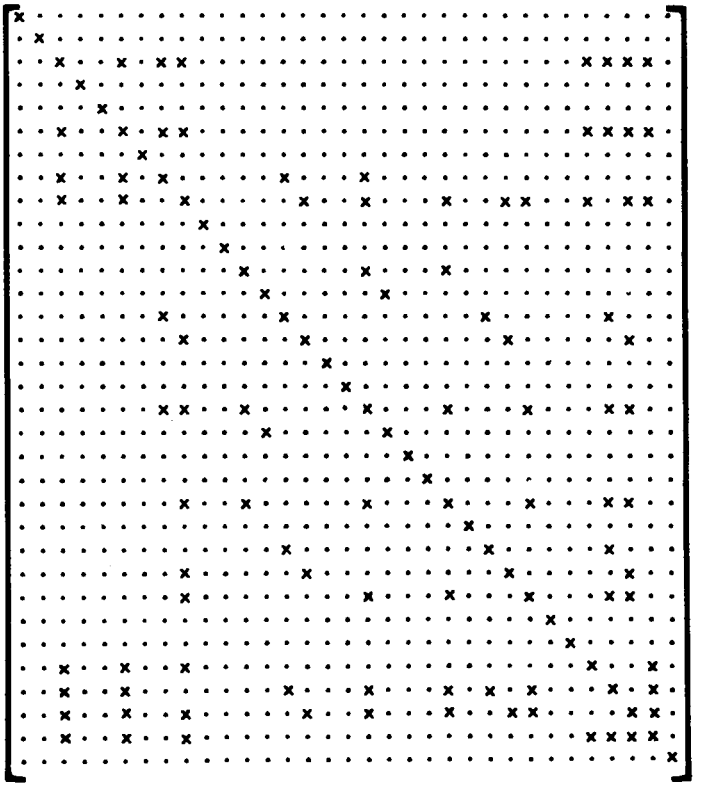

Fig. A-2. Stiffness matrix of Blaw-Knox dynamic model given in Ref. 2

The program does not require any tapes or sense switch settings. The output of this program is the listings of the mass and stiffness matrices and the BCD punched cards which contain the upper halves of the matrices. These cards are compatible with the Eigenvalue program.

The 33-deg of freedom system is reduced to a 32-deg of freedom system by deleting the 29th row and column in this program, since information about the 29th entry of the eigenvectors is known from the input given in Ref. (2). A program listing follows:

A4J	J^A (4)
A5J	J^A (5)
A6J	J^A (6)
A11K	k^A_{11}
A22K	k^A_{22}
A33K	k^A_{33}
A44K	k^A_{44}
A55K	k^A_{55}
A66K	k^A_{66}
AL	l_A
AM	m^A
B4J	J^B (4)
B5J	J^B (5)
B6J	J^B (6)
B11K	k^B_{11}
B22K	k^B_{22}
B33K	k^B_{33}
B55K	k^B_{55}
B66K	k^B_{66}
BL	l_B
BM	m^B
C4J	J^C (4)
C5J	J^C (5)
C6J	J^C (6)

C11K	k^C_{11}
C12K	k^C_{12}
C22K	k^C_{22}
C33K	k^C_{33}
C34K	k^C_{34}
C35K	k^C_{35}
C44K	k^C_{44}
C45K	k^C_{45}
C55K	k^C_{55}
C66K	k^C_{66}
CLK	k^L
CM	m^C
CMK	k^M
CP	$\cos\phi$
CS	$\cos\sigma$
CT	$\cos\theta$
D4J	J^D_4
D5J	J^D_5
D6J	J^D_6
D22K	k^D_{22}
D33K	k^D_{33}
D34K	k^D_{34}

D44K	k^D_{44}
D55K	k^D_{55}
DM	m^D
E4J	J^E (4)
E5J	J^E (5)
E6J	J^E (6)
E22K	k^E_{22}
E23K	k^E_{23}
E25K	k^E_{25}
E26K	k^E_{26}
E33K	k^E_{33}
E35K	k^E_{35}
E36K	k^E_{36}
E55K	k^E_{55}
E56K	k^E_{56}
E66K	k^E_{66}
EL	l_E
EM	m^E
G4J	J^G (4)
G5J	J^G (5)
G6J	J^G (6)
G22K	k^G_{22}
G23K	k^G_{23}

G25K	k^G_{25}
G26K	k^G_{26}
G33K	k^G_{33}
G35K	k^G_{35}
G36K	k^G_{36}
G55K	k^G_{55}
G56K	k^G_{56}
G66K	k^G_{66}
GL	l_G
GM	m^G
P	ϕ
PI	π
PL	l_P
Q66K	k^Q_{66}
QL	l_Q
R55K	k^R_{55}
RL	l_R
S	σ
SK	k^S
SP	$\sin\phi$
SS	$\sin\sigma$
ST	$\sin\theta$
T	θ
ZK	k^Z

Fig. A-3. Symbols used in the Program generating mass and stiffness matrices of the Blaw-Knox dynamic model and their counterparts in Ref. 4

```
*       JPL,J007000,01113330      D ALEMBERT EQS. BARONDESS-UTKU
*       XEQ
*       LABEL
*       LIST8
C       BLAW-KNOX DYNAMIC MODEL   MASS AND STIFFNESS MATRICES
        DIMENSION A(33,33),B(33,33),SKA(5),ELA(5),TT(5)
        READ INPUT TAPE 5,10,A11K,A22K,A33K,A44K,A55K,A66K,C11K,C22K,C33K,
       1C12K,C44K,C55K,C66K,C45K,C34K,C35K,E22K,E33K,E23K,E55K,E66K,E56K,
       2E25K,E26K,E35K,E36K,G22K,G33K,G23K,G25K,G26K,G35K,G36K,G55K,G66K,
       3G56K,D22K,D33K,D44K,D55K,D34K,Q66K,R55K,CMK,B11K,B22K,B33K,B55K,
       4B66K,ZK,A4J,A5J,A6J,B4J,B5J,B6J,C4J,C5J,C6J,D4J,D5J,D6J,E4J,E5J,
       5E6J,G4J,G5J,G6J,AM,BM,CM,DM,EM,GM,P,AL,BL,PL,QL,RL,(SKA(I),I=1,5),
       6(ELA(I),I=1,5)
        PI=3.14159
        TT(1)=0.
        TT(2)=30.
        TT(3)=45.
        TT(4)=60.
        TT(5)=90.
  100   READ INPUT TAPE 5,14,T
        WRITEOUTPUTTAPE 6,15, (A11K,A22K,A33K,A44K,A55K,A66K,C11K,C22K,
       1C33K,C12K,C44K,C55K,C66K,C45K,C34K,C35K,E22K,E33K,E23K,E55K,E66K,
       2E56K,E25K,E26K,E35K,E36K,G22K,G33K,G23K,G25K,G26K,G35K,G36K,G55K,
       3G66K,G56K,D22K,D33K,D44K,D55K,D34K,Q66K,R55K,CMK,B11K,B22K,B33K,
       4B55K,B66K,ZK,A4J,A5J,A6J,B4J,B5J,B6J,C4J,C5J,C6J,D4J,D5J,D6J,E4J,
       5E5J,E6J,G4J,G5J,G6J,AM,BM,CM,DM,EM,GM,P,AL,BL,PL,QL,RL,(SKA(I),
       6I=1,5),(ELA(I),I=1,5),T)
        DO 110 I=1,5
        IF (TT(I)-T) 110,120,120
  110   CONTINUE
        GO TO 1000
  120   IF (I-1) 122,121,122
  121   SK=SKA(1)
        EL=ELA(1)
        GL=EL
        GO TO 125
  122   SK=SKA(I-1)+(SKA(I)-SKA(I-1))*(T-TT(I-1))/(TT(I)-TT(I-1))
        EL=ELA(I-1)+(ELA(I)-ELA(I-1))*(T-TT(I-1))/(TT(I)-TT(I-1))
        GL=EL
  125   T=T*PI/180.
        S=PI/2.-P-T
        CLK=SK*CMK/(SK+CMK)
        ST=SINF(T)
        CT=COSF(T)
        SS=SINF(S)
        CS=COSF(S)
        SP=SINF(P)
        CP=COSF(P)
        DO 20 I=1,33
        DO 20 J=1,33
        A(I,J)=0.
   20   B(I,J)=0.
```

```
B(1,1)=AM+BM+CM+DM+EM+GM
B(1,2)=BM+CM+DM+EM+GM
B(1,3)=(CM+DM+EM+GM)
B(1,7)=DM*ST
B(1,8)=EM*ST
B(1,9)=GM*ST
B(1,27)=-(DM*QL-(EM+GM)*PL)*CT
B(1,28)=B(1,27)
B(1,29)=B(1,27)
B(1,32)=B(1,27)
B(1,33)=-DM*QL*CT
B(2,2)=B(1,2)
B(2,3)=CM+DM+EM+GM
B(2,7)=DM*ST
B(2,8)=EM*ST
B(2,9)=GM*ST
B(2,27)=-(DM*QL-(EM+GM)*PL)*CT
B(2,28)=B(2,27)
B(2,29)=B(2,27)
B(2,32)=B(2,27)
B(2,33)=-DM*QL*CT
B(3,3)=CM+DM+EM+GM
B(3,7)=DM*ST
B(3,8)=EM*ST
B(3,9)=GM*ST
B(3,27)=B(2,27)
B(3,28)=B(2,27)
B(3,29)=B(2,27)
B(3,32)=B(2,27)
B(3,33)=-DM*QL*CT
B(4,4)=AM+BM+CM+DM+EM+GM
B(4,5)=BM+CM+DM+EM+GM
B(4,6)=CM+DM+EM+GM
B(4,7)=DM*CT
B(4,8)=EM*CT
B(4,9)=GM*CT
B(4,27)=BM*AL+(CM+DM+EM+GM)*(AL+BL)+(DM*QL-(EM+GM)*PL)*ST
B(4,28)=(CM+DM+EM+GM)*BL+(DM*QL-(EM+GM)*PL)*ST
B(4,29)=(DM*QL-(EM+GM)*PL)*ST
B(4,32)=B(4,29)
B(4,33)=DM*QL*ST
B(5,5)=BM+CM+DM+EM+GM
B(5,6)=CM+DM+EM+GM
B(5,7)=DM*CT
B(5,8)=EM*CT
B(5,9)=GM*CT
B(5,27)=BM*AL+(CM+DM+EM+GM)*(AL+BL)+(DM*QL-(EM+GM)*PL)*ST
B(5,28)=(CM+DM+EM+GM)*BL+(DM*QL-(EM+GM)*PL)*ST
B(5,29)=(DM*QL-(EM+GM)*PL)*ST
B(5,32)=B(5,29)
B(5,33)=DM*QL*ST
B(6,6)=CM+DM+EM+GM
```

```
B(6,7)=DM*CT
B(6,8)=EM*CT
B(6,9)=GM*CT
B(6,27)=(CM+DM+EM+GM)*(AL+BL)+(DM*QL-(EM+GM)*PL)*ST
B(6,28)=(CM+DM+EM+GM)*(    BL)+(DM*QL-(EM+GM)*PL)*ST
B(6,29)=B(5,29)
B(6,32)=B(6,29)
B(6,33)=DM*QL*ST
B(7,7)=DM
B(7,27)=DM*(AL+BL)*CT
B(7,28)=DM*BL*CT
B(8,8)=EM
B(8,16)=-EM*RL*CT
B(8,17)=B(8,16)
B(8,18)=B(8,16)
B(8,20)=EM*RL*ST
B(8,21)=B(8,20)
B(8,22)=B(8,20)
B(8,27)=EM*(AL+BL)*CT
B(8,28)=EM*BL*CT
B(9,9)=GM
B(9,16)=GM*RL*CT
B(9,17)=B(9,16)
B(9,18)=B(9,16)
B(9,20)=-GM*RL*ST
B(9,21)=B(9,20)
B(9,22)=B(9,20)
B(9,27)=GM*(AL+BL)*CT
B(9,28)=GM*BL*CT
B(10,10)=AM+BM+CM+DM+EM+GM
B(10,11)=B(10,10)-AM
B(10,12)=B(10,11)-BM
B(10,13)=DM
B(10,14)=EM
B(10,15)=GM
B(10,16)=-B(2,27)
B(10,17)=-B(2,27)
B(10,18)=-B(2,27)
B(10,20)=-(BM*AL+(CM+DM+EM+GM)*(AL+BL))-(DM*QL-(EM+GM)*PL)*ST
B(10,21)=-B(10,12)*BL-(DM*QL-(EM+GM)*PL)*ST
B(10,22)=-B(5,29)
B(11,11)=BM+CM+DM+EM+GM
B(11,12)=B(11,11)-BM
B(11,13)=DM
B(11,14)=EM
B(11,15)=GM
B(11,16)=-B(2,27)
B(11,17)=-B(2,27)
B(11,18)=-B(2,27)
B(11,20)=-(BM*AL+(CM+DM+EM+GM)*(AL+BL)+(DM*QL-(EM+GM)*PL)*ST)
B(11,21)=-B(11,12)*BL+B(10,22)
B(11,22)=B(10,22)
```

```
 B(12,12)=B(11,12)
 B(12,13)=DM
 B(12,14)=EM
 B(12,15)=GM
 B(12,16)=B(10,16)
 B(12,17)=B(10,16)
 B(12,18)=B(10,16)
 B(12,20)=-(B(12,12)*(AL+BL)+(DM*QL-(EM+GM)*PL)*ST)
 B(12,21)=-(B(12,12)*BL+(DM*QL-(EM+GM)*PL)*ST)
 B(12,22)=B(11,22)
 B(13,13)=DM
 B(13,16)=DM*QL*CT
 B(13,17)=B(13,16)
 B(13,18)=B(13,17)
 B(13,20)=-(DM*((AL+BL)+QL*ST))
 B(13,21)=-(DM*(BL+QL*ST))
 B(13,22)=-DM*QL*ST
 B(14,14)=EM
 B(14,16)=-EM*PL*CT
 B(14,17)=B(14,16)
 B(14,18)=B(14,16)
 B(14,20)=-(EM*(AL+BL-PL*ST))
 B(14,21)=-(EM*(BL-PL*ST))
 B(14,22)=EM*PL*ST
 B(15,15)=GM
 B(15,16)=-GM*PL*CT
 B(15,17)=B(15,16)
 B(15,18)=B(15,16)
 B(15,20)=B(14,20)
 B(15,21)=B(14,21)
 B(15,22)=B(14,22)
 B(16,16)=(A4J+B4J+C4J+(D5J+E5J+G5J)*ST*ST+(EM+GM)*RL*RL+(D4J+E4J+
1G4J+DM*QL*QL+(EM+GM)*PL*PL)*CT*CT)
 B(16,17)=B(16,16)-A4J
 B(16,18)=B(16,17)-B4J
 B(16,19)=D4J*CT
 B(16,20)=-(DM*QL-(EM+GM)*PL)*(AL+BL)*CT-(D4J+E4J+G4J-D5J-E5J-G5J+
1DM*QL*QL+(EM+GM)*PL*PL)*ST*CT
 B(16,21)=-((DM*QL-(EM+GM)*PL)*BL*CT+(D4J+E4J+G4J-D5J-E5J-G5J+DM*QL
1*QL+(EM+GM)*PL*PL)*ST*CT)
 B(16,22)=B(16,21)+(DM*QL-(EM+GM)*PL)*BL*CT
 B(16,23)=D5J*ST
 B(16,24)=E5J*ST
 B(16,25)=G5J*ST
 B(16,26)=(E5J+G5J+(EM+GM)*RL*RL)*ST
 B(17,17)=B4J+C4J+(D5J+E5J+G5J)*ST*ST+(EM+GM)*RL*RL+(DM*QL*QL+(EM+
1GM)*PL*PL+D4J+E4J+G4J)*CT*CT
 B(17,18)=B(17,17)-B4J
 B(17,19)=D4J*CT
 B(17,20)=B(16,20)
 B(17,21)=B(16,21)
 B(17,22)=B(16,22)
```

```
 B(17,23)=B(16,23)
 B(17,24)=B(16,24)
 B(17,25)=B(16,25)
 B(17,26)=B(16,26)
 B(18,18)=B(17,18)
 B(18,19)=B(17,19)
 B(18,20)=B(17,20)
 B(18,21)=B(17,21)
 B(18,22)=B(17,22)
 B(18,23)=B(17,23)
 B(18,24)=B(17,24)
 B(18,25)=B(17,25)
 B(18,26)=B(17,26)
 B(19,19)=D4J
 B(19,20)=-D4J*ST
 B(19,21)=B(19,20)
 B(19,22)=B(19,20)
 B(20,20)=BM*AL*AL+B(12,12)*(AL+BL)**2-B(10,22)*(AL+BL)*2.+(EM+GM)*
1RL*RL+(DM*QL*QL+(EM+GM)*PL*PL+D4J+E4J+G4J)*ST*ST+(D5J+E5J+G5J)*CT*
2CT+A5J+B5J+C5J
 B(20,21)=B(11,12)*(AL+BL)*BL-B(10,22)*BL+(EM+GM)*RL*RL-B(10,22)
1*(AL+BL)+(DM*QL*QL+(EM+GM)*PL*PL+D4J+E4J+G4J)*ST*ST+(D5J+E5J+G5J)
2*CT*CT+B5J+C5J
 B(20,22)=(EM+GM)*RL*RL-B(10,22)*(AL+BL)+(DM*QL*QL+(EM+GM)*PL*PL+
1D4J+E4J+G4J)*ST*ST+(D5J+E5J+G5J)*CT*CT+C5J
 B(20,23)=D5J*CT
 B(20,24)=E5J*CT
 B(20,25)=G5J*CT
 B(20,26)=((EM+GM)*RL*RL+E5J+G5J)*CT
 B(21,21)=(CM+DM+EM+GM)*BL*BL-B(10,22)*BL*2.+(EM+GM)*RL*RL+(DM*QL*
1QL+(EM+GM)*PL*PL+D4J+E4J+G4J)*ST*ST+(D5J+E5J+G5J)*CT*CT+B5J+C5J
 B(21,22)=(EM+GM)*RL*RL-B(10,22)*BL+(DM*QL*QL+(EM+GM)*PL*PL+D4J+E4J
1+G4J)*ST*ST+(D5J+E5J+G5J)*CT*CT+C5J
 B(21,23)=D5J*CT
 B(21,24)=E5J*CT
 B(21,25)=G5J*CT
 B(21,26)=((EM+GM)*RL*RL+E5J+G5J)*CT
 B(22,22)=(DM*QL*QL+(EM+GM)*PL*PL+D4J+E4J+G4J)*ST*ST+(D5J+E5J+G5J)
1*CT*CT+(EM+GM)*RL*RL+C5J
 B(22,23)=B(21,23)
 B(22,24)=B(21,24)
 B(22,25)=B(21,25)
 B(22,26)=B(21,26)
 B(23,23)=D5J
 B(24,24)=E5J
 B(24,26)=B(24,24)
 B(25,25)=G5J
 B(25,26)=B(25,25)
 B(26,26)=((EM+GM)*RL*RL+E5J+G5J)
 B(27,27)=BM*AL*AL+B(10,12)*(AL+BL)**2-B(10,22)*(AL+BL)*2.+(DM*QL*
1QL+(EM+GM)*PL*PL+D6J+E6J+G6J)+A6J+B6J+C6J
 B(27,28)=B(10,12)*(AL+BL)*BL-B(10,22)*BL-B(10,22)*(AL+BL)+(DM*QL*
```

```
1QL+(EM+GM)*PL*PL+D6J+E6J+G6J)+B6J+C6J
 B(27,29)=-B(10,22)*(AL+BL)+DM*QL*QL+(EM+GM)*PL*PL+D6J+E6J+G6J+C6J
 B(27,30)=E6J
 B(27,31)=G6J
 B(27,32)=B(27,29)-C6J
 B(27,33)=DM*QL*((AL+BL)*ST+QL)+D6J
 B(28,28)=B(10,12)*BL*BL-B(10,22)*BL*2.+DM*QL*QL+(EM+GM)
1*PL*PL+D6J+E6J+G6J+B6J+C6J
 B(28,29)=-B(10,22)*BL+DM*QL*QL+(EM+GM)*PL*PL+D6J+E6J+G6J+C6J
 B(28,30)=E6J
 B(28,31)=G6J
 B(28,32)=B(28,29)-C6J
 B(28,33)=DM*QL*(BL*ST+QL)+D6J
 B(29,29)=DM*QL*QL+(EM+GM)*PL*PL+D6J+E6J+G6J+C6J
 B(29,30)=E6J
 B(29,31)=G6J
 B(29,32)=B(29,29)-C6J
 B(29,33)=DM*QL*QL+D6J
 B(30,30)=B(29,30)
 B(30,32)=B(30,30)
 B(31,31)=B(29,31)
 B(31,32)=B(31,31)
 B(32,32)=B(29,32)
 B(32,33)=B(29,33)
 B(33,33)=B(29,33)
 A(1,1)=A11K
 A(2,2)=B11K
 A(3,3)=C11K+CLK*2.*SP*SP
 A(3,6)=C12K-2.*CLK*CP*SP
 A(3,8)=-CLK*SS*SP
 A(3,9)=A(3,8)
 A(3,29)=2.*CLK*(PL+EL)*CS*SP
 A(3,30)=CLK*EL*CS*SP
 A(3,31)=CLK*GL*CS*SP
 A(3,32)=2.*CLK*(PL+EL)*CS*SP
 A(4,4)=A22K
 A(5,5)=B22K
 A(6,6)=C22K+2.*CLK*CP*CP
 A(6,8)=CLK*SS*CP
 A(6,9)=A(6,8)
 A(6,29)=-2.*CLK*(PL+EL)*CS*CP
 A(6,30)=-CLK*EL*CS*CP
 A(6,31)=-CLK*GL*CS*CP
 A(6,32)=-2.*CLK*(PL+EL)*CS*CP
 A(7,7)=D22K
 A(8,8)=E22K+CLK*SS*SS
 A(8,14)=E23K
 A(8,18)=-CLK*RL*CP*SS
 A(8,22)=-CLK*RL*SP*SS
 A(8,24)=E25K
 A(8,26)=-CLK*RL*CS*SS
 A(8,29)=-CLK*(PL+EL)*CS*SS
```

```
A(8,30)=E26K-CLK*EL*CS*SS
A(8,32)=A(8,29)
A(9,9)=G22K+CLK*SS*SS
A(9,15)=G23K
A(9,18)=-A(8,18)
A(9,22)=CLK*RL*SP*SS
A(9,25)=G25K
A(9,26)=-A(8,26)
A(9,29)=A(8,29)
A(9,31)=G26K-CLK*GL*CS*SS
A(9,32)=A(9,29)
A(10,10)=A33K
A(11,11)=B33K
A(12,12)=C33K
A(12,18)=C34K
A(12,22)=C35K
A(13,13)=D33K
A(13,19)=D34K
A(14,14)=E33K
A(14,24)=E35K
A(14,30)=E36K
A(15,15)=G33K
A(15,25)=G35K
A(15,31)=G36K
A(16,16)=A44K
A(17,17)=ZK
A(18,18)=C44K+2.*CLK*RL*RL*CP*CP
A(18,22)=C45K+2.*CLK*RL*RL*SP*CP
A(18,26)=2.*CLK*RL*CS*CP
A(18,30)=CLK*EL*RL*CS*CP
A(18,31)=-CLK*GL*RL*CP*CS
A(19,19)=D44K
A(20,20)=A55K
A(21,21)=B55K
A(22,22)=C55K+2.*CLK*RL*RL*SP*SP
A(22,26)=2.*CLK*RL*RL*CS*SP
A(22,30)=CLK*EL*RL*CS*SP
A(22,31)=-CLK*GL*RL*CS*SP
A(23,23)=D55K
A(24,24)=E55K
A(24,30)=E56K
A(25,25)=G55K
A(25,31)=G56K
A(26,26)=R55K+2.*CLK*RL*RL*CS*CS
A(26,30)=CLK*EL*RL*CS*CS
A(26,31)=-CLK*GL*RL*CS*CS
A(27,27)=A66K
A(28,28)=B66K
A(29,29)=C66K+2.*CLK*(PL+EL)**2*CS*CS
A(29,30)=CLK*EL*(PL+EL)*CS*CS
A(29,31)=CLK*GL*(PL+EL)*CS*CS
A(29,32)=2.*CLK*(PL+EL)**2*CS*CS
```

```
      A(30,30)=E66K+CLK*EL*EL*CS*CS
      A(30,32)=CLK*EL*(PL+EL)*CS*CS
      A(31,31)=G66K+CLK*GL*GL*CS*CS
      A(31,32)=CLK*GL*(PL+EL)*CS*CS
      A(32,32)=A(29,32)
      A(33,33)=Q66K
      DO 25 I=1,33
      DO 25 J=1,33
   25 A(I,J)=-A(I,J)
      DO 30 I=1,33
      DO 30 J=1,I
      A(I,J)=A(J,I)
   30 B(I,J)=B(J,I)
      DO 50 I=1,33
      IF (I-29) 41,50,43
41    M=I
      GO TO 47
43    M=I-1
47    DO 49 J=1,33
      IF (J-29) 44,49,45
44    N=J
      GO TO 48
45    N=J-1
48    A(M,N)=A(I,J)
      B(M,N)=B(I,J)
49    CONTINUE
50    CONTINUE
      WRITE OUTPUT TAPE 7,13, ((A(I,J),J=I,32),I=1,32)
      WRITE OUTPUT TAPE 7,13, ((B(I,J),J=I,32),I=1,32)
      WRITE OUTPUT TAPE 6,11,((A(I,J),J=1,32),I=1,32)
      WRITE OUTPUT TAPE 6,12,((B(I,J),J=1,32),I=1,32)
      GO TO 100
1000  CALL EXIT
   10 FORMAT (6E12.4)
   11 FORMAT ( 57H1THE ELEMENTS OF MATRIX A LISTED ROW-WISE ARE AS FOLLO
     1WS.,/////,(6E20.9))
   12 FORMAT ( 57H1THE ELEMENTS OF MATRIX B LISTED ROW-WISE ARE AS FOLLO
     1WS.,/////,(6E20.9))
   13 FORMAT ((4E16.9))
   14 FORMAT ( F2.0)
   15 FORMAT (32H1THE INPUT DATA FOR THIS CASE IS,/////,(6E20.9))
      END
```

```
*      DATA
  2.64  E  9  1.30  E  9  1.30  E  9  3.85  E 12  7.78  E 12  7.78  E 12      01
  4.18  E  8  5.58  E  7  7.10  E  7 -1.40  E  7  9.10  E 10  7.31  E 11      02
  0.99  E 38  3.06  E 10 -7.83  E  8  4.71  E  9  2.14  E  8  6.15  E  7      03
  4.04  E  7  3.34  E  9  1.25  E 10 -3.08  E  9  3.22  E  7 -2.09  E  8      04
 -4.57  E  7 -3.35  E  8  2.14  E  8  6.15  E  7 -4.04  E  7  3.22  E  7      05
 -2.09  E  8 -4.57  E  7  3.35  E  8  3.34  E  9  1.25  E 10 -3.08  E  9      06
  1.62  E  8  8.74  E  7  1.32  E 11  3.27  E 10  2.82  E  9  1.77  E 11      07
  1.65  E 10  5.00  E  7  1.11  E  9  4.00  E  8  7.00  E  8  1.00  E 12      08
  1.00  E 12  1.50  E 11  3.32  E  8  2.97  E  8  2.97  E  8  1.23  E  8      09
  8.01  E  7  7.67  E  7  4.06  E  7  3.62  E  7  3.00  E  7  4.65  E  7      10
  7.55  E  7  5.06  E  7  8.50  E  6  1.26  E  7  1.30  E  7  8.50  E  6      11
  1.26  E  7  1.30  E  7  3.42  E  5  1.14  E  5  5.54  E  4  3.04  E  4      12
  1.21  E  4  1.21  E  4  0.5236E  0  4.00  E  1  7.80  E  1  3.70  E  1      13
  3.40  E  1  8.00  E  0  3.40  E  7  1.07  E  8  2.60  E  7  1.06  E  8      14
  1.60  E  8  3.70  E  1  5.72  E  0  1.31  E  0  0.00  E  0  5.72  E  0      15
00                                                                           16
30                                                                           17
45                                                                           18
60                                                                           19
90                                                                           20
91                                                                           21
```

APPENDIX B

Fortran Program for Obtaining Eigenvalues and Eigenvectors of Original Problem

The program, written in Fortran for IBM 7090, can be used for mass and stiffness matrices of order 50 or less. The program does not require any tapes or sense switch settings.

The input of this program should be compatible with the Fortran statements:

```
      READ INPUT TAPE 5,9000, N
      READ INPUT TAPE 5,9990, ((B(I, J), J=1, N),
      I=1, N)
      READ INPUT TAPE 5,9990, ((A(I, J), J=1, N),
      I=1, N)
 9000 FORMAT (7I10)
 9990 FORMAT (4E16.9)
```

where N is the order of the mass and stiffness matrices, [B] is the stiffness matrix ([K] in the text and [A] in the program given in Appendix A), and [A] is the mass matrix ([M_a] in the text and [B] in the program given in Appendix A). The output of this program is the listings of the eigenvalues and associated eigenvectors of [K] and the original problem. The natural frequencies are also computed in cycles/sec units.

This program calls a SHARE subroutine, EIGVV, (Distribution No. PA-460 MI HDI 1), to perform the successive rotations in diagonalizing [$\overline{K}$] and [$\overline{M}_a$]. The listing of the calling program is given on the following pages.

```
*       LIST8
*       LABEL
C       EIGENVALUE PROBLEM
        DIMENSION A(50,50),B(50,50),U(50,50),T(50,50),VALU(50),LCOL(51)
        READ INPUT TAPE 5,9000,N
      1 DO 100 I=1,N
        DO 99 J=1,N
        A(I,J)=0.0
        B(I,J)=0.0
        U(I,J)=0.0
        T(I,J)=0.0
     99 CONTINUE
    100 CONTINUE
        DO 105 I=1,N
        VALU(I)=0.0
    105 CONTINUE
        L=1
        DO 106 J=1,50
        LCOL(J)=L
        L=L+1
    106 CONTINUE
        READ INPUT TAPE 5,9990,((B(I,J),J=I,N ),I=1,N )
        READ INPUT TAPE 5,9990,((A(I,J),J=I,N ),I=1,N )
   9990 FORMAT(4E16.9)
      3 K=1
        DO 4 I=1,N
        DO 40J=K,N
        A(J,I)=A(I,J)
     40 CONTINUE
        K=K+1
      4 CONTINUE
   9000 FORMAT(7I10)
   9001 FORMAT(7F10.7)
      6 K=1
        DO 7777 I=1,N
        DO 7777 J=1,N
   7777 A(I,J)=-A(I,J)
        DO 8 I=1,N
        DO 7 J=K,N
        B(I,J)=-B(I,J)
        B(J,I)=B(I,J)
      7 CONTINUE
        K=K+1
      8 CONTINUE
        L=10
        K=1
        KPAGE=1
   2000 WRITE OUTPUT TAPE 6,8000,KPAGE
        WRITE OUTPUT TAPE 6,8004
        WRITE OUTPUT TAPE 6,8002,N,N
        WRITE OUTPUT TAPE 6,8001,(LCOL(J),J=K,L)
        WRITE OUTPUT TAPE 6,8003,((A(I,J),J=K,L),I=1,N)
```

```
      IF(N-L)201,201,200
  200 KPAGE=KPAGE+1
      L=L+10
      K=K+10
      GO TO 2000
  201 KPAGE=1
      L=10
      K=1
 2001 WRITE OUTPUT TAPE 6,8000,KPAGE
      WRITE OUTPUT TAPE 6,8005
      WRITE OUTPUT TAPE 6,8002,N,N
      WRITE OUTPUT TAPE 6,8001,(LCOL(J),J=K,L)
      WRITE OUTPUT TAPE 6,8003,((B(I,J),J=K,L),I=1,N)
      IF(N-L)203,203,202
  202 KPAGE=KPAGE+1
      L=L+10
      K=K+10
      GO TO 2001
  203 CONTINUE
      CALL SETUP(B,U,50,50)
      CALL EIGVV(B,U,VALU,N)
      L=10
      K=1
      KPAGE=1
 2100 WRITE OUTPUT TAPE 6,8000,KPAGE
      WRITE OUTPUT TAPE 6,8007
      WRITE OUTPUT TAPE 6,8002,N,N
      WRITE OUTPUT TAPE 6,8001,(LCOL(J),J=K,L)
      WRITE OUTPUT TAPE 6,8003,((U(I,J),J=K,L),I=1,N)
      IF(N-L)211,211,210
  210 KPAGE=KPAGE+1
      L=L+10
      K=K+10
      GO TO 2100
  211 KPAGE=1
 2110 WRITE OUTPUT TAPE 6,8000,KPAGE
      WRITE OUTPUT TAPE 6,8006
      WRITE OUTPUT TAPE 6,8003,(VALU(I),I=1,N)
      DO 20 I=1,N
      IF(VALU(I))21,20,20
   21 WRITE OUTPUT TAPE 6,9100,I,VALU(I)
 9100 FORMAT(1H0,27HEIGENVALUE IS NEGATIVE    I=,I3,5X,6HVALUE=,E16.8)
      GO TO 1
   20 CONTINUE
   30 DO 35 I=1,N
      DO 34 J=1,N
      B(I,J)=0.0
   34 CONTINUE
   35 CONTINUE
      J=0
      I=0
      DO 36 K=1,N
```

```
      J=J+1
      I=I+1
      B(I,J)=1.0/SQRTF(VALU(K))
   36 CONTINUE
      DO 3010 J=1,N
      DO 3010 I=1,N
 3010 T(I,J)=U(I,J)*B(J,J)
      DO 3020 I=1,N
      DO 3018 J=1,N
      VALU(J)=0.
      DO 3017 K=1,N
 3017 VALU(J)=VALU(J)+T(I,K)*U(J,K)
 3018 CONTINUE
      DO 3019 J=1,N
 3019 T(I,J)=VALU(J)
 3020 CONTINUE
      DO 3040 J=1,N
      DO 3038 I=1,N
      VALU(I)=0.
      DO 3037 K=1,N
 3037 VALU(I)=VALU(I)+T(I,K)*A(K,J)
 3038 CONTINUE
      DO 3039 I=1,N
 3039 A(I,J)=VALU(I)
 3040 CONTINUE
      DO 3060 I=1,N
      DO 3058 J=1,N
      VALU(J)=0.
      DO 3057 K=1,N
 3057 VALU(J)=VALU(J)+A(I,K)*T(K,J)
 3058 CONTINUE
      DO 3059 J=1,N
 3059 A(I,J)=VALU(J)
 3060 CONTINUE
      DO 3100 I=1,N
      DO 3100 J=I,N
      A(I,J)=(A(I,J)+A(J,I))/2.
      A(J,I)=A(I,J)
 3100 CONTINUE
      CALL SETUP (A,B,50,50)
      CALL EIGVV (A,B,VALU,N)
      DO 3090 I=1,N
      DO 3090 J=1,N
 3090 T(I,J)=A(I,J)
      PI=3.141593
      DO 3099 I=1,N
      IF (-VALU(I)) 3091,3091,3092
 3092 A(1,I)=1./(SQRTF(-VALU(I))*2.*PI)
      GO TO 3099
 3091 A(1,I)=-1./VALU(I)
 3099 CONTINUE
      L=10
```

```
 3078 CONTINUE
 3080 CONTINUE
 2400 WRITE OUTPUT TAPE 6,8000,KPAGE
      WRITE OUTPUT TAPE 6,8010
      WRITE OUTPUT TAPE 6,8002,N,N
      WRITE OUTPUT TAPE 6,8001,(LCOL(J),J=K,L)
      WRITE OUTPUT TAPE 6,8003,((T(I,J),J=K,L),I=1,N)
      IF(N-L)241,241,240
  240 KPAGE=KPAGE+1
      L=L+10
      K=K+10
      GO TO 2400
  241 KPAGE=1
      K=1
      L=10
      GO TO 1
      END
*     DATA
```

```
      K=1
      KPAGE=1
 2200 WRITE OUTPUT TAPE 6,8000,KPAGE
      WRITE OUTPUT TAPE 6,8009
      WRITE OUTPUT TAPE 6,8002,N,N
      WRITE OUTPUT TAPE 6,8001,(LCOL(J),J=K,L)
      WRITE OUTPUT TAPE 6,8003,((B(I,J),J=K,L),I=1,N)
      IF(N-L)221,221,220
  220 KPAGE=KPAGE+1
      L=L+10
      K=K+10
      GO TO 2200
  221 KPAGE=1
      WRITE OUTPUT TAPE 6,8000,KPAGE
      WRITE OUTPUT TAPE 6,8008
      WRITE OUTPUT TAPE 6,8003,(VALU(I),I=1,N)
      WRITE OUTPUT TAPE 6,8103,(A(1,I),I=1,N)
 8103 FORMAT (///////////,68H THE EIGENVALUES OF THE ORIGINAL PROBLEMS IN
     1 CYCLES PER SECOND UNITS,///,(10E12.3))
      K=1
      L=10
 2300 WRITE OUTPUT TAPE 6,8000,KPAGE
      WRITE OUTPUT TAPE 6,8011
      WRITE OUTPUT TAPE 6,8002,N,N
      WRITE OUTPUT TAPE 6,8001,(LCOL(J),J=K,L)
      WRITE OUTPUT TAPE 6,8003,((T(I,J),J=K,L),I=1,N)
      IF(N-L)231,231,230
  230 KPAGE=KPAGE+1
      L=L+10
      K=K+10
      GO TO 2300
  231 KPAGE=1
      K=1
      L=10
 8000 FORMAT(1H1,44X,32HTHE SYMMETRIC EIGENVALUE PROBLEM,20X,4HPAGE,I4)
 8001 FORMAT(1H0,10(6HCOLUMN,I3,2X))
 8002 FORMAT(1H0,55X,I3,3H BY,I3)
 8003 FORMAT(10(1H ,E10.3))
 8004 FORMAT(1H0,56X,8HA MATRIX)
 8005 FORMAT(1H0,56X,8HB MATRIX)
 8006 FORMAT(1H0,51X,16HEIGENVALUES OF B)
 8007 FORMAT(1H0,51X,17HEIGENVECTORS OF B)
 8008 FORMAT(1H0,51X,16HEIGENVALUES OF Q)
 8009 FORMAT(1H0,51X,17HEIGENVECTORS OF Q)
 8010 FORMAT(1H0,43X,32HEIGENVECTORS OF ORIGINAL PROBLEM)
 8011 FORMAT(1H0,56X,8HQ MATRIX)
C     EIGENVECTORS OF ORIGINAL
      DO 3080 I=1,N
      DO 3078 J=1,N
      U(I,J)=0.
      DO 3077 K=1,N
 3077 U(I,J)=U(I,J)+T(I,K)*B(K,J)
```