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ENGINEERING REPORT NO. 669 (ACN 123)

RESEARCH & DEVELOPMENT OF
S-1C HEAT SHIELD PANELS

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AERONCA MANUFACTURING CORPORATION

MIDDLETOWN, OHIO

ENGINEERING REPORT NO. 669 (ACN 123)

RESEARCH & DEVELOPMENT OF S-1C HEAT SHIELD PANELS

George C. Marshall Space Flight Center
NASA Huntsville, Alabama

Contract NAS8-5221

MONTHLY REPORT NO. 3

APRIL, 1963

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INTRODUCTION

This report submitted by Aeronca Manufacturing Corporation to the George C. Marshall Space Flight Center, NASA, Huntsville, Alabama covers the work accomplished on Contract NAS8-5221 for the third calendar month of this program, April 1963.

The following items are included:

1. Total engineering hours expended during April 1963 were 293.
2. Transient thermal analysis and parametric studies for SK 60-B20001 and 30M12571 Heat Shield Panels.
3. Structural Analysis for S-1C heat shield panel 30M12571 based on revised loads information.

TRANSIENT THERMAL ANALYSIS FOR S-1C HEAT SHIELD PANELS

I. INTRODUCTION

Presented in this section are the transient heat transfer analyses of the Saturn base heat shield panel for design drawing Nos. 30M12571 and SK 60B-20001. All analyses are three-dimensional in nature and are based on the heating rates and surface temperatures per NASA Huntsville data of ~~2-5-63~~ & 2-11-63, Fig.1.* Included are the maximum temperature profiles of the edge attachment schemes for the two panel designs. The temperature profiles presented are for the condition of 100% brazing alloy node flow in the honeycomb support structure. All other assumptions and ground rules which govern the analysis are presented in Section II.

In addition to the thermal analyses of the two panel concepts, a set of parametric curves is presented illustrating the temperature differentials across the honeycomb support structure as a function of brazing node flow size, cell dimensions and M-31 reinforcement honeycomb dimensions. While these parametric studies are confined to the dimensions bounded for the most part by the dimensions of the panels considered, they do present the possible trade-offs that could be considered for possible panel design optimization from thermal considerations.

* NASA Data, Figure 2, Estimated Temperatures of S-1C Heat Shield Panel Attachments, 2-11-63 M-P&VE PH 26--63.

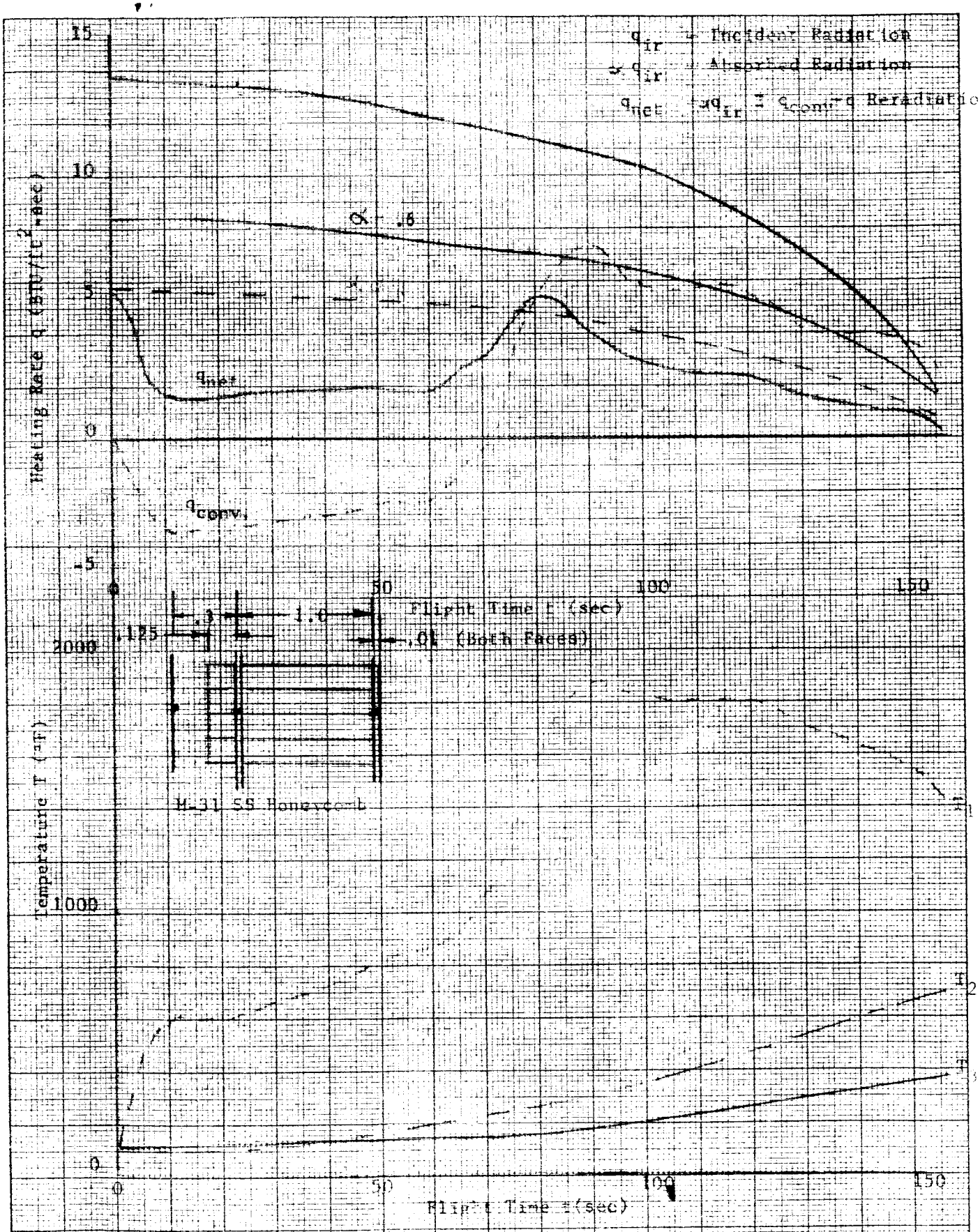

Figure 1 Typical S-1C Heat Shield Heating Rates & Surface Temperature As A Function of Flight Time (NASA supplied data as of 2-5-63)

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II. METHODS OF ANALYSIS

Aeronca has developed a digital computer program which has general applicability to thermal analysis of high temperature structures. For purposes of analysis the structure is represented by a set of spatially distributed points or nodes. The temperature of each node is determined by solving the generalized heat balance equation in finite difference form:

$$T_j' - T_j = \frac{\Delta T}{\rho V C_p}\left[\dot{Q}_j''' + \dot{Q}_j'' - \sum_{i=1}^{n} U_{ij}(T_j' - T_j)\right] \quad (1)$$

T_j' and T_j represent the temperatures at the end and beginning of the time step, respectively. The parameters Q'''_j and Q''_j represent volumetric heating and incident surface flux, respectively. U_{ij} is the thermal conductance between node i and node j. The program does an iteration for T' and re-evaluates temperature dependent functions (radiation coefficient, etc.) on the temperature at the midpoint of the time step. This is the so-called Implicit Method.

The program can use the Explicit Method, in which T' and T are the same as the Implicit Method. The heat balance equation will become explicit in T

$$T_j' - T_j = \frac{\Delta T}{\rho V C_p}\left[\dot{Q}_j''' + \dot{Q}_j'' - \sum_{i=1}^{n} U_{ij}(T_j - T_i)\right] \quad (2)$$

There is now a limit on the length of time step ΔT which the program calculates and is given by $\Delta T = \rho V C_p / U_{ij}$ and taken so that the expression $\rho V C_p / U_{ij}$ is a minimum for the entire nodal system.

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The expression used to evaluate zero-volume nodes (surface) and steady-state calculations is

$$\sum_{i=1}^{n} U_{ij}\left(T_j - T_i\right) - \dot{Q}_j''' - \dot{Q}_j'' = 0 \qquad (3)$$

The parameters Q'''_j and Q''_j are the same as defined previously.

The thermal conductance term, U_{ij} is used in three basic forms: (1) solid to solid conduction, with contact coefficient; (2) solid to solid radiation, with radiation coefficient; and (3) solid to fluid, with conduction and film coefficient.

There are several important features in this computer program. The amount of data necessary to run a problem is quite large and would include such things as: (1) time, boundary temperature tables; (2) time, rate tables; (3) Nusselt number correlation tables; (4) material property tables; (5) node description data for each node; and (6) connection data for each node. Although a large amount of input data is necessary, the engineer requires little or no knowledge of computer programming or techniques to use the program. Use of stacked storage is employed rather than the bulkier reserved storage, so that problems with greater than 1000 nodes can be run.

The program is divided into five chains: Chain 1 accepts data in a form which is easy for the user to prepare and stores it for further processing--in other words, Chain 1 is just to input the problem; Chain 2 processes data from Chain 1 into a form which the program can use; Chain 3 takes

this processed data and performs the computation of Equations (1) or (2); Chain 4 is an editing chain which will, (1) give a time-temperature history if requested, and (2) if the run is pulled for time, punch the current temperature distribution or decimal cards so that the run can be restarted; Chain 5 sets up change cases. A more detailed description of the program is presented in Reference 1.

A second method of heat transfer which was employed was to assemble "n" heat balance equations, one for each temperature point in the panel and for given time intervals and time dependent boundary conditions use the Gauss-Seidel iteration method to determine the unknown temperatures.

This process is repeated for each time interval until the desired transient analysis is complete.

The general form of the heat balance equation for one point in the panel for a single time interval is given below.

Thermal Storage = Convection + Conduction +

$$\frac{\rho VC}{\Delta T}(T-T') = \sum_{n=1}^{6}\left[\frac{A_N}{\frac{\Delta X_N}{K} + \frac{\Delta X_N}{K_n} + \frac{1}{h_N} + \frac{1}{h_c}}\right](T_N - T) +$$

Fluid flow input +

$(W_t/W)WC\ (T_N - T)$ +

Surface Flux +

$A_N Q_F$ +

Solid Radiation +

$$\sum_{h=1}^{7}\sum_{i=1}^{3} \sigma A_h F_i \epsilon_1 \epsilon_2 \left[T_h^4 - T'^4\right]$$

Ref. 1 - ER-638, "A Transient Heat Transfer Computer Analysis for Space Vehicle Application", W. Niehaus, R. Criss, R. Cannizzaro, February 1963

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Basically, the transient analysis was one of using the finite difference technique of dividing the panel geometry into a three-dimensional network of nodes. Each node had a finite volume enclosed by a maximum of six sides. Material properties and states were considered to be uniform within a given node and correspond to the temperature at the center of the node.

III DESIGN POINT ANALYSIS

A. Z-Type Edge Panel (Dwg. [illegible])

The temperature histories at various levels throughout the panel are given in Figure 2 and are based on the effective thermal properties of the various layers as given in Table 1. The temperature histories at various points on the Z-type edge attachment are given in Figure 3.

B. Cup-Type Panel (Dwg. SE [illegible])

The temperature histories at various levels throughout the panel are given in Figure 4 and are based on the effective thermal properties of the various layers given in Table 1.

The temperature histories at various points on the cup-type edge attachment are given in Figure 5.

C. Assumptions Made in the Preceding Analysis

The following assumptions and/or ground rules were made and incorporated in the transient heat transfer analysis:

1. When composite layers existed in the heat shields, (i.e., where parallel heat transfer paths exist), an effective thermal conducti-

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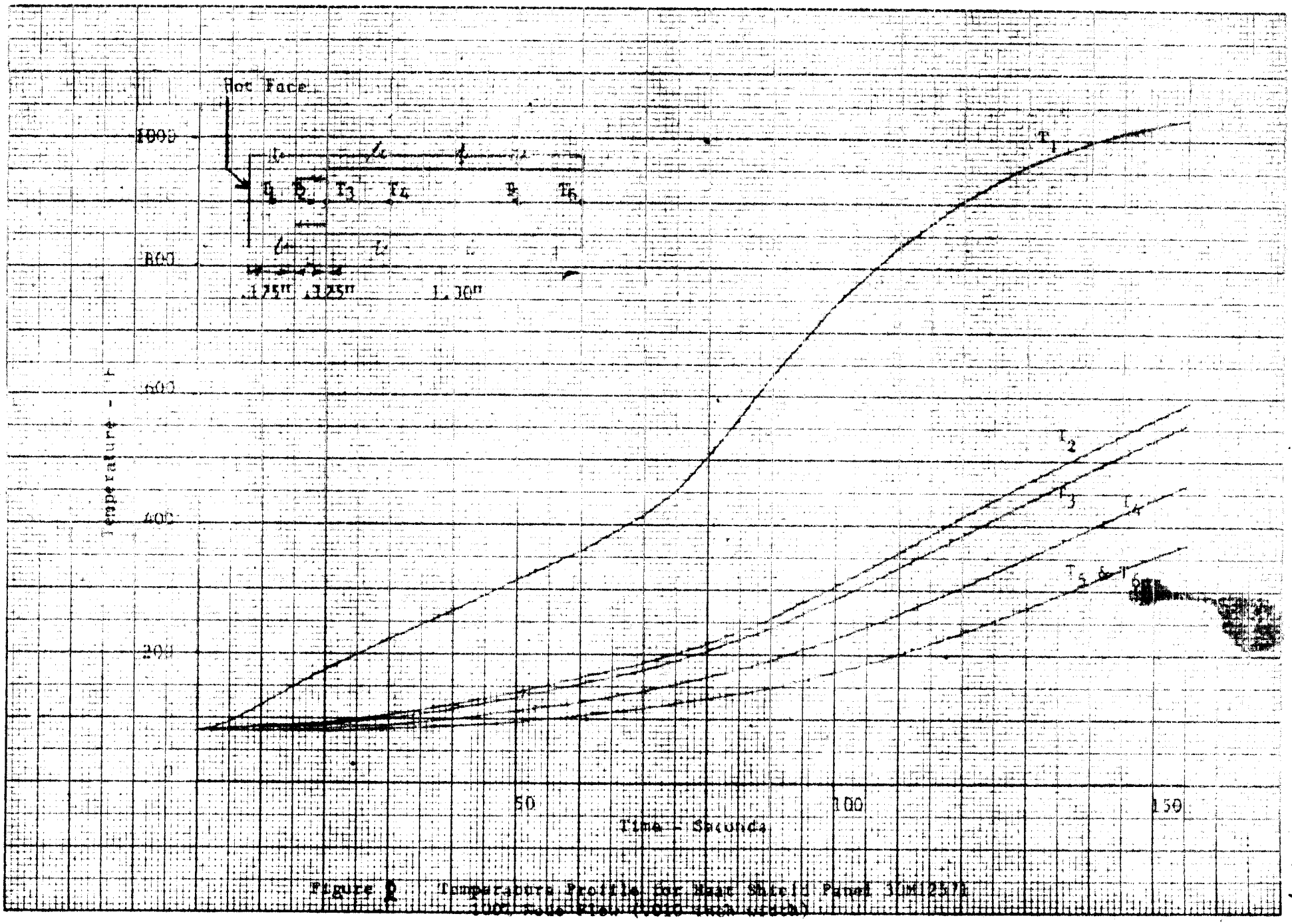

Figure 2 Temperature Profile for Heat Shield Panel 30M12571
100% Node Flow (.010 inch width)

TABLE 3

EFFECTIVE MATERIAL PROPERTIES
BASED ON HOMOGENEOUS LAYERS

Material Layer	T- F	Effective K/Hr-Ft-F	Cp BTU/#/F	Density lbs/cu.ft.
1. Honeycomb Structure (4-15)	[illegible]	.228	.11	[illegible].19
	400	[illegible]	.11	[illegible].19
2. M31 plus Honeycomb Support	[illegible]	.13	[illegible]	50.1
	[illegible]	[illegible]	[illegible]	50.1
3. M-31	[illegible]	[illegible]	.31	47.0
	[illegible]	[illegible]	[illegible]	47.0

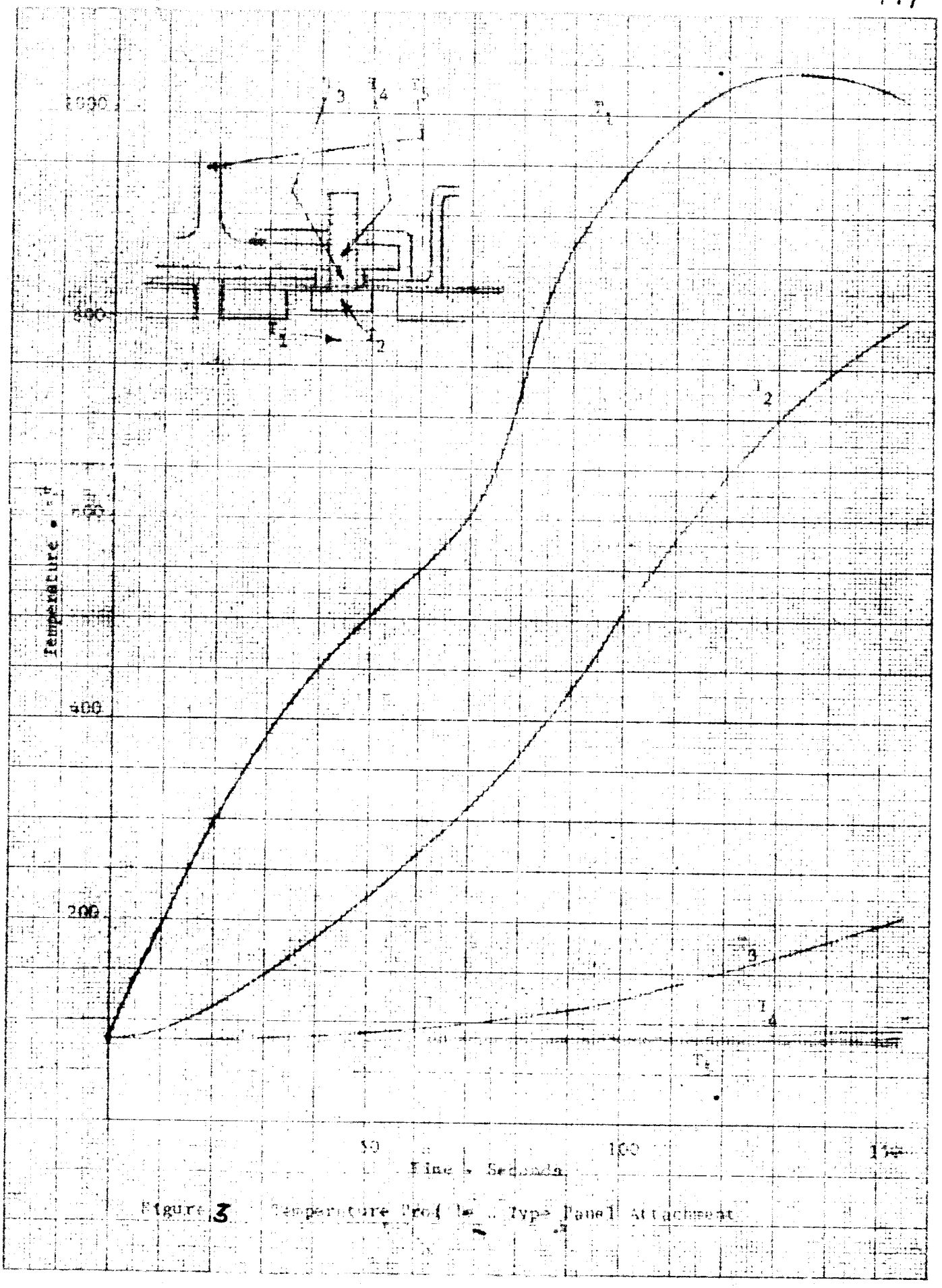

Figure 3 Temperature Profile - Type Panel Attachment

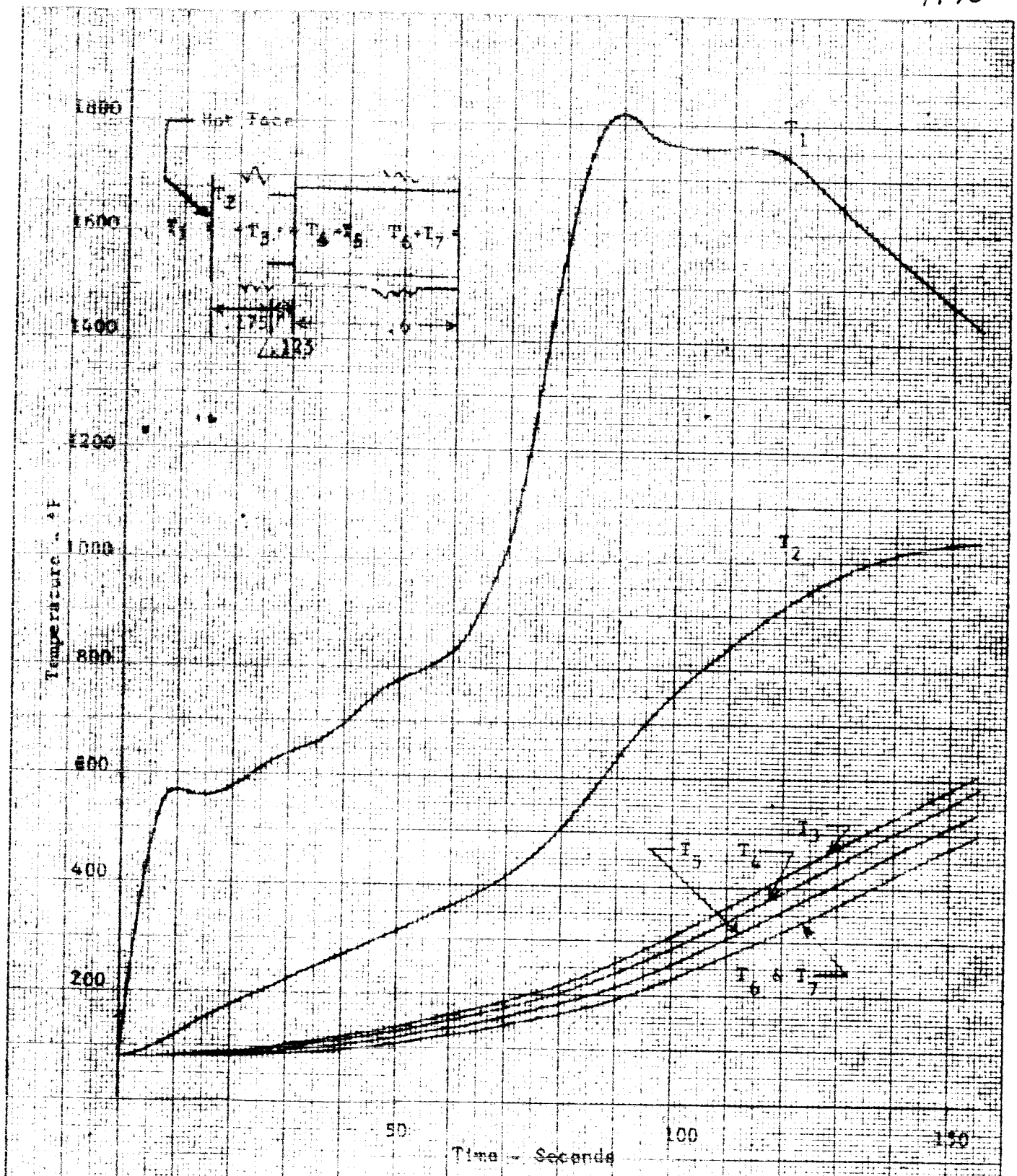

Figure 4 Temperature Profile for Heat Shield Panel SK 60B20001 with 100% Node Flow (.010 inch width)

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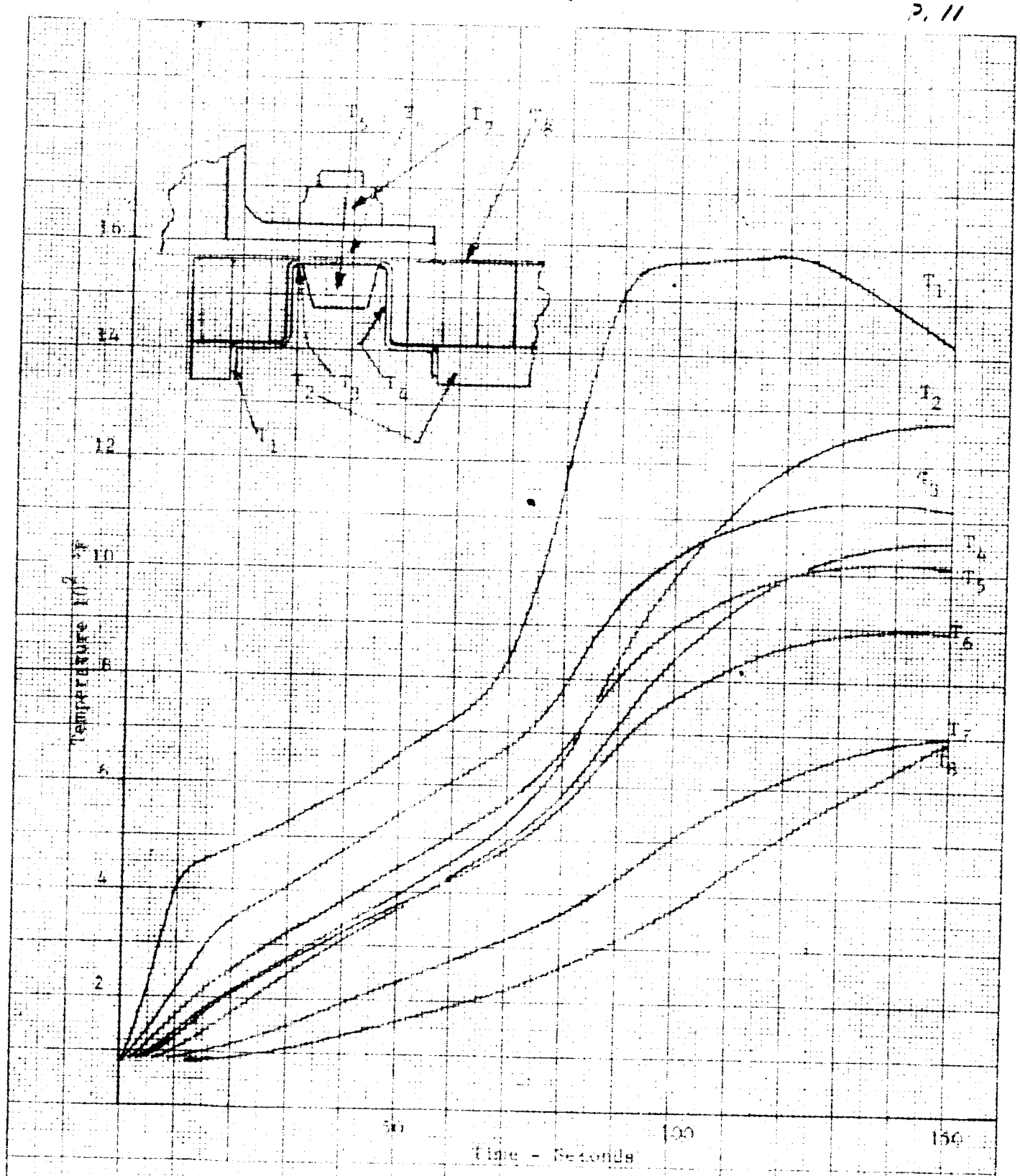

Figure 5 Temperature Profile - Cup-Type Panel Edge Attachment

K+E 10 X 10 TO THE CM 46 1828 KEUFFEL & ESSER CO. MADE IN U.S.A.

vity and density were used. The effective thermal conductivity is equal to the sum of the parallel conductances divided by the total panel heat flow area.

2. The temperature differential across the honeycomb support structure faces was considered small and the unit conductance (K/x) was substituted as a contact coefficient to account for thermal conductance through them. The volumetric heating was neglected.
3. Node flow was included in the thermal conductance of the honeycomb support panel. Two (2) braze nodes per honeycomb cell were considered having an effective cross sectional area equal to that of an equilateral triangle of side 0.010 inch. This dimension was based on measurements of the node flow width made from 30[illegible] panel X-rays.
4. The cold face surface was adiabatic.
5. The radiation exchange and natural convection were both considered negligible in the honeycomb cells.
6. The contact coefficients between the panel edges and the attachment rails were based on a nominal air gap of 0.001".

D. Design Point Temperatures

The maximum temperature differentials to be used for design purposes across the brazed honeycomb sandwich panels are as follows:

Panel Configuration	Braze Alloy Node Flow Condition	°F	Time Duration Seconds
SE 60P20001 0.6" Thick Core	Complete--0.010" Width	80	155
30M12571 1.0" Thick Core	Zero	320	155
30M12571 1.0" thick core	Complete--0.005"	280	155
30M12571 1.0" thick	Complete--0.010" Width	180	155

The beneficial effect of brazing alloy node flow in the load bearing honeycomb core in reducing the thermal gradient and the resultant thermal stress is shown for the 30M12571 heat shield panel design.

IV. PARAMETRIC ANALYSIS

As a result of the thermal analysis performed on the two heat shield panel concepts, a parametric analysis was made which considered the effects of node flow width, cell dimensions, and "-31 reinforcing honeycomb effects on the transient temperature differentials across the honeycomb support structure.

A careful review of the thermal analyses of honeycomb panels in the past has indicated that the presence of good node flow with the use of high thermal conductivity brazing alloy is the probable cause of the high rate of heat conductance through honeycomb panels.

The term "node flow" refers to the phenomenon of the formation of fillets of braze alloy which connect the panel faces, in the corners of the honeycomb core cells. Such metal "ridges" offer conduction paths between the panel faces which are orders of magnitude better than that in the air gap within the cells and which, at reasonably low temperature levels, transfer considerably more thermal energy between panel faces than is transferred by thermal radiation. If the thermal conductivity of the node-flow metal approaches that of silver, which is on the order of 20 times that of a high-nickel-content brazing alloy, the node-flow conductance path will be by far the dominating factor in the transmission of heat through the panel.

Reference 2 presents an analysis of sample test data wherein it is shown, on the basis of a reasonable set of assumptions, that of the total heat passing through the test panel, 5.2 was by radiation between the panel faces, 17.0 was by conduction through the core foil, and the remaining 77.8% was by conduction through the high-silver-content brazing alloy. While the exact magnitude of the numbers may be subject to some questions on the basis of the assumptions used in calculating them, the relative magnitudes are felt to be quite correct.

The parametric analyses presented here are intended to furnish in a limited way the trade--offs to be made in designing a heat shield panel from thermal considerations. The range of honeycomb cell dimensions considered covers the dimensions of the two design panels. As such the material discussed in this section and the feasible trade-offs that can be derived from the curves apply only in the neighborhood of the dimensions and environments of the previously mentioned design panels.

Ref. [illegible]

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A. Node Flow Effect

Figures 6 to 8 show the effects of Node Flow Width on the effective thermal conductivity of the honeycomb, the weight of the honeycomb panel, and the temperature differentials across the honeycomb panel.

Figure 6 shows that in doubling the cell width the effective conductivity of the panel is reduced by a factor of from .5-.4 for node flow widths of about 0.010". Also, it is evident that for a node flow width above 0.010" the effective conductivity increases rapidly. However, with this increase in thermal conductivity, which is desirable from a thermal stress standpoint, there is an increase in panel weight. Figure 7 shows the trade-off between effective honeycomb density and increase in effective conductivity as a function of cell width and node flow width. For a node flow width of 0.010", the per cent increase in density of a honeycomb having a 2-15 cell (.188" cell width, .0015" foil width) when such a braze node flow is added is 21% of the original density with no node flow. The per cent increase in thermal conductivity, however, is 11[illegible]%. In values of density the increase would go from 8.3 to 10.0[illegible] $\#/ft^3$ while the conductivity would increase from .136 BTU/Hr-Ft-°F to .387 BTU/Hr-Ft-°F.

Figure 8 gives the effect of node flow width and cell width on the temperature differentials across a 1.0" thick honeycomb panel (Panel No. 30[illegible]125[illegible]). The temperature differential decreases with an increase in both cell width and node flow width. This is to be expected since an increase in either of these increases the effective thermal conductivity across the panel.

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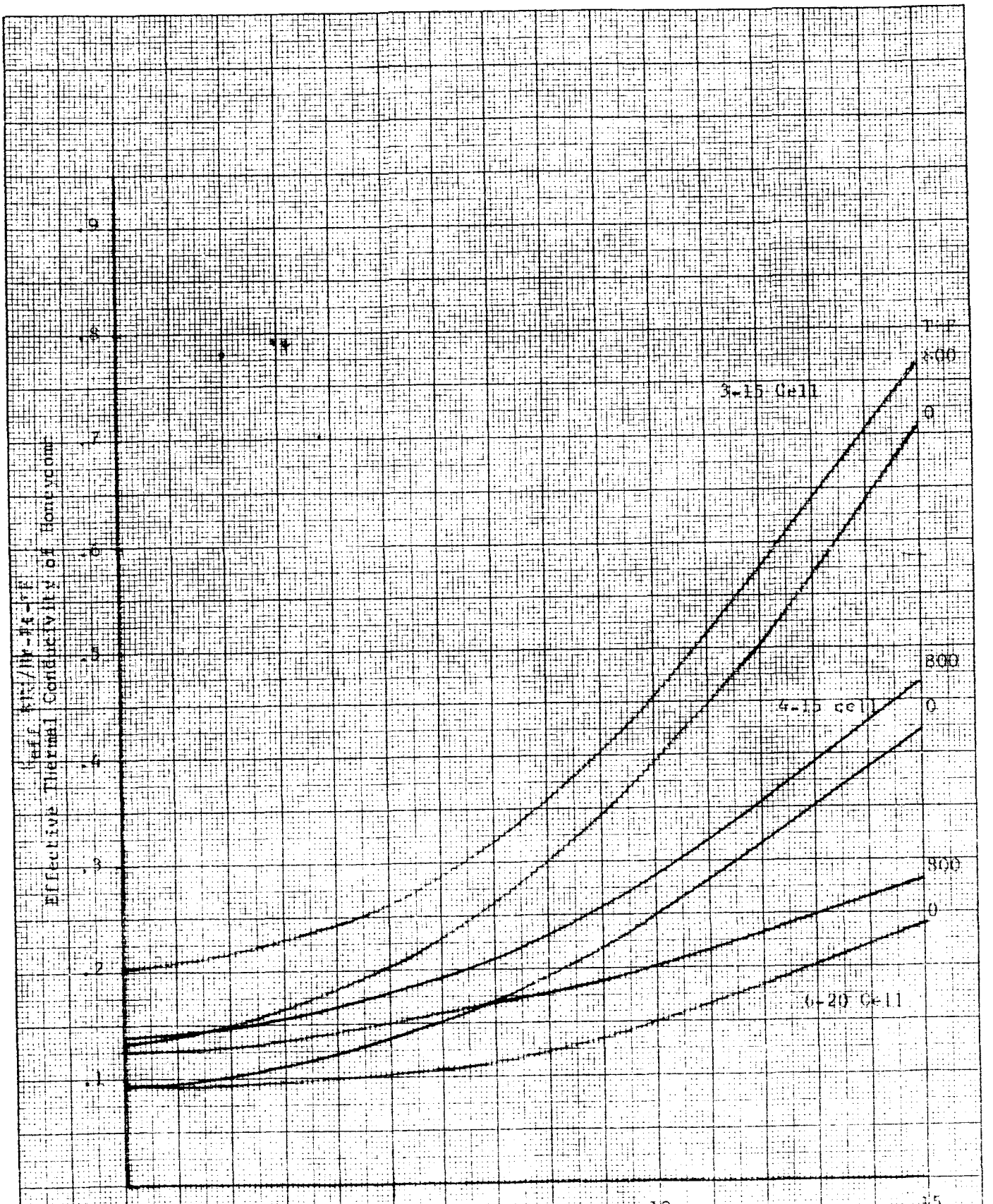

Figure 6 Thermal Conductivity of Heat Shield Panel Vs.

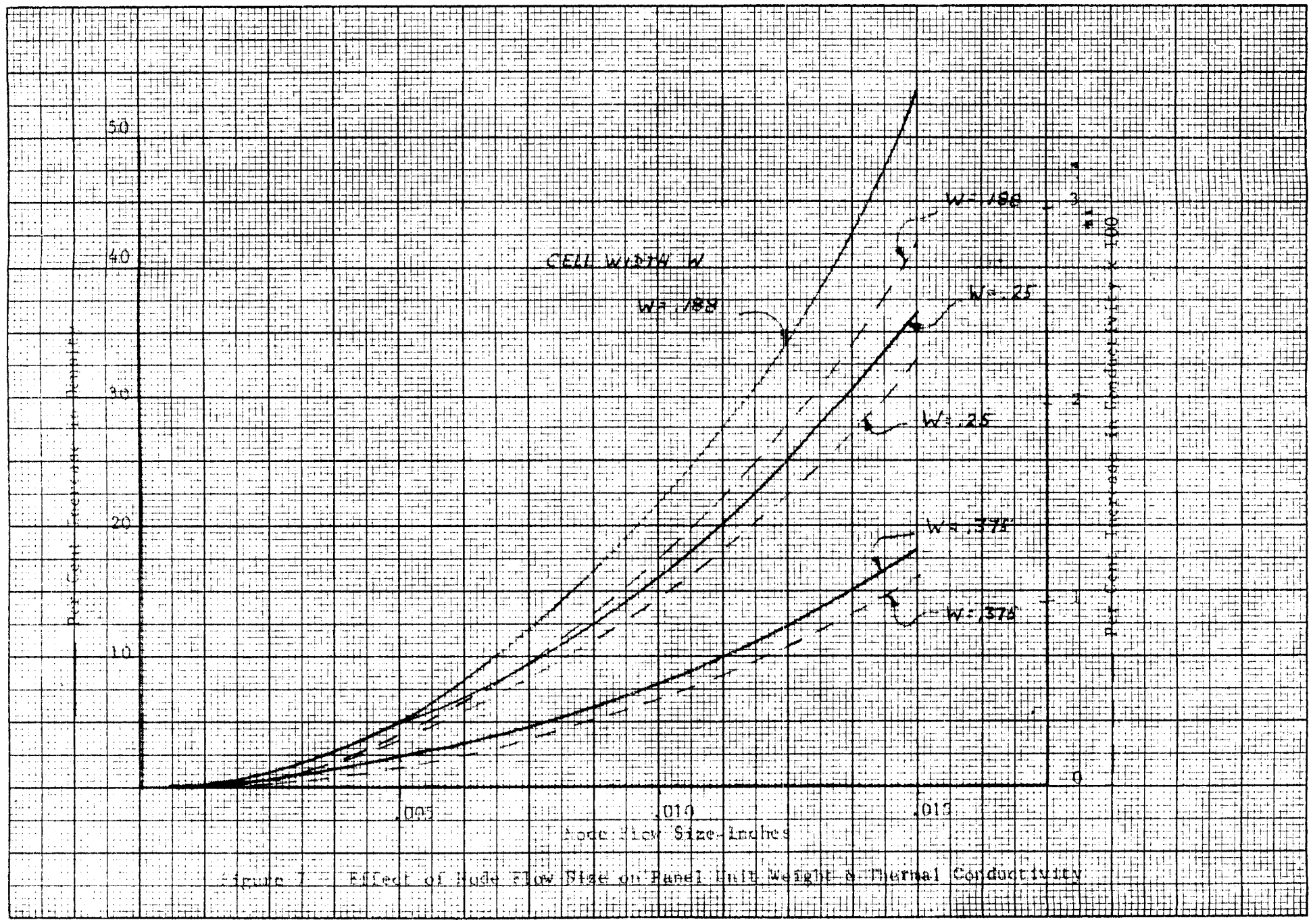

Figure 7 - Effect of Node Flow Size on Panel Unit Weight & Thermal Conductivity

F. Cell Depths

In Figure 9 the temperature differentials are given across the honeycomb as a function of cell depth and node flow width for a constant cell size of 4-15 (½" wide and 0.0015" foil thickness). It is quite evident that for any one cell depth, the temperature drop across the honeycomb decreases with an increase in node flow width. Also, for a constant node flow width the Δ approaches zero as the cell depth is decreased, as would be expected.

It should be pointed out here that holding all parameters constant and decreasing the cell depth increases the back face (cold side) temperature of the honeycomb panel. This is due largely to the fact that the boundary conditions are the same so that there is less total mass to absorb the same amount of heat.

G. M-31 Reinforcing Honeycomb Effects

Figure 10 gives the temperature profiles for the heat shield panel No. 30M12571 as a function of variable M-31 reinforcement cell width. The cell size was varied from 8-15 to 4-15, respectively. As is evident from the curves there was no noticeable effect on the temperature distribution in the honeycomb structure. Although not shown on the curves, a slight variance was noted in the temperatures T_1 and T_2, but was of such a magnitude ($\leq$ 10) as to make it negligible.

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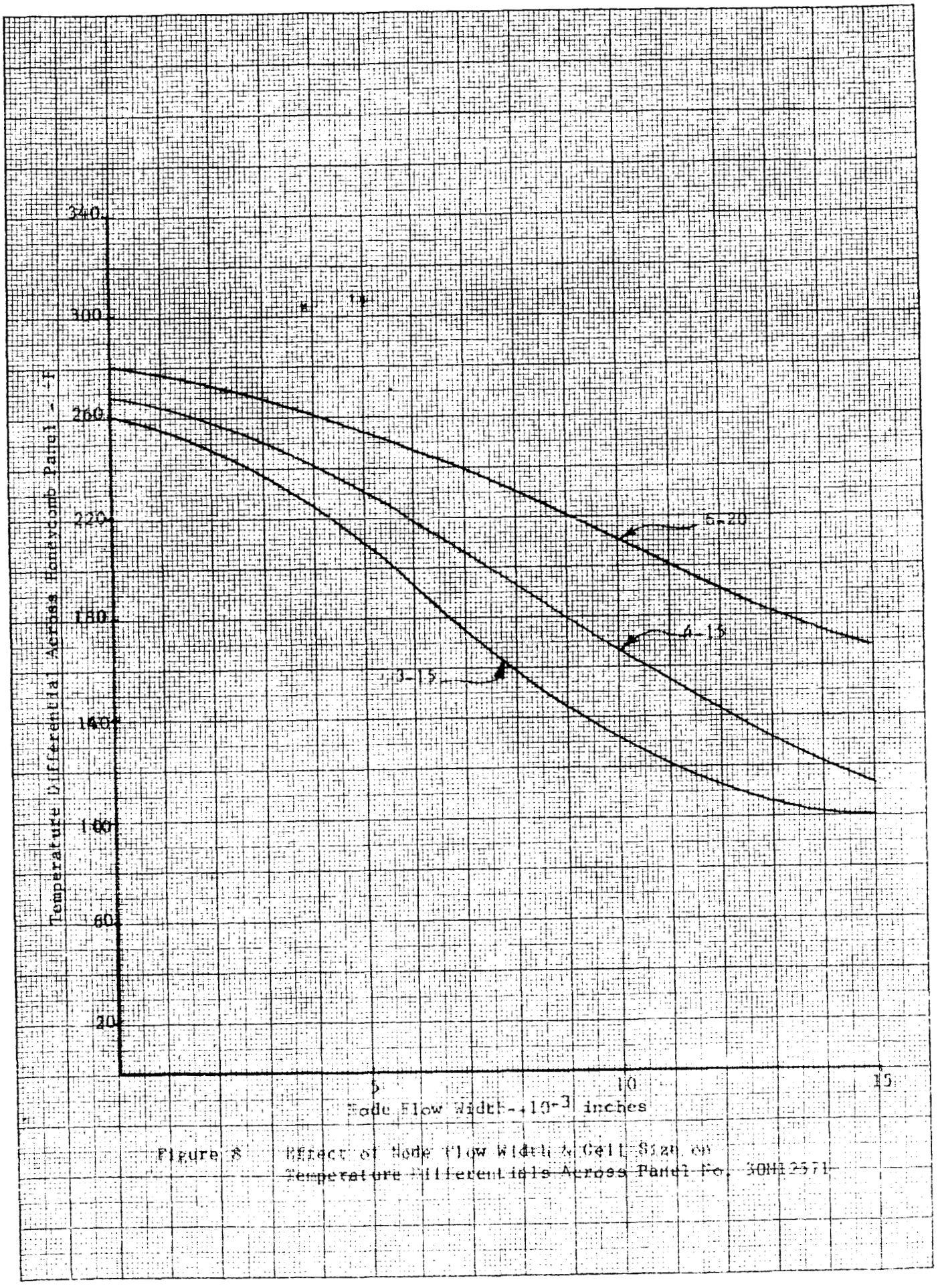

Figure 8 Effect of Node Flow Width & Cell Size on Temperature Differentials Across Panel No. 30H12571

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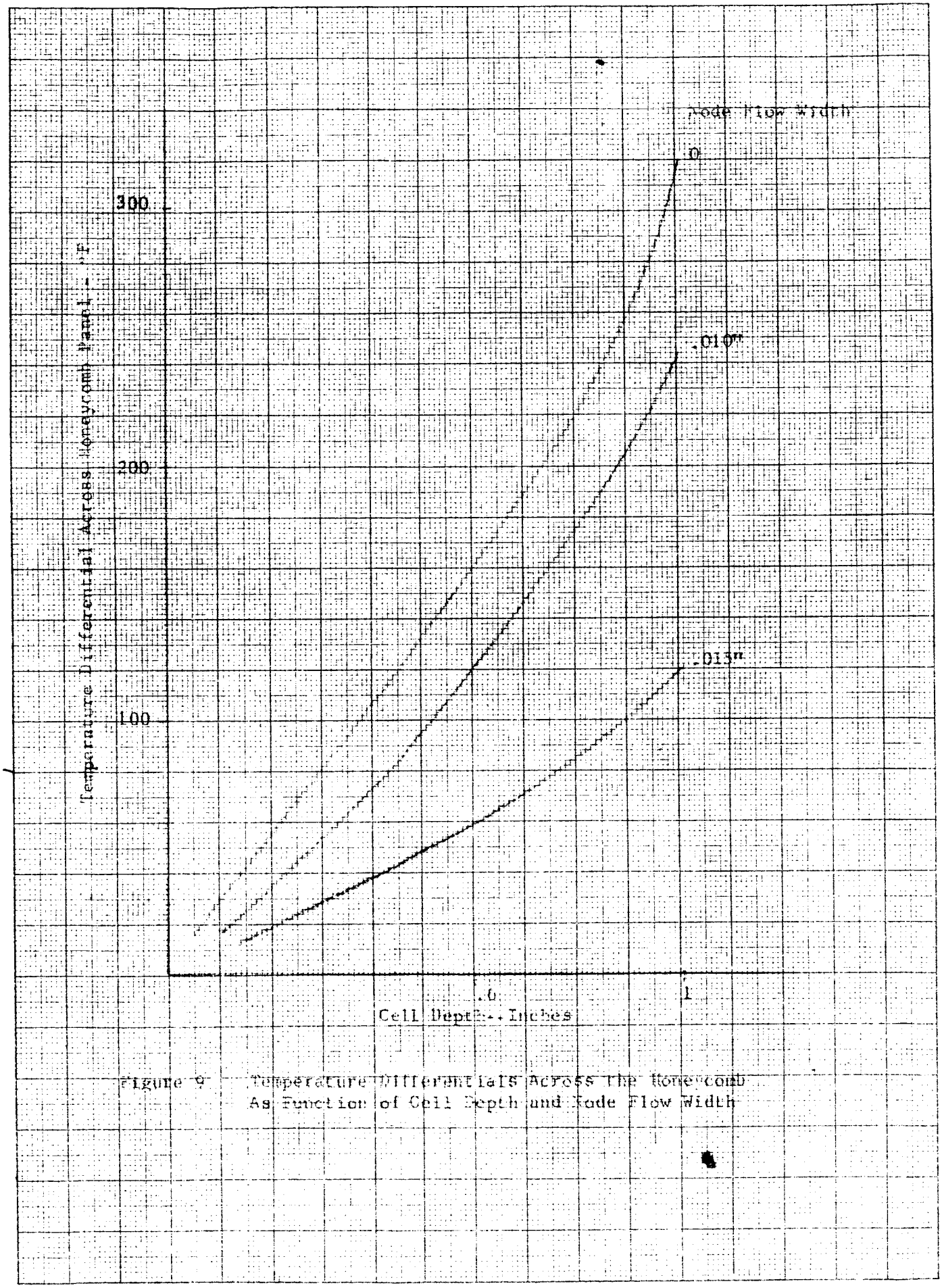

Figure 9 Temperature Differentials Across the Honeycomb As Function of Cell Depth and Node Flow Width

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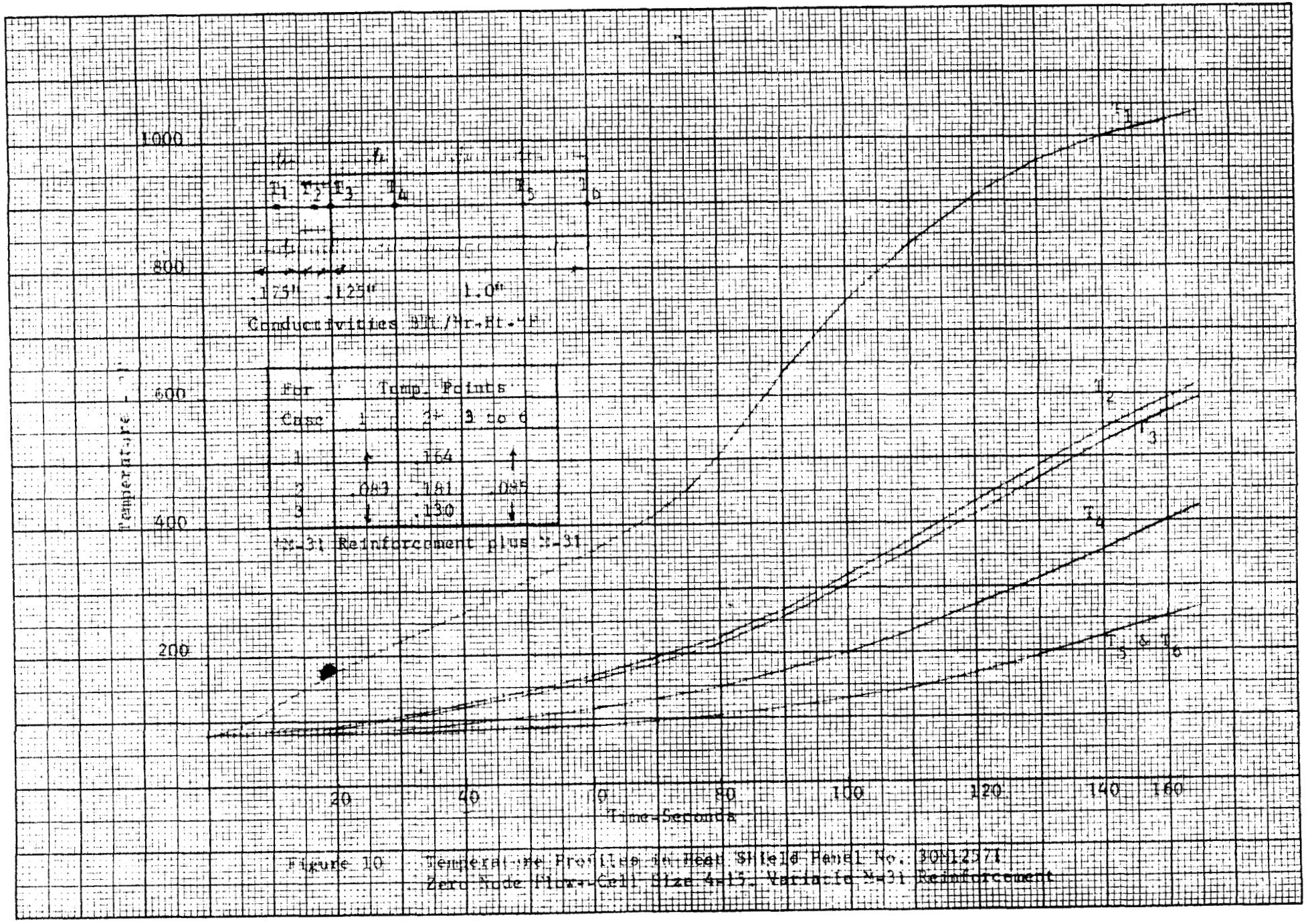

Figure 10 Temperature Profiles in Heat Shield Panel No. 30N12571
Zero Node Flow-Cell Size 4-15, Variable M-31 Reinforcement

D. Supplementary Information Relating to Parametric Studies

The temperature profiles given in Figures 11 to 23 were the basis for the preceding parametric analyses. Figure 24 shows in tabular form the characteristics of the three honeycomb panels that served as the models for the parametric analyses.

V. DISCUSSION

The methods of thermal analysis and the design point analysis presented here have been discussed previously (Ref.) and have been given to consolidate the thermal analysis and its results into a single unit for reference.

In reference to the information given in the section on the parametric analyses, it should be pointed out that the curves can be used most accurately in predicting the trend or trade-offs that occur for any one set of cell dimensions. This is especially so when predicting the effect of node flow width on temperature differentials across the honeycomb panel. It is quite evident from the curves that the node flow size is of prime consideration.

It should also be noted here that the ΔT (158) for the 4-15 cell having 0.010" node flow width in Figure 8 differs from the ΔT given in Section III.3. for the design point (1-0). The difference is due to a refinement in the effective thermal conductivity of the honeycomb panel which was made for the parametric analyses. However since the design point value was conservative, it was not changed.

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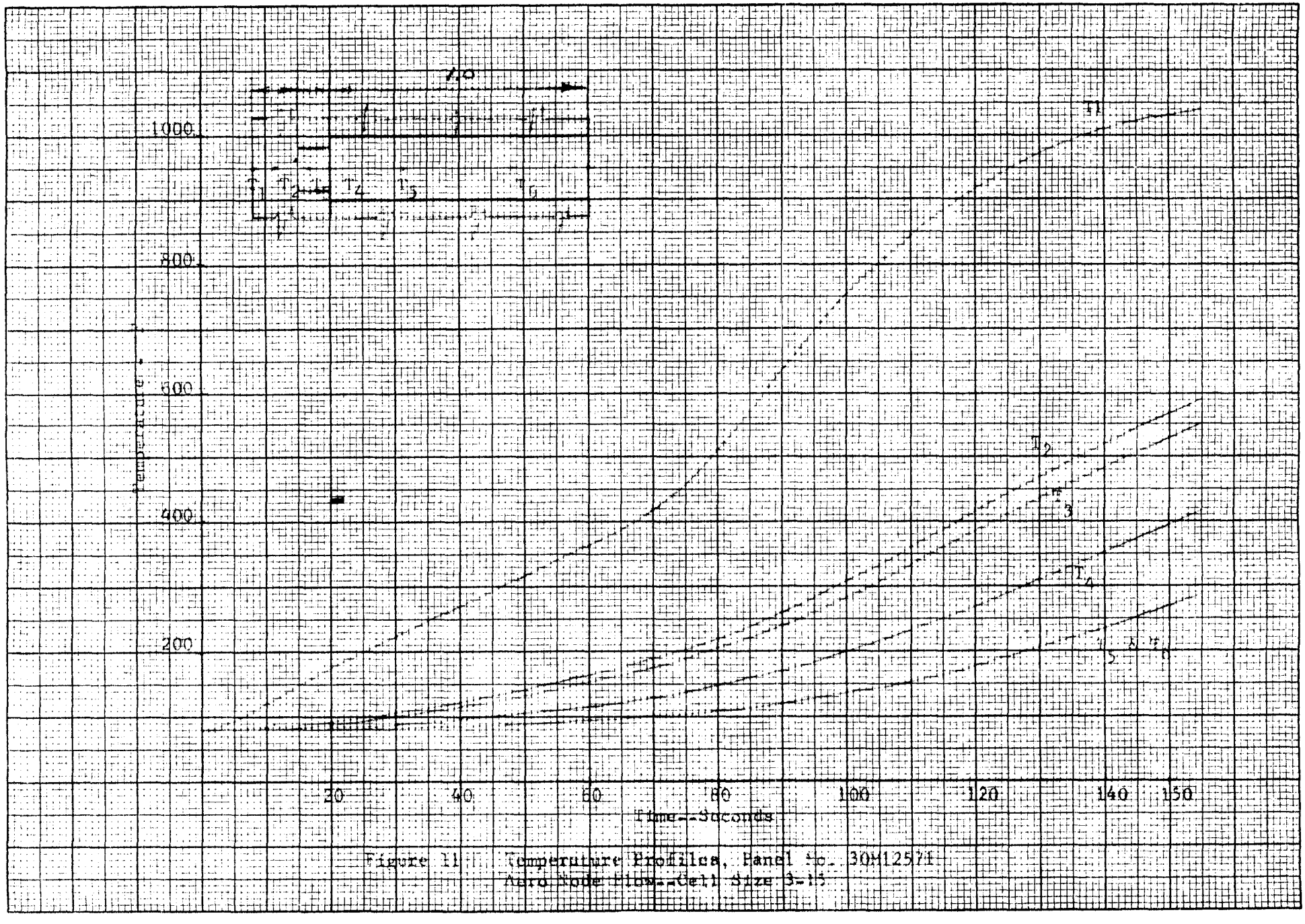

Figure 11 Temperature Profiles, Panel No. 30M12571
Aero Code Flow--Cell Size 3-15

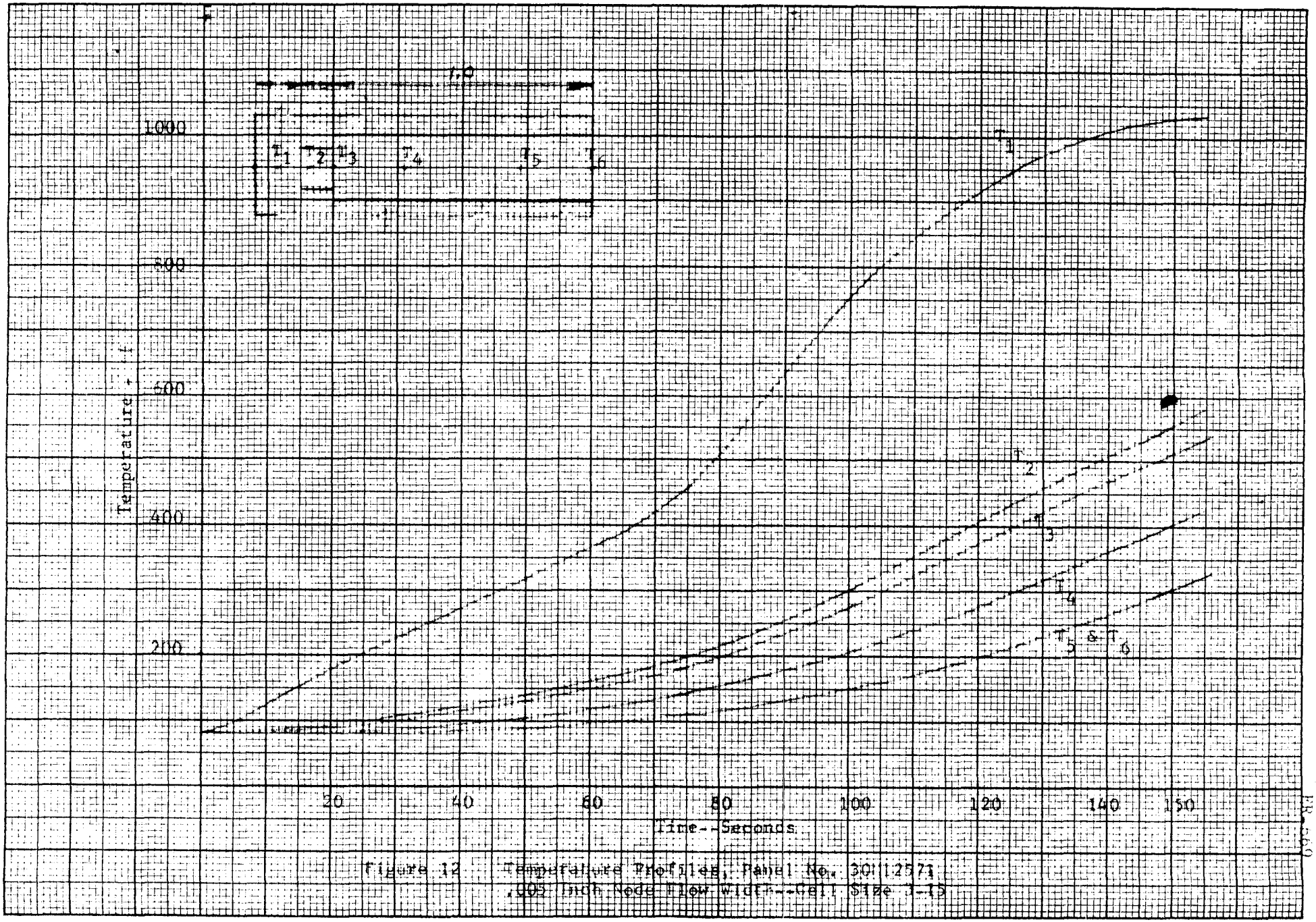

Figure 12 Temperature Profiles, Panel No. 30112571
.005 Inch Node Flow Width--Cell Size 1-15

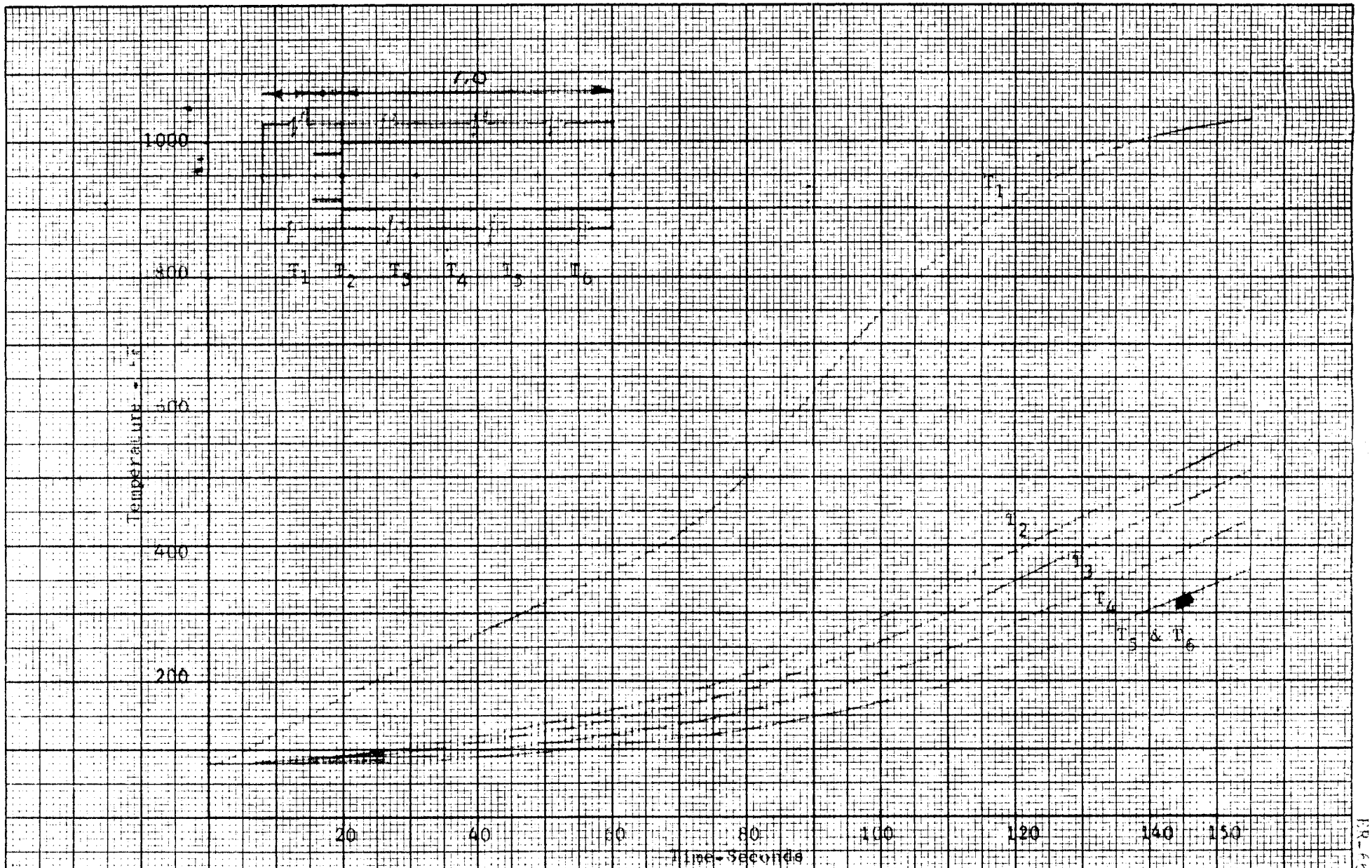

Figure 13 Temperature Profiles, Panel No. 30M12571
.010 Inch Node Flow Width--Cell Size 3-15

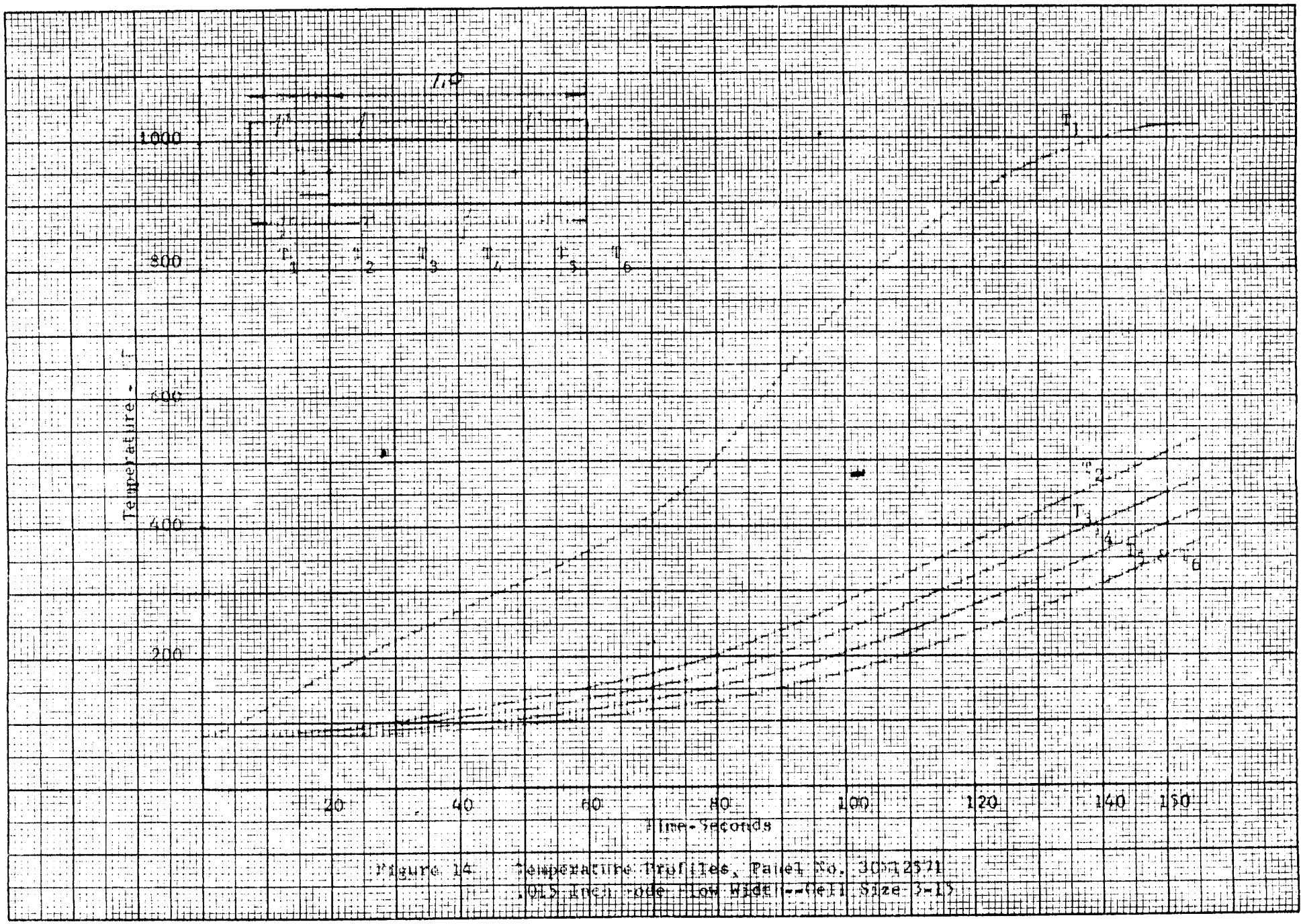

Figure 14 Temperature Profiles, Panel No. 30512571
.015 Inch Mode Flow Width--Cell Size 3-15

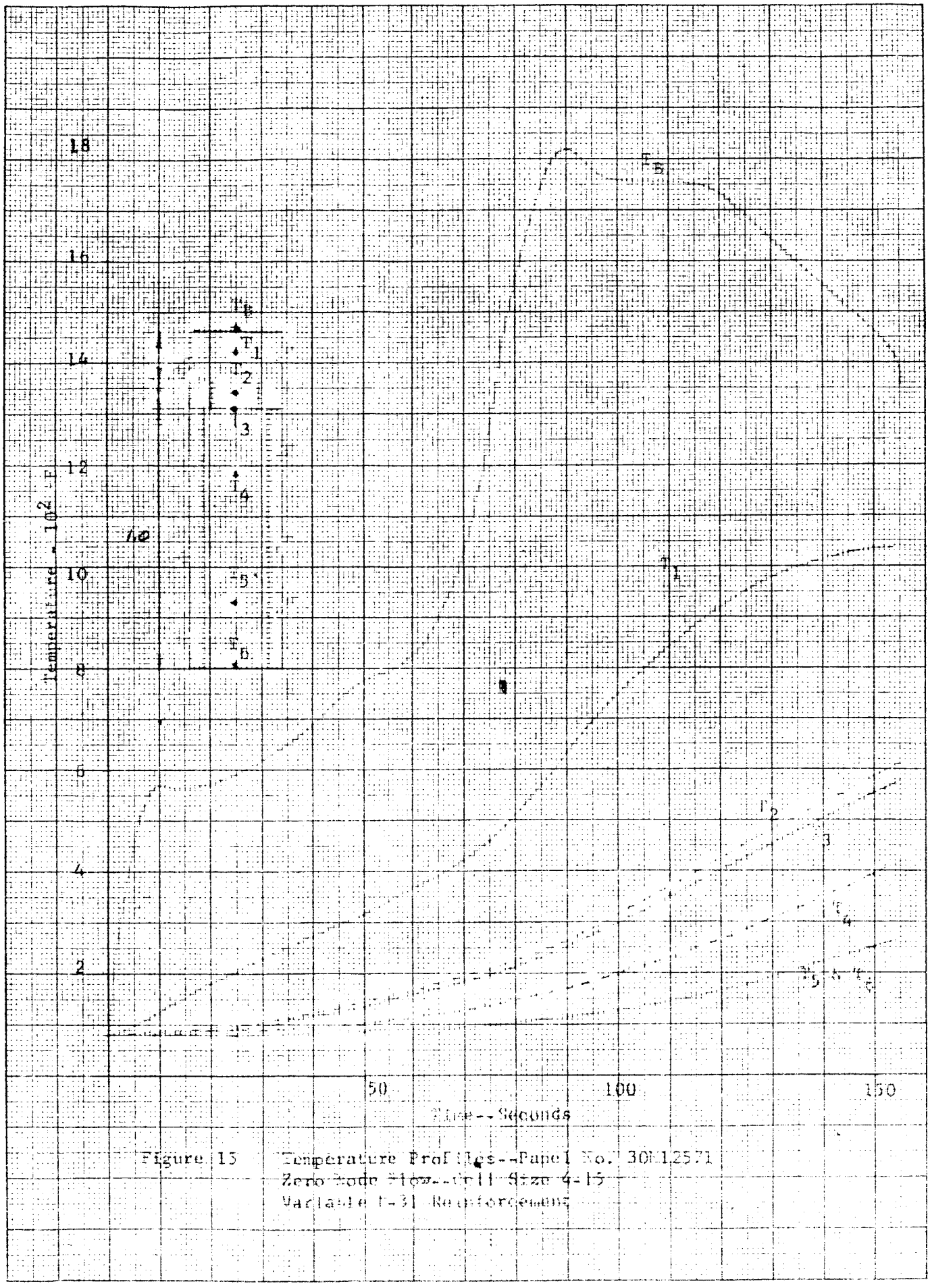

Figure 15 Temperature Profiles--Panel No. 30H12571
Zero Node Flow--Cell Size 4-15
Variable I-31 Reinforcement

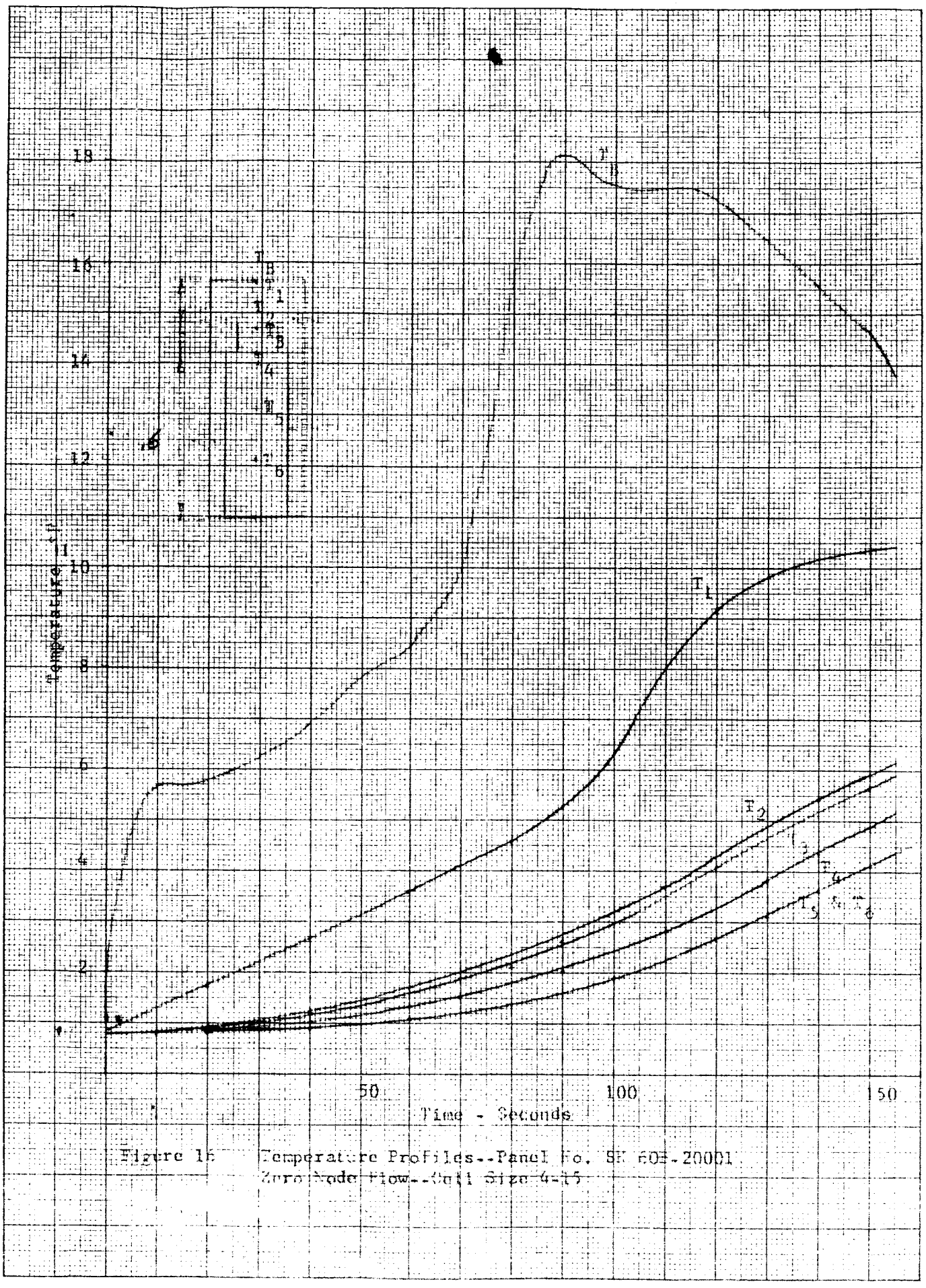

Figure 16 Temperature Profiles--Panel No. SE 60E-20001
Zero Node Flow--Cell Size 4-15

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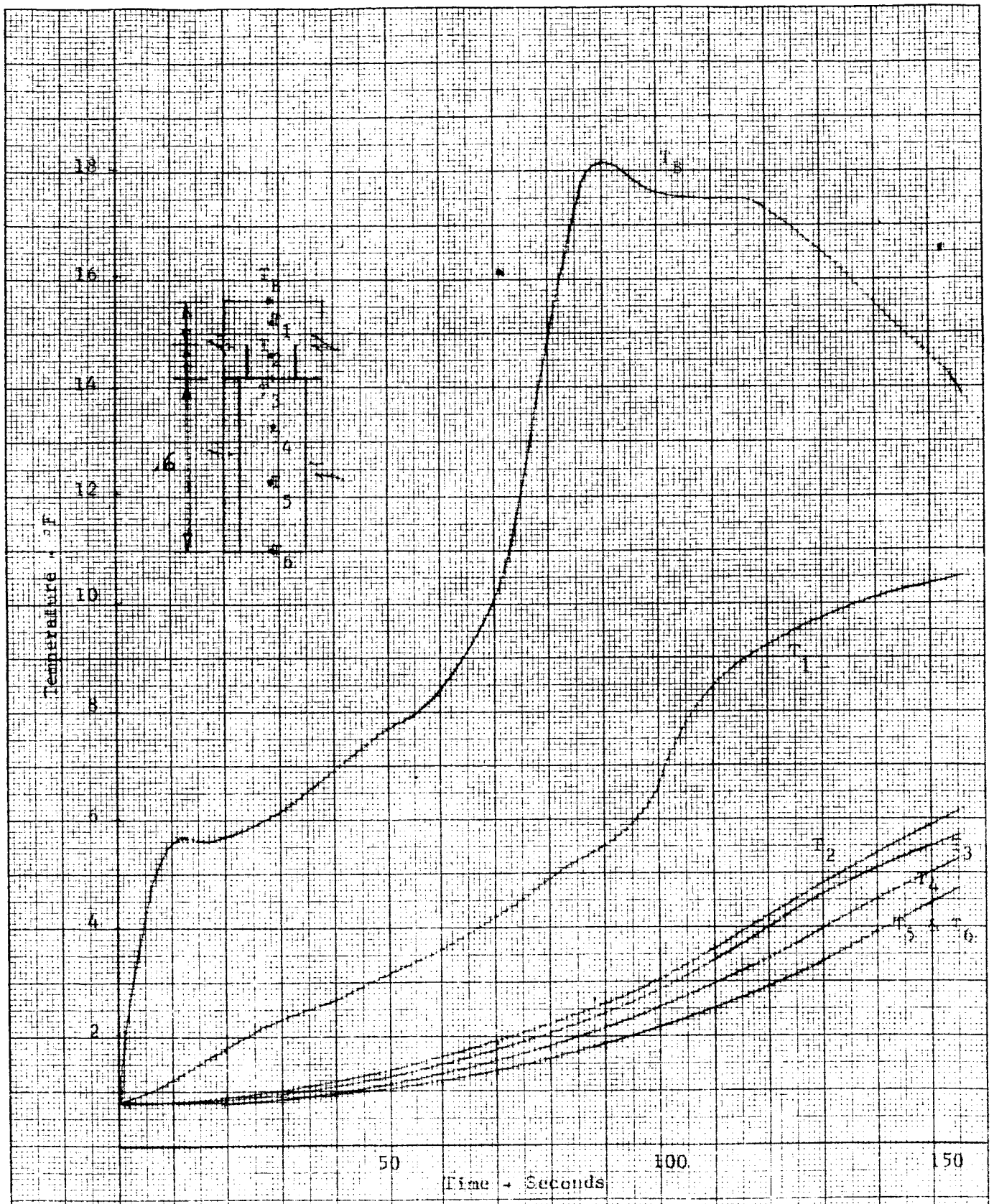

Figure 17 Temperature Profiles--Panel No. SK 60-20001
.003 Inch Node Flow Width--Cell Size 4-15

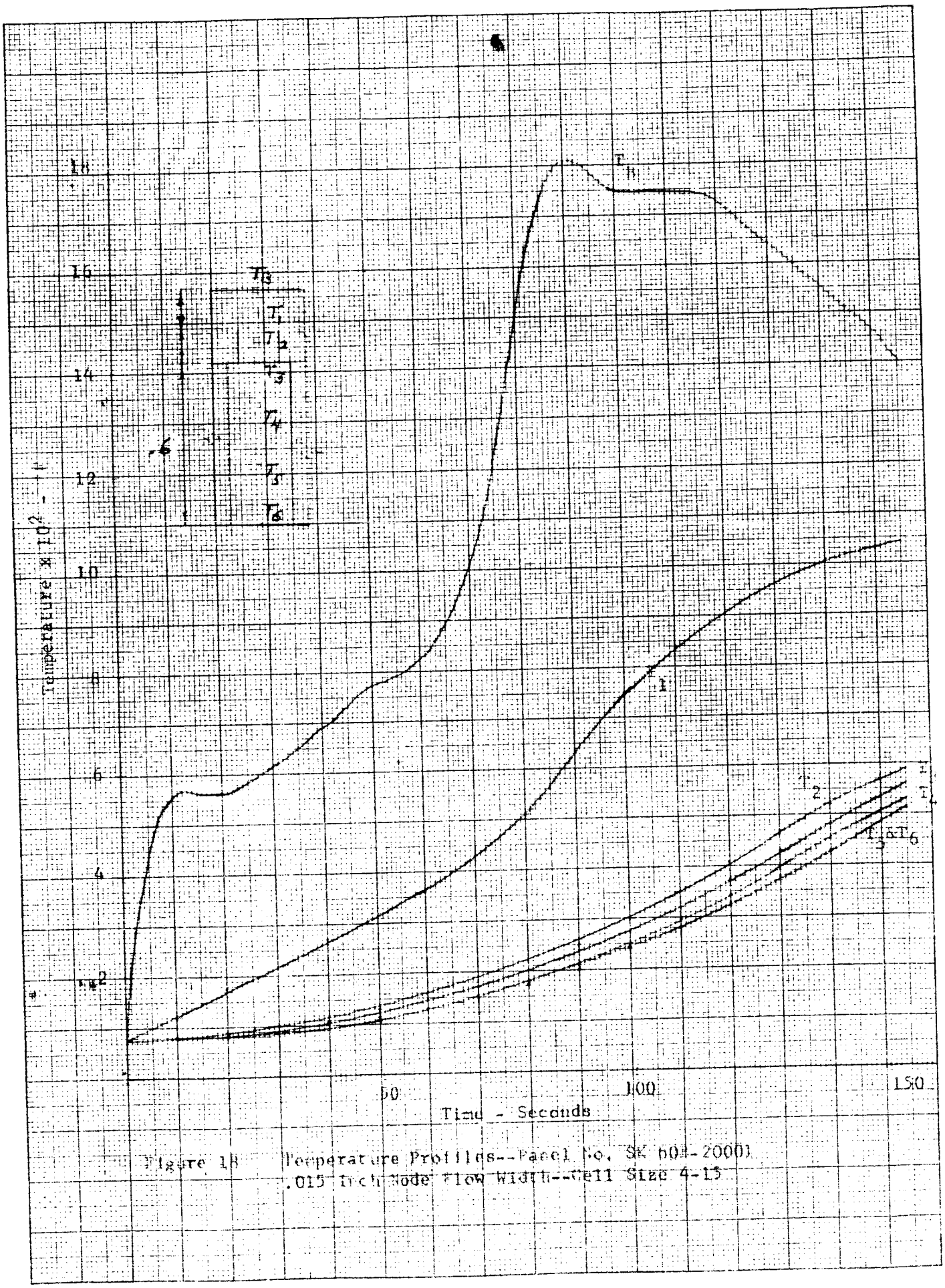

Figure 18 Temperature Profiles--Panel No. SK 604-20001
.015 Inch Node Flow Width--Cell Size 4-15

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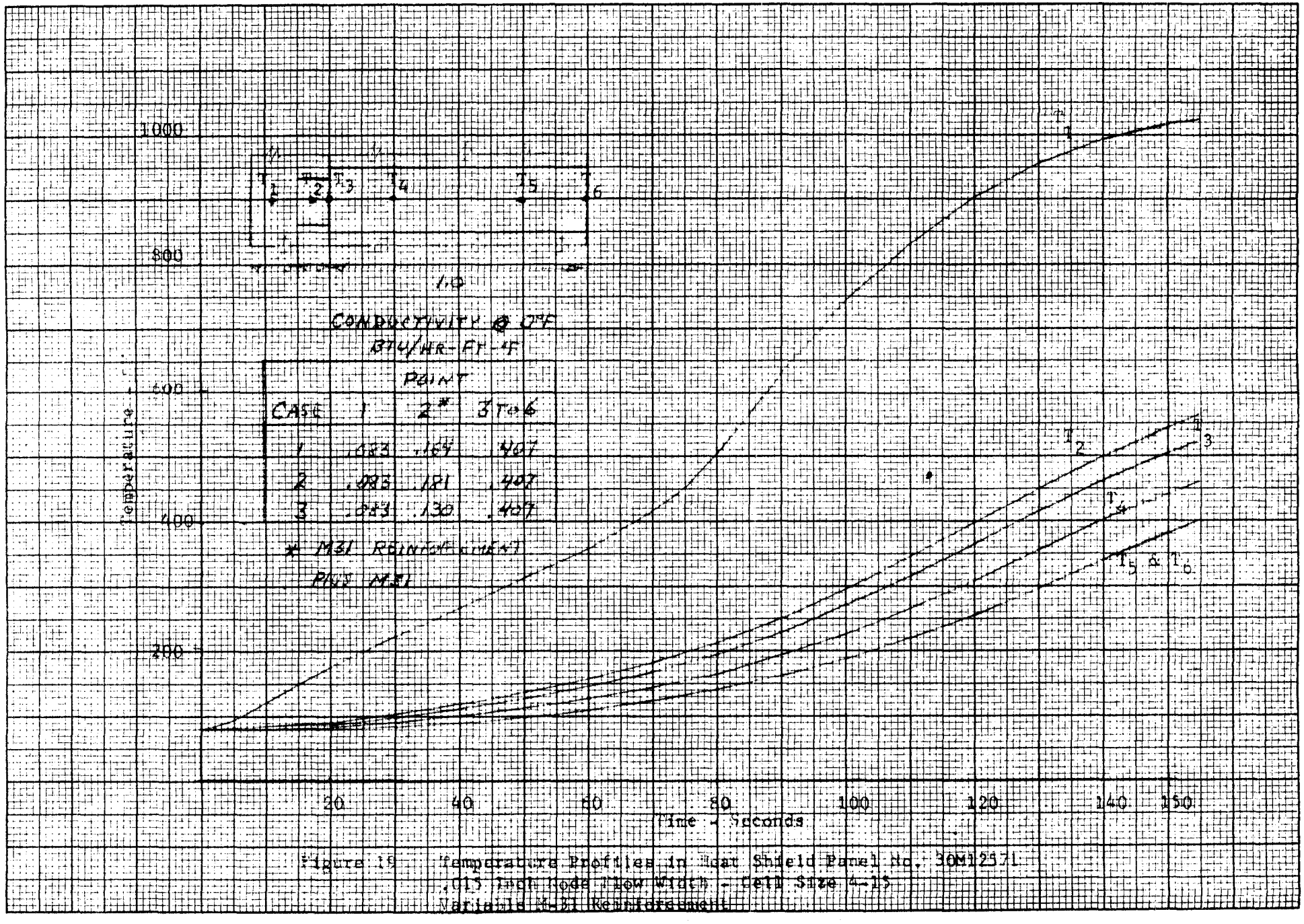


Figure 19 Temperature Profiles in Heat Shield Panel No. 30M12571
.015 Inch Node Flow Width - Cell Size 4-15
Variable M-31 Reinforcement

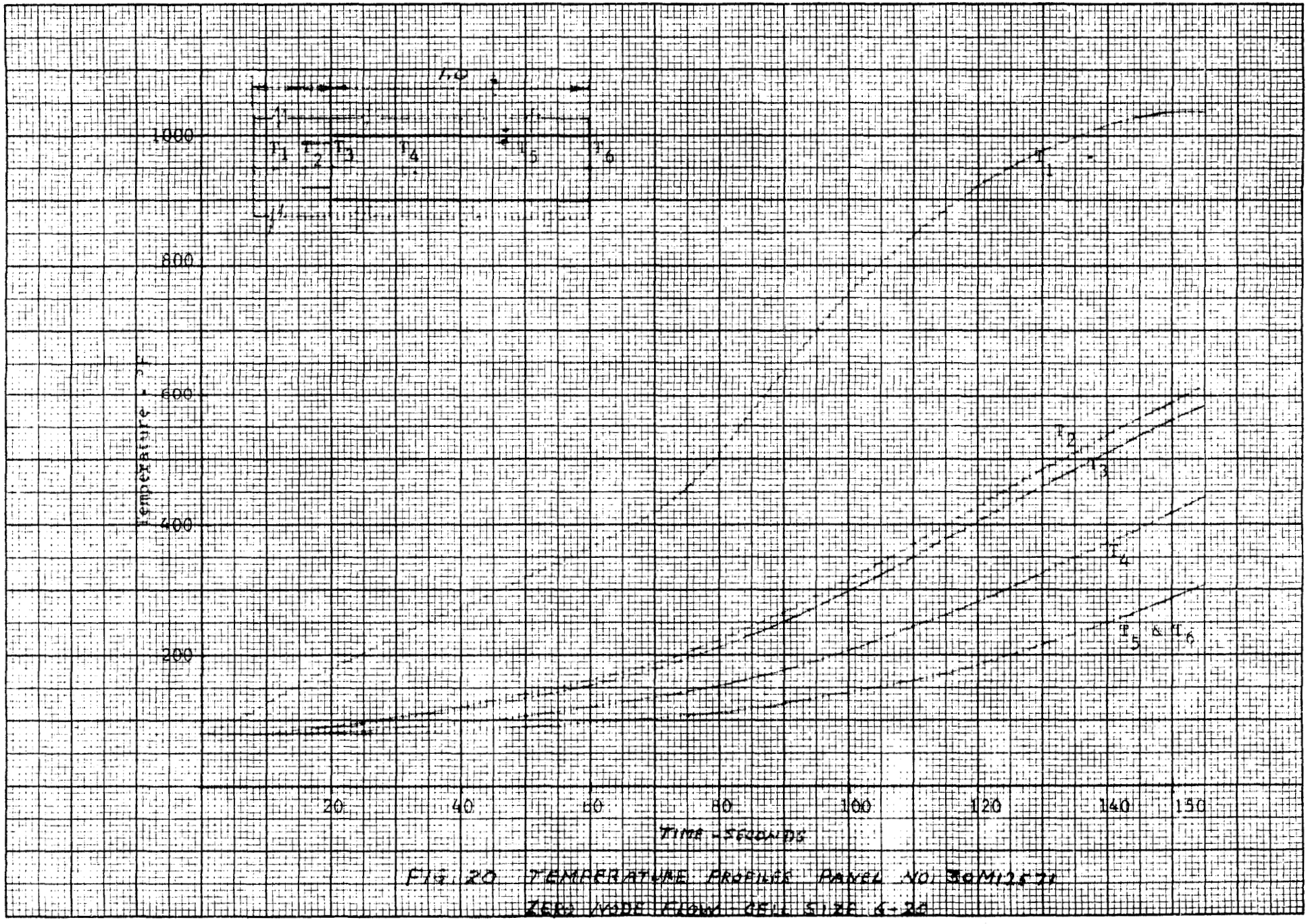


FIG. 20 TEMPERATURE PROFILES PANEL NO. 30M12571
ZERO MODE FLOW CELL SIZE 6-26

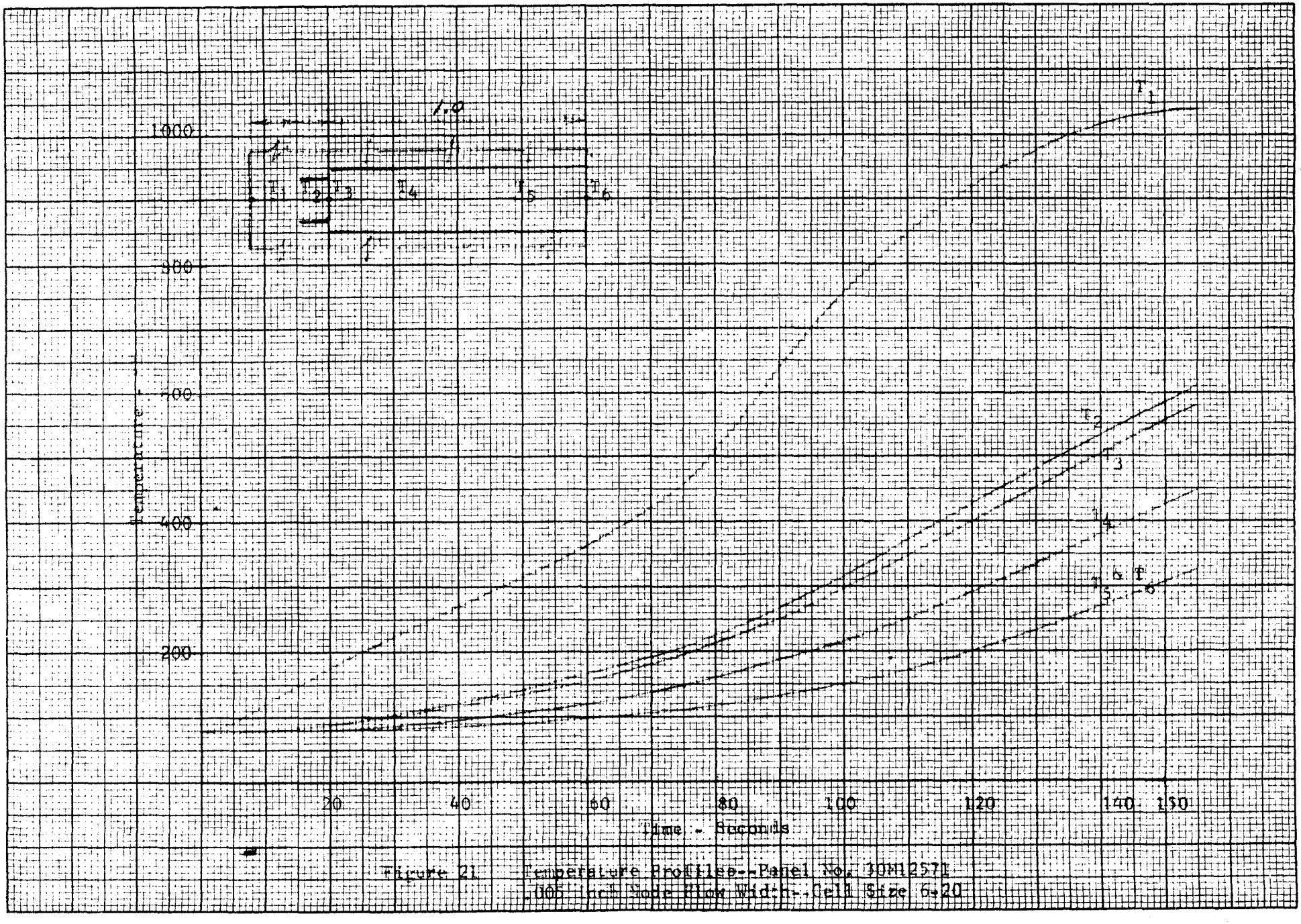


Figure 21. Temperature Profiles--Panel No. 30M12571
.005 Inch Node Flow Width--Cell Size 6-20

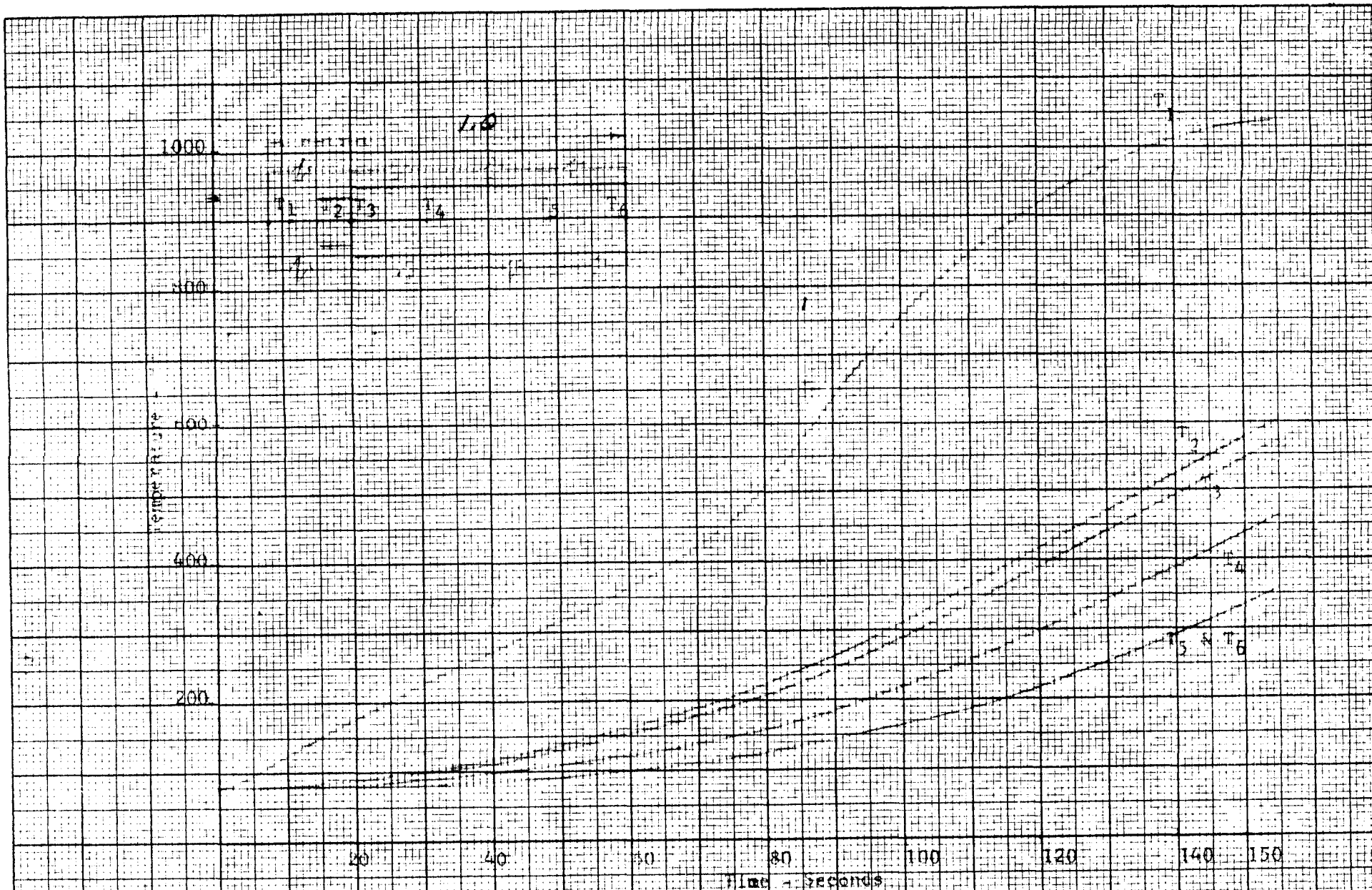


Figure 22 Temperature Profiles--Panel No. 30M12571
.010 Inch Node Flow Width--Cell Size 0-20

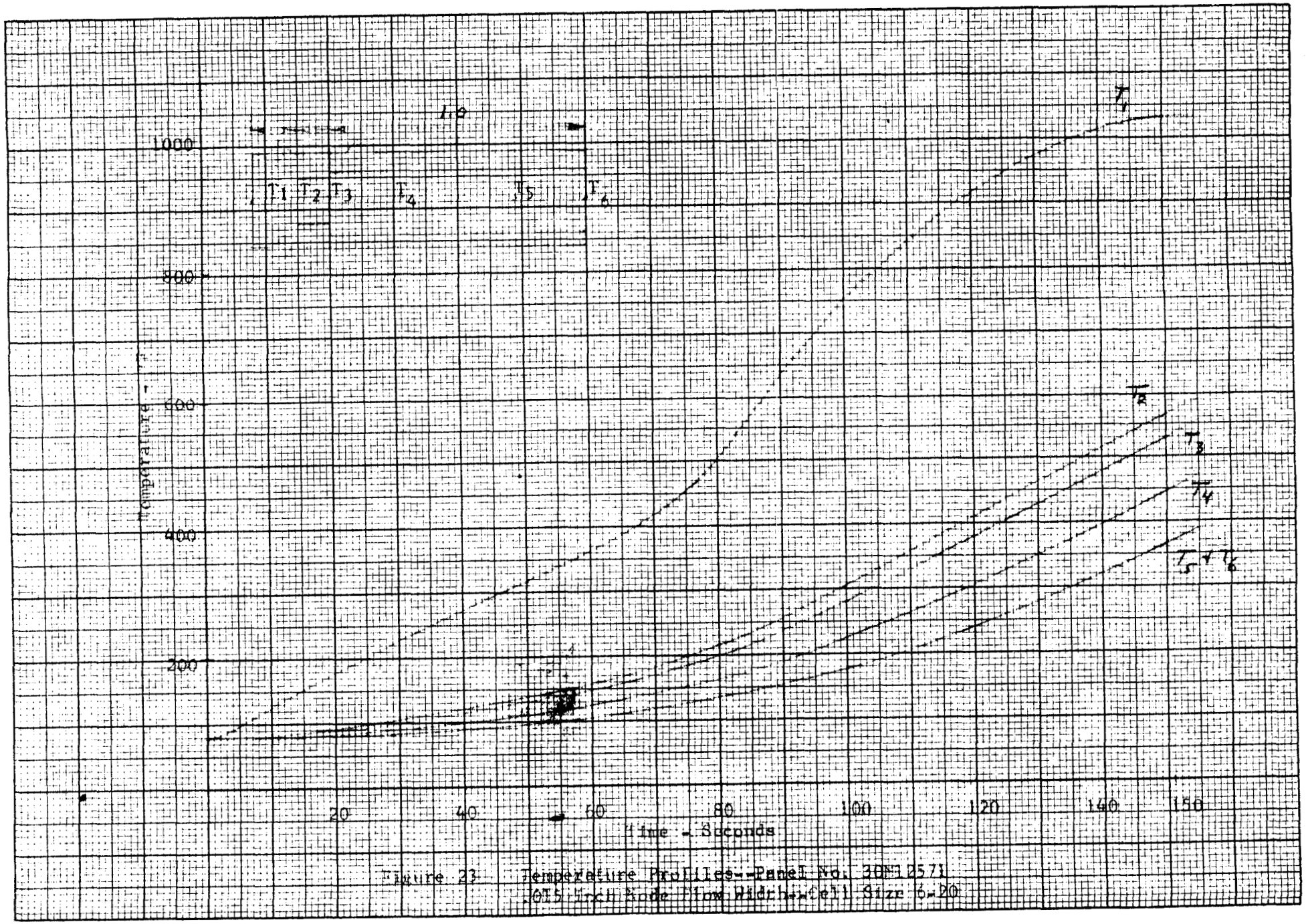


Figure 23 Temperature Profiles--Panel No. 30M12571
.015 Inch Rode Flow Width--Cell Size 6-20

Honeycomb Cell Dimensions		Honeycomb Cells	Honeycomb Foil Area	Node Flow Area A_{NF} (ft^2)			Core Density ρ_{NF}	Node Flow Density NF #/ft^3			Effective Conductivity F			T F
Width	Foil Thick.	ft^2	A_{PH} (Ft^2)	.005	.01	.015	#/ft^3	.005	.01	.015	.005	.01	.015	
.25	.0015	2304	.012	.0004	.0016	.0036	6.2	.248	.99	2.23	.138 .182	.245 .289	.424 .468	0 800
4-15														
.188	.0015	4096	.01[illegible]	.000[illegible]	.0028	.0064	8.3	.433	1.733	[illegible].960	.19[illegible] .25[illegible]	.3[illegible] .446	.[illegible]09 .[illegible]	0 800
3-15														
.375	.002	2048	.011	.0002	.0007	.0016	5.[illegible]	.124	.433	.990	.112 .152	.157 .197	.237 .2[illegible]7	0 800

[illegible]-20

[illegible] 0 Node Flow

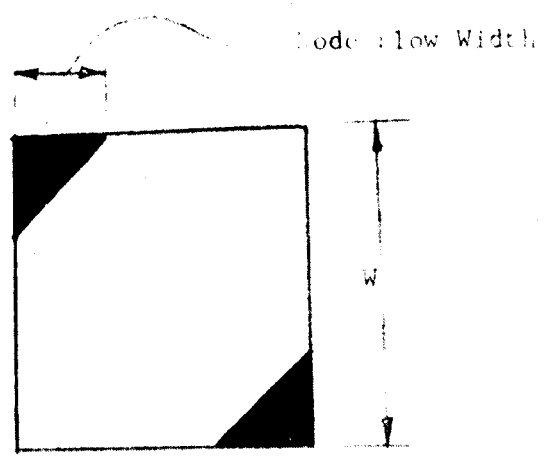


Figure 24 Panel Dimensions and Thermal Conductivities for Parametric Study

[illegible]. STRESS ANALYSIS OF [illegible] PANELS [illegible] NASA (Dwgs 30[illegible]12571)

As a result of the meeting on April 1[illegible], 19[illegible]3 with Messrs. Verble and Lysaght of Nasa, the design conditions were revised as stated below:

(1) Ignition Period- (condition A) assume zero thermal gradient, with a 2.7 psi on the M-31 insulated panel face.

(2) Flight Period -(condition [illegible]) assume linear temperature gradients (Δ [illegible]) through the panel and a 1.0 psi air load on the non-insulated panel face. Thermal and air loads are additive for this condition.

This monthly report will cover the deflections and stresses for (1) revised conditions given above, (2) the effects of a ½" deflection at one corner of a panel (3) 3g acceleration of a 200 l. man jumping on the panel.

The temperature gradients are as follows:

(a) Zero brazing alloy node flow = 320 F (ΔT)

(b) Brazing alloy node flow .005" width = 280 F (ΔT)

(c) Brazing alloy node flow .010" width = 180 F (ΔT)

The minimum ΔT condition (180 F) appears highly representative of actual heat shield panels.

REVISIONS

SUMMARY

Item I - Revised Conditions

A. 2.7 psi air load, zero thermal load for the ignition phase:

Max. deflection 0.55497"

Max. Stress 35,644 psi

B. 1.0 psi air load plus thermal gradients for the flight phase:

DEFLECTIONS

ΔT	W Air Load	W Thermal	W Total
180 F	.20554"	.2[illegible]704"	.4925[illegible]"
280 F	.20554"	.44651"	.65205"
320 F	.20554"	.51030"	.[illegible]1584"

MOMENTS

ΔT	M Air Load	M Thermal	M Total
180 F	133 in.lb.	[illegible]4 in.lb.	207 in.lb.
280 F	1[illegible] in.lb.	115 in.lb.	248 in.lb.
320 F	13[illegible] in.lb.	[illegible] in.lb.	2[illegible] in.lb.

MAXIMUM [illegible] PANEL

ΔT	Total Moment	Bending Stress
180 F	207 in.lb.	20,495 psi
280 F	24[illegible] in.lb.	24,[illegible]54 psi
320 F	[illegible]	[illegible],238 psi

REVISIONS

Item II - The Effects of ½" Deflection at One Corner of A Panel

A. Deflection at Center .2[illegible]05"

. Stress at Center 5050 psi

Part A and B above are additive to the deflections and stresses in Item I.

Item III - 3g Acceleration of a 200 lb. Man Jumping on A Panel

A. Max. Deflection .11[illegible]"

B. Max. Facing Stress 12,925 psi at Center of Panel.

CALCULATIONS FOR 1" ZEE SECTION PANEL

1. Condition A with [illegible] psi - No thermal gradient

2. Condition B with 1.0 psi plus temp. gradients of ΔT [illegible] F
ΔT [illegible]
ΔT 180 F

α 6.1 x $10^{-[illegible]}$ μ 3.0

$\bar{t}$ 1" t' t'' .010"

$$D_o \left[\frac{(\bar{t} - t)^2}{1 - \mu}\right] \left[\frac{t' t''}{1 - \mu}\right] \left[\frac{[illegible]}{[illegible]}\right] \left[\frac{[illegible] \times 10^{[illegible]} \times .01}{1 - .3^2}\right]$$

$$.52894 \times .28[illegible]1 \times 10^{[illegible]} \quad 151.123$$

$$P_e \quad D_o \left(\frac{\pi}{a}\right)^2 \quad 151.123 \left(\frac{\pi}{[illegible]}\right)^2 \quad 535$$

REVISIONS

Condition A Deflection $q = 2.7$ psi

$$W_a = \frac{\pi^2 q b^2}{62.5 \times Pe \times 4} = \frac{\pi^2 \times 2.7 \times \overline{52.778}^2}{62.5 \times 535 \times 4} = .55497''$$

Condition B Deflection $q = 1.0$ psi

$$W_a = \frac{\pi^2 q b^2}{62.5 \times Pe \times 4} = \frac{\pi^2 \times 1.0 \times \overline{52.778}^2}{62.5 \times 535 \times 4} = .20554''$$

$\Delta T = 180$

$$W = \frac{\phi \Delta T (1 - \mu) 16 a^2 \times .5708}{\pi^3 (t + 2t) \times 4}$$

$$\frac{6.1 \times 10^{-6} \times 180 \times 1.3 \times 16 \times \overline{52.778}^2 \times .5708}{\pi^3 (1 + .02) \times 4} = .28704$$

$\Delta T = 280$

$$W = \frac{6.1 \times 10^{-6} \times 280 \times 1.3 \times 16 \times \overline{52.778}^2 \times .5708}{\pi^3 (1 + .02) \times 4} = .44651''$$

$\Delta T = 320$

$$W = \frac{6.1 \times 10^{-6} \times 320 \times 1.3 \times 16 \times \overline{52.778}^2 \times .5708}{\pi^3 (1 + .02) \times 4} = .51030''$$

Condition A Moment

Moment due to air load of square panel. Moment is maximum at the center of panel.

$$M_{air} = .1916 \times q \times \frac{a^2}{4}$$

$q = 2.7$ psi

$$M = .1916 \times 2.7 \times \frac{\overline{52.778}^2}{4} = 360 \text{ in-lbs/in. at center}$$

REVISIONS

Condition B Moments

q 1.0 psi

$M = .191 \times 1. \frac{\overline{\quad}^2}{4}$ 133 in-lb/in, at Center

Moment due to thermal load at the edges for square panel. (At the center the moment is ½ edge moment.)

$$M = \frac{\alpha \Delta T (1-\mu^2)}{(\bar{t} + 2f)}$$

ΔT 180

$M = \frac{6.1 \times 10^{-6} \times 180 \times (1-.3^2)\ 1\ 1.123}{(1 + .02)}$ 148 in-lb/in

ΔT 280

$M = \frac{6.1 \times 10^{-6} \times 280 (1-.3^2)\ 1\ 1.123}{(1 + .02)}$ 230 in-lb/in

ΔT = 320

$M = \frac{6.1 \times 10^{-6} \times 320 (1-.3^2)\ 1\ 1.123}{1 + .02}$ 263 in-lb/in

Condition A Stress

Ignition Air Load [illegible] 2.7 psi

$f_b = \frac{M}{(\bar{t}+f)t'} = \frac{360}{(1+.01).01} = \frac{360}{.0101}$ 35,644 psi

Condition B Stresses

ΔT 180 Max. [illegible] 207 in-lb/in; ΔT 280 [illegible] 248; [illegible]

ΔT 320 [illegible] 265 in-lb/in at center; [illegible] 263 in-lb/in at edge

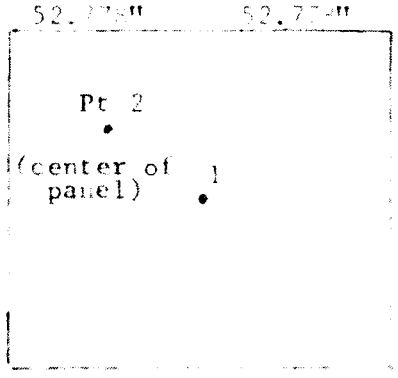
$f_{b180} = \frac{207}{.0101}$ 20,495 psi

REVISIONS

f_{b280} $\frac{248}{.0101}$ 24,554 psi

f_{b320} $\frac{2[illegible]}{.0101}$ 2[illegible],238 psi at Center

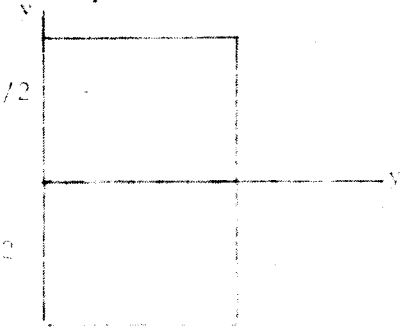
f_{b320} $\frac{2[illegible]3}{.0101}$ 2[illegible],040 psi at Edge



Determine the maximum stress due to ½" deflection at point 1.

1) Assume that these four panels to be one large panel 105.556" square with a deflection of ½" at the Center. Then determine the stress and deflection at point 2 of each panel that is 52.278" square.

(Ref. Theory of Plates and Shells by Timoshenko)



$y = \frac{b}{4}$ $x = \frac{a}{4}$

Moment at Center of Panel (52.278)

$M_x = \frac{qx(a-x)}{2}$ $M_y = \mu\frac{qx(a-x)}{2}$

$$M_x = (1-\mu)qa^2\pi^2 \sum_{m=1}^{\infty} m^2 \left[\frac{-2\left(\frac{m\pi b}{2a}\tanh\frac{m\pi b}{2a} - 2\right)}{\pi^3 m^3 \cosh\frac{m\pi}{2a}} \cosh\frac{m\pi y}{a} + \frac{2}{\pi^3 \cdot m^3 \cosh\frac{m\pi}{2a}}\left(\frac{m\pi}{a}\sinh\frac{m\pi y}{a} - \frac{2\mu}{1-\mu}\cosh\frac{m\pi y}{a}\right)\right]\sin\frac{m\pi x}{a}$$

$\sum = -.00923$

$M_x = \frac{qx(a-x)}{2} - (1-\mu)qa^2\pi^2(-.00923) = \frac{3a^2}{32}q - .0[illegible]377a^2q$

$M_x = -.0299[illegible]a^2q$

REVISIONS

$$M_y = -(1-\mu)\, q a^2 \pi^2 \sum_{m=1}^{\infty} \frac{2}{\pi^3 m^5} \left[\frac{-2\left(\frac{m\pi}{2a} \tanh \frac{m\pi}{2a} - 2\right)}{\cosh \frac{m\pi}{2a}} \cosh \frac{m\pi y}{a} \right.$$

$$\left. \frac{2}{\pi^3 m^5 \cosh \frac{m\pi}{2a}} \left(\frac{m\pi y}{a} \sinh \frac{m\pi y}{a} - \frac{2}{1-\mu} \cosh \frac{m\pi y}{a}\right) \right] \sin \frac{m\pi x}{a}$$

$$\sum = -.00017$$

$$M_y = \mu q x \frac{(a-x)}{2} - .00017\,(1-\mu) a^2 \pi^2 q \qquad x = \frac{a}{4} \qquad y = \frac{a}{4}$$

$$(.02813 - .00117) a^2 q$$

$$M_y = \underline{.02930\, a^2 q}$$

(Ref. Theory of Plates and Shells)

Deflection at the center of panel with a [illegible]" deflection at corner.

$$W = \frac{4qa^4}{\pi^5 D} \sum_{m=1,3,5}^{\infty} \frac{1}{m^5} \left[1 - \frac{\left(\frac{m\pi}{2a} \tanh \frac{m\pi}{2a} - 2\right)}{2 \cosh \frac{m\pi}{2a}} \cosh \frac{2m\pi y}{2a} \right.$$

$$\left. \frac{m\pi}{2 \times 2 \times a \cosh \frac{m\pi}{2a}} \times \frac{2}{\pi} \sinh \frac{2m\pi y}{2a} \right] \sin \frac{m\pi x}{a}$$

$$y = \frac{a}{4} \qquad x = \frac{a}{4}$$

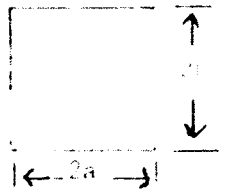
$$\sum_{m=1,3,5}^{\infty} = .1[illegible]288$$

$$W = \frac{4a^4 q \times .1[illegible]2}{\pi^5}$$

REVISIONS

Determine the q load required to give a ½" deflection at center of 105.556" square panel, then the moments and deflection of the 52.778" panel can be calculated.

$W_a = \frac{\pi^2 q}{2.5\,Pe}$ Where [illegible] ½ width



$D = 131.123$

$Pe = D\left(\frac{\pi}{2a}\right)^2 = 131.123\left(\frac{\pi}{1[illegible]}\right)^2$

$Pe = 133.[illegible]345$ $W = .500"$

$.500 = \frac{\pi^2 \times q \times [illegible]^2}{2.5 \times 133.[illegible]345}$

$q = \underline{.1[illegible]12}$ psi air load

Moment

$M_x = .02998a^2q$ a width

$.02998 \times \overline{105.[illegible]}^2 \times .1[illegible]212$

[illegible] in-[illegible]/in at center of 52.[illegible]" panel

Deflection at center of panel

$W = \frac{4a^4q \times .1[illegible]}{\pi^5 D} = \frac{4 \times \overline{1[illegible]}^4 \times .1[illegible] \times .1[illegible]}{\pi^5 \times 131.123}$

$W = \underline{.2[illegible]05"}$

Stress

$f = \frac{[illegible]}{.0101} = 5.050$ psi

The effects on the stress and deflection due to the corner deflection of ½".

REVISIONS

Condition A 2.7 psi ONLY

$M = 360 + 51 = 411$ in-lb at the center of panel

$W_a = .55497 + .26605 = .82102$

$f_b = \frac{M}{(\bar{C}+t)t'} = \frac{411}{.0101} = 40,693$ psi

Condition B 1 psi plus thermal gradient

$\Delta T = 180 \quad M = 133 + \frac{148}{2} + 51 = 258$ in-lb At Center of Panel

$\Delta T = 280 \quad M = 133 + \frac{230}{2} + 51 = 299$ in-lb

$\Delta T = 320 \quad M = 133 + \frac{263}{2} + 51 = 315.5$ in-lb

$\Delta T = 180 \quad W = .20554 + .28704 + .26605 = .75863''$

$\Delta T = 280 \quad W = .20554 + .44651 + .26605 = .91810''$

$\Delta T = 320 \quad W = .20554 + .51030 + .26605 = .98189''$

$f_{b180} = 258/.0101 = 25,545$ psi

$f_{b280} = 299/.0101 = 29,604$ psi

$f_{b320} = 315.5/.0101 = 31,238$ psi

Concentrated load on a simply supported panel, 200 lb, max with a 3g acceleration at the center of panel, for room temperature conditions.

$E = 29 \times 10^{-6}$

$\mu = .3$

$D = \left[\frac{(\bar{C}+t)^2}{1+m}\right]\left[\frac{Et^3}{1-\mu^2}\right] = \left[\frac{(\bar{C}+.01)^2}{2}\right]\left[\frac{29\times10^6\times.01}{1-.3^2}\right]$

$.51005 \times .31868 \times 10^6 = 162,543$

REVISIONS

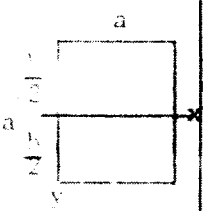
$$W = \frac{Pa^2}{2D\pi^3} \sum_{m=1,3,5}^{\infty} \left(\frac{1}{m^3} \left(\tanh \alpha_m - \frac{\alpha_m}{\cosh^2 \alpha_m} \right) \right)$$

$$\alpha_m = \frac{m\pi b}{2a}$$

$$\sum_{m=1,3,5}^{\infty} \frac{1}{m^3} \left(\tanh \alpha_m - \frac{\alpha_m}{\cosh^2 \alpha_m} \right) = .71870''$$

$$W = \frac{Pa^2 \times .71870}{2 \times D \times \pi^3} \qquad a = \text{width}$$

$$W = \frac{200 \times 3 \times 52.[illegible]^2 \times .71870}{2 \times 162.543 \times \pi^3} = \frac{600,585,378}{5,039,836} = .11916''$$



Moment due to a 200 lb. man stepping on the center of panel with a 3g force.

$$(mx)_{y=0} = \frac{P}{2\pi} \sum_{m=1}^{\infty} \frac{\sin \frac{m\pi c}{a}}{m} \left[(1-\mu) \tanh \alpha_m - \frac{(1-\mu)\alpha_m}{\cosh^2 \alpha_m} \right] \sin \frac{m\pi x}{a}$$

$$(My)_{y=0} = \frac{P}{2\pi} \sum_{m=1}^{\infty} \frac{\sin \frac{m\pi c}{a}}{m} \left[(1-\mu) \tanh \alpha_m - \frac{(1-\mu)\alpha_m}{\cosh^2 \alpha_m} \right] \sin \frac{m\pi x}{a}$$

$$c = \frac{a}{2} \qquad x = \frac{a}{2} \qquad y = 0 \qquad m = \frac{m\pi b}{2a}$$

$$\Sigma_x = 1.01724 \qquad \Sigma_y = 1.36704$$

$$Mx = \frac{P \times 1.01724}{2\pi} = .1[illegible]P$$

$$My = \frac{P \times 1.36704}{2\pi} = .21757P$$

Max. Moment

$$My = .21757P = .21757 \times 600 = 130.542 \text{ in-lb.}$$

REVISIONS

Maximum stress due to the concentrated load.

$$f_b = \frac{M}{(E-1)t^2} = \frac{130.542}{.0101} = 12,925 \text{ psi at center of panel}$$

Deflection .11916

ALTERNATE METHOD TO CHECK THE 600# LOAD CONDITION

Assume a 200# man with a 3g force jumps on the center of panel with 1 sq. ft. area psi $\frac{600}{144}$

4.17

$$M = B\,P \qquad \frac{b}{a} = \frac{12}{52.728} = .22736$$

$B = .20082$

$M = .20082\ P$

where $P = 3 \times 200 = 600$

$M = .20082 \times 600$

$M = 120.492$ in-lb

$$q = \frac{200}{144} = 1.38888 \text{ psi}$$

$$q_o = q\,l = 1.38888 \times 12 = 16.6666$$

$$W_{max} = \frac{.655\ q_o\ a^3}{D\pi^4} = \frac{.655 \times 16.6666 \times 52.728^3}{151,123 \times \pi^4} = \frac{10.61977}{\pi^4}$$

W = .10902" at center of panel

$$f_b = \frac{120.492}{.0101} = 11,930 \text{ psi}$$

NOTE: Flatwise compression strength of 4-15 core 1.0" thick 750 psi.
Ref. [illegible] checked at RC. Sandwich Structures Manual.

REVISIONS

NOMENCLATURE

a, b = lengths of panel in x and y directions

$D_o = \frac{(\bar{t}+f)^2}{1+m} \times \frac{t't'}{1-\mu^2}$ = flexural rigidity (lb-in^2/in)

E = Young's modulus (lb/in^2)

f_b = Stress psi bending

$f = \frac{(t'+t'')}{2}$ in.

$\bar{G}$ = core shear modulus lb/in^2

M = moment in-lb/in

$m = \frac{t'E'}{t''E''}$

$Pe = \frac{(\bar{t}+f)^2}{(1+\mu^2)} \left(\frac{\pi}{b}\right)^2 \frac{t't'}{1+m}$ lbs/in

q = transverse load lb/in^2

T = temperature (°F)

$\bar{t}$ = thickness (in) core

t = thickness of face

$\hat{t} = \frac{\bar{t}-f}{\bar{t}}$

W = panel deflection (in)

α = coefficient of thermal expansion (in/in-°F)

μ = Poisson's Ratio

SUBSCRIPTS

- = core
' = lower face
'' = upper face

REVISIONS