FACILITY FORM 602
N64-30351
(ACCESSION NUMBER)
(THRU)
11
1
(PAGES)
(CODE)
CR-71992
13
(NASA CR OR TMX OR AD NUMBER)
(CATEGORY)

First Monthly Progress Report

on

Thermal Strain Analysis

of

Advanced Manned-Spacecraft Heat Shields

NASA Contract NAS 9-1986
Period 19 August 1963 to 19 September 1963

ARA Report
#25

Prepared by:

Daniel H. Platus, Project Scientist

Approved by:

B. Mazelsky, President, ARA, Inc.

OTS PRICE

XEROX $ 1.00 ph/fs
MICROFILM $ 0.50 mf.

Library Copy
SEP 1964
Manned Spacecraft Center
Houston, Texas

30 September 1963

FIRST MONTHLY PROGRESS REPORT ON THERMAL STRAIN ANALYSIS OF ADVANCED MANNED-SPACECRAFT HEAT SHIELDS

NASA Contract NAS 9-1986

Phase A - Derivation of Basic Equations

The first phase of the study (Derivation of Basic Equations) is proceeding on schedule and is 60% completed. The equilibrium equations in terms of displacements have been derived for the general non-axisymmetric case for both spherical and toroidal coordinates and the equations for the spherical case reduce to the axi-symmetric forms reported in Dr. Morgan's paper, "Thermal Stresses in Missile Nose Cones." The derivation is summarized in Appendix A and the coefficients are presented in tabular form. It is significant to note that, owing to the similarity of spherical and toroidal coordinates, only the toroidal coefficients need be programmed since in the limit as the large radius of the torus reduces to zero a spherical coordinate system is obtained.

It is anticipated that the derivation of stresses in terms of displacements, which constitutes the final portion of this phase of the study, will be completed within the next two weeks.

Phase B - Finite Difference Formulation

The second phase of the study (Finite Difference Formulation) has been in progress for approximately one week in accordance with the program schedule and is 6% completed. The general non-axisymmetric forms of the finite difference analogs

to the partial differential equations are being developed, from which a set of linear algebraic equations in the three displacement components will be obtained. The general forms of the finite difference approximations with respect to three independent variables are being calculated by expanding the displacement functions in power series in the neighborhood of the point under consideration, using unequal increments in each of the three principal directions. In this way formulae will be obtained for the first and second irregular central, forward and backward derivatives at any intersection of grid lines with different increments on a discontinuous grid network.

Phase G - Report Preparation

This first monthly progress report constitutes 3% of the report preparation phase of the study.

APPENDIX A

Derivation of Equilibrium Equations in Terms of Displacements In Spherical and Toroidal Coordinates

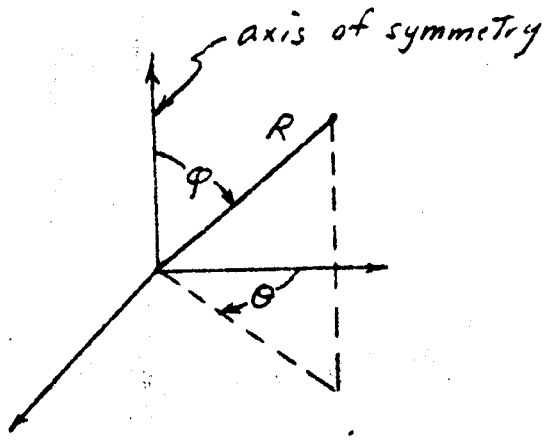
Derivation of Equilibrium Equations in Terms of Displacements in Spherical and Toroidal Coordinates

Orthogonal curvilinear coordinates $(\alpha_1, \alpha_2, \alpha_3)$

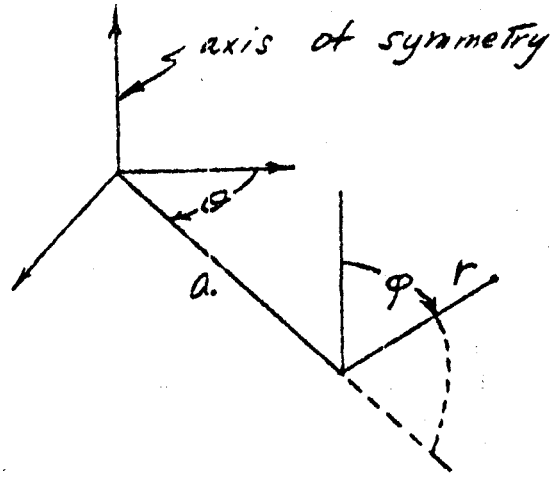
Element of arc ds defined by

$$ds^2 = \sum_{i=1}^{3} g_{ii}\, d\alpha_i^2 \qquad (1)$$

where g_{ii} are the metric coefficients.



Spherical Coordinates



Toroidal Coordinates

	Spherical Coordinates	Toroidal Coordinates
α_1	R	r
α_2	φ	φ
α_3	θ	θ
g_{11}	1	1
g_{22}	R^2	r^2
g_{33}	$R^2 \sin^2\varphi$	$(a + r\sin\varphi)^2$
$g = \sqrt{g_{11} g_{22} g_{33}}$	$R^2 \sin\varphi$	$r(a + r\sin\varphi)$

Note: Toroidal coordinates reduce to spherical coordinates in the limit as $a \to 0$.

Equations of equilibrium with zero body force :

$$\sum_{j=1}^{3}\left[\frac{\partial}{\partial \alpha_j}\left(\frac{g\, g_{ii}\, \tau_{ij}}{\sqrt{g_{ii}\, g_{jj}}}\right) - \frac{1}{2}\cdot\frac{g\,\tau_{jj}}{g_{jj}}\,\frac{\partial g_{jj}}{\partial \alpha_i}\right] = 0 \qquad (2)$$

Where $g = \sqrt{g_{11}\, g_{22}\, g_{33}}$ and τ_{ii} and τ_{ij} are normal and shear components of stress, respectively.

Substituting the respective components of α_i and g_{ii} in Eq. (2) and performing the indicated differentiations and summations, there are obtained the following equilibrium equations in terms of stresses for each coordinate system :

<u>Spherical Coordinates</u>

$$\frac{\partial \tau_{RR}}{\partial R} + \frac{1}{R}\frac{\partial \tau_{R\varphi}}{\partial \varphi} + \frac{1}{R\sin\varphi}\frac{\partial \tau_{R\theta}}{\partial \theta} + \frac{2\tau_{RR} - \tau_{\varphi\varphi} - \tau_{\theta\theta} + \tau_{R\varphi}\cot\varphi}{R} = 0 \qquad (3)$$

$$\frac{\partial \tau_{R\varphi}}{\partial R} + \frac{1}{R}\frac{\partial \tau_{\varphi\varphi}}{\partial \varphi} + \frac{1}{R\sin\varphi}\frac{\partial \tau_{\varphi\theta}}{\partial \theta} + \frac{3\tau_{R\varphi} + (\tau_{\varphi\varphi} - \tau_{\theta\theta})\cot\varphi}{R} = 0 \qquad (4)$$

$$\frac{\partial \tau_{R\theta}}{\partial R} + \frac{1}{R}\frac{\partial \tau_{\varphi\theta}}{\partial \varphi} + \frac{1}{R\sin\varphi}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{3\tau_{R\theta} + 2\tau_{\varphi\theta}\cot\varphi}{R} = 0 \qquad (5)$$

Toroidal Coordinates

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\varphi}}{\partial \varphi} + \frac{1}{(a+r\sin\varphi)}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{(a+2r\sin\varphi)\tau_{rr} - (a+r\sin\varphi)\tau_{\varphi\varphi} - \tau_{\theta\theta} r\sin\varphi + \tau_{r\varphi} r\cos\varphi}{r(a+r\sin\varphi)} = 0 \qquad (6)$$

$$\frac{\partial \tau_{r\varphi}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\varphi\varphi}}{\partial \varphi} + \frac{1}{a+r\sin\varphi}\frac{\partial \tau_{\varphi\theta}}{\partial \theta} + \frac{(2a+3r\sin\varphi)\tau_{r\varphi} + (\tau_{\varphi\varphi}-\tau_{\theta\theta})r\cos\varphi}{r(a+r\sin\varphi)} = 0 \qquad (7)$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\varphi\theta}}{\partial \varphi} + \frac{1}{a+r\sin\varphi}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{(a+3r\sin\varphi)\tau_{r\theta} + 2\tau_{\varphi\theta} r\cos\varphi}{r(a+r\sin\varphi)} = 0 \qquad (8)$$

Hooke's Law including Temperature Terms

$$\tau_{ii} = \lambda\theta + 2\mu e_{ii} - (3\lambda+2\mu)\int_{T_0}^{T}\alpha(T)\,dT \qquad (9)$$

$$\tau_{ij} = 2\mu e_{ij} \qquad (10)$$

$x_1 = r$
$x_2 = \varphi$
$x_3 = \theta$

where

$$\theta = e_{11} + e_{22} + e_{33}$$

and λ and μ are the Lamé constants defined in terms of Poisson's ratio ν and Young's modulus E according to

$$\left.\begin{aligned} \lambda &= \frac{\nu E}{(1+\nu)(1-2\nu)} \\ \mu &= \frac{E}{2(1+\nu)} \end{aligned}\right\} \qquad (11)$$

Strain - Displacement Relations

$$e_{ii} = \frac{\partial}{\partial \alpha_i} \frac{u_i}{\sqrt{g_{ii}}} + \frac{1}{2 g_{ii}} \sum_{k=1}^{3} \frac{\partial g_{ii}}{\partial \alpha_k} \frac{u_k}{\sqrt{g_{kk}}} \tag{12}$$

$$e_{ij} = \frac{1}{2\sqrt{g_{ii} g_{jj}}} \left[g_{ii} \frac{\partial}{\partial \alpha_j}\left(\frac{u_i}{\sqrt{g_{ii}}}\right) + g_{jj} \frac{\partial}{\partial \alpha_i}\left(\frac{u_j}{\sqrt{g_{jj}}}\right)\right], \quad i \neq j \tag{13}$$

Let u, v, w be components of displacement in the three principal directions r or R, φ and θ. Then substitution of these displacements in Eqs. (12) and (13), with the metric coefficients of page ①, yields the strain-displacement relations for the two coordinate systems:

Spherical Coordinates

$$\left.\begin{aligned}
e_{RR} &= \frac{\partial u}{\partial R} \\
e_{\varphi\varphi} &= \frac{1}{R}\frac{\partial v}{\partial \varphi} + \frac{u}{R} \\
e_{\theta\theta} &= \frac{1}{R\sin\varphi}\frac{\partial w}{\partial \theta} + \frac{u}{R} + \frac{v\cot\varphi}{R} \\
e_{R\varphi} &= \frac{1}{2}\left(\frac{1}{R}\frac{\partial u}{\partial \varphi} - \frac{v}{R} + \frac{\partial v}{\partial R}\right) \\
e_{\varphi\theta} &= \frac{1}{2}\left(\frac{1}{R}\frac{\partial w}{\partial \varphi} - \frac{w\cot\varphi}{R} + \frac{1}{R\sin\varphi}\frac{\partial v}{\partial \theta}\right) \\
e_{R\theta} &= \frac{1}{2}\left(\frac{1}{R\sin\varphi}\frac{\partial u}{\partial \theta} - \frac{w}{R} + \frac{\partial w}{\partial R}\right)
\end{aligned}\right\} \tag{14}$$

Toroidal Coordinates

$$
\left.
\begin{aligned}
e_{rr} &= \frac{\partial u}{\partial r} \\
e_{\varphi\varphi} &= \frac{1}{r}\frac{\partial v}{\partial \varphi} + \frac{u}{r} \\
e_{\theta\theta} &= \frac{1}{a+r\sin\varphi}\frac{\partial w}{\partial \theta} + \frac{u\sin\varphi}{a+r\sin\varphi} + \frac{v\cos\varphi}{a+r\sin\varphi} \\
e_{r\varphi} &= \frac{1}{2}\left(\frac{1}{r}\frac{\partial u}{\partial \varphi} + \frac{\partial v}{\partial r} - \frac{v}{r}\right) \\
e_{\varphi\theta} &= \frac{1}{2}\left(\frac{1}{a+r\sin\varphi}\frac{\partial v}{\partial \theta} + \frac{1}{r}\frac{\partial w}{\partial \varphi} - \frac{w\cos\varphi}{a+r\sin\varphi}\right) \\
e_{r\theta} &= \frac{1}{2}\left(\frac{1}{a+r\sin\varphi}\frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w\sin\varphi}{a+r\sin\varphi}\right)
\end{aligned}
\right\} \qquad (15)
$$

Equilibrium Equations in Terms of displacements

Expressing the stresses in terms of displacements using Hooke's law, Eqs. (9) and (10), with the strain-displacement relations, Eqs. (14) and (15), the equilibrium equations, Eqs. (3)-(8), may be written in terms of displacements in the form

$$
\begin{aligned}
& A_k \frac{\partial^2 u}{\partial \alpha_1^2} + B_k \frac{\partial^2 u}{\partial \alpha_2^2} + C_k \frac{\partial^2 u}{\partial \alpha_3^2} + D_k \frac{\partial^2 u}{\partial \alpha_1 \partial \alpha_2} + E_k \frac{\partial^2 u}{\partial \alpha_2 \partial \alpha_3} \\
& + F_k \frac{\partial^2 u}{\partial \alpha_1 \partial \alpha_3} + G_k \frac{\partial u}{\partial \alpha_1} + H_k \frac{\partial u}{\partial \alpha_2} + I_k \frac{\partial u}{\partial \alpha_3} + J_k u \\
& + \bar{A}_k \frac{\partial^2 v}{\partial \alpha_1^2} + \bar{B}_k \frac{\partial^2 v}{\partial \alpha_2^2} + \bar{C}_k \frac{\partial^2 v}{\partial \alpha_3^2} + \bar{D}_k \frac{\partial^2 v}{\partial \alpha_1 \partial \alpha_2} + \bar{E}_k \frac{\partial^2 v}{\partial \alpha_2 \partial \alpha_3} \\
& + \bar{F}_k \frac{\partial^2 v}{\partial \alpha_1 \partial \alpha_3} + \bar{G}_k \frac{\partial v}{\partial \alpha_1} + \bar{H}_k \frac{\partial v}{\partial \alpha_2} + \bar{I}_k \frac{\partial v}{\partial \alpha_3} + \bar{J}_k v \\
& + \bar{\bar{A}}_k \frac{\partial^2 w}{\partial \alpha_1^2} + \bar{\bar{B}}_k \frac{\partial^2 w}{\partial \alpha_2^2} + \bar{\bar{C}}_k \frac{\partial^2 w}{\partial \alpha_3^2} + \bar{\bar{D}}_k \frac{\partial^2 w}{\partial \alpha_1 \partial \alpha_2} + \bar{\bar{E}}_k \frac{\partial^2 w}{\partial \alpha_2 \partial \alpha_3} \\
& + \bar{\bar{F}}_k \frac{\partial^2 w}{\partial \alpha_1 \partial \alpha_3} + \bar{\bar{G}}_k \frac{\partial w}{\partial \alpha_1} + \bar{\bar{H}}_k \frac{\partial w}{\partial \alpha_2} + \bar{\bar{I}}_k \frac{\partial w}{\partial \alpha_3} + \bar{\bar{J}}_k w = \\
& \qquad \frac{(3\lambda + 2\mu)\alpha(T)}{\sqrt{g_{kk}}} \frac{\partial T}{\partial \alpha_k}, \qquad k = 1, 2, 3
\end{aligned}
\qquad (16)
$$

COEFFICIENTS OF EQUILIBRIUM EQUATIONS

SPHERICAL COORDINATES

	$k=1$	$k=2$	$k=3$
A_k	$\lambda+2\mu$	0	0
B_k	μ/R^2	0	0
C_k	$\mu/R^2\sin^2\varphi$	0	0
D_k	0	$(\lambda+\mu)/R$	0
E_k	0	0	0
F_k	0	0	$(\lambda+\mu)/R\sin\varphi$
G_k	$2(\lambda+2\mu)/R$	0	0
H_k	$\mu\cot\varphi/R^2$	$2(\lambda+2\mu)/R^2$	0
I_k	0	0	$2(\lambda+2\mu)/R^2\sin\varphi$
J_k	$-2(\lambda+2\mu)/R^2$	0	0
$\bar{A}_k$	0	μ	0
$\bar{B}_k$	0	$(\lambda+2\mu)/R^2$	0
$\bar{C}_k$	0	$\mu/R^2\sin^2\varphi$	0
$\bar{D}_k$	$(\lambda+\mu)/R$	0	0
$\bar{E}_k$	0	0	$(\lambda+\mu)/R^2\sin\varphi$
$\bar{F}_k$	0	0	0
$\bar{G}_k$	$(\lambda+\mu)\cot\varphi/R$	$2\mu/R$	0
$\bar{H}_k$	$-(\lambda+3\mu)/R^2$	$(\lambda+2\mu)\cot\varphi/R^2$	0
$\bar{I}_k$	0	0	$(\lambda+3\mu)\cot\varphi/R^2\sin\varphi$
$\bar{J}_k$	$-(\lambda+3\mu)\cot\varphi/R^2$	$-(\lambda+2\mu)/R^2\sin^2\varphi$	0
$\bar{\bar{A}}_k$	0	0	μ
$\bar{\bar{B}}_k$	0	0	μ/R^2
$\bar{\bar{C}}_k$	0	0	$(\lambda+2\mu)/R^2\sin^2\varphi$
$\bar{\bar{D}}_k$	0	0	0
$\bar{\bar{E}}_k$	0	$(\lambda+\mu)/R^2\sin\varphi$	0
$\bar{\bar{F}}_k$	$(\lambda+\mu)/R\sin\varphi$	0	0
$\bar{\bar{G}}_k$	0	0	$2\mu/R$
$\bar{\bar{H}}_k$	0	0	$\mu\cot\varphi/R^2$
$\bar{\bar{I}}_k$	$-(\lambda+3\mu)/R^2\sin\varphi$	$-(\lambda+3\mu)\cot\varphi/R^2\sin\varphi$	0
$\bar{\bar{J}}_k$	0	0	$-\mu/R^2\sin^2\varphi$

COEFFICIENTS OF SECOND ORDER EQUATIONS

TOROIDAL COORDINATES

	$k=1$	$k=2$	$k=3$
A_k	$\lambda+2\mu$	0	0
B_k	μ/r^2	0	0
C_k	$\mu/(a+r\sin\varphi)^2$	0	0
D_k	0	$(\lambda+\mu)/r$	0
E_k	0	0	0
F_k	0	0	$(\lambda+\mu)/(a+r\sin\varphi)$
G_k	$(\lambda+2\mu)(a+2r\sin\varphi)/r(a+r\sin\varphi)$	0	0
H_k	$\mu\cos\varphi/r(a+r\sin\varphi)$	$\dfrac{2(\lambda+2\mu)r\sin\varphi+(\lambda+3\mu)a}{r^2(a+r\sin\varphi)}$	0
I_k	0	0	$\dfrac{2(\lambda+2\mu)r\sin\varphi+(\lambda+\mu)a}{r(a+r\sin\varphi)^2}$
J_k	$-(\lambda+2\mu)\left[1/r^2+\sin^2\varphi/(a+r\sin\varphi)^2\right]$	$(\lambda+2\mu)a\cos\varphi/r(a+r\sin\varphi)^2$	0
$\bar{A}_k$	0	μ	0
$\bar{B}_k$	0	$(\lambda+2\mu)/r^2$	0
$\bar{C}_k$	0	$\mu/(a+r\sin\varphi)^2$	0
$\bar{D}_k$	$(\lambda+\mu)/r$	0	0
$\bar{E}_k$	0	0	$(\lambda+\mu)/r(a+r\sin\varphi)$
$\bar{F}_k$	0	0	0
$\bar{G}_k$	$(\lambda+\mu)\cos\varphi/(a+r\sin\varphi)$	$\mu(a+2r\sin\varphi)/r(a+r\sin\varphi)$	0
$\bar{H}_k$	$-(\lambda+3\mu)/r^2$	$(\lambda+2\mu)\cos\varphi/r(a+r\sin\varphi)$	0
$\bar{I}_k$	0	0	$(\lambda+3\mu)\cos\varphi/(a+r\sin\varphi)^2$
$\bar{J}_k$	$-\dfrac{(\lambda+3\mu)r\sin\varphi\cos\varphi+\mu a\cos\varphi}{r(a+r\sin\varphi)^2}$	$-\dfrac{(\lambda+2\mu)r^2+\mu a^2+(\lambda+3\mu)ar\sin\varphi}{r^2(a+r\sin\varphi)^2}$	0
$\bar{\bar{A}}_k$	0	0	μ
$\bar{\bar{B}}_k$	0	0	μ/r^2
$\bar{\bar{C}}_k$	0	0	$(\lambda+2\mu)/(a+r\sin\varphi)^2$
$\bar{\bar{D}}_k$	0	0	0
$\bar{\bar{E}}_k$	0	$(\lambda+\mu)/r(a+r\sin\varphi)$	0
$\bar{\bar{F}}_k$	$(\lambda+\mu)/(a+r\sin\varphi)$	0	0
$\bar{\bar{G}}_k$	0	0	$\mu(a+2r\sin\varphi)/r(a+r\sin\varphi)$
$\bar{\bar{H}}_k$	0	0	$\mu\cos\varphi/r(a+r\sin\varphi)$
$\bar{\bar{I}}_k$	$-(\lambda+3\mu)\sin\varphi/(a+r\sin\varphi)^2$	$-(\lambda+3\mu)\cos\varphi/(a+r\sin\varphi)^2$	0
$\bar{\bar{J}}_k$	0	0	$-\mu/(a+r\sin\varphi)^2$