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APPLICATION OF A BANG-BANG CONTROL SYSTEM TO TWO SPINNING SPACE VEHICLES

by Paul S. Rempfer and Jerrold H. Suddath

Langley Research Center
Langley Station, Hampton, Va.

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APPLICATION OF A BANG-BANG CONTROL SYSTEM TO TWO SPINNING SPACE VEHICLES

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SUMMARY

A study was made of a technique for the synthesis of bang-bang control systems as applied to the attitude control of spinning space vehicles. Emphasis was placed on studying details of the technique when applied to a system with component dynamics. Consideration was also given to the handling of limit cycles.

The investigation was based primarily on analog computer simulations of the attitude control of two different spin-stabilized space vehicles. The report includes the development of the switching logic and of a simple cut-off function to eliminate wasting of fuel in limit cycles. Typical results obtained from computer simulations are included for illustrative purposes.

INTRODUCTION

Reference 1 presented the development and an example application of a technique for the synthesis of bang-bang control systems. Although the technique was sufficiently general to handle system component dynamics, the illustrative example excluded such effects in the interest of clarity. However, the inclusion of system component dynamics and the consideration of limit cycles are practical aspects of the synthesis problem which require attention.

The purpose of this investigation is to study the technique of reference 1 when system component dynamics and limit cycles are considered and to show how the technique might be used in a design study.

The investigation was based primarily on analog-computer simulations of the attitude control of two different spin-stabilized space vehicles. The first of these two vehicles, which had a pencillike configuration and high spin rate, was taken to be representative of some small satellite and/or upper stage configurations. The second vehicle, which had a toroidal configuration and low spin rate, was taken to be representative of some manned space station configurations.

SYMBOLS

$[A]$	$n \times n$ matrix of constant coefficients determined by dynamics of system
$\vec{a}$	constant n-vector determined by dynamics of system
$[B]$	constant positive definite $n \times n$ matrix
$\vec{b}$	n-dimensional gain vector
$\vec{e}$	unit vector in direction of free gyro spin axis, desired orientation of controlled vehicle
I	transverse moment of inertia $I_Y = I_Z$, slug-ft^2
I_X, I_Y, I_Z	moments of inertia of vehicle about principal vehicle X-, Y-, and Z-axes, respectively, slug-ft^2
J	maximum available control acceleration, radians/sec^2
k_1	constant scalar, radians^{-1}
k_2	constant scalar, sec/radian
M_Y	maximum pitch-control moment, ft-lb
p, q, r	angular velocities about principal X-, Y-, and Z-axes, respectively, radians/sec
p_o	positive constant spin rate of vehicle about X-axis, radians/sec
Q	lagging pitch rate to be fed back, radians/sec^2
S	Laplace transform variable
T	time constant for linear jet time lag, sec
T_m	jet-model time constant, sec
$T_1{}^2$	gyro time constant, sec^2
T_2	gyro time constant, sec
t	time, sec
U	linearly lagging scalar control variable
U_n	nonlinearly lagging scalar control variable

u	scalar control parameter
X,Y,Z	principal vehicle-axis coordinates
$\vec{x}$	n-dimensional state vector
x_7	variable defined by equation (A21), radians/sec^2
α,β	gimbal angles of free gyro measured relative to spinning principal body axes, radians
λ	eigenvalue of matrix, defined by equation (A3)
σ	switching function, $b \cdot x$
ω	frequency defined by equation (A8), radians/sec
$\lvert\ \rvert$	absolute expression
det( )	determinant of quantity in parentheses
sgn[ ]	algebraic sign of term within brackets

Subscripts:

m,n	general subscript notation. Subscript m used to differentiate between idealized system ($m = 0$, fourth-order linear control system); system including jet dynamics ($m = 1$, fifth-order linear control system); and system including both jet dynamics and rate-gyro dynamics ($m = 2$, seventh-order linear system and final seventh-order nonlinear system). Subscript n used to refer to particular component of variable when variable is a vector. When only one subscript is used, the variable relates to the m system.

Double subscripts denote the components of a vector. Dots over symbols indicate differentiation with respect to time. An arrow above a symbol denotes a vector. An asterisk denotes the transpose of a matrix.

MOTIVATION

The Idealized Control System

The idealized control system was developed in reference 1. It is idealized in that it does not consider system time lags. For convenience, a brief review of this system is given.

Figure 1 shows the idealized control system. The X-, Y-, and Z-axes are the vehicle principal body axes. The vehicle is spun about the X-axis which is designated as the spin axis.

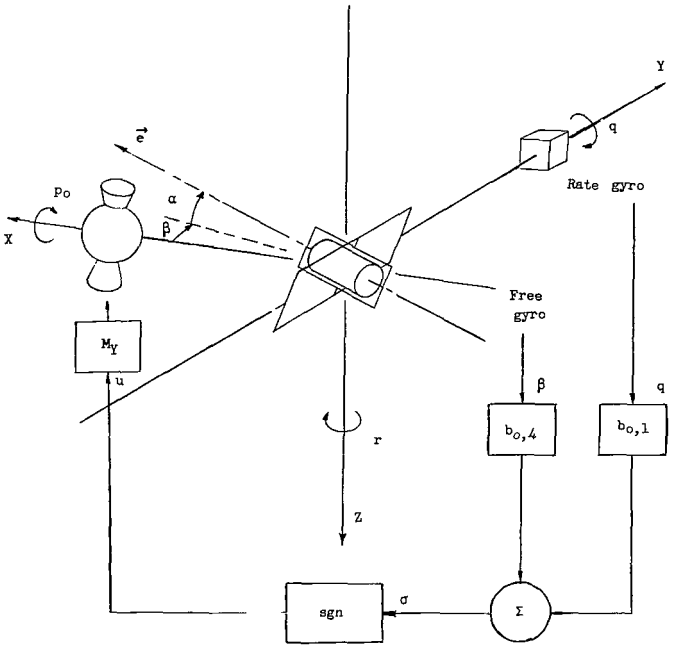


Figure 1.- Original idealized control system.

The control system consists of a free gyro, a rate gyro, a computer, and pitch jets. The free gyro is set so that its rotor axis is parallel with the desired orientation of the vehicle spin axis in inertial space. The angles α and β then determine the misalinement of the vehicle spin axis from its desired orientation. It should be noted that while the misalinement sensing device is considered to be a free gyro, it is by no means essential that this particular instrument be used. Any device capable of measuring the misalinement angle β would be acceptable.

The rate gyro is fixed so as to sense the pitch rate q. Because of gyroscopic coupling in the system, the angle β and the pitch rate q are sufficient to control the system. For the same reason, a pitch control moment is sufficient to control the attitude of the vehicle. The rate q is fed back through the gain $b_{0,1}$ and the angle β is fed back through the gain $b_{0,4}$. The sum of the feedback σ is the switching function. When it is positive, the pitch-control moment is made positive; when it is negative, the moment is made negative.

In reference 1, it was shown that time-dependent gains can be used to obtain good system response and to avoid end points. (For a discussion of end points, see refs. 2 and 3.) The form of time dependence used was

$$b_{m,n} = \hat{b}_{m,n} + \bar{b}_{m,n} \cos \omega_{m,n} t \tag{1}$$

where $\hat{b}_{m,n}$ is the time constant portion of $b_{m,n}$; $\bar{b}_{m,n}$ is the amplitude of the time varying portion of $b_{m,n}$; and $\omega_{m,n}$ is the frequency of the time varying portion of $b_{m,n}$.

Component Dynamics

The pitch jet.- If the control system were implemented, there would be some type of lag in the pitch-control moment due to pitch-jet dynamics. Typically, a jet response is composed of a transport lag, a buildup to peak thrust, a shutdown transport lag, and a decay to zero thrust. Since the main objective of this investigation is the study of a control-synthesis technique, a relatively simple jet response was deemed adequate for study. In practice, complications such as difference between thrust rise and decay dynamics may occur. Such

complications are not considered in the present investigation. The nonlinear jet response considered in this study is shown in figure 2, and its characteristics are:

Initial transport lag, sec	0.014
Linear buildup time constant, sec	0.008
Shutdown transport lag, sec . . .	0.008
Linear decay time constant, sec	0.008

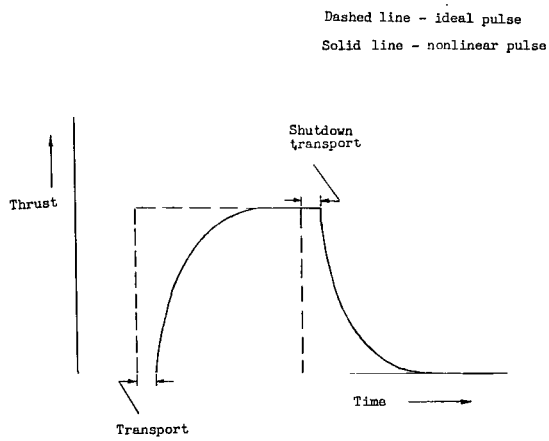


Figure 2.- Simulated nonlinear jet response compared with ideal pulse.

The rate gyro.- In addition to the pitch-jet time lag, there would be a time lag in the pitch-rate feedback due to rate-gyro dynamics. The equations for a rate gyro are derived in reference 4, and the characteristics of the rate gyro considered in this study are:

Maximum pitch rate, g, deg/sec	100
Maximum gimbal angle, deg	3
Gyro time constant, T_1^2, sec^2	0.00036
Gyro time constant, T_2, sec	0.120

The time lag is linear and second order. The equation for the lagging pitch rate Q is given by

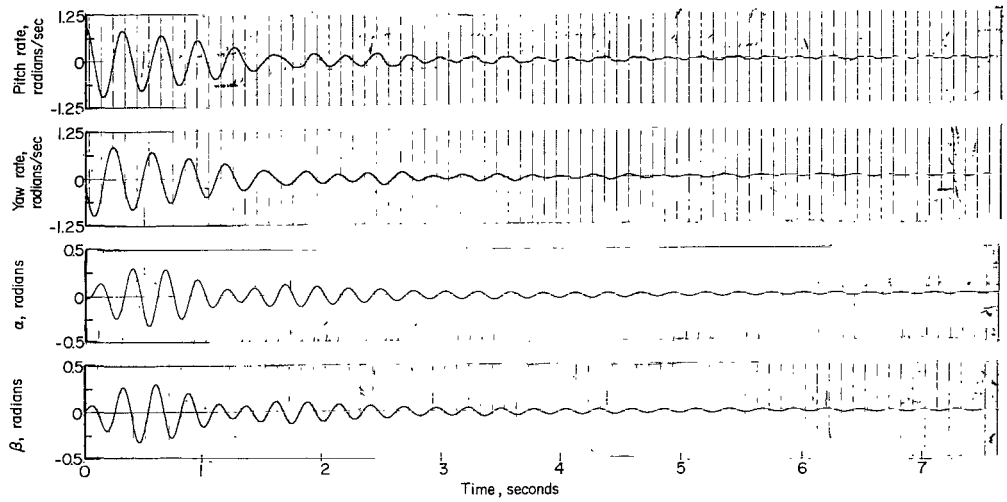
$$T_1^2\ddot{Q} + T_2\dot{Q} + Q = q \tag{2}$$

Effect of Component Dynamics

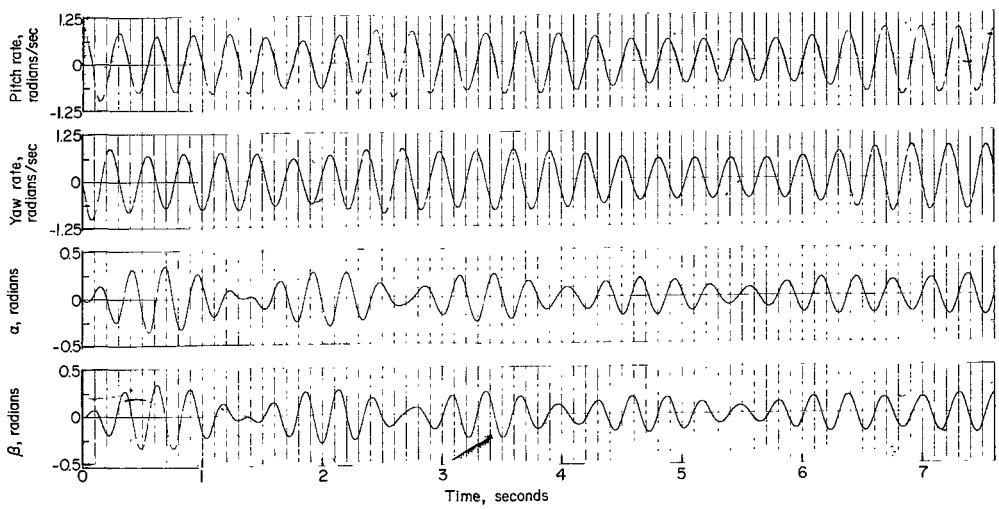
In reference 1, the idealized control system was applied to a rapidly spinning space vehicle with a pencillike configuration. The characteristics of the space vehicle are:

I_X, slug-ft^2	8.0
I, slug-ft^2	40.0
p_o, radians per sec	25.0
M_Y, ft-lb	40.0

Gains were found for which the system performed well; however, the system time lags had not been considered. When the pitch-rate time lag and the pitch-jet time lag were introduced into the idealized system, the system stability deteriorated. This effect is illustrated in figure 3, where a typical system response without time lags is compared with the corresponding response with time lags.



(a) System without time lags.



(b) System with time lags.

Figure 3.- Idealized control system for an initial condition of q = 1.0 radian/sec.

Statement of Problem

The idealized control system for the attitude stabilization of a rapidly spinning space vehicle ceases to be effective with the introduction of time lags. This control system must therefore be modified to perform with time lags. With this modification accomplished, a cut-off switch must be created to eliminate the wasting of fuel in the anticipated limit cycle.

ANALYSIS

Modification for Time Lags

In order to get an indication of the modification necessary in the control system, a brief mathematical analysis in which linear time lags were introduced was considered. This analysis, which is presented in the appendix, indicated that to correct for a jet time lag, the jet output should be fed back. To correct for the rate gyro time lag, only an adjustment of gains was necessary. This approach was used in the analog simulations. It should be pointed out that the nonlinear transport lag in the jet response was included in the analog simulations.

Use of Jet Model

To feed back the actual jet response would require adding thrust sensors to the system. The inclusion of these sensors would mean additional hardware and the possibility of additional component dynamics. It was decided that, rather than feed back the actual jet response, a simpler approach would be to build a model of the jet response by use of analog components and to have its output fed back. A further simplification would be to make the model linear and of first order. The system, with the linear model of the jet response inserted in the control loop, is shown in figure 4.

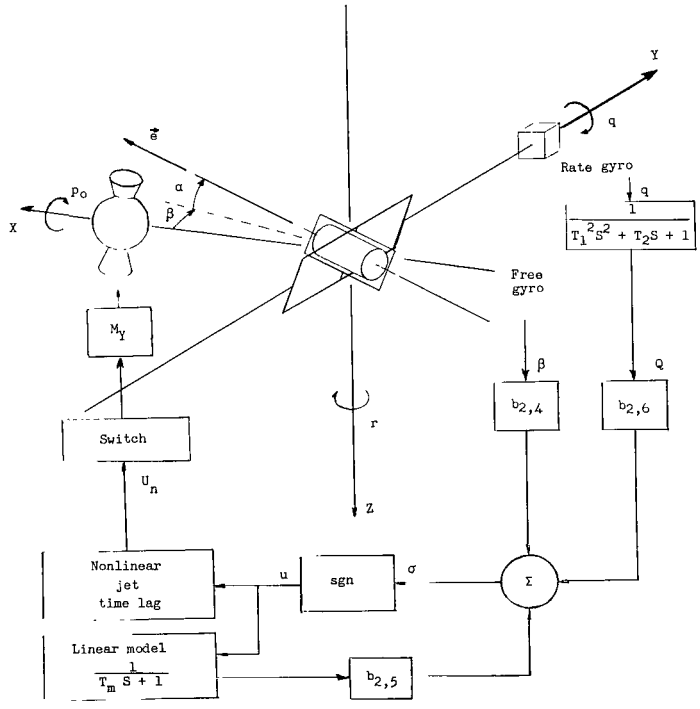


Figure 4.- Modified control system with time lags included and model output fed back.

The system was simulated on an analog computer, and values were found for the gains and for the model time constant for which the system performed well. It is

emphasized that the choice of model can greatly affect the performance of the system and that, in general, this choice may be a critical step in the synthesis procedure. In particular, if the actual jet response is more complicated than the one considered herein (which, as previously noted, may well be the case), then a more sophisticated model might be required in the control loop.

The Cut-Off Switch

Since the uncontrolled system is neutrally stable, the wasting of fuel in a limit cycle can be eliminated by simply turning off the control when the residual rates and misalinement are deemed sufficiently small. Of course, the cut-off function can depend only on the information available to the control system, which in this case was the angle β and the lagging pitch rate Q.

A cut-off function, based only on β and Q, was developed from the following considerations. From the dynamics of the problem, it follows that the magnitudes of β and Q can remain small over relatively long time intervals only if the magnitudes of α, q, and r also remain small over the interval. Therefore, sufficient information on the magnitudes of the misalinement and rates should be contained in the low-frequency component of $k_1|\beta| + k_2|Q|$, where k_1 and k_2 are positive weighting factors to be determined. This weighted sum was passed through a low-pass filter to generate the cut-off function. The control system was then turned on or off depending on the value of this function.

RESULTS AND DISCUSSION

Nominal Control System for Pencillike Configuration

The system performance was judged by its average response time and residual misalinement for the many initial conditions for which the system was tried. Gains were found for which the system gave a good overall performance. These gains were $b_{2,4} = 4.0 + 4.0 \cos 45t$, $b_{2,5} = 0.6 - 0.6 \cos 80t$, and $b_{2,6} = -1.75$. The value for the jet-model time constant T_m was 0.016 second. A good cut-off function was found to be the function $|Q| + 2.5|\beta|$ passed through a linear filter with a time constant of 0.1 second. The system was turned on when the cut-off function was greater than 0.08 and off when it was less than 0.04. This system was then considered to be the nominal system.

A typical response of the nominal system is shown in figure 5. In the results, δ is essentially the magnitude of the error $\left(\text{i.e., } \delta = |q| + |r| + |\alpha| + |\beta|\right)$, and σ is the switching function. In the figure, σ appears to be identically zero after about 1.5 seconds. That σ was not identically zero is evident from the fact that the pitch control switched properly and damped out the error. The system is shown to damp to slightly over 1° in angular misalinement at which time the system would run into a limit cycle if it were not turned off. The average response time was about 6 seconds.

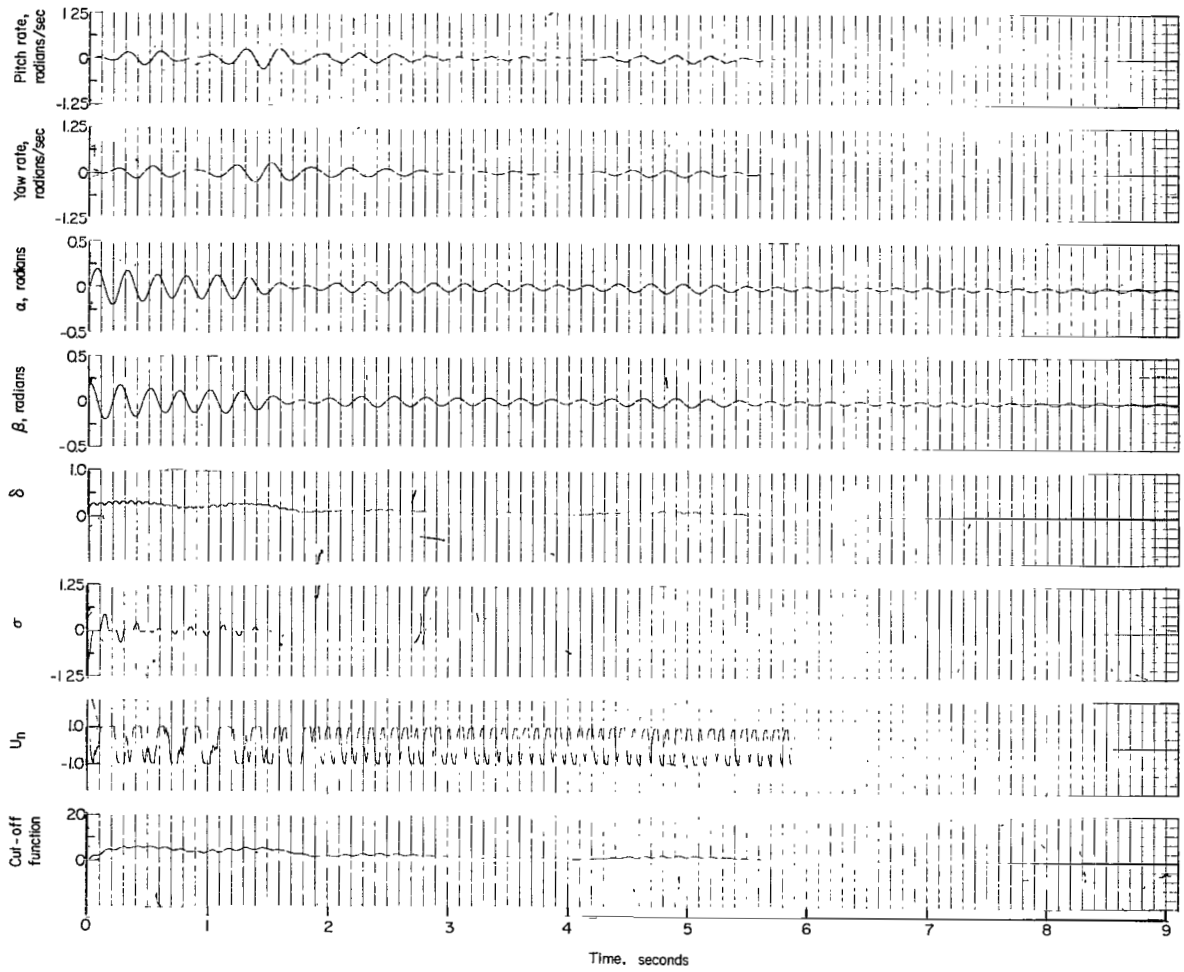


Figure 5.- Time history of response of pencillike configuration to control for initial angular misalinement of $\beta = 0.2$ radian.

A parametric study.- A study of the various parameters of the nominal control system was made to determine their effect on the response. It was expected that system sensitivity to parameter adjustment would increase (decrease) with increased (decreased) spin rate. This idea can be explained by the following simple considerations. Suppose the pitch jets were commanded to give a positive torque at time $t = 0$. Suppose also, because of a time lag, that the positive torque does not develop until $t = \tau > 0$. Then the control is essentially out of phase by the amount $p_o\tau$ (where p_o is the vehicle spin rate). Of course, this phase error is compensated for by putting in a lead of $p_o\tilde{\tau}$, where $\tilde{\tau}$ is an approximation of τ and can be expressed as $\tilde{\tau} = \tau + \delta\tau$. (Clearly, $\delta\tau$ is the error in the approximation.) Hence, the error in the lead compensation, which is equal to $p_o\delta\tau$, is proportional to the spin rate.

The jet-model time constant: The nominal system was studied for its sensitivity to a variation in the model time constant. Deviations as large as ±25 percent were tolerable, but larger deviations resulted in considerably longer response times and larger residual angular misalinements. Figure 6 shows a typical response with a -25-percent deviation in the time constant and can be seen to compare closely with the nominal response shown in figure 5.

It is reemphasized that large differences between the decay and the buildup characteristics of the actual jet response might require the use of a more sophisticated model within the control system.

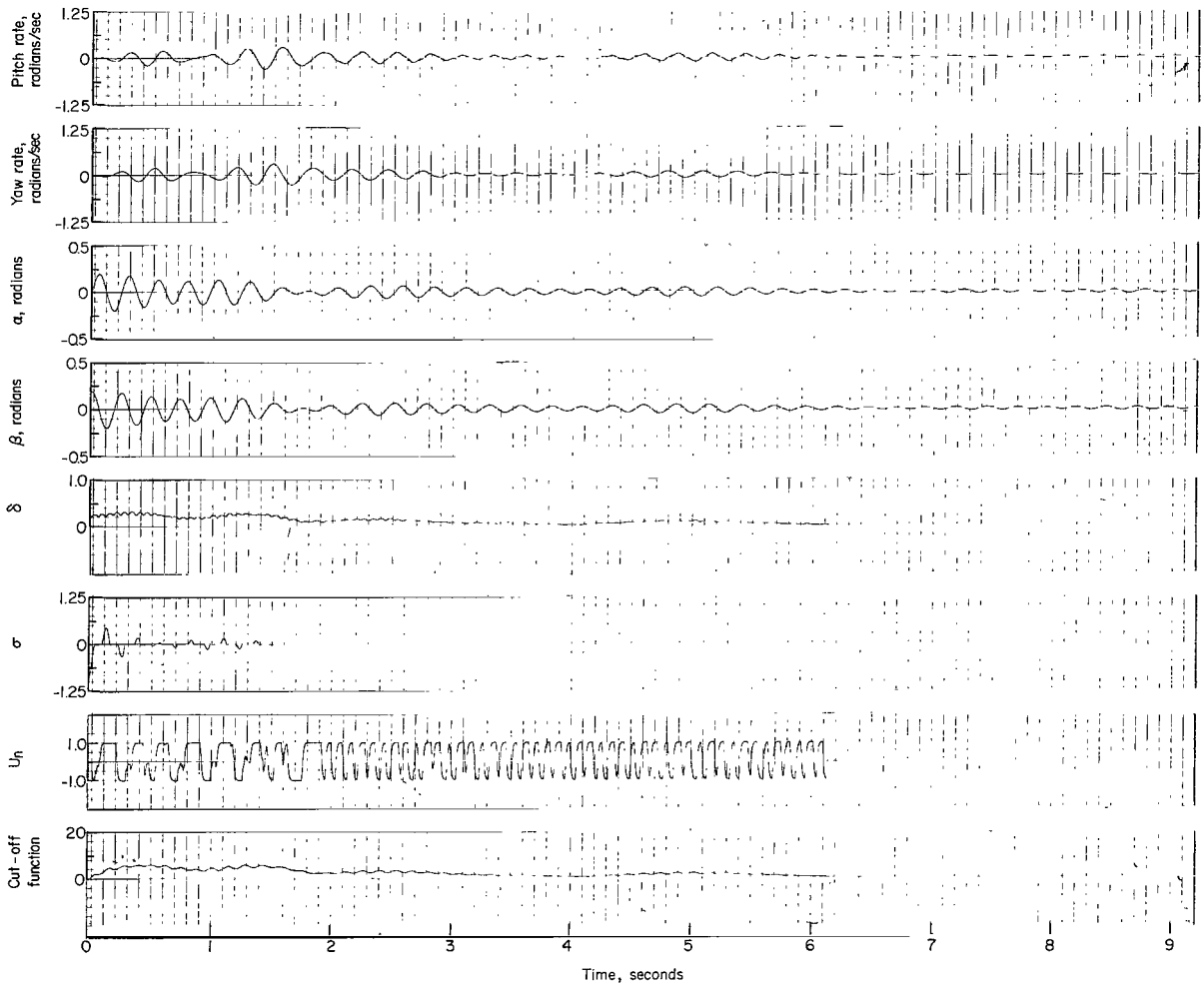
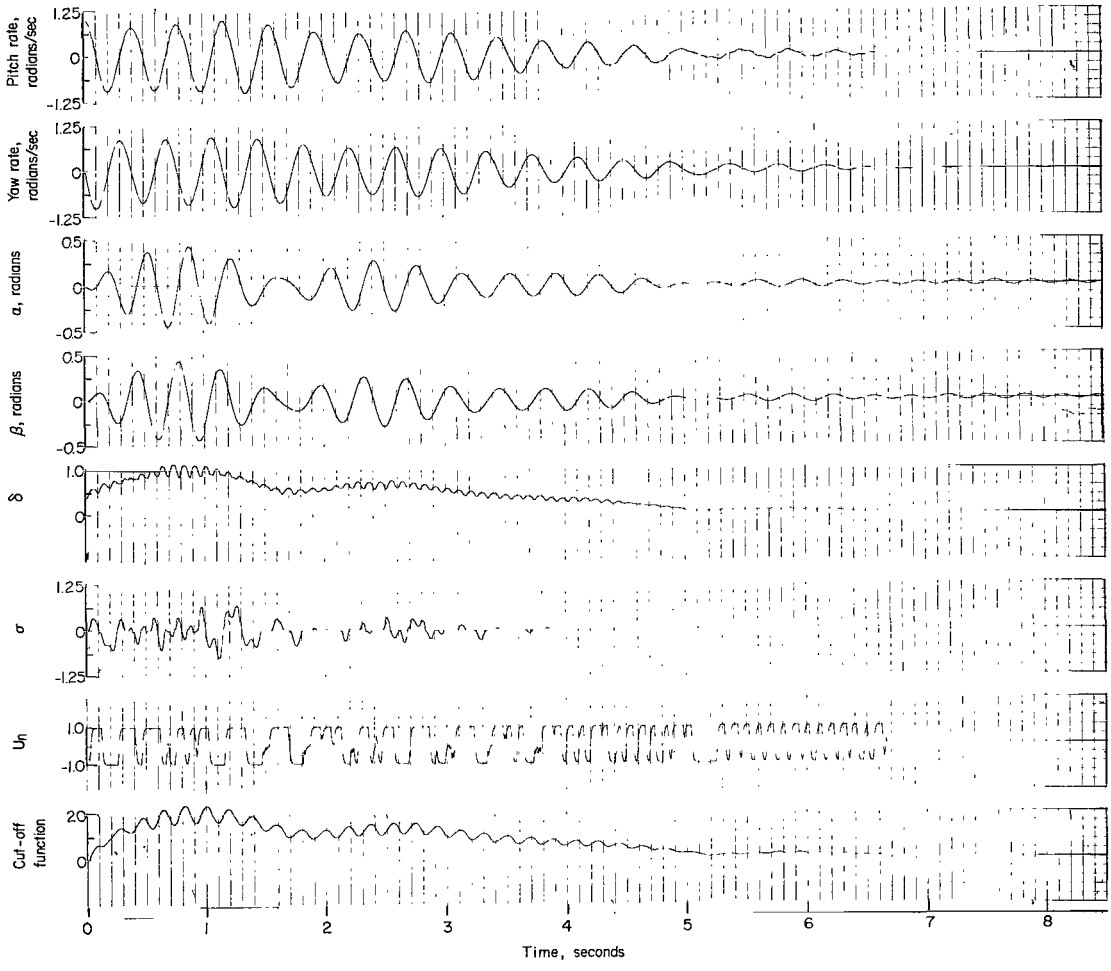


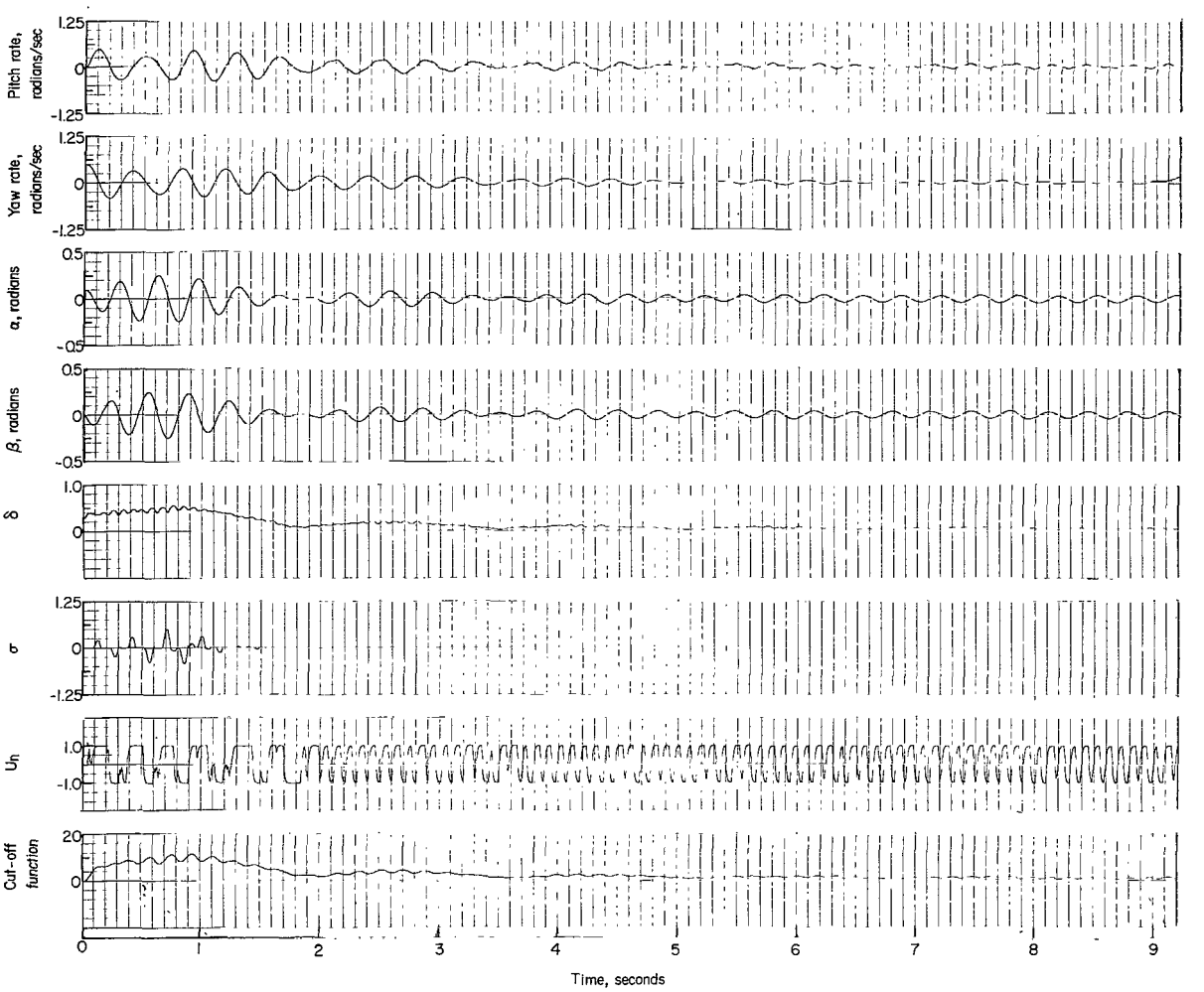
Figure 6.- Time history of response of pencillike configuration to control for initial angular misalinement of $\beta = 0.2$ radian with model time constant decreased 25 percent.

Vehicle spin rate: The nominal system was next studied for sensitivity to spin-rate deviation. This study indicated that spin rate could be decreased as much as 20 percent without resulting in instability. However, from certain initial conditions, the system would run into a limit cycle with a residual angular misalinement between 2° and 3°. Figure 7 illustrates the dependence of system performance on initial conditions for a 20-percent decrease in spin rate. A decrease in spin rate greater than 20 percent resulted in instability for some initial conditions.



(a) Initial condition q = 1.0 radian/sec.

Figure 7.- Time history of response of pencillike configuration to control for initial angular misalinement with vehicle spin rate decreased 20 percent.



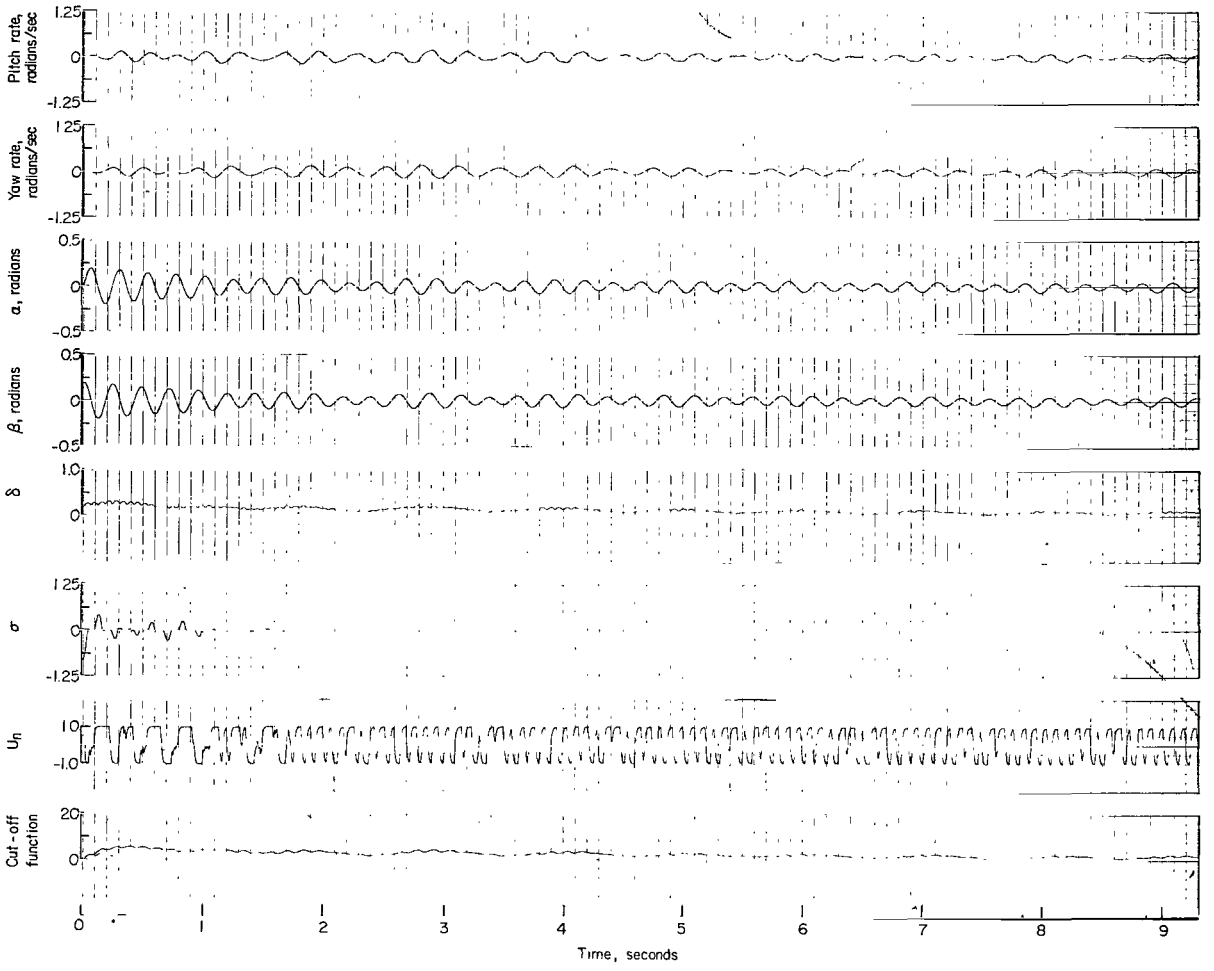
(b) Initial condition $r = 0.5$ radian/sec and $\alpha = 0.1$ radian.

Figure 7.- Concluded.

Similar results were obtained for a spin rate increase up to 5 percent. That is to say that some initial conditions resulted in limit cycles for an increase of 5 percent as illustrated by figure 8. An increase in spin rate greater than 5 percent resulted in instability for some initial conditions.

These results clearly show the expected sensitivity to parameter adjustment at increased spin rates. It is interesting to note that in reference 1, where time lags were not considered, the idealized system was relatively insensitive to spin-rate deviations up to ±50 percent.

Effect of external moment.- The system was next studied to determine the effect of an external moment. For purposes of study, the moment was taken to be constant relative to the spinning principal body axis system. Physically,



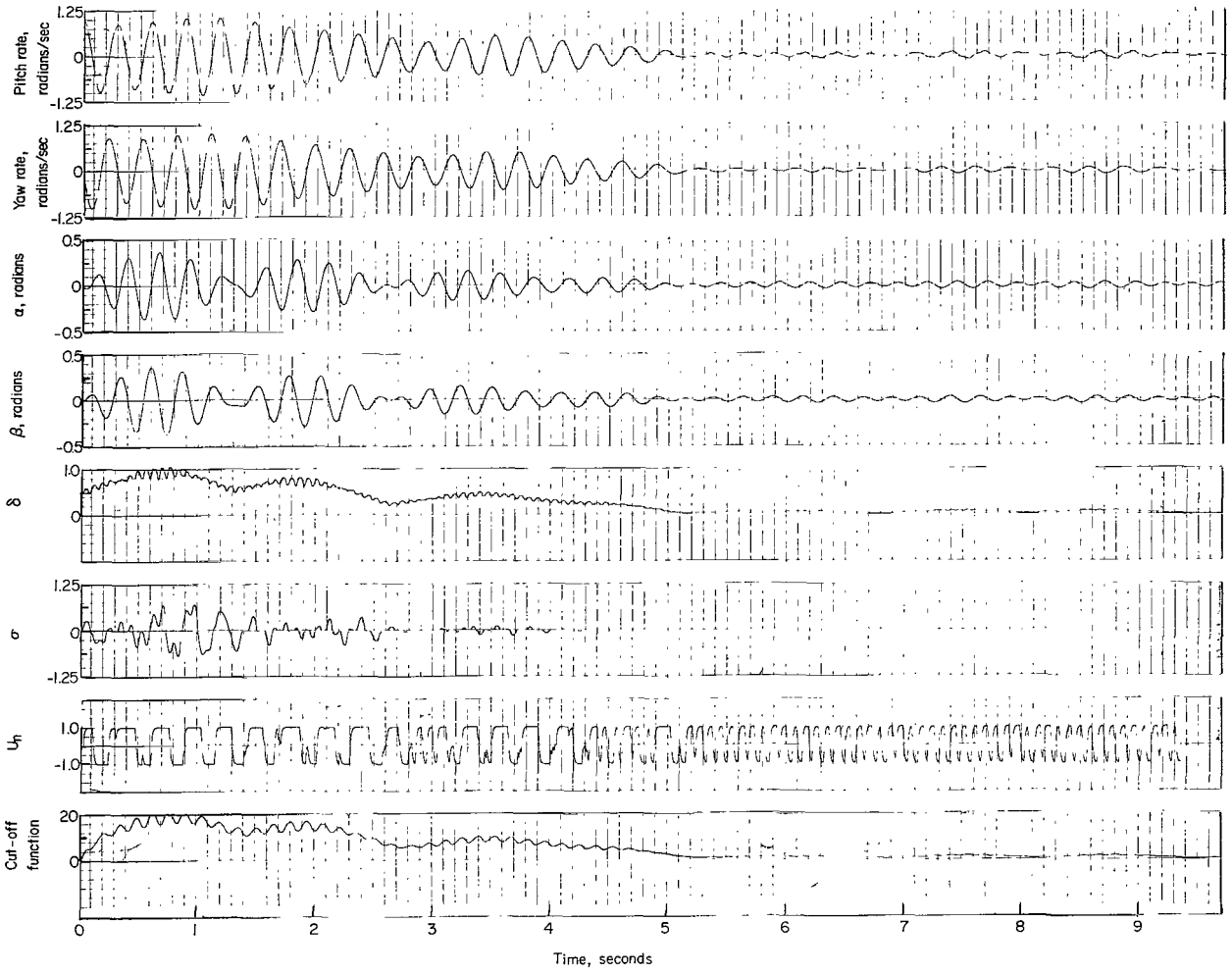
(a) Initial condition $\beta = 0.2$ radian.

Figure 8.- Time history of response of pencillike configuration to control for initial angular misalinement with vehicle spin rate increased 5 percent.

such a disturbance might be due to a thrust misalinement. Pitching moments, yawing moments, and vector sums of the two were considered in combination with various initial conditions. It was found that external torques with magnitudes up to 40 percent of the control torque did not adversely affect system performance. A typical response with external moment applied is shown in figure 9.

Application to a Spinning Space Station

The second vehicle to which the control logic of reference 1 was applied was a large hexagonal slowly spinning space station. The characteristics of this vehicle are given in the following table:



(b) Initial condition q = 1.0 radian/sec.

Figure 8.- Concluded.

I_X, slug-ft^2	1.5×10^7
I, slug-ft^2	1.05×10^7
p_o, radians/sec	0.314
M_Y, ft-lb	3×10^3

Because of its inertia ratio and low spin rate, the vehicle's natural frequency is low. For this reason it was believed that in operation the control jets would fire for relatively long periods of time. If this were the case, the time lags would have negligible effect on system performance. With this in mind, the idealized system was utilized.

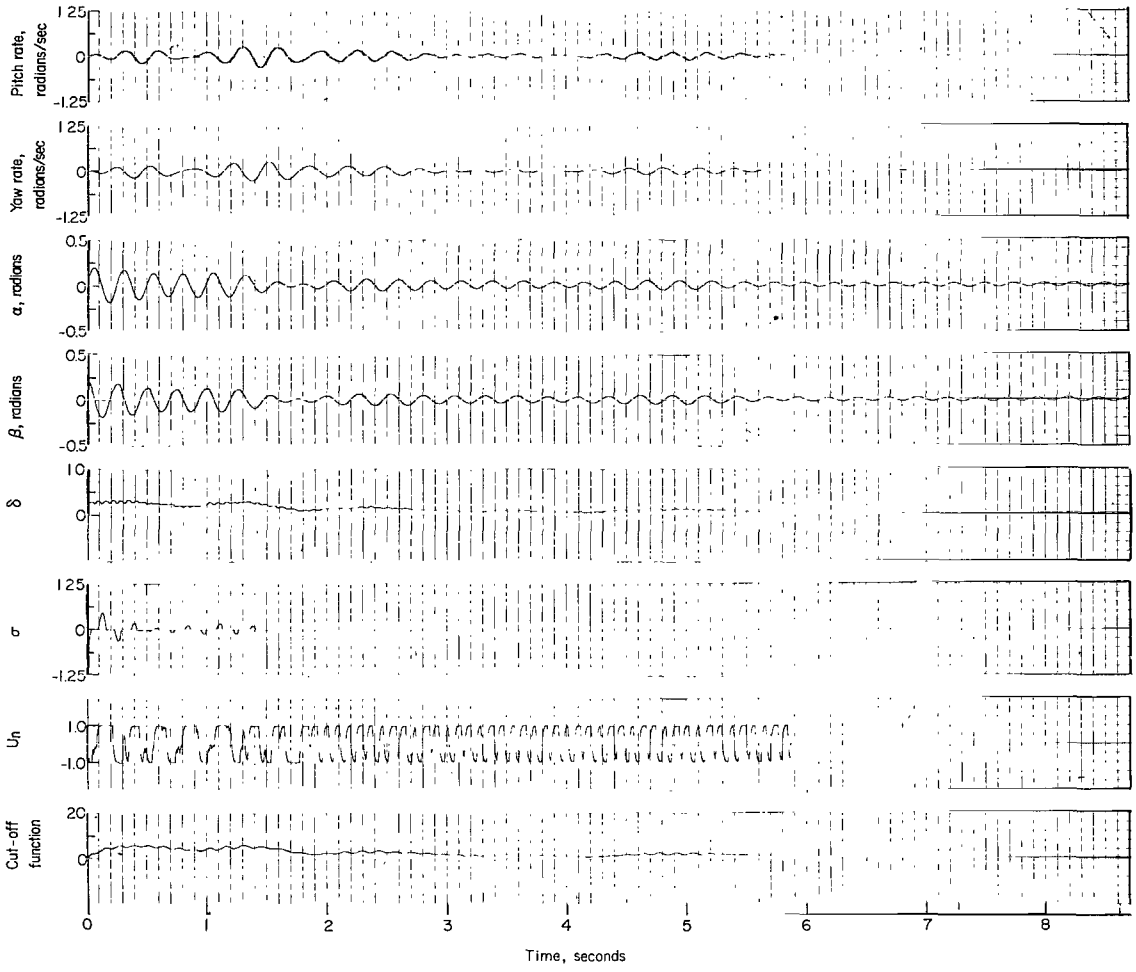


Figure 9.- Time history of response of pencillike configuration to control for initial angular misalinement of $\beta = 0.2$ radian and external pitching moment applied equal to $0.4M_Y$.

With the idealized system being utilized limit cycles were no longer a problem. However, when the control function becomes small and changes sign rapidly the control chatters and operates inefficiently. The problem of when to shut down control operations then still existed. This problem was handled with a cut-off function similar to that used in the study of the pencillike configuration.

The system was simulated on an analog computer and the gains found which gave good overall system response. They were $b_{0,1} = -1.0 - \cos 0.5t$ and $b_{0,4} = 0.2$. A good cut-off function was found to be $25|Q| + |\beta|$ passed through a linear filter with a time constant of 5 seconds. The control system

was then turned on when this function was greater than 0.20 and was cut off when it was less than 0.05. A typical system response is shown in figure 10.

As shown in the figure, jet firing times were on the order of 10 seconds. These long firing times obviously result from the slower spin rate and longer natural periods in the large vehicle. A jet time lag on the order of 10 milliseconds is clearly insignificant in such a slowly moving system. For this reason, utilization of the idealized control logic was considered to be justified. From a number of cases where the initial angular misalinement was 0.1 radian, it was observed that the system would damp to virtually no residual angular misalinement (residual angular rates of about 0.0004 radian/sec), after which it would shut off. The average response time was about 280 seconds.

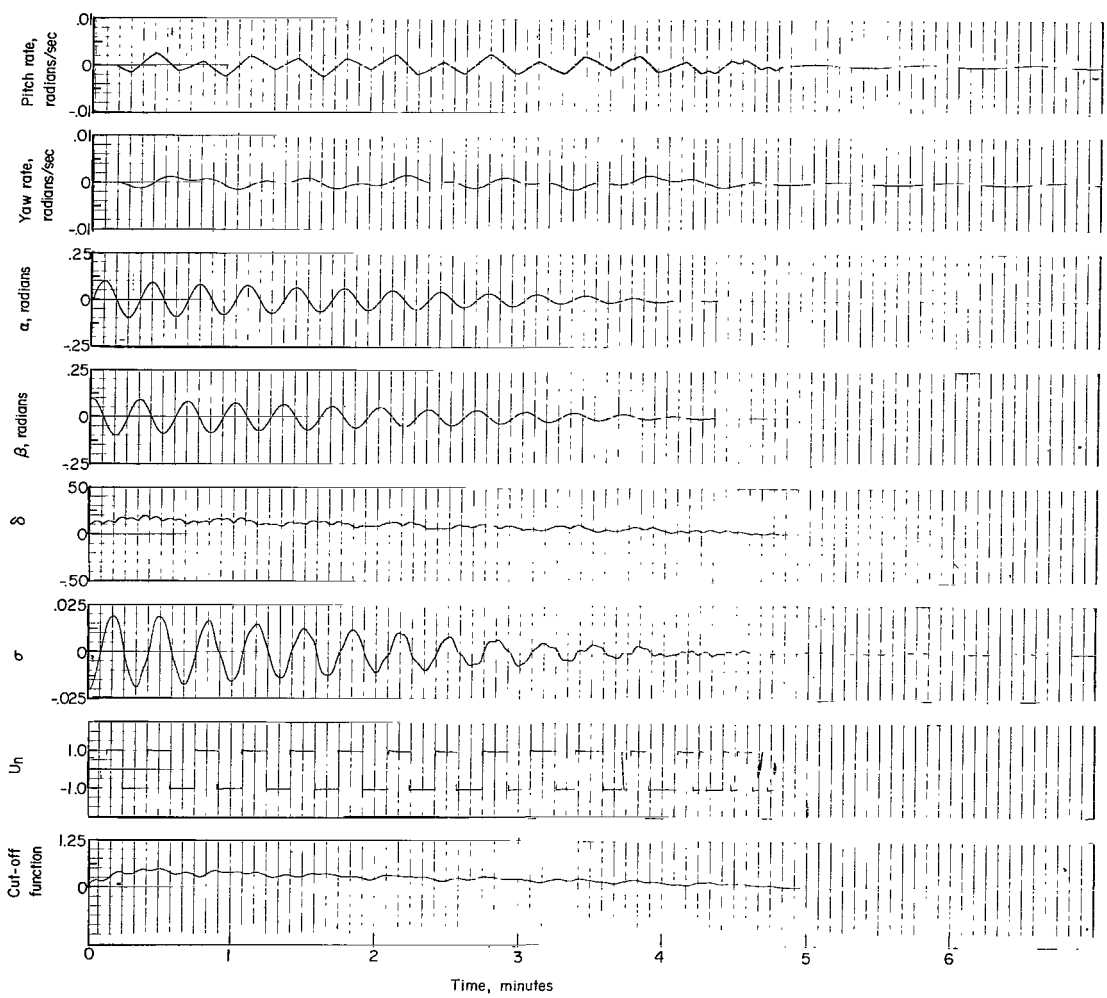


Figure 10.- Time history of response of toroidal configuration to control for initial angular misalinement of $\beta = 0.1$ radian.

CONCLUDING REMARKS

A study was made of a technique for the synthesis of bang-bang control systems as applied to the attitude control of spinning space vehicles. Emphasis was placed on studying details of the technique when applied to a system with component dynamics. While some analytical results were utilized, the investigation was based primarily on analog computer simulations of the attitude control of two different space vehicles.

The first application was to a pencillike vehicle configuration with a high spin rate. It was found that component dynamics cannot be neglected in deriving the switching logic for this type of system. Adequate compensation for a transport plus first-order jet lag was produced by feeding back the output of a linear model of the jet dynamics which was built into the control loop. This feedback had the effect of introducing a lead into the switching to compensate for the lag of the jet. The rate-gyro dynamics were compensated for by a simple adjustment of the gains in the control loop. This system was quite sensitive to increases from the nominal spin rate in that 5-percent increases were very deleterious to system performance. It was, however, less sensitive to decreases in spin rate.

The second application was to a toroidal configuration with a low spin rate and high inertia. The long periods characteristic of the motion of this vehicle permitted neglect of the jet and gyro dynamics. Thus, this study was essentially an application of the type of idealized control system derived in reference 1 to a radically different vehicle. The results showed that this type of vehicle could be very effectively controlled by the idealized control system.

It was found that a relatively simple cut-off function could be used to eliminate the wasting of fuel in limit cycles. Of course, there would be some residual motion after the control system is cut off.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., October 16, 1964.

APPENDIX

MODIFICATION ANALYSIS

In reference 1 the following equations are considered:

$$\dot{\vec{x}} = [A]\vec{x} + u\vec{a} \tag{A1}$$

where

$$u = \operatorname{sgn}\left[\vec{b} \cdot \vec{x}\right] = \frac{\vec{b} \cdot \vec{x}}{\left|\vec{b} \cdot \vec{x}\right|} \tag{A2}$$

where u is understood to be zero when $\vec{b} \cdot \vec{x} = 0$. It was found that the conditions on the vector $\vec{b}$ that make the system stable are:

(a) The vector $\vec{b}$ is of the form

$$\vec{b} = -[B]\vec{a}$$

where

$$[B] = [B]^* > 0$$

(b) All roots of

$$\det\left([A] + \vec{a}\vec{b}^* - \lambda[I]\right) = 0 \tag{A3}$$

have negative real parts. In equation (A3), $[I]$ is the identity matrix.

This relationship was applied to the idealized set of equations to obtain the original idealized gain vector. The equations for the idealized system are written in vector-matrix form

$$\dot{\vec{x}}_0 = [A_0]\vec{x}_0 + \vec{a}_0 u \tag{A4}$$

where

$$\vec{x}_0 = \begin{bmatrix} q \\ r \\ \alpha \\ \beta \end{bmatrix} \tag{A5}$$

$$\left[A_0\right] = \begin{bmatrix} 0 & \omega & 0 & 0 \\ -\omega & 0 & 0 & 0 \\ -1 & 0 & 0 & p_o \\ 0 & -1 & -p_o & 0 \end{bmatrix} \tag{A6}$$

$$\vec{a}_0 = \begin{bmatrix} J \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A7}$$

$$\omega = \left(1 - \frac{I_X}{I}\right)p_o \tag{A8}$$

$$J = \frac{M_Y}{I} \tag{A9}$$

$$u = \operatorname{sgn}\left[\vec{b}_0 \cdot \vec{x}_0\right] \tag{A10}$$

When the conditions are applied to the vector $\vec{b}_0$, the form with the least elements which satisfy these conditions is

$$\vec{b}_0 = \begin{bmatrix} b_{0,1} \\ 0 \\ 0 \\ b_{0,4} \end{bmatrix} \tag{A11}$$

Thus, in the idealized system it is necessary to feed back q, corresponding to $b_{0,1}$, and β, corresponding to $b_{0,4}$.

Now it is desired to modify the original system so that it will handle time lags. The system has a time lag in the pitch feedback due to gyro dynamics and a time lag in the control moment due to the jet dynamics. The jet time lag is considered first.

For purposes of analysis the jet time lag is considered linear and first order. The equation used to describe it is

$$T\dot{U} + U = u \tag{A12}$$

where U is the lagging control and u is the ideal control. This equation may be rewritten

$$\dot{U} = -\frac{1}{T}U + \frac{1}{T}u \tag{A13}$$

If u in the original set of equations (A4) is replaced by U and equation (A13) is added to the original set, a new fifth-order set is formed which may be written in vector matrix form

$$\dot{\vec{x}}_1 = \left[A_1\right]\vec{x}_1 + \vec{a}_1 u \tag{A14}$$

where

$$\vec{x}_1 = \begin{bmatrix} q \\ r \\ \alpha \\ \beta \\ U \end{bmatrix} \tag{A15}$$

$$\left[A_1\right] = \begin{bmatrix} 0 & \omega & 0 & 0 & J \\ -\omega & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & +p_o & 0 \\ 0 & -1 & -p_o & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T} \end{bmatrix} \tag{A16}$$

$$\vec{a}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{T} \end{bmatrix} \tag{A17}$$

$$u = \operatorname{sgn}\left[\vec{b}_1 \cdot \vec{x}_1\right] \tag{A18}$$

Applying the conditions for a stable gain vector to the vector $\vec{b}_1$ gives

$$\vec{b}_1 = \begin{bmatrix} b_{1,1} \\ 0 \\ 0 \\ b_{1,4} \\ b_{1,5} \end{bmatrix} \tag{A19}$$

This application means that with the jet time lag the feedback is q through the gain $b_{1,1}$, β through the gain $b_{1,4}$, and U through the gain $b_{1,5}$. The way to correct for a lagging control is to feed the control output back. The gyro time lag is considered next. The equation used is

$$T_1^2\ddot{Q} + T_2\dot{Q} + Q = q \tag{A20}$$

This equation is broken up into two first-order equations. Let

$$\dot{Q} = x_7 \tag{A21}$$

$$\dot{x}_7 = \ddot{Q} = \frac{1}{T_1^2} q - \frac{1}{T_1^2} Q - \frac{T_2}{T_1^2} x_7 \tag{A22}$$

Adding equations (A21) and (A22) to the fifth-order set given by equation (A14) a seventh-order set is obtained and written

$$\dot{\vec{x}}_2 = \left[A_2\right]\vec{x}_2 + \vec{a}_2 u \tag{A23}$$

where

$$\vec{x}_2 = \begin{bmatrix} q \\ r \\ \alpha \\ \beta \\ U \\ Q \\ x_7 \end{bmatrix} \tag{A24}$$

$$\vec{a}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{T} \\ 0 \\ 0 \end{bmatrix} \tag{A25}$$

$$\left[A_2\right] = \begin{bmatrix} 0 & \omega & 0 & 0 & J & 0 & 0 \\ -\omega & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & p_o & 0 & 0 & 0 \\ 0 & -1 & -p_o & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{1}{T_1{}^2} & 0 & 0 & 0 & 0 & -\frac{1}{T_1{}^2} & -\frac{1}{T_1{}^2} \end{bmatrix} \tag{A26}$$

$$u = \operatorname{sgn}\left[\vec{b}_2 \cdot \vec{x}_2\right] \tag{A27}$$

The condition that $\vec{b}_2$ be of the form $-\left[B_2\right]\vec{a}_2$ can be satisfied with $\vec{b}_2$ of the form

$$\vec{b}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_{2,4} \\ b_{2,5} \\ b_{2,6} \\ 0 \end{bmatrix} \tag{A28}$$

This means that if all the roots of

$$\det\left(\left[A_2\right] + \vec{a}_2\vec{b}_2{}^* - \lambda\left[I\right]\right) = 0 \tag{A29}$$

have negative real parts, the only necessary feedback is β through the gain $b_{2,4}$, U through the gain $b_{2,5}$, and Q through the gain $b_{2,6}$. Rather than solve for the roots of equation (A29) this feedback was assumed sufficient and the problem simulated on an analog computer. It was found that the assumption was correct since the linear problem was made to work with the control output fed back. Modification for the gyro time lag was only an adjustment of gains.

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