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THE DESIGN OF MICROMETEOROID PENETRATION EXPERIMENTS AS SINGLE-SAMPLING LIFE-TEST SAMPLING PLANS

by Arthur G. Holms

Lewis Research Center
Cleveland, Ohio

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SUMMARY

Space vehicles carrying tanks filled with liquid fuels or radiators with liquid coolants are vulnerable to puncture by micrometeoroids and subsequent loss of fluid. Penetration sampling experiments are especially justified when manned missions requiring high reliabilities are considered. Rational design of experiments requires preliminary consideration of the validity of conclusions as determined by the size of the experiments. Adequacy of the size can be judged by proven criteria of the customer-vendor relations of industrial sampling.

The danger of penetration of a vulnerable part of a space vehicle is assumed to be measured by the product of the exposed vulnerable area and the time that the mission will occupy a region of space containing micrometeoroids. The size of the sample is considered to be the factor by which the total sum of area-time products of the experiments exceeds the mission vehicle area-time product. Single binomial sampling plans are presented that cover a wide range of values of the quantities of interest. Procedures are suggested for data analysis following experimentation.

Sampling plans adequate for decisions where low reliability can be tolerated were found to involve experiment area-time products of 10 to 50 times the mission vehicle area-time product. Minimum sampling plans adequate for manned vehicle reliabilities were found to require experiment area-time products 700 to 7000 times the mission vehicle area-time product.

INTRODUCTION

Space vehicles carrying tanks with liquid fuels or radiators with liquid coolants are vulnerable to puncture by micrometeoroids and subsequent loss of fluid. Inferences concerning this hazard have been based on extrapolations of low-velocity ballistic data combined with visual and radar observations of micrometeoroid frequencies. These inferences (ref. 1) have led to the conclusion that micrometeoroids constitute a major hazard deserving direct observation by penetration experiments in space.

Direct observation experiments are reported in references 2 to 5. Weight limitations of available launch vehicles have limited these experiments to small exposed areas. Getting significant data in allowable observation times has

limited the experiments to very thin shells and particles that would be too small to penetrate the shells of contemplated space vehicles. The statistical validity of extrapolating thin-shell penetration data to predict reliability for thick shells has not been investigated.

Penetration sampling experiments are especially justified by missions (such as manned missions) that demand high reliabilities. The suitability of the vehicle meteoroid protection for the mission is assumed to be tested by performing sampling experiments using one or more structures having the same shell material and thickness as the mission vehicle. The rational design of such experiments requires that preliminary consideration be given to the validity of the conclusions as determined by the total exposed area of the sampling structures. One purpose of the present report is to provide a basis for judging the adequacy of the size of proposed experiments.

Irrespective of the statistical adequacy of present weight- and area-limited experiments, the range of uncertainty of conclusions from the data should be assessed. The second purpose of this investigation is to provide a method for the quick calculation of confidence intervals from the acquired data.

The problems of ensuring specified reliability against micrometeoroid penetration and of calculating confidence intervals are reduced to analogous problems in the customer-vendor relations of industrial sampling. Sampling plans are presented in tabular and graphical form to show the consequences, in required sample sizes, of variations in desired reliabilities and allowable decision error probabilities as they vary in ranges of practical interest. After data are acquired, the graphs can be used for quick estimates of confidence intervals. More sophisticated procedures for data reduction are discussed in appendix A.

SYMBOLS

A vulnerable area of mission vehicle

a_i instrumented area

c acceptance number; mission vehicle design is approved if number of penetrated regions of sampling structures each of area a_i does not exceed c

i index

n sample size; number of instrumented areas, each with $a_i t_i$ equal to AT of mission vehicle

P probability

P_1 $P(r > c)$

P_2 $P(r \leq c)$

q unknown constant probability that area a_i is penetrated in time t_i; $1 - q$ is reliability (survival probability)

$\bar{q}$ estimator of q

q_1 designer's penetration probability limit; mission vehicle design should not be rejected if $q < q_1$

$\bar{q}_1$ lower limit of confidence interval

q_2 operator's penetration probability limit; mission vehicle design should not be accepted if $q > q_2$

$\bar{q}_2$ upper limit of confidence interval

r number of penetrated areas

T mission time

t_i time of exposure of ith area

α designer's risk, upper bound on probability that mission vehicle design is rejected with an environment where $q \leq q_1$

β operator's risk, upper bound on probability that mission vehicle design is accepted with an environment where $q > q_2$

CRITERIA FOR SAMPLE SIZES

Analogy with Industrial Sampling

In the planning of micrometeoroid experiments an obvious cost is that of the experiment vehicle and its payload. If such costs are to be justified, they are justified by comparison with those of not having the data. A full quantitative cost analysis needs the elaborate methods of operations research, which are outside the scope of the present paper. This paper presents an elementary analysis which, when related to given conditions, provides some "rule of thumb" criteria on how large the micrometeoroid experiments ought to be. This result is achieved by focusing attention on some basic parameters of the total operations research problem.

The ultimate purpose of micrometeoroid penetration experiments should be to contribute to the accuracy with which decisions are made concerning the adequacy of the micrometeoroid protection designed into important space mission vehicles. Insufficiency of data can contribute to two classes of costs.

One class is the cost of being too pessimistic. Errors of pessimism can result in (1) devoting excessive weight to micrometeoroid protection, with consequent reductions in payload or consequent increases in thrust capacities, or (2) abandoning achievable missions.

A second class of costs is the one resulting from being too optimistic. Associated errors could result in the loss of personnel and the destruction of missions.

The "rule of thumb" criteria for micrometeoroid experiment sizes will now be established by defining some parameters that are related to (1) costs of excessive pessimism and (2) costs of excessive optimism. The discussion will be simplified by personalizing the two classes of costs. The costs of excessive pessimism are to be a burden on the mission vehicle designer in the sense that he suffers the loss caused by excessive shell thickness or delayed missions. The operator is defined as the person who must suffer the loss from excessive optimism, such as the loss of life or missions. The micrometeoroid experimenter must then, in some degree, control the expectation of losses for both the mission vehicle designer and the operator.

The terminology just presented implies that questions as to what constitutes an adequate experiment are amenable to criteria that have been established in industrial sampling, where sampling must satisfy the conflicting interests of producers and consumers. The designer is in a position analogous to that of the producer; the operator is in a position analogous to that of the consumer; and the micrometeoroid experimenter is in a position analogous to that of the sampling inspector.

Just as the consumer specifies a rejectable quality level and a consumer's risk, the operator can specify q_2 as the upper limit on the probability of penetration and can specify that the probability of approving a given mission vehicle design (as based on micrometeoroid experiments) shall be limited to some small number β (the operator's risk) under the condition that the probability of penetration is greater than q_2.

Analogously, just as the producer designates an acceptable quality level and an associated producer's risk, the mission vehicle designer can ask that the micrometeoroid experiments be large enough so that if the true probability of penetration is less than the lower limit q_1 (where $q_1 < q_2$), the probability of the proposed mission vehicle design being rejected will be limited by some small number α.

The procedure leaves decisions uncontrolled under the circumstance that the true penetration probability is between q_1 and q_2. The zone from q_1 to q_2 is called the zone of indifference. A necessary condition for less than infinite sample sizes is that the zone of indifference be greater than zero. The size of the zone of indifference should be adjusted according to the cost of sampling.

In summary, the basis for deciding on sample sizes is achieved by assuming that the object of the experiments is to reach a decision as to whether a particular vehicle is sufficiently penetration resistant for a mission; one of two decisions is reached, namely, (1) safe enough or (2) not safe enough. Safe enough implies that the vehicle design is accepted for the contemplated mission. Not safe enough implies that the vehicle design is rejected with respect to the proposed mission. Because one of only two decisions is to be reached, the decision procedure is based on single sampling and is subject to very elementary methods for determining sample sizes that have been proven satisfactory in industrial sampling. (Sampling procedures such as multiple sampling or sequential sampling are potentially available for problems with three decisions, that is, the choice of (1) safe enough, (2) not safe enough, or (3) get more data and reconsider.

These three-decision procedures are not appropriate to making a rough preliminary estimate of the total amount of sampling that will be needed; but once a program whose size is determined by elementary methods is in progress, the sequential procedures are available for making decisions in advance of completion of the experiments. Some underlying theory of sequential sampling for life testing has been described in refs. 6 and 7.)

Experimentation

The present analysis assumes that the micrometeoroid experiments are to be performed with the same material and shell thickness as are contemplated for the mission vehicle. The sampling structure is to be instrumented to record micrometeoroid penetrations. Its flight path must be programed to sample the environment that will constitute the hazard to the mission vehicle. Questions such as whether an Earth satellite will adequately sample the environment of a deep space mission are not within the scope of the present analysis. The present analysis merely assumes that the sampling structure does sample the same population of micrometeoroid arrivals as will be encountered by the mission vehicle. Other investigations must establish the rules as to what types of flight paths have comparable penetration hazards.

The assumption is made that one or more sampling structures are launched. They need not all be in space at the same time; an underlying assumption is that, although the micrometeoroids may arrive with some degree of clustering, the population is essentially homogeneous with respect to the time span of experiments and related missions. Separate launchings at successive times could provide a test of homogeneity.

The mission vehicle has vulnerable surface area A and hazardous mission time T, so that a measure of the hazard is provided by the area-time product AT. The sampling structures will have other areas and exposure times. The observation of a penetration probability assumes that the sampling will be sufficient to include a fair representation of the micrometeoroids that arrive in clusters. In averaging out the variability associated with clustering, the use of long observation times (and correspondingly small a_i) will be an advantage. The same structure is more likely to be subjected to a series of independent clusters. Increasing the exposed area of a single structure cannot provide a comparable improvement in the averaging of clustering, because the dimensions of the exposed area of a sampling structure are negligible in comparison with the dimensions of a cluster; any practical structure would be too small to receive penetrations by separate clusters at any one time.

The combination of instrumentation and data reduction is assumed to be such that the total sampling area over all the experiment structures is divided into n areas. The observable area-time product $a_i t_i$ of each instrumented area is assumed equal to the area-time product of the mission vehicle. The sample size n is then the ratio of total experimental area-time product to the mission area-time product, and because each $a_i t_i$ is required to be equal to AT,

$$\sum_{i=1}^{n} \frac{a_i t_i}{AT} = n$$

Sampling Plans

A sampling plan consists of specified values of n, α, β, q_1, q_2, and an acceptance number c. If c or less penetrated samples are observed to occur among the a_i during their observation times t_i, then the mission vehicle design is approved. If $c + 1$ or more of the a_i are penetrated during their observation times t_i, the mission vehicle design is rejected. The assumption that the population of micrometeoroid arrivals is essentially homogeneous implies that, with respect to a vehicle with vulnerable area A and mission time T, there exists a constant probability q of its being penetrated. This probability is also valid for any experiment having exposed area a_i and exposure time t_i such that $a_i t_i = AT$. Under these assumptions, the binomial distribution provides the probability theory governing the variables of the sampling plan.

Parametric procedures can be used for constructing sampling plans. The present point of view is that the uncertainties of the micrometeoroid hazard do not justify deciding on sample sizes by accounting for the better efficiencies of parametric or sequential procedures as compared to single-sampling, binomial procedures. Although single-sampling binomial procedures ask for larger sample sizes than do parametric or sequential procedures, the sample sizes must not be viewed as pessimistic. The phenomenon of clustering exists as an unknown that can cause sample sizes, as determined by any method, to be too small.

APPLICATION OF BINOMIAL DISTRIBUTION

Consider n areas a_i exposed to micrometeoroid penetration for times t_i where $a_i t_i = AT$. If q is regarded as a population parameter for areas a_i tested to time t_i leading to either of the observed attributes "penetrated" or "not penetrated", the experiment corresponds to n trials where each trial has a probability of penetration of q and a probability of no penetration of $1 - q$. The binomial distribution gives the probability of r such penetrations, namely,

$$P(r) = \frac{n!}{r!(n - r)!} q^r (1 - q)^{n-r} \tag{1}$$

A sampling plan is considered where n areas a_i are placed on test, and the population is regarded to be acceptable if c or less of them are penetrated within time t_i. The probability that more than c fail within times t_i is then found by summing equation (1) over all values of r for which $r \geq c + 1$. Thus,

$$P_1 = P(r > c) = \sum_{r=c+1}^{n} \frac{n!}{r!(n - r)!} q^r(1 - q)^{n-r} \tag{2}$$

That is, if the probability of one a_i being penetrated in time t_i is q, the probability of observing more than c such areas penetrated is given by equation (2).

The complementary event is the event of c or less areas a_i penetrated in associated observation times t_i. This probability is also obtained by summing equation (1) over the appropriate range of r:

$$P_2 = P(r \leq c) = \sum_{r=0}^{c} \frac{n!}{r!(n - r)!} q^r(1 - q)^{n-r} \tag{3}$$

Sampling Procedures to Control Designer's Risk

The designer has been defined as desiring that the mission vehicle design be approved if the environment provides a probability of penetration q that is less than the specified value q_1. The decision is to be based on a sample of size n of which r areas are observed to be penetrated. The mission vehicle design is to be accepted only if $r \leq c$ (otherwise rejected), where c is specified in advance. The probability of a decision error against the designer is limited to some small probability α by choosing c and n so that

$$P(r > c | q = q_1) \leq \alpha \tag{4}$$

The determination of c and n values satisfying equation (4) is made possible by equation (2). The set of c and n values satisfying equation (4) is the set for which equation (2), with $q = q_1$, results in $P_1 \leq \alpha$. Under the conditions that $P_1 = \alpha$ and $q = q_1$, equation (2) provides a unique functional relation among α, q_1, c, and n, namely,

$$\alpha = \sum_{r=c+1}^{n} \frac{n!}{r!(n - r)!} q_1^r(1 - q_1)^{n-r} \tag{5}$$

If c and n are to be used in sampling plans, they must be chosen as integers. Values of c, n, and q_1 satisfying equation (5) for fixed values of α have been tabulated by several authors (refs. 8 to 11). Values from references 8 to 11 were used to plot figure 1, which shows representations of equation (5) for $\alpha = 0.01$, 0.05, and 0.10. The regions of figure 1 corresponding to values of n in excess of those tabulated in references 8 to 11 were obtained from the Poisson approximation as tabulated in reference 12 and described in appendix B.

In the use of these charts, an experimenter could, for example, begin with specified α and q_1 and then read successively larger values of n corresponding to $c = 0, 1, 2$, etc. Because P_1 of equation (2) is a monotonically increasing function of q, such pairs of values of c and n would then provide assurance that

$$P(r > c | q \leq q_1) \leq \alpha \tag{6}$$

Sampling Procedures to Control Operator's Risk

The operator has been defined as being concerned about acceptance of a mission design when the environment provides a penetration probability greater than the specified value of q_2. The operator asks that the probability of a decision adverse to his wishes be limited to some small probability β. This is to be done by choosing c and n so that

$$P(r \leq c | q = q_2) \leq \beta \tag{7}$$

The determination of c and n values satisfying equation (7) is made possible by equation (3). The set of c and n values satisfying equation (7) is the set for which equation (3), with $q = q_2$, results in $P_2 \leq \beta$. Under the conditions that $P_2 = \beta$ and $q = q_2$, equation (3) provides a unique functional relation among β, q_2, c, and n, namely,

$$\beta = \sum_{r=0}^{c} \frac{n!}{r!(n - r)!} q_2^r (1 - q_2)^{n-r} \tag{8}$$

Values of q_2 for integer values of c and n and fixed values of β are tabulated in references 8 to 11. Values from references 8 to 11 were used as solutions of equation (8) for $\beta = 0.01$, 0.05, and 0.10 in plotting the charts of figure 2, except that the regions of large n were obtained from the Poisson approximation as tabulated in reference 12 and as described in appendix B. Because P_2 of equation (3) is monotonically decreasing with q, the paired values of c and n of figure 2 guarantee that

$$P(r \leq c | q \geq q_2) \leq \beta \tag{9}$$

If a sampling plan is to be selected solely to protect the operator, then corresponding to the given q_2 and β the minimum experiment would correspond to $c = 0$. The mission vehicle design is rejected for one penetration in the experiment and accepted only if no penetrations occur. Such a decision procedure would control the hazard to the operator to the degree specified and at a minimum cost of micrometeoroid experimentation. The cost of making decision errors dangerous to personnel would be controlled, but the cost of making the decision error of being too cautious would be essentially uncontrolled. Control of exces-

sive caution is attained by the specification of q_1 and the designer's risk α.

SAMPLING PLANS FOR JOINT CONTROL OF DESIGNER'S AND OPERATOR'S RISKS

It is possible to select c and n (which must be integers) to satisfy both the designer and the operator. A necessary condition for finite sample sizes is that q_1 be less than q_2. Conditions necessary for consistency are that c and n do not change between the constant α and constant β curves. For example, the operator might state that he does not wish to approve missions with q greater than $q_2 = 0.01$, and that he wants the decision error probability bounded by $\beta = 0.01$. The designer might accede to q_2 being 0.01, but he might require that, if the risk of penetration is 0.001 or less, the probability of the mission design being rejected must be bounded by $\alpha = 0.10$. Values of c and n satisfying both of these requirements can be found from figures 1(c) and 2(a). Minimum cost of experimentation is desired, and therefore figures 1(c) and 2(a) should be searched for the minimum n and associated c that will meet all the specifications. Some values of c and n can be read from figures 1(c) and 2(a) as follows:

Acceptance number, c	Sample size, n	
	Designer's penetration probability limit, q_1, 0.001; designer's risk, α, 0.10 (a)	Operator's penetration probability limit, q_2, 0.01; operator's risk, β, 0.01 (b)
0	104	460
1	530	660
2	1100	850
3	1750	1000

[a]Fig. 1(c).

[b]Fig. 2(a).

The preceding tabulation shows that the n to satisfy α overtakes the n to satisfy β at $c = 2$. With $c = 1$ as the acceptance number, a sample of size $n = 660$ would (from fig. 1(c)) be equivalent to $q_1 = 0.0008$, which is unsatisfactory because it is less than the designer's lower limit. Also, $c = 1$ and $n = 530$ are unsatisfactory because they correspond to $q_2 = 0.0125$, which fails to meet the operator's upper limit. Taking $c = 2$ and $n = 850$ corresponds to $q_1 = 0.0013$ and meets the designer's lower limit. Taking $c = 2$ and $n = 1100$ corresponds to $q_2 = 0.0077$ and thus satisfies the operator's upper limit. The most economical sampling plan that meets all original specifications

occurs for $c = 2$ and $n = 850$, which with $\alpha = 0.10$ and $\beta = 0.01$ corresponds to $q_1 = 0.0013$ and $q_2 = 0.01$.

Interrelations Among Designer's and Operator's Specifications

Interrelations among the parameters of the sampling plans are shown by figures 3(a) and (b), which are cross plots of quantities from figures 1 and 2. The absolute minimum on sample size is always defined by the $c = 0$ line. For example, in figure 3(a) this line shows that, at risks of $\alpha = 0.05$ and $\beta = 0.05$, specifications such as $q_1 = 0.0025$ and $q_2 = 0.14$ can be met with a sample size of 20. At the same q_2, decisions can be based on larger c and correspondingly larger n values for values of q_1 closer to q_2; for example, decisions can be made with the same q_2 and $q_1 = 0.025$ with $c = 3$ and $n = 54$.

The selection of tighter error bounds ($\alpha = 0.01$ and $\beta = 0.01$) produces the results shown in figure 3(b). The general trends are the same for figures 3(a) and (b); however, close comparison of figure 3(a) with (b) (say at $q_1 = 0.002$ and $q_2 = 0.100$) shows that larger sample sizes are required to meet the conditions of the smaller decision error probabilities of figure 3(b).

Typical Sampling Problems

Based on data from figures 1(a), 1(c), 2(a), and 2(c), table I compares sample sizes intended to be appropriate for micrometeoroid experiments related to instrument and to manned payloads. The parameters of table I(a) are based on the assumption that an instrument payload ought to be protected against the financial loss of a mission failure, and a reliability against penetration of 80 percent was assumed to be required. Correspondingly, $q_2 = 0.20$. The α and β risks were set at the largest values that statisticians regard as the transition point between objective experimentation and the realm of the occult, namely, $\alpha = \beta = 0.10$. Corresponding sampling plans are listed in table I(a). The sample sizes (ratios of total exposed area-time products to mission area-time product) vary from 10 to 74 for decisions relevant to $q_2 = 0.20$ and q_1 in the range $q_1 = 0.01$ to $q_1 = 0.10$.

Table I(b) represents a situation intended to apply to manned payloads. The reliability is specified as 99 percent, that is, $q_2 = 0.01$. (Higher reliability is, of course, to be desired. The specification of higher reliability requires larger sample sizes than those listed in table I.) The assumption is made that a manned mission is for a purpose sufficiently important that both the operator's and the designer's risks should be closely bounded. Correspondingly, let $\alpha = \beta = 0.01$. Under these restrictions, the set of sampling plans listed in table I(b) shows sampling sizes from $n = 660$ to $n = 6900$ that provide protection against rejecting the mission from environments that are as safe as $q_1 = 0.00022$ to $q_1 = 0.0052$. (The mission has a probability, bounded by α, of being grounded out of environments that have reliabilities as good as 0.9998 with $c = 1$ and $n = 660$ or environments that have reliabilities as good as 0.995 if $c = 50$ and $n = 6900$.) Experiments of comparable size are also con-

cluded to be appropriate to manned missions in reference 1.

Tabular Presentation of Sampling Plans

The preceding discussion concerned charts that exhibit sampling plans appropriate to specified levels of α and β. The micrometeoroid experimenter might wish to begin with specified values of q_1, q_2, and n and then proceed to investigate the consequences (in terms of α and β) of available choices of c. Table II has been supplied in an attempt to meet such a wish. The values in this table were obtained from reference 13 for values of n of 1000 and less. When interpolation was needed, it was linear. The Poisson approximation, as described in appendix B, was used for larger values of n. When the Poisson approximation was used, the values of α and β were obtained from reference 14. Table II is arranged with the idea that a micrometeoroid experimenter would first consider some number q_2 that represents an approximate maximum allowable probability of penetration. Equivalently, the minimum mission reliability or survival probability is $1 - q_2$.

The section of the table for the given value of q_2 is selected. The experimenter can then scan the values of n and q_1 for what he considers to be reasonable values of these two quantities. In general, table II was prepared with the idea that both α and β should be kept within the limits 0.01 to 0.10. In order that the smallest possible sample sizes can be considered, values of α and β are also presented that correspond to one step in c that would place a sampling plan just beyond these limits on α and β. If for some q_1 and n only one value of c is listed, such an acceptance number should be regarded as the acceptance number that gives the pair of designer's and operator's risks α and β closest to the range 0.01 to 0.10, and these risks are read from the table as criteria of the precision of the sampling plan. In cases where, for selected q_2, n, and q_1, several values of c are listed, the decision point c may be chosen for what may be thought to be the most desirable apportionment of bounds α and β on the decision error probabilities. For example, c might be selected so that the operator's risk β would be smaller than the designer's risk α.

QUICK ESTIMATES OF CONFIDENCE INTERVALS

Some sophisticated procedures appropriate to the analysis of acquired data are discussed in appendix A. These procedures can be avoided, and quick crude estimates of confidence intervals can be computed as follows.

If it is assumed that the results of observing a total of n areas a_i for times t_i where each $a_i t_i = AT$ are that r of the n areas were penetrated, the minimum variance unbiased estimator $\bar{q}$ of the probability q of the penetration of area A in time T is

$$\bar{q} = \frac{r}{n} \tag{10}$$

This estimate is bracketed with a confidence interval as follows. Figure 1 was constructed according to equation (5) so that there is an α probability of observing $c + 1$ or more areas penetrated if $q = q_1$. Let $\overline{q}_1$ be the value of q_1 obtained from a chart of figure 1 for a specified α and with $c + 1$ set equal to the observed number of penetrated areas ($c = r - 1$). Figure 1 provides a transformation of the random variable r to the random variable $\overline{q}_1$. Then, with $\overline{q}_1$ read from the chart,

$$P(\overline{q}_1 \geq q) = \alpha \tag{11}$$

That is, the probability of $\overline{q}_1$ not being less than the parameter q is α.

Figure 2 was constructed according to equation (8) so that there is a β probability of observing c or less areas penetrated if $q = q_2$. Let $\overline{q}_2$ be the value of q_2 obtained from a chart of figure 2 for a specified β and with c set equal to the observed number of penetrated areas ($c = r$). Figure 2 provides a transformation of the random variable r to the random variable $\overline{q}_2$. Then, with $\overline{q}_2$ read from the chart,

$$P(\overline{q}_2 \leq q) = \beta \tag{12}$$

That is, the probability of $\overline{q}_2$ not being greater than the parameter q is given by β. The interval from $\overline{q}_1$ to $\overline{q}_2$ is called a $1 - \alpha - \beta$ confidence interval for the parameter q. In other words, the probability of the range $\overline{q}_1$ to $\overline{q}_2$ covering the unknown parameter q is $1 - \alpha - \beta$. The span of the interval $\overline{q}_1$ to $\overline{q}_2$ is a range of uncertainty associated with the estimator $\overline{q}$ of q.

For example, in a sample of $n = 1000$ it has been observed that $r = 15$ penetrations. The estimator of q is

$$\overline{q} = \frac{15}{1000} = 0.015$$

and the estimate of the penetration reliability is $1 - \overline{q} = 0.985$. An equal-sided, 90-percent-confidence interval is desired. If $\alpha = 0.05$, $\beta = 0.05$, and figures 1(b) with $c = 14$ and 2(b) with $c = 15$, $n = 1000$ are used to read $\overline{q}_1 = 0.0092$ and $\overline{q}_2 = 0.023$, it can be said with 90-percent confidence that the reliability against penetration is between 0.991 and 0.977. From these same data, it is possible to establish just a lower bound on the reliability. If $\beta = 0.01$, it can be observed from figure 2(a) that $\overline{q}_2 = 0.027$ when $c = 15$ and $n = 1000$. It can be said with 99-percent confidence that the lower bound on the reliability against penetration is 0.973.

CONCLUDING REMARKS

The mission vehicle hazard is assumed to be measured by its area-time product, and the sample size is defined to be the factor by which the total sum of

area-time products of the experiments exceeds the mission vehicle area-time product. Single-sample binomial sampling plans have been presented. Emphasis has been on plans that limit decision error probabilities to the range 0.01 to 0.10. This range was selected with the idea that decision error probabilities in excess of 0.10 take the problem out of the range of objective discussion, and that sample sizes for decision error probabilities less than 0.01 are larger than can be justified in view of the instabilities of micrometeoroid arrival rates facing an individual mission.

The sampling plans state that, if the decision maker is concerned with reliability levels that are customarily associated with manned expeditions, say of approximately 0.99, the probability of observing penetrations will be very small. Acquisition of sufficient data for decisions then requires large sample sizes - say in the range 700 to 7000 times that of the mission area-time product. In particular, the probability of approving a mission vehicle design for an environment having a reliability against penetration of less than 0.99 could be limited to 1 percent with a sample of size 660. Having agreed upon such a specification, the rejection of mission vehicle designs, when the environment provides much better reliabilities against penetration, would constitute an undesirable denial of mission opportunities. The range of reliabilities in which unnecessary rejections of mission vehicle designs can occur should be made as small as possible. This range would be excessive for a sample of size 660, where there would be a 1-percent probability of rejecting a mission vehicle design from an environment with a reliability of 0.9998 or better. The region of needless rejections can be closed down to reliabilities of 0.995 by the use of a sample size of 6900. In other words, a sample size as large as 6900 is needed to control decision errors under the conditions that the mission vehicle design should not be approved if the reliability is less than 0.99 and should not be rejected if the reliability is better than 0.995.

On the other hand, limiting the experiments to small sample sizes (area-time products of 10 to 50 times the mission vehicle area-time product) only permits decisions with large error bounds (error probabilities of 10 percent) to be made for much lower reliabilities against penetration (80 percent).

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, July 2, 1963

APPENDIX A

INFLUENCE OF INSTRUMENTATION

Sample Sizes

Instrumentation systems now under development include three types that furnish one of the following three kinds of data:

(1) Total number of penetrations of an area in a given time period of observation

(2) Elapsed time to the first penetration of an area

(3) Elapsed times between penetrations for a succession of penetrations of an area

The procedure of the main section of this report titled SAMPLING PLANS FOR JOINT CONTROL OF DESIGNER'S AND OPERATOR'S RISKS is desirable for initial estimates of the sizes of experiments when no knowledge of the population is at hand. Because that procedure is concerned with only the first penetration of an instrumented area, the above types of instrumentation that record multiple penetrations might be presumed to require smaller sample sizes. In essence, they do not. Most space vehicles (especially manned vehicles) will be designed for high reliability and the chance of an instrumented area being penetrated within the area time product of the mission vehicle is very small. Where q is the probability of such a penetration, the probability of two or more penetrations is

$$q^2 + q^3 + q^4 + q^5 + \cdot \cdot \cdot = q^2(1 + q + q^2 + q^3 + \cdot \cdot \cdot) = \frac{q^2}{1 - q}$$

The preceding equation shows that, when q is small, the probability of more than one penetration is of the order of q^2 and is thus extremely small. These remarks suggest that preliminary estimates of the required size of an experiment can be based on the theory of the main section of the report: the potentially extra information furnished by multiple count instrumentation can be ignored in designing the size of the experiment.

Parametric Estimation

Because the procedure of the main section of this report interprets the data as "no penetration of area a_i occurred during t_i" or "the area a_i received its first penetration during t_i," the procedure might discard information provided by instrumentation systems of the three types just described. After large quantities of data have been received, the nature of the clustering phenomena may be better known, and statistical procedures might be available that would make fuller use of all the information. Data of one of the types just described and some assumptions about the probability distributions of the penetrations permit the fullest use of such information by using a probability equation containing

unknown empirical constants. Values of the unknown constants are estimated by computations from the data. The constants are called parameters, and procedures for evaluating the parameters or statements about the parameters are called parametric statistical procedures.

The arrival of micrometeoroids is described as "clusters plus background" in reference 1. The background arrivals can be expected to fit the Poisson process model, and the arrivals within any cluster might fit a Poisson model with high arrival rate. Taken over a period of time, the totality of such phenomena can be described as a synthesis of heterogeneous populations. As is pointed out in reference 15, the Weibull density has been used to fit collections of heterogeneous populations. It might prove useful in the description of micrometeoroid penetration probabilities at any given thickness.

The correlation of penetration data for several thicknesses under the assumption of a Poisson process is the obvious starting point for efforts to extrapolate thickness data. Non-Poisson phenomena in the range of very high reliability may require other statistical procedures. Non-Poisson high-reliability problems occur in the field of accelerated life tests of electronic parts. Statisticians are developing procedures that deal with the effects of voltage or current and temperature on the life of capacitors and transistors. Extrapolating thickness data for micrometeoroid penetrations might prove to be analogous to extrapolating temperature data for transistors.

APPENDIX B

THE POISSON APPROXIMATION

The values of α and β listed in table II were obtained from reference 13 for sample sizes n up to 1000 and for q values down to 0.01. For larger values of n, the α and β values of table II correspond to small values of q.

The situation consisting of large n and small q permits the use of the Poisson probability function, namely,

$$P(r) = \frac{(qn)^r e^{-qn}}{r!}$$

This approximation is discussed on pages xviii and xix of reference 13. Basic tables for the cumulative Poisson distribution are contained in reference 14. Other Poisson tables that were not available in time for the work of the present investigation are contained in reference 16. The smallest q listed in the Harvard binomial tables (ref. 13) is 0.01. Values of β taken from reference 13 for $q_2 = 0.01$ were compared with values of β for the same conditions as obtained from the Poisson approximation using the tables of reference 14. Differences, for values of n from 150 to 1000, were no greater than 0.002.

Figures 1 and 2 use data for values of n up to the largest values of n contained in references 8 to 11. The cumulative sum tables of the Poisson approximation (ref. 12) were used to plot points for values of q less than 0.10 and values of n larger than 1000 in figures 1 and 2. The Poisson data fell on straight lines in figures 1 and 2. The curves of figures 1 and 2 were then completed by extending smooth curves from $n = 1000$ down to the data points that had been plotted from references 8 to 11.

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TABLE I. - TYPICAL PARAMETERS OF SAMPLING PLANS

(a) Instrument payload vehicles; designer's risk, α, 0.10; operator's risk, β, 0.10; operator's penetration probability limit, q_2, 0.20

Acceptance number, c	Sample size, n	Designer's penetration probability limit, q_1
0	10	0.01
1	18	.03
2	25	.045
5	44	.074
10	74	.097

(b) Manned vehicles; designer's risk, α, 0.01; operator's risk, β, 0.01; operator's penetration probability limit, q_2, 0.01

Acceptance number, c	Sample size, n	Designer's penetration probability limit, q_1
1	660	0.00022
2	850	.00051
5	1320	.00136
10	2000	.0024
20	3300	.0036
50	6900	.0052

TABLE II. - BINOMIAL SAMPLING PLANS

Designer's penetration probability limit, q_1	Sample size, n	Acceptance number, c	Designer's risk, α	Operator's risk, β
Operator's penetration probability limit, q_2, 0.002				
0.0005	1,000	0	0.393	0.135
	1,500	1	.174	.199
	2,500	2	.132	.125
	4,000	3	.143	.042
		4	.053	.100
		5	.017	.191
	6,500	5	.111	.011
		6	.048	.026
		7	.018	.054
		8	.006	.100
	10,000	9	.032	.005
		10	.014	.011
		11	.005	.021
0.001	6,500	9	0.123	0.166
	10,000	13	.136	.066
		14	.083	.105
Operator's penetration probability limit, q_2, 0.005				
0.0005	400	0	0.181	0.135
	650	0	.277	.039
		1	.043	.165
	1,000	0	.393	.007
		1	.090	.040
		2	.014	.125
	1,500	1	.174	.005
		2	.041	.020
		3	.007	.059
	2,500	4	.009	.005
0.001	650	1	0.139	0.165
	1,000	1	.264	.040
		2	.080	.125
	1,500	2	.191	.020
		3	.066	.059
		4	.019	.132
	2,500	4	.109	.005
		5	.042	.015
		6	.014	.035
		7	.004	.070
	4,000	9	.008	.005
0.002	1,500	4	0.185	0.132
	2,500	7	.133	.070
		8	.068	.125
	4,000	11	.112	.021
		12	.064	.039
		13	.034	.066
		14	.017	.105
	6,500	19	.043	.008
		20	.025	.013
		21	.014	.022
		22	.008	.034
	10,000	32	.005	.004

Designer's penetration probability limit, q_1	Sample size, n	Acceptance number, c	Designer's risk, α	Operator's risk, β
Operator's penetration probability limit, q_2, 0.01				
0.0005	150	0	0.072	0.221
	250	0	.117	.081
		1	.007	.286
	400	0	.181	.018
		1	.018	.090
		2	.001	.237
	650	0	.277	.001
		1	.043	.011
		2	.005	.042
	1,000	2	.014	.003
		3	.002	.010
0.001	150	0	0.139	0.221
	250	0	.221	.081
		1	.026	.286
	400	0	.330	.018
		1	.062	.090
		2	.008	.237
	650	1	.139	.011
		2	.029	.042
		3	.005	.111
	1,000	2	.080	.003
		3	.019	.010
		4	.004	.029
0.002	150	0	0.259	0.221
	250	0	.393	.081
		1	.090	.286
	400	1	.191	.090
		2	.047	.237
	650	2	.143	.042
		3	.043	.111
	1,000	3	.143	.010
		4	.053	.029
		5	.017	.066
		6	.005	.129
0.005	1,500	10	0.138	0.118
	2,500	16	.131	.038
		17	.084	.060
		18	.052	.092
		19	.031	.134
	4,000	25	.112	.008
		26	.078	.012
		27	.052	.019
		28	.034	.029
		29	.022	.043
		30	.013	.062
		31	.008	.086
	6,500	46	.010	.008

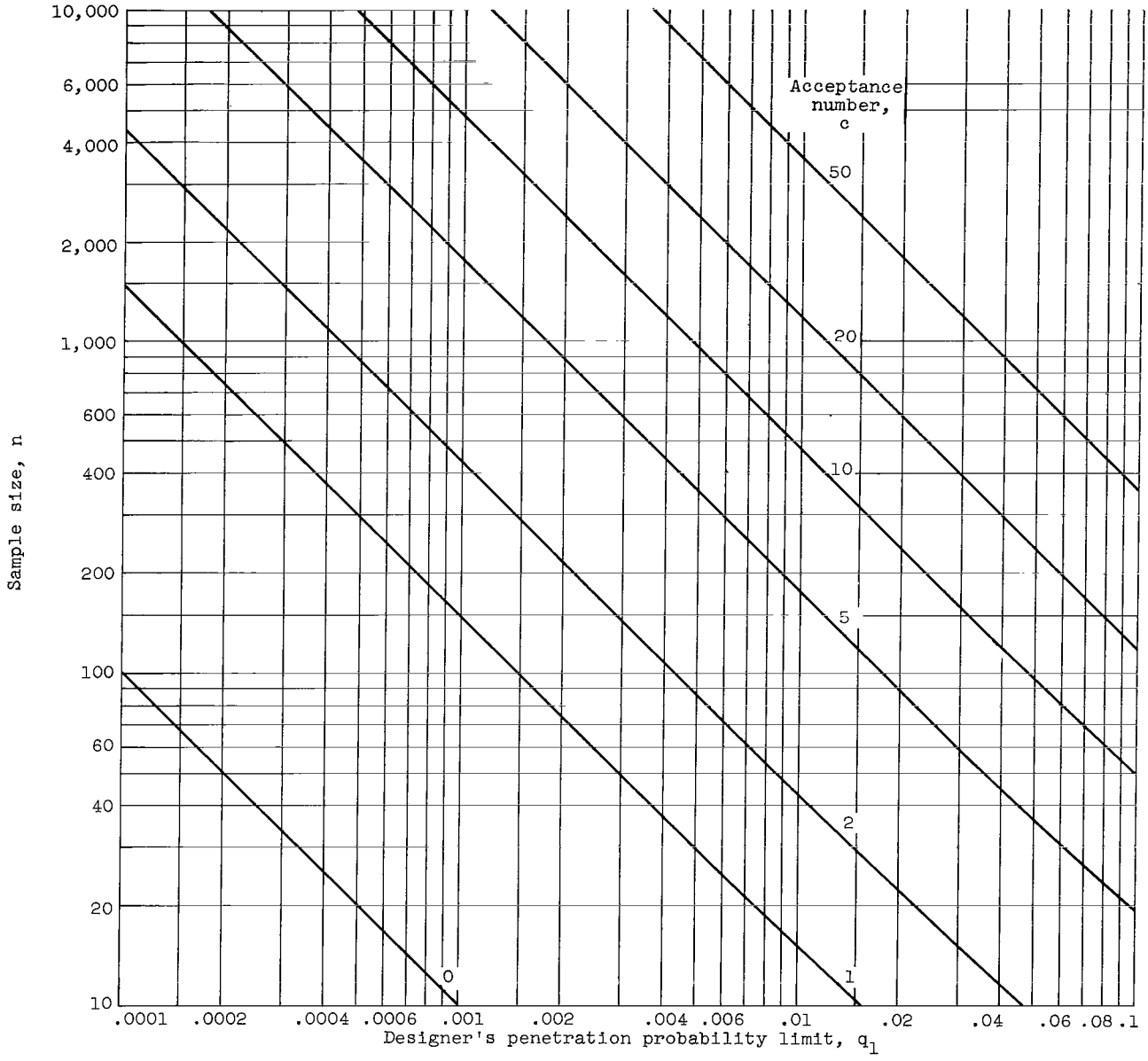
TABLE II. - Continued. BINOMIAL SAMPLING PLANS

Designer's penetration probability limit, q_1	Sample size, n	Acceptance number, c	Designer's risk, α	Operator's risk, β
Operator's penetration probability limit, q_2, 0.02				
0.0005	150	0	0.072	0.048
		1	.003	.196
	250	0	.117	.007
		1	.007	.040
	400	1	.018	.003
		2	.001	.013
0.005	150	1	0.174	0.196
	250	2	.132	.123
	400	3	.143	.041
		4	.053	.097
		5	.017	.188
	650	5	.111	.010
		6	.048	.025
		7	.018	.052
		8	.006	.097
	1,000	9	.032	.005
		10	.014	.010
		11	.005	.020
0.001	150	0	0.139	0.048
		1	.010	.196
	250	0	.221	.007
		1	.026	.040
		2	.002	.123
	400	1	.062	.003
		2	.008	.013
0.01	650	9	0.122	0.163
	1,000	13	.134	.064
		14	.082	.103
0.002	150	0	0.259	0.048
		1	.037	.196
	250	0	.393	.007
		1	.090	.040
		2	.014	.123
	400	1	.191	.003
		2	.047	.013
		3	.009	.041
	650	5	.002	.010
Operator's penetration probability limit, q_2, 0.05				
0.0005	40	0	0.020	0.129
	65	0	.032	.036
	100	0	.049	.006
		1	.001	.037
	150	1	.003	.004
0.001	40	0	0.039	0.129
	65	0	.063	.036
		1	.002	.158
	100	0	.095	.006
		1	.005	.037
	150	1	.010	.004
Operator's penetration probability limit, q_2, 0.05				
0.002	40	0	0.077	0.129
	65	0	.122	.036
		1	.008	.158
	100	0	.181	.006
		1	.018	.037
		2	.001	.118
	150	1	.037	.004
		2	.004	.018
0.005	40	0	0.181	0.129
	65	0	.277	.036
		1	.043	.158
	100	0	.393	.006
		1	.090	.037
		2	.014	.118
	150	1	.174	.004
		2	.041	.018
		3	.007	.055
	250	4	.009	.005
0.01	40	0	0.331	0.129
	65	1	.138	.158
	100	1	.264	.037
		2	.079	.118
	150	2	.191	.018
		3	.065	.055
		4	.018	.126
	250	4	.108	.005
		5	.042	.014
		6	.014	.033
		7	.004	.067
0.02	150	4	0.183	0.126
	250	7	.132	.067
		8	.067	.121
	400	11	.110	.019
		12	.062	.036
		13	.033	.061
		14	.016	.099
		15	.008	.150
Operator's penetration probability limit, q_2, 0.10				
0.01	15	0	0.140	0.206
	25	0	.222	.072
		1	.026	.271
	40	0	.331	.015
		1	.061	.080
		2	.008	.223
	65	1	.138	.009
		2	.028	.036
		3	.004	.100
	100	3	.018	.008
		4	.003	.024

TABLE II. - Concluded. BINOMIAL SAMPLING PLANS

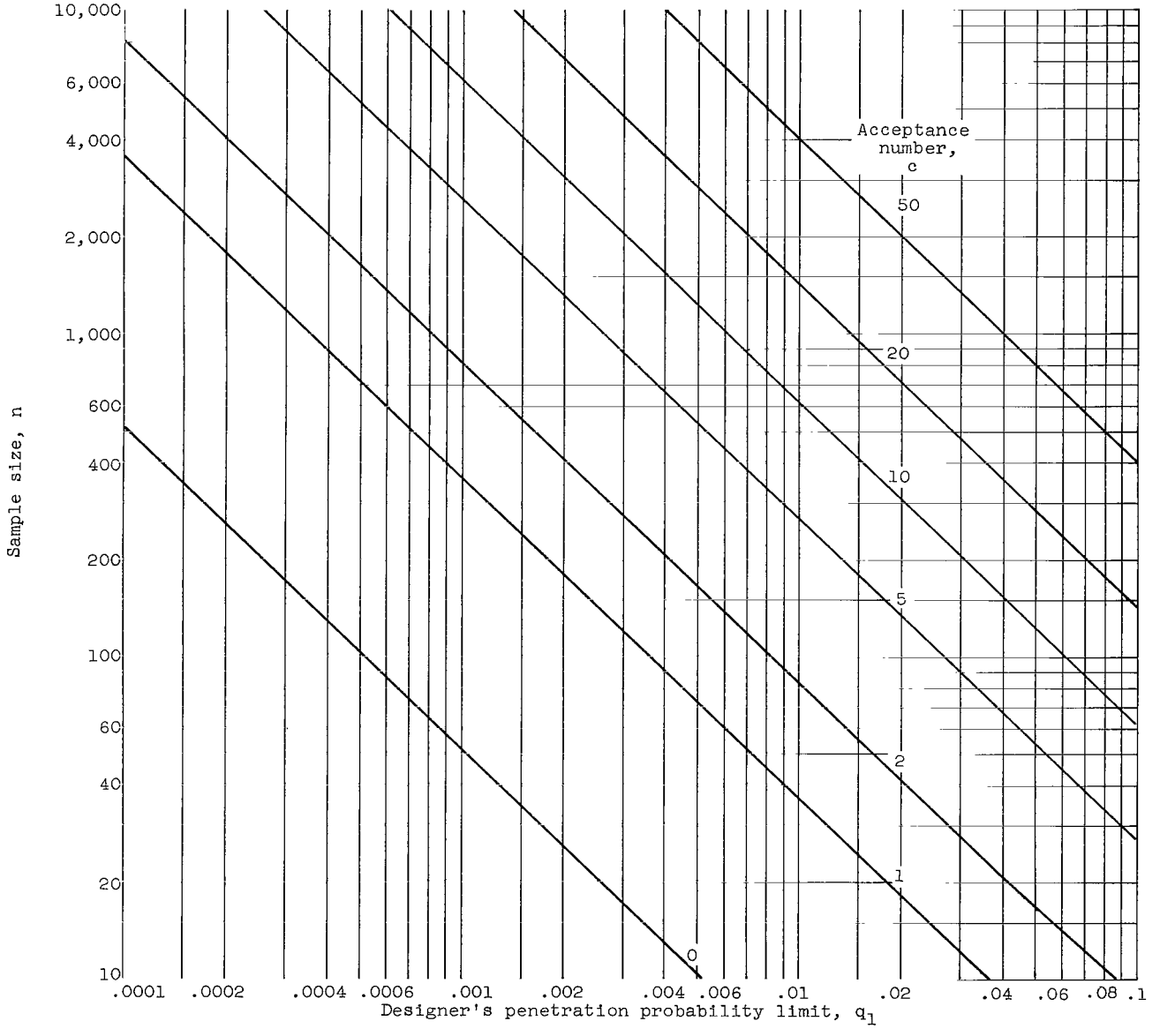
Designer's penetration probability limit, q_1	Sample size, n	Acceptance number, c	Designer's risk, α	Operator's risk, β
Operator's penetration probability limit, q_2, 0.10				
0.02	15	0	0.261	0.206
	25	1	.089	.271
	40	1	.190	.080
		2	.046	.223
	65	2	.141	.036
		3	.041	.100
	100	3	.141	.008
		4	.051	.024
		5	.015	.058
		6	.004	.117
	150	6	.032	.006
		7	.011	.014
		8	.003	.031
0.05	150	10	0.132	0.106
	250	16	.127	.034
		17	.081	.055
		18	.049	.085
		19	.029	.126
	400	26	.073	.009
		27	.048	.015
		28	.031	.023
		29	.019	.036
		30	.011	.052
		31	.007	.075
	650	46	.008	.006
Operator's penetration probability limit, q_2, 0.20				
0.01	10	0	0.096	0.107
	15	0	.140	.035
		1	.010	.167
	25	0	.222	.004
		1	.026	.027
		2	.002	.098
	40	2	.008	.008
0.05	15	1	0.171	0.167
	25	2	.127	.098
		3	.034	.234
	40	4	.048	.076
		5	.014	.161
	65	5	.106	.006
		6	.043	.016
		7	.016	.037
		8	.005	.076
	100	10	.011	.006
		11	.004	.013
0.02	10	0	0.183	0.107
	15	0	.261	.035
		1	.035	.167
	25	1	.089	.027
		2	.013	.098
		3	.001	.234
	40	2	.046	.008
		3	.008	.028

Designer's penetration probability limit, q_1	Sample size, n	Acceptance number, c	Designer's risk, α	Operator's risk, β
Operator's penetration probability limit, q_2, 0.20				
0.10	65	9	0.111	0.137
	100	13	.124	.047
		14	.073	.080
		15	.040	.129
	150	19	.113	.013
		20	.072	.022
		21	.044	.037
		22	.026	.059
		23	.014	.089
		24	.008	.129
	250	34	.030	.007
		35	.019	.011
		36	.012	.017
		37	.007	.026
Operator's penetration probability limit, q_2, 0.50				
0.01	10	0	0.096	0.001
		1	.004	.011
0.10	10	1	0.264	0.011
		2	.070	.055
		3	.013	.172
	15	2	.184	.004
		3	.056	.018
		4	.013	.059
		5	.002	.151
	25	6	.009	.007
0.02	10	0	0.183	0.001
		1	.016	.011
		2	.001	.055
	15	2	.003	.004
0.20	10	3	0.121	0.172
	15	4	.164	.059
		5	.061	.151
	25	7	.109	.022
		8	.047	.054
		9	.017	.115
	40	12	.043	.008
		13	.019	.019
		14	.008	.040
	65	21	.006	.003
0.05	10	1	0.086	0.011
		2	.012	.055
		3	.001	.172
	15	2	.036	.004
		3	.005	.018



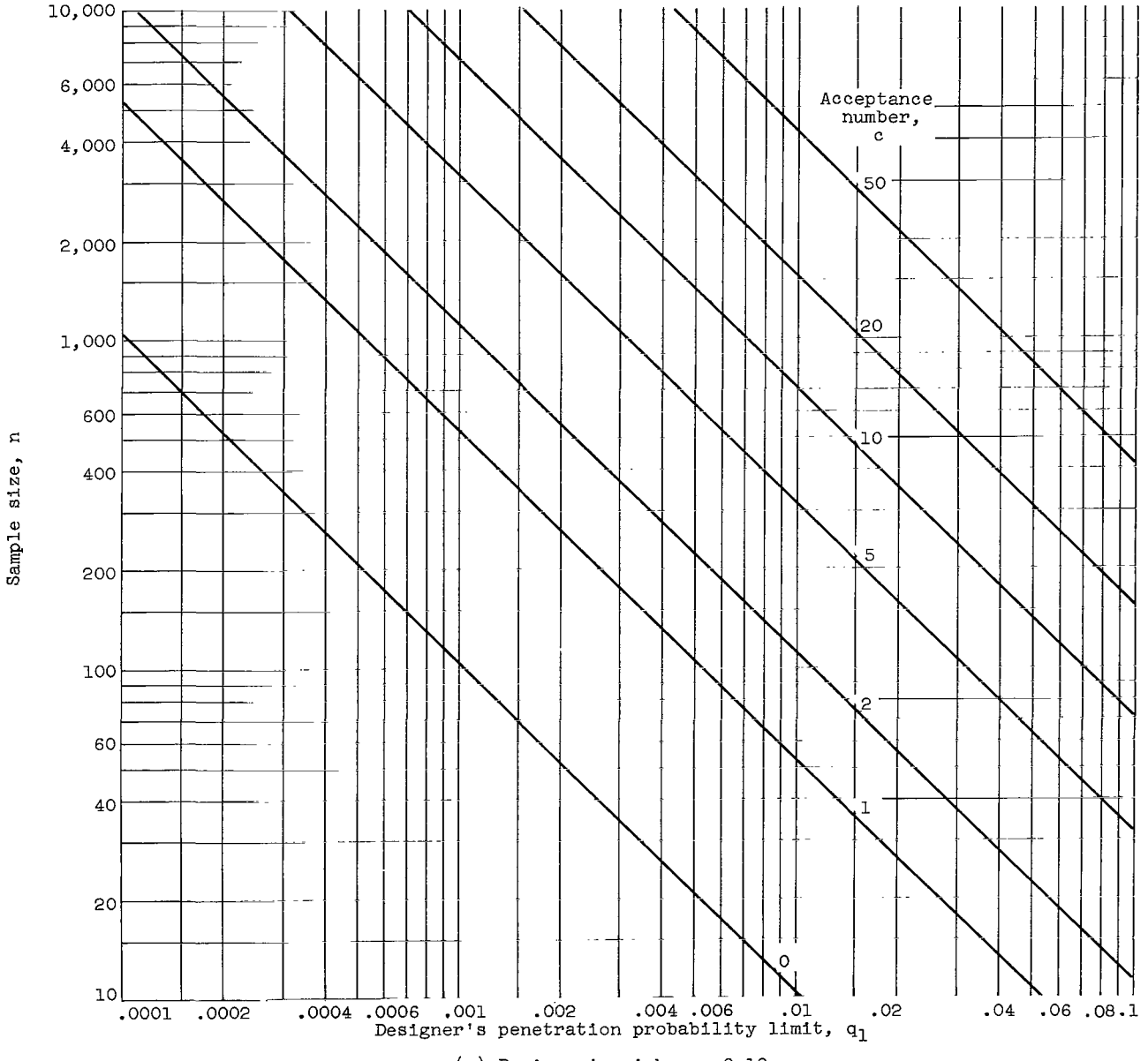
(a) Designer's risk, α, 0.01.

Figure 1. - Binomial sampling plan chart to control designer's risk.



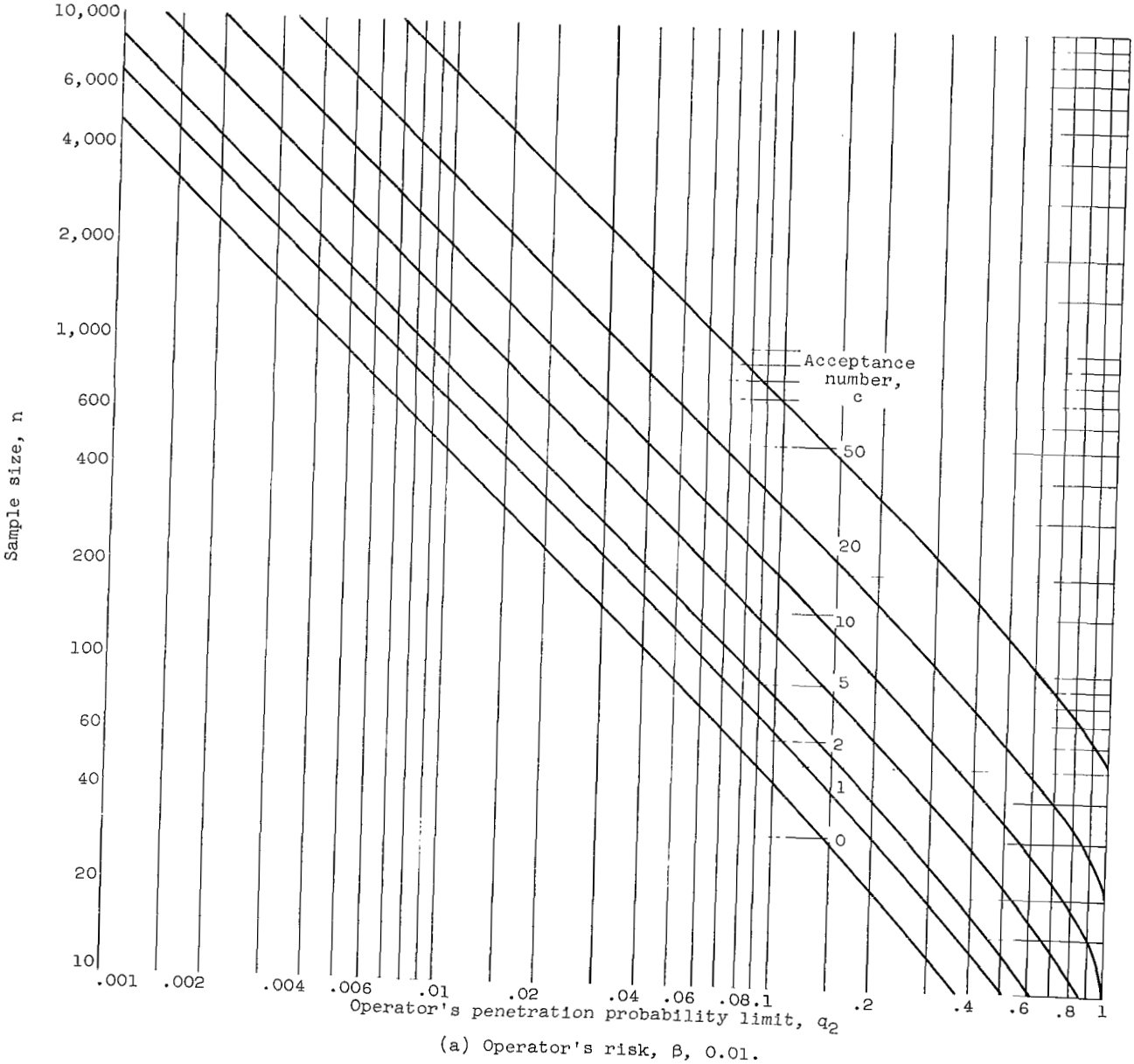
(b) Designer's risk, α, 0.05.

Figure 1. - Continued. Binomial sampling plan chart to control designer's risk.



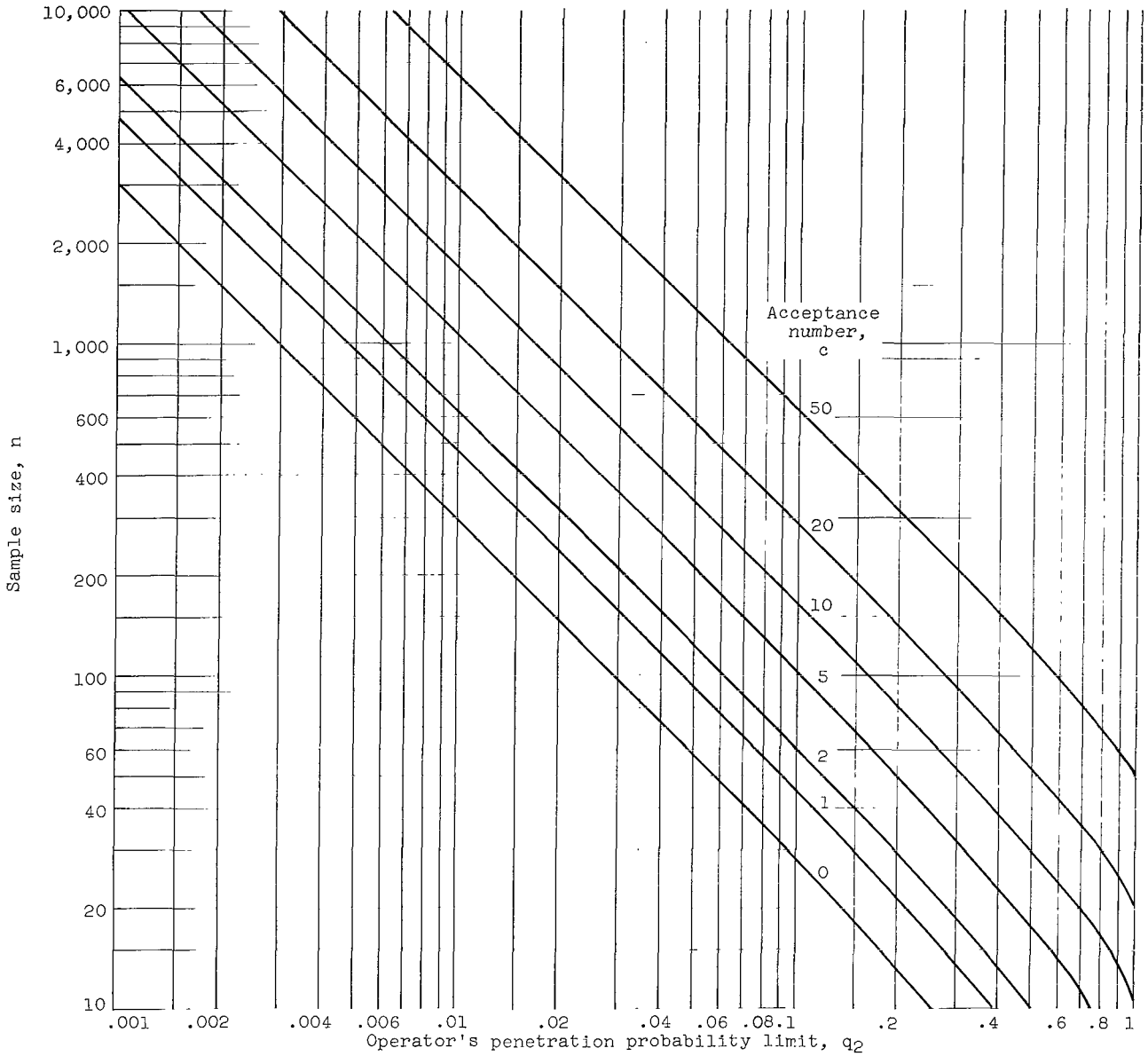
(c) Designer's risk, α, 0.10.

Figure 1. - Concluded. Binomial sampling plan chart to control designer's risk.



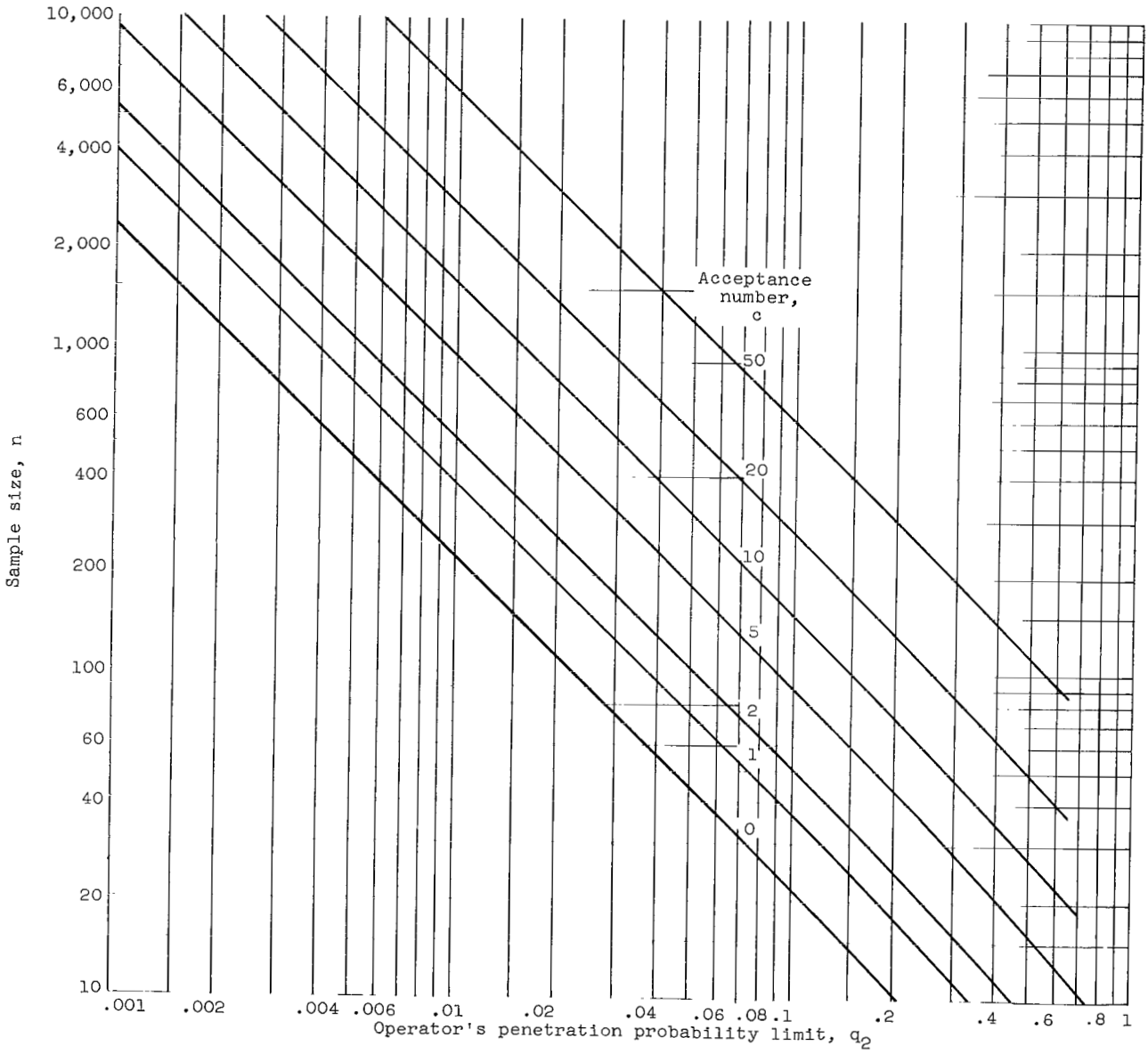
(a) Operator's risk, β, 0.01.

Figure 2. - Binomial sampling plan chart to control operator's risk.



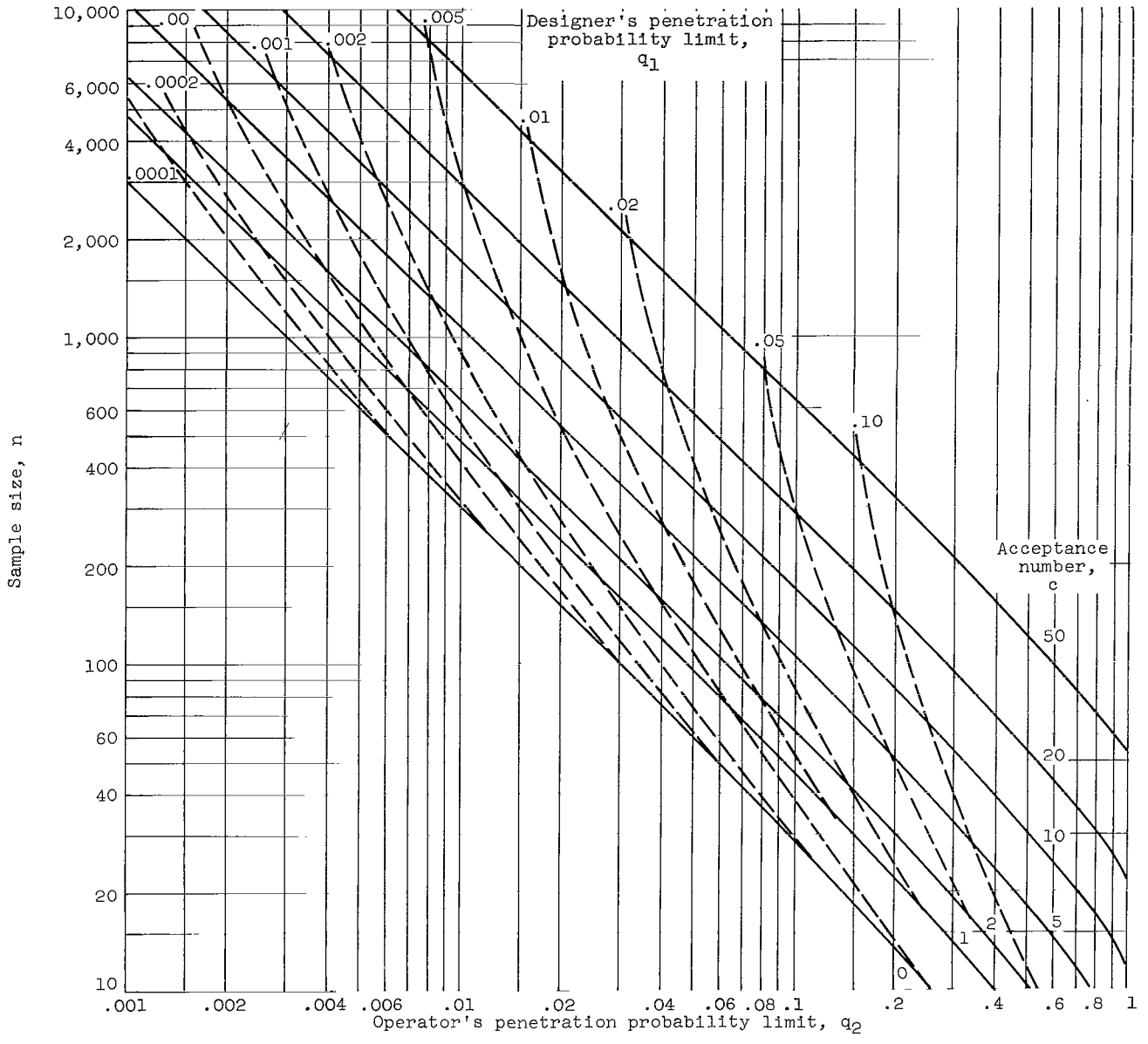
(b) Operator's risk, β, 0.05.

Figure 2. - Continued. Binomial sampling plan chart to control operator's risk.



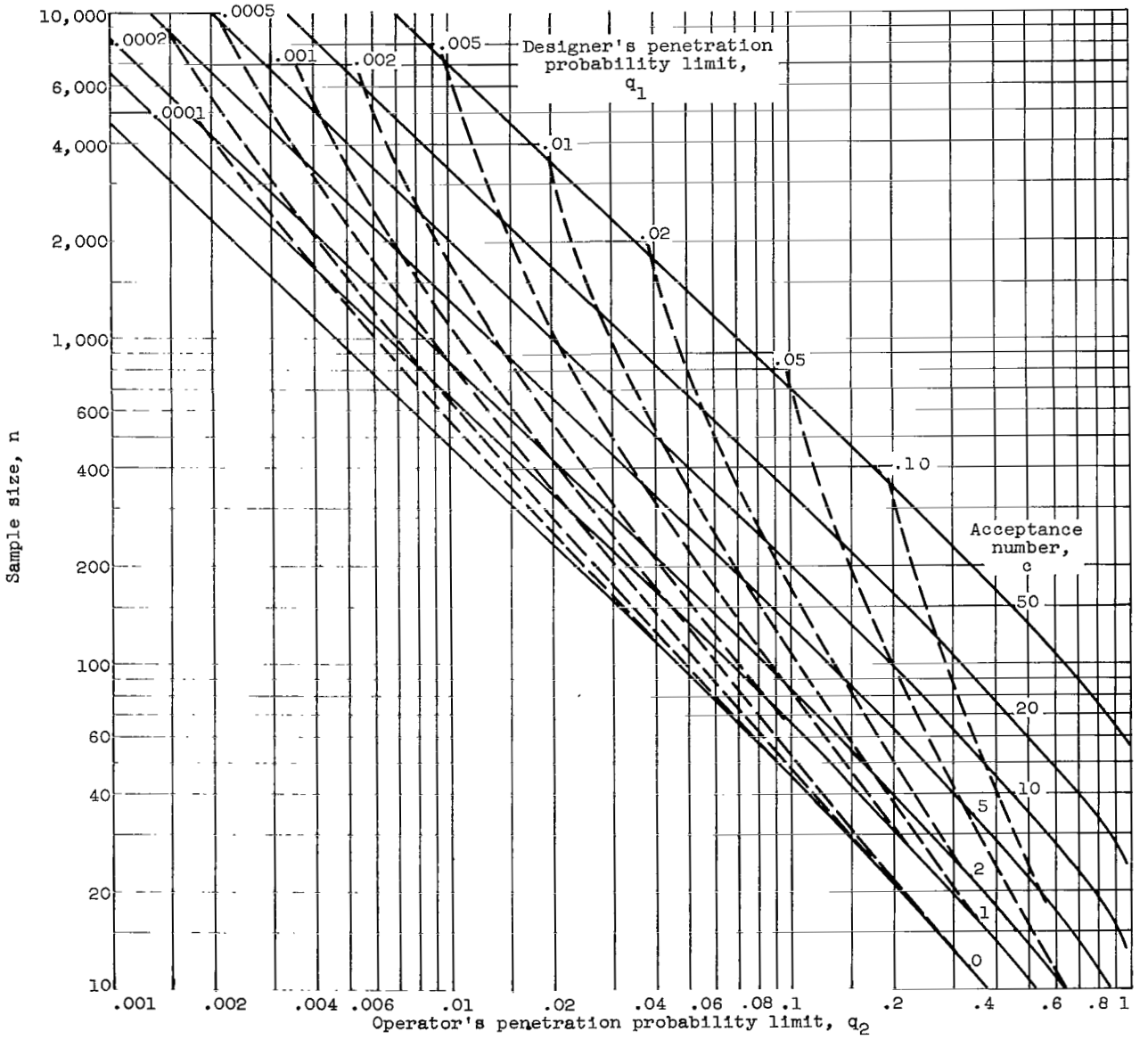
(c) Operator's risk, β, 0.10.

Figure 2. - Concluded. Binomial sampling plan chart to control operator's risk.



(a) Designer's risk, α, 0.05; operator's risk, β, 0.05.

Figure 3. - Sampling plans with constant designer's penetration probability limit.



(b) Designer's risk, α, 0.01; operator's risk, β, 0.01.

Figure 3. - Concluded. Sampling plans with constant designer's penetration probability limit.