FACILITY FORM 602

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TO: Computation Division

FROM: Future Projects Branch
Aeroballistics Division
George C. Marshall Space Flight Center
National Aeronautics and Space Administration

SUBJECT: V-31
CALCULUS OF VARIATIONS
FOR VACUUM FLIGHT
ABOUT ONE ATTRACTING BODY
PHI-INDEPENDENT, FIXED-PLANE INJECTION
THREE DIMENSIONAL CARTESIAN COORDINATES
UTILIZING LUNAR PLANE EXAMINATION
AND IN-PLANE MODIFICATION

I. STARTING VALUES:

F_o (lb.)

W_o (lb.)

I_{sp} (sec.)

$$\bar{X}_o^* = \begin{bmatrix} x_o^* \\ y_o^* \\ z_o^* \end{bmatrix} \quad \text{(m.)}$$

$$\bar{\dot{X}}_o^* = \begin{bmatrix} \dot{x}_o^* \\ \dot{y}_o^* \\ \dot{z}_o^* \end{bmatrix} \quad \text{(m./sec.)}$$

$$\bar{\lambda}_o = \begin{bmatrix} \lambda_{1o} \\ \lambda_{2o} \\ \lambda_{3o} \end{bmatrix}$$

$$\bar{\dot{\lambda}}_o = \begin{bmatrix} 0 \\ \dot{\lambda}_{2o} \\ \dot{\lambda}_{3o} \end{bmatrix}$$

ϕ_L' (deg.)

A_{z_o} (deg.)

i_m (deg.)

Δt_L (sec.)

r^* (m.)

v^* (m./sec.)

t_o

t_c

Dt

t_{SCST}

Dt_{CST}

II. INITIAL COMPUTATION

a) Single Precision Computation

$$F\ (\text{kg.}) = \frac{F_o}{2.2046223}$$

$$m_o = \frac{W_o}{21.627344}$$

$$\dot{m}_o = \frac{F\ (\text{kg.})}{g_o\ I_{sp}}$$

b) Double Precision Computation

$$\Theta_m = 3\pi/2 - \omega\ \Delta t_L$$

$$\bar{n}_{m_e} = \begin{bmatrix} -\cos\ (\omega\ \Delta t_L)\ \sin i_m \\ \sin\ (\omega\ \Delta t_L)\ \sin i_m \\ \cos i_m \end{bmatrix}$$

$$[\phi] = \begin{bmatrix} -\sin \phi_L'\ \cos A_{z_o} & \cos \phi_L' & \sin \phi_L'\ \sin A_{z_o} \\ \sin A_{z_o} & 0 & \cos A_{z_o} \\ \cos \phi_L'\ \cos A_{z_o} & \sin \phi_L' & -\cos \phi_L'\ \sin A_{z_o} \end{bmatrix}$$

$$[\phi]^T$$

$$r = \sqrt{\bar{x}_o^* \cdot \bar{x}_o^*}$$

$$\bar{U} = \bar{\dot{x}}_o^* \times \bar{x}_o^*$$

$$\bar{u} = \frac{1}{\sqrt{\bar{U} \cdot \bar{U}}}\ \bar{U}$$

$$\cos \rho_1 = \bar{u} \cdot \bar{e}_3$$

$$\sin \rho_1 = \sqrt{1 - \cos^2 \rho_1}$$

$$\rho_2 = \tan^{-1} \left(\frac{\bar{e}_2 \cdot \bar{u}}{\bar{e}_1 \cdot \bar{u}} \right) \qquad -\pi \leqslant \rho_2 \leqslant \pi$$

$$[E]' = \begin{bmatrix} \cos \rho_1 \cos \rho_2 & \cos \rho_1 \sin \rho_2 & -\sin \rho_1 \\ -\sin \rho_2 & \cos \rho_2 & 0 \\ \sin \rho_1 \cos \rho_2 & \sin \rho_1 \sin \rho_2 & \cos \rho_1 \end{bmatrix}$$

$$\bar{X}_o' = [E]' \, \bar{X}_o^*$$

$$\sin \psi = -\frac{\bar{e}_1 \cdot \bar{X}_o'}{r}$$

$$\cos \psi = +\frac{\bar{e}_2 \cdot \bar{X}_o'}{r}$$

$$[C] = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[E] = [C] \, [E]'$$

$$[E]^T$$

$$\bar{X}_o = [E] \, \bar{X}_o^*$$

$$\dot{\bar{X}}_o = [E] \, \dot{\bar{X}}_o^*$$

$$\bar{N} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = [E] \cdot [\phi]^T \, \bar{n}_{m_e}$$

$$\dot{\lambda}_1 = - \frac{n_1 \left(\dot{y}_o \dot{\lambda}_{3o} - \dot{z}_o \dot{\lambda}_{2o} - y_o \dot{\lambda}_{3o} + z_o \dot{\lambda}_{2o}\right)}{n_3 y_o - n_2 z_o}$$

$$+ \frac{n_2 \left(\dot{x}_o \lambda_{3o} - \dot{z}_o \lambda_{1o} - x_o \dot{\lambda}_{3o}\right)}{n_3 y_o - n_2 z_o}$$

$$- \frac{n_3 \left(\dot{x}_o \lambda_{2o} - \dot{y}_o \lambda_{1o} - x_o \dot{\lambda}_{2o}\right)}{n_3 y_o - n_2 z_o}$$

$$\sin \chi_{P_o} = \frac{- \lambda_{1o}}{\sqrt{\lambda_{1o}^2 + \lambda_{2o}^2}}$$

$$\cos \chi_{p_o} = \frac{\lambda_{zo}}{\sqrt{\lambda_{xo}^2 + \lambda_{zo}^2}}$$

$$\bar{c}_o^* = [M]^T \begin{pmatrix} -\sin \chi_{p_o} \\ \cos \chi_{p_o} \\ 0 \end{pmatrix}$$

$$c_{xo}^* = \bar{e}_x \cdot \bar{c}_o^*$$

$$c_{yo}^* = \bar{e}_y \cdot \bar{c}_o^*$$

$$c_{zo}^* = \bar{e}_z \cdot \bar{c}_o^*$$

$$\sin \chi_{p_o}^* = \frac{-c_{xo}^*}{\sqrt{c_{xo}^{*2} + c_{yo}^{*2}}}$$

$$\cos \chi_{p_o}^* = \frac{c_{yo}^*}{\sqrt{c_{xo}^{*2} + c_{yo}^{*2}}}$$

$$\chi_{p_o}^* = \tan^{-1}\left(\frac{\sin \chi_{p_o}^*}{\cos \chi_{p_o}^*}\right), \quad -\pi \leq \chi_{p_o}^* \leq \pi$$

$$\sin \chi_{y_o}^* = c_{zo}^*$$

$$\cos \chi_{y_o}^* = \sqrt{c_{xo}^{*2} + c_{yo}^{*2}}$$

$$\chi^*_{y_o} = \tan^{-1}\left(\frac{\sin \chi^*_{y_o}}{\cos \chi^*_{y_o}}\right), \quad -\pi \leq \chi^*_{y_o} \leq \pi$$

$$\dot{\chi}_{p_o} = \frac{\lambda_{1o}\,\dot{\lambda}_{2o} - \dot{\lambda}_{1o}\,\lambda_{2o}}{\lambda_{1o}^2 + \lambda_{2o}^2}$$

$$\bar{a}^*_o = [E]^T \begin{pmatrix} -\dot{\chi}_{p_o} & \cos \chi_{p_o} \\ -\dot{\chi}_{p_o} & \sin \chi_{p_o} \\ & 0 \end{pmatrix}$$

$$\dot{a}^*_{xo} = \bar{e}_1 \cdot \bar{a}^*_o$$

$$\dot{a}^*_{yo} = \bar{e}_2 \cdot \bar{a}^*_o$$

$$\dot{a}^*_{zo} = \bar{e}_3 \cdot \bar{a}^*_o$$

$$\dot{\chi}^*_{p_o} = \frac{a^*_{xo}\,\dot{a}^*_{yo} - \dot{a}^*_{xo}\,a^*_{yo}}{a^{*\,2}_{xo} + a^{*\,2}_{yo}}$$

$$\dot{\chi}^*_{y_o} = \frac{\dot{a}^*_{zo}}{\cos \chi^*_{y_o}}$$

$$\ddot{\chi}_{p_o} = \frac{\lambda_{1o}\,\ddot{\lambda}_{2o} - \ddot{\lambda}_{1o}\,\lambda_{2o}}{\lambda_{1o}^2 + \lambda_{2o}^2} - \frac{2(\lambda_{1o}\,\dot{\lambda}_{1o} + \lambda_{2o}\,\dot{\lambda}_{2o})(\lambda_{1o}\,\dot{\lambda}_{2o} - \dot{\lambda}_{1o}\,\lambda_{2o})}{(\lambda_{1o}^2 + \lambda_{2o}^2)^2}$$

$$\bar{\bar{a}}_o^* = [\Xi]^T \begin{pmatrix} -\ddot{\chi}_{p_o} \cos \chi_{p_o} + \dot{\chi}_{p_o}^2 \sin \chi_{p_o} \\ -\ddot{\chi}_{p_o} \sin \chi_{p_o} - \dot{\chi}_{p_o}^2 \cos \chi_{p_o} \\ 0 \end{pmatrix}$$

$$\ddot{a}_{xo}^* = \bar{e}_1 \cdot \bar{\bar{a}}_o^*$$

$$\ddot{a}_{yo}^* = \bar{e}_2 \cdot \bar{\bar{a}}_o^*$$

$$\ddot{a}_{zo}^* = \bar{e}_3 \cdot \bar{\bar{a}}_o^*$$

$$\ddot{\chi}_{p_o}^* = \frac{a_{xo}^* \dot{a}_{yo}^* - \ddot{a}_{xo}^* a_{yo}^*}{a_{xo}^{*2} + a_{yo}^{*2}} - \frac{[(a_{xo}^* \dot{a}_{xo}^* + a_{yo}^* \dot{a}_{yo}^*)(a_{xo}^* \dot{a}_{yo}^* - \dot{a}_{xo}^* a_{yo}^*)}{(a_{xo}^{*2} + a_{yo}^{*2})^2}$$

$$\xi_x = (x_o^* \cos \chi_{p_o}^* + y_o^* \sin \chi_{p_o}^*)$$

$$\xi_y = (x_o^* \sin \chi_{p_o}^* - y_o^* \cos \chi_{p_o}^* - z_o^* \tan \chi_{y_o}^*)$$

$$M = \frac{3GM}{r^3} \xi_x \cdot \xi_y - \ddot{\chi}_{p_o}^*$$

$$\bar{\chi}_o^* = \begin{pmatrix} -\sin \chi_{p_o}^* \\ \cos \chi_{p_o}^* \\ \tan \chi_{y_o}^* \end{pmatrix}$$

$$\bar{\dot{\lambda}}_o^* = \begin{bmatrix} -\dot{\chi}_{p_o}^* \cos \chi_{p_o}^* - \dfrac{K \sin \chi_{p_o}^*}{2\,\dot{\chi}_{p_o}^*} \\ -\dot{\chi}_{p_o}^* \sin \chi_{p_o}^* + \dfrac{K \cos \chi_{p_o}^*}{2\,\dot{\chi}_{p_o}^*} \\ \dfrac{\dot{\chi}_{y_o}^*}{\cos^2 \chi_{y_o}^*} + \dfrac{K \tan \chi_{y_o}^*}{2\,\dot{\chi}_{p_o}^*} \end{bmatrix}$$

$$\bar{\ddot{\lambda}}_o^* = -\frac{GM}{r^3}\left[\bar{\lambda}_o^* - \frac{3\,\bar{x}_o^* \cdot \bar{\lambda}_o^*}{r^2}\,\bar{x}_o^*\right]$$

c) Values Needed for Future Single Precision Computation

$\bar{x}_o$

$\bar{\dot{x}}_o$

$[E]^T$

$\bar{\lambda}_o$

$\bar{\dot{\lambda}}_o$

$[\phi]$

$\bar{n}_{m_e}$

d) Print Initial and Precomputed Values.

III. UNIT VECTORS:

$$\bar{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \bar{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \bar{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

IV. SUGGESTED ORDER FOR COMPUTING LINE "n".

$\bar{x}$, $\dot{\bar{x}}$, $\bar{\lambda}$, $\dot{\bar{\lambda}}$, λ_7 are brought forward by Runge-Kutta Integration.

$\lambda_i = \bar{\lambda} \cdot \bar{e}_i$, $i = 1, 2, 3.$

$m = m_o - \dot{m}\, t$

F/m

$\chi_p = \tan^{-1} \left[- \frac{\lambda_1}{\lambda_3} \right]$

$\sin \chi_p$

$\cos \chi_p$

$\chi_y = \tan^{-1} \left[\frac{\lambda_2}{\sqrt{\lambda_1^2 + \lambda_3^2}} \right]$

$\sin \chi_y$

$\cos \chi_y$

$$\left[A_\chi\right]^T = \begin{bmatrix} \cos \chi_p & -\sin \chi_p \cos \chi_y & \sin \chi_p \sin \chi_y \\ \sin \chi_p & \cos \chi_p \cos \chi_y & -\cos \chi_p \sin \chi_y \\ 0 & \sin \chi_y & \cos \chi_y \end{bmatrix}$$

$$r = \sqrt{\bar{x} \cdot \bar{x}}$$

$$v = \sqrt{\dot{\bar{x}} \cdot \dot{\bar{x}}}$$

$$v_{cir} = \sqrt{GM/r}$$

$$\ddot{\bar{x}}_g = - GM/r^3 \, \bar{x}$$

$$\ddot{\bar{x}} = 1/m \quad \left[A_\chi\right]^T \quad \bar{F} + \ddot{\bar{x}}_g$$

Integrate $\ddot{\bar{x}}$ for $\dot{x}$ and $\bar{x}$ of next line.

$$\ddot{\bar{\lambda}} = - GM/r^3 \left[\bar{\lambda} - \frac{3\bar{x} \cdot \bar{\lambda}}{r^2} \bar{x} \right]$$

Integrate $\ddot{\bar{\lambda}}$ for $\dot{\bar{\lambda}}$ and $\bar{\lambda}$ of next line.

$$\dot{\lambda}_7 = - F/m^2 \, \bar{\lambda} \cdot \begin{bmatrix} -\sin \chi_p \cos \chi_y \\ \cos \chi_p \cos \chi_y \\ \sin \chi_y \end{bmatrix}$$

Integrate $\dot{\lambda}_7$ for λ_7 of next line.

$$RKCK = \lambda_1 \ddot{x} + \lambda_2 \ddot{y} + \lambda_3 \ddot{z} - \dot{\lambda}_1 x - \dot{\lambda}_2 \dot{y} - \dot{\lambda}_3 \dot{z} + \lambda_7 \dot{m}$$

$$\bar{x}^* = [E]^T \bar{x}$$

$$\dot{\bar{x}}^* = [E]^T \dot{\bar{x}}$$

$$\bar{x}_e = [\phi] \bar{x}^*$$

$$\dot{\bar{x}}_e = [\phi] \dot{\bar{x}}^*$$

$$r = \sqrt{\bar{x}_e \cdot \bar{x}_e}$$

$$v = \sqrt{\dot{\bar{x}}_e \cdot \dot{\bar{x}}_e}$$

$$\Delta r = r - r^*$$

$$\Delta v = v - v^*$$

$$\cos \phi = \frac{\bar{x}_e \cdot \dot{\bar{x}}_e}{rv}$$

$$\sin \phi = \sqrt{1 - \cos^2 \phi}$$

$$\phi = \tan^{-1} \left[\frac{\sin \phi}{\cos \phi} \right], \quad 0 \leq \phi \leq 2\pi$$

$$\sin \delta_r = \frac{\bar{n}_{m_e} \cdot \bar{x}_e}{r}$$

$$\cos \delta_r = \sqrt{1 - \sin^2 \delta_r}$$

$$\delta_r = \tan^{-1}\left[\frac{\sin \delta_r}{\cos \delta_r}\right], \quad -\pi \leq \delta_r \leq \pi$$

$$\sin \delta_v = \frac{\bar{n}_{m_e} \cdot \dot{\bar{x}}_e}{v}$$

$$\cos \delta_v = \sqrt{1 - \sin^2 \delta_v}$$

$$\delta_v = \tan^{-1}\left[\frac{\sin \delta_v}{\cos \delta_v}\right], \quad -\pi \leq \delta_v \leq \pi$$

$$n_e = \frac{\bar{x}_e \times \dot{\bar{x}}_e}{rv \sin \phi}$$

$$\cos \delta = \bar{n}_e \cdot \bar{n}_{m_e}$$

$$\sin \delta = \left|\bar{n}_e \times \bar{n}_{m_e}\right|$$

$$\delta = \tan^{-1}\left[\frac{\sin \delta}{\cos \delta}\right], \quad 0 \leq \delta \leq 2\pi$$