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SATURN S-1C HIGH PRESSURE HELIUM STORAGE BOTTLES

FINAL DESIGN REPORT
CONTRACT NAS 8-5151

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I INTRODUCTION

This report has been prepared under Contract NAS 8-5151 in order to satisfy the requirements of paragraph 3.2.1.4 of NASA-SPEC-20M02008.

The evolution of the helium bottle design, from the original proposal to contract completion, is described in Chapter II.

Chapter III presents summary information on material properties and the qualification test results as well as stress analyses not previously submitted.

Chapter IV discusses the design reliability and presents an analysis based upon the test results.

Chapter V contains the Martin Company's conclusions and recommendations.

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II DESIGN DESCRIPTION

The purpose of this chapter is to briefly review the evolution of the helium bottle design from the time of the original proposal through the qualification program.

A. PROPOSED DESIGN

The helium bottle design, as proposed in response to the original NASA MSFC RFQ, is shown in Reference 1. It was proposed that the bottle be fabricated in one piece from a basic extruded tube. The major characteristics of the design were:

Material:	2219 aluminum alloy
Inside Diameter:	20.26 inches
Wall Thickness:	1.03 inches
Overall Length:	211.13 inches
Weight:	1425 pounds
Internal Volume:	35 cubic feet
End Configuration:	MS 240-24 bosses
	Machined grooves

According to vendors contacted during the preparation of the proposal, the largest extruded tube they could supply would have an inside diameter of 19.5 inches and a wall thickness of 1.125 inches. The Martin Company proposed, therefore, to forge the bottle ends, heat treat the bottle, expand the bottle, prior to aging, by hydraulic bulging in a female die. In addition, the Martin Company suggested in Reference 1 that, since no welding would be required, the higher strength 2014 alloy be used in lieu of 2219.

B. PRELIMINARY APPROVED DESIGN

Shortly after contract award NASA directed that the following design changes be incorporated: change material to 2014, reduce the internal volume to 31.0 cubic feet minimum, 31.5 cubic feet maximum, increase the overall length 0.75 inches, reduce the weight to 1200 pounds maximum.

The preliminary design submitted for approval is shown in Reference 2. The major characteristics of the design were:

Material:	2014 aluminum alloy
Inside Diameter:	19.10 inches
Wall Thickness:	0.90 inches
Overall Length:	211.88 inches
Weight:	1144 pounds
Internal Volume:	31.2 cubic feet
End Configuration:	MC 240-24 [illegible]
	Machined grooves

The design was approved and fabrication of the qualification units was initiated.

C. DESIGN CHANGES

Fabrication of the qualification units was initiated by release of Martin drawing 88-4000500 in November of 1962. During the course of the program, several design changes were incorporated and these are discussed herein.

1. DCN A, February 1963

This revision added product identification data supplied by NASA and changed the protective finish requirement. The original NASA requirement was for sulphuric acid anodizing of both the interior and exterior bottle surfaces. The Martin Company suggested that a chemical conversion coating would be adequate for the interior surface and would reduce the cost. NASA authorized this change and it was incorporated by DCN A and made effective for all units.

2. DCN B, May 1963

NASA directed that the end configuration of the bottle be changed to eliminate the machined grooves and replace them by external threads. This change was incorporated by DCN B and made effective for all units.

3. DCN C, July 1963

NASA directed that the following operations be added to the fabrication process:

a) Change heat treating procedure to add an internal quench simultaneous with the external quench.

b) Add a water solubilized penetrant inspection after heat treat.

c) Add a cleaning operation after penetrant inspection.

d) Add a shot peening operation to be performed on the interior surface after penetrant inspection.

The foregoing additions were made in order to reduce the possibility of stress corrosion and increase the mechanical properties. In order to accomplish the internal quenching, the Martin Company requested and received permission to change the boss in the top end of the bottle from MC 240-24 to MC 240-28.

The other item changed by this DCN was to allow sealing of the anodic coating by using hot water without dichromate. This change was requested in order to avoid the additional expense of a special sealing tank since the anodizing vendor selected did not normally use dichromate. The request was justified on the basis of that panels which had satisfactorily met the corrosion protection requirements.

All of the foregoing changes were incorporated by DCN C and made effective for units 4 and up.

Units 1 through 5 were subjected to qualification testing during the period June 1963 through May 1964. Units 6 through 10 were delivered to NASA-MSFC during November and December 1963.

D. FINAL DESIGN

No bottle failures or design deficiencies were evidenced during the qualification test program. Each bottle passed each test to which it as subjected and therefore no design changes were incorporated. At the conclusion

of the test program drawing 88-4000500 was redrawn on mylar and several additional notes were added to improve the overall documentation for production purposes. This drawing which carries a change letter D and was released in June 1964.

III DESIGN ANALYSIS

The purpose of the preliminary design report, Reference 2, was to provide analytical justification for the design submitted for approval in order to proceed with the fabrication of qualification test units. Reference 2 presented the calculated stress levels for the various test environments and was based upon the expected material mechanical properties. A high probability of survival of the test program was predicted. It is the purpose of this Chapter to update the preliminary report to include the data obtained and additional analyses conducted during the course of the program.

A. Material Properties

The mechanical properties of the extrusions used for the fabrication of the development and qualification units were determined and reported in Reference 3. The values obtained met or exceeded expectations. The transverse tensile strength values are presented, for convenience, herein as Table 1. Since all of the values reported in Reference 3 were from material heat treated by the extrusion fabricator, the question remained as to the effect of the subsequent machining, forging and robert treating operations.

TABLE 1: HELIUM BOTTLE EXTRUSIONS
TRANSVERSE TENSILE STRENGTH

	Temp. (°F)	VALUES IN KSI				
		Max.	Min.	Mean	Std. Dev.	99%
YIELD	250	61.4	56.0	58.6	1.690	54.6
	RT	66.9	62.8	64.4	1.230	61.5
	-320	78.3	73.4	75.7	1.692	71.8
ULTIMATE	250	64.2	57.8	61.0	2.090	56.1
	RT	72.4	69.0	70.8	1.088	68.4
	-320	84.5	80.1	81.9	1.162	79.3

B. Qualifications Test Results

The results of the qualification test program have been reported separately in Reference 4. Qualitatively the pertinent results were:

1. All units passed all tests to which they were subjected
2. No evidence of excessive distortion or permanent deformation was detected at pressure levels up to and including proof (5000 psi)
3. Based upon the pressure-time histories recorded during the burst tests, it is estimated that yield occurred at a pressure of 6000 psi or greater
4. During the burst tests all of the significant plastic deformation took place approximately in the central three to four feet of the cylindrical section of the bottles
5. All of the burst test fractures were ductile and originated in the center of the cylindrical section, the fracture paths were quite similar among the four units burst

Quantitatively the pertinent results are the pressures at rupture recorded during the cryogenic burst tests and presented in Table 2.

TABLE 2: HELIUM BOTTLE BURST TEST RESULTS (-320°F)

Unit No.	S/N	Burst Press. (PSI)
1	0002	6960
3	0004	7040
4	0005	6890
5	0009	7120

C. Failure Analysis

1. Basis for the Analysis

The basic design requirements for the helium bottle are that in a liquid oxygen environment no yield occur at 5000 psi and no rupture

occur at 6660 PSI. Based upon the preliminary analysis contained in Reference 2, it was shown that the rupture requirement was the more critical. The material properties reported in Reference 3 reinforce this conclusion. It is important, therefore, to predict failure (rupture) as accurately as possible. A thorough study of Reference 5 was made, from which it was evident that the factors influencing the selection of a particular rupture formula would be:

a. the geometry of the vessel

b. the stress distribution in the plastic range

c. the plastic stress-strain characteristics of the material

d. The criterion for the initiation of plastic deformation

Analysis of the vessel geometry and the stress-strain curves reported in Reference 3 led to the following conclusions:

a. the length-to-diameter ratio was sufficiently large so that the vessel could be treated as an infinitely long cylinder

b. the diameter-to-thickness ratio was small enough to warrant consideration of the variation of the stresses through the thickness

c. the uniaxial ultimate strain at -320°F was small enough to permit the assumption that the elastic and plastic stress distributions do not differ radically

d. the ratio of ultimate-to-yield strength and the stress-strain curves at -320°F permit the assumption of an ideal elastic-plastic material

The remaining factor to be considered was the criterion for the initiation of plastic deformation. The Tresca or maximum shear criterion was selected because it is the simpler and more conservative of several

available. The foregoing arguments lead one to the conclusion that equation 20 of Reference 5 is the one best suited for predicting rupture of the helium bottles. According to Reference 5, equation 20 was first published by L. B. Turner in 1910 in the Transactions of the Cambridge Philosophical Society. Because this source is not too readily available a rederivation of Turner's result will be presented herein.

2. Derivation of the Burst Pressure Formula

Consider a thick-wall cylinder, of inside radius a and outside radius b, subjected to an internal pressure p. From Reference 6, Table XIII case 29, the principal stresses are:

TANGENTIAL: $$\sigma_\theta = p\,\frac{a^2\,(b^2+r^2)}{r^2\,(b^2-a^2)} \qquad 1)$$

AXIAL: $$\sigma_x = p\,\frac{a^2}{(b^2-a^2)} \qquad 2)$$

RADIAL: $$\sigma_r = -p\,\frac{a^2\,(b^2-r^2)}{r^2\,(b^2-a^2)} \qquad 3)$$

where $$a \le r \le b$$

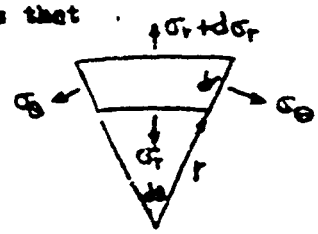
Employing the Tresca criterion, the differences $\sigma_\theta - \sigma_r$, $\sigma_x - \sigma_r$, $\sigma_\theta - \sigma_x$ are examined and plastic deformation starts when the algebraically largest of the three differences reaches a critical value say Y. The critical quantity is

$$\sigma_\theta - \sigma_r = p\,\frac{2a^2b^2}{b^2-a^2}\,\frac{1}{r^2} \qquad 4)$$

and this quantity is largest at $r = a$. Therefore plastic deformation initiates at the inside surface of the cylinder when

$$Y = p\,\frac{2b^2}{b^2-a^2} \qquad 5)$$

The elastic-plastic interface must now move outward from $r=a$ to some radius $r=d<b$. Referring to the sketch below, equilibrium in the r-direction requires that



$$(\sigma_r+d\sigma_r)(r+dr)d\theta - \sigma_r r d\theta - 2\sigma_\theta dr \frac{d\theta}{2} = 0$$

and, neglecting second order small quantities this reduces to

$$\sigma_r dr + r d\sigma_r - \sigma_\theta dr = 0$$

or

$$\frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} \qquad 6)$$

Now, since the zone from $r=a$ to $r=d$ is plastic, $\sigma_\theta - \sigma_r = Y$, a constant. Therefore, equation 6) can be integrated to give

$$\sigma_r = Y \log r + c$$

and the constant of integration, c, is evaluated from the condition at $r=a$.

$$c = -p - Y \log a$$

and so

$$\sigma_r = Y \log \frac{r}{a} - p \qquad 7)$$

The zone between $r=d$ and $r=b$ is elastic except at the interface $r=d$ where equations 3), 4) and 7) require that

$$\sigma_d = -p \frac{a^2}{d^2} \frac{b^2-d^2}{b^2-a^2}$$

$$Y = p \frac{2a^2b^2}{b^2-a^2} \frac{1}{d^2}$$

$$\sigma_d = Y \log \frac{d}{a} - p$$

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Eliminating σ_d

$$Y = -\sigma_d \frac{2b^2}{b^2-d^2} = -\left(Y \log\frac{d}{a} - p\right)\frac{2b^2}{b^2-d^2}$$

and dividing through by Y and solving for $\frac{p}{Y}$

$$\frac{p}{Y} = \log\frac{d}{a} + \frac{1}{2}\left(1 - \frac{d^2}{b^2}\right)$$

When the plastic interface reaches the outside of the cylinder (d=b), we have

$$p = Y \log\frac{b}{a} \qquad 8)$$

Since we have assumed an ideal elastic-plastic material

$$\sigma_y = \sigma_u = Y$$

and the maximum pressure capability of the cylinder, in other words, the burst pressure, is

$$p_B = \sigma_u \log\frac{b}{a} \qquad 9)$$

where σ_u is the minimum ultimate tensile strength of the material.

3. Comparison of Actual and Predicted Burst Pressures

Assuming that the helium bottles are fabricated within the tolerances specified, the inside diameter will be within the range 19.10 - 19.12 inches and the thickness will be within the range 0.90 - 0.89 inches. Since

$$b = a + h$$

$$\frac{b}{a} = 1 + \frac{h}{a}$$

and the least value of b/a occurs when

$$h = .89$$

$$a = 9.56$$

and so

$$\frac{b}{a} = 1.093096$$

and therefore

$$P_B = .08902\,\sigma_u$$

The minimum ultimate tensile strength of each of the extrusions occurs in the transverse grain direction and the values are recorded in Reference 3. These values, along with the calculated and actual burst

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are presented in Table 3.

TABLE 3: COMPARISON OF CALCULATED AND ACTUAL BURST PRESSURES

Unit No.	S/N	σ_u (PSI)	P_n(PSI) CALC.	P_n(PSI) ACTUAL	ACTUAL / CALC.
1	0002	80,200	7140	6960	.975
3	0004	80,500	7160	7040	.983
4	0005	80,100	7130	6890	.966
5	0009	82,500	7340	7120	.970
				AVERAGE	.974

The results shown in Table 3 indicate that the model selected for the failure analysis is sound and that the material used and fabrication controls imposed will result in vessels of consistent strength. Units 4 and 5 incorporated DCN 6, which added internal quenching and shot peening, while units 1 and 3 did not. The internal quenching would, if anything, tend to increase the basic mechanical properties while the shot peening, because it adds residual compressive stresses, would tend to decrease the burst strength. One could conclude from the data in Table 3 that the net effect was a decrease in strength but the effect is small in any event. Another interpretation is that, regardless of any possible effects due to shot peening and internal quenching, the forging and reheat treatment restored 96.6% to 98.3% of the original material strength. Stated another way, the average of four burst tests indicates that the conversion of the basic extrusions into finished pressure vessels results in a loss of basic material strength of only 2.6%.

As pointed out in Reference 4, units 3 and 4 had small local areas which were under the minimum wall thickness of 0.89 inches. Unit 3 had a one square inch area which was 0.87 inches thick and unit 4 had a 16 square inch area 0.88 inches thick. These local thickness deviations did not affect the burst strength.

D. YIELD ANALYSIS

In order to compute the value of the internal pressure at which yielding will initiate, it is necessary to know the distribution of elastic stresses along the length, and throughout the thickness of the helium bottle. Knowing the elastic stress distribution, it is then a matter of selecting and applying a yield criterion. The elastic stresses consist of two parts; the membrane stresses and the discontinuity stresses arising from changes in the geometry of the vessel. For analysis purposes, the geometry of the vessel is as shown in Figure 1.

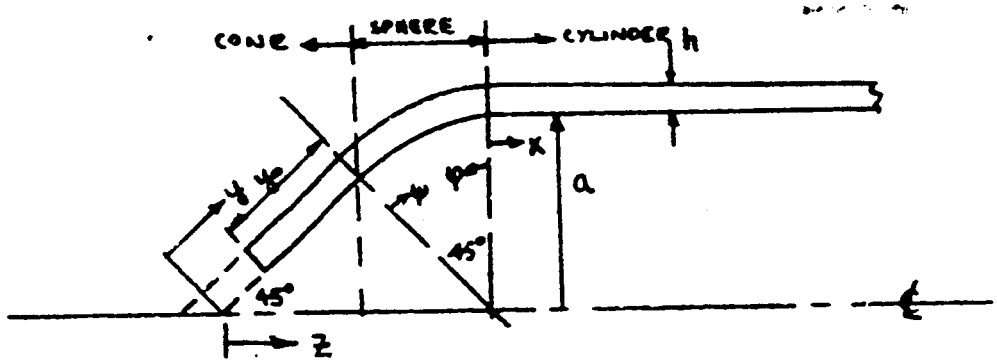


Fig. 1: Vessel Geometry for Analysis Purposes

1. DISCONTINUITY STRESSES

The discontinuity stresses arising from the cylinder-sphere junction and the cone-sphere junction can be analyzed using the equations presented in Reference 7. The complete analysis carried out during the course of the program showed that yielding initiates at the inner surface. Therefore, the equations presented herein, are for the inner surface only.

a) CYLINDER-SPHERE JUNCTION

For the cylinder

$$\sigma_{x_D} = \frac{6}{\beta h^2} H_o \Omega(\beta x)$$

$$\sigma_{\theta_D} = \left[2\beta \frac{a}{h} \theta(\beta x) + \frac{6\nu}{\beta h^2} \Omega(\beta x) \right] H_o$$

For the sphere

$$\sigma_{\varphi_0} = -\left[\frac{1}{h}\tan\varphi\,\Psi(\lambda\varphi) + \frac{6a}{\lambda h^2}\,\Omega(\lambda\varphi)\right]H_0$$

$$\sigma_{\theta_0} = -\left[\frac{2\lambda}{h}\,\Theta(\lambda\varphi) + \frac{6\nu a}{\lambda h^2}\,\Omega(\lambda\varphi)\right]H_0$$

where

$$H_0 = -\frac{pa}{8\lambda}$$

$$\lambda^4 = 3(1-\nu^2)\frac{a^2}{h^2}$$

$$\beta^4 = \frac{3(1-\nu^2)}{a^2 h^2}$$

$$\Theta(\,) = e^{-(\,)}\cos(\,)$$

$$\Omega(\,) = e^{-(\,)}\sin(\,)$$

$$\Psi(\,) = e^{-(\,)}\left[\cos(\,) - \sin(\,)\right]$$

ν) = POISSON'S RATIO

The coordinates x and y are identified in Figure 1 and the functions Θ, Ω, Ψ are tabulated on page 356 of Reference 8.

b) CONE-SPHERE JUNCTION

For the cone

$$\sigma_{y_0} = \frac{1}{hy}\left\{C_1\left[\mathrm{ber}_2\xi - \frac{3}{m^2}\left(\xi\,\mathrm{bei}'_2\xi + 2\nu\,\mathrm{bei}_2\xi\right)\right] + C_2\left[\mathrm{bei}_2\xi + \frac{3}{m^2}\left(\xi\,\mathrm{ber}'_2\xi + 2\nu\,\mathrm{ber}_2\xi\right)\right]\right\} + \frac{3p}{4(1-\nu)} + \frac{pt}{2h}\left(1-\tan\alpha\right)$$

$$\sigma_{\theta_D} = \frac{1}{hy}\left\{ C_1\left[\tfrac{1}{2}\xi\, ber_2'\xi - \frac{3}{m^2}\left(2\, bei_2\xi + \nu\xi\, bei_2'\xi\right)\right] + C_2\left[\tfrac{1}{2}\xi\, bei_2'\xi + \frac{3}{m^2}\left(2\, ber_2\xi + \nu\xi\, ber_2'\xi\right)\right]\right\} + \frac{3p}{4(1-\nu)} + \frac{p\psi}{h}\left(1 - \tan\alpha\right)$$

For the sphere

$$\sigma_{\varphi_D} = \left[\frac{2\lambda}{ah}\left(\frac{1-\tan\psi}{1+\tan\psi}\right)\Omega(\lambda\psi) + \frac{6}{h^2}\,\phi(\lambda\psi)\right]M_\alpha + \left[\frac{1}{h}\left(\frac{1-\tan\psi}{1+\tan\psi}\right)\Psi(\lambda\psi) - \frac{6a}{\lambda h^2}\,\Omega(\lambda\psi)\right]\left[\frac{H_\alpha}{\sqrt{2}} - \frac{pa}{4}\right]$$

$$\sigma_{\theta_D} = \left[\frac{2\lambda^2}{ah}\,\Psi(\lambda\psi) + \frac{6\nu}{h^2}\,\phi(\lambda\psi)\right]M_\alpha - \left[\frac{2\lambda}{h}\,\Theta(\lambda\psi) + \frac{6\nu a}{\lambda h^2}\,\Omega(\lambda\psi)\right]\left[\frac{H_\alpha}{\sqrt{2}} - \frac{pa}{4}\right]$$

where

$$C_1 = \left\{\left(H_\alpha y_0 h \sin\alpha - \tfrac{1}{2}ph\, y_0^2 \sin^2\alpha \tan\alpha\right)\left(\xi_0\, ber_2'\xi_0 + 2\nu\, ber_2\xi_0\right) - 2m^2 y_0\, bei_2\xi_0\left[H_\alpha - \frac{ph^2\tan^2\alpha}{8(1-\nu)}\right]\right\} \div h\left(C + 2\nu G\right)$$

$$C_2 = \left\{\left(H_\alpha y_0 h \sin\alpha - \tfrac{1}{2}ph^2 y_0^2 \sin^2\alpha \tan\alpha\right)\left(\xi_0\, bei_2'\xi_0 + 2\nu\, bei_2\xi_0\right) + 2m^2 y_0\, ber_2\xi_0\left[H_\alpha - \frac{ph^2\tan^2\alpha}{8(1-\nu)}\right]\right\} \div h\left(C + 2\nu G\right)$$

$$C = \xi_0\left(ber_2\xi_0\, ber_2'\xi_0 + bei_2\xi_0\, bei_2'\xi_0\right)$$

$$G = (\mathrm{ber}_2\,\xi_0)^2 + (\mathrm{bei}_2\,\xi_0)^2$$

$$H_\alpha = \left(\sin\alpha - \frac{1}{4\lambda\cos\alpha}\right)\frac{pa}{2}$$

$$M_\alpha = -\frac{(2-\nu)\sin\alpha\cos\alpha}{8\lambda^3}\,pa^2$$

$$m^4 = 12(1-\nu^2)$$

$$\xi = 2\mu\sqrt{y}$$

$$\mu^4 = \frac{12(1-\nu^2)}{h^2\,\mathrm{Tan}^2\alpha}$$

$$\phi(\;) = e^{-(\;)}\left[\cos(\;) + \sin(\;)\right]$$

The coordinates y and ψ are identified in Figure 1 and α is the half-angle of the cone. The function ϕ is tabulated on page 394 of Reference 6. The functions ber_2 and bei_2 are the Bessel-Kelvin functions of order two and the prime denotes differentiation with respect to ξ. In order to use these functions they must first be transformed to functions of zero order. From Reference 9

$$\mathrm{ber}_2\,\xi = \frac{2}{\xi}\,\mathrm{bei}'\,\xi - \mathrm{ber}\,\xi$$

$$\mathrm{bei}_2\,\xi = -\left(\frac{2}{\xi}\,\mathrm{ber}'\,\xi + \mathrm{bei}\,\xi\right)$$

$$\mathrm{ber}_2'\,\xi = -\left(\mathrm{ber}'\,\xi + \frac{2}{\xi}\,\mathrm{ber}_2\,\xi\right)$$

$$\mathrm{bei}_2'\,\xi = -\left(\mathrm{bei}'\,\xi + \frac{2}{\xi}\,\mathrm{bei}_2\,\xi\right)$$

and now the tables of Reference 10 can be used.

Calculations have been carried out for

$a = 9.55$ inches

$h = 0.90$ inches

$\nu = 0.30$

$\alpha = 45^\circ$

and the results are presented in Figure 2.

2. MEMBRANE STRESSES

The most accurate calculation of the membrane stresses at the inner surface can be made using the Lame' thick wall equations for the cylinder and the sphere. Unfortunately, comparable equations for the cone are not available but, as it turns out, the cone is not critical. Thin wall equations are used for the cone. The necessary equations are given in Reference 6, Table XIII.

For the cylinder

$$\frac{\sigma_{\theta M}}{p} = \frac{b^2 + a^2}{b^2 - a^2} \qquad b = a + h$$

$$\frac{\sigma_{xM}}{p} = \frac{a^2}{b^2 - a^2}$$

$$\frac{\sigma_{rM}}{p} = -1$$

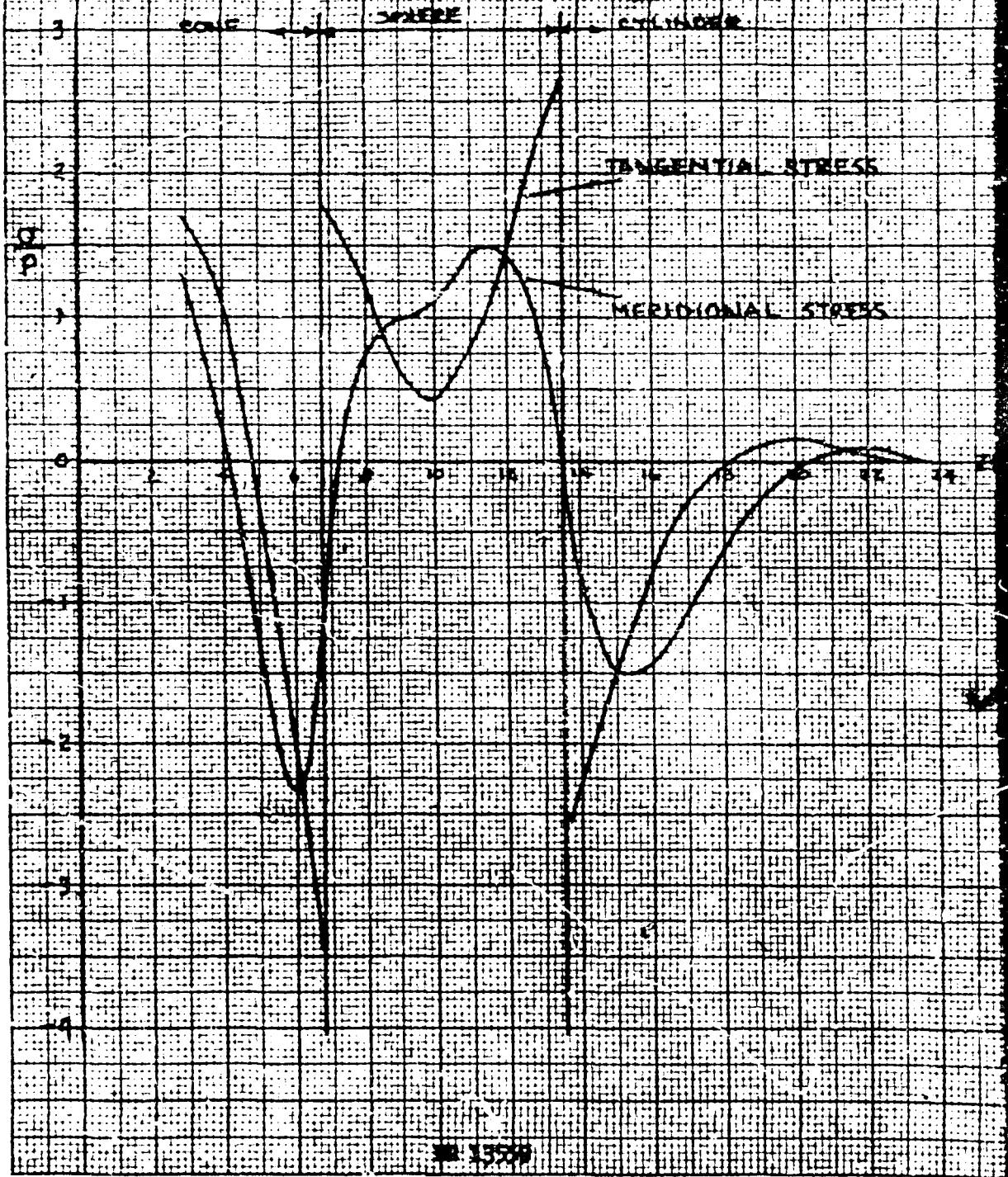
For the sphere

$$\frac{\sigma_{\theta M}}{p} = \frac{1}{2}\,\frac{b^3 + 2a^3}{b^3 - a^3}$$

$$\frac{\sigma_{\varphi M}}{p} = \frac{\sigma_{\theta M}}{p}$$

$$\frac{\sigma_{rM}}{p} = -1$$

FIG 2: DISCONTINUITY STRESSES



For the cone

$$\frac{\sigma_{\theta M}}{p} = \frac{Z \tan\alpha}{h \cos\alpha}$$

$$\frac{\sigma_{yM}}{p} = \frac{1}{2}\frac{\sigma_{\theta M}}{p}$$

$$\frac{\sigma_{rM}}{p} = 0$$

These stresses have been computed for a = 9.55 inches, h = 0.90 inches and α = 45 degrees and are shown in Figure 3.

3. YIELD CRITERION

For ductile materials, a satisfactory criterion for the initiation of yielding is the well-known Von Mises hypothesis which states that yielding occurs when the strain energy causing distortion reaches a critical value. In terms of an effective stress computed from

$$2\sigma_e^2 = (\sigma_\theta - \sigma_r)^2 + (\sigma_x - \sigma_r)^2 + (\sigma_\theta - \sigma_x)^2$$

yielding occurs when the effective stress equals the yield stress for the material.

The total principal stresses are found by summing the membrane and discontinuity stresses. The effective stress can then be computed. The resulting effective stress is shown in Figure 4. Figure 4 shows that yielding will start at the inside surface at a point on the cylinder about six inches from the cylinder-sphere junction. Since, at this point

$$\frac{\sigma_e}{p} = 10.8$$

the pressure at which yielding will start is

$$p_y = \frac{\sigma_y}{10.8} = .0926\,\sigma_y$$

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FIG. 3: MEMBRANE STRESSES

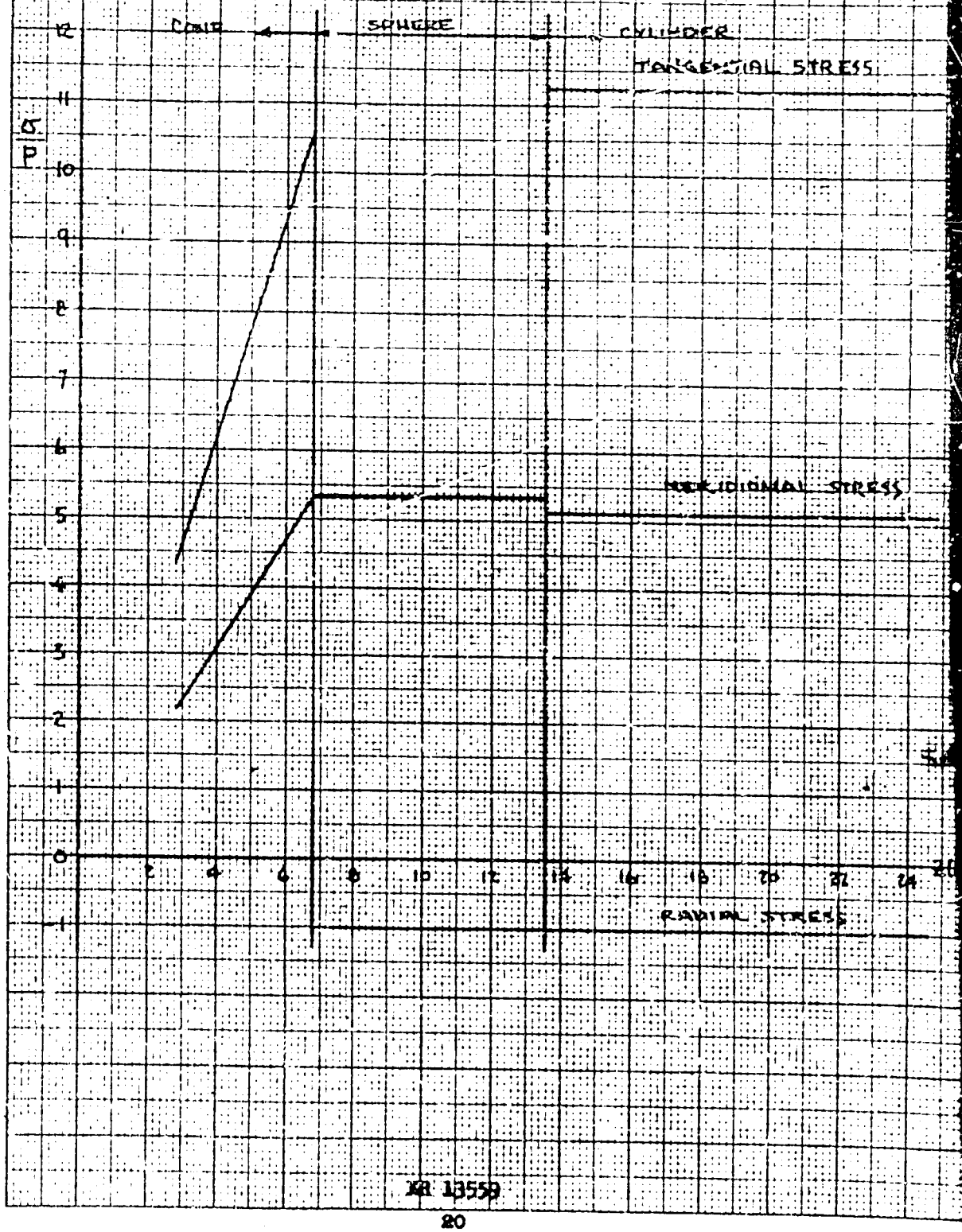

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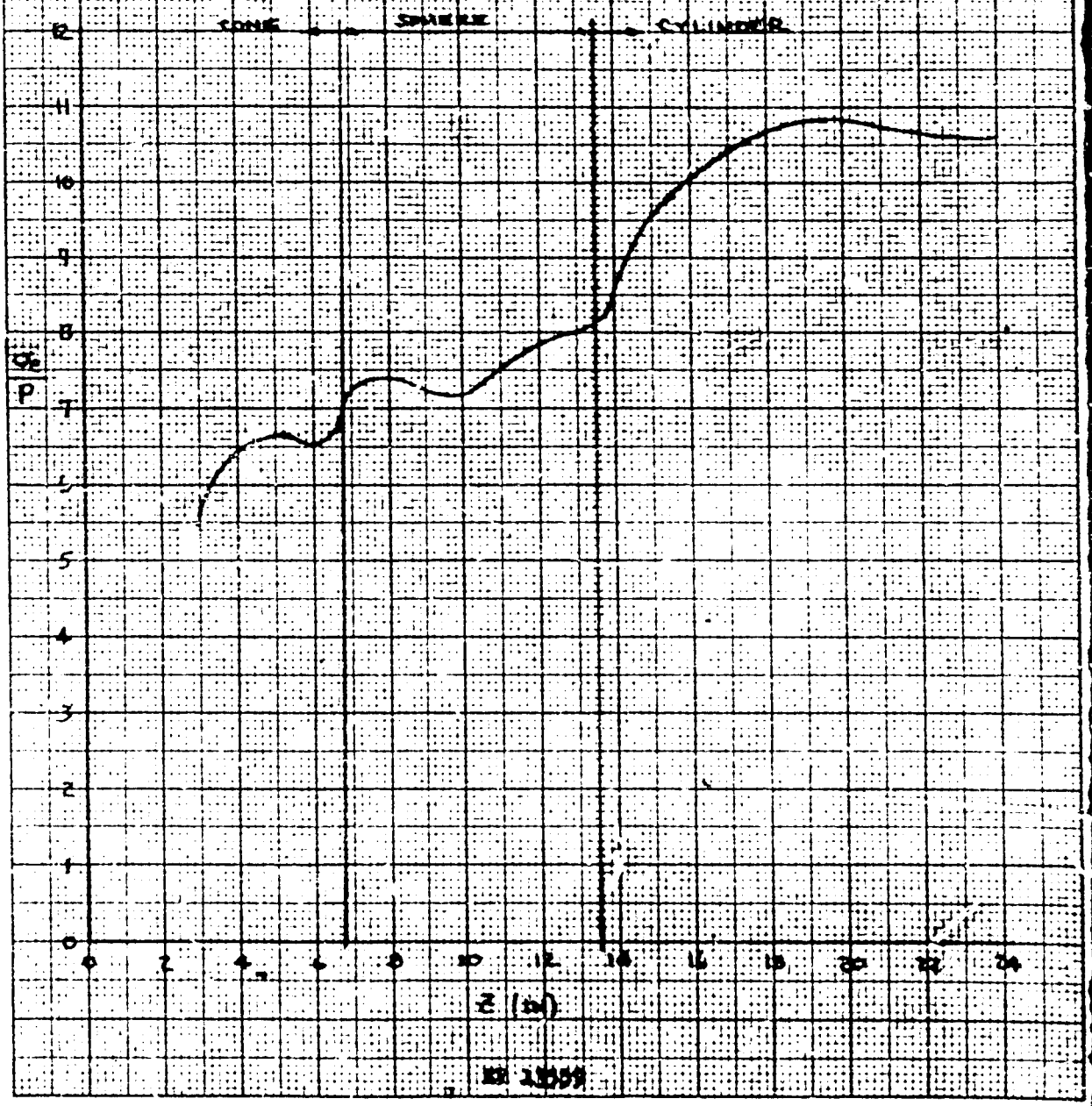
FIG. 4: EFFECTIVE STRESS BASED ON MAXIMUM DISTORTION ENERGY

$$p\,\sigma_e^2 = (\sigma_\theta - \sigma_r)^2 + (\sigma_x - \sigma_r)^2 + (\sigma_\theta - \sigma_x)^2$$

σ_θ – TANGENTIAL STRESS

σ_x – MERIDIONAL STRESS

σ_r – RADIAL STRESS



From Table 1, the minimum yield strength at -320°F is 73,400 psi and therefore the yield pressure is 6790 psi. Even allowing for some loss of strength due to reheat treatment area of the qualification units would be expected to yield at 5000 psi. This result was confirmed during the test program but the actual yield pressures were not determined.

E. END ATTACHMENT LOAD CAPABILITY

The vehicle installation of the helium bottle is such that all loads are taken out at the ends of the bottle. The upper fitting does not carry any load in the flight direction. In order to evaluate the installation from a structural standpoint, the yield and ultimate load capability of the lower end is calculated herein. Four separate conditions are considered; axial load (tension), lateral load (shear), bending, twisting (torsion).

1. SECTION PROPERTIES

The critical section is the 1/8 inch undercut at the end of the external threads. Taking the tolerances such that the minimum section results, we obtain the following section properties:

Cross-Sectional Area

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

$$d_o = 3.744 - 2(.125) = 3.494 \text{ IN.}$$

$$d_i = 1.63 \text{ IN.}$$

$$A = 7.50 \text{ IN.}^2$$

Bending Moment of Inertia

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right)$$

$$I = 6.97 \text{ IN}^4$$

Twisting Moment of Inertia

$$J = \frac{A}{2}\left(r_o^2 + r_i^2\right)$$

$$2r_o = d_o$$

$$2r_i = d_i$$

$$J = 13.94 \text{ IN.}^4$$

2. STRESS CONCENTRATION FACTORS

The appropriate stress concentration factors for a semicircular groove in a round shaft can be found in Reference 11.

Tension

From Figure 24 of Reference 11, for

$$\frac{D}{d} = \frac{3.744}{3.494} = 1.072$$

$$\frac{r}{d} = \frac{.125}{3.494} = 0.036$$

$$K_t = 2.64$$

Bending

From Figure 43 of Reference 11, for

$$\frac{r}{d} = 0.036$$

$$\frac{d}{D} = 0.933$$

$$K_{tb} = 2.30$$

Torsion

From Figure 47 of Reference 11, for

$$\frac{d}{D} = 0.933$$

$$\frac{r}{d} = 0.036$$

$$K_{ts} = 1.80$$

3. CALCULATED LOADS

The load and moment capacities are calculated from elementary formulas.

Axial Load, $P = \frac{\sigma_t A}{K_t}$, POUNDS

Shear Load, $V = \frac{\sigma_s A}{K_{ts}}$, POUNDS

Bending Moment, $M = \frac{2\sigma_t I}{d_o K_{tb}}$, INCH - POUNDS

Twisting Moment, $T = \frac{2\sigma_s J}{d_o K_{ts}}$, INCH - POUNDS

Both the yield and ultimate capabilities have been computed using the 99% strength values given in Table 1. The shear strength was assumed to be 60% of the tensile strength. The loads and moments are tabulated in Table 4.

TABLE 4: END ATTACHMENT LOAD CAPACITIES

LOAD	YIELD			ULTIMATE		
	+250°F	RT	-320°F	+250°F	RT	-320°F
P	155,100	174,700	205,900	159,300	194,300	225,200
V	—	—	—	140,500	170,000	198,500
M	94,500	112,600	124,200	97,100	118,300	137,200
T	—	—	—	149,600	182,000	211,300

IV RELIABILITY ANALYSIS

A reliability analysis was presented in the Preliminary Design Report, Reference 2. This analysis was based upon an estimate of the statistical parameters associated with the helium bottle extrusion mechanical properties. It was shown therein, that the probability of the bottle surviving the qualification test program was 0.9997. It is the purpose of this Chapter to use the actual material and test data generated to show the design margins of safety and the inherent design reliability.

A. Margins of Safety

The design yield and ultimate pressures are computed from the formulas developed in Chapter III and the design (99%) mechanical properties presented in Table 1.

$$P_y = .0926\,\sigma_y \qquad P_B = .08902\,\sigma_u$$

$$\sigma_y = 71{,}800 \text{ psi at } -320°F \qquad \sigma_u = 79{,}300 \text{ psi at } -320°F$$

$$P_y = 6649 \text{ psi} \qquad P_B = 7059 \text{ psi}$$

The allowable yield and ultimate pressures at -320°F are 5000 and 6660 psi respectively. The margins of safety are therefore

$$MS = \frac{[illegible]}{5000} - 1 = +.329 \quad \text{YIELD}$$

$$MS = \frac{7059}{6660} - 1 = +.0599 \quad \text{ULTIMATE}$$

Based upon the results of four burst tests (Table 2) the ultimate margin of safety ranged from +.0345 to +.0691.

E. Reliability

The basic reliability requirement as stated in paragraph 3.2.12 of NASA-SPEC-20M02006 is that the probability of the helium bottle surviving the operational environment shall be .99994. As will be demonstrated, the inherent reliability, based upon the qualification test results, far exceeds this requirement.

From the data in Table 2, the mean burst pressure is 7002 psi and the range of burst pressures for the four samples was 230 psi. For a small number of samples, such as four, the standard deviation can be computed using the method developed in Reference 12. The probability distribution for the least of n values drawn at random from a normal population must satisfy the equation

$$(n-1)\frac{dP}{dX} - PX = 0$$

where

n = number of samples

P = probability of exceeding a given value

X = the normal fraviate

$\frac{dP}{dX}$ = the normal frequency

This equation is solved by trial and error using standard probability tables. For

n = 4

P = .825

$\frac{dP}{dX}$ = .2977

X = .933

the equation is satisfied. The standard deviation of the burst pressures is therefore

$$\sigma = \frac{230}{2(.933)} = 123 \text{ psi}$$

Assuming that the four samples are representative of the population

mean burst pressure	=	7002	=	1.8000
proof pressure	=	5000	=	.7143
operating pressure	=	3000	=	.4286
standard deviation	=	139	=	.02

The probability of surviving the proof test, P_{SP}, is the probability corresponding to the normal deviate X_{SP} where

$$X_{SP} = \frac{1.8000-.7143}{.02} = 54.28 \text{ STANDARD DEVIATIONS}$$

P_{SP} is extremely high. The probability of surviving the operating pressure without a proof test is P_{SO} and P_{SO} corresponds to the normal deviate X_{SO} where

$$X_{SO} = \frac{1.8000-.4286}{.02} = 28.58 \text{ STANDARD DEVIATIONS}$$

The probability of surviving the operating pressure after having survived the proof pressure is much higher than P_{SO} because of the possible failures which are eliminated by the proof test. The required reliability (.99994) corresponds to a normal deviate of 3.22 standard deviations so it can be seen that the inherent reliability far exceeds the requirement.

The implication of the foregoing analysis is that the required reliability could be achieved with a smaller safety factor (lower burst pressure) and thus permit a smaller wall thickness and less weight. This can be accomplished because of the inherently small scatter in the actual as-fabricated strength.

V CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

1. The helium bottle design has been demonstrated to be adequate in all respects
2. The helium bottle fabrication process has been demonstrated to be adequate in all respects
3. The helium bottle inherent reliability far exceeds the design objective.

B. Recommendations

Based upon the analysis and data presented herein it is recommended that:

1. Preflight Certification of the helium bottle be granted
2. Consideration be given to a reduction in the specified proof and burst pressures thereby permitting a weight reduction at no sacrifice in the required reliability.

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