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DRIFT EFFECTS ON PARAMETERS OF THE UPPER F-REGION UNDER DAYTIME EQUILIBRIUM CONDITIONS

by

H. Gregory Tornatore

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IONOSPHERE RESEARCH LABORATORY

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Abstract

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A height independent drift which represents the motion due to electrodynamic forces is inserted into the transport term of the continuity equation for atomic oxygen ions in the upper F-region. Solutions in the form of exponential series are obtained under daytime equilibrium conditions. The factors of diffusion, attenuation of solar radiation, and lack of temperature equilibrium between ions and electrons are included in the solution.

Above the maximum the series converge very rapidly and are well approximated by the first few terms. In this way, simple solutions are developed which allow experimentally observed electron-or ion-density profiles to be related to the basic physical processes of production, loss, and transport of ionization.

Modified solutions to include the effects of helium-and hydrogen-ions on the electron density profile are developed.

Author

1. INTRODUCTION

1.1 Origin of the Problem

The electron density in the F-region of the daytime ionosphere is known to be controlled by several processes which may be listed as production of ionization by solar radiation, loss by recombination, and transport of ionization by electrodynamic forces and plasma diffusion.

At present, this region is undergoing intensive experimental study through the use of rockets, satellites, and incoherent back-scatter radar. Of particular interest are the "topside sounder" satellites which provide measurements of the electron density from the peak to the top of the layer.

It is the purpose of this study to relate these measurements to the basic physical processes of ion production, loss, and transport, and to the temperature and relative densities of the ionic constituents.

1.2 Previously Related Studies

1.2.1 Layer Formation

The fundamental equation which is used to include the production, loss, and transport of ionization is the continuity equation. As the various terms in this equation are quite complicated and not fully understood, no general solution has been found. Some understanding of the process of layer formation has, however, been obtained by simplified treatments restricted to special cases.

Chapman (1931 a,b) showed how a layer can be formed by the absorption of monochromatic radiation in an isothermal atmosphere of uniform

composition. Effects due to the transport of ionization were neglected and the loss rate (through recombination) was assumed to be quadratic in the electron density. The production was found to have a maximum at some height, and decreased monotonically above this height because of decreasing density of absorbing particles. It decreased monotonically below the height of the peak because of the decrease in unabsorbed radiation. The electron density would then roughly follow this curve as a function of altitude, with the maximum electron density occurring near the peak of production.

Chapman's theory of ionization production was found to be successful in explaining many features of the E-region, however it was unable to account for observed diurnal and seasonal variations, and the geographical distribution of the maximum electron density in the F-region.

Yonezawa (1956, 1958) obtained solutions to the continuity equation including the effects of drift. These solutions for a particular model atmosphere were obtained in terms of analytic functions. Solutions for more general cases of atmospheric conditions were obtained by the method of perturbation. In both cases, the solutions involved rather complex integrals which were evaluated numerically. Yonezawa found that the thickness of the F_2-layer was thinner than that predicted by Chapman's theory. When the effects of a vertical electron-ion drift were included it was found that both the maximum ionization and the height at which it occurs increase for upward drifts and decrease for downward drifts. This effect was found to be very pronounced at lower latitudes, becoming less so at middle and high latitudes.

Rishbeth and Barron (1960) and Gliddon and Kendall (1960) obtained numerical solutions to the continuity equation by means of digital computers. Rishbeth and Barron concluded that the magnitudes of the production, loss, and diffusion terms are approximately equal. The effect of a vertical drift velocity, assumed independent of height, on the maximum electron density and its level was found to be similar to the results of Yonezawa. The shape and thickness of the equilibrium layers were found to be very slightly altered by moderate drifts.

Gliddon and Kendall extended the analysis to non-equilibrium conditions. It was found that the electron density reached its peak slightly after noon and that in middle latitudes, the height of the electron density maximum increased from about 250 km. at noon to about 320 km. at midnight.

Bowhill (1962) simplified the analysis by neglecting attenuation of the ionizing radiation by the atmosphere. Analytic solutions to the continuity equation were obtained for equilibrium conditions in the absence of vertical drifts of ionization. Bowhill considered more general atmospheric models than those treated by Yonezawa. With regard to both the height and magnitude of the maximum electron density, the results were found to be consistent with those of Rishbeth and Barron and Yonezawa.

Nisbet (1963) obtained analytic solutions to the continuity equation in the upper F-region under daytime equilibrium conditions. The attenuation of ionizing radiation and a scale height for the ionic species different from that for the neutral constituent controlling

the diffusion were considered. Different scale heights for the molecular constituent responsible for the recombination, lack of temperature equilibrium between ions and electrons, and a non-divergent vertical ion drift were also taken into account. It was found that for increasing geographic latitude at a given time of the year, an increase in the neutral atmospheric temperature produced a decrease in layer thickness. The effect of the electron ion temperature ratio on the shape of the layer in the region of the maximum was found to be very small.

The height and density of the layer were found to decrease as latitude was increased, and the non-divergent vertical drift was found to have little effect on the density of the peak. In general, small upward drifts caused both the height and density of the peak to increase; however, for downward drifts of the same magnitude, while the density decreased only slightly, the decrease in the height of the layer maximum was much more pronounced.

1.2.2 Vertical Drifts

The concept of ionization transport was introduced by Martyn (1947 a,b) in an attempt to account for the peculiar structure of the F-region. At the present, it is generally felt that this motion is due to the diffusion of ionization under the influence of gravity and its own partial pressure gradient, and to electrodynamic forces and winds associated with the ionospheric current system. According to the theory proposed by Baker and Martyn (1953), currents in the lower ionosphere produce a polarization field which is communicated along the geomagnetic lines of force to the F-region where it produces vertical drifts of ionization.

Duncan (1956) showed that the divergence of the drift velocity resulting from this polarization field is small, so that the vertical drifts of ionization may be taken as independent of height.

Chandra, Gibbons, and Schmerling (1960) calculated the velocity by integrating the continuity equation from a reference level up to a level at which the drift was to be determined. A velocity which included the effects of diffusion, was found to be of the order of magnitude of 25 meters/second.

1.3 Statement of the Problem

In this work a velocity which represents the drift due to electrodynamic forces is included in the transport term of the continuity equation. This velocity is taken to be constant with height. Expressions for the production, loss, and diffusion terms are derived. Analytical solutions are obtained under daytime equilibrium conditions. These solutions are in the form of exponential series.

The effect of a height-independent vertical drift of ionization on the electron density profile in the upper F-region is determined. The variation in thickness and shape of the layer is examined by means of the atmospheric parameter known as SCAT. At higher altitudes, helium and hydrogen ions become important. Modified equations to include their effect on the electron density profile are derived.

2. THEORETICAL DEVELOPMENT OF THE PROBLEM

2.1 Region Where Atomic Oxygen Ions Predominate

In accordance with present day ideas, atomic oxygen, O, is the major ionized constituent in the F-region of the ionosphere. This atom is generally thought to be ionized by a quantum of solar radiation, $h\nu$, in the extreme ultra-violet range according to the reaction

$$O + h\nu \rightarrow O^{+} + e \qquad (2.01)$$

thus producing a free electron and an atomic oxygen ion. Rocket measurements using ion mass-spectrometers have well established that O^{+} ions predominate in the height range above 200 km. Johnson, Meadows, and Holmes (1958) measured the ion composition of the upper atmosphere over Fort Churchill, Canada. Their measurements revealed the presence of positive ions of 32, 30, and 16 atomic mass units corresponding to O_2^{+}, NO^{+}, and O^{+} respectively. The height distribution of these ions was determined in three separate experiments and it was found that the predominate ion was O^{+} near the peak of the layer.

2.2 Continuity Equation

The continuity equation for atomic oxygen ions in the F-region during the day may be written

$$\frac{\partial n_i}{\partial t} = Q_z - L_z - \text{div } (n_i v) \qquad (2.02)$$

where n_i is the ion density, Q_z is the rate of ion production at height z (production function), L_z is the rate of ion recombination at height z (loss function), and the term div $(n_i v)$ takes into account the outward

flux of ionization with a vertical velocity, v, and z is the height measured from the ion density maximum.

2.2.1 Production Term

The production term in this work is the same as that of Nisbet (1963) in which the non-monochromaticity of the radiation responsible for the ionization of atomic oxygen in the F-region was taken into account. The independence of the photon absorption rates and the wavelength is also considered.

Under these conditions, the production at height z may be represented by

$$Q_z = N(O)_z \; p_z \tag{2.03}$$

where N(O) is the atomic oxygen density and p is some factor that depends on the solar flux components and the appropriate cross-sections of atomic oxygen. Since some of the radiation entering the ionosphere will be absorbed, p will decrease with decreasing altitude.

Nisbet (1963) found that above approximately 150 km., experimental data could be fitted adequately to theoretical profiles if

$$p_z = p_\infty \left\{1-\kappa \sec \psi \int_z^\infty N(O)_m \exp\left[(1-a)\frac{z}{H}\right]dz\right\} \tag{2.04}$$

where N(O) is the atomic oxygen density at the maximum of the ionization density, H is the scale height of atomic oxygen, a and κ are empirically determined constants, and ψ is the solar zenith angle.

The scale height is an important parameter in aeronomic work which determines the rate at which the number density of unionized air

particles decreases with height. It is defined as

$$H_j = \frac{kT}{m_j g} \tag{2.05}$$

where k is Boltzmann's constant, T is the temperature in degrees Kelvin, m_j refers to the mass of the jth constituent and g is the gravitational acceleration. In an atmosphere where the neutral constituents are in diffusive equilibrium it is necessary to allow each constituent to have its own scale height.

If the neutral atomic oxygen is assumed to follow a diffusive equilibrium distribution, equation 2.04 may be combined with equation 2.03 to give

$$Q_z = p_\infty N(0)_m \exp\left[-\frac{z}{H}\right] + \kappa\, p_\infty N(0)_m^2 \frac{H}{(1-a)} \sec\psi \exp\left[-\frac{az}{H}\right] \tag{2.06}$$

provided that a is greater than 1. Equation 2.06 may be written

$$Q_z = q_m \exp\left[-\frac{z}{H}\right] - \rho_m \exp\left[-\frac{az}{H}\right] \tag{2.07}$$

where

$$q_m = p_\infty N(0)_m \tag{2.08}$$

and

$$\rho_m = p_\infty \kappa N(0)_m^2 \frac{H}{a-1} \sec\psi \tag{2.09}$$

2.2.2 Loss Term

The O^+ ions are lost by a two-stage process involving the formation and subsequent dissociative recombination of molecular ions: the molecular gas participating in this process may be either molecular

nitrogen, N_2, or molecular oxygen, O_2. The formation of molecular ions is accomplished by an ion-atom interchange process first proposed by Bates and Massey (1947) in which the atomic ion, produced in the reaction of equation 2.01, interacts with an unspecified molecular species XY,

$$O^+ + XY \rightarrow XO^+ + Y \tag{2.10}$$

to produce a molecular ion XO^+ and a neutral atom Y. This reaction is then followed by dissociative recombination of the molecular ion XO^+ with a free electron,

$$XO^+ + e \rightarrow O' + X' \tag{2.11}$$

into excited atoms (indicated by the primes on X′ and O′).

The rate of recombination is assumed to depend on the rate of the reaction in equation 2.11. Under these conditions, the loss rate is given by

$$L_z = \gamma\ N(XY)_m\ n_i\ \exp\left[-\frac{bz}{H}\right] \tag{2.12}$$

or

$$L_z = \beta_m\ n_i\ \exp\left[-\frac{bz}{H}\right] \tag{2.13}$$

where

$$\beta_m = \gamma\ N[XY]_m \tag{2.14}$$

and b is a parameter that depends on the concentration of the molecular gases,

$$b = \frac{m(XY)}{m(O)} = \frac{H(O)}{H(XY)} \tag{2.15}$$

where H(O) is the scale height of atomic oxygen and H(XY) is the scale height of the molecular gases. If XY is presumed to be molecular oxygen, b would be 2.0; if molecular nitrogen, b would be 1.75.

2.2.2 Transport

The movement term in the continuity equation is due to changes in temperature, diffusion, and electrodynamic drifts in the F-region. In this work only vertical movements shall be considered so that the drift term may be written

$$\mathrm{div}(n_i v) = \frac{\partial}{\partial z}\left[n_i(v_i + v_f)\right] \tag{2.16}$$

where v_i is the vertical component of the drift velocity due to diffusion, and v_f is the upward component due to electrodynamic forces and which is assumed independent of height. Since we are considering the continuity equation under equilibrium conditions, motions due to the thermal expansion and contraction of the atmosphere are neglected.

The equation of motion for the ions in this region is, Nisbet (1963),

$$\frac{m_i \nu_{in} v_i}{\sin^2 I} = - m_i g + qE - \frac{kT_i}{n_i}\frac{\partial n_i}{\partial z} \tag{2.17}$$

and that for the electrons is

$$\frac{m_e \nu_{en} v_e}{\sin^2 I} = - m_e g - qE - \frac{kT_e}{n_e}\frac{\partial n_e}{\partial z} \tag{2.18}$$

where I is the geomagnetic dip angle, m_e and m_i are the electron and ion masses, n_e and n_i are the electron and ion number densities or concentrations and which will be assumed equal, v_e and v_i are the vertical diffusion velocities of the electrons and ions with respect

to the neutral atmosphere, ν_{en} and ν_{in} are the collision frequencies for the electrons and ions with neutral particles, q is the charge of an electron, T_e and T_i are the equivalent electron and ion temperatures, and E is the electric field.

Since $m_e \ll m_i$, the ratio of the collision frequencies $\frac{\nu_{en}}{\nu_{in}}$ is small, and since the ion and electron drift velocities are approximately equal, the approximation

$$m_e \nu_{en} v_e \ll m_i \nu_{in} v_i \tag{2.19}$$

may be made. Substituting equations 2.18 and 2.19 into equation 2.17 gives the ion diffusion velocity

$$v_i = -g \frac{\sin^2 I}{\nu_{in}} \left[1 + \frac{k(T_e + T_i)}{n_i m_i g} \frac{\partial n_i}{\partial z} \right] \tag{2.20}$$

The collision frequency between ions and neutral particles is assumed to decrease exponentially with height within the region of interest, so that in general,

$$\nu_{in} = \nu_{in_m} \exp\left[- \frac{cz}{H} \right] \tag{2.21}$$

The electron and ion temperatures may be different, so a parameter d is defined as

$$d \equiv \frac{T_i}{T_i + T_e} , \tag{2.22}$$

which may have a value less than 0.5.

Nisbet (1963) defines a diffusion coefficient at the ion density maximum

$$D_m = g \frac{H \sin^2 I}{\nu_{in_m} d} \tag{2.23}$$

Equation 2.20 may thus be written

$$n_i v_i = - D_m \exp\left[\frac{cz}{H}\right] \left\{ \frac{n_i d}{H} + \frac{\partial n_i}{\partial_z} \right\} \tag{2.24}$$

which is just the upward flux of ions due to diffusion.

Substituting equation 2.24 into equation 2.16 and performing the differentiation gives the transport term in the continuity equation as

$$\mathrm{div}(n_i v) = v_f \frac{\partial n_i}{\partial z} - D_m \exp\left[\frac{cz}{H}\right] \left\{ \frac{\partial^2 n_i}{\partial z^2} + \left(\frac{c+d}{H}\right) \frac{\partial n_i}{\partial z} + \frac{cd}{H^2} n_i \right\} \tag{2.25}$$

2.3 Solution of the Equation

Substituting equations 2.07, 2.13, and 2.25 into equation 2.02 gives

$$\frac{\partial n_i}{\partial t} = q_m \exp\left[-\frac{z}{H}\right] - \rho_m \exp\left[-\frac{az}{H}\right] - \beta_m n_i \exp\left[-\frac{bz}{H}\right]$$

$$- v_f \frac{\partial n_i}{\partial z} + D_m \exp\left[\frac{cz}{H}\right] \left\{ \frac{\partial^2 n_i}{\partial z^2} + \left(\frac{c+d}{H}\right) \frac{\partial n_i}{\partial z} + \frac{cd}{H^2} n_i \right\} \tag{2.26}$$

For conditions of equilibrium where $\frac{\partial n_i}{\partial t} = 0$ at all altitudes, equation 2.26 may be written

$$0 = \frac{\partial^2 n_i}{\partial z^2} + \left(\frac{c+d}{H}\right) \frac{\partial n_i}{\partial z} + \frac{cd}{H^2} n_i + \frac{q_m}{D_m} \exp\left[-\left(\frac{1+c}{H}\right)^z\right]$$

$$- \frac{\rho_m}{D_m} \exp\left[-\left(\frac{a+c}{H}\right) z\right] - \frac{\beta_m}{D_m} \exp\left[-\left(\frac{b+c}{H}\right)^z\right] n_i$$

$$- \frac{v_f}{D_m} \exp\left[-\frac{cz}{H}\right] \frac{\partial n_i}{\partial z} \tag{2.27}$$

The solution to equation 2.27 may be obtained in the form of exponential series,

$$n_i = \sum_{\lambda} \left\{ \sum_{n=0}^{\infty} B_{\lambda+n(b+c)} \exp\left[-(\lambda+n(b+c)) \frac{z}{H}\right] + \sum_{n=1}^{\infty} B_{\lambda+nc} \exp\left[-(\lambda+nc) \frac{z}{H}\right] \right\} \tag{2.28}$$

where λ is a floating index to be determined and n is an integer. By substituting equation 2.28 into equation 2.27, it may be shown that equation 2.27 is satisfied if

$$\lambda = 1 + c \text{ and } \lambda = a + c$$

and by values of λ that satisfy the equation

$$\lambda^2 - (c+d)\lambda + cd = 0 \tag{2.29}$$

namely, $\lambda = c$ and $\lambda = d$. With these values of λ substituted into equation 2.28, the solution may be written

$$\begin{aligned} n_i = \sum_{n=0}^{\infty} \Big\{ & B_{d+n(b+c)} \exp\left[-(d+n(b+c)) \frac{z}{H}\right] \\ & + B_{c+n(b+c)} \exp\left[-(c+n(b+c))\frac{z}{H}\right] + B_{1+c+n(b+c)} \exp\left[-(1+c+n(b+c))\frac{z}{H}\right] \\ & + B_{a+c+n(b+c)} \exp\left[-(a+c+n(b+c))\frac{z}{H}\right] \Big\} \\ & + \sum_{n=1}^{\infty} \Big\{ B_{d+nc} \exp\left[-(d+nc)\frac{z}{H}\right] + B_{c+nc} \exp\left[-(c+nc)\frac{z}{H}\right] \\ & + B_{1+c+nc} \exp\left[-(1+c+nc)\frac{z}{H}\right] + B_{a+c+nc} \exp\left[-(a+c+nc)\frac{z}{H}\right] \Big\} \end{aligned} \tag{2.30}$$

Equation 2.30 may be written

$$\begin{aligned} n_i = {} & B_d [S_1 + S_5] + B_c [S_2 + S_6] + B_{1+c} [S_3 + S_7] \\ & + B_{a+c} [S_4 + S_8] \end{aligned} \tag{2.31}$$

where S_1 through S_8 are series of exponential terms that are functions of z.

By substituting equation 2.30 into equation 2.27 and comparing coefficients in the n = 0 term, it may be shown that

$$B_{1+c} = -\frac{q_m H^2}{D_m} \frac{1}{(1+c-d)} \tag{2.32}$$

and

$$B_{a+c} = \frac{\rho_m H^2}{D_m} \frac{1}{a(a+c-d)} \tag{2.33}$$

Recursion relations for the general coefficients B_Π and B_Λ in each series are

$$\frac{B_\Pi}{B_{\Pi-(b+c)}} = \frac{H^2\beta_m}{D_m} \frac{1}{(\Pi-c)(\Pi-d)} \tag{2.34}$$

and

$$\frac{B_\Lambda}{B_{\Lambda-c}} = \frac{v_f H}{D_m} \frac{1}{(\Lambda-d)} \tag{2.35}$$

respectively. Three parameters of equation 2.31 remain to be determined; B_d, B_c and $\frac{H^2\beta_m}{D_m}$.

2.3.1 Location of the Ion Density Maximum

Equation 2.30 may be differentiated with respect to z to give the rate of change of the ion density. The peak is located where $\frac{\partial n_i}{\partial z} = 0$, so that at the maximum

$$\begin{aligned} 0 = -H\frac{\partial n_i}{\partial z} = \sum_{n=0}^{\infty} \Big\{ & [d+n(b+c)]\, B_{d+n(b+c)} + [c+n(b+c)]\, B_{c+n(b+c)} \\ & + [1+c+n(b+c)]\, B_{1+c+n(b+c)} + [a+c+n(b+c)]\, B_{a+c+n(b+c)} \Big\} \\ + \sum_{n=1}^{\infty} \Big\{ & [d+nc]\, B_{d+nc} + [c+nc]\, B_{c+nc} + [1+c+nc]\, B_{1+c+nc} \\ & + [a+c+nc]\, B_{a+c+nc} \Big\} \end{aligned} \tag{2.36}$$

which may be written

$$0 = B_d\,[S_9 + S_{13}] + B_c\,[S_{10} + S_{14}]$$

$$+ B_{1+c}\,[S_{11} + S_{15}] + B_{a+c}\,[S_{12} + S_{16}] \qquad (2.37)$$

2.3.2 Upper Boundary Condition

This boundary condition was obtained by Nisbet by imposing the restraint that the net downward transport of ionization through a level at which $z = H/b+c$ is equal to the total ion production minus the total ion recombination above that level. This may be expressed as

$$-n_i\left(v_i+v_f\right) = \int_{\frac{H}{b+c}}^{\infty} Q_z\,dz - \int_{\frac{H}{b+c}}^{\infty} L_z\,dz \qquad (2.38)$$

Substituting equations 2.07, 2.13, and 2.24 into equation 2.38 gives

$$n_i v_f = -\,q_m \int_{\frac{H}{b+c}}^{\infty} \exp\left[-\frac{z}{H}\right] dz + \rho_m \int_{\frac{H}{b+c}}^{\infty} \exp\left[-\frac{az}{H}\right] dz$$

$$+ \beta_m \int_{\frac{H}{b+c}}^{\infty} n_i \exp\left[-\frac{bz}{H}\right] dz + D_m \exp\left[\frac{cz}{H}\right] \left\{\frac{n_i d}{H} + \frac{\partial n_i}{\partial z}\right\}\Bigg|_{z = \frac{H}{b+c}} \qquad (2.40)$$

Substituting for n_i from equation 2.30 and performing the integrations, one obtains

$$n_i v_f = -\,H\,q_m \exp\left[-\frac{1}{b+c}\right] + \frac{H}{a}\,\rho_m \exp\left[-\frac{a}{b+c}\right]$$

$$+ \beta_m H \sum_{n=0}^{\infty} \frac{B_{d+n(b+c)}}{[d+b+n(b+c)]} \exp\left[-\left(\frac{d+b}{b+c} + n\right)\right]$$

$$- \frac{D_m}{H} \sum_{n=0}^{\infty} [n(b+c)]\,B_{d+n(b+c)} \exp\left[-\left(\frac{d-c}{b+c} + n\right)\right]$$

$$+ \beta_m H \sum_{n=0}^{\infty} \frac{B_{c+n(b+c)}}{[c+b+n(b+c)]} \exp\left[-\left(\frac{1}{b+c} + n\right)\right]$$

$$- \frac{D_m}{H} \sum_{n=0}^{\infty} [c-d+n(b+c)] B_{c+n(b+c)} \exp\left[-n\right]$$

$$+ \beta_m H \sum_{n=0}^{\infty} \frac{B_{1+c+n(b+c)}}{[1+c+b+n(b+c)]} \exp\left[-\left(\frac{1}{b+c} + n + 1\right)\right]$$

$$- \frac{D_m}{H} \sum_{n=0}^{\infty} [1+c-d+n(b+c)] B_{1+c+n(b+c)} \exp\left[-\left(\frac{1}{b+c} + n\right)\right]$$

$$+ \beta_m H \sum_{n=0}^{\infty} \frac{B_{a+c+n(b+c)}}{[a+c+b+n(b+c)]} \exp\left[-\left(\frac{a}{b+c} + n + 1\right)\right]$$

$$- \frac{D_m}{H} \sum_{n=0}^{\infty} [a+c-d+n(b+c)] B_{a+c+n(b+c)} \exp\left[-\left(\frac{a}{b+c} + n\right)\right]$$

$$+ \beta_m H \sum_{n=0}^{\infty} \frac{B_{d+nc}}{d+b+nc} \exp\left[-\left(\frac{d+b+nc}{b+c}\right)\right]$$

$$- \frac{D_m}{H} \sum_{n=1}^{\infty} [nc] B_{d+nc} \exp\left[-\left(\frac{d-c+nc}{b+c}\right)\right]$$

$$+ \beta_m H \sum_{n=1}^{\infty} \frac{B_{c+nc}}{c+b+nc} \exp\left[-\left(\frac{c+b+nc}{b+c}\right)\right]$$

$$- \frac{D_m}{H} \sum_{n=1}^{\infty} [c+d+nc] B_{c+nc} \exp\left[-\left(\frac{nc}{b+c}\right)\right]$$

$$+ \beta_m H \sum_{n=1}^{\infty} \frac{B_{1+c+nc}}{1+c+b+nc} \exp\left[-\left(\frac{1+c+b+nc}{b+c}\right)\right]$$

$$- \frac{D_m}{H} \sum_{n=1}^{\infty} [1+c-d+nc] B_{1+c+nc} \exp\left[-\left(\frac{1+nc}{b+c}\right)\right]$$

$$+ \beta_m H \sum_{n=1}^{\infty} \frac{B_{a+c+nc}}{a+c+b+nc} \exp\left[-\left(\frac{a+c+b+nc}{b+c}\right)\right]$$

$$- \frac{D_m}{H} \sum_{n=1}^{\infty} [a+c-d+nc] B_{a+c+nc} \exp\left[-\left(\frac{a+nc}{b+c}\right)\right] \tag{2.40}$$

By examining only the n = 0 term and using the recursion relation given in equation 2.34, it may be shown that equation 2.40 reduces to

$$\begin{aligned} n_i v_f = & - H \; q_m \exp\left[-\frac{1}{b+c}\right] + \frac{H}{a} \rho_m \exp\left[-\frac{a}{b+c}\right] \\ & - \frac{D_m}{H} (c-d) B_c - \frac{D_m}{H} (1+c-d) B_{1+c} \exp\left[-\frac{1}{b+c}\right] \\ & - \frac{D_m}{H} (a+c-d) B_{a+c} \exp\left[-\frac{a}{b+c}\right] \end{aligned} \tag{2.41}$$

Using the relations for B_{1+c} and B_{a+c} that were derived in equations 2.32 and 2.33, equation 2.41 reduces to

$$B_c = - n_i v_f \frac{H}{D_m (c-d)} \tag{2.42}$$

2.3.3 Total Production and Loss in the Layer

Under equilibrium conditions, the total ion production and loss in the layer must be equal. This is written

$$\int_{-\infty}^{+\infty} Q_z \, dz = \int_{-\infty}^{+\infty} L_z \, dz \tag{2.43}$$

Nisbet writes the total production as

$$\int_{-\infty}^{+\infty} Q_z \, dz = \Sigma \, \Phi_{\infty} \, \sigma \, \eta \tag{2.44}$$

where $\Sigma \, \Phi_{\infty} \, \sigma$ is the sum of the products of the solar EUV flux components and the relevant ionization cross sections, and η is an efficiency factor related to the absorption of ionization by other constituents.

From equations 2.13 and 2.30, the loss term may be written as

$$\begin{aligned}
\int_{z}^{\infty} L_z \, dz = \beta_m H \Bigg\{ \sum_{n=0}^{\infty} \Bigg[& \frac{B_{d+n(b+c)}}{d+b+n(b+c)} \exp\left[-(d+b+n(b+c))\frac{z}{H}\right] \\
& + \frac{B_{c+n(b+c)}}{c+b+n(b+c)} \exp\left[-(c+b+n(b+c))\frac{z}{H}\right] + \frac{B_{1+c+n(b+c)}}{1+c+b+n(b+c)} \exp\left[-(1+c+b+n(b+c))\frac{z}{H}\right] \\
& + \frac{B_{a+c+n(b+c)}}{a+c+b+n(b+c)} \exp\left[-(a+c+b+n(b+c))\frac{z}{H}\right]\Bigg] \\
& + \sum_{n=1}^{\infty} \Bigg[\frac{B_{d+nc}}{d+b+nc} \exp\left[-(d+b+nc)\frac{z}{H}\right] + \frac{B_{c+nc}}{c+b+nc} \exp\left[-(c+b+nc)\frac{z}{H}\right] \\
& + \frac{B_{1+c+nc}}{1+c+b+nc} \exp\left[-(1+c+b+nc)\frac{z}{H}\right] + \frac{B_{a+c+nc}}{a+c+b+nc} \exp\left[-(a+c+b+nc)\frac{z}{H}\right]\Bigg\}
\end{aligned} \tag{2.45}$$

which may be written

$$\begin{aligned}
\int_{z}^{\infty} L_z \, dz = \Big\{ & B_d[S_{17}(z) + S_{21}(z)] + B_c[S_{18}(z) + S_{22}(z)] \\
& + B_{1+c}[S_{19}(z) + S_{23}(z)] + B_{a+c}[S_{20}(z) + S_{24}(z)]\Big\} \, \beta_m H
\end{aligned} \tag{2.46}$$

where the variable z is to be evaluated at minus infinity. It may be shown that in the limit as z becomes large and negative, $S_{21} \ll S_{17}$, $S_{22} \ll S_{18}$, $S_{23} \ll S_{19}$, and $S_{24} \ll S_{20}$.

Substituting equations 2.46 and 2.44 into equation 2.43 under this approximation gives

$$\frac{\Sigma\Phi_{\infty}\,\sigma\,\eta}{\beta_m\,H\,S_{17}(-\infty)} = B_d + B_c\,\frac{S_{18}(-\infty)}{S_{17}(-\infty)} + B_{1+c}\,\frac{S_{19}(-\infty)}{S_{17}(-\infty)} + B_{a+c}\,\frac{S_{20}(-\infty)}{S_{17}(-\infty)} \tag{2.47}$$

In the limit as $z \to -\infty$, series S_{17}, S_{18}, S_{19} and S_{20} become infinite, however their ratio remains finite to give

$$\begin{aligned} B_d = {} & -B_c\,\frac{\Gamma\left(\frac{c-d}{b+c}+1\right)}{\Gamma\left(\frac{d-c}{b+c}+1\right)}\left[\frac{H^2\beta_m}{D_m}\right]^{\frac{d-c}{b+c}}[b+c]^{2\left(\frac{c-d}{b+c}\right)} \\ & - B_{1+c}\,\frac{\Gamma\left(\frac{1+c-d}{b+c}+1\right)\Gamma\left(\frac{1}{b+c}+1\right)}{\Gamma\left(\frac{d-c}{b+c}+1\right)}\left[\frac{H^2\beta_m}{D_m}\right]^{\frac{d-1-c}{b+c}}[b+c]^{2\left(\frac{1+c-d}{b+c}\right)} \\ & - B_{a+c}\,\frac{\Gamma\left(\frac{a+c-d}{b+c}\right)\Gamma\left(\frac{a}{b+c}+1\right)}{\Gamma\left(\frac{d-c}{b+c}+1\right)}\left[\frac{H^2\beta_m}{D_m}\right]^{\frac{d-a-c}{b+c}}[b+c]^{2\left(\frac{a+c-d}{b+c}\right)} \end{aligned} \tag{2.48}$$

Equations 2.37 and 2.48 are both functions of B_d and $\frac{H^2\beta_m}{D_m}$. Taking the values for B_c, B_{1+c}, and B_{a+c} previously calculated, these two equations may be solved simultaneously for B_d and $\frac{H^2\beta_m}{D_m}$. It has been found convenient to solve these equations by having a digital computer search for values of $\frac{H^2\beta_m}{D_m}$ that give the same value of B_d for the two equations.

3. EFFECT OF HELIUM AND HYDROGEN IONS

At levels of more than two scale heights above the maximum, the atomic oxygen ion density given by equation 2.30, for a typical model atmosphere, is approximated to within 1 percent using the first terms in series S_1. With this approximation, equation 2.30 becomes

$$n_i = B_d \quad \exp\left[- \frac{dz}{H}\right] \tag{3.01}$$

This approximation is also made by Nisbet because at these particular levels, the production and loss terms in the continuity equation are small and the low-values of ion-neutral particle collision frequencies allow large diffusion velocities to result for very small departure from a diffusive equilibrium profile.

For a diffusive equilibrium profile equation 2.17 may be written for each ionic constituent

$$k\ T_i \frac{\partial n_i}{\partial z} = - m_i n_i g + qE\ n_i \tag{3.02}$$

When more than one type of ion is present, equation 3.02 may be summed over the various ionic species to give

$$k\ T_i \sum_i \frac{\partial n_i}{\partial z} = - g \sum_i m_i n_i + qE \sum_i n_i \tag{3.03}$$

For the electrons, using the same approximation, equation 2.18 may be written

$$k\ T_e \frac{\partial n_e}{\partial z} = - g\, m_e n_e - qE\ n_e \tag{3.04}$$

It may be assumed in this region that

$$\frac{\partial n_e}{\partial z} = \sum_i \frac{\partial n_i}{\partial z} \tag{3.05}$$

and also that

$$n_e = \sum_i n_i \tag{3.06}$$

Combining equations 3.03, 3.04, 3.05, and 3.06 with the approximation that $m_e \ll m_i$, we obtain

$$\frac{1}{n_e}\frac{\partial n_e}{\partial z} = -\frac{g \sum_i m_i n_i}{n_e k(T_i + T_e)} \tag{3.07}$$

which may be written

$$\frac{1}{n_e}\frac{\partial n_e}{\partial z} = -\frac{g m_+ d}{k T_i} \tag{3.08}$$

where $m_+ \equiv \sum_i m_i n_i \Big/ \sum_i n_i$.

Equation 3.08 may be integrated to give

$$n_e = n_{e_0} \exp\left[-\int_{z_0}^{z} \frac{g m_+ d}{k T_i}\, dz\right] \tag{3.09}$$

Substituting equation 3.02 into equation 3.04 gives

$$\frac{1}{n_i}\frac{\partial n_i}{\partial z} = -\frac{m_i g}{k T_i} - \frac{T_e}{T_i}\frac{1}{n_e}\frac{\partial n_e}{\partial z} \tag{3.10}$$

Substituting equation 3.08 into equation 3.10 gives

$$\frac{1}{n_i}\frac{\partial n_i}{\partial z} = -\frac{m_i g}{k T_i} + \frac{g m_+}{k T_i}(1-d) \tag{3.11}$$

which may be integrated to give

$$n_i = n_{i_0} \exp\left[- \int_{z_0}^{z} \frac{m_i g}{kT_i} dz + \int_{z_0}^{z} \frac{gm_+(1-d)}{kT_i} dz\right] \tag{3.12}$$

From equation 3.05, $H_i = \frac{kT_i}{m_i g}$, so that

$$\sum_i m_i n_i = \exp\left[- \int_{z_0}^{z} \frac{gm_+(1-d)}{kT_i} dz\right]$$

$$\cdot \sum_i n_{i_0} \exp\left[- \left(\frac{z-z_0}{H_i} \right) \right] \tag{3.13}$$

Similarly,

$$\sum_i n_i = \exp\left[- \int_{z_0}^{z} \frac{gm_+(1-d)}{kT_i} dz\right]$$

$$\cdot \sum_i n_{i_0} \exp\left[- \left(\frac{z-z_0}{H_i} \right) \right] \tag{3.14}$$

With equations 3.13 and 3.14, we can calculate the mean molecular mass,

$$m_+ = \frac{\sum_i m_i n_i}{\sum_i n_i} \tag{3.15}$$

or

$$m_+ = \frac{\sum_i m_i n_{i_0} \exp\left[- \left(\frac{z-z_0}{H_i} \right)\right]}{\sum_i n_{i_0} \exp\left[- \left(\frac{z-z_0}{H_i} \right)\right]} \tag{3.16}$$

If we define a dummy variable

$$A \equiv \sum_i n_{i_0} \exp\left[- \left(\frac{z-z_0}{H_i} \right)\right] \tag{3.17}$$

Then

$$\frac{dA}{dz} = - \frac{g}{kT_i} \sum_i m_i n_{i_0} \exp\left[- \left(\frac{z-z_0}{H_i} \right) \right] \tag{3.18}$$

and

$$- \frac{gm_+}{kT_i} = \frac{dA}{dz} /A \tag{3.19}$$

Equation 3.19 may be integrated to obtain

$$- \int_{z_0}^{z} \frac{gm_+}{kT_i} \, dz = \log_e A \Big|_{z_0}^{z} \tag{3.20}$$

which becomes

$$- \int_{z_0}^{z} \frac{gm_+}{kT_i} \, dz = \log_e \left\{ \sum_i n_{i_0} \exp\left[-\left(\frac{z-z_0}{H}\right)\right] - \sum_i n_{i_0} \right\} \tag{3.21}$$

If we make the approximation given in equation 3.16, equation 3.21 may be written

$$\exp\left[- \int_{z_0}^{z} \frac{gm_+}{kT_i} \, dz\right] = \sum_i \frac{n_{i_0}}{n_{e_0}} \exp\left[- \left(\frac{z-z_0}{H_i} \right)\right] \tag{3.22}$$

Substituting equation 3.22 into equation 3.09 gives

$$n_e = n_{e_0} \left\{ \sum_i \frac{n_{i_0}}{n_{e_0}} \exp\left[- \left(\frac{z-z_0}{H_i}\right)\right] \right\}^d \tag{3.23}$$

In regions where helium and hydrogen ions are of importance the first term in series S_1, given in equation 3.01, may be modified to give the electron density

$$n_e = B_d \exp\left[-\frac{dz_0}{H}\right] \left\{ \exp\left[-\left(\frac{z-z_0}{H}\right)\right] \right.$$

$$\left. + \frac{N(H_e^+)_0}{N(O^+)_0} \exp\left[-\left(\frac{z-z_0}{4H}\right)\right] + \frac{N(H^+)_0}{N(O^+)_0} \exp\left[-\left(\frac{z-z_0}{16H}\right)\right] \right\}^d \tag{3.24}$$

where $N(O^+)_0$, $N(H_e^+)_0$, and $N(H^+)_0$ are the atomic oxygen-, helium-, and hydrogen-ion densities at the datum level z_0, provided that

$$R_1 \equiv \frac{N(H^+)_0}{N(O^+)_0} \ll 1$$

and

$$R_2 \equiv \frac{N(H_e^+)_0}{N(O^+)_0} \ll 1$$

and z_0, the height of the datum level above the maximum, is sufficiently large that the first term is series S_1 predominates.

The complete solution for the equilibrium electron density may thus be obtained by combining equations 2.32 and 3.24 to give

$$n_e \approx B_d \left\{ \exp\left[-\frac{dz}{H}\right] \left[1 + R_1 \exp\frac{3(z-z_0)}{4H} + R_2 \exp\frac{15(z-z_0)}{16H}\right]^d \right.$$

$$\left. + S_1(z) - \exp\left[-\frac{dz}{H}\right] + S_5(z) \right\}$$

$$+ B_c \left\{S_2(z) + S_6(z)\right\} + B_{1+c} \left\{S_3(z) + S_7(z)\right\}$$

$$+ B_{a+c} \left\{S_4(z) + S_8(z)\right\} \tag{3.25}$$

4. RESULTS OF THE ANALYSIS

4.1 Profiles

Equation 3.25 is solved using the relations for B_d, B_c, B_{1+c}, B_{a+c}, and $\frac{H^2\beta_m}{D_m}$ that were derived in the previous sections. The equation is programmed for computation by an IBM 7074 digital computer. The computer calculates the electron density and plots it as a function of both geopotential height z and true height.

In order to illustrate the effect of a height-independent vertical drift on the electron-density profile, several plots with different velocities were made, then superimposed on a zero-drift plot. Figure 1 illustrates the form of the results for several velocities. For this figure the following values for parameters were assumed:

$$a = 2.00,\ b = 1.75,\ c = 1.00,\ d = 0.25,\ \frac{\rho_m}{q_m} = 0.100$$

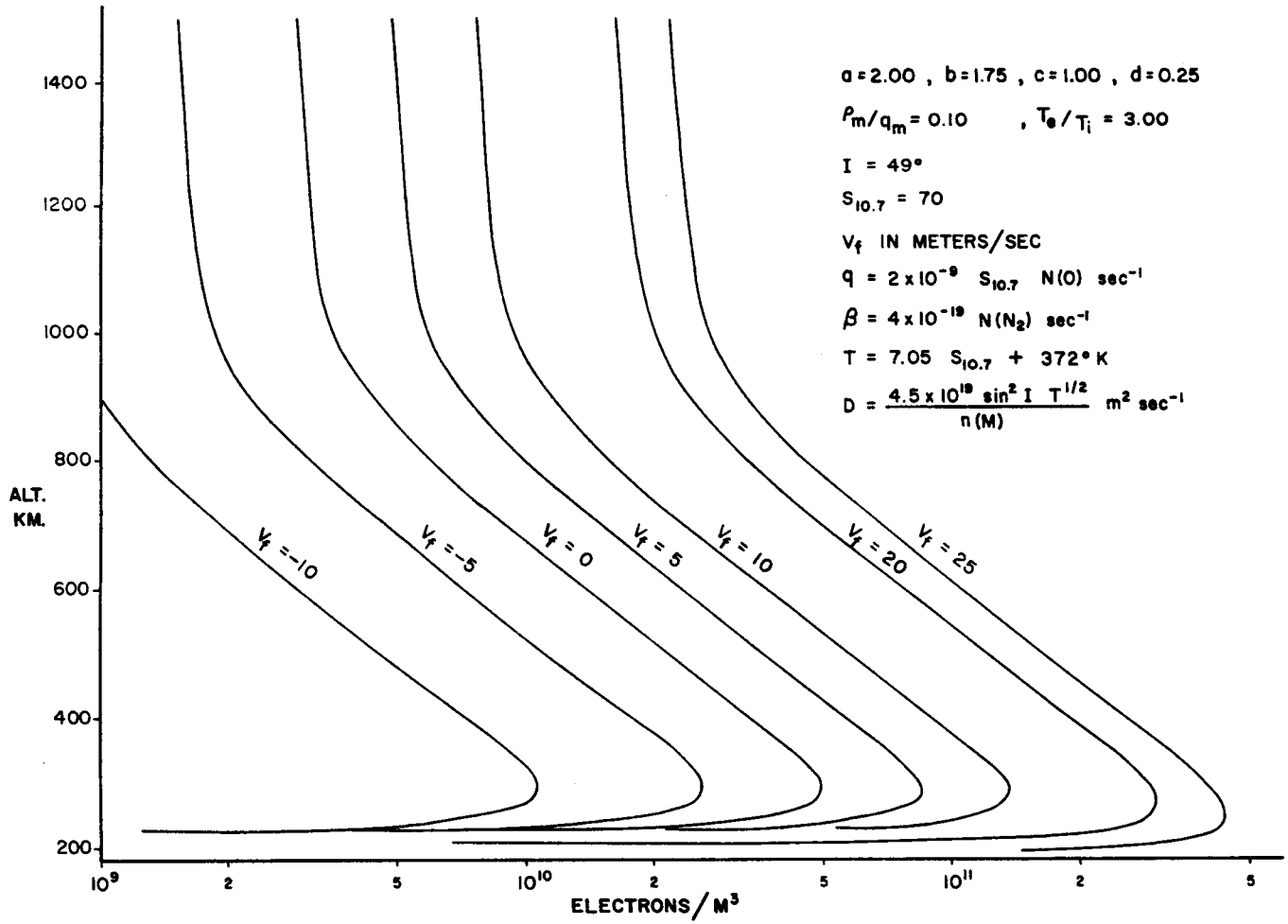
$$q = 2 \times 10^{-9}\ S_{10.7}\ N(O)\ \ \mathrm{sec}^{-1} \qquad \text{Nisbet (1963)}$$

$$T = 7.05^{\circ}\ S_{10.7} + 372^{\circ}\ K \qquad \text{Jacchia (1963)}$$

$$\beta = 4 \times 10^{-19}\ \ N(N_2)\ \ \mathrm{sec}^{-1} \qquad \text{Nisbet and Quinn (1963)}$$

$$D = \frac{4.5 \times 10^{19}\ \sin^2 I\ T^{1/2}}{n(M)}\ m^2\ \mathrm{sec}^{-1} \qquad \text{Shimazaki (1959)}$$

where $S_{10.7}$ is the monthly mean value of the 10.7 cm. solar flux number and n(M) is the number density of the main neutral constituent. Neutral atmospheric densities are taken from atmospheric models due to Nicolet (1961). The helium- and hydrogen-ion density ratios are those used by Nisbet (1963) as calculated from models due to Bauer (1963).



EXAMPLES OF PROFILES FOR SEVERAL VELOCITIES OF VERTICAL DRIFT

FIGURE I

The effect of an upward drift of the electron-density profile is seen to slightly increase the content of the layer. This increase is less than one order of magnitude for drifts up to 20 meters per second. Downward drifts tend to decrease the content of the layer, however, their effect is much more pronounced than for upward drifts of the same magnitude. The height of the maximum electron density remains fairly constant for both upward and downward drifts, decreasing slightly for strong upward drifts.

4.2 Approximate Profiles

Above the maximum, series S_1 through S_8 in equations 2.30 and 3.25 are well approximated by the first few terms. In general, in this re-tion if the height-independent drift is neglected a total of only five terms is required to approximate the electron-density to within 1 percent. When the drift is present, it is necessary to include thirteen terms to make this approximation.

Thus, equation 3.25 may be written

$$n_e \approx \frac{q_m}{\beta_m} \Big\{ K_1 \exp\left(-\frac{L_1 z}{H}\right)\left[1 + R_1 \exp\left[\frac{3(z-z_0)}{4H}\right] + R_2 \exp\left[\frac{15(z-z_0)}{16H}\right]\right]^{L_1}$$

$$+ K_2 \exp\left[-\frac{L_2 z}{H}\right] + K_3 \exp\left[-\frac{L_3 z}{H}\right] + K_4 \exp\left[-\frac{L_4 z}{H}\right]$$

$$+ K_5 \exp\left[-\frac{L_5 z}{H}\right] + K_6 \exp\left[-\frac{L_6 z}{H}\right] + K_7 \exp\left[-\frac{L_7 z}{H}\right]$$

$$+ K_8 \exp\left[-\frac{L_8 z}{H}\right] + K_9 \exp\left[-\frac{L_9 z}{H}\right] + K_{10} \exp\left[-\frac{L_{10} z}{H}\right]$$

$$+ K_{11} \exp\left[-\frac{L_{11} z}{H}\right] + K_{12} \exp\left[-\frac{L_{12} z}{H}\right] + K_{13} \exp\left[-\frac{L_{13} z}{H}\right] \Big\} \qquad (4.01)$$

where

$$K_1 = B_d^{\beta_m/q_m}, \quad L_1 = d$$

$$K_2 = B_c^{\beta_m/q_m}, \quad L_2 = c$$

$$K_3 = B_{1+c}^{\beta_m/q_m}, \quad L_3 = 1+c$$

$$K_4 = B_{a+c}^{\beta_m/q_m}, \quad L_4 = a+c$$

$$K_5 = B_d \frac{\beta_m}{q_m} \frac{H^2\beta_m}{D_m} \frac{1}{(b+c)(b+d)}, \quad L_5 = d+b+c$$

$$K_6 = B_c \frac{\beta_m}{q_m} \frac{H^2\beta_m}{D_m} \frac{1}{(b+c)(b+2c-d)}, \quad L_6 = b+2c$$

$$K_7 = B_{1+c} \frac{\beta_m}{q_m} \frac{H^2\beta_m}{D_m} \frac{1}{(1+b+c)(1+c+2c-d)}, \quad L_7 = 1+b+2c$$

$$K_8 = B_d \frac{\beta_m}{q_m} \frac{v_f H}{D_m} \frac{1}{c}, \quad L_8 = d+c$$

$$K_9 = B_c \frac{\beta_m}{q_m} \frac{v_f H}{D_m} \frac{1}{(2c-d)}, \quad L_9 = 2c$$

$$K_{10} = B_{1+c} \frac{\beta_m}{q_m} \frac{v_f H}{D_m} \frac{1}{(1+2c-d)}, \quad L_{10} = 1+2c$$

$$K_{11} = B_{a+c}^{\beta_m/q_m} \frac{v_f H}{D_m} \frac{1}{(a+2c-d)}, \quad L_{11} = a+2c$$

$$K_{12} = B_d^{\beta_m/q_m} \left(\frac{v_f H}{D_m}\right)^2 \frac{1}{(2c)^2}, \quad L_{12} = d+2c$$

$$K_{13} = B_c^{\beta_m/q_m} \left(\frac{v_f H}{D_m}\right)^2 \frac{1}{(3c-d)^2}, \quad L_{13} = 3c$$

A computer program has been written to calculate these parameters from equations 2.31, 2.32, 2.33, 2.34, 2.42, and 2.49, and to print

them in the form of tables. For this program it is necessary to read in the parameters of the atmospheric model a,b,c,d, ρ_m/q_m, and two parameters related to the height-independent vertical drift. These parameters are defined as

$$GIM = n_i v_f / q_m H \tag{4.02}$$

and

$$VF = v_f H / D_m \tag{4.03}$$

The computer also prints out three parameters of the peak electron density: $\frac{H^2\beta_m}{D_m}$, $\frac{n_m\beta_m}{q_m}$, and SCAT/H. The parameter, $\frac{H^2\beta_m}{D_m}$, as suggested by Rishbeth and Barron, specifies the height of the layer. SCAT is a thickness parameter that is determined by taking points from a reduced true-height profile and fitting a curve to them. In this work, the parameter SCAT/H is the quarter-thickness of a parbola fitted to the peak of the electron density distribution. SCAT/H is defined as, Nisbet (1963)

$$SCAT/H = 1/H \left[\frac{n_m}{2(\partial^2 n/\partial z_m^2)} \right]^{1/2} \tag{4.04}$$

Table 1 in the appendix gives values of these parameters and coefficients for several atmospheric models. Included in these models are ranges of the parameter for the attenuation of solar radiation. Values of the parameters are given for each model under the effects of both upward and downward drift.

5. SUMMARY AND CONCLUSIONS

5.1 Statement of the Problem

The purpose of this work was to develop solutions to the F-region continuity equation, taking into account the effect of a height-independent vertical drift, in order that experimentally observed ion- or electron-density profiles may be related to the basic physical processes of production, loss, and transport of ionization.

5.2 Review of the Procedure

A velocity which represents the drift due to electrodynamic forces was inserted into the transport term of the continuity equation for atomic oxygen ions. This velocity was taken to be constant with height. Expressions for the production, loss, and diffusion of ionization were derived. The production term used was

$$Q_z = q_m \exp\left[-\frac{z}{H}\right] - \rho_m \exp\left[-\frac{az}{H}\right] \tag{5.01}$$

where Q_z is the rate of atomic oxygen-ion production at height z, H is the scale height of atomic oxygen, q_m, ρ_m and a are chosen to fit assumed profiles of atomic oxygen-ion production.

The loss term used was,

$$L_z = \beta_m n_i \exp\left[-\frac{bz}{H}\right] \tag{5.02}$$

where L_z is the rate of recombination of atomic oxygen-ions at height z, and β_m is the linear recombination coefficient effective at the ion-density maximum.

Transport of ionization was given by,

$$\mathrm{div}(n_i v) = v_f \frac{\partial n_i}{\partial z} - D_m \exp\left[\frac{cz}{H}\right] \left\{\frac{\partial^2 n_i}{\partial z^2} + \left(\frac{c+d}{H}\right) \frac{\partial n_i}{\partial z} + \frac{cd}{H^2} n_i\right\} \quad (5.03)$$

where v_f is the drift velocity due to electrodynamic forces and D_m is a diffusion coefficient, defined as

$$D_m = \frac{g H \sin^2 I}{\nu_{in_m} d} \quad (5.04)$$

ν_{in} is the ion-neutral particle collision frequency, defined as

$$\nu_{in} = \nu_{in_m} \exp\left[- \frac{cz}{H}\right] \quad (5.06)$$

where $\nu_{in} = \nu_{in_m}$ at the ion density maximum. c is a factor chosen to fit the assumed neutral atmospheric model being used.

The non-equilibrium of electron-ion temperatures is as considered in the factor d, defined as

$$d = \frac{T_i}{T_i + T_e} \quad (5.07)$$

where T_i is the ion temperature and T_e the electron temperature.

Solutions to the continuity equation under equilibrium conditions were obtained in the form of exponential series. Above the maximum, these series were found to converge very rapidly and may be approximated by the first few terms.

With these approximations the electron density was found to be given by

$$n_e \approx \frac{q_m}{\beta_m} \left\{ K_1 \exp -\left[\frac{L_1 z}{H}\right] \left[1 + R_1 \exp\left[\frac{3(z-z_0)}{4H}\right] + R_2 \exp\left[\frac{15(z-z_0)}{16H}\right]\right]^{L_1} \right.$$

$$+ K_2 \exp -\left[\frac{L_2 z}{H}\right] + K_3 \exp\left[-\frac{L_3 z}{H}\right] + K_4 \exp\left[-\frac{L_4 z}{H}\right]$$

$$+ K_5 \exp\left[-\frac{L_5 z}{H}\right] + K_6 \exp\left[-\frac{L_6 z}{H}\right] + K_7 \exp\left[-\frac{L_7 z}{H}\right]$$

$$+ K_8 \exp -\left[\frac{L_8 z}{H}\right] + K_9 \exp\left[-\frac{L_9 z}{H}\right] + K_{10} \exp\left[-\frac{L_{10} z}{H}\right]$$

$$\left. + K_{11} \exp\left[-\frac{L_{11} z}{H}\right] + K_{12} \exp\left[-\frac{L_{12} z}{H}\right] + K_{13} \exp\left[-\frac{L_{13} z}{H}\right] \right\} \qquad (5.08)$$

where

$$R_1 = \frac{N(H^+)}{N(O^+)}$$

and

$$R_2 = \frac{N(He^+)}{N(O^+)}$$

at level $z = z_0$.

5.3 Conclusions

The effect of a height-independent drift on the electron-density profile for various assumptions about neutral atmospheric models and ionizing radiation may be studied using the values given in Table 1.

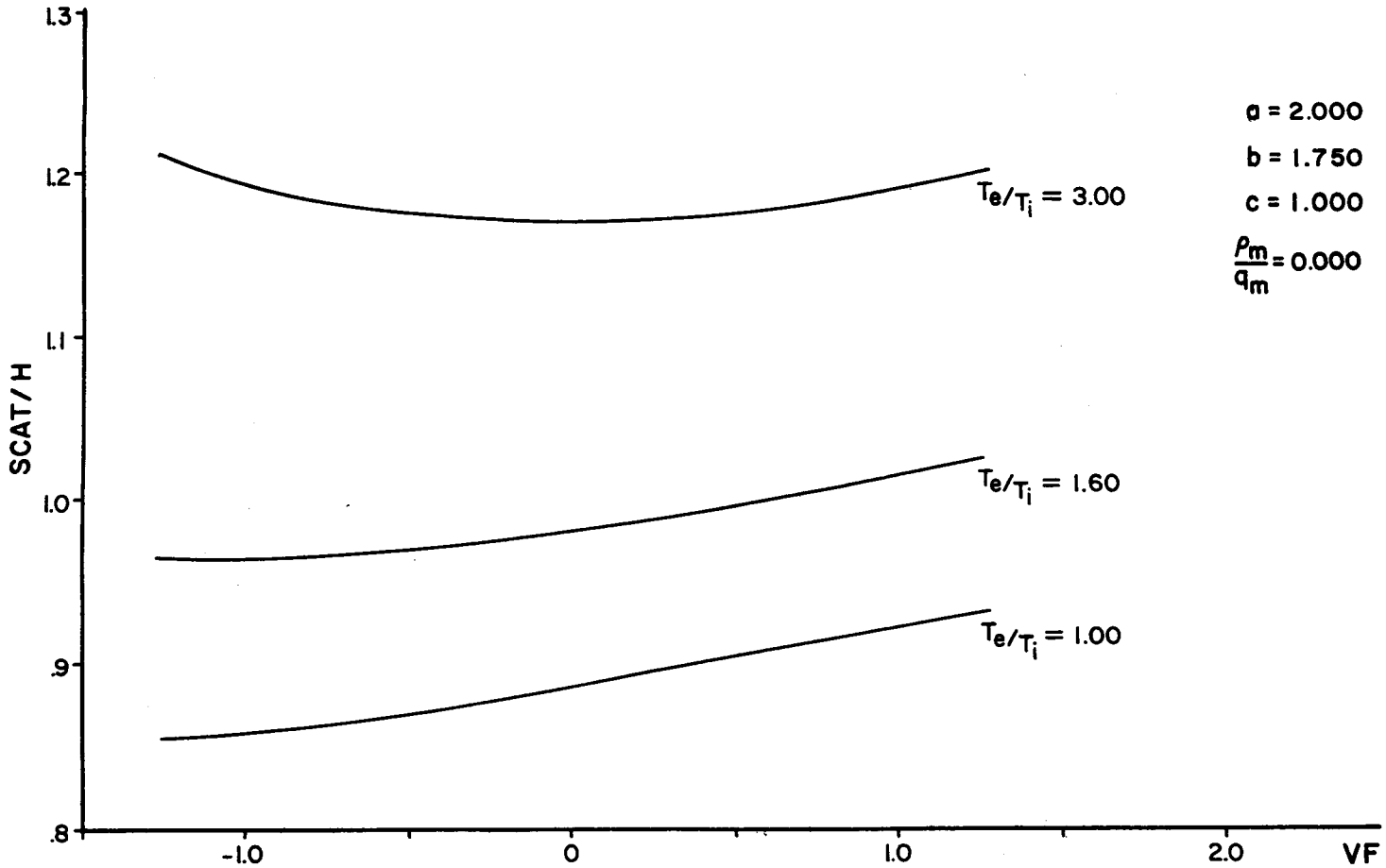
The parameter SCAT/H is of considerable practical interest because its relation to the profile does not depend on assumptions about production, recombination, or diffusion coefficients; and because of its effect on calculations of neutral atmospheric temperatures from true-height profiles near the peak of the layer.

The variation in layer thickness for different values of vertical velocity was examined by observing the effect of the drift on the parameter SCAT/H. Figure 2 illustrates this effect for different values of the temperature parameter d. In the region above 200 km the electron-ion temperature ratio may be as high as 3.00. Under these conditions, vertical drifts have little effect on the shape of the layer as is evidenced by only a 4% difference in the SCAT/H values for moderate upward and downward drift velocities. Increasing ρ_m/q_m and a correspond to increasing both geographic latitude for specific time of the year and neutral atmospheric temperature. The SCAT/H values in Table 1 indicate that both these effects produce a decrease in the thickness of the layer.

The parameter $\frac{n_m \beta_m}{q_m}$ is changed only slightly for moderate drifts, increasing for upward drifts and decreasing for downward drifts. Unless the drift is very strong the peak electron density may be approximated to within an order of magnitude by the relation $n_m = q_m/\beta_m$.

The value of $\frac{H^2 \beta_m}{D_m}$, however, is greatly affected by both upward and downward drifts. The value of this parameter increases by as much as 70% for upward drifts of the order of 25 meters per sec. This corresponds to a decrease in the height of the peak of approximately one scale height under sunspot minimum conditions. Downward drifts of the same magnitude tend to slightly increase the level of the peak.

The effect of vertical drift on the layer and its relation to magnetic latitude may be studied by examining the effects of variations



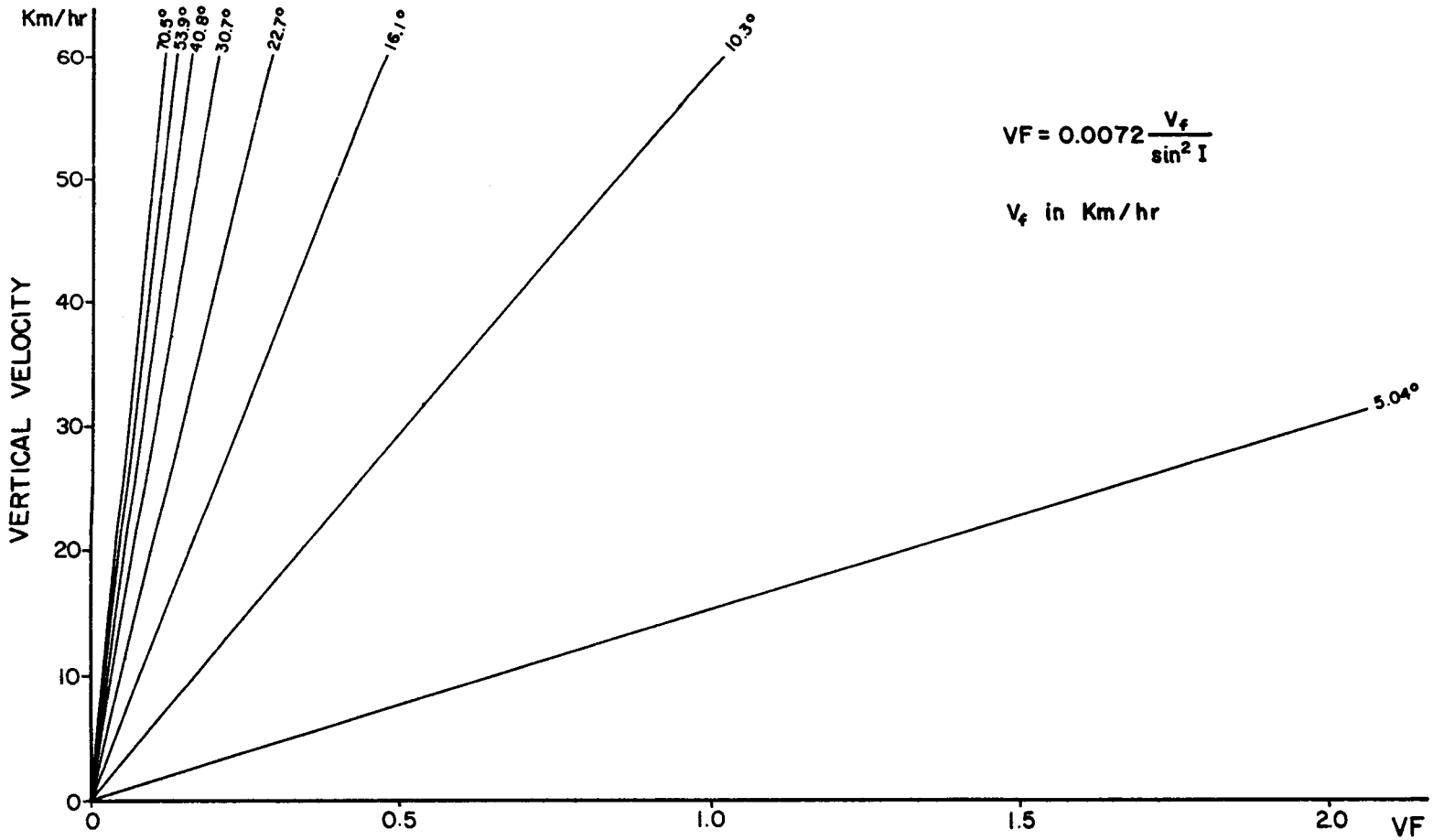
VARIATION IN LAYER THICKNESS FOR DIFFERENT VALUES OF DRIFT

FIGURE 2

in ρ_m/q_m and of $\sin^2 I$ on D_m in the drift parameter, VF, defined in equation 4.03. Relations between VF and vertical velocity are plotted in Figure 3 for several latitudes. It is apparent that in low latitudes even small values of vertical velocity correspond to relatively large values of VF, while in middle and high latitudes only very large values of vertical velocity correspond to relatively large values of VF. This means that in low latitudes the electron density distribution is more sensitive to vertical motions of ionization than at middle and high latitudes.

Increasing solar zenith angle corresponds to increasing ρ_m/q_m and a. In both cases, the effect on the electron density profile is to reduce the value of $\frac{n_m \beta_m}{q_m}$. For increasing latitude, the diffusion coefficient increases causing the height of the layer to decrease and β_m/q_m to increase. These effects combine to reduce the electron density for increasing latitude the same time that it is becoming more invariant to the effects of vertical drift.

Experimental measurements of photon fluxes as a function of altitude are given by Hinteregger and Watanabe (1962). These measurements may be used to investigate the effect of nonmonochromatic, ionizing radiation on the electron density profile in the presence of height-independent drifts. The electron production in the F-region is due primarily to ultraviolet radiation in the range 170-900 Å. Table 2 in the appendix gives values for parameters of the electron density distribution with the values of ρ_m/q_m and a chosen to fit photon flux measurements. The table contains values for these parameters under the



RELATION BETWEEN DRIFT PARAMETER VF AND VERTICAL VELOCITY FOR SEVERAL LATITUDES

FIGURE 3

effects of both upward and downward drift. It is apparent that the effect on the thickness of the F-region peak of the differing absorption rates is less than 1% for both upward and downward drifts of moderate magnitude.

APPENDIX

Tables of Coefficients

Table 1

General Atmospheric Models

Model	1	2	3	4	5
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.3477	0.5177	0.7466	1.0466	1.4264
$n_m\beta_m/q_m$	0.5659	0.6970	0.8453	1.0123	1.2001
SCAT/H	0.8550	0.8670	0.8860	0.9090	0.9320
K_1	0.8022	0.9914	1.2159	1.4854	1.8127
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0403	0.0290	0.0000	-0.0586	-0.1655
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.2318	-0.3451	-0.4977	-0.6977	-0.9509
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	3.0000	3.0000	3.0000	3.0000	3.0000
K_5	0.0451	0.0830	0.1467	0.2512	0.4179
L_5	3.2500	3.2500	3.2500	3.2500	3.2500
K_6	0.0016	0.0017	0.0000	-0.0069	-0.0264
L_6	3.7500	3.7500	3.7500	3.7500	3.7500
K_7	-0.0051	-0.0112	-0.0233	-0.0458	-0.0851
L_7	4.7500	4.7500	4.7500	4.7500	4.7500
K_8	-0.1011	-0.0625	0.0000	0.0936	0.2284
L_8	1.5000	1.5000	1.5000	1.5000	1.5000
K_9	-0.0034	-0.0012	0.0000	-0.0025	-0.0139
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{16}	0.0117	0.0087	0.0000	-0.0176	-0.0479
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	4.0000	4.0000	4.0000	4.0000	4.0000
K_{12}	0.0032	0.0010	0.0000	0.0015	0.0072
L_{12}	2.5000	2.5000	2.5000	2.5000	2.5000
K_{13}	0.0001	0.0000	0.0000	0.0000	-0.0004
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	6	7	8	9	10
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.1000	0.1000	0.1000	0.1000	0.1000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.2612	0.4031	0.5984	0.8604	1.2011
$n_m\beta_m/q_m$	0.4415	0.5643	0.7047	0.8647	1.0466
SCAT/H	0.8400	0.8500	0.8670	0.8890	0.9150
K_1	0.6234	0.7985	1.0070	1.2590	1.5677
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0303	0.0226	0.0000	-0.0482	-0.1393
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.1741	-0.2687	-0.3989	-0.5736	-0.8007
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0052	0.0081	0.0120	0.0172	0.0240
L_4	3.0000	3.0000	3.0000	3.0000	3.0000
K_5	0.0263	0.0520	0.0974	0.1751	0.3043
L_5	3.2500	3.2500	3.2500	3.2500	3.2500
K_6	0.0009	0.0010	0.0000	-0.0040	-0.0187
L_6	3.7500	3.7500	3.7500	3.7500	3.7500
K_7	-0.0029	-0.0068	-0.0150	-0.0310	-0.0603
L_7	4.7500	4.7500	4.7500	4.7500	4.7500
K_8	-0.0785	-0.0503	0.0000	0.0793	0.1975
L_8	1.5000	1.5000	1.5000	1.5000	1.5000
K_9	-0.0025	-0.0009	0.0000	-0.0020	-0.0117
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0088	0.0068	0.0000	-0.0145	-0.0404
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	-0.0002	-0.0001	0.0000	0.0003	0.0009
L_{11}	4.0000	4.0000	4.0000	4.0000	4.0000
K_{12}	0.0025	0.0008	0.0000	0.0012	0.0062
L_{12}	2.5000	2.5000	2.5000	2.5000	2.5000
K_{13}	0.0001	0.0000	0.0000	0.0000	-0.0004
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	11	12	13	14	15
a	3.0000	3.0000	3.0000	3.0000	3.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.3477	0.5177	0.7466	1.0466	1.4264
$n_m\beta_m/q_m$	0.5659	0.6970	0.8453	1.0123	1.2001
SCAT/H	0.8550	0.8670	0.8660	0.9090	0.9320
K_1	0.8022	0.9914	1.2159	1.4854	1.8127
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0403	0.0290	0.0000	-0.0856	-0.1655
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.2318	-0.3451	-0.4977	-0.6977	-0.9509
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	4.0000	4.0000	4.0000	4.0000	4.0000
K_5	0.0451	0.0830	0.1467	0.2512	0.4179
L_5	3.2500	3.2500	3.2500	3.2500	3.2500
K_6	0.0016	0.0017	0.0000	-0.0069	-0.0264
L_6	3.7500	3.7500	3.7500	3.7500	3.7500
K_7	-0.0051	-0.0112	-0.0233	-0.0458	-0.0851
L_7	4.7500	4.7500	4.7500	4.7500	4.7500
K_8	-0.1011	-0.0625	0.0000	0.0936	0.2284
L_8	1.5000	1.5000	1.5000	1.5000	1.5000
K_9	-0.0034	-0.0012	0.0000	-0.0025	-0.0139
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0117	0.0087	0.0000	-0.0176	-0.0479
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.0000	5.0000	5.0000	5.0000	5.0000
K_{12}	0.0032	0.0010	0.0000	0.0015	0.0072
L_{12}	2.5000	2.5000	2.5000	2.5000	2.5000
K_{13}	0.0001	0.0000	0.0000	0.0000	-0.0004
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	16	17	18	19	20
a	3.0000	3.0000	3.0000	3.0000	3.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0250	0.0250	0.0250	0.0250	0.0250
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.2804	0.4372	0.6512	0.9353	1.3002
$n_m\beta_m/q_m$	0.4858	0.6221	0.7746	0.9453	1.1367
SCAT/H	0.8360	0.8470	0.8650	0.8880	0.9120
K_1	0.6850	0.8793	1.1059	1.3756	1.7018
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0325	0.0245	0.0000	-0.0524	-0.1508
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.1869	-0.2915	-0.4341	-0.6235	-0.8668
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0007	0.0010	0.0016	0.0022	0.0031
L_4	4.0000	4.0000	4.0000	4.0000	4.0000
K_5	0.0310	0.0621	0.1164	0.2079	0.3576
L_5	3.2500	3.2500	3.2500	3.2500	3.2500
K_6	0.0010	0.0012	0.0000	-0.0055	-0.0219
L_6	3.7500	3.7500	3.7500	3.7500	3.7500
K_7	-0.0033	-0.0080	-0.0177	-0.0366	-0.0707
L_7	4.7500	4.7500	4.7500	4.7500	4.7500
K_8	-0.0863	-0.0554	0.0000	0.0867	0.2144
L_8	1.5000	1.5000	1.5000	1.5000	1.5000
K_9	-0.0027	-0.0010	0.0000	-0.0022	-0.0127
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0094	0.0073	0.0000	-0.0157	-0.0437
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.0000	5.0000	5.0000	5.0000	5.0000
K_{12}	0.0027	0.0009	0.0000	0.0014	0.0068
L_{12}	2.5000	2.5000	2.5000	2.5000	2.5000
K_{13}	0.0001	0.0000	0.0000	0.0000	-0.0004
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	21	22	23	24	25
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.2531	0.3844	0.5662	0.8136	1.1442
$n_m\beta_m/q_m$	0.4707	0.5977	0.7466	0.9210	1.1254
SCAT/H	0.8290	0.8360	0.8500	0.8720	0.9010
K_1	0.6617	0.8394	1.0550	1.3207	1.6537
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0294	0.0215	0.0000	-0.0456	-0.1327
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.1687	-0.2563	-0.3774	-0.5424	-0.7628
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	3.0000	3.0000	3.0000	3.0000	3.0000
K_5	0.0223	0.0430	0.0796	0.1433	0.2523
L_5	3.5000	3.5000	3.5000	3.5000	3.5000
K_6	0.0007	0.0008	0.0000	-0.0035	-0.0145
L_6	4.0000	4.0000	4.0000	4.0000	4.0000
K_7	-0.0024	-0.0055	-0.0119	-0.0245	-0.0485
L_7	5.0000	5.0000	5.0000	5.0000	5.0000
K_8	-0.0834	-0.0529	0.0000	0.0832	0.2084
L_8	1.5000	1.5000	1.5000	1.5000	1.5000
K_9	-0.0025	-0.0009	0.0000	-0.0019	-0.0111
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0085	0.0065	0.0000	-0.0137	-0.0384
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	4.0000	4.0000	4.0000	4.0000	4.0000
K_{12}	0.0026	0.0008	0.0000	0.0013	0.0066
L_{12}	2.5000	2.5000	2.5000	2.5000	2.5000
K_{13}	0.0001	0.0000	0.0000	0.0000	0.0003
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	26	27	28	29	30
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.1000	0.1000	0.1000	0.1000	0.1000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.1947	0.3062	0.4636	0.6817	0.9785
$n_m\beta_m/q_m$	0.3649	0.4811	0.6191	0.7826	0.9764
SCAT/H	0.8210	0.8260	0.8390	0.8600	0.8890
K_1	0.5121	0.6741	0.8721	1.1178	1.4285
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0226	0.0171	0.0000	-0.0382	-0.1135
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.1298	-0.2041	-0.3090	-0.4544	-0.6523
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0039	0.0061	0.0093	0.0136	0.0196
L_4	3.0000	3.0000	3.0000	3.0000	3.0000
K_5	0.0133	0.0275	0.0539	0.1016	0.1864
L_5	3.5000	3.5000	3.5000	3.5000	3.5000
K_6	0.0004	0.0005	0.0000	-0.0025	-0.0106
L_6	4.0000	4.0000	4.0000	4.0000	4.0000
K_7	-0.0014	-0.0035	-0.0080	-0.0172	-0.0355
L_7	5.0000	5.0000	5.0000	5.0000	5.0000
K_8	-0.0645	-0.0425	0.0000	0.0704	0.1800
L_8	1.5000	1.5000	1.5000	1.5000	1.5000
K_9	-0.0019	-0.0007	0.0000	-0.0016	-0.0095
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0065	0.0051	0.0000	-0.0115	-0.0329
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	-0.0001	-0.0001	0.0000	0.0002	0.0007
L_{11}	4.0000	4.0000	4.0000	4.0000	4.0000
K_{12}	0.0020	0.0007	0.0000	0.0011	0.0057
L_{12}	2.5000	2.5000	2.5000	2.5000	2.5000
K_{13}	0.0001	0.0000	0.0000	0.0000	-0.0003
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	31	32	33	34	35
a	3.0000	3.0000	3.0000	3.0000	3.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0250	0.0250	0.0250	0.0250	0.0250
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.2084	0.3302	0.5009	0.7356	1.0524
$n_m\beta_m/q_m$	0.4028	0.5318	0.6820	0.8576	1.0630
SCAT/H	0.8170	0.8230	0.8360	0.8570	0.8860
K_1	0.5644	0.7437	0.9591	1.2228	1.5525
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0242	0.0185	0.0000	-0.0412	-0.1221
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.1389	-0.2201	-0.3339	-0.4904	-0.7016
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0005	0.0008	0.0012	0.0018	0.0025
L_4	4.0000	4.0000	4.0000	4.0000	4.0000
K_5	0.0157	0.0327	0.0640	0.1199	0.2179
L_5	3.5000	3.5000	3.5000	3.5000	3.5000
K_6	0.0005	0.0006	0.0000	-0.0029	-0.0122
L_6	4.0000	4.0000	4.0000	4.0000	4.0000
K_7	-0.0016	-0.0040	-0.0093	-0.0200	-0.0410
L_7	5.0000	5.0000	5.0000	5.0000	5.0000
K_8	-0.0711	-0.0469	0.0000	0.0770	0.1956
L_8	1.5000	1.5000	1.5000	1.5000	1.5000
K_9	-0.0020	-0.0008	0.0000	-0.0017	-0.0103
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0070	0.0055	0.0000	-0.0124	-0.0354
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0001
L_{11}	5.0000	5.0000	5.0000	5.0000	5.0000
K_{12}	0.0022	0.0007	0.0000	0.0012	0.0062
L_{12}	2.5000	2.5000	2.5000	2.5000	2.5000
K_{13}	0.0001	0.0000	0.0000	0.0000	-0.0003
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	36	37	38	39	40
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.4528	0.6282	0.8448	1.1040	1.4033
$n_m\beta_m/q_m$	0.6638	0.7816	0.9085	1.0452	1.1934
SCAT/H	0.7940	0.8080	0.8250	0.8430	0.8600
K_1	0.8910	1.0562	1.2414	1.4509	1.6906
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0438	0.0293	0.0000	-0.0515	-0.1357
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.2516	-0.3490	-0.4693	-0.6133	-0.7796
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	3.3000	3.3000	3.3000	3.3000	3.3000
K_5	0.0588	0.0967	0.1528	0.2334	0.3457
L_5	3.5500	3.5500	3.5500	3.5500	3.5500
K_6	0.0017	0.0016	0.0000	-0.0048	-0.0162
L_6	4.3500	4.3500	4.3500	4.3500	4.3500
K_7	-0.0058	-0.0112	-0.0202	-0.0345	-0.0557
L_7	5.3500	5.3500	5.3500	5.3500	5.3500
K_8	-0.0864	-0.0512	0.0000	0.0703	0.1639
L_8	1.8000	1.8000	1.8000	1.8000	1.8000
K_9	-0.0026	-0.0009	0.0000	-0.0015	-0.0081
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0102	0.0071	0.0000	-0.0125	-0.0317
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	4.6000	4.6000	4.6000	4.6000	4.6000
K_{12}	0.0021	0.0006	0.0000	0.0009	0.0040
L_{12}	3.1000	3.1000	3.1000	3.1000	3.1000
K_{13}	0.0001	0.0000	0.0000	0.0000	-0.0002
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model	41	42	43	44	45
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.1000	0.1000	0.1000	0.1000	0.1000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.3551	0.5048	0.6932	0.9234	1.1953
$n_m\beta_m/q_m$	0.5398	0.6514	0.7727	0.9046	1.0487
SCAT/H	0.7820	0.7930	0.8090	0.8270	0.8470
K_1	0.7221	0.8765	1.0504	1.2484	1.4766
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0343	0.0236	0.0000	-0.0431	-0.1155
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.1972	-0.2804	-0.3851	-0.5130	-0.6640
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0063	0.0090	0.0124	0.0165	0.0213
L_4	3.3000	3.3000	3.3000	3.3000	3.3000
K_5	0.0374	0.0645	0.1061	0.1680	0.2572
L_5	3.5500	3.5500	3.5500	3.5500	3.5500
K_6	0.0010	0.0010	0.0000	-0.0034	-0.0118
L_6	4.3500	4.3500	4.3500	4.3500	4.3500
K_7	-0.0036	-0.0072	-0.0136	-0.0241	-0.0404
L_7	5.3500	5.3500	5.3500	5.3500	5.3500
K_8	-0.0700	-0.0425	0.0000	0.0605	0.1431
L_8	1.8000	1.8000	1.8000	1.8000	1.8000
K_9	-0.0021	-0.0007	0.0000	-0.0013	-0.0069
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0080	0.0057	0.0000	-0.0104	-0.0279
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	-0.0002	-0.0001	0.0000	0.0003	0.0007
L_{11}	4.6000	4.6000	4.6000	4.6000	4.6000
K_{12}	0.0017	0.0005	0.0000	0.0007	0.0035
L_{12}	3.1000	3.1000	3.1000	3.1000	3.1000
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0002
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model	46	47	48	49	50
a	3.0000	3.0000	3.0000	3.0000	3.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.3343	0.4730	0.6495	0.8695	1.1352
$n_m\beta_m/q_m$	0.5691	0.6871	0.8179	0.9632	1.1253
SCAT/H	0.7710	0.7800	0.7930	0.8110	0.8330
K_1	0.7581	0.9182	1.1013	1.3131	1.5613
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0323	0.0221	0.0000	-0.0406	-0.1097
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.1857	-0.2627	-0.3608	-0.4827	-0.6306
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	4.3000	4.3000	4.3000	4.3000	4.3000
K_5	0.0307	0.0526	0.0867	0.1383	0.2148
L_5	3.8000	3.8000	3.8000	3.8000	3.8000
K_6	0.0008	0.0008	0.0000	-0.0026	-0.0092
L_6	4.6000	4.6000	4.6000	4.6000	4.6000
K_7	-0.0028	-0.0057	-0.0107	-0.0191	-0.0326
L_7	5.6000	5.6000	5.6000	5.6000	5.6000
K_8	-0.0735	-0.0445	0.0000	0.0636	0.1513
L_8	1.8000	1.8000	1.8000	1.8000	1.8000
K_9	-0.0019	-0.0007	0.0000	-0.0012	-0.0066
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0075	0.0053	0.0000	-0.0098	-0.0256
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.6000	5.6000	5.6000	5.6000	5.6000
K_{12}	0.0018	0.0005	0.0000	0.0008	0.0037
L_{12}	3.1000	3.1000	3.1000	3.1000	3.1000
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0002
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model	51	52	53	54	55
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.3850	0.3850	0.3850	0.3850	0.3850
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.2135	0.3456	0.5262	0.7647	1.0684
$n_m\beta_m/q_m$	0.5152	0.6487	0.7967	0.9611	1.1444
SCAT/H	0.9650	0.9680	0.9810	1.0020	1.0250
K_1	0.6764	0.8454	1.0382	1.2622	1.5271
L_1	0.3850	0.3850	0.3850	0.3850	0.3850
K_2	0.0230	0.0180	0.0000	-0.0398	-0.1151
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.1322	-0.2140	-0.3258	-0.4735	-0.6615
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	3.0000	3.0000	3.0000	3.0000	3.0000
K_5	0.0246	0.0498	0.0930	0.1644	0.2779
L_5	3.1350	3.1350	3.1350	3.1350	3.1350
K_6	0.0005	0.0007	0.0000	-0.0033	-0.0133
L_6	3.7500	3.7500	3.7500	3.7500	3.7500
K_7	-0.0017	-0.0045	-0.0105	-0.0221	-0.0432
L_7	4.7500	4.7500	4.7500	4.7500	4.7500
K_8	-0.0852	-0.0533	0.0000	0.0795	0.1924
L_8	1.3850	1.3850	1.3850	1.3850	1.3850
K_9	-0.0018	-0.0007	0.0000	-0.0016	-0.0090
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0064	0.0052	0.0000	-0.0114	-0.0319
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	4.0000	4.0000	4.0000	4.0000	4.0000
K_{12}	0.0027	0.0008	0.0000	0.0013	0.0061
L_{12}	2.3850	2.3850	2.3850	2.3850	2.3850
K_{13}	0.0001	0.0000	0.0000	0.0000	-0.0003
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	56	57	58	59	60
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.3850	0.3850	0.3850	0.3850	0.3850
ρ_m/q_m	0.1000	0.1000	0.1000	0.1000	0.1000
GIM	-0.1740	-0.0840	0.0000	0.0480	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.1516	0.2589	0.4099	0.6147	0.8828
$n_m\beta_m/q_m$	0.3829	0.5090	0.6497	0.8076	0.9852
SCAT/H	0.9490	0.9490	0.9600	0.9790	1.0040
K_1	0.5014	0.6607	0.8423	1.0538	1.3052
L_1	0.3850	0.3850	0.3850	0.3850	0.3850
K_2	0.0163	0.0135	0.0000	-0.0320	-0.0951
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.0938	-0.1603	-0.2538	-0.3806	-0.5466
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0029	0.0050	0.0078	0.0118	0.0169
L_4	3.0000	3.0000	3.0000	3.0000	3.0000
K_5	0.0129	0.0291	0.0588	0.1103	0.1962
L_5	3.1350	3.1350	3.1350	3.1350	3.1350
K_6	0.0003	0.0004	0.0000	-0.0021	-0.0091
L_6	3.7500	3.7500	3.7500	3.7500	3.7500
K_7	-0.0009	-0.0025	-0.0064	-0.0143	-0.0295
L_7	4.7500	4.7500	4.7500	4.7500	4.7500
K_8	-0.0632	-0.0416	0.0000	0.0664	0.1645
L_8	1.3850	1.3850	1.3850	1.3850	1.3850
K_9	-0.0013	-0.0005	0.0000	-0.0012	-0.0074
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0045	0.0039	0.0000	-0.0092	-0.0263
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	-0.0001	-0.0001	0.0000	0.0002	0.0006
L_{11}	4.0000	4.0000	4.0000	4.0000	4.0000
K_{12}	0.0020	0.0007	0.0000	0.0010	0.0052
L_{12}	2.3850	2.3850	2.3850	2.3850	2.3850
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0002
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	61	62	63	64	65
a	3.0000	3.0000	3.0000	3.0000	3.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.3850	0.3850	0.3850	0.3850	0.3850
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.1509	0.2503	0.3903	0.5822	0.8390
$n_m\beta_m/q_m$	0.4154	0.5427	0.6894	0.8587	1.0550
SCAT/H	0.9410	0.9370	0.9440	0.9610	0.9880
K_1	0.5427	0.7014	0.8872	1.1087	1.3784
L_1	0.3850	0.3850	0.3850	0.3850	0.3850
K_2	0.0163	0.0130	0.0000	-0.0303	-0.0904
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.0934	-0.1550	-0.2416	-0.3605	-0.5195
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	4.0000	4.0000	4.0000	4.0000	4.0000
K_5	0.0114	0.0245	0.0484	0.0902	0.1616
L_5	3.3850	3.3850	3.3850	3.3850	3.3850
K_6	0.0002	0.0003	0.0000	-0.0016	-0.0070
L_6	4.0000	4.0000	4.0000	4.0000	4.0000
K_7	-0.0008	-0.0021	-0.0051	-0.0114	-0.0236
L_7	5.0000	5.0000	5.0000	5.0000	5.0000
K_8	-0.0684	-0.0442	0.0000	0.0698	0.1737
L_8	1.3850	1.3850	1.3850	1.3850	1.3850
K_9	-0.0013	-0.0005	0.0000	-0.0012	-0.0071
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0045	0.0037	0.0000	-0.0087	-0.0250
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.0000	5.0000	5.0000	5.0000	5.0000
K_{12}	0.0022	0.0007	0.0000	0.0011	0.0055
L_{12}	2.3850	2.3850	2.3850	2.3850	2.3850
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0002
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	66	67	68	69	70
a	3.0000	3.0000	3.0000	3.0000	3.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.3850	0.3850	0.3850	0.3850	0.3850
ρ_m/q_m	0.0250	0.0250	0.0250	0.0250	0.0250
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.1148	0.2057	0.3356	0.5162	0.7607
$n_m\beta_m/q_m$	0.3321	0.4653	0.6156	0.7876	0.9861
SCAT/H	0.9270	0.9220	0.9280	0.9440	0.9700
K_1	0.4328	0.5993	0.7890	1.0120	1.2814
L_1	0.3850	0.3850	0.3850	0.3850	0.3850
K_2	0.0124	0.0107	0.0000	-0.0268	-0.0820
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.0711	-0.1273	-0.2078	-0.2196	-0.4710
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0003	0.0005	0.0008	0.0012	0.0018
L_4	4.0000	4.0000	4.0000	4.0000	4.0000
K_5	0.0069	0.0172	0.0370	0.0730	0.1362
L_5	3.3850	3.3850	3.3850	3.3850	3.3850
K_6	0.0001	0.0002	0.0000	-0.0013	-0.0057
L_6	4.0000	4.0000	4.0000	4.0000	4.0000
K_7	-0.0004	-0.0014	-0.0038	-0.0089	-0.0194
L_7	5.0000	5.0000	5.0000	5.0000	5.0000
K_8	-0.0545	-0.0378	0.0000	0.0638	0.1615
L_8	1.3850	1.3850	1.3850	1.3850	1.3850
K_9	-0.0010	-0.0004	0.0000	-0.0010	-0.0064
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0034	0.0031	0.0000	-0.0077	-0.0227
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.0000	5.0000	5.0000	5.0000	5.0000
K_{12}	0.0017	0.0006	0.0000	0.0010	0.0051
L_{12}	2.3850	2.3850	2.3850	2.3850	2.3850
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0002
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	71	72	73	74	75
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.3850	0.3850	0.3850	0.3850	0.3850
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.1993	0.3088	0.4509	0.6299	0.8488
$n_m\beta_m/q_m$	0.5062	0.6279	0.7604	0.9054	1.0660
SCAT/H	0.8710	0.8730	0.8820	0.8980	0.9190
K_1	0.6321	0.7809	0.9457	1.1316	1.3449
L_1	0.3850	0.3850	0.3850	0.3850	0.3850
K_2	0.0181	0.0135	0.0000	-0.0276	-0.0771
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.1040	-0.1613	-0.2355	-0.3289	-0.4432
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	3.3000	3.3000	3.3000	3.3000	3.3000
K_5	0.0160	0.0306	0.0542	0.0906	0.1450
L_5	3.6850	3.6850	3.6850	3.6850	3.6850
K_6	0.0003	0.0003	0.0000	-0.0013	-0.0047
L_6	4.6000	4.6000	4.6000	4.6000	4.6000
K_7	-0.0009	-0.0022	-0.0047	-0.0092	-0.0168
L_7	5.6000	5.6000	5.6000	5.6000	5.6000
K_8	-0.0613	-0.0378	0.0000	0.0548	0.1304
L_8	1.6850	1.6850	1.6850	1.6850	1.6850
K_9	-0.0010	-0.0004	0.0000	-0.0008	-0.0044
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0041	0.0032	0.0000	-0.0064	-0.0174
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	4.6000	4.6000	4.6000	4.6000	4.6000
K_{12}	0.0015	0.0005	0.0000	0.0007	0.0032
L_{12}	2.9850	2.9850	2.9850	2.9850	2.9850
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0001
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model	76	77	78	79	80
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.3850	0.3850	0.3850	0.3850	0.3850
ρ_m/q_m	0.1000	0.1000	0.1000	0.1000	0.1000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.1540	0.2469	0.3698	0.5274	0.7237
$n_m\beta_m/q_m$	0.3053	0.5083	0.6324	0.7695	0.9223
SCAT/H	0.8640	0.8650	0.8720	0.8870	0.9090
K_1	0.4931	0.6311	0.7850	0.9595	1.1608
L_1	0.3850	0.3850	0.3850	0.3850	0.3850
K_2	0.0140	0.0108	0.0000	-0.0231	-0.0658
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.0804	-0.1289	-0.1931	-0.2754	-0.3779
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0026	0.0042	0.0063	0.0090	0.0124
L_4	3.3000	3.3000	3.3000	3.3000	3.3000
K_5	0.0096	0.0198	0.0369	0.0643	0.1067
L_5	3.6850	3.6850	3.6850	3.6850	3.6850
K_6	0.0002	0.0002	0.0000	-0.0009	-0.0034
L_6	4.6000	4.6000	4.6000	4.6000	4.6000
K_7	-0.0006	-0.0014	-0.0032	-0.0065	-0.0122
L_7	5.6000	5.6000	5.6000	5.6000	5.6000
K_8	-0.0478	-0.0306	0.0000	0.0465	0.1125
L_8	1.6850	1.6850	1.6850	1.6850	1.6850
K_9	-0.0008	-0.0003	0.0000	-0.0007	-0.0037
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0032	0.0025	0.0000	-0.0054	-0.0148
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	-0.0001	-0.0001	0.0000	0.0001	0.0004
L_{11}	4.6000	4.6000	4.6000	4.6000	4.6000
K_{12}	0.0012	0.0004	0.0000	0.0006	0.0027
L_{12}	2.9850	2.9850	2.9850	2.9850	2.9850
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0001
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model	81	82	83	84	85
a	2.0000	2.0000	2.0000	2.0000	2.0000
b	1.7500	1.7500	1.7500	1.7500	1.7500
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.2500	0.2500	0.2500	0.2500	0.2500
ρ_m/q_m	0.0000	0.0000	0.0000	0.0000	0.0000
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.0753	0.1630	0.2913	0.4671	0.6961
$n_m\beta_m/q_m$	0.4090	0.5598	0.7165	0.8842	1.0667
SCAT/H	1.2090	1.1780	1.1700	1.1790	1.2000
K_1	0.4951	0.6648	0.8407	1.0327	1.2494
L_1	0.2500	0.2500	0.2500	0.2500	0.2500
K_2	0.0075	0.0078	0.0000	-0.0224	-0.0692
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	-0.0430	-0.0931	-0.1664	-0.2669	-0.3978
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0000	0.0000	0.0000	0.0000	0.0000
L_4	3.0000	3.0000	3.0000	3.0000	3.0000
K_5	0.0068	0.0197	0.0445	0.0877	0.1581
L_5	3.0000	3.0000	3.0000	3.0000	3.0000
K_6	0.0001	0.0001	0.0000	-0.0011	-0.0050
L_6	3.7500	3.7500	3.7500	3.7500	3.7500
K_7	-0.0002	-0.0009	-0.0029	-0.0074	-0.0164
L_7	4.7500	4.7500	4.7500	4.7500	4.7500
K_8	-0.0624	-0.0419	0.0000	0.0651	0.1574
L_8	1.2500	1.2500	1.2500	1.2500	1.2500
K_9	-0.0005	-0.0003	0.0000	-0.0008	-0.0050
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0020	0.0021	0.0000	-0.0061	-0.0182
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	4.0000	4.0000	4.0000	4.0000	4.0000
K_{12}	0.0020	0.0007	0.0000	0.0010	0.0050
L_{12}	2.2500	2.2500	2.2500	2.2500	2.2500
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0001
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Model	86	87	88	89	90
a	3.0000	3.0000	3.0000	3.0000	3.0000
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.0000	1.0000	1.0000	1.0000	1.0000
d	0.2500	0.2500	0.2500	0.2500	0.2500
ρ_m/q_m	0.0250	0.0250	0.0250	0.0250	0.0250
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.0005	0.0808	0.1672	0.2929	0.4677
$n_m\beta_m/q_m$	0.3169	0.3386	0.5034	0.6801	0.8768
SCAT/H	2.1300	1.1270	1.1090	1.1100	1.1270
K_1	0.3594	0.3996	0.5848	0.7830	1.0080
L_1	0.2500	0.2500	0.2500	0.2500	0.2500
K_2	0.0000	0.0039	0.0000	-0.0141	-0.0465
L_2	1.0000	1.0000	1.0000	1.0000	1.0000
K_3	0.0000	-0.0462	-0.0955	-0.1674	-0.2672
L_3	2.0000	2.0000	2.0000	2.0000	2.0000
K_4	0.0000	0.0002	0.0004	0.0007	0.0010
L_4	4.0000	4.0000	4.0000	4.0000	4.0000
K_5	0.0000	0.0048	0.0145	0.0340	0.0698
L_5	3.2500	3.2500	3.2500	3.2500	3.2500
K_6	0.0000	0.0000	0.0000	-0.0004	-0.0019
L_6	4.0000	4.0000	4.0000	4.0000	4.0000
K_7	0.0000	-0.0002	-0.0008	-0.0026	-0.0066
L_7	5.0000	5.0000	5.0000	5.0000	5.0000
K_8	-0.0453	-0.0252	0.0000	0.0493	0.1270
L_8	1.2500	1.2500	1.2500	1.2500	1.2500
K_9	0.0000	-0.0001	0.0000	-0.0005	-0.0033
L_9	2.0000	2.0000	2.0000	2.0000	2.0000
K_{10}	0.0000	0.0011	0.0000	-0.0038	-0.0122
L_{10}	3.0000	3.0000	3.0000	3.0000	3.0000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.0000	5.0000	5.0000	5.0000	5.0000
K_{12}	0.0014	0.0004	0.0000	0.0008	0.0040
L_{12}	2.5000	2.5000	2.5000	2.5000	2.5000
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0001
L_{13}	3.0000	3.0000	3.0000	3.0000	3.0000

Table 2

Models Chosen to Fit Photon Flux Measurements

Model			238 Å		
a	3.7300	3.7300	3.7300	3.7300	3.7300
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0040	0.0040	0.0040	0.0040	0.0040
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1630
$H^2\beta_m/D_m$	0.3195	0.4564	0.6310	0.8486	1.1131
$n_m\beta_m/q_m$	0.5510	0.6703	0.8023	0.9486	1.1116
SCAT/H	0.7670	0.7760	0.7890	0.8070	0.8290
K_1	0.7332	0.8949	1.0790	1.2915	1.5402
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0309	0.0213	0.0000	-0.0396	-0.1076
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.1775	-0.2536	-0.3506	-0.4714	-0.6184
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0001	0.0001	0.0001	0.0002	0.0003
L_4	5.0300	5.0300	5.0300	5.0300	5.0300
K_5	0.0284	0.0495	0.0825	0.1328	0.2078
L_5	3.8000	3.8000	3.8000	3.8000	3.8000
K_6	0.0007	0.0007	0.0000	-0.0025	-0.0089
L_6	4.6000	4.6000	4.6000	4.6000	4.6000
K_7	-0.0026	-0.0053	-0.0101	-0.0182	-0.0314
L_7	5.6000	5.6000	5.6000	5.6000	5.6000
K_8	-0.0711	-0.0434	0.0000	0.0626	0.1493
L_8	1.8000	1.8000	1.8000	1.8000	1.8000
K_9	-0.0019	-0.0006	0.0000	-0.0012	-0.0065
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0072	0.0052	0.0000	-0.0096	-0.0251
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	6.3300	6.3300	6.3300	6.3300	6.3300
K_{12}	0.0017	0.0005	0.0000	0.0008	0.0036
L_{12}	3.1000	3.1000	3.1000	3.1000	3.1000
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0001
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model			335 Å		
a	2.6600	2.6600	2.6600	2.6600	2.6600
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0350	0.0350	0.0350	0.0350	0.0350
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.2891	0.4178	0.5832	0.7910	1.0454
$n_m\beta_m/q_m$	0.5065	0.6235	0.7535	0.8981	1.0595
SCAT/H	0.7630	0.7710	0.7840	0.8010	0.8230
K_1	0.6732	0.8310	1.0113	1.2198	1.4642
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0279	0.0195	0.0000	-0.0369	-0.1011
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.1606	-0.2321	-0.3240	-0.4304	-0.5808
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0011	0.0016	0.0022	0.0030	0.0040
L_4	3.9600	3.9600	3.9600	3.9600	3.9600
K_5	0.0236	0.0421	0.0715	0.1170	0.1855
L_5	3.8000	3.8000	3.8000	3.8000	3.8000
K_6	0.0006	0.0006	0.0000	-0.0022	-0.0078
L_6	4.6000	4.6000	4.6000	4.6000	4.6000
K_7	-0.0021	-0.0044	-0.0086	-0.0159	-0.0277
L_7	5.6000	5.6000	5.6000	5.6000	5.6000
K_8	-0.0653	-0.0403	0.0000	0.0591	0.1419
L_8	1.8000	1.8000	1.8000	1.8000	1.8000
K_9	-0.0017	-0.0006	0.0000	-0.0011	-0.0061
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0065	0.0047	0.0000	-0.0089	-0.0236
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0001
L_{11}	5.2600	5.2600	5.2600	5.2600	5.2600
K_{12}	0.0016	0.0005	0.0000	0.0007	0.0034
L_{12}	3.1000	3.1000	3.1000	3.1000	3.1000
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0001
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model			584 Å		
a	3.2400	3.2400	3.2400	3.2400	3.2400
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0120	0.0120	0.0120	0.0120	0.0120
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1630
$H^2\beta_m/D_m$	0.3071	0.4412	0.6128	0.8272	1.0886
$n_m\beta_m/q_m$	0.5343	0.6535	0.7854	0.9317	1.0947
SCAT/H	0.7650	0.7730	0.7870	0.8040	0.8260
K_1	0.7106	0.8716	1.0552	1.2670	1.5149
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0297	0.0206	0.0000	-0.0386	-0.1052
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.1706	-0.2451	-0.3404	-0.4596	-0.6047
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0003	0.0004	0.0006	0.0008	0.0010
L_4	4.5400	4.5400	4.5400	4.5400	4.5400
K_5	0.0264	0.0466	0.0784	0.1270	0.1999
L_5	3.8000	3.8000	3.8000	3.8000	3.8000
K_6	0.0007	0.0007	0.0000	-0.0024	-0.0085
L_6	4.6000	4.6000	4.6000	4.6000	4.6000
K_7	-0.0024	-0.0049	-0.0095	-0.0173	-0.0300
L_7	5.6000	5.6000	5.6000	5.6000	5.6000
K_8	-0.0689	-0.0422	0.0000	0.0614	0.1468
L_8	1.8000	1.8000	1.8000	1.8000	1.8000
K_9	-0.0018	-0.0006	0.0000	-0.0012	-0.0063
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0069	0.0050	0.0000	-0.0093	-0.0246
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.8400	5.8400	5.8400	5.8400	5.8400
K_{12}	0.0017	0.0005	0.0000	0.0007	0.0036
L_{12}	3.1000	3.1000	3.1000	3.1000	3.1000
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0001
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model			770 Å		
a	2.9400	2.9400	2.9400	2.9400	2.9400
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0160	0.0160	0.0160	0.0160	0.0160
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1630
$H^2\beta_m/D_m$	0.3072	0.4406	0.6113	0.8248	1.0850
$n_m\beta_m/q_m$	0.5334	0.6517	0.7828	0.9284	1.0908
SCAT/H	0.7660	0.7740	0.7870	0.8050	0.8260
K_1	0.7096	0.8694	1.0519	1.2627	1.5096
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0296	0.0206	0.0000	-0.0385	-0.1049
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.1707	-0.2448	-0.3396	-0.4582	-0.6028
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0004	0.0006	0.0009	0.0012	0.0016
L_4	4.2400	4.2400	4.2400	4.2400	4.2400
K_5	0.0264	0.0464	0.0779	0.1262	0.1985
L_5	3.8000	3.8000	3.8000	3.8000	3.8000
K_6	0.0007	0.0007	0.0000	-0.0023	-0.0084
L_6	4.6000	4.6000	4.6000	4.6000	4.6000
K_7	-0.0024	-0.0049	-0.0095	-0.0172	-0.0298
L_7	5.6000	5.6000	5.6000	5.6000	5.6000
K_8	-0.0068	-0.0421	0.0000	0.0612	0.1463
L_8	1.8000	1.8000	1.8000	1.8000	1.8000
K_9	-0.0018	-0.0006	0.0000	-0.0012	-0.0063
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0069	0.0050	0.0000	-0.0093	-0.0245
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.5400	5.5400	5.5400	5.5400	5.5400
K_{12}	0.0017	0.0005	0.0000	0.0007	0.0035
L_{12}	3.1000	3.1000	3.1000	3.1000	3.1000
K_{13}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

Model			865-885 Å		
a	2.9400	2.9400	2.9400	2.9400	2.9400
b	2.0000	2.0000	2.0000	2.0000	2.0000
c	1.3000	1.3000	1.3000	1.3000	1.3000
d	0.5000	0.5000	0.5000	0.5000	0.5000
ρ_m/q_m	0.0090	0.0090	0.0090	0.0090	0.0090
GIM	-0.1740	-0.0840	0.0000	0.0840	0.1740
VF	-0.1260	-0.0630	0.0000	0.0630	0.1260
$H^2\beta_m/D_m$	0.3191	0.4548	0.6280	0.8441	1.1070
$n_m\beta_m/q_m$	0.5493	0.6674	0.7983	0.9438	1.1060
SCAT/H	0.7680	0.7760	0.7900	0.8080	0.8290
K_1	0.7311	0.8910	1.0737	1.2849	1.5324
L_1	0.5000	0.5000	0.5000	0.5000	0.5000
K_2	0.0308	0.0212	0.0000	-0.0394	-0.1070
L_2	1.3000	1.3000	1.3000	1.3000	1.3000
K_3	-0.1773	-0.2527	-0.3489	-0.4689	-0.6150
L_3	2.3000	2.3000	2.3000	2.3000	2.3000
K_4	0.0003	0.0004	0.0005	0.0007	0.0009
L_4	4.2400	4.2400	4.2400	4.2400	4.2400
K_5	0.0283	0.0491	0.0817	0.1315	0.2056
L_5	3.8000	3.8000	3.8000	3.8000	3.8000
K_6	0.0007	0.0007	0.0000	-0.0025	-0.0088
L_6	4.6000	4.6000	4.6000	4.6000	4.6000
K_7	-0.0026	-0.0052	-0.0100	-0.0180	-0.0310
L_7	5.6000	5.6000	5.6000	5.6000	5.6000
K_8	-0.0709	-0.0432	0.0000	0.0623	0.1485
L_8	1.8000	1.8000	1.8000	1.8000	1.8000
K_9	-0.0019	-0.0006	0.0000	-0.0012	-0.0064
L_9	2.6000	2.6000	2.6000	2.6000	2.6000
K_{10}	0.0072	0.0051	0.0000	-0.0095	-0.0250
L_{10}	3.6000	3.6000	3.6000	3.6000	3.6000
K_{11}	0.0000	0.0000	0.0000	0.0000	0.0000
L_{11}	5.5400	5.5400	5.5400	5.5400	5.5400
K_{12}	0.0017	0.0005	0.0000	0.0008	0.0036
L_{12}	3.1000	3.1000	3.1000	3.1000	3.1000
K_{13}	0.0000	0.0000	0.0000	0.0000	-0.0001
L_{13}	3.9000	3.9000	3.9000	3.9000	3.9000

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