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PREFACE

The name of the great scientist K. E. Tsiolkovskiy is well known both in this country and abroad. Today, his original research, in the course of which he not only predicted the development of jet aircraft and long-range rockets but theoretically confirmed their feasibility, is necessarily of great interest to the entire aerospace community. He was the founder of the science of astronautics (cosmonautics), which is concerned with the problems of interplanetary travel, and he derived the classical formulas of rocket flight.

Long before the first jet aircraft were actually built, he had prepared sketches and worked out the theory. K. E. Tsiolkovskiy was much preoccupied with the problems of constructing metal dirigibles and put forward a series of original designs. He also pointed out the economies of building large airships. Tsiolkovskiy was a pioneer in the field of experimental aerodynamics. In 1897 in Kaluga he built one of the world's first wind tunnels and for five years he used it consistently to investigate models of airfoils, dirigibles, various geometrical shapes, etc. His powers of analysis and theoretical reasoning enabled Tsiolkovskiy to derive a number of highly important laws of aerodynamics. This work was highly valued by N. Ye. Zhukovskiy.

In his "Aeroplan, ili ptitsepodobnaya (aviatsionnaya) letatel'naya mashina" (The Aeroplane, or Bird-Like (Aerial) Flying Machine), 1894*, Tsiolkovskiy produced the first aerodynamic aircraft design and proposed a workable scheme for an airplane, far in advance of contemporary work in other countries. His proposals became the basis for the construction of the aircraft of those times -- a monoplane with unbraced streamlined wings, wheeled landing gear, and coaxial rotation of the propellers. In the same paper he also proposed for the first time an electrically driven gyroscopic autopilot for the elevators.

In addition to his engineering and technical studies, Tsiolkovskiy wrote a series of papers on the problems of natural science. Most of them reflect his materialistic views of nature, of the structure of the cosmos, and of the future of mankind.

Before the Revolution Tsiolkovskiy led a difficult life. A self-taught man, half-deaf since childhood, he had to pursue his studies alone relying on his own unaided resources. He was plagued

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*Nauka i zhizn' (Science and Life), No. 43-46, 1894.

by poverty for most of his life. Only under Soviet rule did he receive the recognition, care and attention that he deserved.

Although his health was strongly undermined by the privations and hardships of the pre-Revolutionary period, he worked with exceptional zeal and productivity. Whereas during a period of 26 years before the Revolution his publications numbered less than 50 (with about 80 papers in manuscript form), under Soviet rule more than 20 papers (most of which he prepared for publication) issued annually from his pen. In 1934 two volumes of his "Izbrannyye trudy" (Selected Works) on all-metal dirigibles and reaction propulsion appeared. His work is permeated with boldness of spirit and faith in the bright future of humanity.

K. E. Tsiolkovskiy worked long and fruitfully for the good of the Socialist Homeland, the Soviet State, which provided him with all the amenities needed for this purpose. In 1932, on his 75th birthday, the Government bestowed on him a distinguished award, the Order of Labor Red Banner, for his scientific work and inventions.

Before his death, in a letter addressed to Comrade I. V. Stalin, he bequeathed all his life-work to "the party of the Bolsheviks and the Soviet authorities -- the true leaders of civilization."

The great leader of our nation I. V. Stalin sent him a warm reply in which he thanked Tsiolkovskiy for his accomplishments in behalf of the Homeland.

Tsiolkovskiy's works have repeatedly been published in new editions, but only in an incomplete, poorly organized form.

In connection with the decision of the Government to transfer the entire archives on K. E. Tsiolkovskiy to the Academy of Sciences USSR, a committee for editing his work and preparing it for publication was established under the Department of Technical Sciences of the Academy. It has scheduled the publication of the material by volumes: Volume I. Aerodynamics, Volume II. Reaction Propulsion, Volume III. K. E. Tsiolkovskiy's Inventions and Miscellaneous Works, Volume IV. Problems of Natural Science, Volume V. The Autobiography of K. E. Tsiolkovskiy. Biographical Data. Correspondence. Bibliography.

In addition, a special volume containing Tsiolkovskiy's popular writings and science fiction will be published separately. It will contain certain material that has never been published before.

May this entire publishing project be another memorial to the accomplishments of the scientific genius who anticipated the development of present-day technology and contributed so much to it in his work!

Academician B. Yur'yev

K. E. Tsiolkovskiy and his Works on Aerodynamics

The problems of aerodynamics held considerable interest for K. E. Tsiolkovskiy throughout his many-sided life as a scientist. His first work connected with the problems of aerodynamics ("The Theoretical and Practical Aspects of an Aerostat with a Shape Elongated in the Horizontal Direction") was published in 1885 and his last ("The Pressure on a Plane in a Normal Flow") appeared in 1934. In the course of the intervening 20 years Tsiolkovskiy returned repeatedly to the study of aerodynamics, a subject to which he devoted a considerable portion of his time.

For Tsiolkovskiy aerodynamics was not merely of theoretical interest. As distinct from many of the early investigators, who based their designs for flying machines on guesses and assumptions, the great Russian scientist clearly understood that without reliable knowledge concerning the nature of drag, lift, and other characteristics of the interaction between an aircraft and the surrounding medium no progress would be possible either in aviation or in aeronautics. He repeatedly stressed the practical value of the laws of aerodynamics that he established, and he himself gave some excellent examples of their application to the design of aircraft and dirigibles.

In his creative work in the field of aerodynamics, Tsiolkovskiy always subjected his theoretical results to experimental verification. Whenever he introduced any assumption, it was always with the reservation that this assumption and the conclusions derivable from it were only valid to the extent that they could be justified by experiment.

Tsiolkovskiy was a great master of the art of experimentation. He knew how to set up an experiment so as to get a clear answer to a specific question with the maximum simplicity and economy. Moreover, he was an indefatigable experimenter, exhausting all the possible variations of the experimental parameters. However, he never left his experimental results in the form of a random accumulation of facts; he always sought and, with striking intuition, invariably found general theoretical relations between the quantities observed.

Such are the typical features of the aerodynamic investigations of K. E. Tsiolkovskiy, one of the founders of modern aerodynamics.

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The first of Tsiolkovskiy's works wholly devoted to the problems of aerodynamics: "On the Problem of Winged Flight" was written in 1890/91. In this paper he employed completely original and inde-

pendent methods to study the air pressure on a flat plate.

He based the theoretical part of this investigation on the following general picture of the displacement of the fluid particles when a plate moves in a direction perpendicular to its own plane: "The greatest velocity," he writes, "is acquired by the particles lying close to the moving plate; the direction of motion of the particles in front of and behind the plate is the same: the particles in front are driven ahead, those behind are dragged in the same direction. The particles lying close to the plate push against those lying further away, but since the space or, as it were, the channel gradually expands, the velocity of the particles decreases the further they are from the moving plate. Similarly, the particles behind the plate entrain the part of the air lying behind them, the velocity diminishing in the same proportion. Finally, the line of motion of the particles in front of and behind the plate constitutes a single closed curve, so that the fluid particles in front of the plate are directed along these curves beyond the plate."

This picture is considerably closer to reality than that based on the Newtonian impact theory generally accepted at that time. The above quotation illustrates the remarkable clarity of Tsiolkovskiy's fundamental physical concepts.

Apart from the question of the fluid pressure on a flat plate moving in a direction perpendicular to its own plane, Tsiolkovskiy offers a theoretical solution of the problem of the pressure on a similar plate moving simultaneously in two directions: perpendicular to and parallel to its own plane. He explains that the pressure is proportional to the sine of the angle of attack; as is known, at small angles of attack this is completely in accordance with the facts. But perhaps the most interesting and valuable result obtained by Tsiolkovskiy in his first published contribution to aerodynamics consists in his explanation of the effect on the pressure of the aspect ratio of the plate. He established theoretically that, other things being equal, the pressure exerted on the plate by the flow is proportional to the square root of the aspect ratio. This remarkable law anticipated the subsequent development of airfoil theory by about 30 years. Tsiolkovskiy was the first aerodynamicist to draw attention to the significance of the aspect ratio of aircraft wings, explain the reason for the elongated form of the wings of birds and insects, and give a relation between lift and aspect ratio qualitatively close to that established by modern airfoil theory. He had an excellent understanding of the enormous scientific importance of his discovery: later (February 1893), writing about "On the Problem of Winged Flight", he remarks: "When I wrote this article, it was as if no one had ever done anything along these lines before. In fact, since I had no access to a library, I worked completely independently, apart from my use of the most elementary scientific data. Then

Professor Zhukovskiy drew my attention to various investigations that had led to conclusions similar to mine."

Nevertheless, the law expressing the relation between lift and aspect ratio was something new to science. Much later, reviewing his own work in 1932, Tsiolkovskiy, writing about himself in the third person, noted: "He was the first to establish and verify by experiment the law of drag in relation to elongated plates." However, Tsiolkovskiy's priority in solving this extremely important problem in airfoil theory was undeservedly, but thoroughly forgotten. It is to be hoped that the present edition of Tsiolkovskiy's works will put an end to this injustice.

An important part of "On the Problem of Winged Flight" is devoted to the experimental verification of the theoretical conclusions. Tsiolkovskiy invented a clever, but very simple device for determining the pressure on a wing (a device that was "within my means," as he remarks). In this device a pair of wings, mounted on a shaft, could be rotated for a certain time by means of a string unwinding under the action of a weight. If this device was carried, with the wings rotating, in a direction parallel to the shaft, the rotation of the wings was slowed, by an amount depending on the rate of translational motion. By observing the time taken for the string to unwind, it was possible to determine the pressure on the wings for different angles of attack, flow velocities, and aspect ratios.

This device greatly pleased Professor N. Ye. Zhukovskiy, who himself demonstrated it at the mechanical exposition in Moscow in 1894. Later, the apparatus was presented by Zhukovskiy to the Student Aeronautical Club at the Moscow Higher Technical School and is now in the Club's museum. After reading the manuscript of "On the Problem of Winged Flight," Professor Zhukovskiy expressed his keen and general approval. He writes (to Professor A. G. Stoletov): "Tsiolkovskiy's paper makes an agreeable impression, since the author, with sparing use of analysis and simple apparatus, arrives for the most part at the right results.... The author's original method of investigation, his reasoning and ingenious experiments are not without interest and, in any event, mark him out as a talented investigator.... The author's arguments relating to the flight of birds and insects are valid and coincide perfectly with modern views on the subject."

This paper was not printed in its original form. Tsiolkovskiy himself considerably abridged it and revised it for publication. It then appeared in 1891 in Volume IV of the Transactions of the Division of Physical Sciences of the Society of Amateurs of Natural Science under the title "Fluid Pressure on a Plane in Uniform Motion." An extract from the same paper "On the Problem of Winged Flight" relating to the flight of birds and insects was published much later (1912) in the journal of the Seventh Aeronautical Division of the Russian Tech-

nical Society, Tekhnika vozdukhoplananiya (Aeronautical Engineering), No. 2, pp. 138-147, 1912, under the title "The Organization of the Flying Apparatus of Insects and Birds and Their Modes of Flight." As Tsiolkovskiy himself points out in a footnote to this article, the work had not become outdated nor did it conflict with more recent experiments and investigations. It was merely necessary to supplement it somewhat. Both these papers -- "Fluid Pressure on a Plane in Uniform Motion" and "The Organization of the Flying Apparatus of Insects and Birds" -- are included in the present edition, which, moreover, is the first to include Tsiolkovskiy's note on "The Nature of Eddy-Like Motions in Air and Water," which also formed part of "On the Problem of Winged Flight." In this short but fundamentally important note, Tsiolkovskiy points out that the presence in a fluid of two opposing flows (e.g., in flow around the edges of a plate) is one of the causes of eddies. This, of course, coincides perfectly with modern notions concerning the conditions of eddy formation. Thus, the present edition contains all the main parts of Tsiolkovskiy's first work on aerodynamics "On the Problem of Winged Flight."

Tsiolkovskiy was not slow to apply the chief conclusions to be drawn from these studies: characteristically, he always strove to use theory as a means towards advances in technology. In this case, the results of his aerodynamic investigations enabled him to work out a rational scheme for a flying machine and lay the foundations of sound aerodynamic design. Tsiolkovskiy's study of these problems -- "The Airplane or Bird-Like Flying Machine" (1894) -- undoubtedly marks an epoch in the development of aviation engineering. After first disproving the pessimistic theory of flight, according to which "to keep itself in the air a goose would have to do the work of a horse," with the aid of some simple reasoning and equally simple experiments Tsiolkovskiy established that the weakness of the theory lay in its underestimation of the part played by the translational motion. When wing flapping is combined with translational motion, the energy required to remain airborne must be considerably reduced. The most important section of this work is Chapter II, which bears the title "A Sketch of an Elementary Form of Flying Machine." Unlike many foreign aircraft designers of the time, who, inspired by the example of birds, showed a preference for apparatus with mechanically flapping wings, Tsiolkovskiy realized that "the imitation of birds is technically very difficult owing to the complexity of the movements of wings and tail and the equally complex organization of these elements." "We," writes Tsiolkovskiy, "shall only partly imitate the bird, that is to say, as far as we can." He put forward a plan for a flying machine that in all its main features anticipated the modern monoplane with cantilevered wings. "Our apparatus," he writes, "has the shape of a rigid, soaring bird, but instead of a head we envisage

two propellers rotating in opposite senses and developing a pull in one direction.... The place of the muscles will be taken by neutral combustion engines. They do not require a large supply of fuel (gasoline) nor do they need heavy boilers and large supplies of water.... Instead of a tail there will be a dual control surface, consisting of a vertical and a horizontal plane." "The dual control surface, dual propellers, and the rigidity of the wings," he continues, "are not conditioned by any particular advantages or economy in operation, but solely by the feasibility of the design." Tsiolkovskiy paid special attention to the design of the wings. He was the first to advocate an unbraced cantilever wing of uniform strength constructed of hollow metal tubes, and the first to envisage the streamlined fuselage and two possible means of connecting it with the wings: the mid-wing and gull-wing systems. Thus, Tsiolkovskiy must be given credit for inventing and investigating the modern cantilever-wing monoplane.

A considerable part of the work is devoted to the "derivation of the laws of flight" -- to use Tsiolkovskiy's expression -- or, in modern terminology, the aerodynamic design of aircraft. Tsiolkovskiy determined the speed required to keep a machine in the air, the engine power, and a number of other aerodynamic characteristics of aircraft. These passages constitute the first exposition of the elements of aerodynamic aircraft design.

Also in "The Airplane or Bird-Like Flying Machine," Tsiolkovskiy pursues a detailed discussion of the problems of controlling an aircraft both in flight and during takeoff and landing. In particular, he suggests the use of a gyroscope and automatic electromagnetic operation of the controls. Somewhat earlier (in 1893) he had proposed an automatic electromagnetic control system for a dirigible. Thus, Tsiolkovskiy can fairly claim to have been the first to introduce the idea of automatic electrical aviation equipment.

In a later evaluation of his research into the theory of the airplane (1929), Tsiolkovskiy himself wrote in the preface to an article called "The New Airplane": "My first work on the airplane appeared in the journal Nauka i zhizn' (Science and Life) in 1895. Up to that time no one had developed the theory of the airplane in such detail or drawn such concrete conclusions, which are only now being finally vindicated."

From his work on the theory of the airplane and the dirigible it was patently obvious to Tsiolkovskiy that the further development of the theory was being impeded by the almost total absence of reliable, experimentally verified data on the resistance of the medium. The ideas about the interaction between aircraft and medium current at that time were largely based on speculation and of doubtful validity. Thus, for example, an eminent member of the VII (Aeronautical) Division of the Russian Technical Society, M. M. Pomortsev,

believed, as Tsiolkovskiy mentions, that the shape of the rear part of a body made no contribution to the drag and, moreover, that the air friction on the surface of a body could be neglected even in the case of elongated bodies. Of course, these notions are completely false. Another prominent aeronautical engineer of the day, Ye. S. Fedorov, who carried on a polemic with Tsiolkovskiy in connection with his paper "On the Problem of Winged Flight," claimed that the latter was wrong in assuming a fall in pressure behind a plate moving in a direction perpendicular to its own plane equal to the increase in pressure in front of it. "It is usual to assume," wrote Fedorov, "that the pressure drop behind the plate is 0.1 of the excess pressure in front of it." However, Tsiolkovskiy's conclusions proved to be much closer to reality than the generally accepted view.

Now Tsiolkovskiy, of course, could not refute all these erroneous ideas about aerodynamics without appealing to experimentally verified facts. His own experiments with a whirling arm, referred to above, only enabled him to determine the air pressure on a flat plate; the question of the effect of the tail part of a body on its drag, the question of air friction, and many other matters remained unresolved. Accordingly, Tsiolkovskiy was constantly seeking new ways of solving the problems of aerodynamics by experimental means. In 1894 he designed an apparatus for comparing the drag of any body with that of a flat plate. This device consisted of a symmetrical pivot arm with the test model attached to one end and a flat plate to the other. The latter was cut away until equilibrium was just established when the device was in motion through the air. Experiments with this apparatus, the results of tests made on solids of revolution with different degrees of elongation, and the determination of the air friction are described by Tsiolkovskiy in a paper dating from 1896: "A 200-Man Maneuverable Iron Aerostat as Long as a Large Steamship;" The corresponding chapter of this work: "Experiments and Formulas Used to Calculate the Independent Speed of My Aerostat" is reproduced in this edition. In the same work Tsiolkovskiy also describes an apparatus for testing in a natural wind. In this case, the test model and the plate are mounted inside tubes which are arranged with their axes in the direction of the wind. A photograph of this apparatus and the models for it, published for the first time in this edition, gives a clear idea of the tremendous variety of geometric shapes investigated by Tsiolkovskiy employing the natural force of the wind.

By systematically investigating the resistance of the medium, first on apparatus in which the test model had to be set in motion in stationary air and then on apparatus utilizing the natural force of the wind, Tsiolkovskiy arrived at the notion of the wind tunnel. Concerning his invention of the wind tunnel, Tsiolkovskiy writes as follows (article entitled "Independent Horizontal Motion of a Maneuver-

able Aerostat" in the journal Vestnik opytnoy fiziki i elementarnoy matematiki (Herald of Experimental Physics and Elementary Mathematics) No. 258 and 259, Odessa, 1897: "Recently, while conducting certain experiments, I got the idea of using a completely new method involving an artificial wind (a vaned blower -- a kind of large winnower) (the italics here and below are ours. - N. F.). So far I have experimented only with a model 42 cm. long. The experiments confirmed our formulas, and for the model in question I obtained drag coefficients that gradually decreased with increase in the velocity of the artificial airflow. The new method makes it possible to carry out investigations at any time and with considerable accuracy; it is also very convenient for demonstration purposes."

Apart from this first published mention of a new method of testing models in an artificial flow of air, there exists a number of important documents that throw light on the creation by Tsiolkovskiy of the first wind tunnel in Russia and definitely establish his priority in this matter. Among these documents is Tsiolkovskiy's first description of his wind tunnel, published in this volume, which formed part of a letter to Professor A. L. Gershun dated 12 October 1897 (discovered in 1937 by B. N. Vorob'yev). This letter, together with drawings of the wind tunnel and wind tunnel balance and the above-mentioned photograph of the apparatus for testing in a natural airflow, was intended for transmittal to the Committee of the Russian Physico-Chemical Society, which was to consider Tsiolkovskiy's program of experiments for the study of air resistance. Among other things, the letter contains a short, but significant request: "Above all, I humbly beg the esteemed members of the Committee, after having communicated their opinion to the Society, not to divulge anything to anyone concerning my work and plans until they have been completed and published" (author's italics. - N. F.). Clearly, Tsiolkovskiy was already well acquainted with the morals of capitalist society.

Annexed to the letter were the first description and drawings of the wind tunnel, the completely original floating-type wind tunnel balance, the method of preparing the models, and the experimental procedure. It is worth noting the high sensitivity and great accuracy of the measuring equipment, much of which Tsiolkovskiy built with his own hands; thus, for example, the needle of his floating balance begins to deflect from the initial position when the model is acted on by a force of as little as one milligram. In spite of the low flow velocities available (in the first version the fanwheel was rotated by hand) and the low drag of the models tested, Tsiolkovskiy was able to make highly accurate measurements and establish some extremely important laws of drag.

The Committee of the Russian Physico-Chemical Society recognized the desirability of Tsiolkovskiy's program of experiments, but found the sum requested for financing the work (100-200 rubles) too

small and on these "grounds" saw fit to turn down his request. Tsiolkovskiy continued his work with the aid of his own meager resources, which had already been eaten into by the construction of the wind tunnel and its equipment. He published the results of his investigations in 1898 in an historic article "The Air Pressure on Surfaces Introduced into an Artificial Airflow," which marked the beginning of modern experimental aerodynamics. In this article, Tsiolkovskiy described how, for the first time in the history of science, using novel apparatus and a novel technique, he had made a systematic experimental investigation of the frontal drag and lift forces acting on bodies of different geometric shapes, including models of airfoils and the hulls of dirigibles. It was in connection with this research that he first established a number of the fundamental laws of aerodynamics (the effect of the elongation of a solid of revolution on the drag, the effect of the aspect ratio of a wing on its aerodynamic characteristics, the relation between friction, the surface area of a body and the flow velocity, etc., etc.). Tsiolkovskiy was the first to draw attention to the value of the wind tunnel as a means of solving problems in aerodynamics ("this device," he writes, "should be an essential part of the equipment of every university and physics laboratory"). Thus, there is every justification for calling Tsiolkovskiy the founder of modern experimental aerodynamics.

At the end of his article on "The Air Pressure on Surfaces Introduced into an Artificial Airflow" Tsiolkovskiy makes a fervent appeal for further advances in the study of the laws of aerodynamics: "...how important," he writes, "is the correct formulation of the laws of drag and friction! What an enormous contribution to the theory of the aerostat and the airplane! In fact, is there any region of science and technology in which the laws of the resistance of an elastic medium play no part? Thus we are ardently desirous that these laws be established and will do all we can toward carrying out the necessary experiments."

In order to evaluate Tsiolkovskiy's personal achievement, it is necessary to recall the great physical and mental effort required of him. His wind tunnel ("vaned blower," as he called it) was driven by a weight which had to be raised to the ceiling before each experiment. During the short period of operation of the tunnel he had to take the readings, and then analyze the results and plot the graphs himself. He worked on his experiments every day for more than four years, whenever he had time to spare from his teaching duties.

It is astonishing that in spite of the imperfections of his wind tunnel, the low flow velocities and the sheer hard work involved in his experiments, Tsiolkovskiy was able to determine with remarkable accuracy such a small quantity as, for example, the air friction on the surface of a model and to establish the fundamental laws by which

it is governed.

Tsiolkovskiy's exceptional experimental skill and intuition are also evident in his treatment of his observational data. Having decided that power laws governed the dependence of friction on the flow velocity and the dimensions of the model, Tsiolkovskiy definitely established that in the expression for the coefficient of friction the velocity and the size of the model occur with the same exponent, as the theory of dynamic similarity requires. In this connection, it should be noted that the well-known English scientist Froude, who was able to experiment under much better conditions than Tsiolkovskiy, arrived at conclusions relating to the friction accompanying motion through water that were in complete contradiction to the theory of similarity.

The expenditures on building a wind tunnel and the corresponding instruments and test models exhausted Tsiolkovskiy's never very extensive funds, and in 1899 he decided to turn to the Russian Academy of Sciences for financial aid. Tsiolkovskiy's work, including the results of his experiments on air resistance, was handed over to Academician M. A. Rykachev for review. At a meeting of the Physico-Mathematical Division of the Academy Rykachev gave a very favorable report on Tsiolkovskiy's experiments. In this report he stated: "These experiments deserve the close attention of the Academy, for the sake of both the idea behind them and their variety. In spite of the author's primitive home-made apparatus, he has been able to determine the velocity when the air blower is driven by different loads: ... the author has determined the relation between the drag and the flow velocity and the geometry of the shape. Some very interesting experiments relate to the effect of the tail section; .. . the extent to which the author has been able to obtain reliable results is indicated by the agreement with the data provided by tests of the pressure on a flat plate; ... the changes are almost exactly what the theory would lead one to expect.... For all these reasons it would be extremely desirable to repeat the experiments on a larger scale and with more accurate instruments, and I respectfully suggest that this Division grant the author's request and extend to him material support from the fund intended for the promotion of science." On the basis of Rykachev's report, the Academy of Sciences decided to assist Tsiolkovskiy and granted him 470 rubles to continue his experiments. Tsiolkovskiy proceeded to build a new wind tunnel, considerably larger than the first. In building and adjusting this tunnel he gave careful attention to the flow created by the fan. In order to make sure that the flow was always uniform and in the same direction, Tsiolkovskiy employed, for the first time, a screen consisting of oriented baffle plates. Almost throughout the year 1900 Tsiolkovskiy was occupied in building his new wind tunnel and the auxiliary equipment for it. In his article "Air Resistance and Aero-

nautics" (1902) he describes it as follows: "The Academy entrusted Academician M. A. Rykachev with the task of reviewing my work." As a result of his favorable report and intercession with the Academy, the latter decided to give me a grant (470 rubles) for carrying out further experiments, in accordance with a program which I also put before the members. This was in May 1900. I then set about building a large wind machine ... to give a flow an arshine (71.12 cm) high and wide. The measuring instruments were also made -- and all this was redone and readjusted as many as six times before a really satisfactory flow of air was achieved.

"Toward the end of 1900 I was able to start assembling the models and performing experiments. Toward the end of the following year (16 December 1901) I had already completed part of the proposed program and submitted a report to the Academy on the progress I had made. This report was simply a summary of my notes and consisted of 80 written pages of text and 58 tables and drawings."

In spite of the fact that the report was very bulky and contained an enormous amount of experimental material and corresponding generalizations, the program originally proposed by Tsiolkovskiy was far from completion. "There is a still a great deal to be done," he writes in the same article "Air Resistance and Aeronautics," "and God knows when I shall finish it and solve the problems that preoccupy me. In all probability it will be necessary to undertake new experiments with better instruments in order to get more accurate results in certain instances." The problems posed by Tsiolkovskiy were so broad and varied that it became necessary to set up a special aerodynamics institute to solve them, a suggestion which he actually made in one of his letters to the Presidium of the Academy of Sciences, a letter that remained unanswered.

Tsiolkovskiy's report to the Russian Academy of Sciences, which he had intended for inclusion in the Transactions of the Academy, was never published during the pre-Revolutionary period; it appears for the first time in this edition of Tsiolkovskiy's collected works. Apparently, the reason for the non-publication of the report was the petty fault-finding attitude of Academician Rykachev, to whom the report was assigned for review. While acknowledging that the experiments were "numerous, diversified, and interesting and deserve attention," Rykachev made what, in our opinion, were excessive demands on the author of the report, expressive of his distrust. He writes: "In connection with the publication of Mr. Tsiolkovskiy's work in the transactions of the Academy, it will first be necessary for the author to present his experimental material in more orderly form, grouped so that each conclusion given in the text is accompanied by all the observations from which this conclusion is drawn, with an indication, at least, of the day on which these observations were made ...; moreover, observations that have been disregarded should

also be included and the reasons for disregarding them explained. The numbers under which each experiment is entered in the table should be noted in the raw material of the report." For the sake of these missing numbers Academician Rykachev overlooked the main contents and conclusions of Tsiolkovskiy's work, which included the following important findings: experimentally established relations between the pressure on a plate and its aspect ratio, between the pressure on a plate and the angle of attack (over the range from 0 to 90°), between the drag of a cylinder and its proportions, angle of inclination to the flow, and the flow velocity, between the lift and frontal drag on airfoils and their concavity, and many others. In the same report, laying the foundations of the aerodynamic design of aircraft, Tsiolkovskiy established that the engine power required is proportional to the frontal drag coefficient and inversely proportional to the lift coefficient to the power 3/2; the corresponding formula is now a feature of all textbooks on aerodynamics. On the basis of his experiments on surfaces elongated in the direction of flow, Tsiolkovskiy was also able to give the principal equations of turbulent friction. In this report and the later "Air Resistance and Aeronautics," which included its more important conclusions, Tsiolkovskiy showed that the drag coefficient of all bodies depends not only on their shape but also on the size of the body and the velocity of the flow. In the article "Air Resistance and Aeronautics" he writes: "Up to now ... it has always appeared that the drag coefficient depends only on the shape of the body and not on its size or the flow velocity; in other words, that the drag of similarly varying bodies is proportional to the square of the flow velocity and the area of the body projected on a plane perpendicular to the flow. Now we can see just how ill-founded these conclusions of the generally accepted theory are."

In this same article "Air Resistance and Aeronautics," and in his report to the Academy of Sciences, Tsiolkovskiy arrives at an important conclusion concerning the incorrectness of the prevailing views on the nature of the resistance exerted by a medium and theoretical methods of determining the drag. He formulates this conclusion as follows: "The basis for determining the drag of any shape was assumed to be the pressure on an inclined plane. The surface of any body can be visualized as consisting of a multiplicity of plane faces. By double integration we obtain the total pressure on the body. The first mistake lay in neglecting friction, since it always appeared that with the elongation of the body the drag diminished ad infinitum, though this has never been verified experimentally.... Moreover, there are many formulas for the pressure on an inclined elementary surface.... From the consequences of applying these and other formulas it is clear that the accepted method is, in general, never applicable and this constitutes the second error: new ideas

about the phenomenon of drag are needed together with experiments, which are the only means of verifying them." These fundamental results obtained by Tsiolkovskiy working under the auspices of the Academy of Sciences remained unacknowledged by the official scientific circles of the pre-Revolutionary period.

Tsiolkovskiy's subsequent works on aerodynamics belong to the Soviet period of his life. He deals with a number of aerodynamic questions in "The New Airplane," a work dating from 1929. Here he developed the idea of an aircraft without a fuselage, the stability of which was to be created by an internal overpressure. In view of the absence of a body, struts, wheels, pontoons, and braces, the drag would be much reduced.

Another work by Tsiolkovskiy belonging to this period is entitled "Pressure on a Plane in a Normal Airflow" (1930). This is devoted to the question of the pressure and temperature of air at high speeds. It remained unfinished, like a number of other incomplete fragments of a larger work that Tsiolkovskiy was planning: "Fundamentals of the Construction of Gas Engines, Motors, and Flying Devices."

* *
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As we have seen, Tsiolkovskiy's legacy in the field of aerodynamics is both varied and extensive. His work embraces a wide circle of problems, many of which he was the first to pose and solve. He was a genuine pioneer in aerodynamics, to which he brought new and original methods of investigation. However, in autocratic Tsarist Russia the ruling classes and the representatives of aeronautical engineering of the 7th Division of the Imperial Russian Technical Society, who were closely linked with them, despised and ridiculed, brushed aside and suppressed the work of the remarkable Russian scientist. The slavish predilection for all things foreign was so ingrained in these people, they were so cut off from the national Russian soil, that they had little faith in the prophetic words of the genius Lomonosov: "The Russian land can give birth to its own Platos and brilliant Newtons." In official circles the attitude to this self-taught teacher, working in a remote spot in the provinces on the problems of aviation and aeronautics, was one of haughty arrogance; at best, he was looked upon as an inventor with fantastic ideas.

Only with the advent of Soviet rule, from the very first months, did the official attitude to the great scientist radically change. The state institutions connected with civil and military aviation invited him to participate in their work; his scientific findings were systematically published and became widely known. The

creative force of Soviet patriotism gave Tsiolkovskiy a new lease on life, which enabled him to overcome the obstacles of age and poor health and labor fruitfully for his native country. The following lines from an article "The Future of Flight" printed just before his death in the Kaluga newspaper "Kommuna" are permeated with Tsiolkovskiy's highly patriotic spirit: "All this is but a feeble attempt to predict the future of aviation, aeronautics, and rocket flight. Of one thing I am firmly convinced -- the Soviet Union will lead the world. The capitalists are also working on these problems, but the capitalist system is inimical to everything new. Only in the Soviet Union is there a powerful aviation industry, a wealth of scientific institutions, public interest in the problems of aeronautics, and workers endowed with a love of their country so extraordinary that it can not fail to ensure the success of our undertakings."

Tsiolkovskiy's high sense of patriotism also dictated the appeal he addressed shortly before his death to the great leader of the nation Comrade Stalin: "All my work on aviation, rocketry and interplanetary flight," he wrote, "I bequeath to the Bolshevik party and the Soviet Government -- the genuine leaders of the progress of human culture. I am convinced that they will successfully complete it."

The development of aviation science in the Soviet Union and the broad popularization of Tsiolkovskiy's ideas constitute the fulfillment of the testament of the great Russian scientist.

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I would like to take this opportunity of thanking the secretary of the Commission for the Publication of the Works of Tsiolkovskiy, B. N. Vorob'yev, for his invaluable assistance in sifting through the extensive archive material containing Tsiolkovskiy's writings on aerodynamics, in editing the text, and in compiling the notes. I am also greatly indebted to B. N. Vorob'yev for his detailed bibliographic survey that enabled me to find my way among the profusion of scientific documents that Tsiolkovskiy left behind.

N. Ya. Fabrikant

A E R O D Y N A M I C S

FLUID PRESSURE ON A PLANE SURFACE IN UNIFORM MOTION*

I wish to express my warm thanks to A. G. Stoletov and N. Ye. Zhukovskiy for pointing out to me the significance of my own work in relation to other work of the same kind[1**].

K. Tsiolkovskiy

Let us resolve the uniform and rectilinear motion of a flat plate into two components, one parallel to the plate itself, which, for the sake of brevity, we shall call the parallel component, and one perpendicular to the plate, the normal component.

In the presence of steady normal motion the force exerted by fluid pressure on a flat plate may be expressed by the equation:

$$F = \frac{K \cdot S \cdot d}{g} \cdot V_n^2, \qquad (1)$$

where F denotes the force, K is a correction factor, less than unity, S the area of the plate, d the volumetric weight of the fluid, g the acceleration due to gravity, and V_n the normal velocity of the plate.

The determination of F from this formula gives values very close to those obtained by means of experiments (the error is not greater than 1/10) employing rotary apparatus.

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*This article was first published in Volume IV of the Transactions of the Division of Physical Sciences of the Society of Amateurs of Natural Science, 1891.

**The numerals refer to the correspondingly numbered notes grouped at the end of the book (pages /265-268). (Ed.)

However, equation (1) cannot be used to determine F, unless the motion of the plate and the surrounding medium is steady.

Let us first determine the work that the plate must do to impart steady motion to the medium. We shall have to adopt some hypothesis regarding the nature of the fluid motion, which, of course, will appear the more likely the more closely the conclusions that follow from it are confirmed by experiment.

Here is one such hypothesis concerning the motion of an incompressible medium when the plate in question is circular in shape (Fig. 1).

The greatest velocity is acquired by the particles lying close to the moving plate, the direction of motion of the particles in front of and behind the plate being the same: the particles in front are driven ahead, those behind are dragged along in the same direction owing to the rarefaction of the air. We shall assume that a sphere of fluid surrounding the plate, like a great circle, has a uniform velocity equal to the velocity of the plate; the particles of this sphere push against and entrain other more remote parts of the fluid; but since the space or, as it were, the channel gradually expands, the velocity of the particles decreases the further they are from the moving plate.

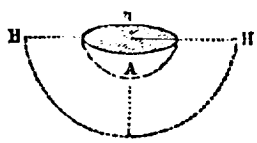

Fig. 1

Thus, denoting (Fig. 1) the radius of the plate by r, the radius of a certain spherical layer of fluid by H, its velocity by V, and the velocity of the plate by V_n, we find:

$$V = \frac{V_n \cdot \pi \cdot r^2}{2\pi \cdot H^2} = V_n \cdot \frac{r^2}{2H^2}, \qquad (2)$$

for clearly the velocity of the layer H is as many times less than the velocity of the plate as the surface of the layer is greater than the surface of the plate.

As for the entrainment of the parts of the fluid lying behind the plate in a manner analogous to the motion of the particles driven in front of the plate, the notion is one that is easy to verify both experimentally and theoretically. Suppose the plate undergoes a certain forward displacement; then, if the fluid had no elasticity and lay in a gravitationless medium -- if a void were left behind the plate corresponding to its displacement -- there would be no suction or entrainment of fluid; but since, in ordinary circumstances, the

fluid has weight, not to mention elasticity, it will immediately occupy the vacuum left behind the plate. A second layer is then entrained behind the first and so on, just as the layers of fluid in front of the plate push against each other.

The differential of the work T done in moving the fluid

$$dT = dm \cdot \frac{V^2}{2g}, \tag{3}$$

where dm is the mass of a spherical layer of thickness dH; however,

$$dm = 4\pi \cdot H^2 \cdot dH \cdot d$$

and since

$$V = V_n \cdot \frac{r^2}{2H^2},$$

$$dT = \frac{\pi \cdot d}{2g} \cdot V_n^2 \cdot \frac{r^4}{H^2} \cdot dH. \tag{4}$$

Integrating this equation, we find

$$T = \frac{-\pi \cdot d \cdot V_n^2}{2g} \cdot \frac{r^4}{H} + C, \tag{5}$$

where C is the constant of integration.

If H = r, the work T is equal to that necessary to impart a constant velocity V_n to the sphere A (Fig. 1); then

$$\frac{3}{4}\pi \cdot r^3 \cdot d \cdot \frac{V_n^2}{2g} = \frac{-\pi \cdot d \cdot V_n^2}{2g} \cdot r^3 + C; \tag{6}$$

from the last two equations we get

K. Tsiolkovskiy

FLUID PRESSURE ON A PLANE

SURFACE IN UNIFORM MOTION

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1891

[Reproduction of title page]

$$T = \frac{\pi \cdot d}{2g} r^3 . V_n^2 \left(1 - \frac{r}{H}\right) + \frac{2\pi \cdot d}{3g} \cdot r^3 \cdot V_n^2. \qquad (7)$$

In order to determine the work corresponding to the whole of the infinite fluid surrounding the plate, we must put $H = \infty$; we then find

$$T = \frac{7}{6} \cdot \frac{\pi \cdot d}{g} \cdot r^3 \cdot V_n^2. \qquad (8)$$

Knowing the work T and the displacement of the plate, it is possible to find the average pressure on it.

However, before the latter equation can be used to determine the pressure on a flat plate in complex motion (i.e., parallel and normal motion), it is necessary to establish the following propositions:

a) Both theory and experiment show that in the presence of normal motion the pressure on a plate is almost strictly proportional to its area, irrespective of its shape.

b) In exactly the same way, both theory and experiment show that at the initial moment of normal motion of a plate the work T is expressed by equation (8) irrespective of the shape of the plate, provided that it is not very elongated; given these two propositions, the theory of which I must omit for lack of space, equation (8) can also be applied to a rectangle with sides a and b, provided it is not too elongated;

Accordingly, let us substitute for $\pi \cdot r^2$ and r in (8) as follows:

$$\pi \cdot r^2 = S \quad \text{and, since } \pi \cdot r^2 = a.b, \; r = \sqrt{\frac{ab}{\pi}};$$

whence

$$T = \frac{7}{6} \cdot \frac{d}{g} \cdot S \sqrt{\frac{ab}{\pi}} \cdot V_n^2. \qquad (9)$$

We shall now determine the pressure of the air or any fluid on a rectangular plate, one side a of which is perpendicular and the other b parallel to its direction of parallel motion.

If the plate had only the normal velocity V_n, the work done in acquiring this velocity (in a small interval of time) would be expressed by equation (9). However, a plate that also had a parallel velocity V_p would in unit time pass through the space V_n in a direction perpendicular to the former, and therefore its work per second would be as many times greater than the work given by equation (9) as the parallel velocity V_p is greater than the width b of the rectangle, in the direction of which this movement is executed. Actually, in the presence of a normal velocity alone, a rectangle imparts a known motion to the air close to the area $a \cdot b$; in the presence of translational motion the same rectangle imparts to the air in one second a motion close to a surface of length V and width a, i.e., an area $V_p \cdot a$,

$$\frac{V_p \cdot a}{a \cdot b} = \frac{V_p}{b} \tag{10}$$

times greater than the previous one. To each part of this strip of air the rectangle imparts a certain motion.

Thus, denoting the force exerted by the normal pressure on a flat plate due to this cause by F, we find that on the basis of equations (9) and (10) the work done per second

$$F \cdot V_n = \frac{7 \cdot d}{6 \cdot g} \cdot S \cdot V_n^2 \sqrt{\frac{ab}{\pi}} \cdot \frac{V_p}{b}; \tag{11}$$

whence

$$F = \frac{7 \cdot d \cdot S}{6 \sqrt{\pi g}} \cdot \sqrt{\frac{a}{b}} \cdot V_n \cdot V_p. \tag{12}$$

Introducing the correction factor K into the right-hand side of this equation, in view of the fact that (10) is not rigorously derived and proposition (10) is limited, and adding to the pressure of

(12) the pressure of (1) due solely to the normal steady motion, we get

$$F = K \cdot \frac{S \cdot d}{g} \cdot V_n^2 \left\{ 1 + \frac{7 \cdot K}{6\sqrt{\pi}} \cdot \sqrt{\frac{a}{b}} \cdot \left(\frac{V_p}{V_n} \right) \right\} \qquad (13)$$

Before proceeding to point out the scientific value of this equation, I consider it necessary to verify it by experiment.

The apparatus designed to verify the theory consists of a small stand with a horizontal shaft revolving in it. Two thin wires are attached to the shaft at right angles. At the ends of these wires it is possible to mount various thick paper rectangles of different sizes and proportions, so that the shaft supports two rectangular, symmetrically mounted wings (Fig. 2) lying in the same plane. The shaft and the wings can be made to revolve for a certain length of time by means of a string which is unwound from the shaft under the action of a weight.

With this apparatus it is easy to see how the air resistance increases with increase in the parallel velocity of the wings, which, apparently, acting on the thin edge of the wings, ought not to have any effect on the rate of rotation of the device.

To demonstrate this, I take the apparatus in my hands and allow the weight to fall, so that the wings revolve at a known speed; if now, during their rotation, I begin to walk, still holding the apparatus, in a direction parallel to the shaft, the rotation clearly becomes slower and may even almost stop, if the rate of translational motion is increased (the weight was 2 grams, the width of the wings about 4 cm., the length about 5 cm., the same as the shortest distance from the wings to the shaft).

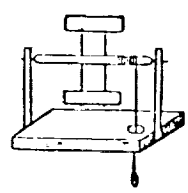

Fig. 2

However, in order to obtain accurate results, it is necessary to formulate a few auxiliary equations applicable to our apparatus (Fig. 2).

Denoting the air pressure on one wing by F, the distance between the center of pressure and the shaft by R, the radius of the shaft by r, and the load by P, we find the condition of uniform motion of the wings

$$2 \cdot F \cdot R = rP. \qquad (14)$$

If in the course of time t the wings make n complete revolutions, while the string unwinds through the length L, then

$$2\pi \cdot R \cdot n = V_n \cdot t \tag{15}$$

and

$$2\pi \cdot r \cdot n = L. \tag{16}$$

Using these equations to eliminate R and r from (14), and equation (13) to eliminate F, we get

$$\frac{4S \cdot \pi \cdot d}{g \cdot t} \cdot R^2 \cdot n \left\{ \frac{2\pi \cdot R \cdot n}{t} + \frac{7 \cdot K}{6\sqrt{\pi}} \sqrt{\frac{a}{b}} \cdot V_p \right\} = \frac{P \cdot L}{2\pi \cdot n}. \tag{17}$$

Knowing that S = ab and solving this equation for t, we find the time for a string of length L to unwind under the action of different loads:

$$t = \frac{14 \cdot dn^2 \cdot R^2}{3 \cdot P \cdot L \cdot g} \cdot \sqrt{\pi^3 \cdot a^3 \cdot b} \cdot V_p \cdot K + \\ + \sqrt{\pi^3 \cdot a^3 \cdot b} \cdot \sqrt{\frac{196 \cdot d^2 \cdot n^4 \cdot R^4}{9 \cdot P^2 \cdot L^2 \cdot g^2} V_p^2 K^2 + \frac{16 \cdot d \cdot n^3 \cdot R^3}{P \cdot L \cdot g \cdot a^2}} \tag{18}$$

We shall use this formula for surfaces that are not too elongated, when the factor K may be assumed equal to unity; for example, let us put:

$$a = 5 \text{ cm}, \; b = 5 \text{ cm}, \; d = 0.0013, \; n = 10, \; R = 10 \text{ cm},$$

$$V_p = 100 \frac{cm}{sec}, \; L = 21 \text{ cm}, \; g = 980 \text{ cm/sec}^2 \tag{19}$$

and compute a series of times corresponding to a series of assumed loads P.

Then, rotating the wings of an apparatus (Fig. 2), satisfying conditions (19), by means of a series of weights, while at the same time giving the apparatus a translational velocity of 100 cm/sec in the direction of the axis of rotation of the wings, and noting the time for the string to unwind, we get another series of times, in this case determined experimentally. The computed and the experimentally determined times are compared in the following table (20):

P	$t_{comp.}$	$t_{exp.}$
1	90.6	91
2	47.2	47
3	34.8	35
4	27.6	28
5	23.0	23
6	20.1	20
7	18.0	18
8	16.3	16
9	15.0	15
10	13.9	14
20	8.6	9

(20)

The first column gives the load in grams, the second the computed time, the third the time found experimentally. It is clear

that part of our theory at least is well-founded.

It remains to verify the influence of the aspect ratio of the wings a/b on the force F, as expressed by equation (13).

From equation (18), representing the consequences of this law, it is clear that for a constant load P and a surface that is not too elongated, when K = 1, the time taken by the string to unwind is proportional to the square root of the width of the wings b, or, if the length a is constant, inversely proportional to the aspect ratio a/b of the wings. We shall use this deduction from equation (18) to check equation (13) with respect to the aspect ratio of the plate. To this end, I cut out plates with different aspect ratios and mounted them successively on the spokes of the apparatus. Out of numerous experiments I present a single table (21).

b	$1^1/_4$	$2^1/_2$	5	10	15	20
$\frac{a}{b}$	4	2	1	$^1/_2$	$^1/_3$	$^1/_4$
$t_{exp.}$	13	24	35	53	67	80
$t_{comp.}$	$17^1/_2$	25	35	50	60	70

(21)

The first row gives the width of the plate b (or the length in the direction of the shaft) in centimeters, the dimension in the perpendicular direction being fixed (5 cm.); the second gives the aspect ratio a/b of the rotating rectangles: the third the time t in seconds determined experimentally for a constant load P of 3 grams and, in all other respects, in accordance with conditions (19); finally, the fourth row gives the time taken for the string to unwind as computed from equation (18) assuming that K = 1. As the table shows, the agreement between the experimental and the computed results is the greater the less the elongation of the plate or, more correctly, the more the shape of the plate approaches a square.

By substituting successively in equation (17) the dimensions of the plate and the corresponding experimental times from table (21) and assuming that P = const. = 3 grams and, in all other respects, conditions (19), we obtain from (17) a series of values of the factor K, which it is then easy to express by means of an empirical formula.

This formula is very simple, namely:

$$K = \frac{9}{7 + 2 \cdot \left(\frac{a}{b}\right)} . \tag{22}$$

Computing the times from equation (18) and taking K from (22), we obtain a series of values: 13, 23, 35, 53, 68, 79 very close to those obtained by experiment[2].

I must mention one of my experiments that very clearly indicates the enormous influence of the aspect ratio on the air pressure, or on the time the string takes to unwind. The basis for this experiment is equation (18) or equation (13), which may be simplified when the load P is small or when the rotational velocity V_n is small compared with the translational velocity V_p; then, instead of equation (18), we get:

$$t = \frac{28 \cdot d \cdot n^2 \cdot R^2}{3 \cdot P \cdot L \cdot g} \cdot a \cdot \sqrt{\pi^3 \cdot (a \cdot b)} \cdot V_p \cdot K. \tag{23}$$

From the latter equation it is clear that, when the area a·b of the rectangle is constant and the other conditions are the same, the time the string takes to unwind is proportional to a, i.e., to the length in a direction perpendicular to the shaft. Accordingly, if, for example, we have a rectangle one side of which is twice as long as the other, and if in two successive experiments this rectangle is arranged in two ways -- once with the long side at right angles to the shaft and once with the long side parallel to the shaft, then in the first case the unwinding time will be twice what it is in the second, even though the center of pressure is in both instances the same distance R from the shaft.

And, in fact, this theoretical conclusion is completely borne out by experiment.

Thus, a single rectangle of area a·b, acted on by the same load, will rotate at different speeds depending on its orientation.

From equation (12) it is also clear that in these two cases the pressure will differ by a factor of two. In fact, in the first

case $\frac{a}{b} = 2$, in the second $\frac{a}{b} = \frac{1}{2}$; the ratio is equal to 4, the square root of which is 2. Consequently, F changes by a factor of two.

Considering equation (13) and using the empirical expression (22) for K to solve the problem of the maximum of the pressure F, we find that, if the area a·b is constant and other things are equal, the pressure will be greatest when the aspect ratio of the rectangle is equal to $3\frac{1}{2}$.

Equation (13) may be written in another form:

$$F = K \frac{d}{g} \cdot S \cdot V^2 \cdot \sin^2 i \left\{1 + \frac{7 \cdot K}{6\sqrt{\pi}} \sqrt{\frac{a}{b}} \operatorname{ctg} i\right\}, \qquad (24)$$

and it will then express either the wind pressure on a stationary rectangle or the pressure of a fluid on a rectangle moving in it. In this equation V denotes the velocity of the wind or of a plate, one side a of which is perpendicular to the velocity V, and i is the angle between the velocity vector and the plane of the plate.

If the angle i is small, equation (24) becomes:

$$F = K \cdot \frac{d}{g} \cdot S \cdot V^2 \cdot \sin i \left\{\frac{7K}{6\sqrt{\pi}} \cdot \sqrt{\frac{a}{b}}\right\}; \qquad (25)$$

from this formula it is clear that if the angle i is small, the pressure is proportional to sin i, as is now assumed by all authors concerned with drag.

THE NATURE OF EDDY-LIKE MOTIONS IN AIR AND WATER[3*]

My hypothesis concerning the motion of a fluid under the influence of the normal motion of a flat plate makes it easy to understand the reason for the formation of eddies.

In accordance with this hypothesis, the motion of air under the influence of the first impact may be represented by the sketch in Fig. 1**.

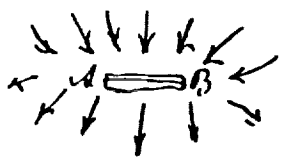

Fig. 1

At the edges of the plate AB two opposing flows meet and create a circular motion. It is only necessary to pull an oar through still water, as in rowing, to convince oneself of the truth of this theoretical conclusion.

Conversely, whenever flowing water encounters an obstacle, a bridge, rapids, and so on, the funnel-like depressions characteristic of eddies are inevitably formed.

Similarly, air, when it strikes a building, mountain, or large obstacle, must form eddies, and indeed it is in these circumstances that eddies are mainly observed. Powerful cyclones and tornadoes are born in the mountains and carried for extremely long distances by the general flow before they die out. Of course, there may be other possible causes of eddy formation.

From these arguments you may judge for yourselves whether my hypothesis is a likely one.

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*Taken from a manuscript dated by the Author 19 January 1892.

**For the most part, the sketches reproduced in this edition are Tsiolkovskiy's own. (Editor)

THE ORGANIZATION OF THE FLYING APPARATUS OF INSECTS AND BIRDS AND THEIR MODES OF FLIGHT*

I

Stationary atmosphere

Insects equipped with a single pair of wings offer an example of an elementary form of flying machine.

Each wing is a more or less elongated ellipsoidal plate.

How is this simple apparatus used to achieve flight? How does an insect stay in the air, move, turn, rise and fall?

Let us consider the moment when the wings descend. At this point, of course, the air exerts a pressure on their underside, the intensity of which depends on their rate of motion, i.e., may be

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*This article is actually an extract from my manuscript "On the Problem of Winged Flight." This was transmitted to Professor Zhukovskiy by Professor Stoletov. The former, having reviewed it favorably, suggested that I pick out what was, in my opinion, the most important part for publication in the Transactions of the Society of Amateurs of Natural Science.
At that time I chose the computations and experiments on the resistance of plates.
Although the manuscript was written not later than 1890, i.e., 22 years ago, on re-reading it I did not find it outdated, nor have the findings been contradicted by subsequent research along the same lines.

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* First published in the journal Tekhnika vozdukhoplavaniya (Aeronautical Engineering), No. 3, 1912. The article is reproduced here with additions by the author which, for technical reasons, were omitted from the version published in the above-mentioned journal. (Editor)

equal to the weight of the insect or greater or less. In the first case there is no change in the vertical position of the insect; in the second it rises; and in the third it falls.

I am assuming that the surfaces of the wings are parallel to their horizontal axis of rotation, and at the same time parallel to the longitudinal axis of the insect (Fig. 1).

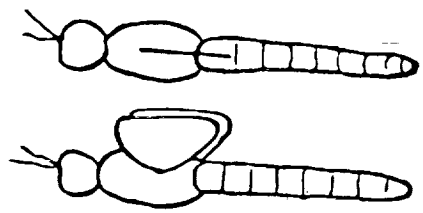

Fig. 1.

When the wings are raised they should assume a vertical position, so that the air offers no resistance and no downward-directed pressure is exerted.

Thus, with the beating of the wings, the body of the insect will now rise, now fall. If the amount by which the body rises is equal to the subsequent fall, the average level of flight will remain the same. In view of the extremely high rate at which the wings beat, the oscillating motion of the insect's body is scarcely discernible. This form of flight is comparatively rare, since the atmosphere is hardly ever completely still. However, the body of the insect may rise even if the wings are inclined to the vertical during the upward beat, since the upward pressure may still be much greater than that in the opposite direction.

However, in this case the insect will not have any horizontal motion, and the work it does in maintaining itself in the air or hovering without support will be much greater than in rapid translational flight.

In order for there to be such motion, it is necessary for the surfaces of the wings to be inclined to their horizontal axis of

rotation. Then the normal air pressure on the wings can be resolved into two parts (Fig. 2): one -- almost vertical -- will counteract the weight of the creature, the other -- horizontal -- will thrust it forward in the horizontal direction; this force will balance the air pressure on the body and wings of the insect.

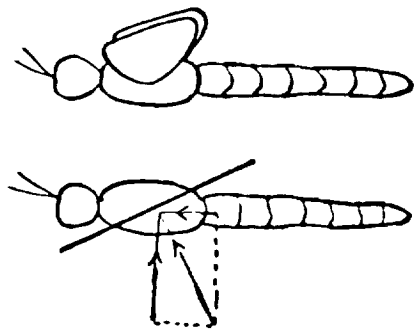

Fig. 2.

For the sake of brevity, I shall describe as positive any inclination that produces a force acting on the wings to thrust the insect forward.

In the absence of translational motion, any, even the slightest inclination of the forward part of the wings may be called positive. But in the presence of translational motion this is not so (Fig. 3). Fig. 3 shows the two components of the relative velocity of the air, due to the translational motion of the insect and the descent of the wings. Being parallel to the resultant of the velocity, the surface of the wings does not develop either a forward or a backward force, and hence does not give a positive inclination, though the actual inclination may be very great, 30°, for example.

The inclination shown in the figure may be described as the zero inclination*. Any, even the slightest reduction in this inclination will make it positive, because the relative flow will then

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*Because it neither supports the weight of the body nor imparts translational motion.

be directed against the underside of the wings.

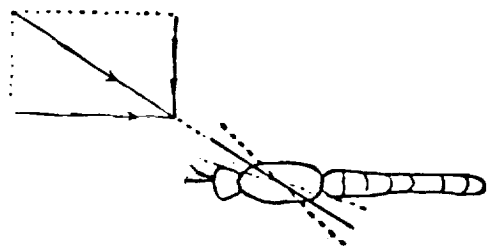

Fig. 3.

The horizontal position is another limiting position of the wing, in which the forward thrust is a minimum.

In view of the fact that the rear part of the wings of flying creatures is thinner and more flexible, it is deflected upwards by the air pressure on the down beat, thus independently acquiring a positive inclination.

A positive inclination not only thrusts the insect forward, but also prevents it from falling (Fig. 3).

If the deviation from zero inclination (Fig. 3) is reversed, i.e., if the angle of inclination increases, the relative flow will be directed at the upper surface of the wing, producing a downward pressure that will brake the translational motion and deprive the insect of its sustaining vertical force, i.e., will cause it to fall.

Now let us consider what happens when the wings move in the other direction, i.e., on the up beat. Here again there are three important possibilities (Fig. 4). If the relative counterflow, composed of the translational motion of the body and the beat of the wings, coincides with the changed inclination of the plane of the

wings, then the counterflow* will slide along the wings without exerting pressure on them. This inclination is the reverse of the previous one, i.e., in other words, the leading edge of the wing is higher than the trailing edge. In spite of this, the inclination can also be described as zero in relation to the propulsive force. The second case is obtained whenever this zero inclination increases (Fig. 4). Then the flow is directed against the underside of the wing and not only maintains the insect in the air but retards its flight, since the horizontal component is opposed to the direction of motion of the insect's body. We shall call this kind of inclination negative. Thus, even when the wings are raised, they can be given an inclination that causes a lift force to act on the insect.

Naturally, this lift force depends on the amount by which the inclination increases and on the translational velocity of the insect. It may be smaller than, greater than or equal to the weight of the insect, so that the insect may either fall, rise or remain at the same level.

The same holds true for the down beat, i.e , for a positive inclination (Fig. 3).

In the case of a positive inclination (Fig. 3), the translational velocity of the insect increases, decreases or remains unchanged, depending on whether the resistance of the air is less than, greater than or equal to the horizontal component of the pressure on the wings.

In the case of a negative inclination (Fig. 4), the horizontal component counteracts the horizontal flight of the insect, so that the translational motion is always retarded. The body of the insect is maintained in the air, during the brief moment taken to raise the wings, by the previously accumulated vital force contained in the body of the rapidly moving creature; but the retardation is fairly slight in view of the high translational velocity and the rapidity with which the wings are raised.

In general, then, the flight of an insect is now faster -- on the down beat -- now slower -- on the up beat.

The kind of flight described is the most favorable kind for an

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*These arguments concerning the pressure of the counterflow relate to the intersection of the wing by a plane normal to the longitudinal axis of the wing and parallel to the long axis of the insect's body (see figures).

insect, since the wings will support the body not only on the down beat but also on the up beat, which reduces the demands on the insect's strength. But even in this case the wing mechanism may be fairly simple: suppose that, before they start to beat, the wings have a negative inclination (Fig. 4); when they are raised they will balance the rapidly moving body; but, by virtue of their flexibility, during the rapid down beat their negative inclination becomes positive, and, thanks to the positive component, the flight of the insect will be accelerated.

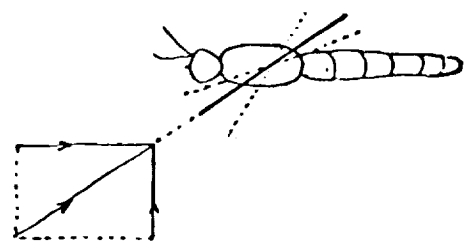

Fig. 4.

In view of their primitiveness and the rapid beating of their wings, insects can and probably do manage without a negative inclination: the flexibility of the wings alone is sufficient for sustained flight.

The velocity of the different parts of the beating wing is not the same. This means that the shape of the wing may not be perfectly plane nor may all the parts of the wing have the same flexibility.

Even if the wings of a flying insect are motionless relative to its body, there may still be pressure on them, and this pressure may be greater than, less than or equal to the weight of the creature, depending on the magnitude of the negative inclination. And therefore, in general, whatever the state of the wings -- whether they beat or not -- by maintaining a known inclination the insect can remain at the same height, rise or fall at will, if only it can preserve a sufficient horizontal velocity. If not, it can re-acquire such a velocity either by powerfully beating its wings at a positive inclination or by allowing itself to fall, i.e., in inclined flight

with the wings in motion or relatively motionless.

Conversely, when it is necessary to slow down or lose horizontal velocity, the insect, desiring to settle on the ground or the branch of a tree, without beating its wings, gradually increases their negative inclination, until it loses almost all its translational velocity and can cling to some perch; the increase in the negative inclination of the wings must completely balance the force supporting the weight of the insect's body, which decreases owing to the fall in horizontal velocity. Of course, there is a limit to this fall in velocity, beyond which no inclination is of any avail; this is why landing is impossible without a slight jolt.

The above, however, is more applicable to birds, which have a more complex flying apparatus. An insect can scarcely change the inclination of its wings in relation to its body freely as desired. Nevertheless, thanks to the flexibility of its elongated body it can change their inclination in relation to the horizon. The body acts like two control surfaces -- a horizontal and a vertical.

The deflection of the body changes the inclination of the wings with respect to the horizon, turning the insect to right or left.

The elongation of the wings, noticeable in all creatures that fly, increases the vertical sustaining force, for the same wing area.

The organization of the wings of birds and their mechanism of flight can be identified with the mechanism employed by insects if the bird is in rapid flight and its feathers are all interlocked to form a single whole as a result of the air pressure against the underside of the wing, which in this case is equivalent to a single elementary plate. The only difference is that the wings of a bird, driven by the muscles of the relatively solid leading edge, can be made to assume the necessary inclination at will and, in general, by means more complicated than those available to the insect.

But the situation is different at the beginning of a bird's flight or when, in a still atmosphere, it desires to hover motionlessly in the air, a feat that requires intense effort and is therefore more often reserved for windy days. Then every element of the wing, every feather performs its function independently, exactly like the wing of an insect; the bird becomes a multiwinged creature, the number of wings depending on the number of feathers.

When the wing is raised, if the bird itself is not moving and there is no wind, the air presses against the upper surface of the wing. To prevent the air pressure from forcing the bird down, each feather of the wing is capable, under the influence of this pressure, of orientating itself almost vertically. On the down beat the feathers are quickly rotated by the reversal of the pressure and align themselves horizontally with the edges overlapping to form

a single whole -- a continuous plate.

Anyone who has observed a bird hovering above its nest on a still day will have noticed how it is possible to see through the wings, which open to let through light and air as they are raised.

The curvature of the lower surface of the wings does very little to intensify the pressure on the wings in the form of flight just described and hence only slightly reduces the rate of beating and the work done in rising into the air. More important is the fact that the complexity of the wings and their convexity enable the bird to fold them neatly and snugly against its body. In fact, the bird uses its wings as a top coat to keep it warm when the fire that gives it the energy to fly is banked. But there is another, more deep-seated reason for the curvature of the wings.

In rapid horizontal flight the band of air below and above the wings acquires a known velocity as a result of their pressure. In order for this process to develop uniformly and smoothly, during the short time the air takes to pass beneath the wing, and in order for the air pressure on the wing to be uniformly distributed, the wing is curved. Without this curvature the air would not be given a smooth, uniform acceleration and would not exert the maximum * pressure on the wing. The curvature is similar to that of the blades of a turbine wheel that utilizes the force of moving water. The wings of creatures that fly rapidly are less convex than those of creatures that fly more slowly. But the more rapid the flight the sharper the body should be and the less its lateral curvature; thus, in rapidly flying creatures the low degree of curvature of the wings corresponds to a low degree of lateral curvature of the body, which may perhaps help the wings to fit more snugly to the flanks of the bird and keep it warm.

The independent action of each wing, i.e., the independent control of its inclination, and the rapidity with which the wings beat make lateral stabilization easy. Longitudinal stabilization, of course, is controlled by the tail.

If airplanes are ever replaced by ornithopters, in order to design the latter rationally it will be necessary to make a closer study of the flight of birds and insects.

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*Much later this was repeatedly confirmed by myself and others.

II

Uniform horizontal motion of the medium

So far we have considered only flight in a stationary atmosphere.

A uniform flow of air, which is at all points in uniform rectilinear motion in a single horizontal direction, can not, however much ingenuity is employed, have any effect on the motion of wings or the other organs necessary for flight. Everything will be exactly the same as if the air were absolutely still. The energy needed to fly will be the same, and the translational velocity will be the same, if it is referred to the air and not to the ground. Otherwise the velocity will change in magnitude and direction. This change in the velocity and direction of the flow is well known and we shall not dwell on it here.

Having derived the formulas for the motion of birds, insects, airplanes and airships, we can apply them, without introducing any error, to the case of a medium in uniform, horizontal motion.

In fact, everything depends on the relative, not on the absolute velocity, which can never be determined. Thus, the velocity of the medium relative to the earth is 30 meters per second; in relation to the sun the same air has a velocity of about 28 versts (= 28 km) per second; in relation to the stars we obtain yet another figure, and so on, because the universe is infinite. If the calculations pertaining to flight or other phenomena were to contain the absolute velocity of the medium, mechanics would have no useful application, since the absolute velocity can never be known.

Of course, no benefit and no work, for sustaining flight or other purposes, can be derived by a natural or artificial flying body or projectile from a mass of air, water or anything else moving uniformly in a horizontal straight line, albeit at infinite speed -- this makes absolutely no difference*. Only the velocity in relation to the earth will change.

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*I have retained these lines because even today it is not unusual to encounter muddy thinking about the law of relative motion even in the literature.

Thus, migrating birds make use of favorable winds, but this only increases their speed; the energy they expend per unit of time and even the movements of the wings and tail are not affected one jot.

If, for example, a bird flies 50 versts per hour in still air, then in a favorable wind blowing at the same speed it will fly 100 versts in the same period of time, doing the same amount of work and going through literally the same motions. Only in this sense can the uniform motion of the medium be any advantage to the bird or airplane. A wind will also facilitate takeoff from the ground.

III

An irregular flow of air

It is another matter when the air flow is nonuniform, as it is in practice. This nonuniform flow, this gustiness, is used by birds to fly without expending energy -- it can also be used by a flying machine to reduce the load on the engines.

Of course, it is obvious why an ascending flow* will maintain a body in the air without it having to do work. The same properties are shared by an inclined ascending flow. Such a flow is unavoidable when a medium moves over a surface inclined to the horizon or meets a steep mountain.

A vertical oscillating movement of the air may also cause a body to ascend, but it is more usual to encounter a horizontal oscillation of the medium, since there are fewer obstacles to this kind of motion.

We often see hawks flying almost effortlessly, without using their wings, carried along in their spiral ascent by the general flow of air. The reason for their soaring flight may be precisely this horizontal motion of the air.

It is rare that we observe a wind that blows first from one side and then from the other, changing direction every few minutes.

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*For example, the central ascending part of a cyclone or whirlwind.

It is more usual to note the speed of the wind now increasing by a certain amount a as compared with its average speed b, now decreasing with respect to b by a similar amount (a).

Let us consider this very simple case of oscillating motion. Actually, it is not only a and b that change but also the direction of the wind. Let us start by examining the phenomenon in a simplified form.

We shall assume that the hawk is carried in one direction by the wind at an average speed b. Then to the bird it will seem as if the wind is blowing alternately from in front and behind at the speed a; and relative to the hawk the motion of the air will in fact be thus. You will recall that everything depends on the relative, not on the absolute velocity.

What does the hawk do? He always keeps the underside of his wings facing into the wind, so that every gust raises him by a certain amount, depending on the intensity of the gust a. He is forced to describe circles, rising or falling, depending on the strength of the gusts, even flapping his wings at critical moments to assist the air in its work. Each half circle corresponds to a change in the relative direction of the air flow by the amount a; a complete circle corresponds to a full cycle of the oscillation of the atmosphere. This spiral trajectory is carried along, together with the hawk, by the general air flow at the speed b.

It is quite possible that in reality the phenomenon described is much more complicated -- my only object is to demonstrate the possibility of soaring, under certain conditions, simply by utilizing the strength of a fluctuating wind.

This example, of course, will also apply to a rationally controlled or suitably designed flying machine.

Many birds soar, others merely make some use of irregular air currents, i.e., employ them to facilitate flight. It is doubtful whether insects, with their uncomplicated flying mechanisms, are capable of the wing movements that make soaring possible.

However, it is not difficult to visualize a machine that would maintain itself in the air and fly, without any link with the ground, thanks to its own oscillating motion.

Imagine a sort of aerial aerometer with a wing attached, supported by a balloon inflated with a light gas (Fig. 5).

This system will always occupy a vertical position. So will the wing, until a wind begins to blow from the side. Then, rotating about its axis, it will assume a position slightly inclined to the horizon. The air pressure due to the lateral gust will force the machine to rise for a certain time.

But when the wind begins to blow from the opposite side, the plate, i.e., the wing, will again be slightly deflected from the

vertical under the influence of the new current, and again the machine will rise, and so on.

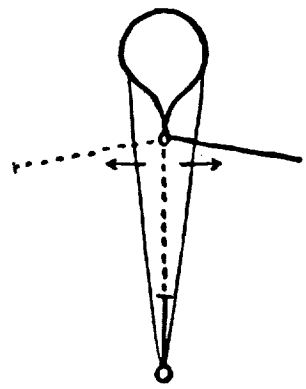

Fig. 5.

If the machine had weight, the ascent could be controlled or rather the attraction of gravity could be neutralized, and the machine would remain at the same height.

The design of such a machine, its efficiency, the load it could raise, and other pertinent computations must be based on a careful study of the irregular motion of the wind, i.e., its velocity and direction as a function of time. Once some such study of this motion has been made, we shall be able to design a craft that will stay in the air without any link with the ground and exploit its various advantages.

THE AEROPLANE OR BIRD-LIKE (AERIAL) FLYING MACHINE*

I

A General View of Winged Flight Wings, Based on The Most Elementary Experiments and Arguments**

Imagine a small bench (Fig. 1) with a horizontal shaft; two wings are mounted symmetrically on the shaft and in the same plane; the wings can be rotated through the weight of a load attached to a cord wound around the shaft.

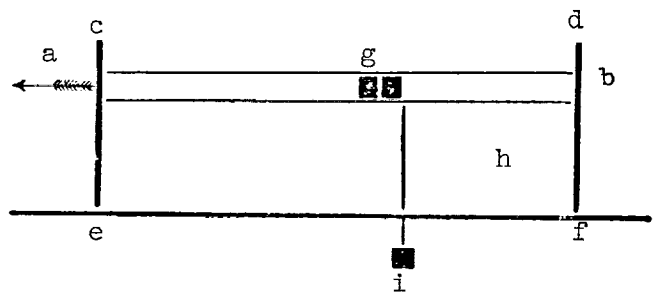

Fig. 1.

ab - the shaft about which the cord gi is wound, the cord being pulled downward by the load i; ce and df are struts supporting the shaft. The arrow to the left indicates the direction of motion.

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*This work was first printed in Nos. 43-46 of the periodical Nauka i zhizn' [Science and Life] (Moscow), 1894. It was printed as a separate edition, in the form of the book, from the same font in 1895.

**This article was written as early as February 1894; the major inferences had been drawn even earlier (1890). (Note by K. E. Tsiolkovskiy.).

Let us take up this device and carefully observe the speed with which the cord unwinds, or the speed with which the wings move.

Before the cord has become completely unwound, let us carry the apparatus in the direction its axis (shaft) points, and we shall immediately observe the slowing down of the rotation of the wings.

If we start running, the wings of the apparatus will spin at an extremely slow rate. In general, the faster the apparatus moves in the direction of its axis, the slower will be the rotation.

Clearly, the resistance of the air to the flapping of the wings will increase as the translational motion of the wings partakes of the overall motion.

Now I take some large lightweight wings built to resemble bird wings: as they are raised they pass freely through the air, and as they are lowered the air exerts a certain amount of pressure on them -- from below upward -- and the weight of my body is thereby diminished. Remaining in one spot I wave the wings like a bird; the faster I do that the greater the pressure that the air will exert on them. Thus, in accord with the well-known laws governing the resistance of fluids, when I double the speed with which I flap the wings, the pressure on the wings will increase fourfold, and when I triple the speed with which I flap the wings, the pressure will increase ninefold, and so on.

It is clear from the foregoing that, at a certain rate of wing flapping, the pressure on the wings from below will be equal to the weight of my body plus the weight of the wings. In that case, I will no longer be pressing on the ground, I will no longer be touching the ground -- and it is from that instant that my flight will begin.

Fairly straightforward calculations reveal, however, that under such elementary conditions, with a wing surface of 11 square meters ($2^1/_2$ square sagenes) and at a weight, of the wings and the human body combined, of 100 kg (6 poods), the work per second required to balance out the force of gravity will be about 10 horsepower (750 kilogram-meters), i.e., the human driver will have to exert an energy at least 75 times greater than he is usually capable of (according to LeBon, the average work of a healthy human adult is even 4 times less than what we presuppose here).

All of this information is generally known, and scientists have long ago proved that a goose maintaining itself in the air under the conditions described has to expend the energy of a horse*. But

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*This view of flight conditions has given rise to pessimism in aviation thinking.

let us assume that I am actually endowed with such a gigantic energy enabling me to hover motionless in the air by flapping wings. The work I do will of course be proportional to the resistance experienced by the wings and to the speed with which the wings move. The first quantity is invariable, since it is always equal to the body (which we assume here to include the weight of the wings). Consequently, the work I do will depend exclusively on the speed at which the wings are flapped. But, on the basis of the preceding experiment (Fig. 1), should I chance upon the notion of running while flapping the wings, then the flapping speed would decrease, given the same pressure on the wings. The work required to achieve flight would be reduced by the same factor.

Now suppose I should run at an extremely fast speed, and suppose I should begin the flight from the roof of an express train or during a very powerful head wind, and here let the energy required to achieve flight be comparable to the energy actually available to me. The former might be even still less: it all would depend on the rate of the translational motion. The experiments carried out with the rotor (Fig. 1) leave no room for doubt; a missile of this type has been demonstrated publicly and, on the initiative of N. Ye. Zhukovskiy [Joukowsky], was recently placed on display at the Moscow Mechanical Exposition (January 1894). Moreover, there have been other such projectiles and other such experiments (Marey, Lilienthal, etc.), which, though more complicated, confirmed the same basic truth: to wit, that as the translational velocity of flying machines is increased, the work required to keep the machines airborne varies inversely to the rate of the motion; nay, we can state further that, on the basis of both experimental and theoretical evidence, when the speeds are appreciable this decrease will be almost proportional to the rate of translational motion.

The problem is simply that rapid motion in fluids, even such low-density fluids as air, is fraught with great difficulties, for it requires work proportional to the cube of the velocity. For example, when the velocity is increased fourfold, the work done in cutting through the air will increase by a factor of 64 (or 4^3 fold), while the work done in maintaining the aerial projectile aloft will decrease by only a factor of 4.

The resistance of the medium, which is barely perceptible and even negligible at low velocities, becomes an unmanageable burden at a very fast translational motion.

These conclusions necessarily dampen our ardor somewhat and cool our enthusiasm for aviation machines. Nevertheless, we cannot refrain from making some exact calculations in order to ascertain the degree of advantage involved and the conditions propitious for flight.

The problem of flying by means of wings is one which is now

fashionable. On all sides, we hear pronouncements made on a "new" theory of soaring flight based on the inertia of the medium, i.e., on the inertia of air, in the case in point. The flying ability of birds is now ascribed not so much to the strength of their muscles as to the inertia of the medium or, expressed more simply, to the increased resistance presented by the air in response to the fast translational motion of the flying organisms. Now what is the "essence" of this phenomenon, and how are we to account for the experiment with the "rotor" (Fig. 1) and with similar contraptions (Marey, Maxim, and others)?

Imagine that you are standing on a viscous soil, something like pitch (black tar which becomes a semi-solid at a certain temperature) covered by grease, so that the soles of your feet cannot possibly adhere to it.

The longer you stand on this ground, the deeper you will sink down. If you walk or run over this ground, your shoes will leave imprints which will be the shallower the faster you move. Clearly, the work done in your movement will decrease in proportion to your speed, for as the speed increases you will have to make your way out of tracks which become shallower and shallower. No matter how soft this ground is and no matter how light it is in weight, its inertia in response to a sufficiently rapid motion will hinder your body from sinking into it, and you will then be moving as if there were a perfectly solid support underneath. We would have arrived at the same conclusions if we decided to traverse the sticky ground on a bicycle or some other vehicle, instead of walking or running. I once had occasion to skate in the autumn over very thin ice which had just frozen over; it did not break or even crack so long as I was skating vigorously, but I had only to slow down or come to a stop for the ice to start cracking, with water beginning to flow in the cracks around the skates.

II

Outline of an Elementary Design of an Aviation Vehicle

We begin by selecting a system appropriate to a flying projectile and make certain assumptions convenient to expedite our calculations. It is a most natural and best approach to imitate a bird, insofar as possible, as the most perfect mechanism most perfectly adapted for speed of flight and maneuverability. My first calculations therefore pertained to such a bird-like projectile capable of flapping its wings. These calculations are highly interesting from

the standpoint of clarifying the phenomenon of flying by animals; but they do not suit our purposes, since imitating a bird is extremely difficult to achieve in the engineering sense because of the complexity of the motion of the wings and tail, as well as the structural complexity of these organs.*

We shall seek to imitate the bird only partially, to the degree feasible.

Consider a soaring bird, i.e., a bird flying without flapping its wings.

Sometimes the bird soars or glides on the speed which it had acquired earlier; for example, if it wishes to come to a stop, say to perch on a tree or to land on ground, it will cease flapping its wings for a while, since it will have to lose its translational velocity, and gliding flight will be achieved for several seconds as a result.

Sometimes it glides owing to a slow fall in an inclined position, and sometimes it soars on gusts of wind; in the first case the flight is accomplished by the force of gravity, and in the second case it is accomplished by the energy of the air flowing at a variable speed.

As an example, Lilienthal recently succeeded in executing soaring flight with his artificial wings, and a gust of wind raised the renowned researcher several meters high on one occasion. Even though this braked his fall, it was still quite hazardous, for there was a serious danger of the experimenter being turned over (in this context, I feel it my duty to express my deep sympathy on the occasion of the news of the accident which befell him in a flight, as reported in the press).

Sometimes the soaring flight of birds can be prolonged for an indefinite span of time on account of continual gusts of wind, i.e., enormous air waves. This phenomenon is a complex one, and the scope of this work does not permit us to go into an explanation of its causes here; we need only mention that in general soaring flight in a uniform stream of the atmosphere, requires a certain thrust, without which it cannot be prolonged. In birds, this thrust is brought about by beating the wings against the air. But what if there are no such beats, i.e., if our bird were to stop still in the midst of its soaring flight! What would then hold it up?

Let our projectile be given the shape of a soaring bird frozen in mid-flight, except that, in the place of the bird's head, we put

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*However, Truve was successful in constructing a model which flapped wings and flew several meters in this manner.

two propellers rotating in opposed directions and producing a thrust in a single direction coinciding with the flight of the projectile. This thrust of the propellers would be capable of sustaining the translational velocity of the vehicle and could occur, under these conditions, at a single height, or could occur in gaining height. The propellers are rotating in opposed directions so that they will not tend to spin our artificial bird about its longitudinal axis[5]. We replace the animal's muscles by impulsive neutral engines. These will not require a large fuel reserve (gasoline) and will not require heavy steam boilers or substantial water supplies[6]; the power cylinders can be cooled by air, as is usually done in practice*.

Instead of the tail, we can construct a double rudder -- in the vertical and in the horizontal planes. The first plane will control the inclination of the projectile to the horizon, while the second rudder will control the inclination to the meridian. The first rudder will chiefly maintain the longitudinal axis of the projectile in the horizontal direction, while the second will chiefly allow it to turn to one side. It is desirable that both rudders operate automatically, as is now being attempted in the case of steamships.

A double rudder, double propeller, and immobility of the wings, are features which we have hit upon not for convenience and economy in work expenditure, but simply because the design is feasible; all these departures from the natural flight organs actually only increase the amount of energy required and are adopted here because of bitter necessity[7].

Let us not consider this any great tragedy, however, since a locomotive is not an exact simulacrum of a horse, any more than a steamship is an exact similacrum of a fish.

Nevertheless, in the form described here, our projectile will greatly resemble the structure of a flying coleopterous insect such as a May-bug or beetle.

During flight, its rigid outer wings are extended and motionless, as in our imitation of a bird. The other wings are slender semi-transparent wings which the insect flaps to maintain its thrust, and which correspond to the beating blades of our propellers. The insect's rudder is provided by its own articulated and flexible body.

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*It is true that by utilizing very thin tubes on the boilers, as in the Serpollet system, it may be possible, theoretically, to construct steam engines of extremely high power output.

III

Basic Air Resistance Formulas and Derivation of the Laws of Flight, Disregarding the Resistance of the Medium to the Translational Motion of the Projectile[8]

We may begin by first finding some of the laws pertaining to the flight of an airborne vehicle, using a simplified design of such a vehicle (Fig. 2) in order to render the calculations more convenient.

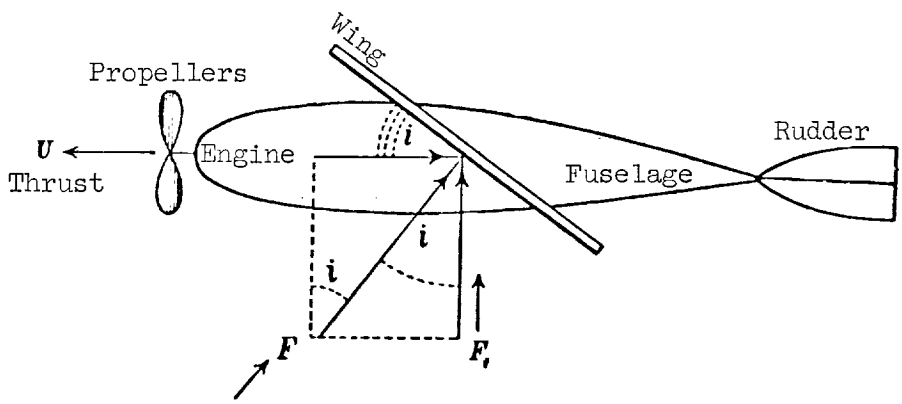

Fig. 2

When the projectile moves rapidly in the horizontal direction and at a small angle of inclination i of the wings to the horizon, we have, in an approximation*:

*Cf. Vol. IV of the Proceedings of the Division of Physical Sciences of the Imperial Society of Amateurs of Natural Science [Trudy otdel. fiz. nauk imp. obshch. lyubitelei estestvoznaniya] Moscow, 1891; formulas (22) and (25) from the article "Fluid pressure on a plane moving through the fluid."

$$F = \frac{d}{g} \cdot S \cdot V^2 \cdot \sin i \cdot \frac{21 \cdot \sqrt{\frac{a}{b}}}{2 \cdot \sqrt{\pi}\left(7 + \frac{2a}{b}\right)} \qquad (1)$$

Here F is the force of the pressure of the oncoming air normal to the wing, a force which, because of the low value of the inclination angle i, may be assumed equal to the vertical component F_1 of that force (Fig. 2); it will keep the projectile from falling and is equal to the weight of the projectile and all its contents; d is the volume weight of the air; g is the acceleration of the force of the earth's gravitational attraction; S is the area presented by one of the wings, which are assumed to be equal, rectangular in shape and symmetrically aligned; a and b are the length and the width of each of the wings; V is the horizontal translational velocity of the projectile with respect to the direction of the length of the projectile or with respect to the direction of the width of the wing; i is the angle of inclination of the wings to the horizon -- this is not a large angle; π is the ratio of the perimeter of a circle to its diameter. We learn from the formula that the pressure of the oncoming air on the wings is proportional to the sine of the angle of deviation of the wings from the direction of the air stream. This law is confirmed by many scientists and was discovered in comparatively recent times (Duchmin, 1842; Otto Liliental, 1889; Marey, 1890; these scientists derived it experimentally; it has been confirmed theoretically by Lord Rayleigh in 1876). We also realize from the formula that the force F of the pressure on the wings depends not solely on the amount of surface they present but also on the aspect ratio $\frac{a}{b}$ of the wings. We put, in equation (1):

$$\frac{21 \cdot \sqrt{\frac{a}{b}}}{2g\sqrt{\pi}\left(7 + \frac{2a}{b}\right)} = f\left(\frac{a}{b}\right); \qquad (2)$$

whereupon we obtain

$$F = d \cdot S \cdot V^2 \cdot \sin i \cdot f\left(\frac{a}{b}\right). \qquad (3)$$

On putting the ratio $\frac{a}{b}$ equal to x in formula (2) and seeking to maximize the expression $f\left(\frac{a}{b}\right)$, we find that this expression obtains its highest value at $x = 3^1/_2$. Indeed, the force exerted by the pressure on the wings, with their area S unaltered, will be greatest when the length of the wing exceeds its width by a factor of $3^1/_2$. The force of the pressure will then be increased by only 1/5 compared to the force of the pressure on square-shaped wings, where

$$\frac{a}{b} = 1.$$

At an aspect ratio in (2) equal to

$$\frac{a}{b} = 2,$$

the force F will be increased by a factor of 1.16, i.e., almost as much as (1/8). We realize then that the aspect ratio of the wings can be profitably increased only in a very modest way, at most by a factor of two.

Equation (1) does not make entirely explicit those favorable conditions which we assumed for our calculations of bird-like flying machines. I therefore consider it necessary to derive here some other formulas which are in essence identical to those cited earlier, and which constitute a corollary or a particular case of the preceding ones.

Consider the equation*

$$F = \frac{S \cdot d}{g} \cdot V_n^2 \left\{ 1 + \frac{21 \cdot \sqrt{\frac{a}{b}}}{2 \cdot \sqrt{\pi}\left(7 + \frac{2a}{b}\right)} \cdot \left(\frac{V_p}{V_n}\right) \right\}, \qquad (4)$$

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*Cf. Ibid: formulas (13) and (22)

where V_n is the normal velocity at which the plate is driven; this motion is perpendicular to the wing surface; V_p is the velocity parallel to the wing plane. If the velocity parallel to the wing plane is absent ($V_p = 0$), then the force of the pressure on the wing will become equal to

$$F = \frac{S \cdot d}{g} \; V_n^2. \tag{5}$$

Clearly, the factor in the brackets [in equation (4)] is equal to

$$\left\{ 1 + \frac{2l \cdot \sqrt{\frac{a}{b}}}{2 \cdot \sqrt{\pi}\left(7 + \frac{2a}{b}\right)} \cdot \left(\frac{V_p}{V_n}\right) \right\}, \tag{6}$$

and demonstrates by how many times the pressure on the plate (or on the wing) will be increased as a result of a translational motion at velocity V_p compared to that force (4) when the translational motion is absent. At an aspect ratio of say two:

$$\frac{a}{b} = 2,$$

the factor (6) will revert to

$$\left\{ 1 + 0.75 \left(\frac{V_p}{V_n}\right) \right\}, \tag{7}$$

from which we infer that the pressure force F will increase at an extremely rapid rate as the velocity of the translational motion of the plate increases. Let us cite an example: suppose I flap wings, while remaining motionless, at a speed of 10 centimeters a second; this will

cause a certain air pressure to be exerted on the wings; now suppose that, flapping the wings like a bird, I at the same time proceed to run forward at a speed of 10 meters a second (1000 cm/sec); then the velocity V_p of my translational motion will exceed the speed at which the wings are flapped by 100 times ($V_p : V_n = 100$); and this suffices for the force of the pressure on the wings to increase 77 times, in accord with formula (7).

Are these not conditions favorable to flight!? The factor (6) shows that as the speed of my running increases the air will, as it were, thicken, apparently turning into a mass of steadily increasing density. In the limit, i.e., when the translational velocity becomes infinite, the light gas will have transformed, as it were, into a completely motionless, straight, and strikingly slippery steel track for flying projectiles. If we neglect the friction of the wings on the air (the wings will offer no other resistance to the oncoming air, being, theoretically at least, infinitely thin and flat) and if we neglect the resistance of the air to my body or to the projectile, then nothing will prevent the projectile from moving forward at an arbitrarily fast speed, and the work done in achieving this motion will tend to zero in the limit; in other words, a very rapid motion through the air would constitute the most convenient and the cheapest means of communication.

I repeat that equation (1) which serves as the basis for our calculations of the flight conditions is only a particular case of equation (3), which yielded us such promising results. By transforming this equation (3), we may obtain the same entirely propitious conclusions.

After this rather brief digression, let us again resume our efforts toward compiling the formulas determining the laws and conditions for flight. One of the most important problems to solve is to find the energy of the engines at which a projectile of known weight and design will remain airborne. Clearly, the weight P_v of such a projectile (Fig. 2) involved in uniform and horizontal translational motion must be equal to the sum of the pressures exerted on both wings (Fig. 2), i.e.,

$$2 \cdot F_i = P_v \tag{8}$$

or approximately, when the angle i is small,

$$2 \cdot F = P_v. \tag{9}$$

But the weight P_v of an airship is composed of many parameters including: the weight $2P_a$ of the wings; the weight P_c of the fuselage of the projectile and of its skin; the weight P_p of the passengers; the weight P_m of the engines, which consists of two terms: the first P_1 refers to the part responsible for keeping the projectile from falling, i.e., the part that copes with gravity, and the second P_2 refers to the part involved in overcoming friction and the resistance of the air[9]; we thus have

$$P_m = P_1 + P_2; \tag{10}$$

and, further, the weight P_b of the gasoline or other fuel, in other words, the weight of the material supply of potential energy for the liberation of that energy by means of engines, if needed; finally, there is still the special load P, the weight of all small accessories on the projectile and of sundry unforeseen items.

We then have

$$P_v = 2P_a + P_c + P_p + P + P_1 + P_2 + P_b. \tag{11}$$

Even though there is a single motor, it can be conceived of as divided in two parts: the first, a weight P_1, to hindering the projectile from losing height, and a second, of weight P_2, cutting through the surrounding medium and thereby blazing the trail ahead.

Dividing both parts of the preceding equation by the weight P_v of the entire projectile, we have

$$1 = \frac{2P_a}{P_v} + \frac{P_c}{P_v} + \frac{P_p}{P_v} + \frac{P}{P_v} + \frac{P_1}{P_v} + \frac{P_2}{P_v} + \frac{P_b}{P_v}. \tag{12}$$

Naturally, each of the ratios in the second part of this equation is less than unity; we denote these by coefficients C: C_a for the wings, C_c for the fuselage, C_p for the passengers, C for the load, C_1 and C_2 for the load represented by the engines and C_b for the fuel load; we now write:

$$1 = 2 \cdot C_a + C_c + C_p + C + C_1 + C_2 + C_b, \tag{13}$$

where

$$C_a = \frac{P_a}{P_v}; \tag{14}$$

$$C_c = \frac{P_c}{P_v}; \tag{15}$$

$$C_p = \frac{P_p}{P_v}; \tag{16}$$

$$C = \frac{P}{P_v}; \tag{17}$$

$$C_1 + C_2 = C_m; \tag{18}$$

$$C_1 = \frac{P_1}{P_v}; \tag{19}$$

$$C_2 = \frac{P_2}{P_v}; \tag{20}$$

$$C_b = \frac{P_b}{P_v}. \tag{21}$$

Suppose two engines are present in the air ship; we denote the power developed by the first engine as T_1, and the power developed by the second as T_2, with the total power of the two engines as T_m; then

$$T_1 + T_2 = T_m. \tag{22}$$

By bringing the propellers into motion, the motors develop a horizontal thrust f of two types: f_1 keeps the projectile from falling, and f_2 overcomes the inertia of the air and its friction against the wings and against the fuselage of the projectile (Fig. 2). The amount of thrust of the second type depends exclusively on the velocity of the projectile, its design, and the surface area of the wings; the first type of thrust in turn depends on the weight of the ship and on the angle of inclination i of the wings to the horizon (Fig. 2); and of course

$$f = f_1 + f_2. \tag{23}$$

As far as the first type of thrust is concerned, for the time being, and examining Fig. 2, we find the following expression for that thrust when the projectile is maintaining a constant velocity and a constant height above sea level:

$$f_1 = 2 \cdot F \cdot \sin i. \tag{24}$$

If the angle i is small, then, from equation (9) we obtain, in place of this last equation (24), the formula

$$f_1 = P_v \cdot \sin i. \tag{25}$$

On the other hand, we have

$$f_1 \cdot V = T_1 \cdot C_h; \tag{26}$$

here C_h, or the propeller coefficient, shows how great a part of the total power of the engine of the first type T_1 is utilized as a result of the propeller action. Note that C_h will approach unity as the surface of the propeller increases; however, because of the friction of the propeller blades against the medium, C_h will not exceed 3/4. The last equation is used to express the work done per second by the thrust of the 1st type f_1 at a horizontal projectile velocity V.

Now expressing the fact that the power T_1 of the engine is proportional to the weight P_1 of the engine and to the specific energy E, we write

$$T_1 = E \cdot P_1. \tag{27}$$

Here the energy E is the work done per weight unit of engine per time unit, or the quotient from the division of the power of the motor (together with its generator, e.g., a boiler in the case of steam engines) by its weight.

From the three last equations and from equation (19), by eliminating f_1, T_1, and P_1 in succession, we obtain

$$V \cdot \sin i = C_1 \cdot E \cdot C_h. \tag{28}$$

On substituting in equation (3)

$$F = \frac{P_v}{2}$$

according to equation (9), and then eliminating sin i from the last formula (28) by means of equation (9), we obtain

$$E = \frac{P_v}{2 \cdot d \cdot S \cdot V \cdot C_1 \cdot f\left(\frac{a}{b}\right) C_h}. \tag{29}$$

This formula contains the expression for the energy E required from the engines of the flying projectile to sustain it in the air at a certain height and at a translational velocity V. If this velocity is not large, for example if it does exceed the velocity of birds, and also if the wings of the projectile are not particularly large, and if accordingly the friction against the oncoming air is not large, then the power T_2 of the second engine may be neglected as a comparatively small quantity, and we shall then have information on the total power T_m of the engines, in accord with equations (18) and (26); namely, we have

$$T_m = T_1 = E \cdot C_1 \cdot P_v. \tag{30}$$

The energy E of the engines, the energy required for flying, is in general inordinately high, and therefore let us analyze the conditions under which it would attain its least value. From equation (29), it is clear that the specific energy E of the engines is inversely proportional to the translational velocity V of the projectile, i.e., the greater the velocity of the projectile, the less the energy required to propel it; hence, we must impart to the

projectile a shape as slender and smooth as possible (as in the case of birds and fish) and we must avoid giving the wings very large dimensions, in order to avoid an inordinately great increase in the friction and in the resistance of the medium. Furthermore, we must transfer as large a portion as possible of the total lifting force of the projectile to the engine of the second type, designed to cut through the air. Formula (28) confirms the concepts we expressed earlier, to a partial extent, on the ease of flying, such as it was conceived of in the first days of speculation on this subject. In fact, this formula reveals that when the velocity V of the translational motion of the projectile is very high, the energy E and accordingly the work of its engines [equation (30)] is extraordinarily small; it becomes zero at infinite horizontal velocity, i.e., the air presents an entirely motionless support for the wings, for instance in the manner of rails on a railroad. As for the labor involved in cutting through the air with a small and sharp projectile, it seems to be so trivial to us! The horizontal velocity seems little short of infinite to our imagination! Pursuing this analogy, the work of flying resembles the motion of a projectile over a firm and very smooth ground.

The following important inference from equation (29) consists in the notion that the energy E is directly proportional to the ratio P_v:S, i.e., to the ratio of the weight of the projectile to its wing area; now, if the weight of the projectile increases in proportional to the increase in the wing area, then no great energy will be required. Put briefly, the energy will be independent of the mass of the projectile, provided the wing area is proportional to the weight of the projectile. It turns out, apparently, to make no difference whether the flying machine is built to carry one man aloft or a thousand men aloft. As we become better acquainted with the theory of airfoils, we see that this encouraging conclusion is drastically restricted in practice.

Further, it is clear from formula (29) that the energy E is inversely proportional to the wing area. It is accordingly useful, to a certain extent, to have the greatest possible wing coefficient (C_a).

The energy is inversely proportional to the volume weight of the air medium d, i.e., as the flying altitude increases, the energy required from the engines propelling the aerial machines will become greater.

The energy E is inversely proportional to the weight of the machine, or to its coefficient C_1. This means that the greater the part of the total lifting force that we assign to the engine, the smaller the engine power may be.

The energy E is inversely proportional to the value of the function $f(\frac{a}{b})$. This function of the aspect ratio $\frac{a}{b}$ of the wings will have a maximum when the wing aspect ratio is $3^1/_2$. But an oblong wing will be heavier than a square wing of the same area, so that, in practice, taking other conditions into consideration, we can have a $\frac{a}{b}$ ratio of at most two. Then

$$f\left(\frac{a}{b}\right) = f\ (2) = 0.76:g,$$

and in the case of a square wing

$$f\ (1) = 0.66:g,$$

i.e., less by only 1/6 to 1/7.

All of the conclusions presented here are, of course, only approximate. Actually, the variables chosen are interdependent as well. For example, when the projectile rises into the rarefied layers of the atmosphere the energy will have to increase; but the velocity of the projectile in a rarefied medium will increase because of the decreased resistance presented by the medium, and the energy will decline for that reason. Again, flying projectiles of large size will cut through the air with comparatively greater ease and therefore will have a greater translational velocity V, so that the engine energy will be reduced [in accordance with equation (29)].

In sum, we see fairly clearly that the problem of how to determine the energy of an airborne craft and the laws governing its motion is a highly complicated question and still requires a great deal of work. For the time being, we have only been feeling out our ground in a rough way and now, with some idea of what it is like, we can proceed with greater boldness.

IV

Theory of the Strength of the Wings on an Aerial Vehicle

We are obliged to copy the shape of the wing and its position relative to the projectile's hull from the example set by birds. Let us assume to start, for the sake of simplicity in our calculations, that the surface of the wing has a rectangular shape and a horizontal position, and that the long side a of the wing is normal to the longitudinal axis of the fuselage.

Assuming the wing to be made of some material of uniform density, we find the following condition for equal resistance to breaking strain: the thickness h of any portion of the wing will be directly proportional to its distance from the thin terminal portions of the wing.

We consider it superfluous to prove this theorem here. A wing designed in obedience to this law for economy of weight will have a transverse cross section as shown in Fig. 3 or, more accurately, one such as shown in Fig. 4[10].

Note that, in the derivation of the preceding theorem, we neglect the actual weight of the wing itself, as a quantity rather small compared to the force of pressure exerted on the wing [equation (9)].

The resistance to breaking strain at the point where the wing is attached, where it is of thickness h, is expressed, in the case of a force acting on the middle of the wing (or on the center of air pressure of the oncoming stream) by the following ratio:

$$\frac{I \cdot h^2 \cdot K \cdot D}{3L \cdot m \cdot D_1}. \tag{31}$$

We shall not bother to prove this theorem either, in view of its elementary nature; clearly, from this formula we see that the wing's resistance to breaking strain is proportional to the width I of the wing, to the square of its thickness h^2, to the strength of the wing material, and to the density ratio $\frac{D}{D_1}$ (D is the natural density of

the wing material, D_1 is the mean density of the same material when subjected artificially to compression or thinning; we shall explain this shortly); moreover, the wing's resistance to breaking strain is inversely proportional to the desired strength m of the wing (for which the modulus of rupture K decreases m times) and to the length L of the wing.

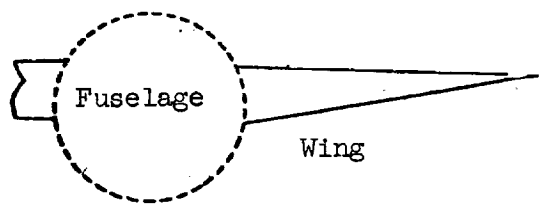

Fig. 3

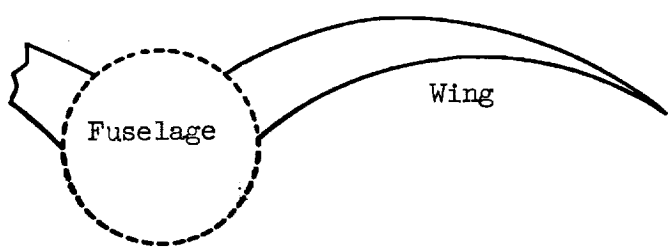

Fig. 4

Let us select the best material, for instance steel of density D_1 and of strength K; the rupture modulus (K) for the best grades of steel will not be greater than 100 kg per square mm of the cross

section. If we fashion a solid wing of this material, the weight will be large, and the resistance to breaking strain will be quite small. How are we to deal with this situation? If we could but rarefy the material to any arbitrary extent, the weight of such a low-density wing would be less and its strength would be greater. This flows from the last formula; actually, by leaving the surface and weight of the wing unaltered, but thinning out the material to one-half its density, we are afforded the possibility of also doubling its thickness h, so that the resistance to breaking strain will be increased by 4 times; the decrease in the density D, on the other hand, will mean a decrease of only twofold in the resistance to breaking strain; the net result then, in general, is that this resistance will be doubled. At first glance, changing the density of the material seems to be perfectly inconceivable, at least over a wide range, and in fact altering the density of steel by working the steel, is feasible only to an extremely small degree. It can be diminished far more drastically by melting the steel down with some low-density materials, for example with aluminum, but all of this will still fail to meet the problem.

Let us try another approach: we now vary not the physical density but rather the average overall density of the wing, making it of small high-density components with the greatest possible distances between the components. If, for example, we were to construct the wings of hollow metal tubes coated with some overall surface, then the average density of this type of wings would be far less than the density of steel. There exist many practicable ways to reduce the average density of structural materials. Nature has not neglected this approach either; for example, the bones of many animals are either hollow (i.e., filled with air, as in the case of birds), or are composed of a multiplicity of crisscrossing rods or plates covered by an overall osseous surface (the skull of humans and other animals, the heads of the large arm and foot bones). Our giant bridges and various huge structures that have gone up in recent times are also composed of a whole veritable labyrinth of beams and cross-beams, even though not always covered by some general surface. If we were to use such a covering on the Eiffel Tower, for instance, as a casing, we would find the average density of the tower to be similar to the density of the wings we are referring to.

In expression (31), the ratio $\frac{K}{D_1}$ is an important one. This is the strength of the material per unit density D_1. If we consider a long list of the materials used in industry, and determine the $\frac{K}{D_1}$ ratio for each of these, we shall find aluminum alloys to be most

suitable and toughest (cobalt, nickel, and other expensive and little-used materials are not considered here). Without going into the technical details of these alloys, we shall take them as materials for our wings and we shall reserve the possibility of arbitrarily reducing the average density D, assuming here that the tensile strength $\frac{K}{m}$ of the material will decrease in proportion to the decrease in the density. It will decrease by $\frac{D_1}{D}$ times, and the factor $\frac{D}{D_1}$ will appear in formula (31).

Decrease in density is achieved in practice only by the extremely artificial construction of a "porous mass." This is a particularly difficult task, when the elements of this mass have to be very fine. And the wings of the projectile will be such. A slight change in the density is of course not all that difficult, but it is no easy matter to thin out a material by 100 to 1,000 times.

For the wing, a sufficient resistance to breaking strain must be equal to the force of the pressure F exerted on the wing by the oncoming air stream, i.e., to half the weight P_v of the projectile. On the basis of formula (31), then, we write

$$\frac{I \cdot h^2 \cdot K \cdot D}{3 \cdot L \cdot m \cdot D_1} = \frac{P_v}{2}. \tag{32}$$

The weight P_a of the wing as a triangular prism (Fig. 5) is clearly equal to

$$P_a = 1/2 \cdot L \cdot I \cdot h \cdot D. \tag{33}$$

We saw that the most favorable ratio of the length L of the wing to its width I did not exceed two

$$\frac{a}{b} = \frac{L}{I} = 2.$$

We consequently have, in addition, the equation

$$L = 2I. \tag{34}$$

On eliminating L from equations (32) and (33), we find

$$\frac{h^2 \cdot K \cdot D}{3 \cdot m \cdot D_1} = P_v, \tag{35}$$

$$P_a = I^2 \cdot h \cdot D. \tag{36}$$

Now, on eliminating from equation (35) the thickness h of the wing, by means of equation (36), and on determining the weight P_a of the wing, we obtain

$$P_a = I^2 \cdot \sqrt{\frac{3 \cdot D_1}{K}} \; P_v \cdot D \cdot m. \tag{37}$$

Clearly, from this formula, the weight P_a of the wing will be proportional to I^2 at the same weight P_v of the projectile, i.e., proportional to the wing area, since $S = L \cdot I$, or, according to equation (34), $S = 2 \cdot I^2$.

Again, clearly, from formula (37), the weight of the wing will be inversely proportional to the square root of the strength of the material $\frac{K}{m}$, and will be directly proportional to the square root of the average density of the porous mass forming the wings D or inversely proportional to the square root of the thinning out $\frac{D_1}{D}$ of this material, e.g., steel, aluminum, etc.

Thus, the more porous we make the wings, the lower their weight will be, for the same projectile and the same wing area. But there is a limit, of course, to this thinning-out process: namely, the

fraction $\frac{h}{I}$ must not be close to unity, for otherwise the wings would turn into spherical masses and since during their translational motion, they would thereby present a great resistance to the oncoming air, they would be extremely ill adapted to performing their function -- that of keeping the projectile airborne. It might even occur to us, and not without reason either, to fill the porous mass of the wings with hydrogen; then the internal partition members of the wings might no longer be necessary, but in that case our aerial vehicle would have become transformed into an aerostat.

Let us transform equation (37). We saw that the energy of the projectile remained unchanged when the weight increased in proportion to the wing surface, i.e., when the ratio $\frac{P_v}{S}$ remained constant. We will assume

$$\frac{P_v}{S} = A,$$

or, since $S = 2I^2$, then

$$\frac{P_v}{2I^2} = A.$$

Now, on eliminating I^2 from (37) by means of this equation, and then on eliminating P_a or the weight of the wings from the same equation, since this constitutes, according to equation (14), a small part C_a of the projectile weight P_v, we find

$$C_a = \sqrt{\frac{3 \cdot m \cdot P_v \cdot D_1 \cdot D}{4 \cdot A^2 \cdot K}}. \qquad (38)$$

Hence, it is clear that the wing coefficient C_a will increase in proportion to the square root of the total weight P_v of the aerial

vehicle. Let me clarify this conclusion: since we possess a certain maximum energy E in the engines, we may construct a series of flying projectiles of different mass. All of our projectiles will fly when the area of their wings corresponds to their weight [equation (29)]. But the relative weight of the wings

$$C_a = \frac{P_a}{P_v}$$

will then increase more and more as the projectile itself is enlarged. Briefly then, in small vehicles the relative weight of the wings will be small, in large vehicles it will be large and will swallow up the projectile proper. For example, if the weight of the projectile increases successively by 4, 9, 16, 25, 36 times, then the relative weight of the wings on these projectiles will increase successively by 2, 3, 4, 5, 6 times, but the absolute weight of the wings will then be given by the numbers: $8P_a$, $27P_a$, $64P_a$, $125P_a$, $216P_a$, where P_a is the absolute weight of the wings on the smallest projectile in question.

This conclusion following from equation (38) presupposes an unaltered mean density D of the wings. If it is altered, the relative weight C_a of the wing may remain constant. This requires that the product P_v D in formula (38) remain constant. By putting

$$P_v \cdot D = B, \tag{39}$$

we see that the mean density D of the wings is inversely proportional to the weight P_v of the projectile; then the relative weight C_a of the wing will be the same for both large and small aerial vehicles.

Consequently, the wings will have to be made increasingly more porous to the degree that the weight of the projectile is larger, until, at a certain projectile size, they are bound to turn into impracticably bulky shapes which function poorly.

Thus, as the weight of the projectile is increased, we face one of two possibilities: either the wings absorb an increasingly greater share C_a of the total weight of the projectile, while their

density D remains unchanged, or else their density D can decrease in proportion to the increased weight of the projectile but they will become transformed into a porous mass incapable of cutting through the air and incapable of fulfilling its function.

Either alternative shows that the size of the flying vehicles is limited.

With the aid of the last equations given, we can also draw more specific conclusions.

Let us calculate the size and the area of a wing made of the best available steel. In equation (13), we put all the coefficients equal to each other, with the exception of the reserve coefficient C which, we shall assume, is twice the value of the others, i.e.,

$$C = 2 \cdot C_a \text{ and } 1 = 9 \cdot C_a,$$

and, since one of the coefficients, to be specific the passenger coefficient C_p, may not be less than 50 kg (3 poods being the weight of a lean man of moderate height), then

$$P_a = 50 \text{ kg, and } P_v = 450 \text{ kg (27 poods);}$$

in addition, we put in equation (37)

$$D_1 = 7;\ m = 5;\ K = 700{,}000 \text{ kg/square decimeter,}$$

or 70 kg per square mm of cross section; D = 7 (i.e., a wing of solid steel with no cavities); we then compute the width I of the wing at 0.85 meter and, consequently, the length of 1.7 meter. Quite a low figure! But we see from the formula that the width I of the wing is proportional to the fourth-power root of 1/D. Therefore, if the mean density D of the wing is reduced by 16 times, say, by making the wing of hollow tubes, hollow crosspieces and such, then the width I of the wing and its length L will be doubled, the area will in turn increase

from 1.4 square meters to 5.8 square meters. But would not a wing of that type be inordinately "bulky," i.e., would it not take on a bulky form? From equation (36) we find that

$$\frac{h}{I} = \frac{P_a}{I^3 D}, \tag{40}$$

or, in other words, we have an expression for the "bulkiness" of the wing which reveals what part is accounted for by the thickness h of the wing with respect to its width I. Assuming here, according to the above conditions and calculations,

$$P_e = 50 \text{ kg}; \; I = 1.7 \text{ meter}; \; D = \frac{7}{16}$$

(close to the density of dry spruce wood), we compute

$$\frac{h}{I} = \frac{1}{44};$$

which is not at all "bulky." If we again decrease the mean density D of the wing by 16 times (such physically low-density bodies are unknown to us), the dimensions of the wing would be increased twice again, and the "bulkiness" would now be expressed by the fraction 1/22; the wing area would now be 32 square meters,

$$I = 3.4 \text{ meters}; \; L = 6.8 \text{ meters}.$$

We may offer some general laws concerning the "bulkiness" $\frac{h}{I}$. From equation (37), we find

$$I = \sqrt[4]{\frac{P_a^2 \cdot K}{3 \cdot D \cdot D_1 \cdot P_v \cdot m^2}} \tag{41}$$

and on eliminating I from formula (40), we now find

$$\frac{h}{I} = \sqrt[4]{\frac{27 \cdot m^3 \cdot D_1^3 \cdot P_v^3}{D \cdot K^3 \cdot P_a^2}} \tag{42}$$

which means that the "bulkiness" $\frac{h}{I}$ of the wing is inversely proportional to the fourth-power root of the mean wing density D.

We now proceed to determine the maximum value I and area S of the wing, assuming the maximum possible $\frac{h}{I}$ ratio of the wing and maximum strength of the wing material. To do this we begin by eliminating P_a from equations (40) and (42) by means of the formula

$$C_a = \frac{P_a}{P_v} ;$$

and then we find the ratio $\frac{P_v}{D}$, which we eliminate from equation (40); after all that, we find

$$I = \frac{1}{3} \left(\frac{h}{I}\right) \frac{C_a \cdot K}{D_1 \cdot m}. \tag{43}$$

It is now clear that the dimension I of the wing is proportion-

al to the $\frac{h}{I}$ ratio of the wing*, to the strength of the material in its customary dense form, $\frac{K}{m}$, and to the wing coefficient C_a.

We assume, for example:

$$\frac{h}{I} = \frac{1}{20} \text{ (bulkiness)}; \ \frac{K}{D_1} = 100{,}000;$$

$C_a = 1/9$ and $m = 5$ (firmness); then we compute: $I = 3.6$ meters, $L = 7.4$ meters, $S = 26.64$ square meters.

Now let us mentally construct a broad variety of flying machines differing greatly in weight P_v; the largest dimension I of the wing will be independent of the weight of the projectile, according to formula (43), and therefore we shall impart to all of the machines wings of the same dimension I and shape, although of different density, and accordingly of different weight. The weight of the wings will be proportional to the weight of the projectile.

Then, on the basis of equation (29), we may conclude: the engine energy E required for winged flight is, other conditions being equal, proportional to the weight P_v of the projectile. The tremendous differences in the translational velocity V of the projectiles are not taken into consideration in this conclusion; in fact, the velocity of larger, heavier projectiles would be far greater, so that the energy will not increase so rapidly with the weight of the projectile, as we had surmised earlier.

Once we know the area of the wings for a given aerial vehicle, we will be able to use equation (29) to determine the absolute value of the engine energy E for a given projectile weight and different coefficients; but the difficulty now lies in the fact that we still do not know their values, nor the magnitude of the translational velocity of the vehicle.

We shall now proceed to a solution of these problems.

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*This serves to confirm the preceding inference that "bulkiness" is proportional to the dimension I of the wings.

V

Horizontal Translational Velocity of the Aeroplane and the Energy of its Engines

Equation (29) implies that: the faster the translational motion of the flying vehicle, the less will be the energy E of the engines of the first type (the energy required to sustain the projectile aloft). But in response to that motion the air stream will also be exerting a pressure on the fuselage of the projectile and on its wings, offering resistance to the medium even at zero angle of inclination to the horizon. In order to overcome this new sum of the pressures on the fuselage and wings, a thrust of the second kind f_2 is required in addition, as well as the engines capable of developing that thrust.

We saw [equation (43)] that fairly "bulky" wings (with the $\frac{h}{I}$ ratio not below 1/20) are mandatory for large projectiles, i.e., their thickness h constitutes an appreciable portion of their width and length; so that they experience not only friction with the air as they move but also resistance of a different kind dependent upon the degree of their "bulkiness" and on the inertia of the air. Which "bulkiness" is most advantageous for the wings and what relative wing area is most practicable is something we will not undertake to solve here.

Imitating the flying organisms, we might say: the wing should not be very "bulky" and the amount of its surface area need not differ greatly from the surface area of the fuselage itself.

If the wings are not very "bulky,"

$$\frac{h}{I} < \frac{1}{20},$$

and their area is equal to the area of the fuselage, then clearly the resistance they present to the oncoming air stream will be approximately equal to the resistance of the fuselage and in any case will not be greater than the latter. For the projectiles we have in mind, the pressure on the fuselage can be determined and then multiplied by several times (n + 1) in accord with the number of times the total resistance of both wings and fuselage is greater than the resistance

of the fuselage alone. Now, if the wing area is double (n) the fuselage area, then we assume that the resistance presented by the wing area is as many times greater than the resistance of the fuselage; the total resistance (n + 1) will then be 3 times greater.

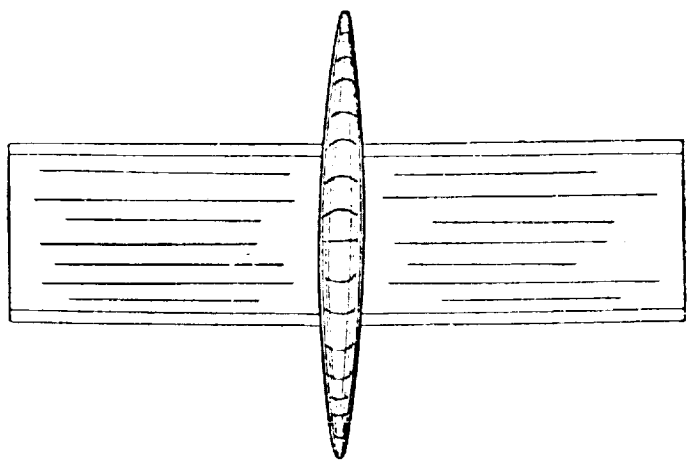
The pressure exerted by the air on a round plate (of radius r) in rectilinear motion normal to the plate at velocity V, will be approximately equal to

$$\frac{\pi \cdot r^2 \cdot d}{g} \cdot V^2, \tag{44}$$

where d is the volume weight of the air, g is the acceleration due to gravity, and r is the radius of the mean cross section through the projectile. The pressure on the fuselage is several times less (U times less) than would appear from formula (44), because of its bird-like shape (only three times more sharply tapered); it is thus equal to

$$\frac{\pi \cdot r^2 \cdot d \cdot V^2}{g \cdot U}; \tag{45}$$

where U is the utilization factor of the fuselage shape.



But the resistance of the wings increases this pressure by $(n + 1)$ times, since the surface area and the resistance of the wings alone is n times greater than the surface area and the resistance of the fuselage (Fig. 5).

Thus, the resistance of the projectile, being independent of the inclination of the wings, or in other words the thrust of the second kind f_2, is equal to

$$f_2 = \frac{\pi \cdot r^2 \cdot d \cdot V^2 (n + 1)}{g \cdot U}. \tag{46}$$

The work of the second kind, T_2, done by the projectile in cutting through the air, per unit time, is expressed by the formula

$$T_2 = f_2 \cdot V. \tag{47}$$

But since the propeller does not have a fixed support, as the wheels of a locomotive have, for instance, a part of the total work T_2 done by the engines of the second kind is performed fruitlessly and only the remaining portion of the work (denoted by the coefficient C_h) is available for continuing the path of the projectile; we thus have

$$T_2 \cdot C_h = f_2 \cdot V. \tag{48}$$

Hence, by eliminating f_2 by means of equation (46), and then determining T_2 from the resulting equation, we find

$$T_2 = \frac{\pi \cdot r^2 \cdot d \cdot (n + 1) \cdot V^3}{g \cdot C_h \cdot U}, \tag{49}$$

or the expression for the power developed by engines of the second kind. Here we may put

$$T_2 = E \cdot P_2, \tag{50}$$

since engine work is expressed in terms of the product of its unit energy E and the weight P_2; then, by eliminating P_2 from the resulting equation with the aid of (20), we find

$$E \cdot P_v C_2 = \frac{\pi \cdot r^2 \cdot d \cdot (n + 1) \cdot V^3}{g \cdot C_h \cdot U},$$

and hence

$$V = \sqrt[3]{\frac{g \cdot C_h \cdot U \cdot E \cdot C_2 \cdot P_v}{\pi \cdot r^2 \cdot d \cdot (n + 1)}}\ . \tag{51}$$

We cannot use this formula to determine the translational velocity V of the projectile, for we do not know the energy E of its engines. But we can determine E once we have eliminated V from equation (29). On doing this, we find

$$E = \frac{\sqrt{P_v \cdot r} \cdot \sqrt[4]{n \cdot (n + 1)}}{C_h \sqrt{d} \cdot \sqrt{8 \cdot g \cdot S^3 \cdot U\ (C_1^3 C_2) \cdot \left[f\left(\frac{a}{b}\right)\right]^3}}. \tag{52}$$

Let us recall that $f\left(\frac{a}{b}\right)$ here is expressed by the equation

$$f\left(\frac{a}{b}\right) = \frac{21\sqrt{\frac{a}{b}}}{2g\sqrt{\pi}\left(7 + \frac{2a}{b}\right)}$$

where $\frac{a}{b}$ is the ratio of the length L of the wing to its width I.

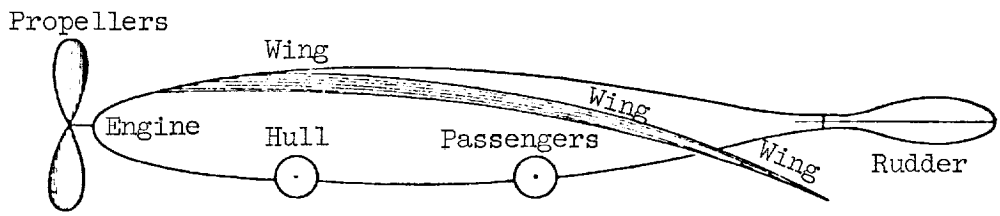


Fig. 6

Longitudinal vertical cross section.

Clearly, the hull of a projectile carrying a man aloft cannot be very small: the width and the depth (2r) must be at least 1 meter, and the length must be at least 10 meters. Only with such an aspect ratio, combined with the smooth shape of a fish, could it easily cut through the air and achieve rapid motion. If we were to fashion the hull in an elliptical shape leaving the same area of the average cross section through the hull but squeezing it in from the sides and raising it slightly upward, room could be made inside fairly easily for even 8 to 10 people. Doubling these dimensions, we find that an accommodation space of 10 sagenes (~ 20 meters) in length and one sagene (~ 2 meters) in width and depth could comfortably accommodate as many as 40 passengers (Fig. 6). Thus, the weight P_v of the projectile, given the same dimensions r of the hull, is a variable, and in that case we may state, by using the last equation, that the energy E of the engines must be proportional to the square root of the weight of the entire projectile plus all its contents, i.e., pro-

portional to P_v.

Formula (52) was derived under the condition that the resistance and area of the wings are n times greater than the resistance or area of the hull to the oncoming air stream. But the hull surface, assuming it to be derived from rotating about a chord an arc of a parabola taken near the apex, is approximately equal to

$$\frac{2}{3}\pi\,(2r)\cdot(20r) = \frac{80}{3}\pi\cdot r^2, \tag{53}$$

or, still more roughly, to $80\ r^2$; consequently, the area of the wings (2S) will be expressed by the formula

$$2S = 80\ r^2,$$

whence

$$n = \frac{S}{40\cdot r^2}. \tag{54}$$

We assume that all the coefficients C are equal, so that the coefficients of engines of the first and second kind, C_1 and C_2 in equation (52), may be assumed equal; but would this be an advantage? The energy of the engines is minimized when the product $C_1^3 C_2$ in equation (52) is maximized. Leaving the sum of the coefficients unchanged and equal to

$$\frac{1}{9} + \frac{1}{9} = \frac{2}{9},$$

we now write

$$C_m = C_1 \cdot C_2.$$

We now have to seek the maximum of the function

$$F = C_1^3 \cdot C_2, \tag{55}$$

in which we eliminate C_2 by means of equation (18); we then put the derivative of the function so obtained equal to zero; this yields us

$$\frac{dF}{dC_1} = 3\ C_m \cdot C_1^2 - 4C_1^3 = 0; \tag{56}$$

and on solving this equation we have

$$C_1 = \frac{3}{4} \cdot C_m \text{ and } C_2 = \frac{1}{4}\ C_m. \tag{57}$$

Thus, in order to limit ourselves to the least energy E of the engines, we must distribute their total weight P_m over 2 parts, one of which, designed to cut through the medium by means of the hull and wings, will have to be 3 times as great as the other, which is designed expressly to keep the projectile aloft. In other words, the work done by the engines of the 1st type must be triple that done by engines of the 2nd type, or the work done to maintain a certain optimal velocity of the aerial vehicle requires only 1/4 of the total work done by the engines.

We can now analyze in detail the value of all the variables we find in equation (52), and we can also compute the energy required to sustain flight.

The weight P_v of the projectile must be at least 450 kilograms (27 poods); but this weight may increase to a certain limit at which the mean density of the wings will be equal to the density of steel or of some other substance; in response to this increase all the coefficients will also increase proportionally, whereupon the energy E will be proportional to the square root of the weight. For example, if a series of aeroplanes carries 1, 4, 9, and 16 passengers, respectively, then the energy of the engines will have to be increased respectively as 1, 2, 3 and 4.

The diameter of the mean cross section of the hull (2r) must not be less than 1 meter; if the area of this circle is squeezed in from the sides without altering the magnitude, to form an oblong or elliptical shape, then the resistance to the air will be barely affected at all, while the height of the resulting cabin will be adequate for comfortable seating (about 2 arshins [1.5 meter]).

The propeller coefficient C_h has a value not exceeding 3/4.

The energy E also depends on the volume weight d of the medium. Indeed, engine energy in a projectile rising high up in the air will increase in proportion to the square root of the rarefaction $\frac{1}{d}$ of the upper layers of the atmosphere. At a height of 10 versts [11 km], where the air is rarefied to one-fourth its density at sea level, twice the energy expenditure will be required from a projectile of the same weight; at a height of 5 versts, one and a half times the energy will be required; at a height of one verst or less, the energy change will be as insignificant as in the case of meteorological fluctuations in the density.

The area of a single wing (S) is expressed in the formula $S = 2I^2$, and since we have, from equation (43):

$$I = \frac{1}{3} \cdot \left(\frac{h}{I}\right) \cdot \frac{C_a \cdot K}{D_1 \cdot m},$$

then the area S is

$$S = \frac{2}{9} \cdot \left(\frac{h}{I}\right)^2 \cdot \frac{C_a^2 \cdot K^2}{D_1^2 m^2}. \qquad (58)$$

The coefficient n depends on S and, actually, is just as constant a quantity.

The coefficient U is the utilization factor of the oblong shape of the hull. U will be close to 50 when the shape of the hull is well chosen and when the aspect ratio is 10.

The product $(C_1^3 C_2)$ in the radicand in equation (52) is shown on the basis of formula (57) to be

$$C_1^3 \cdot C_2 = \frac{27}{256} \cdot C_m^4; \tag{59}$$

so that the energy E will be inversely proportional to the coefficient C_m of the engines.

We now have all the data needed to compute the energy E required to sustain flight. We put:

P_v = 450 kg (27 poods) = 2r = 1 meter;

$U = 50$; $d = 0.0013$; $\frac{a}{b} = 2$;

$g = 9.8$ meter/sec^2; $S = 27$ square meters; $C_m = \frac{2}{9}$; $C_h = \frac{3}{4}$.

We then compute E = 18.6 meter/sec, i.e., on the average the engine (together with a power generator) must generate work of $18^1/_2$ kg-meter per second for each kilogram of its weight, or it must lift itself $18^1/_2$ meters per second. This corresponds to $2^1/_2$ horsepower work units per 10 kg-m of weight of the machine, or about 1 horsepower per 10 pounds of weight.

For a long time I mistrusted this conclusion and continued to search for some errors in the formulas, but unfortunately never turned up any such errors. According to the drawing (Fig. 7) below, we can visualize the relative dimensions of the wings. The true

length of the hull is 10 meters, the width with the wings extended is about 15 meters; 2/9 of the total lifting force of the aerial vehicle is absorbed by the weight of the wings, and the engines account for an equal amount (6 poods); there remains 5/9 of the lifting force (15 poods), which is distributed over the hull (3 poods), fuel (3 poods), passengers (3 poods), various aeroplane accessories (3 poods), and finally the 3 poods of supplies. The engine energy required for flight and for lifting one man aloft, even though less than the energy which Maxim claims to have achieved, is nevertheless 18.6 times greater than the most powerful Yakovlev gas and kerosene engines, 40 times greater than the Otto gasoline or kerosene engines, and 186 times greater than the energy of conventional small steam engines.

By the way, we may cite other calculations for the same projectile. For example, formula (36) yields us information on the mean wing density; it turns out to be 2/99, or about 1.50 the density of water; actually, the average wing density should be 20 times less than the density of dry spruce wood (0.4). It would be a bit difficult to construct such a wing.

But if the weight of the projectile and of all of its parts were several times greater, then the wing density would have to be increased by the same number of times, even though its dimensions could not be increased. Actually, by eliminating the weight P_a of the wing from equation (42) by means of equation (14), we find

$$\frac{Pv}{D} = \left(\frac{h}{I}\right)^4 \cdot \frac{K^3 \cdot C_a^{\,2}}{27\, D^3 \cdot m^3}, \qquad (60)$$

i.e., the wing density D must increase in proportion to the weight P_v of the projectile, in order to avert any change in the "bulkiness" $\frac{h}{I}$ of the wing and in the ratio of the weight of the wing to the weight of the projectile. Let us now compute the translational velocity of our projectile lifting aloft a single man and capable of accommodating not more than four. To do this, we make use of equations (51) and (57), from which we find: V = 35.4 meters/sec, or 128 km/hour. Not bad, this speed! In 3 hours, we could fly from Moscow to Nizhniy-Novgorod.

The volume of the hull of the projectile is such that it could easily accommodate up to 4 persons. We shall therefore cite here the data referable to this projectile capable of lifting aloft one, two, or four persons, respectively. Its dimensions remain the same: hull length 10 meters, average width 1 meter (height of mid-section about 2 arshins, width about 1 arshin); wing width 3.67 meters, wing length 7.34 meters, wing surface 27 square meters, surface of the two wings 54 square meters, hull volume at least 4 cubic meters.

Number of passengers	Projectile weight, kg	Engine weight, kg	Engine energy, kg-m/sec*	Engine power, HP	Velocity, m/sec	Weight of both wings, kg	Weight of 1 square meter of wing	Mean wing density	Flying time, hr
2	900	200	26.2	52.5	49.9	200	3	$\frac{1}{50}$	6
1	450	100	18.6	18.6	35.4	100	1.8	$\frac{1}{25}$	4
4	1800	400	37.2	148.8	70.8	400	7.2	$\frac{1}{12^1/_2}$	3

The coefficients also remain unchanged, but the absolute masses of the engines, wings, hull, etc., increase in proportion to the number of passengers, and this proportionality is a direct and inevitable consequence of the increase in the number of passengers.

*per kilogram of engine weight.

Hence we see that the weight of the wings is equal to the weight of the engines, and ranges from 3 to 24 poods. The energy of the engines per second per kilogram of engine weight ranges to 37 kilogram-meters. This energy (for lifting 4 persons aloft) is 37 times greater than the energy developed by the Yakovlev engines, for which a single steam horsepower corresponds to 4 poods of weight under favorable conditions. The engine power reaches 149 HP; about 40 HP would be expended per person on the average. The wing is lightweight, and each square meter of the wing weighs on the average from 2 to 7 kg; the mean density ranges from 1/50 to 1/12 of the density of water, i.e., is 20.5 times lighter than spruce wood. It is left to the reader to judge the extent to which such a wing could be realized in practice. My personal opinion is that such a wing could be constructed indeed, albeit only at great pains. The non-stop flying time of the projectile extends from 3 to 6 hours. As a basis, we assume gasoline engines requiring only 1/2 kg gasoline per horsepower; if a steam engine is used, an appreciable supply of water will be required, and this will reduce the flying time by several times; the gasoline supply we assumed to be only half the weight of the engines; the tensile strength of the steel is assumed at 70 kg per square mm cross section and the rigidity, at 5.

These results do not appear particularly satisfactory to us from the standpoint of feasibility; let us stretch our point to the ultimate: as we shall assume the wing surface to be far larger.

We see from formula (43) that the size of the wing is proportional to K/m, i.e., it is proportional to the strength of the material and is inversely proportional to the desired rigidity of the wings. If we were to double the strength K of the material, or if we were to risk decreasing the rigidity m of the wings by the same number of times, then the dimensions of the wings would end up twice as great, and the wing area would be 4 times as great. On reducing the rigidity to $2^1/_2$ (a coefficient inadmissible in practice), we bring the surface area of the two wings to 216 square meters. Reports indicate this to have been the wing surface area in Maxim's flying machine. I am somewhat astonished by the fact that he was successful in building wings of such enormous dimensions; they were probably fitted with various masts and braces, like Lilienthal's wings, and accordingly had to offer a great resistance to the oncoming air stream. For that reason the velocity of the projectile could not be at its greatest, and consequently the energy E could not be at its least value.

Disregarding all that, let us assume that our projectile is equipped with just such wings but of smooth shape and with no parts protruding outwards, so that we shall not have to take into account the resistance presented by such parts. According to equations (52) and (54), if we at the same time double the space occupied by

the hull, i.e., if we double r or increase the projectile volume by 8 times, then n will remain the same but the energy E of the engines will be reduced to one-third. The spacious interior by the hull will enable us to accommodate many passengers, as many as 64, in flight.

Herewith are the general data for such projectiles, similar to those outlined, but of different density and weight. The height and width of the hull at the midsection will be 2r = 2 meters, the length 20 meters, the surface area about 80 square meters, the volume not less than 32 cubic meters (varying weight); the surface of the two wings 216 square meters in area, the average "bulkiness" ratio $\frac{h}{I} = 1/20$, the length 14.68 meters, the width 7.34 meters, the wing rigidity $2^1/_2$.

Let the passenger capacity for eight projectiles be, respectively: 1, 4, 9, 16, 25, 36, 49, and 64; the weight of each passenger plus supplies will be assumed at 100 kg, so that their weight total will be, respectively: 100, 400, 900, 1600, 2500, 3600, 4900, and 6400 kilograms, i.e., up to $6^1/_2$ tons. The same figures apply to the weights of the engines in the various projectiles; the same figures likewise apply to the masses of the hulls and the necessary appurtenances for them and for the passengers; and the same will apply, respectively, with regard to the weight of the two wings. The weight of the gasoline to power the engines will also be assumed proportional to the number of passengers; it is expressed by half the figures listed. In sum, we have now determined the total weight of the projectiles, which will be, respectively: 450, 4050, 7200, 11,250, 16,200, 22,050, and 28,800 kg, i.e., from 1/2 to 28 tons or from 27 poods to 1,728 poods with some to spare.

We compute the energy E of the engines to be, respectively: 9.3; 18.6; 27.9; 37.2; 55.8; 65.1; 74.4; in kilogram-meters per second. Thus, the energy increases in proportion to the square root of the number of passengers; for the last projectile, this figure will be 8 times as large as for the first projectile. The same figures show how many horsepower are required to carry a single passenger on the airship; namely, for the eighth projectile a 74 HP machine is required per passenger. Clearly, then, it will be difficult to construct airships carrying a sizable number of air travelers, whereas the most enormous theoretically conceived aerostat can carry as many as 600 passengers under far less exacting conditions, requiring no more than 1 HP per passenger and moving at a speed of 62 km/hr. With respect to the ratio of the largest aerostat to the aeroplane No. 8, we may state the following: one hundred times as many passengers and one-hundredth the amount of engine

energy; one hundred times the possibility of doing the job and one-hundredth the traveling cost. (Cf. Tsiolkovskiy, "Aerostat metallicheskiy, upravlyaemyy, vyp. 1 i 2 [Dirigible metal aerostat, Nos. 1 and 2]).

We shall soon see that our seemingly exorbitant conclusions are fully confirmed by the data on Maxim's and Lilienthal's experiments.

The absolute power of the eight aerial vehicles will be, respectively (in horsepower): 9.3; 74.4; 251.1; 595.2; 1164.5; 2008.6; 3190.9; and 4761.6; it is obvious that the power of the engines increases faster than the weight of the projectile: it is proportional to the weight of the projectile multiplied by the square root of the weight. The speeds of the airplanes will be, respectively: 64, 128, 192, 256, 320, 384, 448, and 512 km/hr; starting from a rather insignificant value equal to the speed of larger balloons (aerostats), it attains an astonishing magnitude: 500 versts per hour [533 km/hr]. But a velocity such as this is both necessary and advantageous; if we were to think of lowering it, the engine energy would have to be increased still further, and the design of the aerial vehicle would be less feasible. Only 1/4 of the total engine power would have to be used, however, to develop this staggering speed; the remaining 3/4 would be expended strictly on keeping the projectile airborne, i.e., combatting gravity. The law of velocities is explicit: the speed is proportional to the square root of the weight of the projectile or of the number of passengers.

The mean wing density D for the different projectiles will be expressed by the following figures, respectively: 1/400; 4/400; 9/400; 16/400; 25/400; 36/400; 49/400; 64/400; this means, from 1/400 to 1/6 of the density of water*; in the case of the heaviest of the projectiles enumerated above, it would be $2^1/_2$ times less than the density of dry spruce wood. This illustrates the difficulties of constructing wings of great surface area and low weight. One square meter of such wings would weigh, on the average, (in kilograms): 1/2; 2; $4^1/_2$; 8; $12^1/_2$; 18; $24^1/_2$, and 32, respectively, and actually would be even slightly less; note that the wing must withstand a substantial force of pressure almost equal to half the weight of the projectile; the center of this pressure coincides approximately with the geometrical center of the wing. For each

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*Because of the convex wing surface, the mean wing density may be appreciably larger, a comforting fact.

square meter of wing surface, we find, on the average, a pressure on the part of the oncoming air 8 times in excess of the weight values specified.

The gasoline supply would make it possible for the aeroplane to cruise over a considerable distance without having to replenish the fuel stores. This cruising range is the same for all aeroplanes, assuming, as earlier, for aeroplanes one-half as large, 1/2 kg gasoline per horsepower per hr (which is very little), whereupon we find the longest trip to be about 700 km; but the time it takes totravel this distance will be different for the various projectiles, namely, it will be the reciprocal of their velocities; so that these times will be, respectively: 11, $5^1/_2$, $3^2/_3$, $2^3/_4$, $2^1/_5$, $1^5/_6$, $1^4/_7$, $1^3/_{11}$ hours.

In view of the high engine energy, it is quite probable that the utilization of the fuel will be far from the ideal, as we assume here, so that both the distance of 700 km and the flight time should in practice be 2 to 3 times shorter.

We will now confirm our conclusions by Maxim's experiments. His flying projectile weighed, together with two passengers and fuel and water stores, 2725 kg; clearly then, its weight lay between our two projectiles carrying 4 and 9 passengers. The wing surface of Maxim's vehicle, as well as its design, came quite close to what we assumed for our aeroplanes (Fig. 2). The power of its engines should consequently be somewhere between 74 and 251 HP. The power of the engines on Maxim's projectile was, as we know, between 90 and 225 HP. We see now that our conclusions are a bit plausible. Making more precise calculations on the basis of available formulas and laws, we find that a projectile similar to Maxim's vehicle, with a wingspread of 216 square meters, and weighing, like Maxim's vehicle, 2725 kg, must have an engine of 137 HP. Is this not close to Maxim's conclusions?! The speed of his projectile reached 150 km/hr, which again is not in contradiction with our calculations (from 128 to 192 km/hr). Nevertheless, on flying its 100 meters, it could hardly have achieved such velocities because of the imperfections in the design of its wings. As for the future prospects and significance of the aerial vehicle, Maxim himself acknowledges that its applicability is limited exclusively to military purposes, and that it is unsuitable for carrying cargoes and passengers, i.e., the renowned inventor expresses the same thoughts which we voiced over a year ago concerning aeroplanes ("Aerostat", no. 2, 1893, page 83).

But the energy that Maxim managed to extract from his engines is remarkable! His engine (together with the power generator, of course) weighed about 420 kg. Its power ranged from 90 to 225 HP; thus, the energy E per second per kg of engine weight was somewhere between 21.4 and 53.5 kg. Applying this energy to our aerial

vehicles with wingspread of 216 square meters, we find that this is enough energy to raise aloft a projectile carrying a crew of from 5 to 35 persons.

By the way, we also find confirmation for our conclusions in the work of still another renowned investigator, Otto Lilienthal, who only recently suffered so terribly for his devotion to science.

From equation (4), we find

$$V_n = \sqrt{\frac{21^2 \cdot \left(\frac{a}{b}\right) \cdot V_p}{4\pi\left(7 + \frac{2a}{b}\right)^2} + \frac{F \cdot g}{S \cdot d}} - \frac{21\sqrt{\frac{a}{b}} \cdot V_p}{2\sqrt{\pi}\left(7 + \frac{2a}{b}\right)}.$$

Using this formula, we are able to calculate the velocity of the vertical fall of Lilienthal's parachute. It is known that the latter dropped through the air, from his vehicle, in an inclined direction from a hill 80 meters high; the time of the descent was about 40 seconds; there was a head wind with a velocity of 7 meters per second, while he flew 320 meters in the horizontal direction. The translational velocity amounted to about 8 meters/sec, and the velocity of the fall was about 2 meters/sec.

Therefore, in the preceding formula we put

$$V_p = 7 + 8 = 15 \text{ meters/sec},$$

i.e., the relative velocity (relative to the air) is made up of the sum of the velocities of the projectile (relative to the earth) and of the wind; F or the weight of the vehicle together with the aeronaut was about 90 kg, as we know; the surface S of the wings was 14 square meters; $a/b = 1$; we then compute $V_n = 3.5$ meters, whereas the speed of the fall (in the presence of wind) was about 2 meters/sec. At first, this contradiction presented me with a dilemma. But it was readily resolved on attributing it to the ascending current of air due to the inclination of the hill from which the experimenter

launched into flight (against the wind).

In fact, the tangent of the angle of inclination of the hill to the horizon amounted to at least 1/4, so that the air blowing against the hillside acquired an ascending velocity amounting to 1/4 to 1/5 the wind velocity; this ascending current was what lifted the wings, thereby almost reducing in half the speed of the wings' descent.

VI

Some Additional Features of the Design of an Aeroplane, and its Operation

Were we to judge by the energy of now available engines, we would have to abandon all dreams of flying by means of bird-like projectiles (or, rather, insect-like projectiles, since we have indicated that our aeroplane resembles most closely a coleopterous insect, as for example a ladybug). I have valid theoretical reasons, however, for believing in the possibility of constructing extremely lightweight yet powerful gasoline or petroleum engines fully capable of meeting the problem of flight. That is why I refuse to draw back from probing further into this interesting question.

We shall continue to hope that this problem will be favorably resolved, sooner or later.

In my view, it is not at all out of place to communicate here some very brief additional information on a flying machine.

Earlier, we provided only a schematic design for the sake of simplifying the calculations (Chapter III, Fig. 2); but how a flying projectile is to be actually designed, how it is to be put into motion, how it can be landed on the ground and is to retain its equilibrium -- we said nothing about these things earlier. To be brief, I suggest that, first of all, we make an attentive examination of the accompanying drawings (Fig. 6 and Fig. 7).

Fig. 6 gives a lateral view of the projectile; the following drawing gives the cross section transverse to the hull and the vertical cross section.

It is clear from these figures that the wings need not have the form of rectangular plates: they are more similar to bird wings, except for being motionless relative to the hull (i.e., they are not flapped in the manner of birds). The form is rounded everywhere; the wing cross sections are bounded in every direction by smooth lines, so that the wings cut through the air with ease; the pressure on the entire bottom surface of both wings is fairly uniform. During flight the front portion of the hull is raised slightly as if the machine were attempting to climb a mountain; this inclination aids somewhat the performance of the engines; but owing to considerations of space we shall not go into all these details and many others.

We also see some wheels jutting from underneath the hull. I shall explain their function at this point.

The projectile climbs into the air only on achieving a certain translational velocity. How is this velocity to be imparted to it? The propeller turns rapidly, forming a fairly strong thrust, but the

projectile will remain motionless if the wheels are braked; gravity and the friction on the wheels will hold the projectile fast. By making things a hundred times easier with the use of the wheels and a smooth horizontal roadbed in the form of a cast iron platform or rail track, we find that our projectile will roll rapidly on its wheels, drawn along by the action of its propellers (in the manner of a steamship); as the speed increases the pressure on the wings from the oncoming air stream will be increased and, on the other hand, the pressure of the projectile weight on the launch platform will decrease, and very soon the instant will come when the projectile wheels will no longer be in contact with the platform, and when the flying machine will rise smoothly into the air, under the powerful action of its engines.

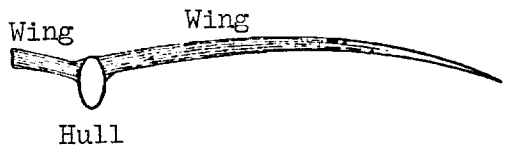

Fig. 7

The wheels are also needed in landing the projectile on the earth. The machine cannot simply set down just any place at hand; it requires a smooth and fairly extensive platform, the braking action of which will enable the machine to safely lose its translational velocity. This reveals certain additional complications in the use of the aeroplane. It is also possible that the aeroplane can descend safely onto the level surface of a snow field in the wintertime, and onto the smooth surface of the water in a river or in a lake in the summertime.

Some other birds, the swifts, for example, acquire the appropriate horizontal speed by diving; imitation of this maneuver would be rather risky for a man-made projectile, except for the case of rolling off a launching hill built expressly for the purpose, and even this does not in any way eliminate the difficulties.

The climb is greatly facilitated when a wind is blowing, provided the projectile is launched into motion against the wind. Birds sometimes soar to great heights without the slightest effort by stretching their wings to meet the wind. Later, of course, they

have to begin flapping their wings in order not to lose the relative translational velocity of their bodies. In the face of an uneven air stream, both birds and specially designed aeroplanes could, theoretically, keep themselves aloft almost exclusively by riding gusts of wind. Uniform, even if rapid, motion of the atmosphere, on the other hand, would not reduce the work the engines have to do, i.e., whether the wind blowing is a head wind or a tail wind, whether it is blowing hard or soft, the amount of energy spent per hour by the engines would not vary in the slightest on that account. It is merely that a tail wind would accelerate the motion, while a head wind would slow it down. Moreover, it is doubtful that a gusty wind (an uneven air stream) could, in practice, appreciably facilitate the performance of engines on large man-made flying projectiles capable of carrying passengers. To ride the gusts of wind, birds must accomplish complex maneuvers, which could hardly be imitated by the simply designed projectile of ours. And moreover then the bird becomes a plaything of the wind. It is true that the bird is capable of rising and falling as it wishes while it hunts its prey, but it is constrained at the same time to execute circles while passively abandoning itself to the overall air current.

Lilienthal's experiments reveal that it is by no means so difficult as it seems to maintain the longitudinal axis of the flying projectile in a horizontal position. Nevertheless, even Lilienthal, who has learned to steer his own machine so dexterously, had to acknowledge that a sudden gust of wind (an accelerated motion of the air) lifting him several meters unexpectedly during his descent (for, he was making an oblique landing) was not without danger to him*. Simple control by a rudder, similar to a bird's empennage, would therefore not only require experience and continual alertness, but would not even be sufficiently reliable. An automatic control device would be required. Its operation could not be based exclusively on the laws of gravity (such as a double-legged tube with mercury or similar instrument), as the operation of a horizontal level control which I suggested earlier for a maneuverable aerostat ("Aerostat," No. 2, page 47). Actually, when an aeroplane is tilted, it acquires an accelerated motion in the direction of the tilt (as when rolling downhill), so that the effect of gravity becomes more complicated and the mercury in the double-legged tube would not perform its function, i.e., it would retain its inclined level for some time in unison with the projectile. On an aerostat, however, a tilt would not produce any accelerated

*It is reported that a famous aeronaut and investigator recently plummeted during one such flight, breaking his leg.

motion, so that a controller of this type would work correctly, actuating an electrical current and re-establishing the equilibrium of the airship. It seems to be that an aeroplane would require a tiny rapidly rotating disk coupled to the axes in such a way that its plane could consistently retain its position no matter what the rotation and tilt of the vehicle, as a suitable horizontal level control. When the disk (gyroscope) is maintained in continual rapid rotation, its plane will be fixed with respect to the projectile, but as soon as the projectile changes direction, the relative position of the rotating disk will also change. Of course, a change of this type could provide the reason for switching on or off the electrical current acting on the electromagnet and on the rudder, which constrain the projectile deviating ever so slightly from its correct position to resume its horizontal position[11].

K. E. TSIOLKOVSKIY'S FIRST EXPERIMENTS ON AIR RESISTANCE (1895-1896) BEFORE THE CONSTRUCTION OF HIS WIND TUNNEL*

1. Fig. 8 shows the apparatus which I employed to investigate the drag of elongated bodies (of the type of our aerostat) moving at a velocity of one meter per second.

2. The test objects were bounded by surfaces of revolution obtained by rotating an arc of a circle about its chord. The diameter

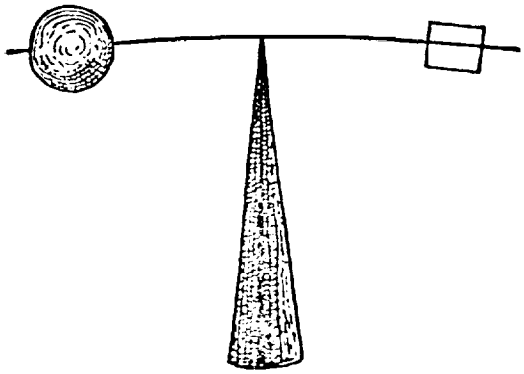

Fig. 8

of the center cross section of all these bodies was about 10 cm., and the cross-sectional area between 80 and 82 cm^2.

3. The test shape (made of paper) was mounted at one end of

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*This article is actually a chapter from Tsiolkovskiy's "A 200-Man Maneuverable Iron Aerostat as Long as a Large Steamship" which he had published in Kaluga in 1896. In this chapter, entitled "Experiments and Formulas Used to Calculate the Independent Speed of My Aerostat," Tsiolkovskiy describes his first experiments in aerodynamics, dating from the years 1895-1896. (Editor).

a steel rod (Fig. 8) and a small plate at the other. The direction of motion of the apparatus coincided with the direction of the axis of the elongated test shape and was normal to the direction of the horizontal rod and the vertical plate. Before the translational motion began, I made sure that the rod was free of the slightest rotation about the pivot point. In the course of numerous experiments, the plate was cut away or replaced until the air pressure on it equalled the longitudinal pressure on the shape. This occurred when the translational motion no longer caused the rod to rotate (the centers of pressure were the same distance from the pivot point). In this case, I shall call the ratio of the area of the plate to the area of the largest cross section of the test shape the drag coefficient of the shape.

Below I give the characteristics of the test shapes and the results of experiments on them:

4.	Length of shape.....	21	32	42	52	62 cm
5.	Surface area of shape..	440	670	880	1080	1300 cm^2
6.	Area of plate.....	222	18	19	21	24 cm^2
7.	Drag coefficient.....	0.272	0.222	0.235	0.259	0.296

The last row of figures was obtained by dividing the area of equal resistance (6)* by the area of the greatest cross section, i.e., by 80 or 82 (2).

8. An inspection of the drag coefficients shows that the least resistance corresponds to a shape, the length of which is almost three times greater than its height. Thus, with increase in the slenderness of the body the drag first falls and then rises again. This becomes understandable if we assume the existence of friction between the air and the body.

9. This apparatus not only demonstrated the existence of friction, but also made it possible to determine the corresponding coefficient. In this case the elongated shape was removed and

*The figures in parentheses refer to the corresponding paragraph in the text. (Editor).

replaced with something like a flag or wind vane, always oriented in the direction of motion.

10. In the presence of motion at the rate of one meter per second, the ratio of the area of the plate of equal resistance to twice the area of the larger friction surface (both sides were taken into account) was 1/58.

11. Multiplying the surface area (5) of each test object by the coefficient of friction (1/58) and subtracting this value of the friction from the area of equal resistance (6), we arrive at the following figures:

12. 14.41 6.45 3.83 2.21 1.59 cm^2.

From these figures we can derive the approximately valid law:

13. The force needed to part the air (not including friction) is inversely proportional to the square of the aspect ratio of the body.

14. These further experiments (9) with the apparatus convinced me that the coefficient of friction is inversely proportional to the velocity of the friction surface, i.e., may be expressed by the formula $\frac{1}{58V}$, where V is the velocity of the surface in meters per second.

15. Knowing the law (13) characterizing the force needed to part the air or the resistance due to inertia and the law of friction (14), it is easy, even on a purely empirical basis, to construct a formula for the total air resistance to bodies of the simple shape described in (2). Thus, we get:

$$\frac{Kd\pi b^2}{2g} V^2 \left\{ \frac{18}{23} \left(\frac{b}{a}\right)^2 + \frac{a}{\frac{3}{8}\, 58bKV} \right\}$$

where K is the ratio of the true resistance of the surface to the theoretical $\frac{d\pi b^2}{2g} V^2$; according to the experiments of Cailletet and

Colardo (which deserve special attention since the motion of the plate was rectilinear), K = 1.06; a and b are the half-length and half-height of the aerostat; V is its speed; and g is the acceleration of gravity, the other quantities being obtained from the experiments described.

With this formula I computed the longitudinal pressure on the aerostat due to the airflow.

16. The term in brackets $\frac{18}{23}\left(\frac{b}{a}\right)^2 + \frac{a}{\frac{3}{8}\,85bKV}$ is the total drag coefficient.

17. For example, putting the aspect ratio equal to 7 ($\frac{a}{b} = 7$; this is the value adopted for our proposed aerostat), we find the following drag coefficients for different velocities:

Velocity m/sec	Velocity km/hr	Drag coefficient
1	3.6	1/3.1
2	7.2	1/6.0
3	10.8	1/8.5
4	14.4	1/10.9
5	18.0	1/13.0
6	21.6	1/15.0
8	28.8	1/18.5
10	36.0	1/21.5
12	43.2	1/24.2
24	86.4	1/34.9

18. Determining in the usual way the aspect ratio of the aerostat corresponding to minimum drag, we obtain the formula:

$$\frac{a}{b} = \sqrt[3]{36V}.$$

19. From this we can compute the most favorable aspect ratio for different velocities V and the corresponding minimum drag coefficients:

Velocity of aerostat..	1	2	4	8	10	12	m/sec
Aspect ratio.....	3.3	4.16	5.24	6.6	7.11	7.56	
Drag coefficient.....	0.217	0.135	0.085	0.054	0.046	0.041	

20. From these formulas we see that, given a minimum drag coefficient, the friction is equal to twice the drag due to the inertia of the air. Accordingly, the friction of the air plays an even more important part than its inertia.

21. In the light of these laws, my previous calculations ("Aerostat," No. 2, pp. 73-75) relating to the drag on a slowly moving aerostat (for example, one driven by man-power) are incorrect.

In regard to the independent motion of an airship traveling at high speeds (from 20 to 40 versts per hour), I was quite right.

22. In order to determine directly the drag coefficients of elongated bodies (2) moving at high speeds, I constructed an apparatus (Fig. 9) consisting of two horizontal tubes, mounted on a tripod; these tubes were about 75 cm. long and had openings about 25 cm. wide. In one of these I mounted the test shape on a rod (Figs. 8 and 9), and in the other a plate; the rod, of course, was introduced into the tubes through special openings and, as usual, was free to rotate about a central pivot point. The tubes were taken out on the roof and set up in the direction of the wind. I stood to one side and observed the rod in the gap between the tubes to determine on which half the air pressure was the greater, i.e., which half was outbalanced.

The test shape I employed was 62 cm. long (4). The wind speed at the observation point changed quickly and constantly, vary-

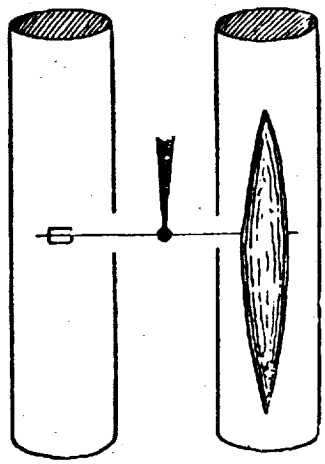

Fig. 9

ing between 0 and 5 meters per second. As plates of equal resistance (6), I used successively copper coins with areas of 11.6, 8, and 6.2 cm^2, respectively. When the velocity of the wind was small, the special shape prevailed, but as soon as the velocity reached 2-3 meters per second, the balance swung in favor of the plate (area 11.6 cm^2; corresponding coefficient 1/7). At a velocity of about 4 meters per second an area of 8 cm^2 was sufficient (corresponding coefficient 1/10). At velocities of more than 5 meters per second the shape was outbalanced even by the coin with an area of 6.2 cm^2, the corresponding coefficient being 1/13.

It is readily seen that all these experiments are in approximate agreement with our formulas based on other experimental results.

The drag coefficients of elongated bodies moving at slow speeds are surprisingly high (cf. 7). Thus, for a body with an aspect ratio of 5.2 moving at 1 m/sec the drag coefficient is 0.259 or about 1/4. However, to some extent we observe the same thing when elongated bodies move slowly through water. Thus, experiments with a wooden body having an aspect ratio of 5 and the same shape as that previously adopted gave a coefficient of about 1/3; in this experiment the rate of motion was not determined, but was less than 0.5 m/sec.

I have made many other experiments with surfaces of different shapes. Thus, for a sphere and a cylinder, at a velocity of about 1 m/sec, I obtained coefficients of 4/9 and 0.6. At high speeds the drag coefficient of a sphere is close to 0.4.

In time I hope to publish a detailed description of all my experiments on drag which I am ready to repeat in public.

K. E. TSIOLKOVSKIY'S FIRST DESCRIPTION OF HIS WIND TUNNEL SUBMITTED TO THE RUSSIAN PHYSICOCHEMICAL SOCIETY IN PETERSBURG ON 12 OCTOBER 1897*

First of all, I must humbly beg the much-esteemed members of the reviewing committee not to communicate to anyone anything concerning my work and plans, before they have been brought to completion and published.

All the drawings submitted are rough and schematic, because they are not yet ready for publication.

I enclose a photograph of the apparatus used for my early experiments. HP is a vaned blower (a sort of winnowing machine). A similar device which I have constructed is 150 cm tall and 40 cm wide. The stream of air, which gradually expands and weakens, emerges at P and initially, at the mouth of the blower, is about 40 cm square. P denotes a series of horizontal baffles intended to regulate the flow, which otherwise would not be perfectly uniform below the opening. H denotes the shaft of the stand and the bearings (not shown) together with the surrounding clamp (a sort of Proni clamp). This clamp has a handle (black circle), by means of which the vanes Л are made to rotate. The clamp is fitted with two nuts and bolts. Depending on how tightly we screw these up, we can vary the friction between the iron shaft and the two wooden blocks of the clamp. By turning the vanes faster and faster, we eventually arrive at a moment when the clamp slides over the shaft and the velocity of the air flow becomes maximal and constant. In all the experiments the air pressure on the test model is determined at this maximum velocity, which corresponds to the degree of clamping and depends on the experimenter. In each individual experiment the velocity is found from the pressure exerted on a plate using the coefficients of Cailletet and Colardo (or Langley), assuming that the barometric pressure and the temperature of the air are known.

Я is a rectangular tin vessel containing water, in which floats a second smaller vessel. The test model, the pressure on

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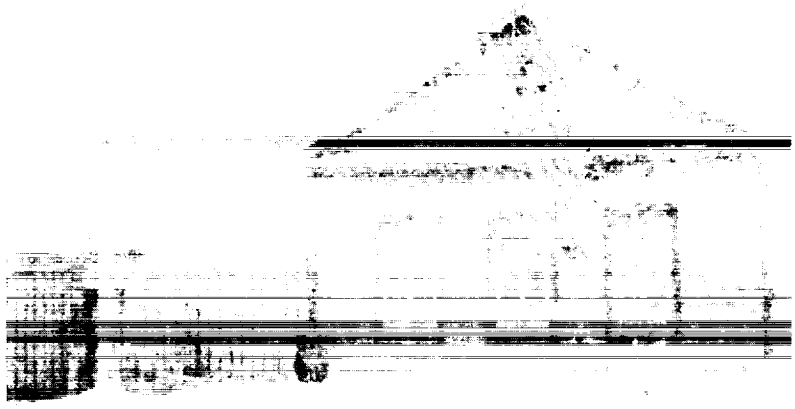
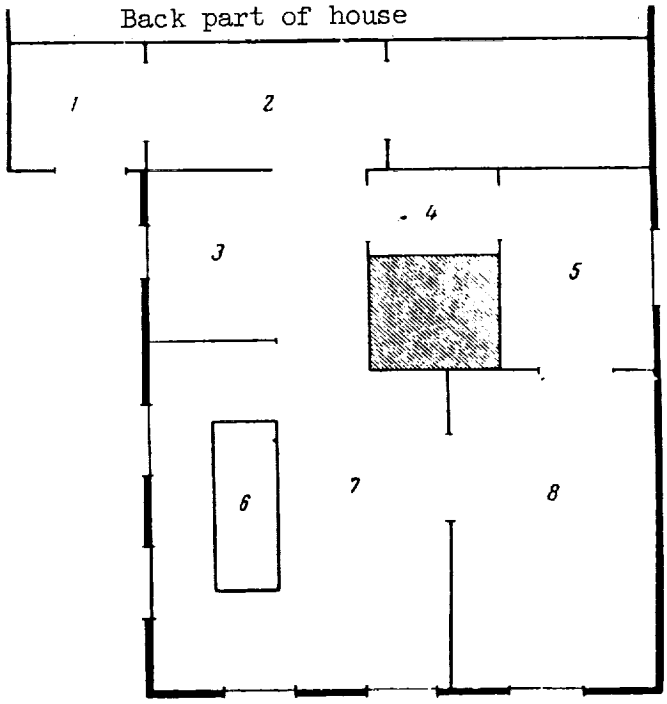
*Letter from K. E. Tsiolkovskiy addressed Kaluga, 5 October 1897 to A. L. Gershun. (At that time Professor Gershun was a member of the Presidium of the Russian Physicochemical Society which met at Petersburg University.) The letter is preserved in the Moscow Division of the Archives of the Academy of Sciences, No. 555.





Street of the Year 1905, formerly Georgiyevskaya Street

Scale 1 m = 2.3 cm 0 1 2 3 м

[Legend for figure on next page]

[Legend for figure]

General view of the house and floor plan of the Speranskaya's House in Kaluga on Street of the Year 1905 (formerly Georgiyevskaya Street). In this house Tsiolkovskiy performed his experiments with the wind tunnel he had invented (1896-1903).

1 - porch, 2 - veranda, 3 - hall, 4 - passage, 5 - kitchen, 6 - wind tunnel, 7 - living room, where the experiments were performed, 8 - bedroom.

which we propose to determine, is fastened to the latter by means of 4 vertical rods or in some other manner. To the same platform as that on which the vessel filled with water is placed there is attached a stand, from which a long, thin iron wire is freely suspended. At the top the wire is fastened to a thread on which the floating model pulls, tending to drag it in the direction of the air flow. This causes the wire to deflect from the vertical. In the experiments, the deflection is determined as the tangent of the angle of inclination by means of a scale divided in millimeters. The intensity of the air pressure will be proportional to the number of scale divisions indicated by the wire. By means of a previous calibration experiment I determined how many milligrams corresponded to a deflection of the wire through 1 mm. Thus, the deflections and the corresponding air pressures can all be expressed in milligrams.

The inner vessel is equipped with two light arms which make its motion almost exactly parallel to the direction of the air flow (Fig. 2). During the experiment the outer vessel is covered with a lid, in which narrow slots are formed for the free passage of the rods supporting the test model.

I made the models very light, out of paper. When it became necessary to build a model in the form of a surface of revolution, I first carefully traced the curve of the principal longitudinal section of the model. Following this curve, I turned half the model in wood on a lathe, up to the principal cross section (Fig. 3). (Actually, this was done for me at the local railway school.)

Around the wooden half-model I stuck strips of wet paper and wrapped (bandaged) the whole thing tightly with broad tape (swaddling it like a baby). I then allowed the paper to dry out thoroughly, stripped off the tape and carefully removed the paper, which retained

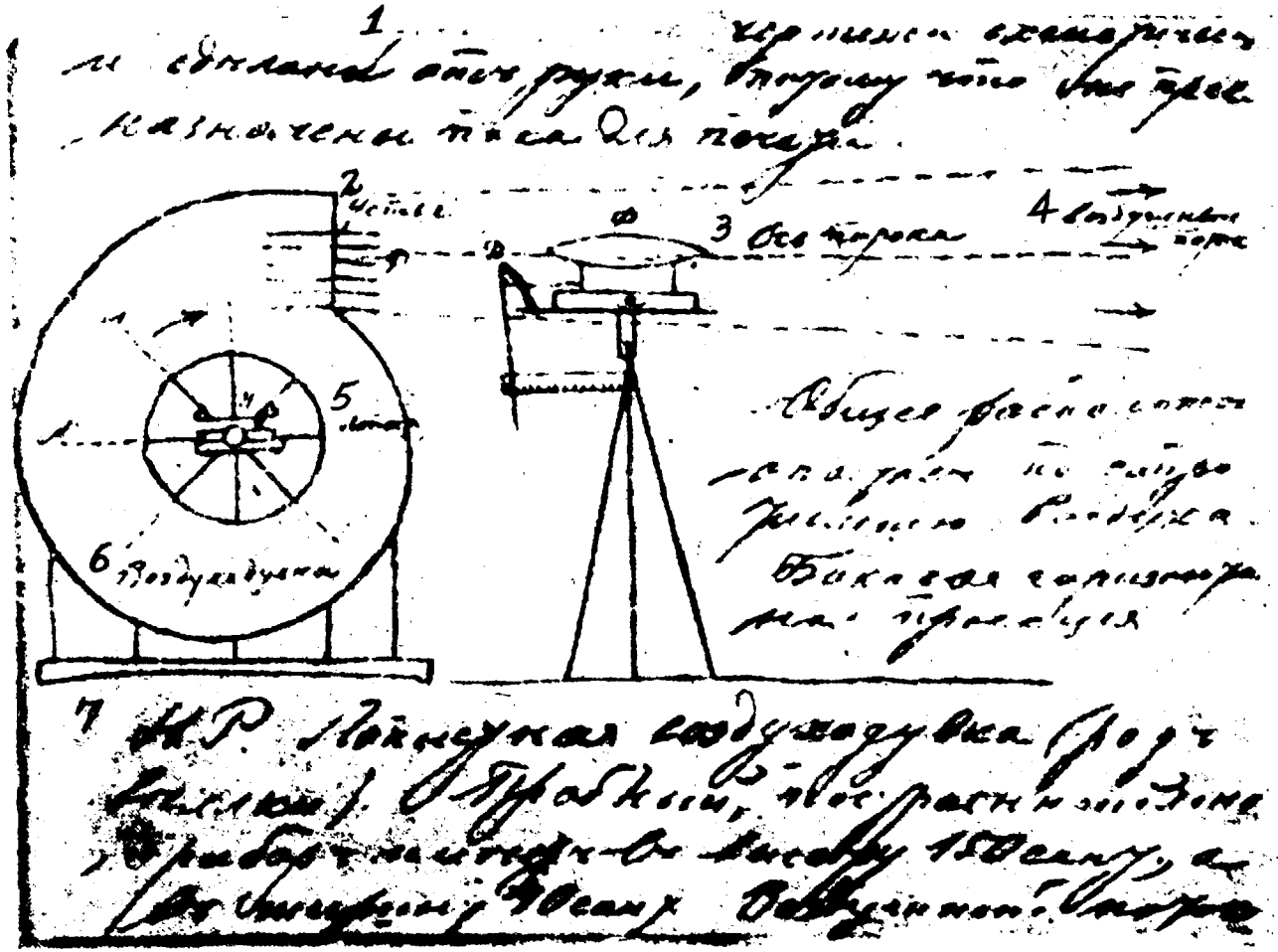


Fig. 1

General arrangement of apparatus used to measure air resistance. Side elevation.

[Key for Fig. 1 on next page]

[Key for Fig. 1]

1 - ...drawings are rough and schematic, since they are not yet ready for publication; 2 - outlet; 3 - flow axis; 4 - air flow; 5 - vane; 6 - air blower; 7 - HP - vaned air blower (a sort of winnowing machine). This device which I have constructed, is 150 cm tall and 40 cm wide. The stream of air ....

the convex shape of the wooden form. After that, it only remained to glue the pieces of paper together. After the paper shell had been removed, the broader opening was fitted with a paper hoop (Fig. 3). If the shape was very elongated, several such hoops were glued inside the shell. The other half of the model was prepared in the same way, though sometimes it was not the same shape as the first. When the two halves had been glued together, the model was complete.

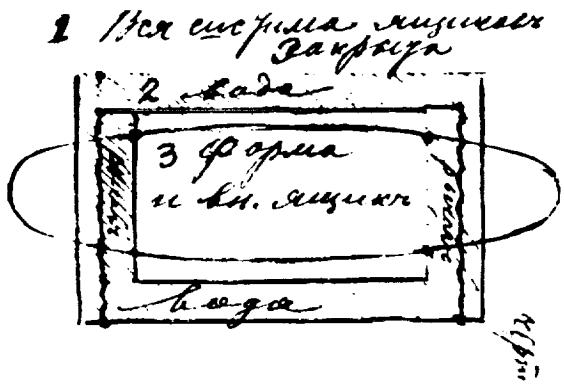


Fig. 2

1 - Cover; 2 - Water; 3 - Model and inner vessel.

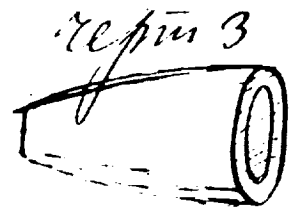


Fig. 3

Turning to the experiments, it was first necessary to confirm that the floating vessel and the model attached to it were sufficiently mobile. Accordingly, I placed the system of vessels on the table and observed what pressure had to be exerted on the model in order to obtain an appreciable displacement. For this purpose I constructed a device capable of exerting very small pressures on the model. It consists of a rod A (Fig. 4), mounted in a more or less vertical position, and a thin wire bent at right angles and rotating about A.

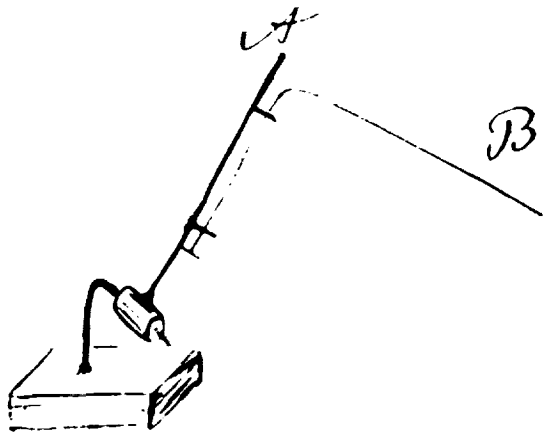


Fig. 4

The lighter the wire B, the more perpendicular the axis of A, and the less B is deflected from the equilibrium position, the smaller the force exerted by B on any obstacle it encounters. With the aid of this device it is easy to exert a pressure of a few milligrams. The model begins to move appreciably even under a pressure of 1 milligram. And since the air pressure in the experiments reaches 5 grams and more, the sensitivity of my apparatus would appear to be perfectly satisfactory.

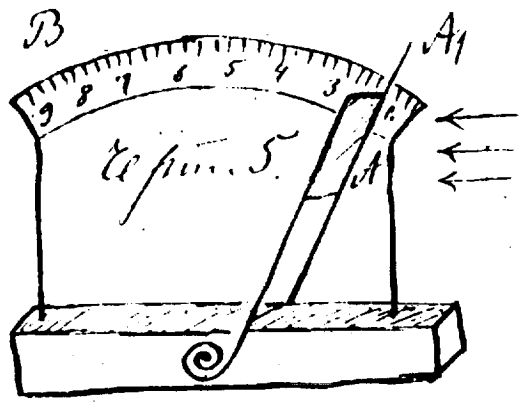


Fig. 5

It is also important to investigate the artificial air flow (Fig. 1). It is necessary to know the velocity of the flow at different distances from the mouth of the blower and from its central horizontal axis. Fig. 5 shows a device for finding the velocity of the air flow directly. It consists of a plate A mounted on two spring legs; the deflection of this plate from the equilibrium position depends on the velocity of the flow.

Oscillation of the plate indicates a nonuniform, gusty flow. I obtained the scale B as follows. First, I computed the pressure on the plate A (from the Cailletet and Colardo coefficient) for different flow velocities; then I exerted these same pressures on the plate A by means of a pulley and weights, marking off the readings of the pointer along the arc BA and engraving the corresponding values

of the velocity.

Alternatively, we can use an apparatus similar to but smaller than that shown in Fig. 2. The place of the test model is taken by a vertical plate (Fig. 6).

Fig. 6

The flow of air exerts pressure on B, forcing it to move and deflect the spring A by an amount proportional to the air pressure. The scale shows neither the exact velocity nor the pressure, but only their constancy or nonuniformity in different parts of the flow.

Fig. 7

Thanks to the water, the readings of this instrument are less sensitive to vibrations.

The experiment is set up as shown in Fig. 1. The model is mounted in the middle of the flow, facing in the same direction and not too close to the mouth of the blower, so that the difference in velocity at the ends of the model is not too great. The dimensions of the model, particularly in the vertical direction, should be small compared with the width or cross-sectional area of the flow. In the tests the ratio of the maximum cross-sectional area of the model (80 sq. cm) to the cross-sectional area of the flow did not exceed 1/20. However, it is desirable to have a much smaller value. Theoretically the flow should be infinite. Nevertheless, I shall have occasion to show (by various methods) how little the drag coefficients are affected when an infinite flow is replaced with a finite one.

Fig. 8

The experiment begins with the bolts H relatively slack (Fig. 1), which corresponds to a low degree of friction on the shaft and a low flow velocity. All the models are tested with the nuts in the same position, at the same velocity of the artificial flow. When the pressure on the last model has been determined and recorded, it is replaced with a plate of known area mounted normal to the flow. From the pressure on the plate, in conjunction with the readings of the barometer and temperature, we determine the velocity of the flow. The centers of the models and the plate must lie at the same point in the flow. The pressure on the plate is best determined at the beginning and end of a series of experiments, in order to verify that the velocity of the air has not changed in the course of the experiment. Each of the figures obtained is converted into the drag coefficient of the corresponding model, when we compare, by division, the pressure on the model with the pressure on an area equal to its greatest cross section (this is computed beforehand; the pressure obtained for the plate enables us to determine the pressure on any area at the same flow velocity).

Upon tightening the clamp H by several turns of the nuts and thus increasing the velocity of the artificial air flow, we accurately obtain a series of different pressures, from which we derive a series of other coefficients for the same models at a different speed, calculated, as mentioned above, from the Cailletet and Colardo coefficients.

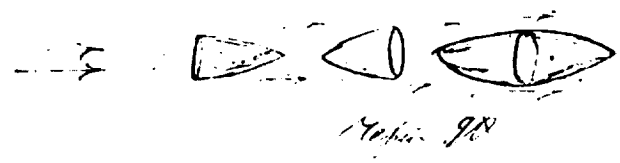


Fig. 9

On performing a further series of experiments with the same models, after further increasing the friction and hence the flow velocity, we obtain a third group of pressures and coefficients. Thus, by observing, say, a score of models at ten different speeds, we obtain 200 figures expressing the pressures on different models at different speeds. It then remains to express graphically, or by means of empirical equations, the relation between the pressure and the flow velocity, and sometimes between the pressure, the flow velocity, and the variable shape of the body. At any rate, apart from the derivation of certain laws of resistance, the drag coefficients I have obtained for various bodies have definite value as factual or verificational material.

I propose to carry out experiments with the following models at different speeds:

1. With bodies of different aspect ratio, obtained by revolving an arc of a circle about its chord.

This group of observations will also include a spherical surface and its parts.

So far I have conducted experiments with surfaces of this kind only at a speed of 1 meter per second and in the wind. Experiments of the latter type are inconvenient and difficult, not to mention their inaccuracy (I shall send the committee a published descrip-

tion of these experiments).

2. It would also be interesting to determine the pressure on the same models with the longitudinal axis inclined to the direction of the flow, but for this an extremely large blower would be required.

3. With an ellipsoid of revolution of different degrees of elongation (Fig. 7).

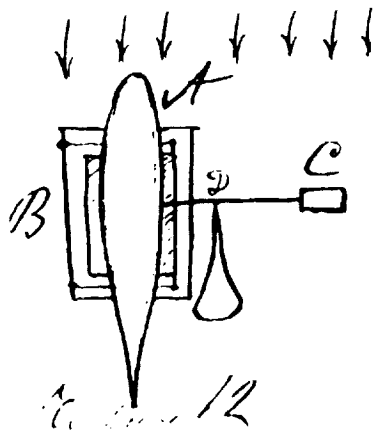


Fig. 12

4. With pancake-shaped, fish-shaped and egg-shaped bodies (in the article the shape will be given in the form of a drawing or equation).

5. With various kinds of cylindrical and conical surfaces (Fig. 8).

6. With polyhedra.

7. Certain writers on air resistance (Pomortsev) neglect the influence of the tail end of the model and the air friction (even with elongated bodies). I shall conduct experiments to clarify

this disputed point.

8. With a square plate normal to the direction of flow but at different distances from a horizontal plane (platform) (Fig. 10).

9. With the same plate but mounted at an angle to the direction of flow. In this case, the vessel with the water and the plate must be arranged so that the direction of the plate coincides with the direction of the flow; this happens when the indicator (dynamometer, Fig. 1) reads zero. Then, in relation to a circle previously traced on the platform, the vessel and the plate are rotated successively through 5, 10, 15, 20, etc. degrees. (Verification of the formulas of Duchmin, Langley, and Lord Rayleigh.)

10. Experiment with an elongated rectangular plate mounted in the direction of the flow and slightly inclined to it. The elongation or aspect ratio of the plate is varied. Derivation of a law expressing the influence of the aspect ratio and area.

11. Experiment with the same plate but with the long edge normal to the flow. Derivation of same law.

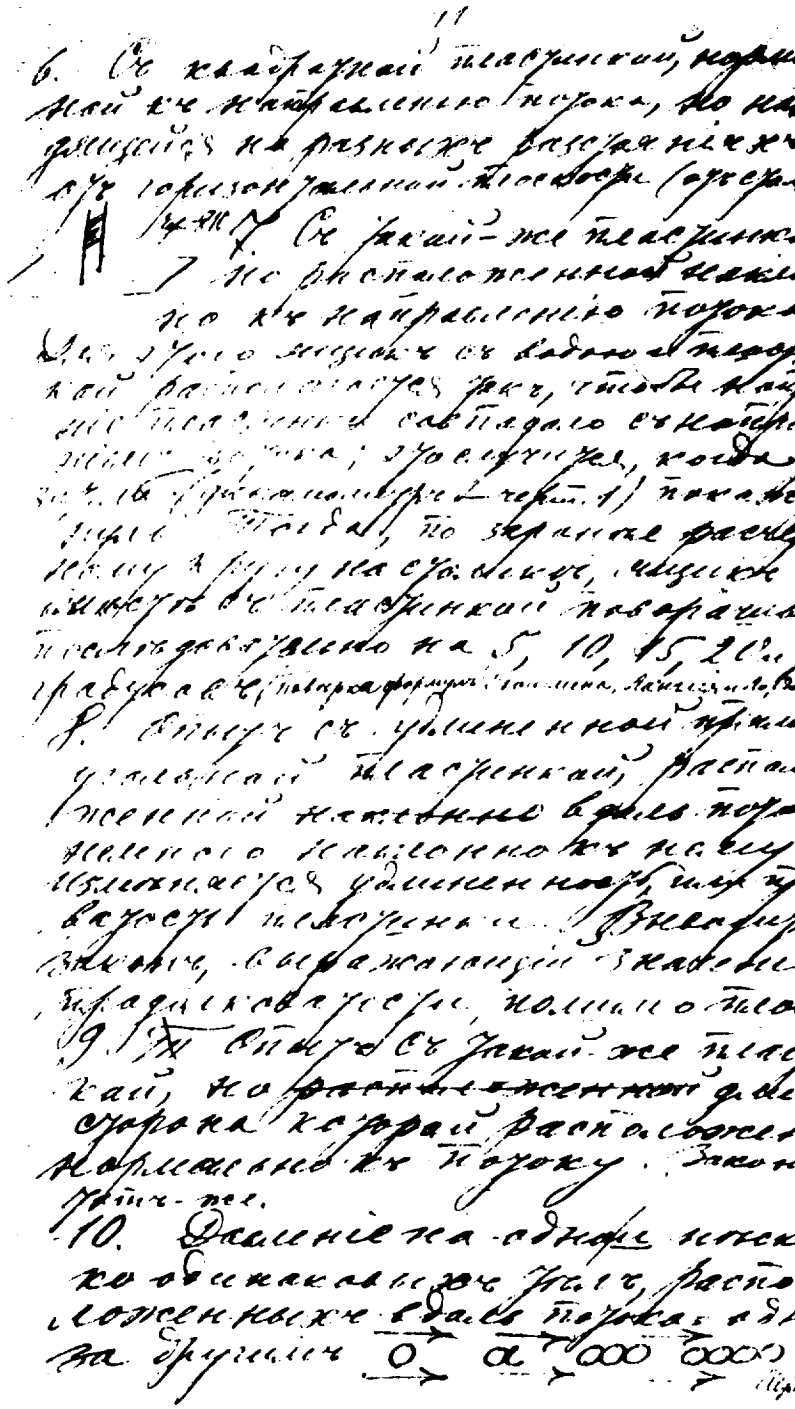
12. The pressure on one body and a series of identical bodies arranged one behind the other in the direction of the flow (Fig. 11).

Application to the problem: which cuts through the air more easily -- a single airship or several airships linked together in a chain?

Note that experiments on relatively blunt bodies can be conducted at the same or at two different velocities, since the drag coefficients of such bodies are almost independent of the flow velocity, provided it is not less than 1.2 meters per second.

In order to demonstrate the experiment to a considerable number of persons it can be set up in the following form (Fig. 12): A is a test model, the low resistance of which we wish to demonstrate graphically to the public; B is the vessel filled with water previously described (Fig. 2); D is the fulcrum of the lever arm BC; C is a plate, the air pressure on which tends to outbalance the model A.

If, for example, we take a plate, the area of which is 1/10 of the cross-sectional area of the model, and bring into action the blower (Fig. 1) with the clamp tightly screwed down, we see that as the velocity of the air flow gradually increases there arrives a moment of equilibrium, after which the balance shifts in favor of the plate. From this we may conclude that the drag coefficient decreases with increase in the flow velocity, and at the moment of equilibrium



Figs. 10, 11

[English translation of the above points 6-10 given in present chapter.]

may be expressed by the fraction 1/10. I have already performed a preliminary experiment of this type.

Kaluga, 12 October 1897
Georgiyevskaya ul., dom Speranskoy

AIR PRESSURE ON A SURFACE SUSPENDED IN AN ARTIFICIAL AIR FLOW*

Notes for Reading in Conjunction with the Article

The speed of rotation of the blower blades is proportional to the square root of the weight of the load (5 and 6)**.

Deflection of the needle by 1 mm corresponds to 1/80 g (about 12 dynes). The resistance of the body is expressed as mm of deflection, i.e., as eightieths of a gram (24).

The reading of the needle was verified with a weight before the start of each experiment (23 and 24).

The resistance measurements were made at air densities close to 0.0012 of the density of water.

The resistance of the stands, supports, and strips was constantly checked; in most cases they were as follows:

Load	1/2	1	2	4	8	16	pounds
Resistance	3	6	11.5	21.5	42	82	mm

The resistances quoted in the paper are after subtraction of the resistance of stands and so forth.

The resistance of a plate set normal to the air flow is proportional to the flow (26, 27, and 28).

The force on 1 cm^2 for various loads was as follows:

Load	1/2	1	2	4	8	16	pounds
Force	0.325	0.65	1.3	2.6	5.2	10.4	mm

*First printed in the journal Vestnik opytnoy fiziki i elementarnoy matematiki (Herald of Experimental Physics and Elementary Mathematics) in 1899.

**The numbers in parentheses refer to the corresponding paragraphs in the text. (Editor.)

The force on 80 cm^2 = 26, 52, 104, 208, 416, 832 mm (see 38).

The air flow speed is proportional to the square root of the load (29).

The ratio of velocities for the various loads is given by (30):

1 1.4 2 2.8 4 5.7

The absolute speeds (m/sec) for the same loads were (35):

1/2	1	2	4	8	16	pounds
0.756	1.069	1.512	2.138	3.024	4.276	m/sec

The projection of a body is here defined as the area of the shadow of the body on a plane perpendicular to the air flow direction on the assumption that parallel light rays run parallel to the wind direction. In short, this is the area of the projection of the body on a plane normal to the air flow (102).

The resistance of the projection is the force from the wind pressure on a plate perpendicular to the flow and having the area of the projection.

The resistance coefficient is a term I use frequently; it is the ratio of the resistance of the body to the resistance of the projection for the same wind speed. It indicates the resistance of the body as a fraction of the resistance of the projection for the same wind speed (the projection is sometimes the area of the largest cross section of the body). The shape utilization or shape utility (shape factor) is the reciprocal of this ratio, namely the ratio of the resistance of the projection to the resistance of the body at the same wind speed. It indicates by how much the resistance of the body is reduced by virtue of its shape relative to the resistance of the projection for the same wind speed. The shape factor is usually larger than one and the resistance coefficient less than unity, but the reverse can occur.

The law of relativity of motion makes it a matter of indifference whether the body moves in still air or the air moves against the fixed body. The resistances must be strictly equal in the two cases, if the conditions of motion are identical.

The <u>coefficient of friction</u> between a plane and the air is the ratio of the absolute force of friction on one side of the frictional surface to the resistance of the same surface when moving in

air with the same speed along the normal to itself.

The aspect ratio is the ratio of the length of the body to the mean diameter of its largest cross-section (or to the width).

Longitudinal curved surfaces I constructed very simply from paper. Shapes taking the form of surfaces of rotation I made by first carefully drawing the principal curve of the longitudinal cross-section. I then turned half of the shape (up to the largest cross-section) from wood on a lathe, according to this curve. This half former I covered with strips of wetted paper and bound them tightly into place with broad tape (swaddled, as a baby). The paper was allowed to dry thoroughly; then the tape was unwound and the paper was carefully removed, this having very closely the convex form of the surface of the wooden former. It remained merely to glue the pieces of paper on the shape. After this paper shell had been removed, the wide end was covered with a paper support (of rice paper). The other half of the shape, which was sometimes unequal to or different from the first, was made in the same way. The two halves were lightly glued if necessary.

The blower consisted of a wooden box bolted together. The walls were lined on the inside with cardboard, while the curved surface was made of tin-plate. The axle and bearings of the fan were of metal, while the vanes were of thin cardboard. I did not weigh the blower, but I think that it weighed not more than 50 pounds.

Most of the shapes for resistance test I glued together from rice paper.

I

DESCRIPTION OF APPARATUS AND CONDUCT OF EXPERIMENTS [12]

1. The artificial air flow was produced by means of an apparatus resembling a winnowing machine (Fig. 1).

2. B is the blower, whose height was about 150 cm and width 45 cm. The vanes b were driven by loads varying from 1/2 to 16 lb. The diameter of the fan (12 vanes) was 100 cm. The load acted as follows: a cord was wound on the shaft S by means of a handle (not shown) and passed through a fixed pulley screwed to the frame; its other end was attached to a hook also attached to the frame next to this pulley. The moving pulley carried the load via two hooks. There was also an additional load (1/2 lb, not more) intended to

counteract friction (which it overcame at small loads) and so caused the rotation to correspond more closely to the principal load (see below).

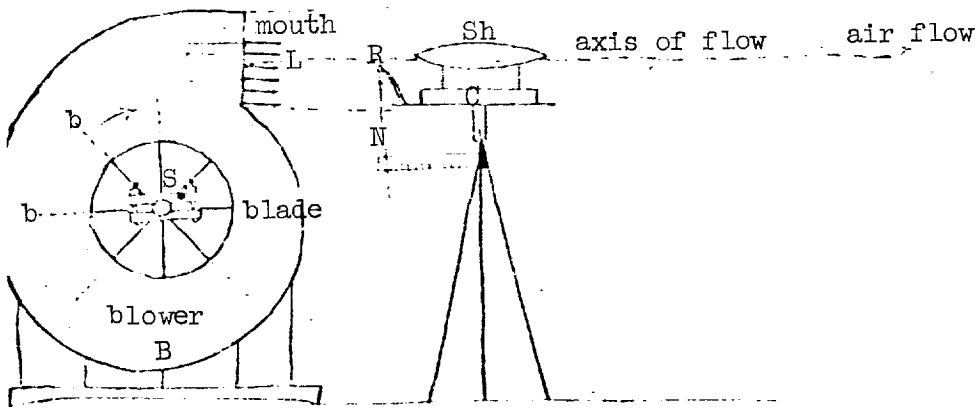


Fig. 1

3. To the load L there was attached also a cord in constant contact with the floor, to ensure that the weight of the cord acting on the apparatus was constant.

4. The cord could make up to 18 turns around the shaft, while the time of observation of the air flow and of the forces it produced was not less than 11 sec (for a load of 16 lb).

5. The additional load ensured that the time for the cord to unwind completely was strictly inversely proportional to the square root of the weight of the load. For instance, the time of unwinding for a load of 1/2 lb was 62 sec, which, relative to the least time (see 4) was increased by a factor 62/11 = 5.64, whereas the law of inverse proportion to the square root of the weight gives

$$\sqrt{(16/1/2)} = \sqrt{32} = 5.66.$$

6. We may conclude that the speed of the air flow is proportional to the square root of the principal load.

7. The maximal speed can be deduced from the diameter of the fan. A load of 16 lb caused the outside of the fan to travel in 11 sec a distance 100 × 3.1416 × 18 = 5655 cm, which in 1 sec is 515 cm or 5.15 m.

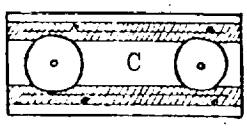


Fig. 2

8. The wind speed in these tests was therefore close to 5.15 m/sec.

9. This speed was somewhat reduced by the action of the lattice that renders the flow uniform (L in Fig. 1) and by imperfection in the equipment. This lattice is a box open on two opposite sides divided up by 11 thin strips into 12 equal sections, which in turn are divided into 48 equal apertures by 3 vertical strips. The lattice suppresses eddies and equalizes the speed (makes the wind less gusty).

10. The area of the lattice is only slightly less than that of the blower outlet: height and width about 35 cm, length in flow direction 25 cm.

11. The shape Sh to be tested is set up on rods attached to an open tin box, which floats in another container C filled with water.

12. This C is closed with a sectional lid having slots to allow the 4 rods carrying the shape Sh to move freely.

13. Between the rods (Fig. 2) along the flow there are attached two parallel tin strips; between these are two horizontal light circles on the lid, which turn freely on vertical needles. These circles have a diameter very slightly less than the distance between the tin strips; their function is to ensure that the rods move freely without rubbing on the sides of the slots. If the shape moves, one of the strips bears lightly on the circles and rolls almost without friction.

14. The length, width, and height of the outer container (cm) were 30, 15, and 4; those of the inner, 20, 10, and 2.5.

15. The floating box could carry (including its own weight) up to 500 g (i.e., over a pound).

16. The sensitivity, even when loaded with the heaviest shape, was more than adequate; a horizontal force of 1 mg (about 1 dyne) was sufficient to move the floating box. It was necessary merely to add sufficient water and to remove air bubbles adhering to the bottom of the floating box.

17. For the latter purpose it was necessary to press the box down to the bottom and rotate it slightly. However, this does not eliminate the bubbles permanently, for they continue to be produced on account of warming-up of the water and other reasons, which causes them to coat the walls of the vessels. These bubbles reduce the mobility of the box, so they had to be removed from time to time, as stated.

18. Container C was set horizontally on the tripod head T to bring the shape Sh to the center of the flow and to make the direction of motion of the inner box coincident with that of the air flow.

19. Under the tripod head T there was a horizontal ruler graduated in mm pointing along the flow direction.

20. In the same vertical plane as this fixed ruler there hung a light lever or needle N, whose axis was horizontal and fixed. All these were attached to T.

21. A very light and freely moving rod R joined needle N to the rods in the floating box. The wind tends to move the shape together with the needle when the blower is working; it is deflected to the right from the vertical and shows the force exerted by the air flow on the shape and rods.

22. The reading deviates more widely from proportionality to the force the larger the deflection.

23. This was readily demonstrated by deflecting the needle with a weight instead of by air pressure. For this purpose a very light pulley made of paper was used to bring the gravitational force on the weight to the horizontal. One end of a thread was attached to the rods, the thread running over the pulley and carrying a paper pan at its far end. This pan was loaded with weights from a decigram upwards. The deflection of the needle was at first almost proportional to the load, but the larger deflections were less than this would imply.

24. I bent the needle as shown in Fig. 1 to give almost strict proportionality in the readings. The apparatus was such that a deflection of N by 1 mm corresponded to 1/80 gram (about 12 dynes).

25. The pressure force on the rods, strips. needle N, and so on, expressed in mm, was as follows for six different loads:

1/2	1	2	4	8	16	pounds
3	6	11.5	21.5	42	82	mm

26. These resistances had always to be subtracted from the resistances of the shapes.

27. The speed of the flow was determined by fixing normal to the flow on two diagonally placed rods a pair of almost square plates of total area 14 cm^2. The forces on these, as corrected for the resistance of the rods (25) for the same loads, were

4.5	9	18	36.5	73	145	mm

28. From this we see that the force on the plate is proportional to the load, which is expected.

29. But the force on a plate is proportional to the square of the flow speed, or the flow speed is proportional to the square root of the force, so the speed is proportional to the square root of the load.

30. Then the ratios of flow speeds for the various loads are as follows

$$1 \quad \sqrt{2} \quad 2 \quad 2 \cdot \sqrt{2} \quad 4 \quad 4\sqrt{2},$$

so the highest speed (load 16 lb) was 5.66 times the least (load 1/2 lb).

31. The pressure on a plate normal to the flow is not dependent on the density of the air for a given load, as theory and experiment show; a fall in air density causes an increase in flow speed, so any fall in pressure is nullified.

32. The series of 27 therefore remains the same no matter what the barometer reading.

33. But the absolute speed alters, and hence so does the resistance of slim bodies, for which air friction plays a large part.

34. The force of (24) on a plate readily gives the flow speed; for this we use Cailletet and Colardo's formula $0.071V^2$, which gives in kg the wind force on a plate 1 m^2 in area from a wind speed V in m/sec. It is assumed that the atmospheric pressure is 1 kg/cm^2 and the temperature is 10°C, or that the density of air is constant at 0.0012 of that of water.

35. This formula gives the following speeds:

0.756 1.069 1.512 2.138 3.024 4.276 m/sec.

These speeds are only 1/5 less than the speed at the outside of the fan in the blower (7).

36. The present air flow had a restricted cross-sectional area (about 1200 cm^2), which was larger than that in Maxim's apparatus*. The wider the flow in relation to the model, the closer the force should be to the theoretical value.

37. However, tests with plates of area up to 80 cm^2 or even 100 cm^2 showed no appreciable deviation. On this basis we can assume

*Hiram Maxim. Natural and artificial flight. The Aeronautical Annual, 1896, Boston.

that the air flow is quite adequate (acts as unbounded) for shapes whose cross-sectional area does not exceed 80 cm^2.

38. In view of the fact that I frequently used such areas, I give the force on a plate of area 80 cm^2 for the various flow speeds (see 35). The force on 1 cm^2 was 0.325, 0.65, 1.3, 2.6, 5.2, and 10.4 mm.

II

FORCE ON A FLAT PLATE NORMAL TO THE FLOW

39. The flow speeds quoted in (35) are based on the experiments of Cailletet and Colardo, because they determined the pressure on a plate in rectilinear motion, whereas others have measured the pressure during motion in a circle (on rotating machines), in which the circular pressure is the larger the less the radius of the circle.

40. The following values (in kg of pressure from a wind moving at 1 m/sec per m^2 of plate normal to the flow, subject to condition 34) show the great variation in the values for the relative resistance obtained by various workers:

41. Goupil and Marey: 0.130.

42. International wind-speed scale: 0.123.

43. Piobert, Morin, Renard, and Langley: about 0.085.

44. Cailletet and Colardo: 0.071.

45. Poncelet derived from theory the expression d/2g, in which d is the weight of unit volume of air and g is the acceleration due to gravity. This gives for the metric atmosphere (735.5 mm) and 10°C only 0.0612.

46. Cailletet and Colardo's value (0.072) agrees most closely

with this; it is only 1/16 larger than this theoretical value.

47. It was inconvenient to determine the flow speed directly on my apparatus, but it was easy to measure the force on a plate, from which I deduced the speed in accordance with Cailletet and Colardo's results. The probability of the results is thus strengthened by (35).

III

FORCE ON AN INCLINED PLATE

48. The forces on plates placed at various angles to the air flow have also been measured or deduced theoretically by various workers, but here there is no less disagreement (41, 44, 45).

49. Let P_1 denote the wind pressure on a plate normal to the flow and let P be the pressure on the same plate placed at an angle to the flow; then we have:

50. From Newton's laws, $\frac{P}{P_1} = \sin^2 i$. The gross incorrectness of this is now quite clear from theory and experiment.

51. Leslie gives $\frac{P}{P_1} = \sin i$*.

52. Lord Rayleigh's formula is**

---

*All recent workers on resistance agree that the pressure at acute angles is proportional to the sine of the angle of inclination. Duchmin's experiments (Recherches experimentales sur les lois de la resistance des fluides (Experimental Investigations of the Laws of Resistance of Fluids), Paris, 1842) and (the late) Lilienthal's (Der Vogelflug als Grundlage der Fliege-Kunst (Bird Flight as the Basis of the Art of Flying), Berlin, 1880) lead to the same conclusion.

**Lord Rayleigh. On the resistance of fluids. Phil. Mag., 2, 1876.

$$\frac{P}{P_1} = \frac{(4 + \pi) \sin i}{4 + \pi \sin i}.$$

53. Langley's experiments* with rotating machines give

$$\frac{P}{P_1} = \frac{2 \sin i}{1 + \sin^2 i}.$$

54. I made experiments with an inclined square plate of area 100 cm^2.

First I determined a series of horizontal forces with successive turns of the plate through 5° about a horizontal axis. The initial position was with the plate normal to the flow; the measurements extended up to an angle of inclination of 45°.

55. Then the position of stand (Fig. 1) and box was altered to perpendicular; the plate lay along the air flow and was rotated about a vertical axis. The zero position was taken as that in which the flow exerted no force on the plate and did not deflect the needle of the dynamometer (Fig. 1).

56. Combining the series of 54 and 55 into one, and eliminating the effects of the rods in 55, reading the angles from the direction of the flow, and converting the measured forces to ones normal to the surface of the plate, we get (for a flow speed close to 1.5 m/sec, produced by a load of 2 lb) the following results:

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*Langley. Experiment in Aerodynamics. 1891.

Angle	0°	5°	10°	15°	20°	25°	30°	35°
Force	0	18	37	56	81	105	123	135
Angle		40°	45°	50°	55°	60°	65°	
Force		140	143	144	143.5	142	139	
Angle			70°	75°	80°	85°	90°	
Force			136	134	133	132	132	

57. These forces normal to the plate have been derived from those recorded in the tests of (54) by division by the cosine of the angle between the plate and the flow direction (the angle in the table) and from those recorded in the tests of (55) by division by the sine of the angle[13].

58. The table shows that the force is almost exactly proportional to the angle for angles of 0 to 15°; then there is a somewhat more rapid rise up to 20°, and then a slower one to a maximum at an angle of 50° to the flow.

59. In this position the force normal to the plate is 12 mm larger than that when the angle between plate and flow is 90°. In general, this is so for angles between 35 and 85°.

60. None of the formulas of 50-53 predicts this feature; they all merely show that the force is at first proportional to the sine of the angle of deviation from the flow direction, then rises <u>less</u> rapidly, but always continuously, to 90°.

61. The following are the $\frac{P}{P_1}$ given for various angles by the formulas of 51, 52, and 53, and by my experiments.

62. The table shows that my results for small angles are somewhat less than those given by Langley and Rayleigh; subsequently they are closest to Langley's results and differ most widely from Leslie's.

63. Tests with different loads and plates of identical inclination showed that the force is proportional to the load or to the

square of the flow speed.

	5°	20°	35°	50°	60°	90°
Leslie	0.087	0.342	0.574	0.766	0.866	1
Rayleigh	0.145	0.470	0.708	0.819	0.920	1
Langley	0.166	0.612	0.864	0.984	0.990	1
My results	0.137	0.614	1.023	1.091	1.076	1

IV

FORCE ON AN OBLONG INCLINED FLAT PLATE

64. None of the formulas given above indicates the effect of the length of the plate on the force.

In fact, the force is scarcely dependent on the length as such if the plate is perpendicular to the flow; it is dependent solely on the area; but the length has a marked effect on the force if the plate is inclined*.

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*Amans. Sur un appareil destiné à mesurer la force propulsive de diverses palettes (An Apparatus Designed to Measure the Propulsive Force of Different Bladings). L'Aeronaute, 1890.

This paper states that length in an inclined plate placed perpendicular to the flow increases the force exerted by the wind; but formulas are not given. Langley's experiments point to the same conclusion.

67. I did a series of measurements with rectangular plates 4 cm wide and of lengths 4, 8, 12, 16, 20, 24, and 32 cm; these were placed along the flow and across it at the same acute angle of 20°.

68. I give only the results for a wind speed of about 3 m/sec; here also the effect of 63 was observed.

With the transverse position (long edge of rectangle perpendicular to flow direction) I found:

Length	4	8	12	16	20	24	cm
Force	22	43	64	90	110	136	mm

This shows that the force is almost proportional to the area for the transverse position.

69. However, with an acute angle to the wind there is a clear increase in the force per unit area. For an angle of 12° and a speed of 3 m/sec we have

Length	4	8	12	16	20	24	cm
Force	6	14.5	25	38	47.5	59.5	mm

The forces on 16 cm^2 are then respectively 6, 7.3, 8.3, 9.5, and 9.9, so the pressure increases continuously, in this case by a factor rather less than 2.

70. The force is also not proportional to the plate area when the plate is placed longitudinally, even at 20°. The test for a speed of 3 m/sec gave

Length	4	8	12	16	20	24	32	cm
Force	22	33	42	53	62	73	85	mm

The force increases by only 50% when the length of the plate is doubled.

71. These three tables give the force in the flow direction; the forces perpendicular to the plates may be found by dividing these forces by the sine of the angle of inclination of the plate to the wind.

V

TESTS ON FRICTION

72. To elucidate the laws governing the friction of air with the surfaces of solids I used three circular cylindrical surfaces all of diameter 10.3 cm and circumference 32.5 cm but of different lengths (12.5, 25, and 50 cm).

73. These were placed along the flow direction (generator parallel to that direction) by raising or lowering an end or by rotating the cylinder to give the minimum resistance as read from the needle of the dynamometer (1, c) for a given wind.

The cylinders were, of course, open (had no bases) to allow the air to pass freely through.

74. To simplify the deductions, all measurements were made at air densities close to 0.0012 of the density of water (see 34).

75. Tests were done with the cylinders with loads of 1/2 to 16 lb; correcting for the resistance of the rods, we have

Area	Length	1/2	1	2	4	8	16
813	12.5	4	7	12 12	21.5	36	60
1625	25	6	11	19	32.5	56	94
3250	50	9	16	29	50	87	160

The surface areas of the cylinders have been doubled because the friction was developed on the outer and inner surfaces. The resistance is expressed in millimeters (1 mm corresponds to 1/80 g).

76. The results reveal the following fairly clear-cut laws of friction.

77. The friction is not proportional to the square of the speed; a doubling of the speed causes only a threefold (average 3.04-fold) increase in friction, or an increase in speed by 1.41 times caused an average increase in resistance of 1.7434 times. The deviations were slight and occurred in either direction.

78. The friction is not porportional to the length; on average, doubling of the length caused an increase in friction by a factor 1.546.

79. The frictional force T for cylinders of this diameter (about 10 cm) is given by

$$T = A \cdot v^a l^b,$$

in which v is wind speed in m/sec, l is the length of the cylinder, and the other quantities are constants.

80. For a particular case we have $T_1 = Av^a l_1^b$ and $T_2 = Av^a l_2^b$, so:

81. $$\frac{T_2}{T_1} = \left(\frac{l_2}{l_1}\right)^b, \quad \text{or} \quad b = \log\left(\frac{T_2}{T_1}\right) : \log\left(\frac{l_2}{l_1}\right).$$

83. But we have seen that when the ratio of lengths is $2\left(\frac{l_2}{l_1} = 2\right)$ the ratio of the forces is $1.546\left(\frac{T_2}{T_1} = 1.546\right)$, so b =

= log (1.546) : log (2) = 0.629, or about 0.63.

84. Then from b we can find a. For a given length of cylinder we have

$$T_3 = A \cdot v_2^a \cdot l_3 \quad \text{and} \quad T_4 = A \cdot v_3^a \cdot l_3^b.$$

85. Then we have

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_2}\right)^a \quad \text{and} \quad a = \log\left(\frac{T_4}{T_3}\right) : \log\left(\frac{v_3}{v_2}\right).$$

86. But we have seen that a doubling of the speed $\left(\frac{v_3}{v_2} = 2\right)$ increases the friction by nearly a factor 3 $\left(\frac{T_4}{T_3} = 3.04\right)$, so a = = log (3.04) : log (2) = 1.604.

87. This then gives us A. From (79) we have

$$A = \frac{T}{v^a l^b} = \frac{T}{v^{1.604} \cdot l^{0.63}}.$$

Insertion of any set of values for v, l, and T from the table of (75) then gives us A. For example, we take from the table of (75) the friction value 32.5 (mean about 32.3), which corresponds to a speed of 2.138 m/sec and a length of 50 cm. Then A as found from the above formula is: A = 1.264.

88. The experimental results of (75) are then represented by the empirical formula

$$T = 1.264 \cdot V^{1.604} \cdot l^{0.63}.$$

89. We apply this equation to the deduction of a general formula for the frictional force on a rectangle of length L meters and of width 1 m moving along the direction of side L with a speed V m/sec.

Considering the friction on one side only, we have

$$\frac{1.264 \times 100}{32.5 \times 2 \times 80 \times 1000} \cdot V^{1.604} \cdot (L \cdot 100)^{0.63}.$$

This formula is derived from the preceding one by converting cm to m and my mm (arbitrary unit of resistance equal to 1/80 g) to kg. Here the frictional force is in kg and the length L of the rectangle in m. Then we have

$$T = 0.0004423 \cdot V^{1.604} \cdot L^{0.63}.$$

90. The coefficient of friction (ratio of the frictional force to the air resistance for movement of the same plate at the same speed but in the direction normal to itself) is given by Cailletet and Colardo's formula (34)

$$F = (1 \times L) \cdot 0.071 \cdot V^2.$$

91. Dividing the previous formula by this, we have

$$\frac{T}{F} = 0.00623 \cdot V^{-0.396} \cdot L^{-0.37}.$$

The coefficient of friction thus decreases as the speed and length L in the flow direction increase; moreover, the power (about 0.4) is nearly the same for both[14].

92. The last formula gives us the inverse relation

$$\frac{F}{T} = 160.5 \cdot V^{+0.396} \cdot L^{+0.37},$$

which indicates by how much the frictional force on a rectangular plate is less than the resistance of the same plate in normal motion at the same speed.

93. The following table is based on formula 92, which is simplified for use by reduction to

$$\frac{F}{T} = 160.5 \cdot V^{0.4} \cdot L^{0.37}.$$

The laws of (77) and (78) have not been verified by experiment for very long surfaces or high speeds, so the values in table (93) may be considered undoubtedly correct only up to lengths of 50 cm and speeds up to 5 m/sec; formula (93) is correct within the same limits. I very much regret that I could not carry out more extended tests with longer surfaces or speeds above 5 m/sec. Analogy with Frood's experiments for water (see: Dislere, Exposé sommaire des expériences faites à Amsterdam sur la resistance des carenes (A Brief Report on Experiments on the Resistance of Keels, Performed in Amsterdam), Paris, 1878) would indicate that the indices in (93) gradually decrease as L and V increase.

The coefficients of friction are found from the values in the table by taking the reciprocals.

94. For example, the coefficient of friction is 1/655 for a surface 10 m long moving at a speed of 4 m/sec.

m/sec	1	2	4	8	10	20	40
1/10 m	68	90	119	157	172	227	299
1	160	212	272	369	403	532	702
10	376	496	655	864	945	1247	1645
100	882	1164	1536	2026	2216	2923	3857
200	1139	1503	1983	2616	2861	3775	4981
1000	2068	2728	3600	4750	5194	6853	9043

The table shows that the friction of bodies of moderate length in the flow direction can be neglected if their dimensions in this direction are more than 0.1 m.

However, the friction in my models, which were not very long in the flow direction, may also be neglected, for it was not more than 1/20 of the resistance from inertia.

Previously (Vestnik opytnoy fiziki, No. 259) I gave the formula

$$\frac{T}{F} = \frac{1}{58V},$$

for the coefficient of friction on the assumption that the surface is reasonably extensive in the flow direction (Vestnik opytnoy fiziki, No. 258, p. 146, footnote). Now we can determine the extent to which we must increase the surface, together with the speed, in order to obey Hagen's law (see previous formula) and so to make true the deductions made in my paper "Independent motion of an aerostat" (Vestnik opytnoy fiziki, Nos. 258 and 259).

95. We put

$$58V = 160 \cdot V^{0.4} \cdot L^{0.37}$$

(from the previous formula and 93). Then

$$L = \left(\frac{58}{160}\right)^{2.7} \cdot V^{1.62} = 0.0646 \cdot V^{1.62} \text{ m.}$$

Hagen's law will be obeyed if the length of the rectangle in the flow direction increases as $V^{1.62}$.

Then for the velocities in question (Vestnik opytnoy fiziki, No. 258, p. 150) we have the following lengths:

V =	1	4	12	20	40	m/sec.
L =	0.065	0.610	2773	8285	25.44	m.

This shows that even very high speeds (e.g., 40 m/sec or 144 km/hr) do not require very long surfaces (ones ten times less than those of my proposed aerostats). This means that the friction is even less than that implied by my previous work.

96. For instance, in example 74 (Vestnik opytnoy fiziki No. 259) the limiting speed of the aerostat was 328 m/sec, so the coefficient of friction was 1/19,024 from the previous work, whereas from (91) of the present paper it is 1/32,800. The friction in these new tests is thus only two-thirds in relation to my cost extended calculation, so the speed of the aerostat would be higher than that previously calculated.

This means that the favorable nature of my deductions on the aerostat is not altered; only the accuracy is affected, and that not very much.

VI

REGULAR PRISMS AND CIRCULAR CYLINDER

100. These were placed with their axes perpendicular to the flow direction. The axes were about 10 cm long.

101. Triangular prism, side of triangle 4 cm. The force on any body of small extent in the flow direction is found to be directly proportional to the load or to the square of the wind speed, so the forces on such bodies will be given for one load only (although I did tests with various loads).

102. Load 2 lb; one face of prism parallel to flow direction (wing). The wind force corresponded to 44 mm. The force on a plate normal to the flow and having the projection of the prism* is calculated as 45 mm, so the resistance coefficient is only very slightly less than unity.

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*By this is meant the projection on a plane normal to the flow. (Editor).

103. One face perpendicular to the flow and facing it. Force 58 mm; force on plate 53 mm.

104. The same, but edge facing the wind. Force on prism 31.5 mm, force on projection 53 mm. The resistance is then substantially less than that of a plate (by a factor of 1.7) and the resistance coefficient is 0.59 (almost as for a cylinder).

105. Square prism. Side of base 4 cm; face normal to wind; force on prism 54 mm, on projection calculated as 53 mm. The pressure therefore scarcely differs from that on a plate equal to the face.

106. The same prism, but placed with edge facing wind (symmetrically). Force on prism 58 mm, on projection 74 mm. Resistance coefficient 0.78, but it is actually better to have the face normal to the wind, because the absolute resistance is less.

107. Hexagonal prism. Side of base 3 cm. Placed with corner leading (minimum projection), giving a minimum force on projection of 69 mm; force on prism 61 mm. Resistance reduced relative to a plate by only a factor 1.13; resistance coefficient 0.89.

108. Maximum projection (two faces normal to wind). Force on projection 78 mm, on prism 52 mm. Resistance coefficient 0.65 (close to that for a cylinder).

109. Ratio of largest resistance (2 faces parallel to wind) to least (2 faces perpendicular to wind) is 1.2, which is not large.

110. Regular octagonal prism. Minimum projection: resistance reduced in ratio 65:53 = 1.23; maximum projection: reduced in ratio 69:42.5 = 1.62. Resistance coefficients respectively 0.82 and 0.61. Ratio of largest resistance to least 53:42.5 = 1.24.

111. Right circular cylinder. Length of axis 10 cm (as for prism); base diameter 5 cm. Resistance strictly proportional to square of speed (41 mm for load of 2 lb; resistance of projection 65 mm, resistance reduced by a factor 1.59, resistance coefficient 0.63). It seems odd that the resistance of a cylinder is somewhat higher than that of an octagonal prism of the same projection. It would seem that the leading edge, which divides the air, may play some part.

112. Formulas (50)-(53) have been used by some scientists to determine the forces on polyhedral and other bodies. For instance,

Leslie's formula (51), which gives forces about half the true ones, gives 0.785 for the resistance coefficient of a cylinder, which is grossly different from the experimental value. On the other hand, the more correct formula (53) gives an even larger error, while Newton's invalid formula gives less error. This shows that the formulas of (50)-(53) cannot be used to calculate the forces on curved or polyhedral surfaces.

113. We have shown by experiment that these formulas are not even suitable for flat plates, if these are elongated and lie at acute angles to the flow (see 64-70).

VII

RIGHT ELLIPTIC CYLINDERS

114. I used four elliptic cylinders, with the major axis at various angles to the flow direction. The following results were obtained with a load of 2 lb:

Axis		Axial ratio	Inclination of major axis to flow			
Major	Minor		0°	30°	60°	90°
70	56 mm	1.26	40	45	56.5	60
68	49	1.39	33.5	43	55	62
65	43	1.51	28	37	54	62
62	31.4	1.98	17.5	34	55	65

The length (height) of each cylinder was 10 cm.

115. The principal positions (0 and 90°) are used in the calculations. A cylinder having its major axis along the wind may be considered as one of aspect ratio greater than one; one with its minor axis so, less than one. A circular cylinder has an aspect ratio of unity. Then, including the circular cylinder of (111), we have 9 measurements on elliptic cylinders of various fineness ratios.

116. From the table of 114 we have

1.98	1.51	1.39	1.25	1.00	0.80	0.72	0.66	0.51
2.34	2.00	1.9	1.83	1.59	1.52	1.42	1.37	1.25

The first line is the aspect ratio (ratio of length of axis along flow direction to length of axis perpendicular to flow); the second is the reduction factor relative to the resistance of the projection (shape factor).

117. Then I used a cylinder whose base took the form of a fish; the front part had the shape of an ellipse, and the rear ended in an acute angle. This shape was very smooth. The base had a length of 18 cm and a greatest width of 5.8 cm, so the aspect ratio was 3.1.

118. Loads of 2, 4, and 8 lb were used, with the cylinder having on one occasion its elliptic end towards the wind and on the other its acute-angled end. The following resistances (mm) were recorded:

	2	4	8	16 lb
Blunt end toward wind	24	42	75	135 mm
Sharp end toward wind	26.5	52	104 mm	-

These show that the resistance when the wind strikes the rounded end is much less than when it strikes the sharp one. Moreover, the resistance in the first case does not increase as rapidly as the load (or as the square of the speed), whereas in the second case it is proportional to the load. The least resistance coefficient for a cylinder with its circular end facing the wind is 0.224; the largest, 0.316. The corresponding values for the sharp end facing the wind are both 0.342. The resistance is reduced (the shape factors are) 4.47, 3.14, and 2.92.

VIII

REGULAR POLYHEDRA AND SPHERES

119. Tetrahedron (regular figure with four faces). Length of edge 10 cm, resistance proportional to load, so only the resistance for a load of 2 lb is given.

120. One face parallel to flow, one edge symmetrically placed with respect to air; resistance 31 mm. Edge symmetrically placed with respect to air and normal to the flow: resistance 43 mm. One face normal to flow and apex facing flow: resistance 25 mm, resistance coefficient 0.442. Face perpendicular to wind and facing it: resistance 61 mm, resistance coefficient 1.08. The rear pyramid merely increases the resistance. The minimum resistance coefficient differs only slightly from that of a sphere.

121. Octahedron (regular eight-faced figure). Length of edge 6 cm; axis passing through vertices normal to flow.

a) Four-faced corner facing into flow: resistance 30.5 mm, resistance of projection 47, resistance coefficient 0.65;

b) Edge normal to flow and facing into wind: resistance 25 mm (less than the previous by a factor 1.22), resistance coefficient 0.61.

122. Cube; length of edge 8 cm.

a) One face normal to flow: resistance 64 mm, resistance of projection 84 mm, resistance coefficient 0.76 (resistance of cube

less by a factor 1.3 than the resistance of a plate perpendicular to the flow and having the dimensions of the projection of the cube). Dubois and Duchmin give the same ratio for water as varying between 1.22 and 1.27.

b) Two faces parallel to flow, others identically inclined to flow; one edge faces into flow. Resistance 84 mm, relative to resistance on projection 0.71.

123. Regular icosahedron (polyhedron bounded by 20 equilateral triangles). Length of edge slightly over 6 cm. Axis through opposed vertices normal to wind. Rotation about this axis has little effect on the resistance (variation from 69 to 71 mm). Area of projection less than 100 cm^2, force on it 130 mm. Resistance coefficient not less than 59:130 = 0.53.

124. Sphere. Area of great circle 63 cm^2; force on projection 82 mm; resistance of sphere (same load, 2 lb) 35 mm, resistance coefficient 0.43.

125. Pomortsev has used formula (53), the most nearly correct one, and finds the resistance coefficient for a sphere as 0.86. This is yet another proof of how incorrect it is to use the formulas for plane surfaces to determine the resistance of curved ones.

I did my tests with the sphere (as usual) at all available flow speeds and found that the resistance was proportional to the square of the speed, as for all bodies of small fineness ratio. I found almost exactly the same coefficients previously for sphere and cylinder by the use of unbounded air flows (see Tsiolkovskiy, "A maneuverable iron aerostat," Vestnik opytnoy Fiziki [i elementarnoy matematiki], No. 259, p. 278).

This agreement indicates that my flow was wide enough for the models used.

IX

ELONGATED BODIES OF ROTATION

126. Consider a body or surface generated by rotating a circle about a chord. I made paper surfaces of this type of four lengths:

21, 32, 42, and 62 cm. The diameter (Fig. 2) of the largest cross-section was close to 10 cm, and the area to 80 cm^2.

The resistance with the long axis (axis of rotation) along the flow was as follows:

	Load	1/2	1	2	4	8	16 lb
Length, cm	21	6	12	24	47	79	110
	32	5	10	17	31	48.5	80
	42	4	8	16	28.5	48.5	89
	62	4	8	16	31	55	104

These shapes were tested at various speeds. The first column is the length in cm. The resistance decreases only very slightly as the length increases; it even increases with the length for speeds of 2.14 m/sec (4 lb load) and above. The shape with the least resistance at this speed is the one with a length of 42 cm. A load of 16 lb (speed 4.28 m/sec) gives least resistance for a length of 32 cm. This effect is at first sight surprising; it in part arises from the increased surface area (and hence increased friction) of long bodies.

128. Only at speeds up to 2 m/sec (not more) was the resistance proportional to the load (or square of the speed); above this it rises much more slowly, which is especially clear for bodies of small aspect ratio, for which the friction is small relative to the inertia, on account of the small surface. For this reason, the effect is not completely explicable in terms of friction, which is not proportional to the square of the speed (88). The effect is surprising and attracted my attention particularly, because it points to aspects that would facilitate flight.

Thus, the law that the resistance of a body is proportional to the square of its speed has been repeatedly demonstrated from theory (when friction is neglected) and has been confirmed by experiment (at least for short bodies) -- and now it turns out that, roughly speaking, this law does not hold even for a small body for which friction may be disregarded. In fact, the table of 126 shows that a length of 21 cm (and width of 10 cm) gives an increase in resistance by only a factor a little over 2, not by the factor 4 expected from that commonly accepted law, when the load increases from 4 to 16 lb (speed increases by a factor 2)!

This at first sight strange result may perhaps be explained as follows.

The same table shows that the law is followed the more closely the lower the speed; gross deviations occur only for speeds between 2 and 4 m/sec.

It may be that the motion extends far around the body when the speed is low, the work of perturbation of the air being comparatively large. Higher speeds do not cause the perturbation to spread so far; the air is merely compressed near the front half of the body and returns some of its work of compression when it expands and exerts a pressure on the rear part. It seems to me that part of the work originally lost by the body thereby returns to it, and that this part is the larger the more rapid the motion.

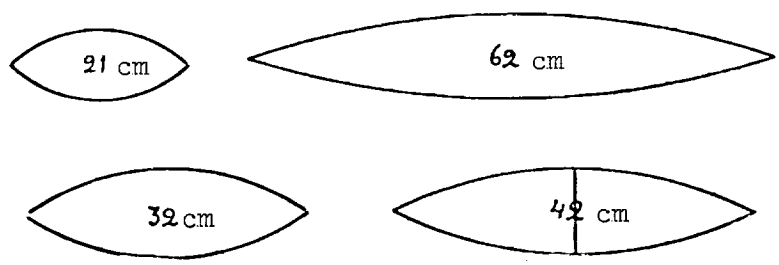

Fig. 3

This return does not occur for longer bodies, for at the largest speed I used (4.3 m/sec) the lateral displacement of the air occurred extremely slowly, the perturbation propagated far, the air

was condensed (compressed) little, and so there was scarcely any return of the work expended. However, at sufficiently high speeds even very long bodies should undoubtedly show this return and hence should show a favorable deviation from the law of increase of resistance in proportion to the square of the speed (of body or medium, both being the same).

This means that my previously assumed (Vestnik opytnoy fiziki No. 259) resistance of air as dependent on inertia must be subtantially reduced, on account of the excellent elasticity of air. The reduction is the larger the greater the speed of the aerostat. Who knows what speeds air vessels may attain in the future as a result of the principle of return of energy! In any case, they will be greater than those I gave in the above paper.

If we calculate the friction for all the bodies in Table 126 (for which purpose special coefficients applicable to the curvature of the body have to be determined) and subtract this from the total resistance, we obtain a table giving the resistance dependent on the inertia and elasticity of the air alone.

Taking a series of bodies for the same speed, we find that the inertial resistance is approximately inversely proportional to the square of the aspect ratio.

This is the case for low speeds (up to 2 m/sec); but we can see that it does not apply for the inertial resistance at 4.2 m/sec. We must consider how to explain this strange feature (see Vestnik opytnoy fiziki No. 259, p. 277).

The fact is that at small speeds there is no compression of the air and associated return of energy; at higher speeds we find lateral compression for blunt shapes but not for sharp-edged ones that move the air aside slowly; so for the latter the inertial resistance is much larger than the aspect ratio law would imply.

129. The relative fall in resistance dependent on the elasticity of the medium is not so prominent for elongated bodies, because friction plays a large part for these; in contrast, the inertia (lateral displacement of the air) is slight, on account of the sharp-edged form. But for large bodies and for speeds at which the friction is not large (91), the favorable reduction of the inertial resistance should be perfectly clear.

130. The following table shows by how much the resistance of the shapes of (126) is reduced relative to the force on the area of largest cross-section (on the projection). These numbers may be called the shape factors; they are the reciprocals of the resistance coefficients. In addition, the speeds are given, while the lengths of the bodies are replaced by the aspect ratio (ratio of length to width).

Load	1/2	1	2	4	8	16 lb
Speed	0.76	1.07	1.51	2.14	3.02	4.28 m/sec
2.1	4.3	4.3	4.3	4.4	5.3	7.6
3.2	5.2	5.2	6.1	6.7	8.6	10.4
4.2	6.5	6.5	6.5	7.3	8.6	9.4
6.2	6.5	6.5	6.5	6.7	7.6	8.0

131. This shows that the least resistance in my experiments occurred with the body of aspect ratio 3.1 at the highest speed. Pomortsev found by means of variational calculus for the same aspect ratio in a body of ideal shape (least resistance) that the resistance coefficient should be 0.4, which would imply a shape factor of 2.5, which is less than the value found in my tests by a factor 4.

132. I stress this point in order to show once again how inapplicable the reasonably reliable resistance formulas for plates (52 and 53) are in the determination of resistance for curvilinear bodies.

X

COMPARISON OF RESISTANCE FOR BODIES OF DIFFERENT SHAPES HAVING ROUGHLY THE SAME LENGTH AND ASPECT RATIO

133. The table below gives the resistances of a) one of the bodies of (126) of length 21 cm, b) an ellipsoid of rotation, c) a body smooth in the longitudinal direction but of square cross-section (built up from four equal plates bounded by arcs of circles), and d) a body composed of two equal conical surfaces (Fig. 4).

Fig. 4

134. Comparison of body a with the ellipsoid of rotation shows that the latter has only slightly less resistance than body a, which had small cones at the ends; in fact, the ellipsoid even had a larger /112

Load (lb)	1/2	1	2	4	8	16	
a	6	12	24	47	79	110	Already tested
b	5.5	11	24	42.5	74	117	Ellipsoid of rotation
c	6.5	13	25	47	80	128	Square section
d	12	24	47.5	94	185	-	Cones

resistance at or about a speed of 4 m/sec and above. This means that it is doubly favorable to use a shape ending in cones for a metal aerostat (as regards construction and as regards resistance).

135. Resistance of body c (square cross-section) also did not differ greatly from that of bodies a and b, which shows that the corners on the cross-section have little effect on the resistance.

136. But corners on the longitudinal cross-section (except at the ends) have a very undesirable effect on the resistance, as is clear from body d, whose resistance is 2-3 times greater and which increases in almost strict proportion to the load or to the square of the speed (so the work of inertia is here much greater than the work of friction).

XI

SIGNIFICANCE OF RESISTANCE OF THE REAR

137. Some deny that the rear part of the body is significant in relation to the resistance to the wind. But the contribution from the rear is very great for long and smooth bodies. The following results illustrate the importance of the rear part for various bodies.

138. I used, among others, elongated bodies divided by a central cross-section into two equal parts. For instance, I used half an ellipsoid of rotation and half a body obtained by rotating a part of a circle around its chord. I placed the bodies in the usual way along the flow, but alternately with the blunt and sharp ends facing into the wind (Figs. 3 and 4).

139.

Load	1/2	1	2	4	8 lb
Half of ellipsoid, convex side to wind, length 11 cm.....	7.5	14	28	55	106
the same, but blunt end to wind.....	30	60	120	238	-
Half of body obtained by rotation of an arc, with sharp end to wind, length 21 cm.....	-	13	26	49	88
the same, but blunt end to wind.....	19	40	81	164	-

140. Comparison of the results in the second line with the resistance of a complete ellipsoid given in (133) shows that the rear part reduces the resistance of an ellipsoid by only about 1/3.

141. The third line in (139) shows that the force is less on a flat plate.

142. Comparison of the forces given in the third line with those for the complete bodies in (126) shows that the rear part here halves the resistance.

143. Finally, comparison of the results in the fourth line with the forces on a flat plate having the area of the central cross-section for the complete body shows that a rear part of that extension reduces the resistance of the plate. From all this we see that we should not neglect the rear part.

I also report here some tests which, although of interest in themselves, also point to the importance of the rear part. I took half an ellipsoid of rotation (Figs. 3 and 4) and half a body (length 42 cm) obtained by rotation of an arc of a circle, i.e., the halves used in the tests of (139). These halves I joined by their planar (and equal) edges to give a body somewhat resembling Krebs and Renard's aerostat (Fig. 5), from which it differs only in having half the aspect ratio.

144. The following table gives the resistances of this body with its two ends turned to the wind.

Load	1/2	1	2	4	8	16
Blunt end to wind	3.5	7	14	25	45	85
Sharp end to wind	6	12	24	46	84	134

145. The most obvious point is that the resistance when the rear is sharp-ended is almost half of that when it is blunt-ended.

146. Comparison of line 2 with the resistance of the complete ellipsoid (139, line 2) shows that the rear halves the resistance at low speeds and that the value of the rear increases with the speed.

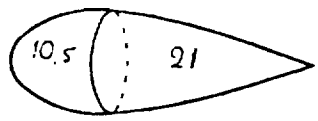

Fig. 5

147. The body of (143) had a length of 32 cm; comparison with a body of the same length and aspect ratio (Table 126) shows that the resistance of the composite body is somewhat less than that of the simple one of (126); but the resistance of the more symmetrical body (with conical ends) becomes less considerable even at speeds of 3-4 m/sec.

148. I deduce that it is best to make an aerostat symmetrical, with the largest cross-section in the middle.

XII

RESISTANCES OF HALF CYLINDER, HEMISPHERE, CONICAL SURFACE, AND SCHWARTZ AEROSTAT

149. The resistance of these bodies is almost exactly proportional to the load (or square of the speed), so I give only the resistance for the speed produced by a load of 2 lb.

150. The resistance of a half cylinder was 23 mm when the axis was normal to the flow and the convex surface faced the wind. The force on the projection was 34 mm (area of projection 32 x 8), so the resistance coefficient is 0.67.

151. The resistance was 43 mm with the cylinder facing the other way (nearly twice as great); the force on the projection was 34 mm, which is less than for the half-cylinder, so the resistance coefficient was 1.26. The resistance of the complete cylinder with the ends open (clear tube) was somewhat larger than that with the ends closed; the same applied to the half-cylinder.

152. The resistance of an open cone with its point facing the wind was 55 mm, or 51 mm with the base closed with a circle. The height of this cone, as also that of the double one used in (133), was 10 cm, the cross-sectional area being 80 cm^2. Comparison with the resistance of the double cone of 133 shows that the latter, in spite of the rear part, gave a resistance somewhat higher than that of an open conical surface.

153. The same conical surface, but with the base facing the wind, gave a resistance of 128 mm with the base open and 108 mm with it closed. The force on the projection was 104 mm, i.e., somewhat less.

154. The following table gives results for short open cones; the first line is the height in mm, the diameter of the base in each case being slightly over 7 cm.

Height, mm	0	11	16	20	23	26	31	35
Base to wind	60	63	64	65	66	67	68	68
Point to wind	60	50	48	45	43	42	39	37

155. I examined whether the resistance of a set of cones is proportional to the number of them if they are placed along the flow. I made 5 equal (double) cones and mounted them in different numbers on a wire with their vertices in contact, which gave the following results:

No. of cones	1	2	3	4	5
Resistance	4	8.5	11.5	16	19

The resistance is thus almost proportional to the number of cones, which shows that a train of aerostats would scarcely move with a speed greater than that of each aerostat separately. In this case the load was 4 lb. The length of a double cone was 14 cm, and the base diameter was 3 1/3 cm.

156. The resistance of a hemispherical surface with its convex side facing the wind was 31 mm, or 32 mm when the rear was closed with a circle. With the base facing the wind the results were 99 mm (open) and 94 mm (closed), so the resistance is increased by a factor 3.2 when a hemisphere is turned to make its base face the wind. Comparison of the least resistance with that of a sphere of the same diameter from (124) shows that the resistance of an open hemisphere is less by a factor 1.13.

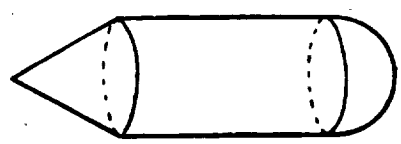

Fig. 6

157. The resistance was 20 mm when the hemisphere had its base parallel to the wind, no matter whether the base was closed or not. This resistance is more than half of the resistance of a complete sphere of the same diameter.

158. Schwarz's aluminum aerostat (Figs. 6 and 7) was tested near Berlin in 1897; the front end had the form of a hemisphere and the rear end that of a circular cone of height 10 m. The rear cone was joined to the front end by a cylinder 12 m in diameter and 24 m long. To determine the resistance of such a body I made a model of it having an area of the circular cross-section of 63 cm^2.

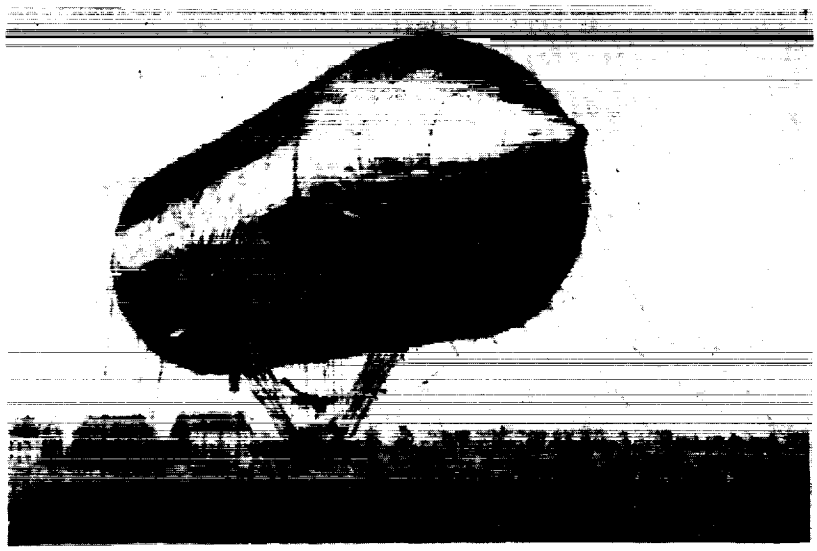

Fig. 7

159. The results were as follows:

Load, lb	2	4	8
Sphere facing wind	26	50	98
Cone facing wind	28	54	107

The resistance is thus almost proportional to the square of the speed.

Further, we see that the resistance with the sphere at the front was slightly less than that with the cone at the front. The force on the projection (load 8 lb) was 328 mm, which means that the resistance coefficient under the most favorable conditions was 98:328 = 0.30, the shape factor thus being 3.34, which is a little more than for a spherical surface and much below the values for elongated bodies of simple shape (Table 130).

160. To conclude I give results obtained with a surface made by joining a sphere to a cone tangential to it.

The length of the generator of the cone was 12 cm; the area of a great circle of the sphere was 63 cm^2 (diameter about 8.9 cm). Loads of 2, 4, and 8 lb were used, the height of the cone being parallel to the flow. The results were as follows:

Load, lb	2	4	8
Sphere facing wind	28	51	90
Cone facing wind	28	55	112

The resistance is thus less when the sphere faces the wind for speeds above 1 m/sec. The largest shape factor was 3.65 and the least 2.93. The least resistance coefficient was 0.27, as against 0.43 for a sphere. The cone at the rear therefore reduces the resistance of the sphere considerably.

CONCLUSIONS

161. Langley's formula gives the best results for the force on an inclined square plate; Lord Rayleigh's formula is also applicable,

but none of the formulas predicts the observed increase in the normal pressure at medium inclinations of the plate to the wind.

162. Both of the above formulas can be used to solve problems relating to the aeroplane (by which is meant a flying machine resembling a bird or insect).

163. The resistance of a body not elongated along the flow direction (or rounded) is proportional to the square of the flow speed. The same may be said for somewhat elongated bodies if rough corners occur in the middle (double cone).

164. There is no formula that gives the wind force on an inclined and elongated plate; the greater the elongation and the inclination to the wind, the less suitable the known formulas for the resistance of a plate. The force on a long plate lying along the flow is considerably less than that they predict, while it is larger than they predict when the plate lies across the flow.

165. The absolute frictional force (in kg) on one side of a rectangle of width h and length L when the motion lies along L and has a speed V is $T = 0.0004423hV^{1.6}L^{0.63}$ and so is not proportional to the square of the speed or to the length L of the rectangle.

166. The frictional force per unit area of a rectangle is

$$T_1 = 0.0004423 \cdot \frac{V^{1.6}}{L^{0.37}}.$$

167. The frictional coefficient is

$$\frac{0.00623}{V^{0.4} \cdot L^{0.37}},$$

and hence decreases almost equally (index 0.4) with increase in speed

and in length of surface along the flow direction (L and V are in m and m/sec respectively).

168. In order to render the laws of friction given in my recent paper (Vestnik opytnoy fiziki No. 259) fully correct, the length of the frictional surface along the flow direction must satisfy $L = 0.0646V^{1.62}$ (must increase with the speed). The friction in relation to the dimensions of my aerostats is incomparably less than that I assumed in my paper. For instance, for an aerostat 200 m long and moving at 12 m/sec the friction is 4 times less than that I previously assumed (Tsiolkovskiy, "A maneuverable iron aerostat").

169. The greater the surface area, the less the elongation, and the more rapid the motion, the more we may reasonably neglect the frictional force relative to the resistance from inertia; but for maneuverable aerostats the frictional force cannot be neglected.

170. None of the formulas for the resistance of a plate can be used for the analytic determination of the resistance of a curved or polyhedral surface, for the results of such calculations are in gross conflict with experiment. Any agreement is purely accidental. For instance, the incorrect formula given by Newton actually gives results closer to the true ones than does the correct formula given by Langley.

171. I am ready to repeat any of the experiments described in this paper for anyone who desires this.

172. The apparatus I have constructed is so cheap, convenient, and simple as to be able to solve rapidly problems insoluble from theory, and it should be considered a necessary requirement for any university or physical laboratory.

Many experiments I have not described here have been done in 2-3 min with this apparatus.

173. The resistance of a smooth elongated body (one resembling a ship) does not increase as rapidly as the square of the speed.

174. The air resistance is made up of two components: air friction and air inertia. Subtraction of the first shows that the inertial resistance also increases less rapidly than as the square of the speed, as is readily seen for bodies of small elongation; this is explained by the elasticity of the air, which is rapidly compressed along the sides at the nose part of the body and which also expands rapidly to press on the rear part and thus to accelerate it (or to

return part of the work done by the body).

175. The inertial resistance is inversely proportional to the square of the aspect ratio (ratio of length to width) for low speeds and slightly elongated bodies or for high speeds and highly elongated ones.

176. If the speed increases in proportion to the elongation, the lateral compression of the air will be roughly constant, and hence two laws governing the inertial resistance will be obeyed approximately: the inertial resistance is a) proportional to the square of the speed and b) inversely proportional to the square of the aspect ratio.

177. The contribution from the rear sometimes increases the resistance and sometimes leaves it unaffected, but mostly reduces it.

The rear part is of great importance for bodies having the shape of ships or for maneuverable aerostats.

178. To conclude, I must express my regret that my experiments are not so accurate, extensive, and numerous as to allow the deduction of empirical formulas for the resistance of elongated bodies. I have already pointed out the limited value of the friction formulas given here.

How important it is to formulate the laws of friction and resistance as accurately as possible! What vast use they have in the theory of the aerostat and aeroplane! In fact, there is no branch of science or technology in which the laws of resistance for an elastic medium are not important. This is why we ardently wish these laws to be established and to facilitate, so far as we are able, the experiments needed for this purpose.

K. E. TSIOLKOVSKIY'S REPORT TO THE RUSSIAN ACADEMY OF SCIENCES ON HIS EXPERIMENTS ON THE RESISTANCE OF AIR PERFORMED UNDER THE AUSPICES OF THE ACADEMY IN 1900-1901*

I have already been engaged in experiments on the resistance of air for 15 years or more. Nine years ago I applied to the Academy of Sciences with a broad program of new experiments on the resistance of air and a request for material aid in carrying out these experiments.

The Academy, in the person of M. A. Rykachev, received my proposals very sympathetically and decided to grant me financial assistance in pursuing my program of research.

In 1901 I sent the Academy a report on the first part of this program.

Academician M. A. Rykachev scrutinized my work very closely and in the margins pencilled numerous, mostly pertinent notes, which I gratefully utilized in correcting my report. Moreover, the esteemed Academician expressed some general comments on my work, which were later printed in one of the Academy's publications. These general comments and certain notes in the margins of my manuscript revealed to me that M. A. Rykachev had formed an exaggerated opinion of the inaccuracy of certain of my experiments.

My report did not mention the humidity, time, temperature and barometric pressure corresponding to the experiments.

Academician Rykachev said that, in fact, my experiments could be independent of density of the air, only if it were assumed that, other things being equal, the resistance of the air is proportional to the square of the flow velocity.

The majority of my experiments were performed precisely when the wind pressure could be assumed proportional to the square of its velocity. Thus, the greater part of my experiments did not depend on the temperature and the atmospheric pressure, and hence measurements of this kind would have been quite superfluous.

But, says Mr. Rykachev, in certain experiments the author

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*Foreward to "Report to the Academy of Sciences on Experiments on the Resistance of Air," written by K. E. Tsiolkovskiy on sending this manuscript to N. Ye. Zhukovskiy, 21 March 1908. Published for the first time, in slightly abridged form.

(that is, I) assumes that the coefficients depend on the flow velocity, this velocity depends on the density of the air, hence during the experiments it is necessary to determine the temperature and the atmospheric pressure.

Qualitatively speaking, the Academician is quite right, but quantitatively he is not.

In fact, the second group of experiments relates not to the velocity cited (1, 2, 3, 4 meters per second), corresponding to an air density 0.0012 times the density of water, but to another velocity corresponding to the various temperatures and pressures of the air during the different experiments.

If the density of the air differed much from 0.0012, I did not perform any experiments at all; I admit, however, that I did depart from this rule to carry out experiments at an air density of 0.0011. Just how should the corresponding flow velocity vary in these circumstances? It is easy to compute that when the air density decreases by 1/12 the original value, the flow velocity must increase by 1/24 of the original velocity. However, as I have remarked, the coefficients, though they do not depend on the velocity, do so only very slightly (in most of my experiments this dependence was not observable at all, although, theoretically, it must exist). And therefore, if we assume that the variation in the air density is a small quantity of the first order, the relative variation in flow velocity will be twice as small, and the variation in the drag coefficient will be a small quantity of the second order. When the resistance is proportional to the square of the velocity, the drag coefficient is expressed by a straight line parallel to the x-axis. Otherwise it is expressed by a curve slightly inclined to the x-axis.

The error, pointed out by the Academician, exists, but it is so insignificant that it is completely swallowed up in a variety of other, equally unavoidable errors.

I only regret that, owing to the misestimation by the esteemed Academician, this theoretical error prevented me from publishing my work promptly in the transactions of the Academy. Of course, Mr. Ryakachev did not bring this about intentionally, and I remain very grateful to him and to the Academy for proffering me their material and moral support.

A very brief report on the first half of my work, without any drawings or description of the apparatus, was published in "Nauchnoye Obozreniye"*.

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* Reproduced in this edition (see following article) - Ed.

But this was only a summary.

K. Tsiolkovskiy

1. The resistance of the air to the passage of various bodies can be demonstrated in various ways.

It is possible to drive a body through stationary air and determine the resistance of the counterflow. If the test bodies are not elongated in shape, it is possible to utilize circular motion (as in the majority of such experiments), otherwise rectilinear motion is unavoidable. Thus, rectilinear motion is required to determine the drag of long, hull-like forms.

This rectilinear motion may be vertical or horizontal. In either case large enclosed spaces are needed; systematic experiments employing this method, though very necessary, are extremely expensive.

In studies of the resistance of air it is customary to use circular motion; rectilinear motion is very rarely employed. Thus, Cailletet and Colardo used it to determine the pressure on a flat plate dropped from the Eiffel Tower.

It is simpler to expose a body to the wind and determine its resistance; but, in the first place, the wind is very irregular, both in direction and in velocity, and therefore requires special apparatus, and, in the second place, its velocity is not easy to determine.

However, all these methods, being irreplaceable under certain conditions, may be regarded as invaluable tools of science*.

2. In my latest experiments (1900-1901) I used a large bladed blower, rather like a winnowing machine.

I had already used a similar, but smaller machine in 1897-1898. I have since quadrupled the cross-sectional area of the flow.

My previous experiments were described in "Vestnik opytnoy fiziki," No. 269-272, 1898, and in a separate publication. Since the published drawings of the wind machine were grossly distorted, I now reproduce an accurate drawing of the blower I used in my experiments of 1897-1898 (Fig. 1).

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*Both methods are used in the Ryabushinskiy Aerodynamics Institute.

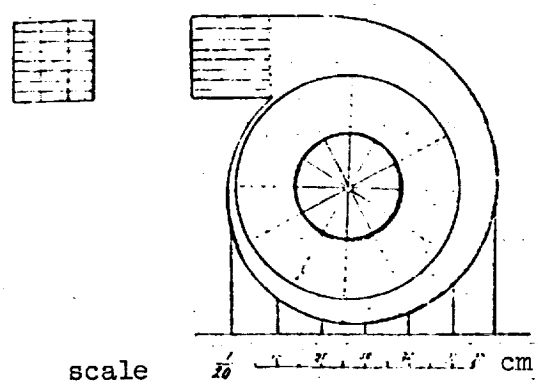

Fig. 1.

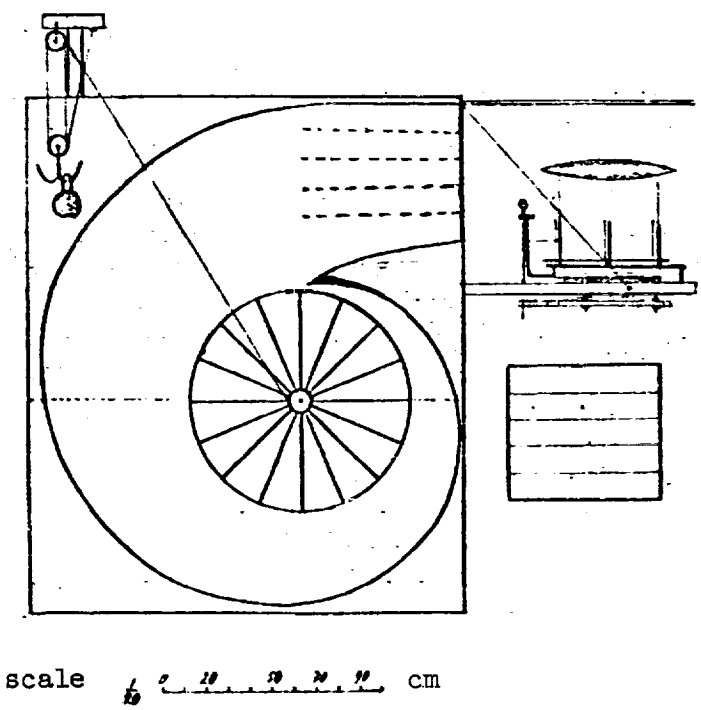

Fig. 2.

The two rectangles represent the cross-sectional area of the air flow. It is clear that the second machine is somewhat taller than the first, while the flow cross section is almost 4 times greater. I achieved this by rapidly widening the air channel around the bladed wheel.

The wheel is the same size in each machine: 1 meter in diameter. In the first blower, however, the circular lateral openings are small compared with the wheel, whereas in the second they are exactly the same size.

I recall that the old machine gave more constant readings than the new one, but in the latter the flow is much more uniform, that is to say that at a specific point in the flow the wind pressure was more constant in the old machine than in the new one, but in the latter this pressure is approximately the same in all parts of the flow.

The rather considerable oscillations of the indicator registering the wind pressure in the latest machine obliged me to devise a special measuring instrument capable of damping these oscillations.

3. The drawing shows a bladed wheel with 16 linen blades. At the periphery these are attached to two parallel and identical rims, and at the center to springs soldered to the axle. Accordingly, my blades are not spokes, although they are always moderately taut. There are simply cross braces at each circular opening to support the two wire rims. The axle of the wheel is made of iron and revolves on cones seated in conical recesses in the ends of the axle.

4. The side walls of the wind machine are made of wood and consist of two identical and parallel rectangles with circular openings. Between these walls a long piece of sheet metal, one arshine (71.12 cm) wide, is bent into an Archimedean spiral.

Thus, the interior of the wind machine forms a right cylinder, the base of which is bounded by Archimedean spirals.

When the blades revolve, the air is sucked into the circular openings, moves along the spiral, and is forced in a straight line through the rectangular mouth of the channel. The size and shape of this opening are indicated on the right of Fig. 2. The emerging flow of air has a slight upward tendency and, moreover, is considerably weaker at the bottom of the opening.

5. In order to overcome these shortcomings, I fitted the mouth of the machine with four <u>almost</u> horizontal linen baffles. These baffles, if sufficiently taut, can easily be inclined to the horizontal and clamped in this position. I also gave them an inclination relative to each other, until I obtained a flow of uniform intensity from each opening[15]. As a result of the almost complete

horizontality of the baffles, the flow, too, was horizontal.

The five openings and the baffles are indicated by dotted lines in Fig. 2. I have also shown their inclination.

6. The bladed wheel is turned with the aid of a four-block pulley. On the right, in front of the mouth of the machine, I have shown the measuring apparatus: the drag of a smooth, elongated shape is being determined.

The air flow is 71 cm wide and 62.4 cm high; accordingly, the cross-sectional area of the flow is 443 sq. cm, or 0.44 sq. m. All the dimensions of the machine can readily be estimated from the scale at the foot of the drawing.

7. Figures 3 and 4 show the measuring apparatus. The former is an elevation (view from the side), the latter a plan view (from above).

Fig. 4 shows how the tank Я filled with water and the indicating needle are displaced in the longitudinal and transverse directions. The frame PP carries the tank Я in the longitudinal direction. In this same frame, when stationary, the measuring apparatus or tank Я can move in the transverse direction. The test shape (Ф, Fig. 3), the drag of which is being measured, moves together with the tank. The shape can be raised or lowered with the aid of wire supports n of different lengths (Fig. 3).

Thus, any shape can be placed in any part of the flow, my intention being to make possible the investigation of its various zones.

8. The water-filled tin tank (see Figs. 3 and 4) is in the shape of a rectangular parallelepiped, except that the rear portion is sharply dropped. Inside floats another similar tank slightly smaller in size and lacking a dropped section (indicated by dotted lines in Fig. 3). To the rear end of this floating tank there is attached a brake m in the form of a tin plate. The dropped part of the larger tank is also intended to permit the free movement of this brake through the water. The object of the brake (the size may vary) is to slow the longitudinal movement of the floating tank and the indicator c connected with it.

In the old machine (Fig. 1) I managed without a brake, from which I conclude that the flow was more uniform. If greater resources and more space had been available, I could have achieved the same uniformity with the new machine, by making the spiral of Archimedes less steep. The less the value of the coefficient a in the equation of the spiral of Archimedes in polar coordinates ($r = a_{\varphi}$), the more uniform the flow, but at the same time the greater the width and height of the machine, even though the

dimensions of the air flow remain the same.

9. To the bottom of the inner floating tank I soldered three iron rods and over these I fitted three flattened tin tubes a.

The tubes pass through a wooden rod k, k or ruler. The ruler itself is attached to two stands, also soldered to the bottom of the floating tank. The lower tank containing the water is covered with two small plates, between which a longitudinal slot runs the whole length of the tank. The tubes a also pass freely through this slot. As they move with the floating tank, they do not rub against or even touch the edges of the slot, thanks to the rod k, k, which rolls between light wheels K, K turning on needle bearings. Since these bearings are not easy to mount perfectly vertically, they must not only pass through the centers of the discs but also through their centers of gravity, otherwise the discs will revolve under the influence of their own weight and move the rod in one direction or the other.

10. The indicating needle or pendulum c, which also rotates on similar, but horizontal bearings, is connected with the nearest tube a by means of a light tie.

When the air flow is directed against the apparatus, the wind exerts pressure on the shape and displaces the floating tank together with the supports (n, a). By means of the horizontal tie (about 0.3 mm thick), this tank deflects the needle c from the vertical, this deflection being the greater the higher the drag of the shape Φ. The deflection can be read off from a scale M, consisting of a ruler graduated in millimeters, mounted horizontally at the foot of the measuring apparatus.

I generally disregard the permanent tubular supports a and their rods. Both these and the needle c are screened from the air flow by plates the length of a finger and 1-1/2 to 2 cm wide.

In Fig. 3 the lower boundary of the air flow is denoted by the broken line, so the greater part of the measuring apparatus is not exposed to the wind. In spite of this, I also used a wide screen ш for the greater protection of the wheels K, K and the measuring apparatus as a whole.

However, this screen, which moves together with the tank, does not extend into the flow and, in fact, barely reaches it (extends to the broken line in Fig. 3).

11. Thanks to these screens, in determining the drag of the test shape it is only necessary to take into consideration the resistance of the upper supports resting in the tubular stands.

These cylindrical supports are normally slightly more than 1 mm thick. If the upper wire supports are removed and the air flow started

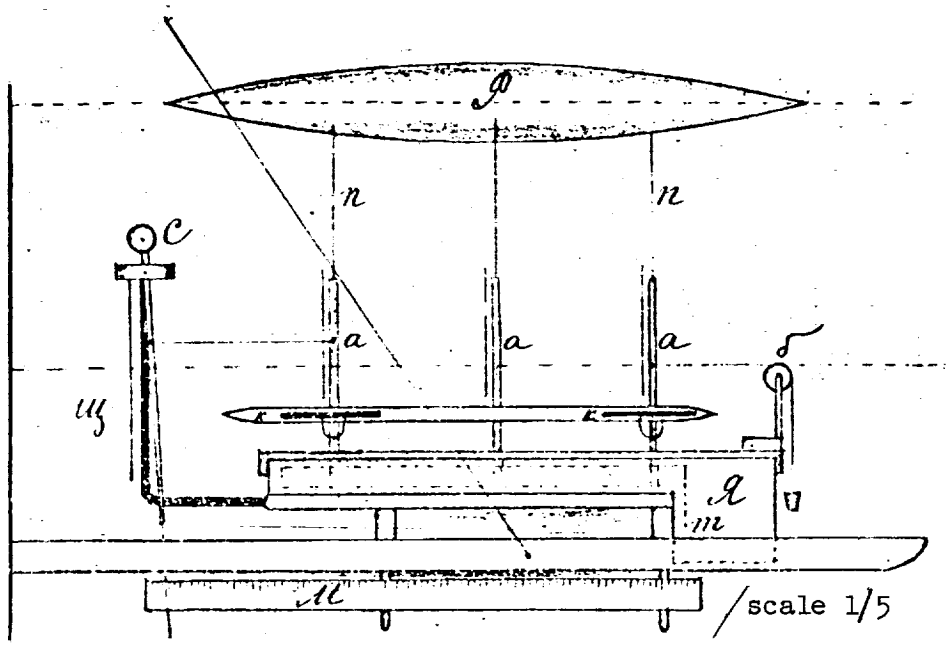

Fig. 3.

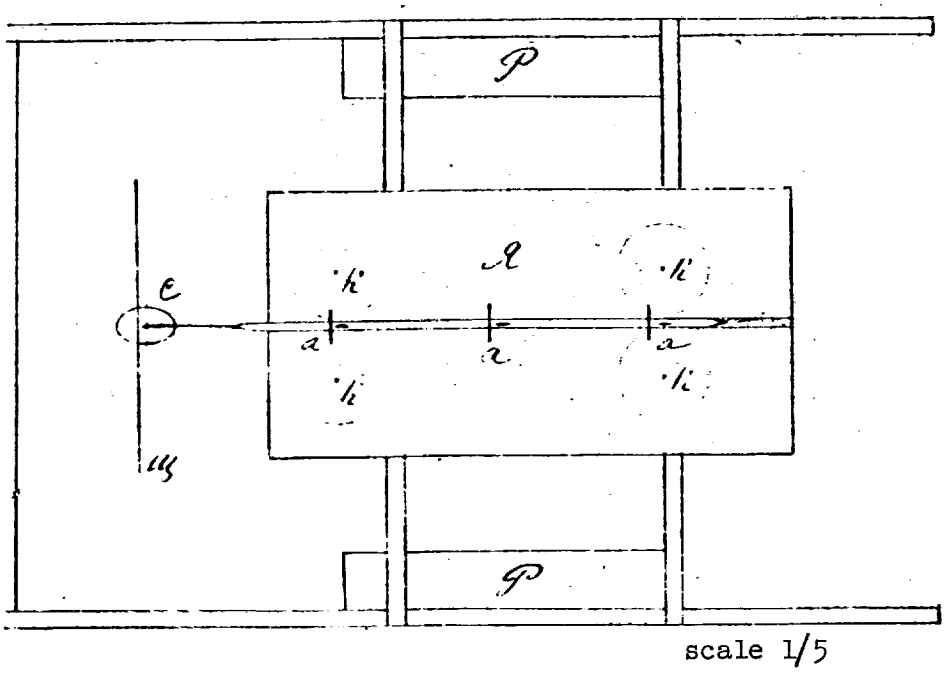

Fig. 4.

up, the needle remains motionless, pointing to zero. When the needle deflects sharply, owing to the pressure on the test shape, the supports are some distance from the screens and apparently less protected, but experiments show that even when the screens are furthest away they are still effective.

I conducted these experiments by removing the screens from the left.

When the wind machine is in action, the needle still points to zero, even when they are removed.

12. In view of the considerable length of the horizontal tie a, which pulls on the needle, and of the needle itself, and, furthermore, in view of the small deflection from the vertical, which does not exceed 100 mm on the scale M (see Fig. 3), I assumed that the wind pressure was proportional to the linear deflection of the needle from the vertical.

13. In order to test this theoretical conclusion in practice and to reduce the readings of the needle to a definite value, I used a light paper pulley block revolving on horizontal bearings (Fig. 3,b)

Over this block I passed a light thread, one end of which I attached to a support a, so that it assumed a horizontal position. From the other end of the thread I suspended a light cardboard box. By virtue of its own weight (plus the weight of the thread) this box caused the floating tank to move to the right, pulling the needle c out of the vertical. Let us assume that needle was deflected through 5 mm. I then placed a gram weight in the box and observed how much the needle deflected from the zero mark. Let us suppose that it deflected 105 mm. Then a gram weight is enough to deflect the needle c through 100 mm, and a reading of 1 mm thus corresponds to a force of 1/100 gram (about 10 dynes).

In this way I confirmed that the readings of the needle were proportional to the intensity of the load, and hence to the intensity of the wind pressure.

14. In order to measure forces of different magnitude I prepared 4 needles of different weight. Details are given in the table: the rows of figures represent (reading from top to bottom) the number of the needle, beginning with the lightest, the length of the arm of the needle, to which the horizontal tie is attached, the deflection of the needle in mm due to the weight of the box and thread b and, of course the weight of the needle and the deviation of its center of gravity from the axis of rotation, the deflection of the needle due to the combined action of the box and the load (see last row of table; this load varied), the deflection in mm due

TABLE

No. of needle	1	2	3	4
Arm (cm)	10	8	8	8
Deflection due to weight of box (mm)	23.5	12	5.7	1.5
Deflection due to weight of box and load (mm)	123.5	112	105.7	101.5
Deflection due to load (mm)	100	100	100	100
Load (grams)	0.5	1.0	2.0	8.0

to the action of the load alone, and, finally, the load itself in grams.

15. From the table it is clear that a deflection of 1 mm corresponds, respectively, to forces of 1/200, 1/100, 1/50, and 1/25 grams.

Knowing the deflection and the number of the needle, it is always possible to compute the intensity of the wind pressure corresponding to this deflection.

The needles had small attachments that could be changed or displaced vertically in order to round off the readings to accord with the above table.

16. In my latest experiments I express all the forces in terms of the readings of needle No. 2 in mm; the deflection of this needle through n mm corresponds to a force of n/100 grams.

Thus, if I use the first needle (No. 1) and get 28 mm, I must divide this figure by two. If needle No. 4 gives me 20 mm, I must multiply this figure by eight, that is, I get the reading that would have been given by needle No. 2, assuming that its readings were unlimited.

17. Thus, the readings of the first needle must be divided by two, those of the second remain unchanged, those of the third must be multiplied by two, and those of the fourth by eight.

As the weights (Fig. 2) are lowered to the floor, the bladed wheel makes 33 complete revolutions. Apart from the weights, the hook carries a special supplementary load. I varied this until the wheel began to rotate under the slightest additional weight (for example, 1/4 pound). On being set very slightly in motion, the wheel comes to rest very slowly, even in the absence of a variable load, if the supplementary (constant, fixed) load is properly chosen.

18. I shall make no further reference to the supplementary load; all my remarks will relate to the variable load of 1 to 130 pounds.

During the first three revolutions the wheel gradually accelerates, but starting from the fourth the time required for a complete revolution is constant for a given load.

At the side of the wheel I arranged a bell, which rings briefly at the beginning and end of each revolution. Observing my stopwatch, I called out the time to an assistant whenever the bell rang. From these figures it was easy to confirm the statement made above.

19. In order to determine the time taken by the cord wound

onto the central shaft to unwind, I considered only uniform rotation of the wheel, i.e., the time taken to perform the last 30 revolutions (33-3). For this purpose, I subtracted the time taken by the first three revolutions from the total duration of the 33 revolutions. I obtained the following table: (see /131).

The first line shows the weight of load in pounds (not including the supplementary, constant load); the second line, the time of the first three (non-uniform) revolutions; the third, the time of 33 revolutions; and the fourth, the time of 30 uniform revolutions.

20. From the table it is clear that the unwinding time is inversely proportional to the square root of the effective load. For example, when the load is made 4 times greater the unwinding time is $\sqrt{4}$ times or half as great.

The peripheral velocity (or angular velocity) of the wheel is proportional to the square root of the load. Thus, when the load is increased by 16 times, the angular velocity of the wheel increases by $\sqrt{16}$ or 4 times.

21. Hence we may deduce, with almost complete certainty, that in my experiments the velocity of the air flow is also proportional to the square root of the load. (Assuming, of course, that the density of the air is constant.)

22. The models and apparatus arranged in the flow have no perceptible effect on the speed of the wheel and, in all probability, on the flow velocity itself, as may be seen from the following experiment.

23. For a rough investigation of the flow in the wind machine and two reduced models of it, I set up a series of horizontal vanes (Fig. 5) and also used thin threads suspended from an horizontal rod (Fig. 6).

The first apparatus indicated the intensity of the flow pressure simultaneously at different points in the flow*. The second apparatus (Fig. 6) showed that only in the center part of the flow is its direction perpendicular to the plane of the mouth of the blower.

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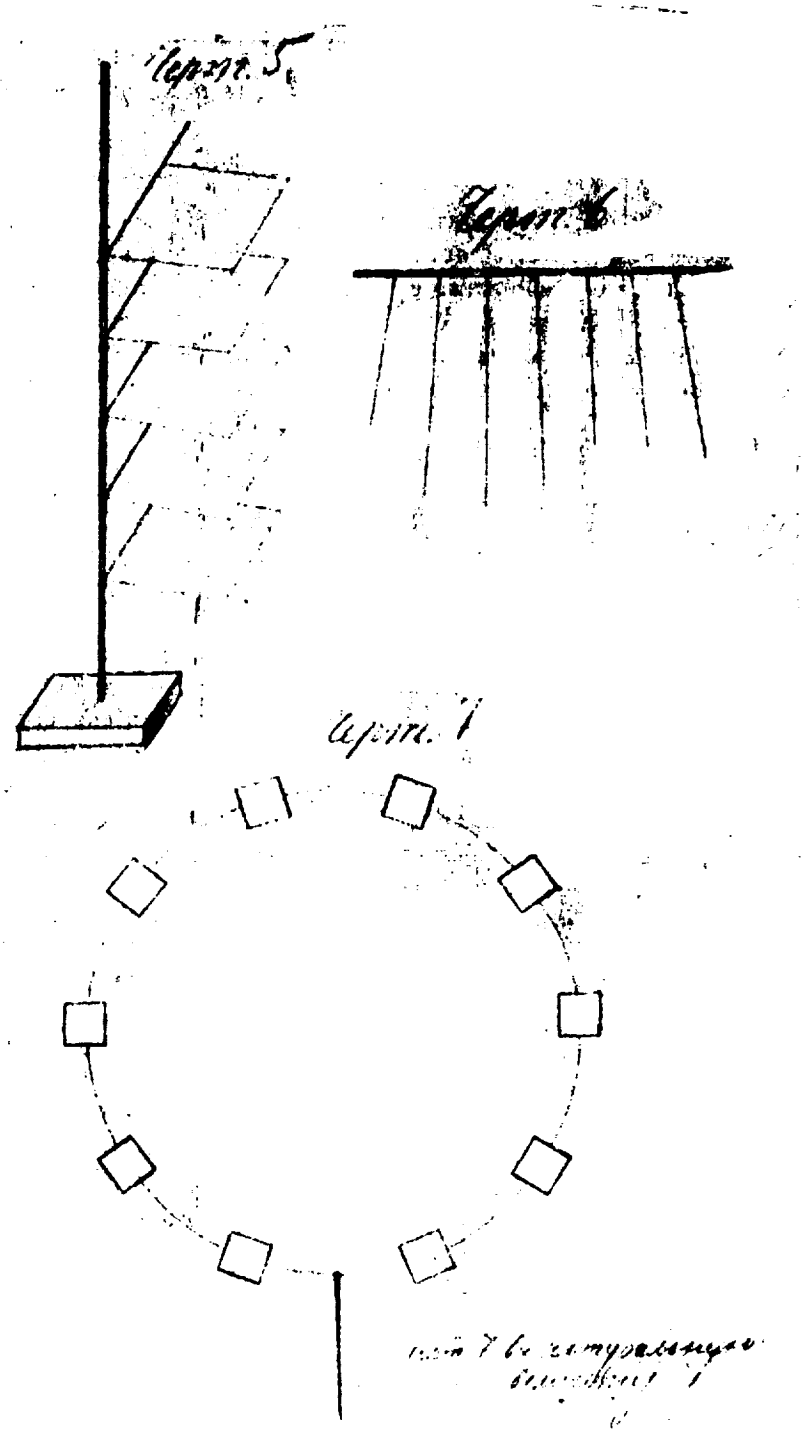
*In fact, the more intense the air flow the greater the deflection of the surface of the vane from the vertical.

Load	1	4	16	32	64 pounds
Seconds	45	19	9	6	$4^1/_2$
"	266	129	64	46	32
"	221	110	55	40	$27^1/_2$



Figs. 5, 6 and 7.

In other parts of the flow it is directed somewhat toward the side, though evidently the deflection is the greater the smaller the load.

The horizontality of the flow may be determined with the aid of a light vane with the horizontal axis of rotation passing through its center of gravity.

Without a system of baffles (Fig. 2) at the mouth of the blower, there is an appreciable upward deflection of the flow. With the baffles it is almost horizontal.

24. For a given load the readings of the needle (c, Figs. 3 and 2) were not generally constant. The needle moved as the wheel revolved, and these fluctuations were the more considerable the greater the measured drag and the more intense the air flow. The greatest fluctuations were registered in measuring the pressure on a flat plate. Only at small deflections and low flow velocities was the needle almost stationary. I observed the needle throughout the time taken by the cord to unwind; an assistant recorded my readings. I made my observations at equal intervals of time (about 2 seconds) with the aid of a metronome, which ticked every $1^7/_8$ seconds.

25. The air flow ordinarily became steady after 5-10 revolutions of the wheel, and it was from this point that the readings were recorded. After the experiment I divided the sum of the readings by the total number of observations, that is, I took the average as the true value.

26. The average readings (sometimes derived from 40 observations) were, for the most part, found to be in agreement. However, this was not always and not completely true. Often I carried out two identical experiments and only if the average readings were sufficiently close did I accept one of them or their arithmetic mean.

Altogether, my experiments involved several tens of thousands of readings.

27. I investigated the flow: in the longitudinal direction, along the flow axis (center-line of flow), and in the transverse horizontal and transverse vertical directions.

My investigation of the flow in the longitudinal direction gave the following results.

In general, these investigations always gave favorable results with respect to the uniformity of the flow.

28. Investigations in the horizontal transverse direction, 15 cm to the right and left of the flow axis, gave values of 34.1 and 35.9, which is somewhat higher than the previous figure. In

general, numerous experiments of this type showed that the pressure increases somewhat in the direction of the edges of a rectangular flow and only falls again at the edges themselves.

Distance from mouth of blower in cm	40	60	80	105
Resistance of 36 sq. cm. square plate and support	34.9	34.7	34.7	34.3

29. In order to investigate the flow in the vertical direction, I took a long wire stand supporting a square that could be moved up and down. This square I arranged to coincide with the five openings of the blower mouth in succession (Fig. 2). Starting with the upper horizontal opening, I obtained the following values:

33 33.5 33.5 33 33.5

I obtained this favorable result by deliberately varying the inclination of the baffles. The plate was 40 cm from the mouth of the blower. I noted that when the plate was arranged not opposite the center of a particular opening but opposite the linen baffle the pressure recorded was somewhat less. However, the majority of the test models were smaller than the openings and were unaffected by the baffles.

30. In my apparatus the force exerted on the plate in no way depends on the temperature and pressure of the air (I refer to the barometric pressure). Actually, the density of air does vary with the temperature and barometric pressure, but even if the density varies there is no change in the wind pressure for a given load.

Let us suppose, for example, that the density of the air in-

creases 4 times; then as a result of the greater resistance the bladed wheel will rotate twice as slowly; hence the flow velocity will be halved. For this reason the pressure on the plate will be 4 times less (proportional to the square of the flow velocity), but it must also be 4 times greater owing to the assumed increase in the density of the air. Thus, generally speaking, the pressure remains as before, in spite of the enormous change in the density of the air, which may also be caused by an arbitrary change in temperature and barometric pressure.

31. The same considerations apply to other surfaces and shapes. However, the law stating that the resistance is proportional to the square of the flow velocity can only be regarded as approximately true. But, bearing in mind the quite insignificant variation in the density of the air in practice, we may assume that in my apparatus, in general, the drag of any shape is independent of the temperature and the barometric pressure.

Thus, we may further assume that all my experiments were conducted at normal barometric pressure and normal temperature.

Nevertheless, I took care to perform my experiments at approximately the same air density, slightly less than 0.0012 the density of water.

32-34. My experiments have led me to the conclusion that the pressure on a given plate arranged normal to the direction of the flow is proportional to the load. The load varied between 5 and 130 pounds. From these experiments I deduced that at a load of 10 pounds the force exerted on a square measuring 100 sq. cm. is on the average 0.92 grams.

35. Of course, the resistance of the supports was deducted.

The supports were wires 1.15 mm thick, all of a length such that the center of the square coincided with the center-line of the flow, an arrangement which I always preserved in connection with the other test shapes.

In the upper parts of the flow, along the edges, I mounted two horizontal rulers, each more than a meter long; over these a horizontal transverse ruler was free to move, and from this was suspended a vertical ruler graduated in centimeters.

The vertical ruler could also be moved, so that it could be placed in any part of the flow and thus used (without a cathetometer) to verify the position of the model.

36-37. We have seen that the flow velocity is proportional to the square root of the load (21), and since the force exerted on a square plate is proportional to the load (33), we can also say that it is proportional to the square of the flow velocity.

38. From the results of the experiments I further conclude that the force exerted on a square plate is proportional to its area. In my experiments this area varied between 1 sq. cm and 400 sq. cm.

39. In order to determine the force exerted on a square with sides 1 cm long, I first determined the force on 10 such squares threaded on a very fine wire ring (Fig. 7), the resistance of which was naturally deducted.

However, the mean pressure on larger squares proved to be somewhat less, partly due to the effect of the baffles, partly due to the small load* required and hence the low flow velocity, as a result of which the flow diverges more strongly toward the sides and loses strength more rapidly with distance. In the extreme case, the difference in pressure reached 3%.

40. Experiments with circular plates and other similar nonelongated shapes without openings showed that the same law is applicable to them too, i.e., their resistance is proportional to their area.

41. Experiments with a ring (Fig. 8) gave a value of 41, whereas the pressure on an equivalent square plate is 35. The pressure on a grill (Fig. 9) was found to be 47, whereas, judging by the area, it should have been 35.

I performed a very careful experiment with a grid of squares, shown in natural size in Fig. 9_1. A corresponding unperforated square had an area of 256 sq. cm. In an experiment on this square the pressure was found to be 0.711 per sq. cm (for a load of 8 pounds). The area of the grid, with the holes deducted, was 192 sq. cm. In this case the experimental pressure was 0.852 per sq. cm.

The ratio of the pressure on the grid to that on the continuous plate is 1.2, i.e., there is an increase in pressure of 20%. The area of the grid was determined before it was cut up, as in all such cases, and, accordingly, the accuracy of the measurements of area can be absolutely guaranteed.[16]

42. Thus, on the basis of my experiments, I can express the force F on a nonelongated continuous plate, mounted normal to the flow, by means of the equation:

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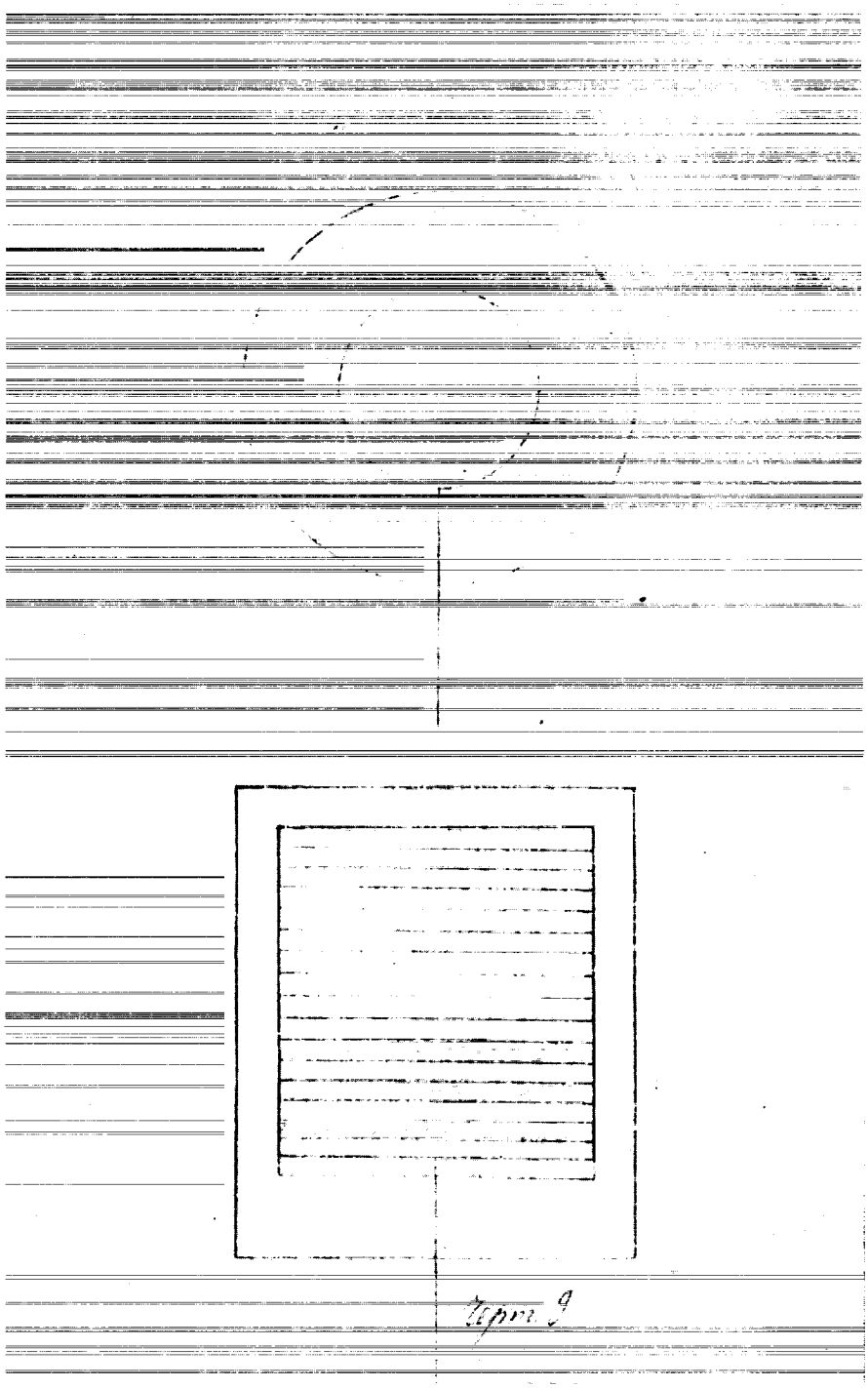
*At low flow velocities the needle oscillates less, giving a more accurate mean result.



Figs. 8 and 9.

$$F = A \cdot S \cdot v^2,$$

where A is a constant, v is the flow velocity, and S is the area of the plate.

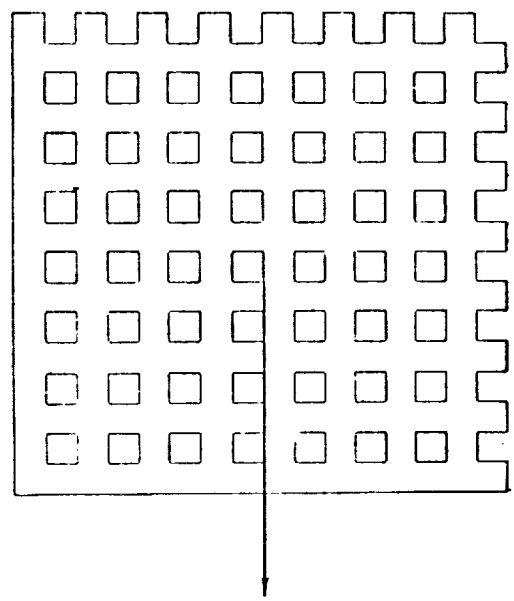


Fig. 9_1.

Here I can point to an analogy between the resistance of water and the resistance of air. In fact, the force exerted by water and air can be expressed by a single equation:

$$F = \psi \cdot \gamma \cdot S \cdot \frac{v^2}{2g},$$

where γ is the weight of 1 cu. meter of fluid (water or gas), S is the area of the plate normal to the flow, g is the acceleration of terrestrial gravity, v is the flow velocity, and ψ is a coefficient.

According to Dubois and Duchmin, ψ for water is equal to 1.43. For air, according to the experiments of Cailletet and Colardo, this value should be less, namely about 1.2; Langley gives a figure close to that of Dubois, namely 1.4. However, Dubois and Duchmin found 1.43 for a plate moving in stagnant water, while for running water (and a stationary plate), as might be expected they found another value, namely 1.86[22].

Sometimes the ψ for air is taken as a variable depending on the size of the square, but, as far as I know, this is not done for the resistance of water. In view of the constancy of this coefficient for water (irrespective of the area) and in view of my own experiments, in varying the area from 1 sq. cm to 400 sq. cm I was disposed, like other investigators, to assume that the coefficient ψ was constant for air too (at least for relatively small areas and low velocities).

Obviously, in my equation

$$A = \frac{\gamma \cdot \psi}{2g}.$$

43. The latest experiments of Cailletet and Colardo deserve particular attention. They determined A for plates dropped from the Eiffel Tower. For a temperature of 10°C and one new atmosphere (735 mm Hg; corresponding air density close to 0.0012 the density of water) they obtained:

$$A = 0.071,$$

i.e.,

$$F = 0.071 \cdot S \cdot v^2 \text{ kg}.$$

44. In my air flow, for a load of 10 pounds, the pressure per

100 sq. cm is 0.92 gram. Hence the pressure per sq. meter will be 0.092 kg. For a load of P pounds the pressure will be 0.0092 P.

From this equation and the previous one, assuming S = 1, we find: $0.0092 \cdot P = 0.071 \cdot v^2$, whence $v = \sqrt{0.12958\ P}$, or, approximately, $v = \sqrt{0.13 \cdot P}$; whence $P = 7.717 \cdot v^2$ or, approximately, $P = 7.72 \cdot v^2$.

47.[17] These equations enable us to compute the load producing a given flow velocity.

48. It is also easy to compute the velocity corresponding to a given number of pounds (see Table on next page).

50. Knowing that the diameter of the bladed wheel is 1 meter, from (19) we can determine the rim velocity of the wheel for any load. Thus, for a load of 1 pound it is 0.857 meters per second. On comparing this rim velocity with the flow velocity for the same load of 1 pound (48), we get: 0.361:0.428 = 0.84, i.e., the flow velocity is 0.84 times the rim velocity or the velocity of the parts of the blades furthest from the axle.

The loss of velocity is equal to 16% or about 1/6 of the velocity at points on the rim.

The figure for the earlier wind machine (Fig. 1) was almost the same (0.83) (see "Air Pressure").

53. For the experiments with elongated rectangles I set up a wire square with four pins soldered to it. To the pins I attached a frame consisting of four strips of cardboard (Fig. 10). The perimeter of the wire was 94 cm, the same length as the center-line of the frame.

54. The following table gives the results of these experiments*.

55. The first row gives the width of the rectangle in centimeters, the second the total area of the frame in square centimeters, the third the aspect ratio or the ratio of the length of one of the four rectangles composing the frame to its width, and the fourth the pressure in millimeters (1 mm = 1/100 g) per 100 sq. cm of the surface of the frame for a load of 10 pounds.

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*Of course, the proximity of the four rectangles could not fail to affect the results of the experiments.

P = 1	...	2	3	5	10	20	30	40 pounds	
v = 0.36	...	0.51	0.62	0.81	1.14	1.61	1.97	2.28 meters per second	
P = 50	...	60	70	80	90	100	110	120	130 pounds
v = 2.55	...	2.79	3.02	3.24	3.42	3.61	3.78	3.95	4.11 meters per second

Width, cm	1/2	1	$1^1/_2$	2	3	4	5	6	8	32
Area, cm^2	47	94	141	188	282	376	470	564	752	800
Aspect ratio	47	23.5	15.7	11.7	7.8	5.9	4.7	3.9	2.9	1.3
Pressure corresponding to a load of 10 lb., per 100 sq. cm	125	123	120	118	111	108	103	103	99	91

The relation between the aspect ratio and the resistance may be expressed by a curve with the aspect ratio plotted along the x-axis and the resistance along the y-axis.

As nearly as possible I drew a smooth line between the points thus obtained. On examining this curve, it occurred to me to express it by means of the simple equation:

$$Y = A + B \log_{10} X,$$

where A and B are constants, Y is the resistance, and X is the aspect ratio of the rectangle.

56. In order to determine the constants, I substituted in this equation two pairs of coordinates obtained experimentally, namely the resistance corresponding to the aspect ratios 23.5 and 1.3.

I then found

$$Y = 88.15 + 25.03 \log_{10} X.$$

Note that the last row of the table was derived from a double series of average observations and that in the experiment with the rectangle having an aspect ratio of 1.3 I, of course, used not a frame, but a single larger rectangle (covering the entire wire square).

The formula is not applicable to aspect ratios of more than 25. According to the formula, the resistance increases continuously, though very slowly, with increase in the aspect ratio; according to the experiments, however, it evidently tends to a known limit, so that whatever the aspect ratio the resistance need not be taken as greater than 125-130.

Even in this case, it is 40% greater than the pressure on an equivalent square plate. This increase can not be attributed to errors in the observations or inaccuracies in the apparatus. However, I would have been more doubtful of my conclusions, if I had not found them quite clearly confirmed by observations on the drag of wires and

and elongated cylinders in general.

Scale 1/2"

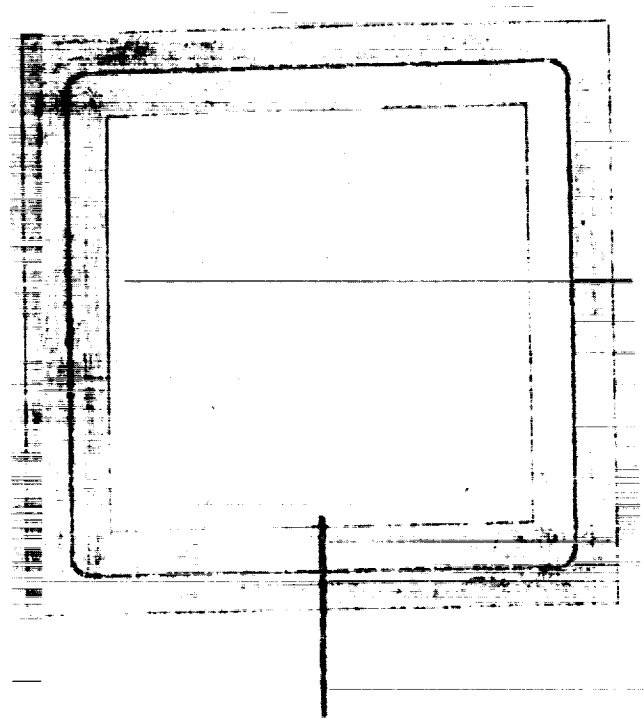

Fig. 10.

The formula also ceases to apply when the aspect ratio of the rectangle is small, less than 1.3, for example; then the pressure is determined in the same way as for a square.

59. It turns out that if several squares are joined to form a long rectangle, the pressure increases. This increase is the greater the closer together the squares. The pressure on a series of closely spaced squares is greater than that on a similar series of squares spaced further apart.

This fairly delicate experiment was conducted with the aid of the model illustrated in Fig. 12. The squares can be slid along the

wires, either toward or away from each other.

Scale 1/4"

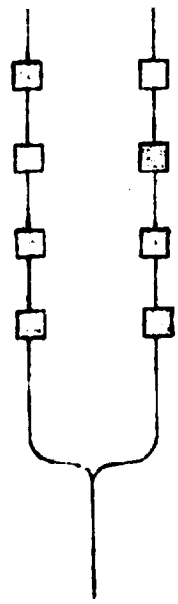

Fig. 12.

61. The question of the drag of various elongated solids of revolution is particularly important in relation to the design of controlled aerostats. Accordingly, though not exactly on the spot, I undertook to determine the drag of such bodies (Fig. 14). The drawing shows the greatest longitudinal cross section. Clearly, this cross section is defined by arcs of two equal circles. As usual, the center of each model was placed on the center-line of the flow or at its axis. The drag was determined with special care in view of its low value for bodies of this kind (cf. Figs. 2 and 3).

There follows a table giving the dimensions of the bodies.

Perimeter	Diameter	Cross-sectional area	Length	Aspect ratio
1	2	3	4	5
32.0	10.2	82.0	23.5	2.30
32.0	10.2	82.0	34.0	3.33
32.3	10.3	83.0	42.8	4.15
33.0	10.5	85.5	52.9	5.04
32.5	10.4	84.0	65.0	6.32
32.5	10.4	84.0	77.6	7.40

From left to right the columns show: the length of the perimeter of the average cross section, its diameter, its area, the length of the body, and its aspect ratio or the ratio of the length to the greatest width (diameter). The linear dimensions are given in centimeters, the areas in square centimeters.

Scale 1/5"

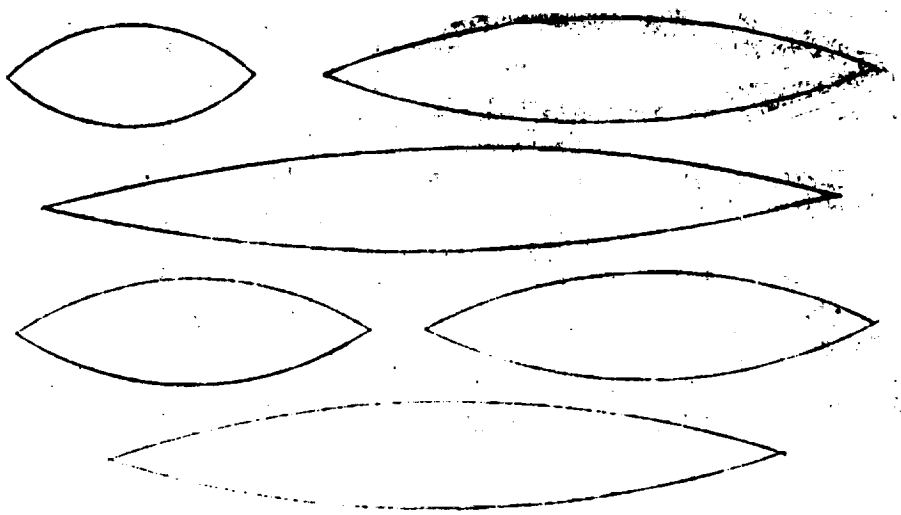

Fig. 14.

65. When the longitudinal axis of the bodies was arranged in the direction of flow, I obtained the drag table given below. In the table the forces are expressed in terms of the deflection of the needle, in millimeters per 100 sq. cm of the greatest cross section. I have divided the values obtained for the drag by the cross-sectional area from the table (64) and then multiplied by 100. The experiments were performed with different loads, as indicated in the first row of the table.

From any row of the table, i.e., for any body, we see that the drag is not proportional to the load or to the square of the flow velocity.

Load in pounds	10	20	30	40	50	60	70	80	90	100	110	120	130
Aspect ratio 2.30	21.1	37.1	45.6	54.3	60.5	69.0	75.1	80.0	86.5	94.0	97.3	105.5	108.8
3.33	12.2	21.0	29.3	37.0	43.0	48.0	53.7	60.0	64.3	71.3	75.9	79.6	83.2
4.15	13.1	23.1	31.1	38.0	46.0	52.3	65.8	57.1	71.9	75.1	84.8	90.2	95.2
5.04	14.6	23.6	32.5	41.3	50.0	58.9	68.9	72.5	83.8	88.3	96.4	107.0	112.0
6.31	14.2	23.0	33.1	43.0	55.2	62.0	72.0	80.0	91.0	101.0	107.6	116.8	125.2
7.40	16.0	29.0	42.4	53.8	65.6	77.8	89.3	100	111.8	121.1	132.7	143.3	157.6
Velocity in meters/second*	1.14	1.61	1.97	2.28	2.55	2.79	3.02	3.24	3.42	3.61	3.78	3.95	4.11

*It is understood that in the experiments the velocity depends on the density of the air or on its temperature and pressure.

Thus, when the load is raised from 10 to 100 pounds or multiplied by 10, the drag is not 10 times greater, in fact, the increase is much less, namely, for bodies with the aspect ratios given in the table,

4.5 5.9 5.8 6.7 7.1 and 7.6 times greater.

It is remarkable that for the bluntest body we get the best result. Thus, the drag of smooth solids of revolution is not at all subject to the same law as the drag of plates.

Let us now take any column of the table, corresponding to bodies with different aspect ratios but the same flow velocity. We see that as the aspect ratio of the body increases, the drag decreases only up to an aspect ratio of 3.33; as the aspect ratio rises further, the drag begins to increase again.

Thus, at any flow velocity, the minimum drag is displayed by a body with an aspect ratio close to 3.33[18].

81. On examining the curves of drag vs. load (Fig. 23), we see that they resemble the arc of an hyperbola, taken at some distance from the vertex. In fact, no equation expresses my experiments with elongated bodies as accurately as the equation of an hyperbola in rectangular coordinates, the origin of which lies on the curve at a certain distance from the vertex.

82. Assuming that the coordinate axes are parallel to the axes of symmetry of the hyperbola, we get an equation of the form:

$$y = \sqrt{Ax^2 + Bx + C} - D,$$

where the ordinate y expresses the drag and the abscissa x the load created by the wind; A, B, C, and D are constant positive numbers. In order to determine these constants, it is necessary to take four pairs of coordinates from the drag curves.

83. Solving the system of four equations for each aspect ratio, we get the following values of A, B, C, and D:

Constants

Aspect ratio	A	B	C	D
2.30	0.0910	99.0	137.0	11.7
3.33	0.0858	82.1	858.5	29.3
4.15	0.22635	80.3	709.67	26.64
5.04	0.4676	116.31	1927.21	43.9
6.31	0.5138	234.74	8742.25	93.5
7.40	1.084	138.84	1616.0	40.2

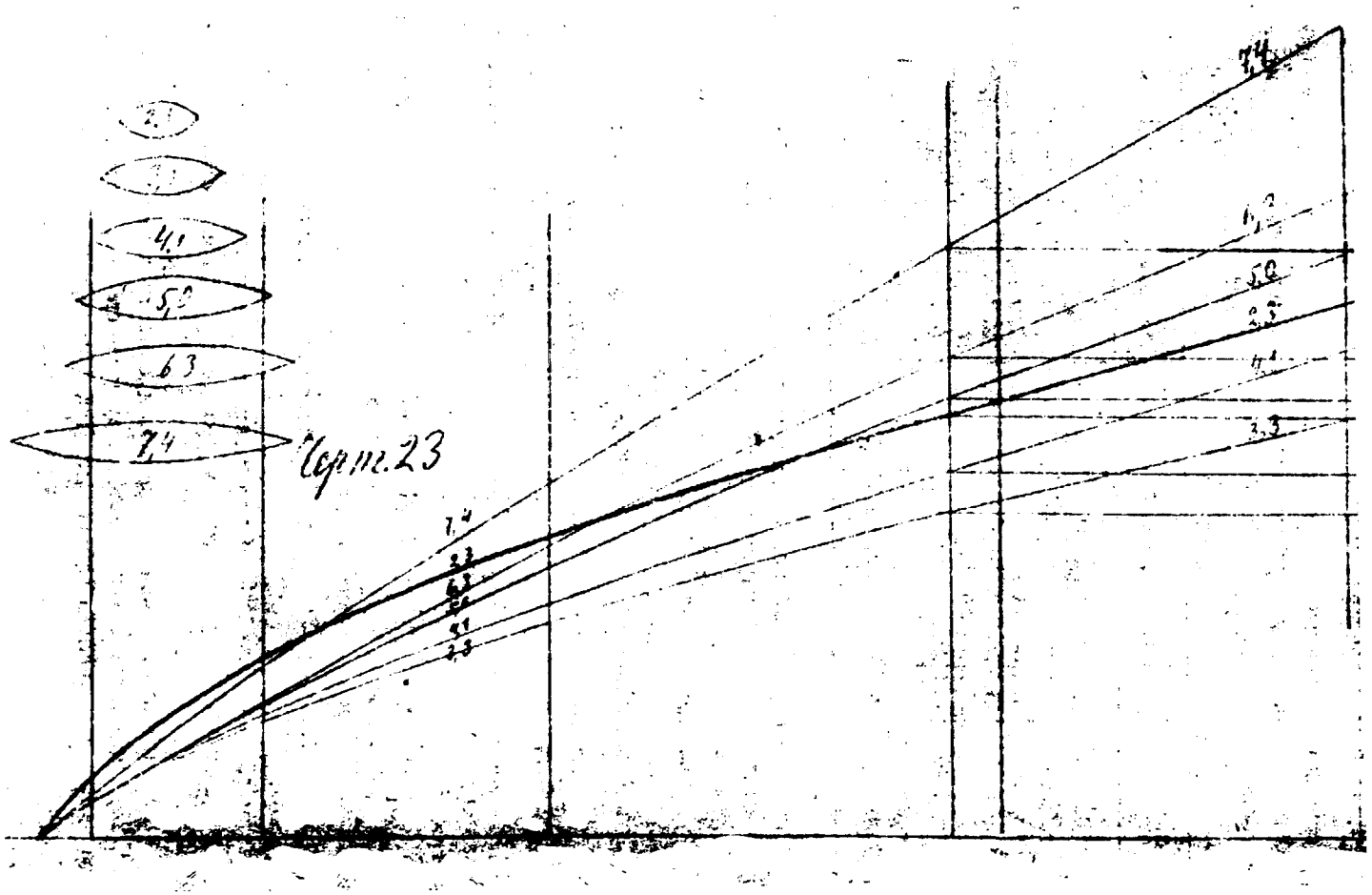

Fig. 23.

90. Hence it appears to follow that the most favorable aspect ratio is close to 3 and that the greatest form factor does not exceed 32. In connection with the latter, I conducted a series of experiments with bodies of the same shape (that is, similar bodies), but different size, which showed that as the size of a body increases so does its form factor.

91. I made three elongated bodies with an aspect ratio of 4.2 (Fig. 24).

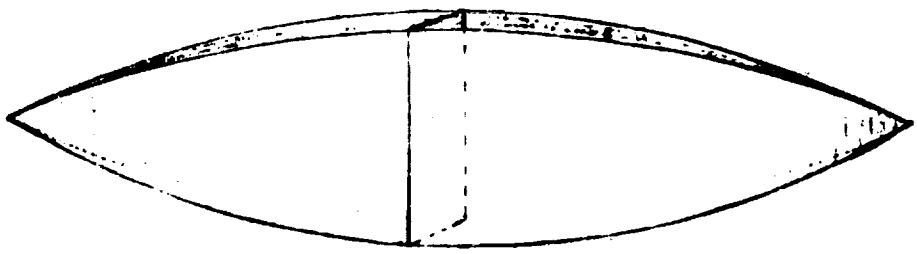

Fig. 24.

The cross section was square. I prepared these models as follows. On cardboard I drew a curve based on the equation

$$y = A \cdot \cos x,$$

where A is a constant less than unity. By means of two such identical curves I defined a cardboard shape, so as to obtain a symmetrical elongated plate. I then glued four of these plates together to form a single body. The biggest of these bodies was 68 cm long, the second was half as long as the first, the third half as long as the second.

92. The corresponding form factors are given in the following table:

Length (cm)	Cross-sectional area in sq. cm.	Load in pounds					
		4	8	16	32	64	128
17	17.6	4.7	4.4	4.9	6.9	7.4	7.9
34	66.0	4.7	5.1	5.4	7.1	8.2	9.5
68	263.0	5.4	6.3	6.9	8.0	8.8	9.9

The rows of the table show how the form factor increases with increase in the load.

The columns show how the form factor increases with the dimensions of the model, though the flow velocity remains the same. These experiments are only rough, but they certainly make it clear that the form factor increases with the absolute dimensions of the body*.

93. The form factors of very elongated solids of revolution, obtained in my experiments, may well increase many times over with increase in size to the dimensions of an airship.

The problem of whether this increase will be more rapid for slightly or strongly elongated bodies is one that I shall leave for a future occasion.

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*I very distinctly observed an analogous phenomenon in similar experiments with water. This must be very familiar to naval architects. Experiments in water to determine the drag of a ship are made only with models of considerable size.

101. In order to determine the resistance of inclined plates, I began by using three squares of different sizes (25, 100 and 400 sq. cm) and two identical rectangles with an aspect ratio of 3 and an area of 48 sq. cm (4 x 12). I arranged the rectangles with their long sides now vertical, now horizontal. I could rotate these rectangles about their vertical axis through a definite number of degrees. In order to measure the angle between the plate and the direction of the air flow, to the back of the rectangle (or square) I attached horizontally a small cardboard protractor. A needle attached to the axis of rotation (i.e., the fixed wire about which the rectangle revolved) marked off on the protractor the inclination of the plate in relation to the direction of flow.

102. At angles of 90 to 50° the measuring apparatus was arranged as usual; at angles between 40 and 0° the apparatus (the water-filled tank, the indicator and everything floating in the tank) was arranged at right angles to the usual position (see Figs. 29, 30 and 4).

103. In the first case, in order to obtain the component of the wind pressure normal to the plate, it is necessary to divide the resistance obtained from the experiment by the sine of the angle of inclination; in the second case, in order to determine the same component perpendicular to the plate it is necessary to divide the resistance by the cosine of the angle formed by the direction of the flow and the surface of the plate[19].

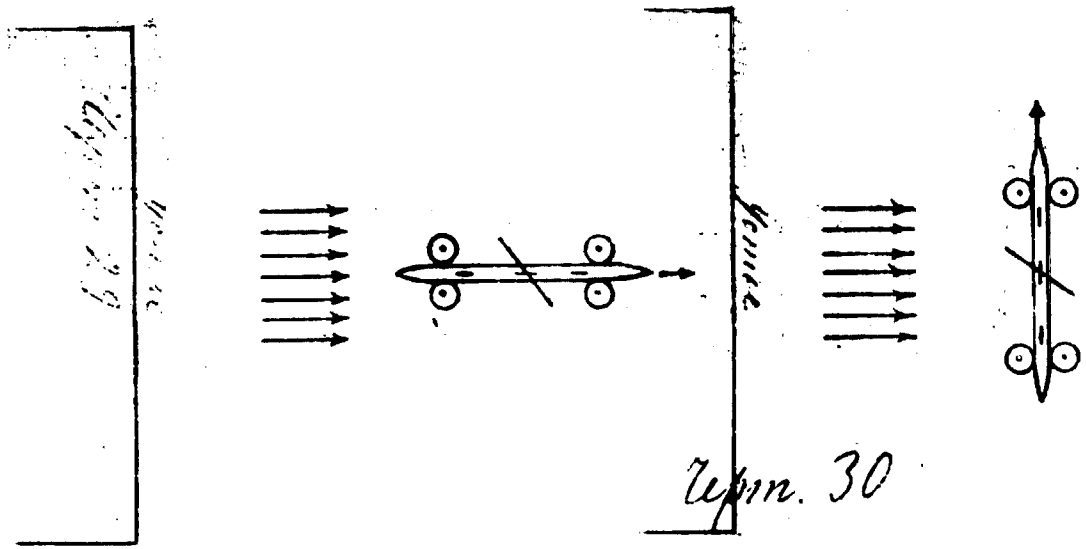
107. As a result of the experiments I was able to compile the following table for the three squares and two rectangles.

108-109. The first row shows the inclination of the plate to the flow. The next three rows show the pressure on the squares of different areas (cf. first column). The last two rows show the pressure on the same rectangle, with an aspect ratio of 3, arranged with first the long side, then the short side perpendicular to the flow. It should be noted that all these pressures are referred to an area of 100 sq. cm and a load of 10 pounds.

112. On comparing the figures for the pressure on the two larger squares (400 and 100 sq. cm), we note that the values are not very different; we also note that at an inclination of 60-50° the pressure has a maximum, which exceeds the perpendicular pressure by 2-3%.

113. Why the small square (25 sq. cm) gave figures considerably different from those for the larger squares I shall not take it upon myself to explain. I shall merely say that the experiment on the 100 sq. cm square was conducted with particular care and many checks.

Angle of inclination of plate to flow	5	10	15	20	25	30	40	50	60	70	80	90
20 × 20 = 400 sq. cm^2	11.5	24.7	41.6	57.0	75.0	86.0	88.3	90.0	90.3	89.2	87.5	87.5
10 × 10 = 100 sq. cm^2	11.6	26.7	38.8	54.8	68.1	78.7	88.0	92.0	93.8	93.4	91.6	92.0
5 × 5 = 25 sq. cm^2	-	17.5	29.2	40.4	-	66.3	80.4	87.8	88.7	90.6	91.9	92.0
Long side of rectangle normal to flow (4 × 12 cm)	20.7	41.3	58.5	66.4	77.3	71.2	75.4	90.0	94.5	99.6	97.2	98.4
Short side normal to flow (12 × 4 cm)	3.0	13.0	22.2	29.0	45.8	61.2	84.5	98.0	97.8	97.1	97.7	99.0



Figs. 29 and 30.

114. It is interesting to compare the pressures on the two identical rectangles at the same angle of inclination to the flow (see last two rows of table).

Thus, at an inclination of 10° the pressure on the rectangle arranged along the flow is three times less than that on the same rectangle arranged transversely to the flow. As the inclination increases, this pressure ratio steadily declines, so that at 30-40° it is close to unity.

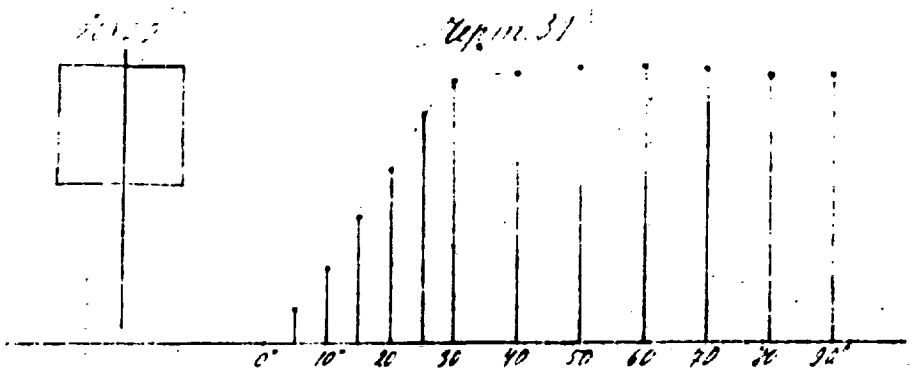


Fig. 31.

115. Other things being equal, the pressure on a square is more or less a mean of the pressures on equivalent rectangles arranged as described.

Curves plotted on the basis of the table (see Figs. 31-35) give a clear idea of the laws of pressure for squares and rectangles.

116. From the first three drawings, for squares, we see that the normal pressure varies little at angles of inclination from 50 to 90°. We also see that at very acute angles the pressure increases more rapidly than the inclination.

117. From Figs. 34 and 35, for the rectangle, we note the following:

When the rectangle is arranged across the flow, the pressure

increases more rapidly than its inclination to the flow; on the other hand, when the rectangle is arranged along the flow, the opposite is true (as with a square); in the former case, the rapid increase in pressure is also accompanied by a fall in pressure between 30 and 40°; in the latter, a slow initial increase is followed by a more rapid rise and a pressure maximum close to an angle of inclination of 50°.

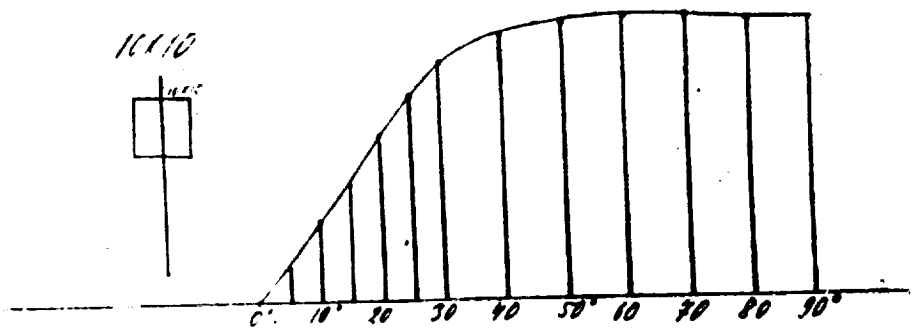


Fig. 32.

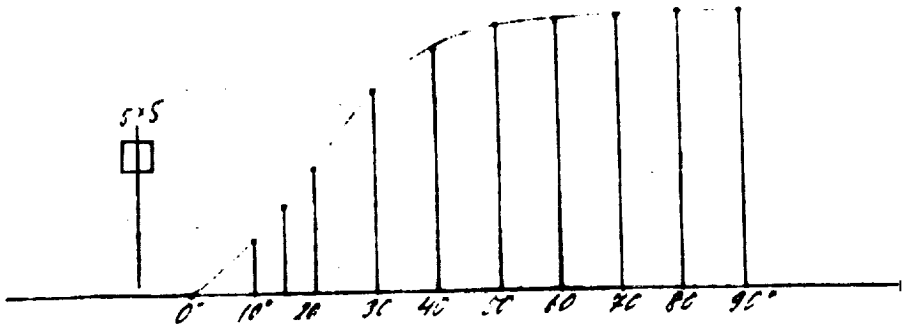


Fig. 33.

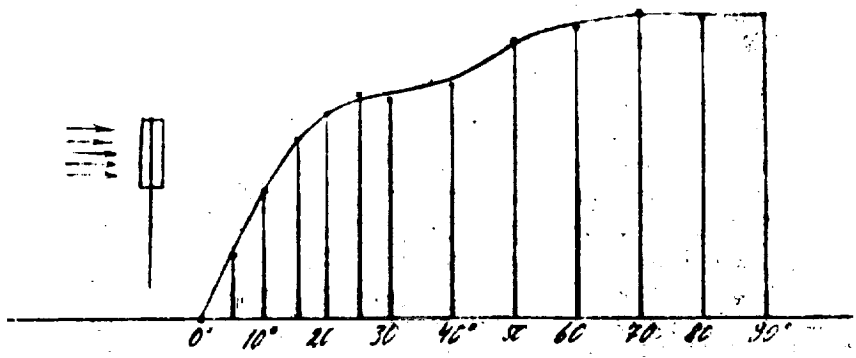


Fig. 34.

118. A comparison of Figs. 34 and 35 also reveals why the wings of birds and insects are arranged transversely to the counter-flow of air and not parallel to it. In fact, at small angles of inclination, this arrangement results in a pressure several times greater than that given by the opposite arrangement of the wings.

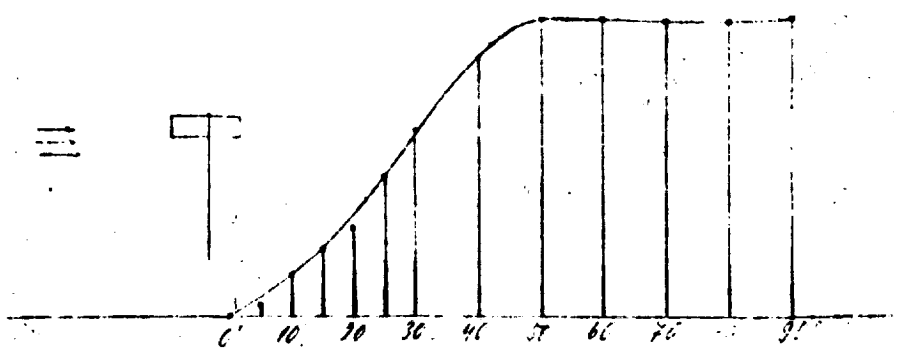


Fig. 35.

119. The above experiments were conducted with cardboard plates somewhat less than 1 mm thick. I next made four pairs of similar rectangles out of tin sheet $\frac{1}{3}$ mm thick.

Each pair consisted of identical plates differently arranged. Their dimensions were as follows:

5 X 10.5 X 15.5 X 20 and 5 X 25 cm.

The experiments were performed as described above.

120. The results are presented in the table (see p. /153).

124. A description of the table would be superfluous, since it is the same as the previous one. It starts with the most elongated rectangle arranged along the flow and ends with the same rectangle arranged in the opposite manner, with the long side perpendicular to the air flow. The center row, of course, relates to a square. Set out in this way, the table clearly shows how, at a given inclination, the pressure increases continuously with increase in the aspect ratio. This can be observed up to 25-30°; beyond that the pressure varies little, in spite of differences in the aspect ratio. Up to the same angle the pressure difference between the two rectangles is the greater the greater their angle of inclination to the flow[20].

134. Taking the rectangle with an aspect ratio of 4 (5 X 20) inclined to the flow at an angle of 10° and arranging it in the two opposite ways described in relation to the flow, I obtained the following results:

When the short side was perpendicular to the flow:

Loads:	10,	40,	80 pounds
Pressure:	5.2,	20.3,	40.0 mm.

With the rectangle in the other position I obtained:

Loads:	5,	10,	20,	30	pounds
Pressure:	12.6,	24.1,	43.9,	65.0	mm.

	Angle of inclination to flow in degrees	Pressure reduced to 100 sq. cm and a load of 10 pounds						
		5	10	15	20	25	30	35
Long side perpendicular to flow	25 × 5	3.2	7.8	16.6	24.2	39.2	49.9	62.5
	20 × 5	6.6	13.6	21.1	31.5	42.8	53.6	69.6
	15 × 5	7.2	14.6	24.5	36.0	50.0	64.1	75.5
	10 × 5	6.4	16.00	32.00	44.00	59.60	74.36	88.50
	Square 10 × 10	11.6	26.7	38.8	54.8	68.1	78.1	-
Short side perpendicular to flow	5 × 10	16.4	32.5	48.0	61.3	66.7	67.9	70.8
	5 × 15	23.7	42.5	40.7	64.1	73.6	76.4	75.9
	5 × 20	23.0	44.3	62.1	70.2	74.4	76.7	72.4
	5 × 25	22.7	42.7	61.9	68.4	69.6	75.0	74.4

[Table cont'd. on next page]

[Table cont'd.]

	Angle of inclination to flow in degrees	Pressure reduced to 100 sq. cm and a load of 10 pounds						
		40	45	50	60	70	80	90
Long side perpendicular to flow	25 x 5	72.7	86.0	101.0	106.5	104.0	103.4	104.9
	20 x 5	78.3	89.1	102.8	103.1	103.5	101.4	102.3
	15 x 5	85.0	91.3	98.7	97.8	100.6	100.8	102.6
	10 x 5	98.2	101.0	102.9	99.5	97.2	98.0	95.8
	Square 10 x 10	88.0	-	92.0	93.8	93.4	91.6	92.0
Short side perpendicular to flow	5 x 10	78.8	82.0	89.0	96.4	97.0	95.6	96.0
	5 x 15	77.3	81.1	86.9	94.1	98.6	100.3	95.3
	5 x 20	80.0	86.0	95.4	98.0	102.0	101.8	99.5
	5 x 25	77.6	81.4	91.5	102.3	105.6	108.5	106.7

135. From these and similar experiments I found that the pressure on any flat plate, however arranged, is approximately proportional to the square of the flow velocity or to the load.

147. The results I obtained with rectangles inclined or perpendicular to the direction of the flow can not be compared with those of other investigators, although it has long been known that the same elongated inclined plate creates greater resistance if the long edge is perpendicular to the flow (Langley, Aman). Perhaps other, more detailed experiments have been performed, but if so I am unaware of them*.

148. However, I can compare my investigations of the wind pressure on an inclined square with the results of others.

At the same time, it is generally assumed that the laws of pressure for a square are equally applicable to any elongated plate, which -- if my experiments are correct -- is a great mistake.

For purposes of comparison, let us take the formulas of Newton, Leslie, Rayleigh (or Gerlach), Langley (or Duchmin) and my previous experiments dating from 1897-1898.

With the aid of these formulas I have compiled the following table of pressures for a nonelongated inclined plate. I was obliged to take the wind pressure perpendicular to the plate as unity, since in this respect the disagreement among scientists is horrifying. I myself, using squares of different sizes, obtained results that were not entirely consistent.

However, since I made especially careful measurements with the square 100 sq. cm in area (the pressure was determined not less than twice, upon rotation through a given number of degrees to right and left of the direction of the flow), I have tabulated the figures relating to this particular square.

In the formulas given in the table, F denotes the force acting on the inclined square; F_1 the force acting on a square normal to the direction of the wind; i the angle of inclination of the plate to the flow in degrees; π the ratio of perimeter to diameter.

149. If we compare, for an inclination of 5°, the figures of Langley and Newton, we find that Langley's pressures are 23 times greater than those of Newton. At more acute angles the difference is even greater.

---

*I now know that the curves obtained by Langley closely resemble mine.

Angle in degrees	5	20	35	50	60	90
Newton $F = F_1 \cdot \sin^2 i$	0.0076	0.117	0.330	0.587	0.750	1
Leslie $F = F_1 \cdot \sin i$	0.0870	0.342	0.574	0.766	0.866	1
Rayleigh and Gerlach $F = F_1 \frac{(4 + \pi) \sin i}{4 + \pi \cdot \sin i}$	0.1450	0.470	0.708	0.819	0.920	1
Langley and Duchmin $F = F_1 \frac{2 \sin i}{1 + \sin^2 i}$	0.1730	0.612	0.864	0.966	0.990	1
My experiments 1897-1898	0.1370	0.614	1.023	0.091	1.076	1
My present experiments 1900-1901	0.1300	0.600	0.925	1.005	1.022	1

In spite of this, Newton's formula still holds an honored place in various engineering books and not only in relation to aeronautical but also in relation to marine engineering (determination of the water pressure on a moving ship).

This formula is still recommended by the Prussian Academy of Structural Engineering for determining the wind pressure on an inclined plane. All this shows how slowly new knowledge is disseminated, even among scientists.

150. On examining the table, we see that the figures for the pressure have increased continuously with the passage of time. Thus, the smallest values are given by Newton's formula, the largest are mine, if we disregard the acute angles, at which there is a great similarity between my figures and those of Rayleigh and Gerlach (theoretical) and Langley (experimental).

There is also a difference between my first and my latest experiments, which were repeated several times; at average angles this approaches 9%.

151. If my experiments are accepted, then at acute angles, less than 10°, it is more correct to use the formula of Lord Rayleigh, whereas at angles of more than 15°, Langley's formula is nearer the truth, as well as being simpler.

152. For acute angles between 5 and 35° it is even simpler to employ Leslie's formula, provided the coefficient is changed. We then get:

$$F = F_1 \cdot 1.7 \cdot \sin i.$$

Using this formula I have computed a series of pressures for angles between 5 and 45° and compared them with the pressures obtained in my experiments.

From the table it is clear that the pressures calculated from the modified Leslie formula differ from my findings by not more than 5%, as the angle varies from 10 to 35°.

153. On examining the first curve in Fig. 36, that corresponding to a square, we see that the initial section of the curve is close to a straight line. Hence, for the pressure on an inclined square up to an inclination of 30° we shall have an even simpler formula, namely:

$$F = A \cdot i,$$

where i is the angle of inclination of the plate to the flow in degrees. Then, in accordance with the experimental data, we find

$$F = 2.1 \cdot i,$$

where F is the force acting on an area of 100 sq. cm at a flow velocity of 1 meter per second.

Angle in degrees	5	10	15	20	25	30	35	40	45
Leslie's formula (modified)	0.148	0.296	0.440	0.581	0.719	0.850	0.976	1.09	1.20
My experiments	0.130	0.290	0.440	0.600	0.745	0.860	0.925	0.97	0.99

Using this formula, I have computed a series of pressures, which I have compared with the pressures found experimentally.

Hence we see that up to 35° the difference is comparatively small.

154. In general, for a square, circular or any other non-elongated plate we can write:

$$F = 2.1 \cdot i \cdot S \cdot v^2,$$

and in practical units (meters, kilograms):

$$F = 0.0021 \cdot i \cdot S \cdot v^2.$$

155. When a rectangle, ellipse or other elongated plate is arranged transversely to the flow, so that the long side or axis is perpendicular to the direction of the flow, we can also take equally simple formulas for the pressure.

Angle in degrees	5	10	15	20	25	30	35	40
Formula	10.5	21	31.5	42	52.5	63	73.5	81.9
Experiment	9	21	32	44	54.5	63	67.7	70.5

However, the more elongated the plate, the less the angle of inclination up to which the formula is applicable.

For aspect ratios of from 2 to 5 I have derived formulas that can be used with confidence up to angles of 10-15°:

Aspect ratio	Formula for pressure
2	$F = 2.7 \cdot i \cdot S \cdot v^2$
3	$F = 3.2 \cdot i \cdot S \cdot v^2$
4	$F = 3.5 \cdot i \cdot S \cdot v^2$
5	$F = 3.6 \cdot i \cdot S \cdot v^2$

In order to obtain practical coefficients it is necessary to

divide these coefficients by 1000. For example, for an aspect ratio of 5 we get: $F = 0.0036 \cdot i \cdot S \cdot v^2$ kilograms.

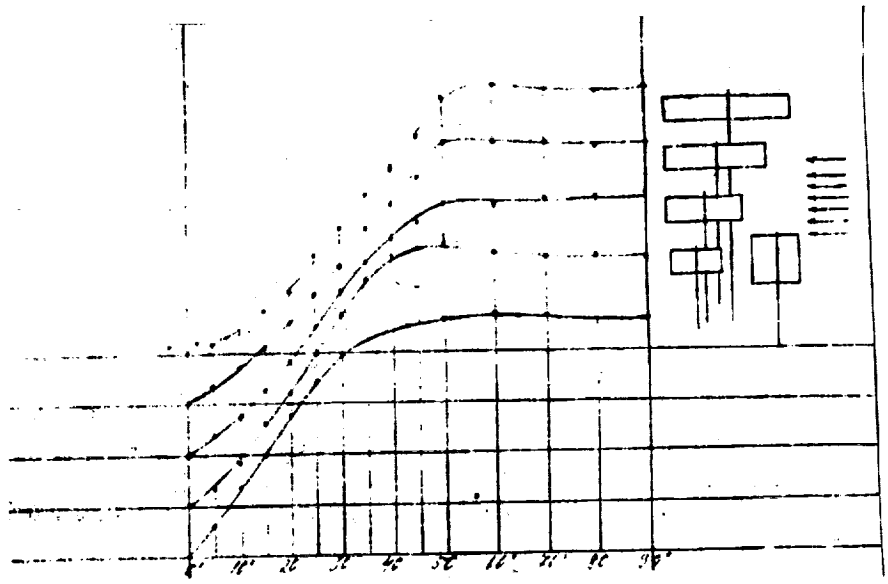


Fig. 36.

For airplanes with plane wings I can recommend the formulas of (155), since, as we shall see, the most favorable angle of inclination for an airplane scarcely exceeds 10-15°. Moreover, to get the full advantage of an airplane design it is necessary to use elongated wings arranged with the long side perpendicular to the wind or to the counterflow of air, exactly the conditions under which these formulas are applicable.

It is often necessary to know the component of the wind pressure in the direction of the wind itself or in a direction perpendicular to the latter.

My formulas give only the pressure normal to the plate, i.e., the approximate (neglecting friction) resultant.

If we denote the latter force by F, the component in the di-

rection of the wind will be equal to $F \cdot \sin i$, where i is the angle of inclination of the wind velocity to the plate. The component in a direction perpendicular to that of the wind velocity may be expressed as $F \cdot \cos i$.

Actually, in my experiments I determined these components, but converted them into the force acting normal to the plate.

156. Let us now consider the resistance of a right circular cylinder. In these experiments I used 13 cylinders of almost the same diameter (5 cm), but different heights (1-25 cm); moreover, 10 cylinders about 10 cm in diameter and 3-50 cm tall; then 5 cylinders 25 cm tall, but differing in diameter (1.05-5 cm); and, finally, two identical cylinders 0.9 cm thick and 39.2 cm long terminating in sharp cones. Here I have given the total length of the cylinder and cones (39.2). I also used round wire from 0.365 to 3 mm thick.

157. By arranging the small cylinders, 5 cm in diameter, with the axes parallel to the direction of the flow and exposing them to the flow of air produced by a load of 32 pounds, I obtained the results presented in Fig. 43.

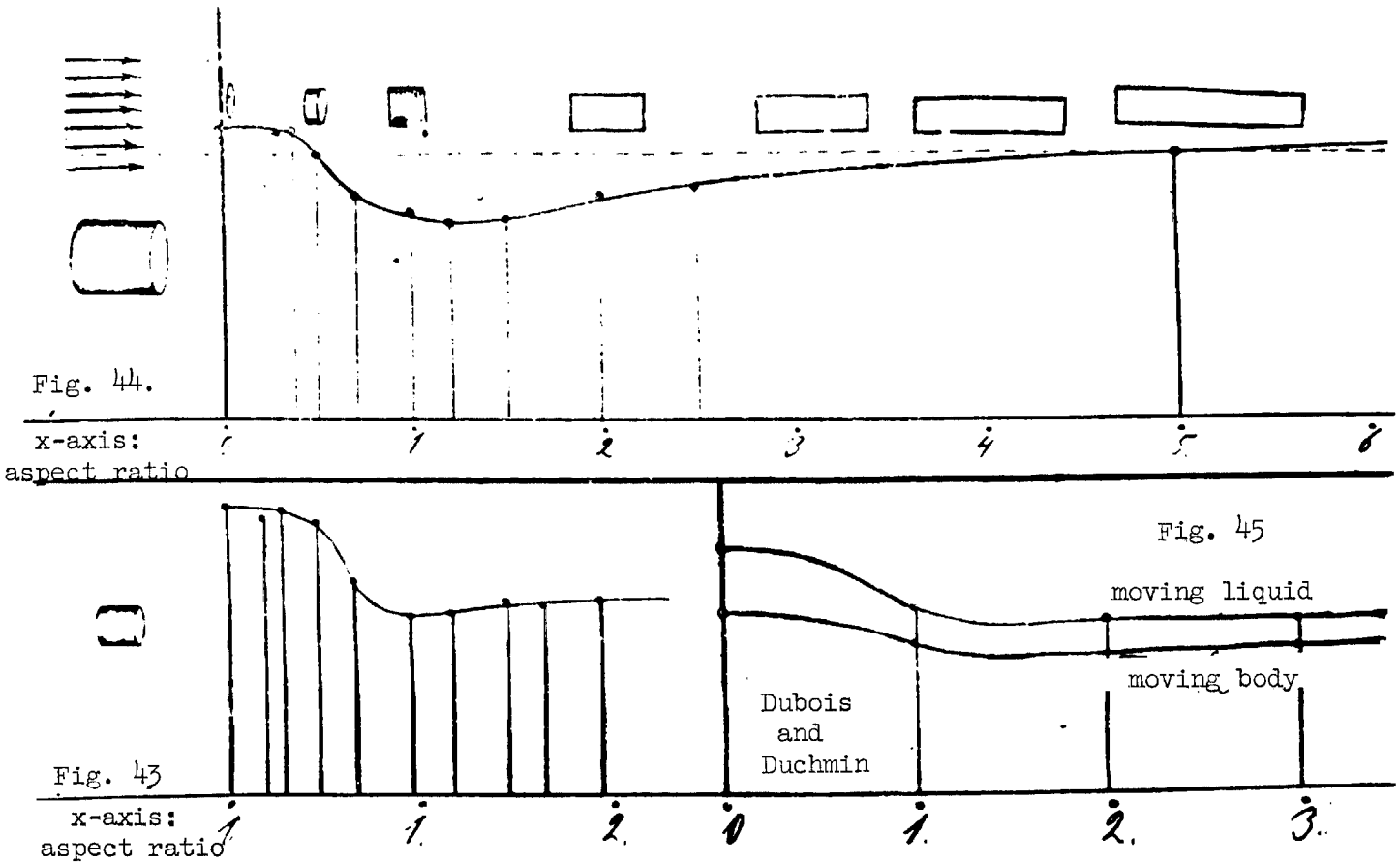
The ordinates, in millimeters of length, express millimeters of pressure, reduced to an area of 100 sq. cm and a load of 8 pounds (or a velocity of 1 m/sec); the abscissas denote the aspect ratio of the cylinder or the ratio of the length to the diameter of the base.

159. The figure shows clearly how the pressure falls, at first very slowly, then very quickly; later the fall is slowed, then stabilized, and finally the curve begins slowly, but continuously to rise.

160. In order to verify these conclusions, I performed equally accurate experiments with better constructed and larger cylinders, namely those 10 cm in diameter.

These experiments were performed with particular care at a load of 8 pounds, that is, at a velocity of 1 m/sec. The results are shown in Fig. 44. This figure permits the same conclusions as the previous one, but the curves are not exactly the same. The ordinate corresponding to an abscissa equal to zero indicates the pressure on an infinitely small cylinder, i.e., on a circular plate.

161. These experiments are very important, since the results are applicable to all right prisms and cylinders with a nonelongated base. (Cf. "Air Pressure," section 122.)



Figs. 43, 44, and 45.

162. The curve of Fig. 44, which in my opinion is of considerable interest, makes it possible to determine the probable pressures on the base of a cylinder for different aspect ratios[21]. For this purpose, it is necessary to express in millimeters the ordinates of the curve corresponding to the required abscissas, i.e., the required aspect ratios of the cylinders. Then we get the pressure in conventional units (1 mm = 1/100 gram) acting on 100 sq. cm of the base of the cylinder for a flow velocity of 1 m/sec (see following table).

163. From this table it is clear that the pressure on the cylinder is a minimum at an aspect ratio close to 1.2.

If we were given a certain right prism with a not too elongated base, we could determine the wind pressure in the direction of the axis either from the table or directly from the curve in Fig. 44.

For this purpose, we compute the diameter of the circle, the area of which is equal to the area of the base of the prism. The ratio of the height of the prism to the diameter of this circle is the aspect ratio of the prism. Suppose it is 1.2; then, from the table, we find that the pressure on the prism is 50.5 mm or in practical units: $F = 0.0505 \cdot S \cdot v^2$ kilograms, where S is the area of the base of the prism in sq. meters.

164. In experiments with water similar to those described, Dubois and Duchmin made certain observations on prismatic bodies which are illustrated in Fig. 45. The upper curve relates to a stationary body and a moving liquid, the lower to a moving body and a stagnant liquid. It is strange that the two curves do not coincide even approximately*[22].

However this may be, the phenomenon repeats itself in water in almost exactly the same way as in analogous experiments in air, as is perfectly evident from a comparison of the curves in Figs. 44 and 45.

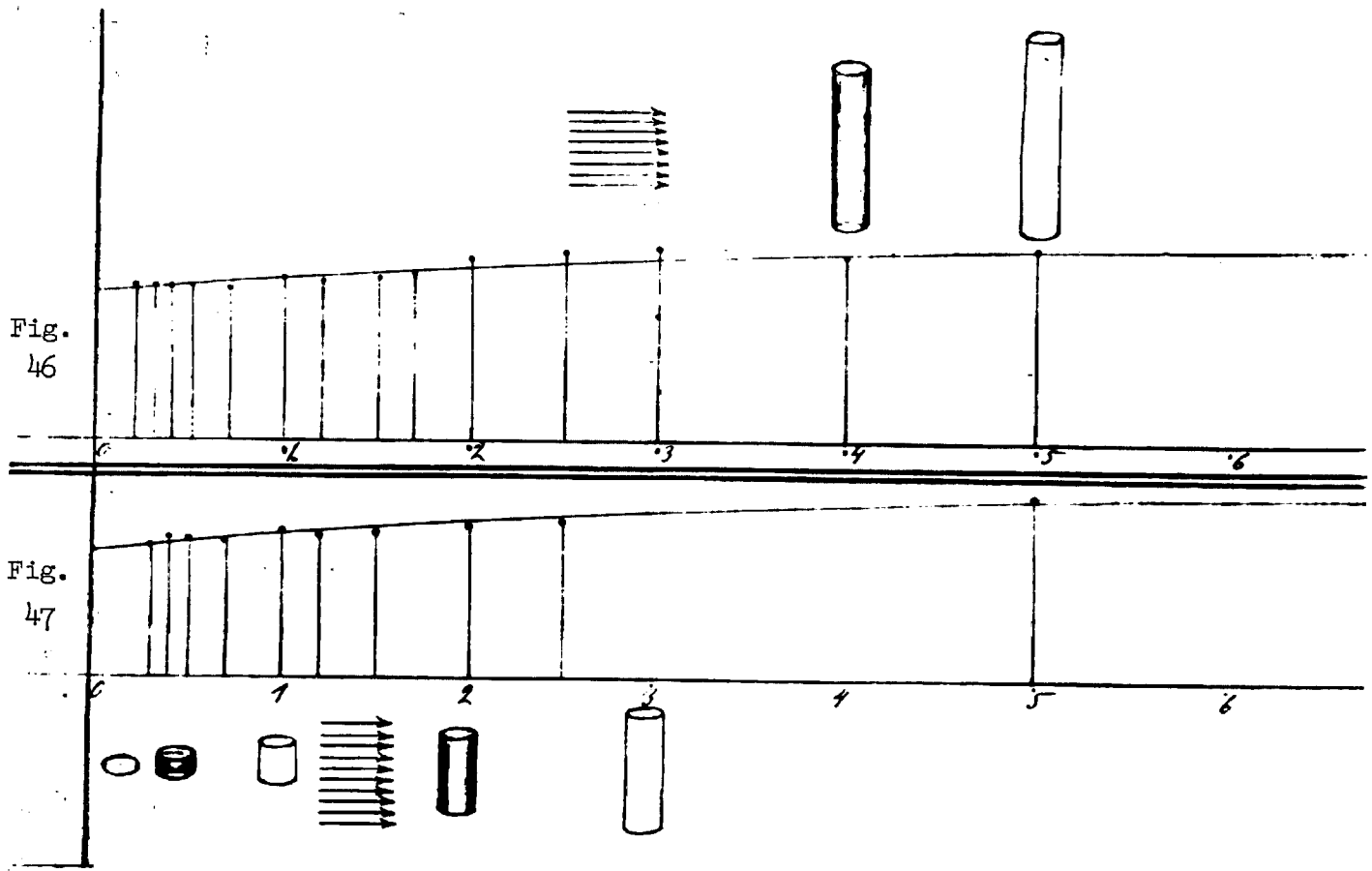
165. In order to determine the transverse drag of a cylinder, i.e., that corresponding to a flow perpendicular to the axis of the cylinder, I performed another two series of experiments (cf. Figs. 46 and 47).

I performed experiments on the small cylinders (5 cm in diameter) at loads of from 8 to 32 pounds. I reduced the pressures obtained to a load of 8 pounds or a velocity of 1 m/sec and to 100 sq. cm of the area of the longitudinal (axial) cross section of the cylinders.

---

*This may depend on the difficulty in determining accurate values of the velocity of the liquid.

Aspect ratio	Pressure on 100 sq. cm of base at a velocity of 1 m/sec.	Aspect ratio	Pressure on 100 sq. cm of base at a velocity of 1 m/sec.
0.0	73.6	1.6	51.8
0.2	73.6	1.8	54.1
0.3	73.4	2.0	56.5
0.4	72.5	2.5	60.0
0.5	68.0	3.0	62.5
0.6	62.5	3.5	64.5
0.7	57.5	4.0	66.0
0.8	54.6	4.5	67.0
0.9	53.0	5.0	68.0
1.0	51.5	5.5	68.5
1.2	50.5	6.0	69.0
1.4	50.7		



Figs. 46 and 47.

166. The results are presented in Fig. 46.

167. For the reasons already mentioned, the experiments with the larger cylinders (10 cm in diameter) necessarily gave better results, as may be seen from Fig. 47; here the experimental points lie closer to the smooth curve than in Fig. 46.

From these two drawings it is clear that the drag increases with increase in the length of the cylinder, tending to a certain limit, so that the curve must have an asymptote parallel to the x-axis*.

168-169. Giving preference to the curve shown in Fig. 47, I shall use it to compile a table of resistances and drag coefficients for a transverse flow acting on cylinders with different aspect ratios. The first columns of the table show the aspect ratio of the cylinders. The second columns show the resistance in conventional units (mm per 100 sq. cm) for a flow velocity of 1 m/sec.

171. The third columns give the drag coefficients or the pressure compared with the pressure of the same flow on the axial section of the cylinder (assuming it to be square).

From the table we see that these coefficients are far from being constant, as it is customary to suppose on the strength of existing theories, for in my experiments they vary from 0.46 to 0.64 (an increase of 39%).

172. The fourth columns give the form factor of the cylinders with different aspect ratios or the ratio of the pressure on a square plate, the area of which is equal to the area of the longitudinal section of the cylinder, to the resistance of the cylinder itself, other things being equal.

Short cylinders have the greatest form factor; then, as the height of the cylinder increases, the form factor diminishes, evidently approaching a certain limit, beyond which it cannot diminish further.

The drag of a cylinder is found to be 1.5-2 times less than that of a square plate.

According to my 1897 experiments ("Air Pressure," section 111), the drag coefficient of a cylinder with an aspect ratio of 2 (length 10 cm, diameter 5 cm) is 0.63, i.e., greater than the tabulated value (0.552). In this case, the difference is due to the larger dimensions of the tabulated cylinders. Given the same dimensions (cf. the ordinate for an aspect ratio of 2 in Fig. 46), we get exactly the same coefficient.

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*Do not forget that, as almost always, the drags are reduced to unit cross-sectional area.

Aspect ratio	Pressure in mm per 100 sq. cm for a velocity of 1 m/sec	Drag coefficient	Form factor	Aspect ratio	Pressure in mm per 100 sq. cm for a velocity of 1 m/sec	Drag coefficient	Form factor
0.2	33.8	0.460	2.18	3.0	43.3	0.588	1.70
0.3	34.2			3.2	44.0		
0.4	34.7			3.4	44.5		
0.5	35.0	0.475	2.13	3.6	45.0		
0.6	35.4			3.8	45.4	0.617	
0.7	35.7			4.0	45.8	0.622	1.67
0.8	36.1			4.2	46.2		
1.0	37.1	0.504	1.983	4.4	46.4		
1.2	37.7			4.6	46.7		
1.4	38.5			4.8	46.9		
1.6	39.3			5.0	47.1	0.640	1.563
1.8	39.8			5.2	47.3		
2.0	40.6	0.552	1.813	5.4	47.4		
2.2	41.1			5.6	47.5		
2.4	41.6			5.8	47.6		
2.6	42.3			6.0	47.7	0.648	1.543
2.8	42.3			6.2	47.7		

173. Apart from the experiments with these cylinders, I performed experiments with cylinders with greater aspect ratios -- up to 44.

Here is the table for these cylinders and the results of experiments with them:

Reduced to a load of 8 pounds	For 8 pounds			For 32 pounds	
	1	2	3	4	5
Diameter of base of cylinder in cm	3.90	2.87	1.97	1.05	0.90
Aspect ratio	6.4	8.7	12.7	23.8	43.6
Pressure per 100 sq. cm for a velocity of 1 m/sec	55.2	59.5	58.4	59.4	61.7
Drag coefficient	0.75	0.81	0.80	0.81	0.84
Form factor	1.333	1.237	1.260	1.237	1.194

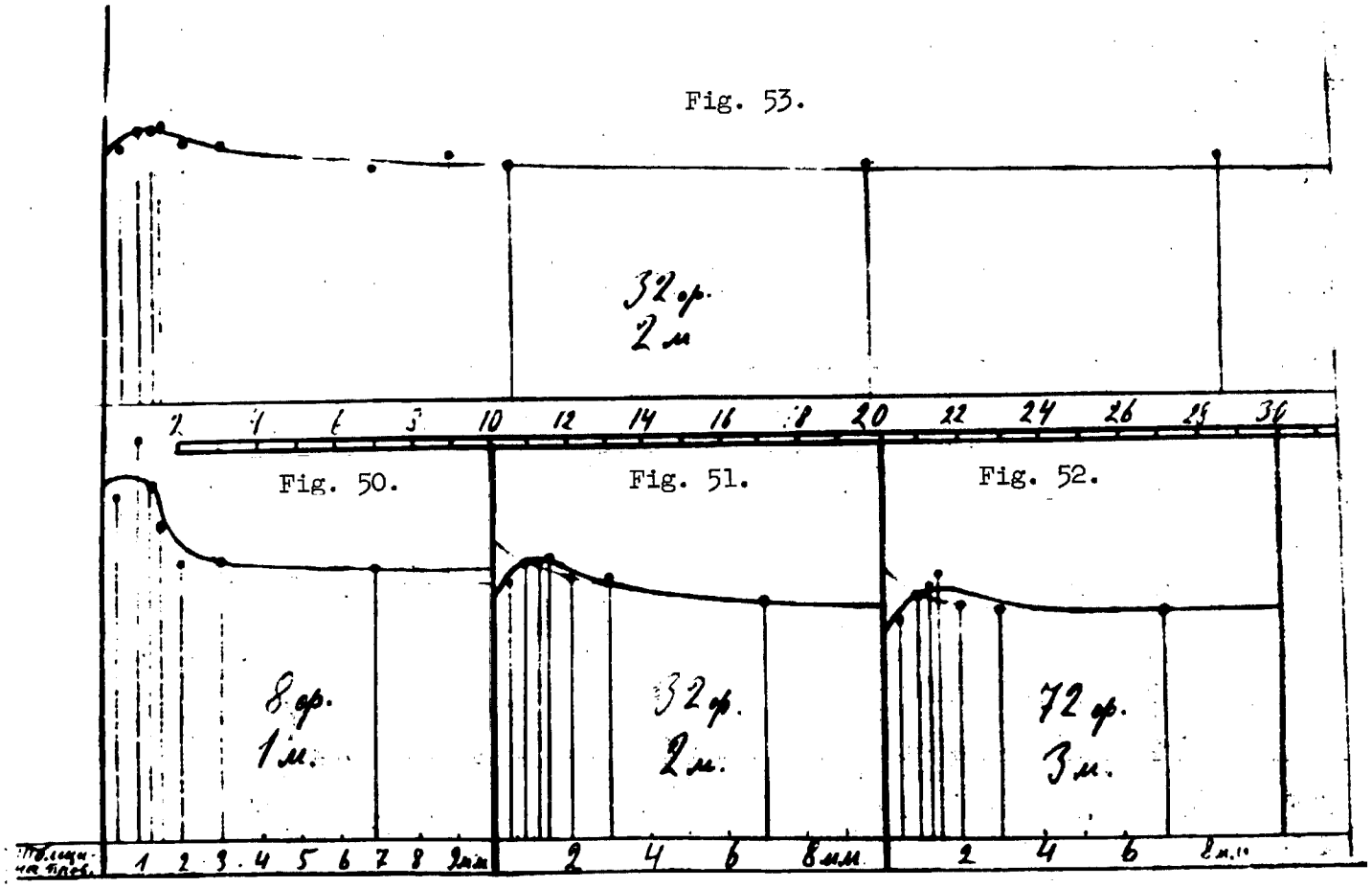
Hence we see that with increase in the aspect ratio the drag coefficient also increases. However, this increase may be due to the small thickness of the cylinders employed.

The drag coefficient for thin wires also increases, as we shall see.

175. A wire is also a cylinder. Therefore I shall introduce here the results of experiments on the resistance of wires. These results are shown in Figs. 50, 51 and 52. The wire used was from 0.365 to 7.1 mm thick.

179. On drawing through the experimental points smooth curves representing the resistance as a function of the thickness of the wire and measuring the ordinates of the curves, we get the following table.

Thickness of wire in mm	Resistance in mm, reduced to an area of 100 sq. cm and a velocity of 1 m/sec. Velocity equal to 1, 2, 3 m/sec Load in pounds:			Drag coefficient Velocity in m/sec.		
	8	32	72	1	2	3
0.5	91	67	57	1.23	0.91	0.77
1	90	69	61	1.22	0.93	0.83
2	73	66	61	0.99	0.89	0.83
3	69	63	58.5	0.93	0.85	0.79
4	68	61	56.5	0.92	0.83	0.76
5	67.5	60	56	0.91	0.82	0.76
6	67	59.5	56	0.91	0.81	0.76
7	67	59	56	0.91	0.80	0.76
8	67	58	56	0.91	0.79	0.76
9	67	58	56	0.91	0.79	0.76
10	67	58	56	0.91	0.79	0.76



Figs. 50, 51, 52, and 53.

From this table it is clear that the drag coefficient decreases with increase in the flow velocity and, in general*, with increase in the thickness of round wires.

The last three columns express the resistance in relation to the resistance of a square plate, the area of which is equal to the longitudinal section of the wire. From these columns it is clear that, at low flow velocities, a thin wire offers even greater resistance than a flat plate under the same conditions.

The conclusion is paradoxical, but may be explained, on the one hand, by the high aspect ratio of the wire cylinder and, on the other, by the laws of friction, which, as we shall see, is very great for small surface areas and small flow velocities.

Though there is no doubt that the round shape of the cross section of the wire is efficient in cutting through the air, this advantage is outweighed by the friction, which is very high for low velocities and short (in the direction of the flow) surfaces.

Thus, the experiments with cylinders of different thicknesses and aspect ratios led me to the conclusion that the transverse drag coefficient of a cylinder may fluctuate, at least between 0.46 and 1.29, whereas, theoretically, any of the existing formulas gives a single drag coefficient for any cylinder.

This clearly reveals the total inadequacy of all the existing formulas and methods of determining the resistance of a cylinder (and practically all other bodies and surfaces).

Newton's formula gives a coefficient of $\frac{2}{3}$ or 0.667 for a cylinder. According to Leslie, we find $\pi/4 = 0.785$. Langley and Rayleigh give even higher values.

Of course, the agreement with my experimental coefficients may be purely a matter of chance.

181. The resistance of a wire is equal to

$$F = k\,SV^2,$$

*If we are to trust the experiments, at very small thicknesses the coefficient decreases; this may be due to the adhesion of air, which, as it were, makes the cross section of the wire more elongated.

where k is a coefficient from the table in (179), divided by 100, S the area of the longitudinal section, and V the velocity. It is clear from the above that the drag coefficient of a cylinder in a flow perpendicular to its axis depends not only on the aspect ratio of the cylinder but also on its absolute dimensions.

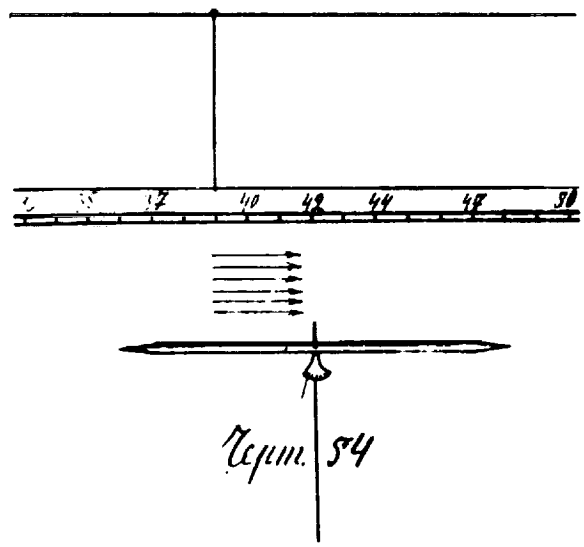


Fig. 54.

Given the same aspect ratio and flow velocity, the drag coefficient will be the greater the more slender the cylinder. The flow velocity also has an influence, and by no means a negligible one, on the value of the coefficient; with increase in velocity the coefficient decreases. General formulas for the drag coefficient K of a cylinder should give the dependence of the coefficient on the aspect ratio of the cylinder L, the flow velocity V, and the diameter of the base D: thus, K = F (L, V, D), where F denotes a function.

Before a thorough study has been made of the laws of friction, upon which the drag of a cylinder (like any other body) partly depends, it would be premature to dream of establishing the exact form of the above function.

183. The determination of the wind pressure on an inclined wire is also a matter of some practical importance.

It is possible to determine the horizontal and vertical components of this force.

For these experiments I constructed a light paper cylinder (Fig. 54) 26 cm long and ending in two sharp cones, each $4^1/_4$ cm long and about 0.9 cm wide at the base.

I regarded this cylinder as an infinite cylinder, since the resistance of the cones in the axial direction (i.e., along the length of the cylinder) could not be very great.

To the cylinder I attached a small quadrant with a needle that indicated the inclination of the cylinder in relation to the flow.

187. I performed the experiment at three flow velocities: 1, 2 and 3 meters per second. The values I obtained for the pressure in the direction of the flow I reduced to a velocity of 1 m/sec, but I did not reduce them to an area of 100 sq. cm of the longitudinal section. From the table (section 179) I saw that the pressures at velocities of 2 and 3 m/sec are not very different, and accordingly I took their average value. I then took the pressure perpendicular to the axis of the cylinder as unity and thus obtained the following table of ratios for the various inclinations of a virtually infinite cylinder.

From this table it is clear that the ratios differ very little at different velocities, starting from an angle of inclination of 10°; therefore we can take the last column of figures as constant and applicable to any flow velocity.

190. In exactly the same way we can obtain the component of the wind pressure in a direction perpendicular to the flow. For this purpose, the measuring apparatus, as always in such cases, was arranged in a position perpendicular to that originally employed.

192. The experiments were performed at flow velocities of 2 and 3 m/sec, and the results are presented in the following table.

The pressure on the cylinder in a flow normal to the axis was taken as unity.

This table shows the component of the resistance perpendicular to the flow, in relation to the maximum resistance with the flow normal to the axis of the cylinder.

If we have a coil of wire and need to know the vertical wind pressure upon it, we can use the table below (see p. 217). In general, the force acting on the wire will be:

$$F = \frac{K}{1000} \cdot K \cdot S \cdot V^2 \text{ kg}$$

Angle of inclination in degrees	Velocity 1 m/sec	Velocity 2 and 3 m/sec (mean value)
	Reduced to 1 m/sec	
0	0.0824	0.06823
10	0.1029	0.09063
20	0.1440	0.1417
30	0.2471	0.2356
40	0.3871	0.3644
50	0.5354	0.5340
60	0.7041	0.7065
70	0.8646	0.8704
80	0.9773	0.9816
90	1.0000	1.0000

where K is the pressure on the cylinder in mm per 100 sq. cm at a velocity of 1 m/sec, K_1 is taken from the table, S is expressed in sq. meters, and V in m/sec.

Angle of inclination in degrees	Coefficient K (mean value from experiments at velocities of 2 and 3 m/sec)
0	0.0000
10	0.06103
20	0.1296
30	0.2034
40	0.2863
50	0.3361
60	0.3263
70	0.2717
80	0.1609
90	0.0000

195. Without having exhausted the question of the drag of right circular cylinders, I shall now turn to right elliptical cylinders.

I made 20 models of cylinders of this type 10 cm tall, about 5 cm wide, and from 5 to 42.3 cm long. The corresponding aspect ratio or ratio of length to width (major and minor axes of the elliptical base of the cylinder) varied between 1 and 8.6.

I also arranged the same cylinders transversely to the flow, making the corresponding aspect ratio less than one, i.e., I took the

reciprocal of the length-to-width ratio, which varied from 0.164 to 1. Thus, altogether I employed aspect ratios varying from 0.164 to 8.6.

196. Below I give a table obtained from observations on cylinders arranged parallel to and across the flow. The experiments were performed at loads of 8, 32 and 72 pounds or at velocities of 1, 2 and 3 m/sec.

The first column indicates the aspect ratio of the cylinder, the second, third and fourth the resistance, reduced to 100 sq. cm of the greatest section of the cylinder perpendicular to the flow and to a load of 8 pounds or a velocity of 1 m/sec.

Finally, the last three columns give the form factor of the cylinder, for the same three flow velocities -- 1, 2 and 3 m/sec.

198. In the case of highly elongated cylinders arranged across the flow, the drag is close to that of a square plate equal in area to the longitudinal section of the shape (see following table).

Moreover, when the aspect ratio is less than 2, the coefficients depend very little on the flow velocity. At high aspect ratios, however, we note that the drag coefficients decrease with increase in the flow velocity and the aspect ratio of the cylinder. At an aspect ratio of about 6 this decrease in the coefficients stops, and with further increase in the aspect ratio the coefficients even begin to increase.

199. It is noteworthy that the moment of minimum drag occurs at the same aspect ratio for all the flow velocities employed. I had already observed this phenomenon in connection with relatively elongated solids of revolution.

The only difference is that with the latter the minimum drag coefficient corresponded to an aspect ratio half as great. For solids of revolution, the maximum form factor, at a velocity of 3 m/sec, is 11.7, whereas with an elliptical cylinder, at the same velocity, it is 4.7, i.e., less than half as much. However, if we take a cylinder bounded by arcs of a circle, i.e., a tapering form, the maximum form factor is almost exactly half as great as that of a body formed by rotating an arc of a circle (see the table in section 219).

204. The following practical table (see pp. 221-222) gives the resistance in mm, reduced to 100 sq. cm of projected area and a velocity of 1 m/sec.

Up to an aspect ratio of 1.4, the coefficients differ very little at different flow velocities, and, accordingly, for any velocity we can use a resistance $F = K \cdot S \cdot V^2$, where K is a coefficient from the table below divided by 1000, and S is the projected area in sq. meters, i.e., the area of a central section of the cylinder formed by intersection with a plane normal to the direction of the flow.

Aspect ratio of elliptical cylinder	Resistance in mm (= 1/100 g) reduced to a velocity of 1 m/sec (8 pounds) and 100 sq. cm of projected area. Velocity in m/sec			Form factor at velocities of (in m/sec)		
	1	2	3	1	2	3
0.164	70	Almost the same	Almost the same	1.051	Almost the same as at 1 m/sec	Almost the same as at 1 m/sec
0.173	70			1.051		
0.222	69			1.067		
0.259	67			1.099		
0.297	64			1.150		
0.425	59			1.247		
0.466	57			1.280		
0.610	50.5			1.458		
0.659	49			1.502		
0.715	46			1.600		
0.791	44			1.673		
0.831	43.5			1.692		
1.000	43			1.701		

[Table cont'd. on next page]

[Table cont'd.]

Aspect ratio of elliptical cylinder	Resistance in mm (= 1/100 g) reduced to a velocity of 1 m/sec (8 pounds) and 100 sq. cm of projected area. Velocity in m/sec			Form factor at velocities of (in m/sec)		
	1	2	3	1	2	3
1.20	42.6	Almost the same	42.40	1.710	Almost the same as at 1 m/sec	1.629
1.27	36.9		32.38	1.996		2.272
1.40	33.2		30.02	2.230		2.44
1.52	32.0		26.89	2.240		2.74
2.00	25.8		23.89	2.850		3.08
2.35	24.8	23.6	22.44	2.97	3.26	3.27
2.96	23.5	19.3	18.16	3.14	3.81	4.06
3.36	22.0	18.6	16.78	3.35	3.96	4.39
3.85	20.0	17.6	16.00	3.70	4.19	4.60
4.51	20.3	16.9	15.33	3.64	4.35	4.78
5.66	18.7	16.9	15.78	3.94	4.35	4.70
6.12	19.0	16.1	15.78	3.90	4.52	4.70
8.63	22.1	19.6	16.29	3.33	3.76	4.515

Aspect ratio	Resistance in mm per 100 sq. cm of cross-sectional area, reduced to 1 m/sec. Velocity in m/sec			Aspect ratio	Resistance in mm per 100 sq. cm of cross-sectional area, reduced to 1 m/sec. Velocity in m/sec		
	1	2	3		1	2	3
0.0	73.6	Almost the same as for 1 m/sec		3.2	22.0	20	18
0.2	70.0			3.4	21.6	19.6	17.5
0.3	65.5			3.6	21.1	19.1	16.8
0.4	60.5			3.8	20.9	18.7	16.4
0.5	55.5			4.0	20.6	18.4	16.0
0.6	51.0			4.2	20.3	18.1	15.7
0.7	46.5			4.4	20.2	17.8	15.4
0.8	44.5			4.6	20.2	17.6	15.1
0.9	44.5			4.8	20.2	17.6	15.0
1.0	43.0			5.0	20.1	17.5	15.0
1.1	42.5			5.2	20.1	17.6	14.9

[Table cont'd. on next page]

[Table cont'd.]

Aspect ratio	Resistance in mm per 100 sq. cm of cross-sectional area, reduced to 1 m/sec. Velocity in m/sec			Aspect ratio	Resistance in mm per 100 sq. cm of cross-sectional area, reduced to 1 m/sec. Velocity in m/sec		
	1	2	3		1	2	3
1.2	41.0	Almost the same as for 1 m/sec		5.4	20.0	17.6	14.9
1.3	35.0			5.6	20.0	17.5	14.8
1.4	33.0	32	31	5.8	19.9	17.5	14.7
1.6	31	29.5	28	6.0	19.9	17.5	14.6
1.8	29	27.5	26	6.5	20.0	17.7	14.7
2.0	28	26.0	24.5	7.0	20.6	18.0	15.2
2.2	26.5	24.5	23	7.5	21.0	18.5	15.6
2.4	25.5	23.5	22.0	8.0	21.4	19.0	15.9
2.6	24.4	22.4	21.6	8.5	22.0	19.5	16.3
2.8	23.5	21.5	19.5	9.0	22.8	20.4	16.6
3.0	22.8	20.6	18.8				

208. I built several cylinders (or right prisms) with bases either polygonal or curved, my object being to determine the drag and compare it with that of an elliptical cylinder, which I had already observed.

The aspect ratio and all the dimensions of the new cylinders were approximately the same as those of an elliptical cylinder, chosen as a standard of comparison, the dimensions of which were as follows: length 24.7 cm, width 5.48 cm, height 10 cm.

The aspect ratio was 4.51*.

The bases of all these cylinders are shown in Figs. 61-75.

Inside each figure I have, in most cases, written three numbers: the one in the middle gives the drag or resistance in mm (1 mm = 1/100 gram) corresponding to 100 sq. cm of projected area, reduced to a load of 8 pounds or a velocity of 1 m/sec (thousandths of these numbers constitute practical coefficients); the second number, on the right, gives the form factor; finally, the third, on the left, is the reciprocal of the form factor, which I have called the drag coefficient with respect to the drag of the projected area.

If the figure is not the same from the front and from the back, I have noted two numbers in the middle, one above the other, the first corresponding to a flow from the right (with respect to the drawing), the second to a flow from the left.

The first 7 cylinders were tested at a flow velocity of 2 m/sec, the rest at 3 m/sec.

209. The large, angular cylinders, which present more resistance to the flow, have a drag almost proportional to the square of the flow velocity. But the other, well-shaped cylinders show much less respect for this law.

210. From Fig. 61 it is clear that a rectangular parallelepiped offers less resistance than one of its faces turned into the flow.

An experiment with model 63 showed that its resistance is almost the same whether the flow be from the left or the right.

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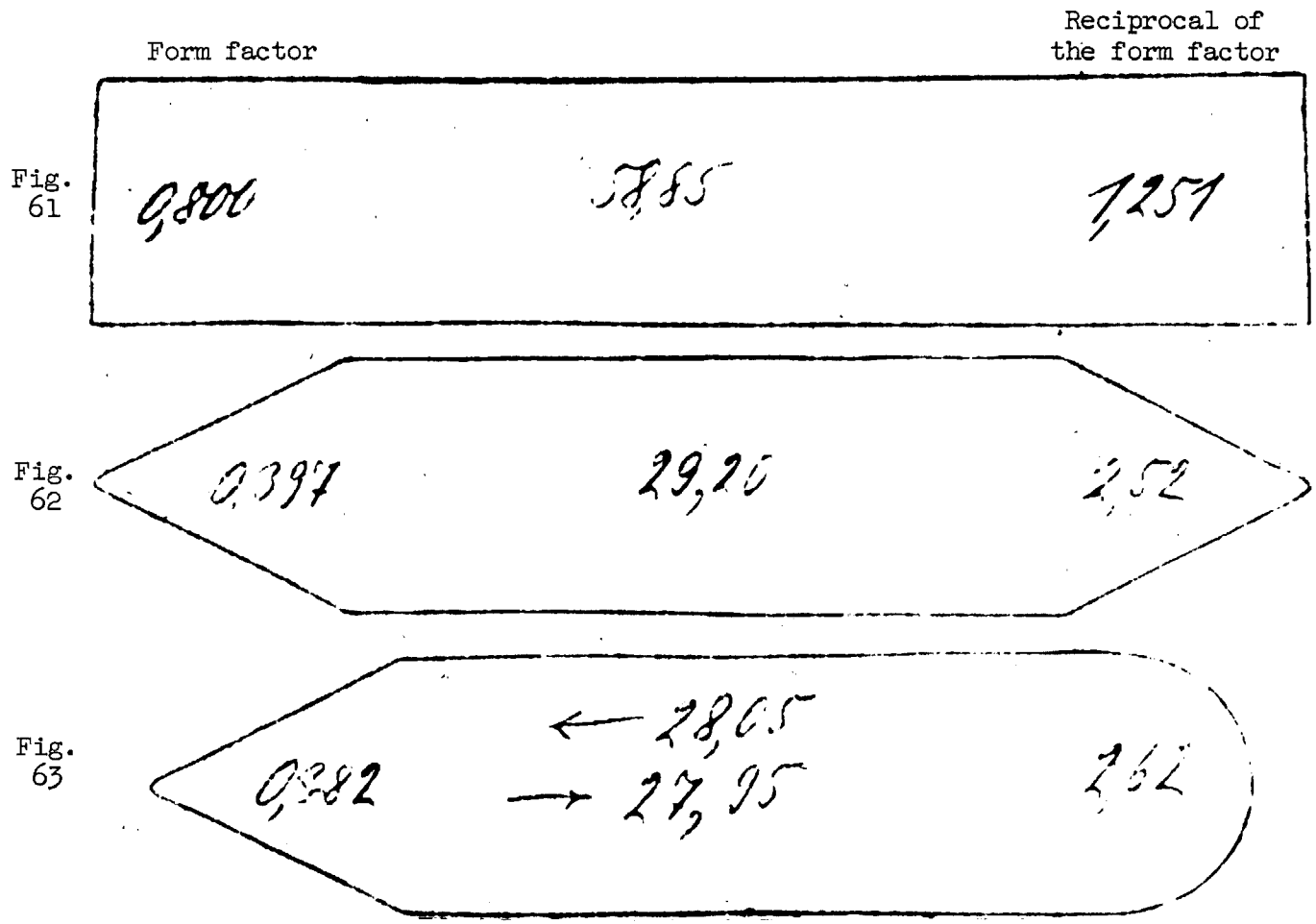
*For an unambiguous investigation of the significance of shape it would be necessary to take a body with an aspect ratio half as great. At high aspect ratios the friction smothers, as it were, the drag due to the inertia of the medium. - Author.

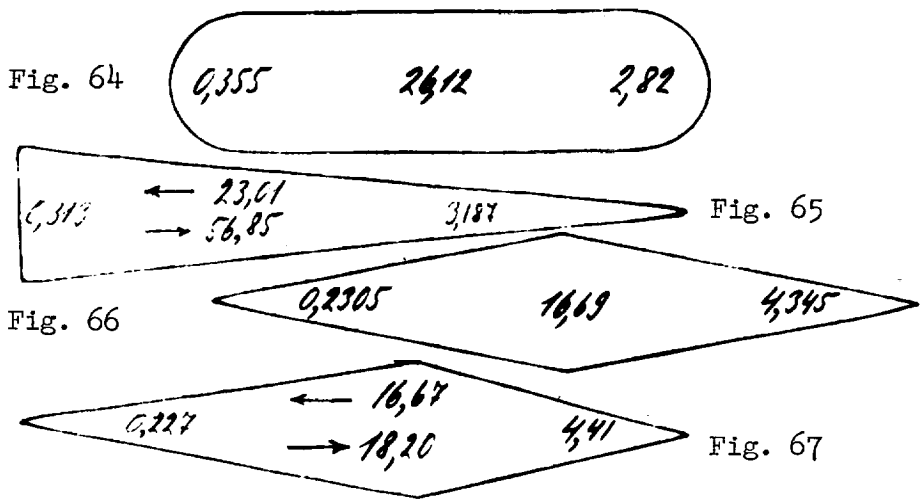


Absolute pressure in mm per 100 sq. cm of cross-sectional area, reduced to 1 m/sec.

Figs. 61, 62, and 63.

211. The same experiment with a wedge (Fig. 65), on the other hand, gave a huge difference; thus, the form factor is 3.187 when the wedge moves forward and only 1.295 when it moves in the opposite direction.

The next two prisms gave a fairly high form factor, up to 4.41. I confess that I did not expect such low resistance from such angular forms. But it must not be forgotten that these form factors increase much less with increase in size and velocity than those of smooth shapes.



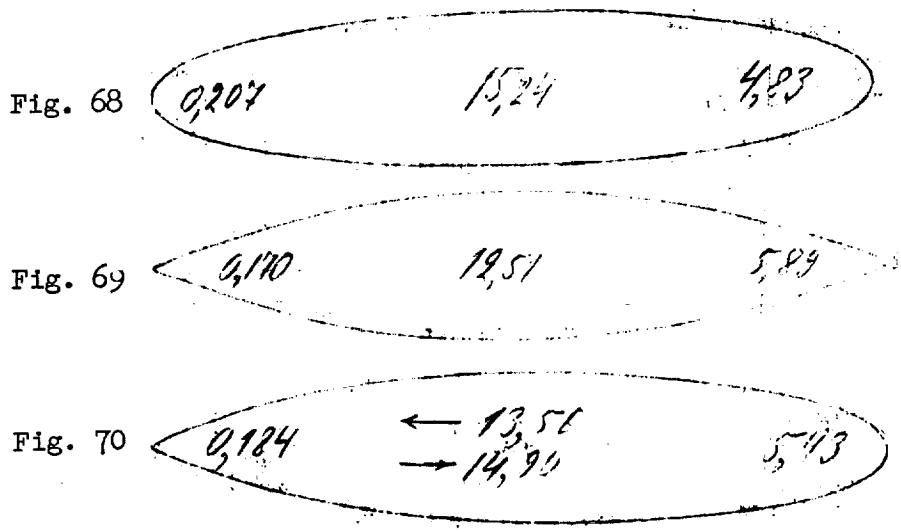
Figs. 64, 65, 66, and 67.

From a comparison of the resistance of an elliptical cylinder (Fig. 68) with that of the models that follow it, I found that this elliptical shape is far from being the best.

Thus, even a cylinder with a base bounded by simple arcs of a circle (Fig. 69) gives a form factor of 5.89, whereas, under the same conditions, an elliptical cylinder gives only 4.83. If we take a shape (Fig. 70) halfway between the shapes of Figs. 68 and 69, i.e., if we leave the front elliptical and form the rear edge with two arcs

of a circle (Fig. 70), we get an average form factor, namely 5.43. But if this shape is reversed in relation to the direction of the flow, we find that the form factor falls to 4.94.

213. The shape shown in Fig. 71, bounded in front by arcs of smaller radius than the shape in Fig. 70, gave a marked increase in the form factor, as might have been expected.



Figs. 68, 69, and 70.

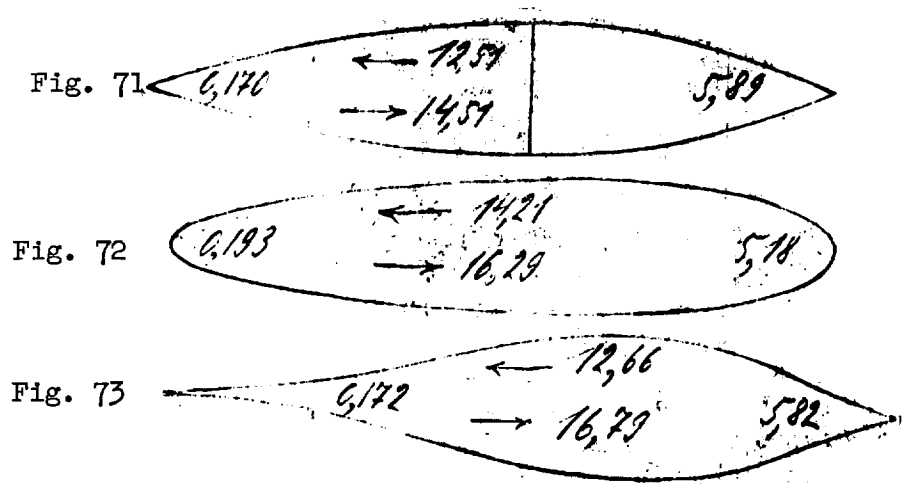
On the other hand, when the model was reversed, the form factor fell sharply to 5.08.

A smooth shape, consisting of two semi-ellipses with different aspect ratios (Fig. 72), did not give a particularly low resistance. The form factor was 5.18, or 4.52 when the shape was reversed with the thinner part in front.

214. The cylinder shown in Fig. 73 gave a form factor of 5.82, which fell to only 4.31 when the model was placed tail first in the flow.

A better performance was given by the shapes in Figs. 74 and 75, which had the same form factor -- 5.94. This is the highest form factor obtained in my experiments on cylinders, at a flow velocity of 3 m/sec. With their tail ends to the flow the same shapes gave the following form factors: the first, with an elliptical head, 4.45; the second, with an angular nose, 4.88.

219. I consider it useful to bring together all my data on various right cylinders and prisms in the form of a single table (cf. Figs. 61-75).



Figs. 71, 72, and 73.

220. It is interesting to compare these figures with data on the resistance of water to flat-bottomed river vessels, which may also be regarded as right cylinders with a curved base, though comparatively very low.

The drag coefficient for well-designed river boats is taken as between 0.16 and 0.20. If, as proposed by Dubois and Duchmin, we take a coefficient of 1.43 for a flat plate moving in a direction perpendicular to its plane, the form factor of the better river boats will vary between 9.0 and 7.15. The average form factor is about 8.1,

which is not so far off the 5.94 of the best of my shapes (Figs. 74 and 75) at a velocity of 3 m/sec. At a velocity of 4 m/sec the form factor even approaches 7.

The form factor for better designed river boats is taken between 6.8 and 5.3, i.e., on the average 6, which is quite close to my experimental values.

Fig. 74

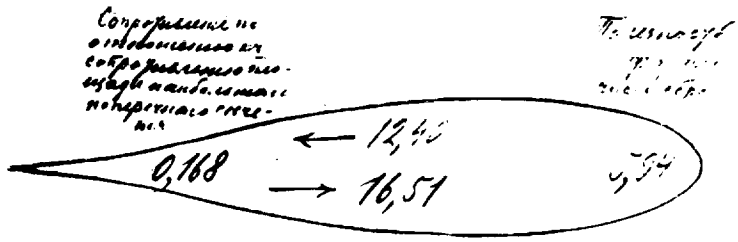
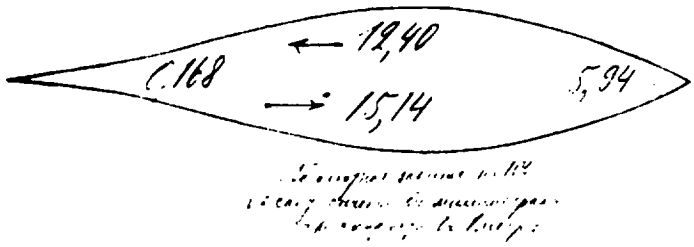


Fig. 75



Figs. 74 and 75.

I am surprised at the similarity between these two different elements. The size of the models is different, it is true, and this should have some effect on the form factor. Actually, I expected large vessels to have a much smaller drag than any assumed in practice; the difference proved to be quite small, and what is more in favor of the large models (ships).

222. I also tested large segments of a right circular cylinder.

No. of figure	Velocity	Resistance per 100 sq. cm of section, reduced to a velocity of 1 m/sec.	Same for reversed flow	Form factor, nose first	Form factor, reversed position	Reciprocal of form factor (nose first)	Same for reversed position
1	2	3	4	5	6	7	8
61	Velocity 3 m/sec (72 pounds); 2 m/sec (32 pounds)	58.85		1.251		0.800	
62		29.20		2.52		0.397	
63		28.05	27.95	2.62	2.63	0.382	
64		26.12		2.82		0.355	
65		23.01	56.85	3.187	1.29	0.313	0.773
66		16.69		4.345		0.230	
67		16.67	18.20	4.41	4.04	0.227	0.248
68		15.24		4.83		0.207	
69		12.51		5.89		0.170	
70		13.56	14.90	5.43	4.34	0.184	0.203
71		12.51	14.51	5.89	5.08	0.170	0.197
72		14.21	16.29	5.18	4.52	0.193	0.222
73		12.66	16.79	5.82	4.31	0.172	0.240
74		12.40	16.54	5.94	4.45	0.168	0.228
75		12.40	15.14	5.94	4.88	0.168	0.206

I took a cylinder with a height equal to the diameter of the base (10 cm) and cut it in two with a plane passing through the axis. One half, the shape of a short trough, was bounded by two semicircular faces; the other did not have plane faces, but a semicircular diaphragm was glued to the middle of the cylindrical surface. These cylinders were arranged with the opening facing into the flow, then reversed, and finally turned sideways. The experiments were performed at a velocity of 1 m/sec.

223. The results are shown below in mm per 100 sq. cm of the projected area of the shape.

	Concave side into flow	Convex side into flow	End face into flow
1. Hemicylinder with two bases	91.68	38.64	48.71
2. Hemicylinder with semicircular diaphragm at center	95.08	46.84	58.61

It is clear that the resistance of the hemicylinder with the central diaphragm is greater, other things being equal, than that of the hemicylinder with the semicircular bases, the difference amounting to 20%.

225. A uniformly curved surface is also part of the surface of a right circular cylinder.

I investigated the resistance of these uniformly (but slightly) curved plates (i.e., plates with a constant radius of curvature).

I took 4 identical tin squares (thickness of tin 0.3 mm) and bent them to different radii.

226. The characteristics of these bent squares, each of which before bending had sides 10 cm long, are given in the table below. To this same table, for the sake of completeness, I have added a hemi-

cylinder with two bases (cf. 222 and 223).

The last column but one shows the ratio of the chord to the altitude of the curve, the last column the reciprocal of this ratio or the degree of bending of the plate.

No.	Chord in mm	Altitude in mm	Ratios	
			$\frac{\text{Chord}}{\text{Altitude}}$	$\frac{\text{Altitude}}{\text{Chord}}$
1	99.3	3.1	32.03	0.031
2	98.0	8.4	11.67	0.086
3	96.0	12.5	7.67	0.130
4	92.3	16.0	5.77	0.173
5	102.5	50.5	2.48	0.493

227. I performed experiments on these cylindrical surfaces exactly like those I performed on rectangular and square plates arranged at an angle to the flow.

However, the pressures I obtained have not been converted to pressures normal to the plates or the chords of the curved surfaces.

If the measuring apparatus is set up as usual, parallel to the flow, the motion of the plate and the floating tank will also be parallel to the flow or into the wind. In this case, by rotating the chord of the plate at different angles to the flow, I could determine the resistance of a cylindrical surface inclined to the direction of the wind itself.

Assuming that the cylindrical surfaces are the wings of a bird or an airplane, with the aid of my experiments, and with the measuring apparatus in the lengthwise position, I could determine the thrust needed to move an isolated wing in, say, the horizontal direction.

228. On the other hand, by arranging the measuring apparatus normal to the flow, I could measure the component of the wind pres-

sure in a direction perpendicular to the flow. Applying this to the wings of a bird flying horizontally, I could find the vertical component of the force exerted by the counterflow, that is, the lift force.

229. The plates were rotated about their vertical axes (as the flat ones had been previously) and exposed to a flow of 1 m/sec.

For the sake of comparison, I have included in the table the data for a flat plate (10 × 10 sq. cm) and determined the value of the required thrust and of the lift force developed (see section 236).

230. It is interesting to note that a lift force (see table in section 226) is generated even when the chord of the normal section of the cylindrical surface coincides with the direction of flow (see table, angle of inclination 0°). This is nothing new, since Lilienthal deduced the same thing from his own experiments, which he performed on a single curved surface (similar to my No. 2), which he found most suitable for an airplane and which, in fact, my experiments, too, revealed to be optimal, since it generates the greatest lift at angles up to 30°.

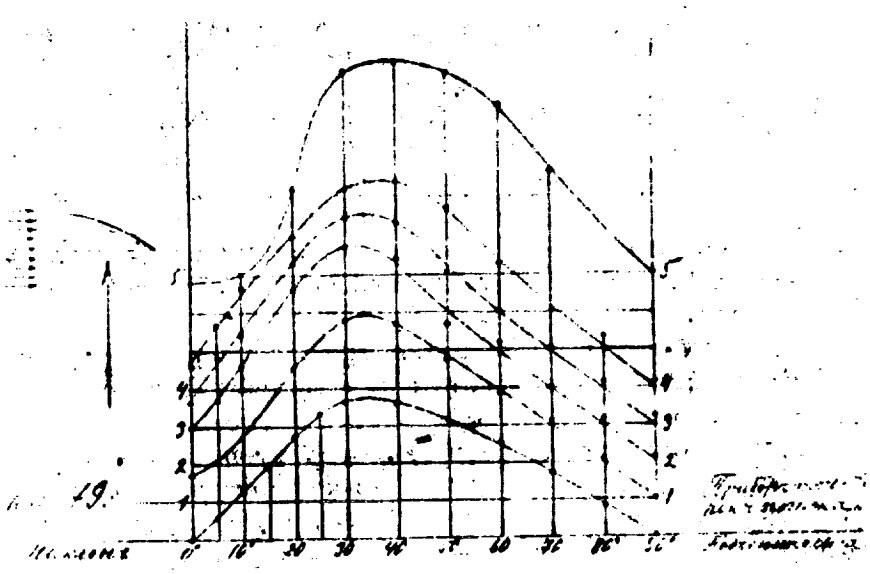


Fig. 79.

231. Before drawing further conclusions from the table, I shall present some curves showing the variation of the lift force (Fig. 79) and the frontal drag (Fig. 80) as a function of the angle of inclination of the chord to the direction of flow.

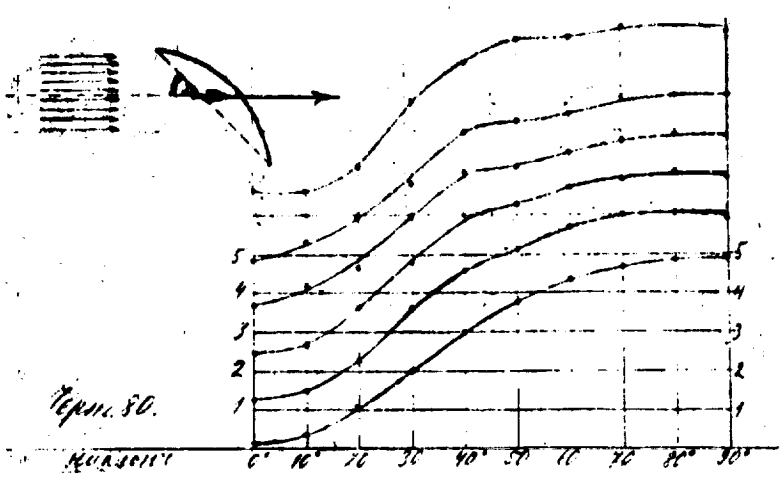


Fig. 80.

The abscissas represent the angle of inclination of the plate to the flow, the ordinates the frontal drag and the lift force at a velocity of 1 m/sec. The y-axis is the same for all the curves; the x-axis varies and is denoted by the corresponding number of the plate (cf. table in section 226).

The lower, unnumbered curve relates to a flat plate, the uppermost to the most highly curved surface, namely a hemicylinder.

At zero inclination all the surfaces develop a lift force (Fig. 79), except the flat plate and the hemicylinder; the latter even, as it were, develops a negative lift force. In any case the lift force is close to zero.

Angle of inclination of chord to flow, in degrees		0	5	10	20	30	40	50	60	70	80	90
Lift force. Load 8 pounds. Velocity 1 m/sec. Concave side into wind.	Flat plate	0	9.28	21.04	41.2	54.6	53.9	47.2	37.5	25.5	12.7	0
	No. 1	11.0	16.4	26.0	53.2	71.8	70.6	56.2	42.8	31.8	16.0	0
	No. 2	14.8	26.0	39.6	69.9	86.4	82.0	55.4	47.8	29.6	18.0	2
	No. 3	10.0	27.6	36.4	65.2	83.2	80.6	61.0	45.6	30.8	17.2	4
	No. 4	9.0	25.4	40.2	60.0	78.8	91.4	72.0	49.6	35.8	20.0	2
	No. 5	-3	. . .	+0.0	+34.9	85.8	84.0	80.3	67.6	41.1	-	-
Convex side into wind	No. 2	14.8	+3.4	+12.2	+29.6	37.4	38.0	38.2	31.8	22.8	13.6	0

[Table cont'd. on next page]

[Table cont'd.]

Angle of inclination of chord to flow, in degrees		0	5	10	20	30	40	50	60	70	80	90
Frontal drag. Load 8 pounds. Velocity 1 m/sec. Concave side into wing	Flat plate	2.0	2.8	5.4	16.0	30.1	45.6	56.3	65.0	70.2	72.2	73.7
	No. 1	3.6	-	7.4	19.0	39.8	54.0	62.2	70.8	75.0	75.2	73.0
	No. 2	7.0	-	9.6	24.6	42.0	60.4	63.8	71.2	74.0	76.5	74.1
	No. 3	9.8	-	13.0	24.0	45.2	62.0	64.0	69.6	74.3	76.0	75.0
	No. 4	12.2	-	19.6	28.4	42.4	62.6	66.8	69.2	75.4	75.8	76.5
	No. 5	24.6	-	24.0	33.7	59.9	74.4	82.9	84.0	91.8	-	86.0

From Fig. 79 it is clear that the lift force quickly increases with increase in the angle of inclination to 30-40°, and then falls, rather less quickly, reaching zero at 90°.

Only the hemicylinder develops hardly any lift force up to 10°.

233. If we continue curves 1, 2, 3 and 4 on the lefthand side, we see that the lift force does not vanish even at a certain small negative angle of inclination, i.e., in relation to the wings of a bird we may say that the front edge of the wings may even be dropped slightly in relation to the rear edge, yet a certain positive lift force will still be developed.

235. The curves of frontal drag for the wing (Fig. 80) show a slight irregularity at 50°; I would not have accepted this, that is, I would not have accepted the results of my experiments, if I had not noted the same irregularity (an unexpected depression) in all the curves: Nos. 1, 2, 3, 4, and 5. Only the curve for a flat plate is without it.

236. By measuring the ordinates of all the curves in Figs. 79, 80 and 81, we get the most probable value of the forces acting on wings arranged in various positions in relation to the wind.

The proposed table (see below) contains 6 horizontal bands, each of which has three rows of figures. The first band relates to a flat plate, the remainder to cylindrical surfaces Nos. 1, 2, 3, 4, and 5, in that order.

The first row in each band gives the lift coefficient K_v, the second the frontal drag coefficient K_f, and the third the lift force corresponding to unit drag, i.e., the last row in each band expresses the quality of the wing.

The row of figures at the bottom of the table gives the lift force for wing No. 2 with the convex side turned into the wind.

237. Let us concentrate our attention on the third row of figures in each band, i.e., on the relative lift force or the lift force corresponding to unit thrust.

It should be borne in mind that all the plates have the same velocity of 1 m/sec and all (except No. 5) have the same area (100 sq. cm). We see that for all the surfaces the relative lift force increases with increase in the angle of inclination only up to a certain point; beyond this it begins to decrease again. Thus, for a flat plate the most favorable angle of inclination to the flow is close to 10° (i.e., the angle at which unit thrust of the engine gives the greatest lift); for cylindrical surfaces Nos. 2 and 1 this angle is not far off

Angle of inclination in degrees		0	2.5	5	7.5	10	12.5	15	17.5	20
K_v - lift coefficient		0	4.4	9.3	14.5	19.5	25.0	30.0	35.5	40.5
K_f - frontal drag coefficient		2	2.2	2.8	4.0	5.5	7.5	10.0	13.0	16.0
Ratio $\frac{K_v}{K_f}$		0	2	3.32	3.50	3.55	3.33	3.00	2.73	2.53
Concave side into wind	No. 1	11 4.0 2.75	13.5 4.5 3.00	16.5 6.0 3.00	21.4 6.0 3.57	26.5 7.5 3.51	33.5 9.5 3.53	39.5 12.5 3.16	46.5 15.5 3.00	53.0 19.0 2.79
	No. 2	14.7 7.0 2.10	20.5 7.0 2.93	27.0 7.5 3.60	34.0 8.5 4.00	40.5 10.0 4.05	48.5 12.5 3.88	55.5 16.0 3.47	63.5 20.3 3.13	70.3 24.5 2.87
	No. 3	11.0 10.0 1.10	18.5 11.0 1.68	25.5 12.5 2.40	32.0 14.0 2.29	39.0 16.5 2.36	46.0 18.5 2.49	53.0 21.0 2.52	60.0 24.0 2.50	66.0 27.0 2.44
	No. 4	10.0 12.3 0.813	18.0 13.3 1.35	24.5 14.7 1.67	31.0 16.5 1.80	37.5 18.5 2.27	44.0 20.5 2.15	50.5 22.8 2.21	56.0 26.0 2.15	61.5 28.6 2.15
	No. 5	12.5 24.0 0.51	12.3 23.8 0.52	12.5 23.5 0.53	13.5 23.8 0.57	15.0 24.5 0.61	18.5 25.5 0.73	25.5 27.5 0.93	35.5 30.5 1.164	49.0 34.0 1.440
No. 2, convex side into wind; K_v		14.8	-	+2	-	+12.5	-	21.5	-	28.8

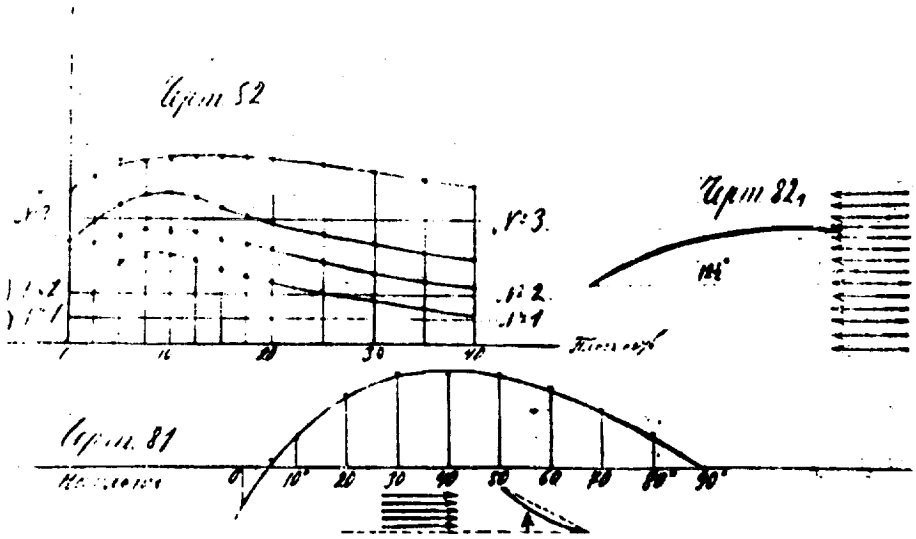
[Table cont'd. on next page]

[Table cont'd.]

Angle of inclination in degrees		25	30	35	40	50	60	70	80	90
K_v - lift coefficient		49.5	55.0	56.0	54.5	47.0	36.6	25.5	13.0	0
K_f - frontal drag coefficient		23.0	30.0	38.0	45.6	56.5	65.0	70.0	72.5	73.5
Ratio $\frac{K_v}{K_f}$		2.15	1.80	1.47	1.19					0
Concave side into wind	No. 1	65.0 28.0 2.32	72.0 39.0 1.85	73.0 47.0 1.56	70.5 53.5 1.32	57.6 62.0	44.0 70.5	30.0 75.0	15.0 75.5	0 74.0 0
	No. 2	81.0 33.5 2.42	86.5 42.0 2.06	84.7 50.5 1.68	84.5 58.3 1.45	60.0 64.0	44.5 71.0	30.0 75.0	15.5 76.0	0 74.0 0
	No. 3	77.0 35.0 2.20	83.3 44.0 1.89	84.0 52.5 1.60	80.5 60.5 1.33	63.0 64.5	44.5 69.5	30.0 74.0	15.0 76.0	0 75.0 0
	No. 4	71.5 30.7 2.33	79.0 45.4 1.74	82.0 54.0 1.52	82.0 62.0 1.32	70.0 61.5	50.4 69.5	32.5 74.5	16.5 76.3	0 76.0 0
	No. 5	82.0 46.5 1.764	95.5 68.3 1.605	99.5 74.5 1.458	100.0 82.5 1.342	95.0 85.7 1.151	91.0 85.7	42.0 87.0	19.3 87.5	0 86.0 0
No. 2, convex side into wind; K_v		34.0	32.5	39.2	39.8	37.5	31.8	23.8	13.5	0

10°; for No. 3 it is equal to 15°; for No. 4 it lies between 10 and 25°; and, finally, for No. 5 the maximum corresponds to 25°. Hence we conclude that, in general, it increases with increase in the curvature of the surface.

Fig. 82 shows the curves of variation of the lift force as a function of the angle of inclination. For a flat plate the maximum corresponds to an angle close to 9°; for Nos. 1 and 2 it is close to 10°; for No. 3 to 15°.



Figs. 81, 82, and 82_1.

238. At the most favorable angle of inclination, the relative lift force for a flat plate is 3.55, and for the other surfaces (Nos. 1-5 in that order) it is 3.57, 4.05, 2.52, and 2.33.

An examination of this series of maxima (236) shows that it has its own maximum, namely for a curvature close to 1/12 or surface No. 2, which coincides with Lilienthal's view on the optimum curvature.

At the optimum angle of inclination and a velocity of 1 m/sec, the flat plate develops an absolute lift force of 19.5, while surface No. 2 (236) develops 40.5, i.e., more than twice as much, and therefore has the same effect as a flat wing with an area more than twice

as great.

In insects the weight of the wings is a negligible part of the weight of the body as a whole and therefore they do not overburden the creature. Here nature refrains from complicating matters and uses the simplest form of wing (mainly flat), since a two- or three-fold increase in the weight of the wings means little to a small insect. At a higher level of creation, economy in the weight of the wings is extremely important, since the weight of the wings of heavy birds, for example, is quite a large proportion of their total weight. Here, too, nature follows the wiser course by making the wings curved and thus reducing their weight and surface area. However, even in insects the wing sheaths are curved, possibly for better shielding and protection.

Apart from these considerations, it is possible that very small surfaces do not obey the same laws as large ones (as in birds).

239. Instead of dividing the lift force on the wing by the thrust needed to move it at a speed of 1 m/sec, we can do the opposite and get the work done by the engine in unit time or, more accurately, the work done by a wing moving horizontally at a speed of 1 m/sec referred to unit lift force. In general, the results obtained may be regarded as the work factor for the flight of a flying machine.

Having presented the table thus obtained (see below), I shall demonstrate its application, and the application of the other coefficients.

240. If we have a curved plate, the forces acting on it are expressed in practical units by the figures of the table in section (236) divided by 1000.

Thus, for example, expressed in kilograms, the forces acting on a square meter of the surface of wing No. 2 moving at a speed of 1 m/sec at an angle of inclination of 10° are:

1. in the direction of motion -- 0.010;

2. in a direction perpendicular to the direction of motion of the plate -- 0.0405.

Let us denote the lift coefficient by K_v, and the thrust coefficient by K_f; then the work factor will be equal to $\frac{K_f}{K_v}$ (see previous table, where the work factors were computed).

241. If the area of the wing is equal to S and the velocity

to V, in general, the lift force P on the wing will be:

$$P = K_v \cdot S \cdot V^2.$$

242. The frontal drag or thrust Q developed by a screw propeller is

$$Q = \frac{K_f}{K_v} \cdot P = K_t \cdot P,$$

where P is the lift force and, at the same time, the weight of the airplane plus load and passengers.

243. The thrust T per unit of time will be

$$T = K_t \cdot P \cdot V.$$

244. From equation 241 we find

$$V = \sqrt{\frac{P}{S \cdot K_v}};$$

now, eliminating V from equation 243, we get

$$T = P \cdot \sqrt{P} \cdot \frac{K_t}{\sqrt{S \cdot K_v}}.$$

Angle in degrees	Work factors relating to the motion of a flying machine					
	0	2.5	5.0	7.5	10.0	12.5
Flat plate	Infinity	0.500	0.300	0.286	0.282	0.300
No. 1	0.364	0.334	0.303	0.280	0.283	0.284
No. 2	0.476	0.342	0.278	0.250	0.274	0.258
No. 3	0.909	0.595	0.490	0.438	0.413	0.402
No. 4	1.23	0.739	0.600	0.532	0.493	0.466
No. 5						

[Table cont'd. on next page]

[Table cont'd.]

Angle in degrees	Work factors relating to the motion of a flying machine					
	15.0	17.5	20	25	30	35
Flat plate	0.333	0.366	0.395	0.465	0.545	0.679
No. 1	0.317	0.333	0.359	0.431	0.542	0.635
No. 2	0.288	0.320	0.349	0.414	0.486	0.596
No. 3	0.396	0.400	0.409	0.455	0.528	0.625
No. 4	0.452	0.465	0.465	0.429	0.575	0.659
No. 5	1.784	0.859	0.694	0.567	0.623	0.686 and at an angle of 40° = 0.745

In these simple equations, based exclusively on experiment, there is, of course, no allowance for the drag on the fuselage and parts of the airplane other than the wings[23].

245. Assuming that one of the quantities on the right hand side of the last equation is varies, while the others remain the same, we may say:

1. The work done by the engine per second must be proportional to the weight of the airplane (with all its contents) to the power 3/2, i.e., proportional to $P^{3/2}$

2. When the ratio $\frac{P}{S}$ is invariable, i.e., when the area of the wings S increases in the same proportion as the weight of the airplane P, the work T done by the engine remains proportional to the weight of the airplane P.

3. The work T is inversely proportional to the square root of the area of the wings S or (assuming square wings) inversely proportional to their linear dimensions.

4. The work T is proportional to the work factor K_t, i.e., to the figures in table (239).

Hence, in order to reduce the expenditure of energy, it is necessary to choose a curvature close to that of surface No. 2.

5. The work T is inversely proportional to the square root of the lift coefficient K_v. Thus, in order to reduce the work to a minimum, it is necessary to take the largest lift coefficient from table (236).

6. The greater the lift coefficient K_v the smaller can be the area of the wings S without detriment to the work T.

7. From the above and directly from equation 244 it is clear that the work T is proportional to the fraction K_t: $\sqrt{K_v}$ or since $K_t = \frac{K_f}{K_v}$, the work is proportional to $\frac{K_f}{\sqrt{K_v^3}}$.

Angle in degrees	5	7.5	10	12.5	15	17.5	20	25	30
From table (239)	0.278	0.250	0.247	0.258	0.288	0.320	0.349	0.414	0.486
K_v	5.20	5.83	6.36	6.96	7.45	7.97	8.38	9.00	9.30
$K_t:K_v$	0.05340	0.04290	0.03883	0.03707	0.03865	0.04015	0.0460	0.05226	0.04165

If there were a mathematical relation between K_f and K_v, it might be possible to find the most favorable ratio of these coefficients.

246. In order to seek a minimum of $\frac{K_t}{\sqrt{K_v}}$, we must take the figures for wing No. 2 and seek particular values of $\frac{K_t}{\sqrt{K_v}}$.

We then get the above figures for different angles of inclination of wing No. 2.

247. From this table we see that the minimum of $\frac{K_t}{\sqrt{K_v}}$, and hence the minimum of the work T done by the engine, corresponds to an inclination of 12.5°. This is also the optimum angle of inclination of the chord of a curved plate to the direction of the relative air flow.

248. Other investigators have arrived at other conclusions concerning the optimum angle of inclination.

Thus, Dzhevetskiy, basing himself on the work of Duchmin and Froude on the resistance of water, gave by analogy 1° 50' as the optimum angle for air, a value very different from mine. The same result was reached by Fedorov (1-2°), who based his conclusions on the latest data on the resistance of air.

Professor Zhukovskiy, however, using Lilienthal's data, finds a value of 15°.

In analyzing this article, Fedorov presents his own calculations, on the basis of which he also finds the optimum angle to be 12-15°. Shirman gives 7° for the same angle.

However, from table 246 it is clear that between 7.5° and 20° the ratio $\frac{K_t}{\sqrt{K_v}}$ varies only slightly (by 16 percent), and therefore at different inclinations within these limits the same value of T is required for flying, so that almost all the above investigators are close to the truth.

For high speeds and large surfaces, however, my result is only approximately applicable. In this case, according to the laws of friction (see "Air Pressure") the optimum angle must be less than 12.5°. The optimum curvature will probably be less, too.

249. If it is advantageous to give the wings of an airplane a certain curvature, then obviously it must be equally advantageous to do the same for the propellers. Thus, I can not agree with the opinion of Hiram Maxim, though based on experiment, that flat propellers are the most favorable.

251. Though some of my conclusions relating to plates are in agreement with those of Lilienthal, the same can not be said in connection with the lift force acting on the wings.

For the forces acting on a curved plate of optimum shape (1/12), Lilienthal gives two equations:

$$N = \eta \cdot 0.13 \cdot S \cdot V^2$$

and

$$T = \zeta \cdot 0.13 \cdot S \cdot V^2.$$

One is for the component normal to the plate, the other gives the component acting parallel to the chord of a normal section of the plate.

In other words, one component is perpendicular to the chord, and the other is parallel to it; η and ζ are coefficients the value of which depends on the angle of inclination of the plate to the flow. At an angle of 90°, $\eta = 1$, i.e., the normal component is equal to $N = 0.13 \cdot S \cdot V^2$. Even from this it is clear that Lilienthal's formula for the wind pressure employs an obsolete and, as we have shown, extremely unreliable coefficient.

In order to avoid this, let us take the normal force at an angle of inclination of 90° as unity. Then the normal force acting on the plate at other angles of inclination, in accordance with Lilienthal's and my own experiments, will be as shown in the table.

At acute angles, such as I have given here, the forces normal to the plate differ only slightly from the lift forces I have calculated myself (third row); see table 236, from which the values given in the last row of this table have been taken.

Actually, Lilienthal has another component φ, but this, being relatively small at all angles, has little effect on the results tabulated. The table also shows that, according to Lilienthal, the lift force on the wings starts to develop at - 9°, whereas according

to my observations it starts at - 4 or - 5°, so that at an angle of 0° my lift force is half as great as his, and in general tends to remain the smaller of the two.

Angle	-9°	-4°	0°	5°	10°	15°	20°	25°	30°	90°
Lilienthal	0	0.200	0.381	0.650	0.825	0.901	0.922	0.922	0.910	1
My experiments	-	0	0.186	0.342	0.513	0.701	0.890	1.025	1.095	1

However, this difference becomes the less the greater the angle; starting from 20° the difference is quite small. Subsequently, my lift force becomes slightly greater than Lilienthal's.

As far as the absolute magnitude of the lift force is concerned, according to my experiments, apart from these corrections, it is almost twice as small, since Lilienthal uses an obsolete coefficient for the force acting on a flat plate normal to the flow ($0.13 \cdot S \cdot V^2$).

253. Equation (244) and table (246) give the thrust for the flight of an airplane (useful work); of course, the propeller, the drag of the fuselage, etc., must add to this work.

On the other hand, however, we see that a fivefold increase in the aspect ratio, in a direction transverse to the flow, can almost double the force acting on the wing, without it being necessary to increase the area.

If this conclusion holds true for curved plates, as it does for flat ones, the lift coefficient K_v must be increased in equation (244), while the values in the last row of table (246) must be reduced.

The constant multiplier in equation (244) is equal to $\frac{K_t}{\sqrt{K_v}}$ or

Angle in degrees	7.5	10	12.5	15	17.5	20	25
K_t	0.286	0.282	0.300	0.333	0.366	0.395	0.465
K_v	14.0	19.5	25.0	30.0	35.5	40.5	49.5
$\sqrt{K_v}$	3.74	4.42	5.00	5.477	5.96	6.36	7.04
$K_t : \sqrt{K_v}$	0.07647	0.063800	0.0600	0.06086	0.06140	0.06211	0.06605

$$T = 0.316 \cdot P \cdot \sqrt{\frac{P}{S}},$$

and according to Lilienthal:

$$T = 0.408 \cdot P \cdot \sqrt{\frac{P}{S}}.$$

I would find this relatively astonishing resemblance between the results more understandable, if the above-mentioned scientists had taken into account the aspect ratio of the wings and made the same assumptions as myself; in fact, Lilienthal, for example, took the coefficient for the force acting on a flat plate as 0.13 instead of 0.071, as I did, in accordance with Cailletet and Colardo.

259. According to Renard, for flat wings $T = 0.932 \cdot P \cdot \sqrt{\frac{P}{S}}$.

In order to compare this formula with my results, we must compile a table for a flat square wing similar to table (246) for cylindrical surface No. 2.

From this it is clear that the minimum work T corresponds to an angle of inclination of the plate to the horizontal of 12.5°, i.e., the same angle as for the optimum curved surface (No. 2).

260. In practical units, in accordance with section 255, the coefficient in the equation for T will be $\sqrt{1000}$ times greater than the value given in this table (0.0600), i.e., it will be 1.8974.

This figure is double Renard's.

For an elongated wing, my coefficient (1.9) -- see section 253 -- must be reduced by $2\sqrt{2}$ times, i.e., by 2.83 times; it will then become 0.6705, which again is not quite in agreement with Renard's coefficient.

This difference is probably attributable, among other things, to the fact that Renard used less elongated wings in his experiments. In general, it would appear that investigators are still little concerned about the aspect ratio of the wings.

equation (255) and the above data, we find: $T = 4658$ kilogrammeters per second or about 62 horsepower. If we allow for the propeller, the drag of the fuselage and other factors, this figure must be tripled.

Actually, for steamships the comparable figure is multiplied (according to Froude) by 2.7; we shall triple it, since we must also bear in mind the drag of the fuselage and the wing braces. We then get an ideally small figure for the work done in flying, or 186 indicated horsepower.

With Maxim's machine the corresponding figure is 363 horsepower or twice as much; and even then the results are doubtful.

256. For Langley's "Aerodrome" $P = 13.6$ kg and $S = 8\ m^2$.

Thus, according to my formula, tripling the figure obtained, we get $3T = 23.9 \cdot 3 = 69.6$ kilogrammeters per second, which is quite close to the actual figure for "Aerodrome," namely, 86.4 kilogrammeters per second (according to Shirman's calculations; see "Vozdukhoplavaniye," No. 4, p. 78).

257. Ader's "Avion" had a wing area of not more than 50 m^2 and a loaded weight of 500 kg. In this case, for "Avion" we find $3T = 1464 \cdot 3 = 4392$ kilogrammeters per second or about 58 indicated horsepower. In fact, it developed about 40 nominal horsepower. Thus, if it had got off the ground with these 40 horsepower, it would have been a miracle of technical efficiency.

258. There is no harm in comparing my formulas with similar ones derived by Shirman ("Vozdukhoplavaniye," No. 4) on the basis of the investigations of Welner, Lilienthal and Renard.

Earlier (255), I have, for ideal wings*, the thrust formula:

$$T = 0.414 \cdot P \cdot \sqrt{\frac{P}{S}}.$$

According to Welner:

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*My subsequent experiments showed that cylindrical wings with a constant curvature are not ideal. - Author.

$\frac{K_f}{K_v \cdot \sqrt{K_v}}$. For the optimum elongated wing, in the best case, we can double the lift coefficient K_v, whence the coefficient in equation (244) is reduced by $2\sqrt{2}$ or 2.83 times, i.e., becomes 0.0131.

254. Hence the formula for the optimal minimal work done by an airplane with fixed wings attached to the fuselage will be

$$T = 0.0131 \sqrt{\frac{P^3}{S}} = 0.0131 \cdot P \sqrt{\frac{P}{S}}.$$

Owing to the effect of the propeller, the drag of the fuselage and its parts, etc., this computed value should be at least tripled.

255. In practical units the coefficient in the latter formula will not be 0.0131. In order to find it, we must turn to table (246); K_t does not change, but $\sqrt{K_v}$ (see 236) must be made equal not to $\sqrt{48.5}$ but to $\sqrt{0.0485}$, since for conversion to practical units K_v must be divided by 1000. Thus, the practical coefficient in equation (254) will be 1000 times greater, namely, 0.414, and the equation will assume the form

$$T = 0.414 \cdot P \cdot \sqrt{\frac{P}{S}}.$$

How far our present airplanes are from satisfying this equation can be seen from the data and computations relating to flying machines.

Thus, for Maxim's airplane P = 3600 kg and S = 370 m^2 (see "Vozdukhoplavaniye," editor M. M. Pomortsev, No. 4, p. 80). Using

261. As I have already pointed out, there is no need to imagine that the optimum inclination of the wings to the flow is something fixed. On the contrary, on the basis of my past ("Air Pressure") and present experiments, with increase in the velocity of the flow or the plate, as well as with increase in its surface area, the coefficient of friction diminishes, and at the same time the optimum angle must diminish also.

262. In the presence of very large surface areas and high velocities we can be guided by the formula (154-155)

$$F = K \cdot i \cdot S \cdot V^2,$$

which I gave for small angles of inclination in relation to a rectangular plate arranged transverse to the flow (long side of rectangle perpendicular to direction of flow). Here the friction is almost completely eliminated.

The formula gives the force normal to the plate; the thrust coefficient or horizontal component is equal to

$$K_f = K \cdot i \cdot \sin i,$$

where K is a coefficient depending on the aspect ratio of the rectangle (154-155), and i is the angle of inclination of the plate to the flow; the lift coefficient or vertical component is equal to

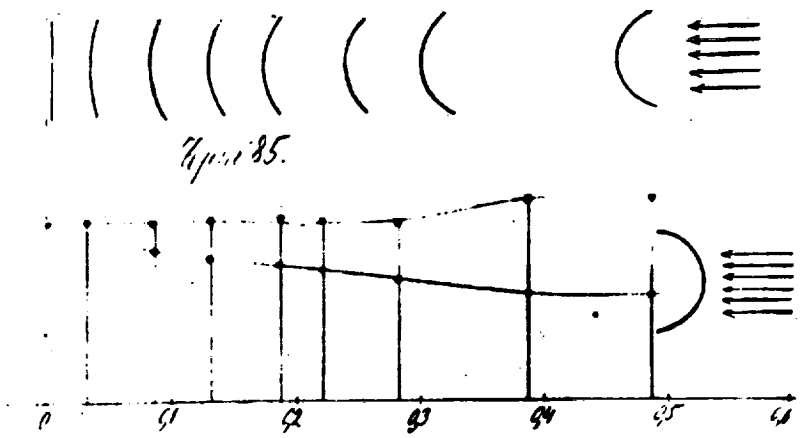
$$K_v = K \cdot i \cdot \cos i.$$

Hence

$$K_t = \frac{K_f}{K_v} = \frac{\sin i}{\cos i} = \tan i$$

and

$$\frac{K_t}{\sqrt{K_v}} = \frac{\tan i}{\sqrt{Ki \cdot \cos i}}.$$



Cylindrical surface

Fig. 85.

Thus, this function or work factor (246) decreases continuously with decrease in the angle of inclination i. Actually, in view of the smallness of the angle i, it is possible to put tan i = i, cos i = 1; hence, $\frac{K_t}{\sqrt{K_v}} = \sqrt{\frac{i}{K}}$, i.e., the work factor is a minimum when i = 0.

Hence in the limit, in the absence of friction and drag, we see that the optimum angle of inclination for an airplane (with flat wings) is equal to zero. In this case, the wings assume an horizontal

Curvature or ratio of altitude to chord	0.000	0.086	0.130	0.185	0.221	0.281	0.3870	0.485
Force per 100 sq. cm of projected area, concave side into flow	74	72.9	74.4	75.6	74.1	72.7	82.6	82.2
Same, convex side into flow	74	61.2	58.8	56.8	54.1	49.7	44.2	42.6
Ratio of forces	1.000	1.192	1.266	1.332	1.371	1.443	1.868	1.930

position, and the work done by the airplane diminishes to zero.

First of all, however, there are various kinds of drag and friction which are quite impossible to avoid; secondly, are we justified in applying the formula for the normal force

$$F = K \cdot i \cdot S \cdot V^2$$

at any, including the smallest, angle?

The extreme limit of experimental application is 2-5°; below this value, as already noted, the pressure falls sharply compared with the value predicted by my formula and those of others, apart from Newton's, which, in this case, may have a certain validity.

Similarly, the optimum curvature of cylindrical wings is scarcely something independent of the size of the wings or the speed at which they move. It is very probable that with increase in the speed and surface area of the wings this curvature, like the optimum angle of inclination, may diminish, and perhaps the work T may diminish at the same time.

267. Upon determining for an inclination of chord to flow of 90° the forces acting on cylindrical surfaces arranged now with the convex, now with the concave side turned into the flow (Fig. 85), we can compile a table (see above) showing the forces corresponding to 100 sq. cm of projected area at a flow velocity of 1 m/sec.

CONTINUATION OF REPORT ON EXPERIMENTS ON THE RESISTANCE OF AIR PERFORMED IN 1901 AND 1902 UNDER THE AUSPICES OF THE IMPERIAL ACADEMY OF SCIENCES*

273. I performed experiments on air friction with the aid of circular cylindrical surfaces, made either out of shiny white tin 1/3 mm thick or drawing (ivory) paper of approximately the same thickness (1/3 mm). The perimeter of the base of all these cylinders was the same, namely, 50 cm.

The height of the hollow cylinders varied; for the tin cylinders with soldered wire legs it was as follows: 2, 3, 4, 5, 7, and 10 cm; the heights of the cylinders made of best ivory paper were: 10, 15, 20, 30, 40, 50, 60, 70, and 100 cm. Altogether there were 15 models. Models up to 15 cm long were mounted on one leg; the longer models on two. The two cylinders 10 cm long -- one of tin and one of paper -- were used to compare the friction at surfaces in different states. The difference proved to be in favor of the tin, but was only small. At high flow velocities the paper models 10 and 15 cm long proved unsuitable, owing to excessive buckling, since they were light, weak, and attached at only one point. Accordingly, they have been omitted from the following tables.

274. The legs of all the cylinders were made of round wire 1.38 mm thick. The tin cylinders had short legs (10.5 cm long), because they were soldered to the bottom of a hoop; the legs of the paper cylinders were 26 cm long, because they passed right through the cylinders from top to bottom. During the experiments, of course, the generator or height of the cylinder was arranged parallel to the flow. By calculation (see 175) I obtained a drag curve for the wire and a drag table for the legs.

275. A direct experimental check on the drag of the legs showed that the figures contained in the table were 9 percent greater than those given by experiment. However, taking into account the finite thickness of the cylinders, i.e., a certain head resistance,

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*"Continuation of Report on Experiments on the Resistance of Air...." was not forwarded to the Academy of Sciences and remained in Tsiolkovskiy's files. It is published here, in slightly abridged form, for the first time. (Ed.)

I left the tabulated figures as they were and deducted them from the total drag of the cylinders.

The drag of the models was also slightly greater than the drag due to friction because it was impossible to eliminate all vibration, which also added to the drag.

The drag was further increased by inaccuracies in the geometry of the cylinders.

Therefore, my values for the drag are, in any case, much higher than the values corresponding to friction alone (see table).

276. The following table shows my figures for the drag of the cylinders, with the drag of the supports deducted as usual.

The first row of the table gives the load in pounds, the first column the height of the cylinder (or the length in the direction of the flow) in centimeters, the second the sum of the inside and outside surface areas of each cylinder in square centimeters.

277. From the table we see that the height or generator of the cylinders varied between 2 cm and 1 meter, and the friction surface between 200 and 10,000 sq. cm or 1 sq. meter.

Taking any row of the table, we see that the friction for any area (same model) increases less rapidly than the load, i.e., the friction is not proportional to the square of the flow velocity, but proportional to V^x, where x is a variable less than two. Thus, from the second row it is clear that when the load increases by 11 times (from 10 to 110 pounds), the drag increases by less than 5 times.

However, this increase becomes more and more proportional to the square of the flow velocity as the length of the cylinder increases. Thus, at a load ratio of 10 (from the second to the eleventh column of the table) for the different cylinders (13 models) we get the following drag ratios: 4.7, 4.5, 4.8, 5.0, 5.2, 4.7, 7.0, 7.2, 7.5, 7.7, 7.6, 7.6, and 7.4*.

In general, there is an undoubted increase in the ratio with increase in the dimensions of the cylinder, the ratio approaching a certain limit, probably 10 (since this is the load ratio). It may be assumed that if the length of the surface were very great (theoretically, infinite), the friction would be proportional to the square of the flow velocity. The departure from this law is the greater the

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*The jump between 4.7 and 7 is due partly to the fact that the length of the cylinder suddenly doubles (see table 276).

Height of cylinder, cm	Total friction surface: inside + outside, sq. cm		Load in pounds						
			10	20	30	40	50	60	70
2	200	Tin	5.3	8.6	11.2	12.3	14.1	16.3	18.4
3	300		6.3	9.4	12.4	15.2	17.2	19.1	21.4
4	400		7.1	11.4	14.1	16.3	19.6	22.3	25.7
5	500		8.1	12.9	16.2	20.1	24.1	26.8	29.1
7	700		9.2	15.4	19.8	23.9	28.5	31.9	36.8
10	1000		11.7	18.6	25.4	31.2	34.6	39.4	44.2
20	2000	Paper	15.0	27.2	38.2	47.4	57.4	67.0	73.4
30	3000		20.0	35.6	50.4	62.4	77.8	92.6	102.8
40	4000		25.6	46.0	64.0	82.8	103.0	117.8	135.4
50	5000		29.4	54.4	77.8	97.6	119.2	141.8	163.6
60	6000		36.0	65.0	92.4	119.2	147.2	172.4	194.0
70	7000		40.0	78.6	106.0	133.6	163.2	197.2	230.0
100	10000		55.0	98.0	140.8	180.2	225.0	259.6	299.6

[Table cont'd. on next page]

[Table cont'd.]

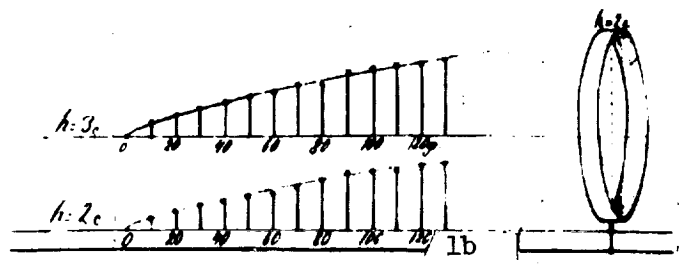
Height of cylinder, cm	Total friction surface: inside + outside, sq. cm		Load in pounds					
			80	90	100	110	120	130
2	200	Tin	21.6	24.6	25.1	25.6	27.7	28.7
3	300		21.6	27.0	28.5	30.1	30.4	31.9
4	400		26.3	32.2	33.9	36.1	38.0	39.4
5	500		31.3	36.5	40.2	42.6	45.1	46.1
7	700		39.0	45.3	47.3	51.0	53.8	58.4
10	1000		49.2	51.3	54.8	59.0	69.2	65.9
20	2000	Paper	84.0	99.6	105.0	115.2	128.8	129.2
30	3000		124.0	134.0	143.4	156.8	168.8	180.4
40	4000		151.0	174.0	191.4	204.8	221.6	237.2
50	5000		191.6	206.8	225.8	247.2	264.5	278.0
60	6000		216.4	246.8	274.6	284.8	306.4	325.4
70	7000		254.0	278.8	304.2	326.4	358.4	380.4
100	10000		332.4	366.0	409.0	429.6	476.0	504.4

Experiments performed 21-24 August, 1904. Temperature 20°C, air pressure 745-760 mm.

less the length of the cylinder in the direction of the flow. Hence, x will be less than two and tend closer to unity the shorter the length of the surface.

Fig. 89

Fig. 88



Figs. 88 and 89.

278. Now let us examine the vertical columns of the table or the increase in friction with increase in length at a given flow velocity.

The length of the cylinder and the area of the friction surface increase by 50 times; however, the friction is far from 50 times as great, the multiplier varying with the flow velocity.

Thus, for various loads from 10 to 130 pounds (between the second and last rows of the table), we get the following ratios (reading from left to right), as the height of the cylinder increases by 50 times: 10.4, 11.4, 12.6, 14.7, 16.0, 16.0, 16.3, 15.4, 14.9, 16.3, 16.8, 17.2, and 17.6.

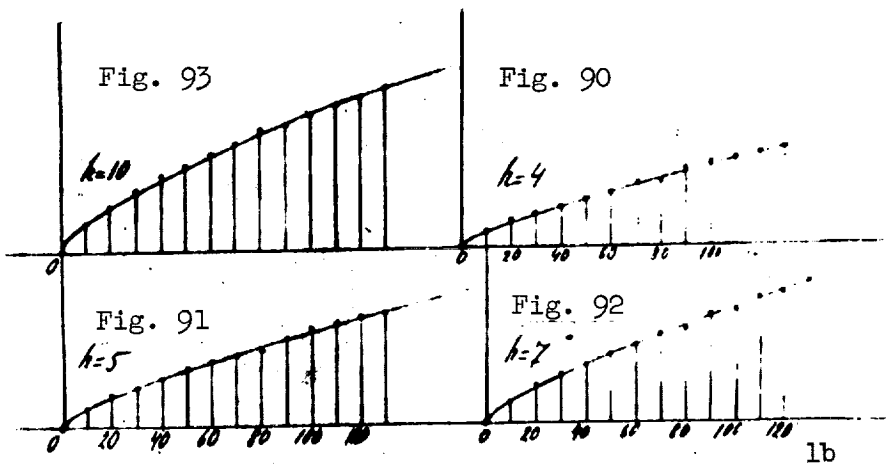
Hence it is clear that at any flow velocity the drag due to friction increases much more slowly than the area of the friction surface or the length of the surface in the direction of the flow, namely, 5-3 times more slowly.

At first, at smaller loads, the ratio increases rather quickly, then it begins to fluctuate and, in general, increases very slowly. Whether the ratio reaches 50, i.e., the ratio of the surfaces (or lengths), at any flow velocity is uncertain, but possible.

Thus, at a given flow velocity, the friction is proportional to l^y, where l is the length of the cylinder, and y is an exponent

less than unity and differing the more from unity the less the velocity of the flow.

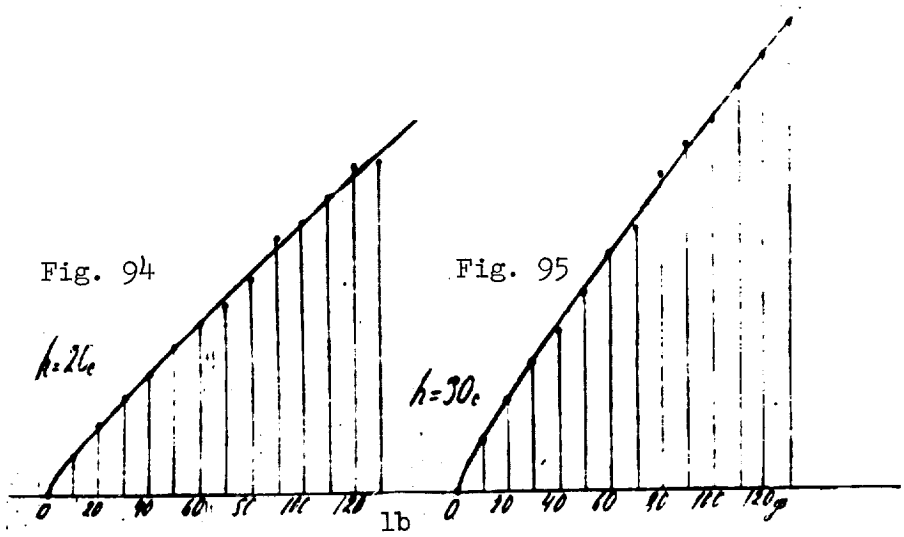
279. Each row of table (276) corresponds to a curve expressing the friction as a function of the flow velocity. The abscissas give the load in pounds, the ordinates give the friction in millimeters. Thus, we obtain Figs. 88, 89, 90, 91, 92, 93, 94 and 95. Starting from a length of 40 cm, the absolute value of the friction for the cylinders is extremely large and accordingly is expressed not in whole millimeters but in fractions of millimeters. As compared with the previous figures the scale of Figs. 96, 97, 98, 99 and 100 (actually, only the ordinates) is as follows: 1/4, 1/5, 1/6, 1/7, and 1/10.



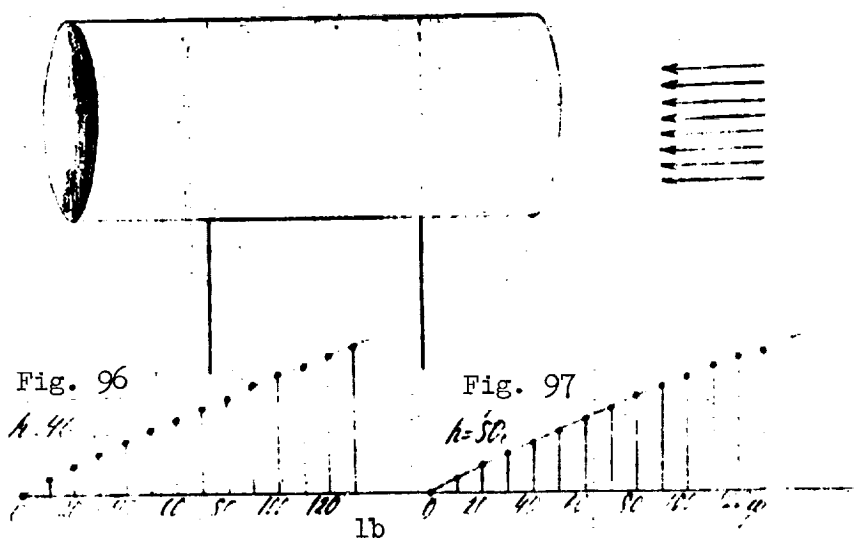
Figs. 90, 91, 92, and 93.

From these figures it is clear that the friction curve for any cylinder may evidently be represented by an arc of an hyperbola.

280. The majority of the curves do not change the sign of the radius of curvature. The latter increases continuously with increase in the abscissas or ordinates. The greater the length of the cylinder,



Figs. 94 and 95.



Figs. 96 and 97.

the closer the arc of the hyperbola to a straight line, i.e., the further the vertex of the hyperbola from the origin of the coordinates. From the closeness of the arc to a straight line it is clear that with long cylinders the friction is proportional to the load or the square of the flow velocity, a conclusion that we have already reached elsewhere (see section 277)[24].

Fig. 100

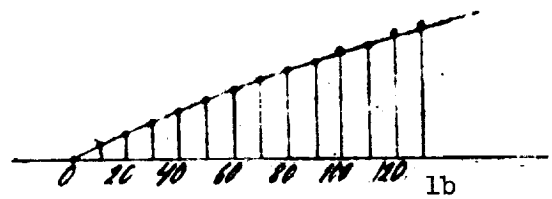


Fig. 98

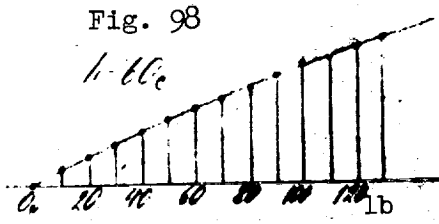
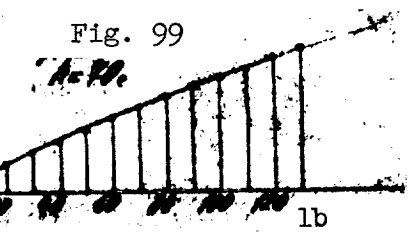


Fig. 99



Figs. 98, 99, and 100.

290. The law I derived in relation to friction from my previous experiments ("Air Pressure," Nos. 88-93) is approximately applicable within the limits of those experiments, but I cannot allow its applicability outside the limits of the experiments (see "Air Pressure," No. 93), since it is far from being in agreement with the results of later experiments on longer surfaces. Let us compare the data of these earlier experiments ("Air Pressure," No. 93) with the more recent ones. In my original table I gave the ratio of the drag on a flat plate in a normal flow to the drag on one side of the same plate moving in the direction of the flow.

These are figures from the table based on my earlier experiments (No. 93). For a surface 100 cm long these figures are already theoretical.

Length of surface in cm	Velocity in m/sec		
	1	2	4
10	68	90	119
100	160	212	279

From the table based on my present experiments I find:

Length of surface in cm	Velocity in m/sec		
	1.14	1.97	3.95
10	80.6	116.5	162.5
100	167.3	193.0	233.7

The difference is considerable, but in experiments of this kind it should not reduce one to despair. In any case, the later figures are closer to the truth, since they are based on more numerous and more accurate experiments.

291. The practical formula for the friction force acting on one side of a rectangle of area S m^2 at a flow velocity of V meters per second has the form:

$$F = \frac{K}{1000} \cdot S \cdot V^2 \text{ kg},$$

where K is a coefficient selected in accordance with the velocity V and the dimension of the rectangle in the direction of flow. For example, for a length of 1 meter and a velocity of 4 m/sec, we get

$$F = \frac{0.315}{1000} \cdot S \cdot V^2 \text{ kg}.$$

If $S = 1 \text{ m}^2$, $V = 4$ m/sec, then $F = 0.00504$ kg or about 5 grams.

292. From the results of the experiments we can plot the curves shown in Fig. 118, which give the drag as a function of the load (in pounds). For all the lengths tested (2 to 100 cm in the direction of flow), these curves resemble arcs of hyperbolas, as in the graphs expressing the drag as a function of length.

294. For this reason it occurred to me to express the friction force as a function of the length of the plate and the load by means of a second-order equation in three variables

$$z^2 + Ax^2 + By^2 + Cz + Dxy + E = 0.$$

Here z is the friction (in millimeter units) on one side of a rectangle 100 cm wide and y cm in length (in the direction of flow); x denotes the load in pounds.

295. In order to determine the constants in equation (294), namely, A, B, C, D, and E, I substituted five times particular values of x, y and z.

I thus obtained five equations with five unknowns (A, B, C, D, and E).

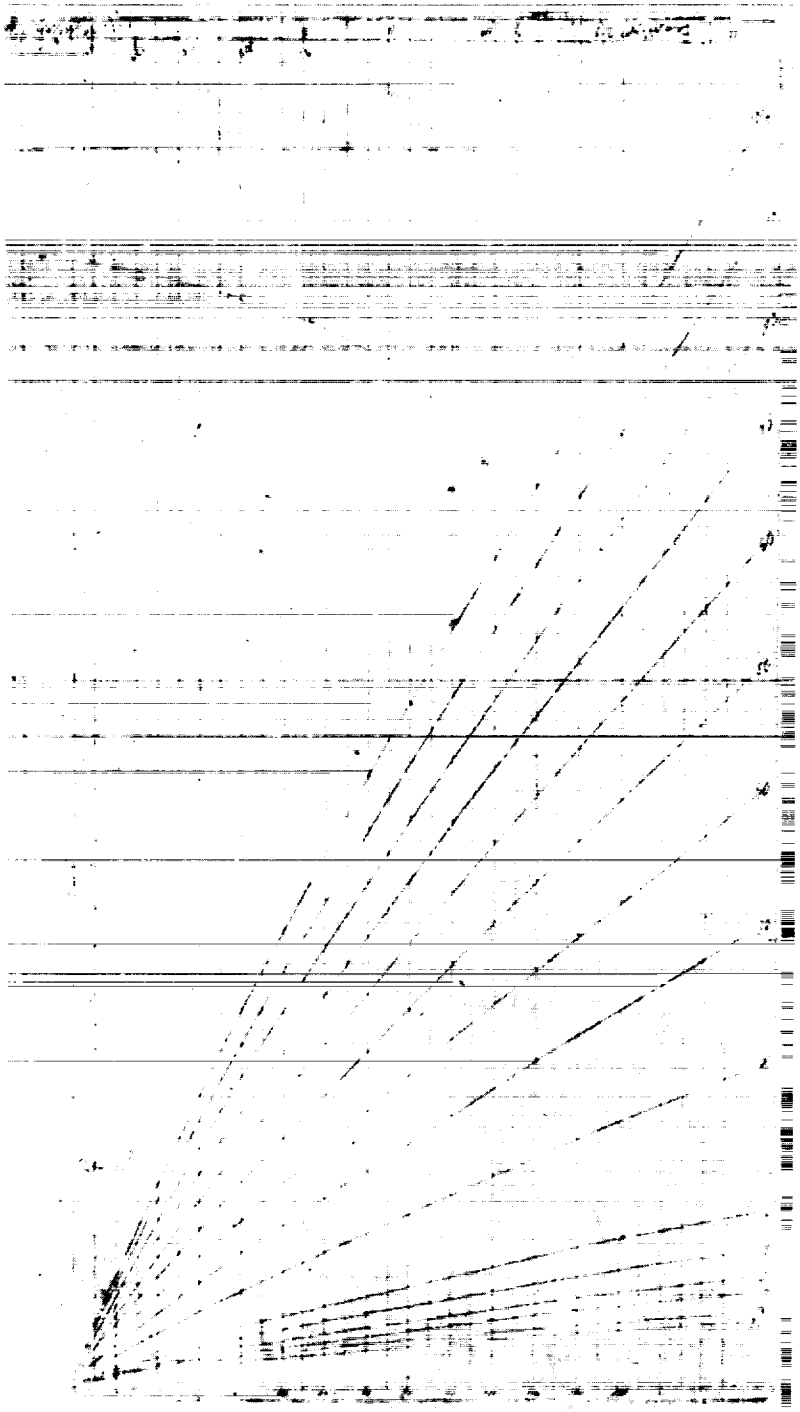


Fig. 118.

Here are the five points I took with their coordinates:

Point	1	2	3	4	5
Load in pounds, x	20	20	60	120	130
Length in direction of flow in cm, y	5	100	40	5	100
Friction in millimeter units, z	12	100	118	44	506

296. Solving these five equations, I found the following values for the constants:

$$A = -1.0911;\ B = -0.2835;\ C = +1760.2;\ D = -85.686;$$

$$E = -11\ 376.56.$$

All the constants except C are negative.

297. From equation (294) I obtain:

$$z = \sqrt{-Ax^2 - By^2 - Dxy - E + \frac{C^2}{4}} - \frac{C}{2}.$$

Substituting numerical values of the coefficients, I find that all the terms under the root sign are positive $\left(\frac{C}{2} = 880.1;\ \frac{C^2}{4} - E = 785.952\right)$.

The difference between experiment and calculations based on the above formula is the greater the smaller x and y. For x and y greater than 10, the formula corresponds fairly closely with experiment, but for values of x and y smaller than 10 cm or 10 pounds, it is necessary to take another formula and other experimental points in order to determine the constants. Thus, it is possible to take the equation of a conical surface, the vertex of which coincides with the origin of the coordinates and which passes through the axes x and y.

The equation of this surface will be

$$zy + Azy + Byx = 0.$$

In order to determine the constants A and B, it is enough to take two experimental points; we then find:

$$A = \frac{3}{7} \text{ and } B = -\frac{11}{7},$$

i.e.,

$$z = \frac{-Byx}{y + A} = \frac{11 \cdot yx}{7y + 3x}.$$

For loads greater than 10 pounds and a length greater than 10 cm the formula gives results in sharp disagreement with experiment and therefore is not applicable in these circumstances.

If y = 10 and v = 10 m/sec, and at somewhat greater values of y and v, K may be expressed approximately as:

$$K = \frac{1}{10^4}\left(\sqrt{\frac{131 \cdot 133}{v^2y}} - \frac{1239}{v^2y}\right) = \frac{0.0362}{v\sqrt{y}} - \frac{0.1239}{v^2y} = 0.00102.$$

298. At a load of x pounds the wind pressure normal to a plate 100 cm wide and y cm long will be equal (in mm units) to

$$0.092 \times 100 \times y \cdot x = 9.2 \cdot xy = f \cdot xy.$$

Accordingly, other things being equal, the coefficient of friction or the ratio of the friction force to the normal force acting on the plate may be expressed by:

$$K = \sqrt{\frac{A}{f^2y^2} - \frac{B}{f^2x^2} - \frac{D}{f^2xy} + \frac{C^2 - 4E}{4 \cdot f^2\ x^2y^2}} - \frac{C}{2fxy}.$$

305. In accordance with sections 297 and 298, the coefficient of friction will be:

$$K = \sqrt{\frac{A_2}{y^2} + \frac{B_2}{v^4} + \frac{D_2}{v^2y} + \frac{C_2}{v^4y^2}} - \frac{E_2}{v^2y},$$

where A_2, B_2, D_2, C_2, and E_2 are also positive and constant.

306. This equation may also assume the form:

$$K = \sqrt{\frac{-A}{f^2y^2} + \frac{-B}{f^2g^2v^4} + \frac{-D}{f^2gv^2y} + \frac{C^2 - 4E}{4f^2g^2v^4y^2}} - \frac{C}{2fgv^2y}.$$

Here $f = 9.2$, $y = 7.72$, $f^2 = 84.64$, $fg = 71.024$, $f^2g = 653.422$, $f^2g^2 = 5075.1$, so that for equation (305) we get: $A_2 = 0.00128913$, $B_2 = 0.00005586$, $D_2 = 0.131133$, $C_2 = 154.868$, $E_2 = 12.392$.

307. If y is expressed in meters, equation (306) will assume the form:

$$K = \frac{1}{10^4}\left\{\sqrt{\frac{12.89}{y^2} + \frac{5586}{v^4} + \frac{131133}{v^2y} + \frac{1548680}{v^4y^2}} - \frac{1239}{v^2y}\right\}$$

i.e., it has a value of about 1/1000. According to the theory derived from previous experiments ("Air Pressure," No. 93), $K = 1/945$. Hence it is clear that the previous equations, though very dissimilar in form from the newer and simpler ones, give results not very different from those of the present computations (at least for not very large values of v and y).

In the last equation it is possible to discard the second term, which is comparatively small, and we then get the even simpler formula:

$$K = \frac{0.0362}{v\sqrt{y}},$$

which is valid for medium values of v and y greater than 10. This equation shows that, in this case, the coefficient of friction is approximately inversely proportional to the flow velocity (Hagen's law) and inversely proportional to the square root of the length y of the

surface in the direction of flow, i.e., the absolute friction will be proportional to the flow velocity v and the square root of the length of the surface y.

309. The drag of elongated solids of revolution (of the dirigible type) is composed of two parts: 1. the drag due to inertia, and 2. the drag due to friction. We now know the latter and have even formulated the coefficient of friction:

$$K = \frac{0.0362}{v\sqrt{y}},$$

where it is understood that the friction is that of a rectangle or cylinder, the length of which in the direction of flow is y meters, while the flow velocity is v m/sec. The absolute value of the friction force (in kilograms) on the outside of a circular cylinder of diameter D and length y will be, in accordance with (43):

$$K \cdot 0.071 \cdot S \cdot v^2 = K \cdot 0.071\ (\pi \cdot D \cdot y)\ v^2 = \frac{0.0362 \cdot 0.071}{v \cdot \sqrt{y}} \cdot \pi \cdot D \cdot y \cdot v^2 =$$

$$= (0.0362 \cdot 0.071 \cdot \pi) \cdot D \cdot \sqrt{y} \cdot v = 0.0080 \cdot 7237 \cdot D \cdot \sqrt{y} \cdot v.$$

If we imagine a hollow circular cylinder of the same length y and the same diameter D, as the solid of revolution, the friction of the cylinder will not be as many times greater than the friction of the solid as the area of the cylinder is greater than the area of the solid. Here it is necessary to introduce (into the above formula) a special correction factor.

310. In order to determine this factor, I performed a series of experiments on the friction of plane surfaces bounded by arcs of

circles (Fig. 119).

Fig. 119a.

There follows a table of these surfaces which were easily bent so that they formed, as it were, a part of the previous experimental cylinders.

311. These surfaces were arranged as usual in the central part of the flow and in the same direction. The supports used were the same as for the cylinders 26 cm long.

312. Deducting the drag of the supports, I obtained the following value for the surface corresponding to 100 cm of the perimeter or double width of the surface (I assume that the friction is proportional to the width of the surface).

313. Assuming that the increase in friction is proportional to the increase in the length of the rectangle (for a small increase the error is negligible), I determined the friction of the rectangles for

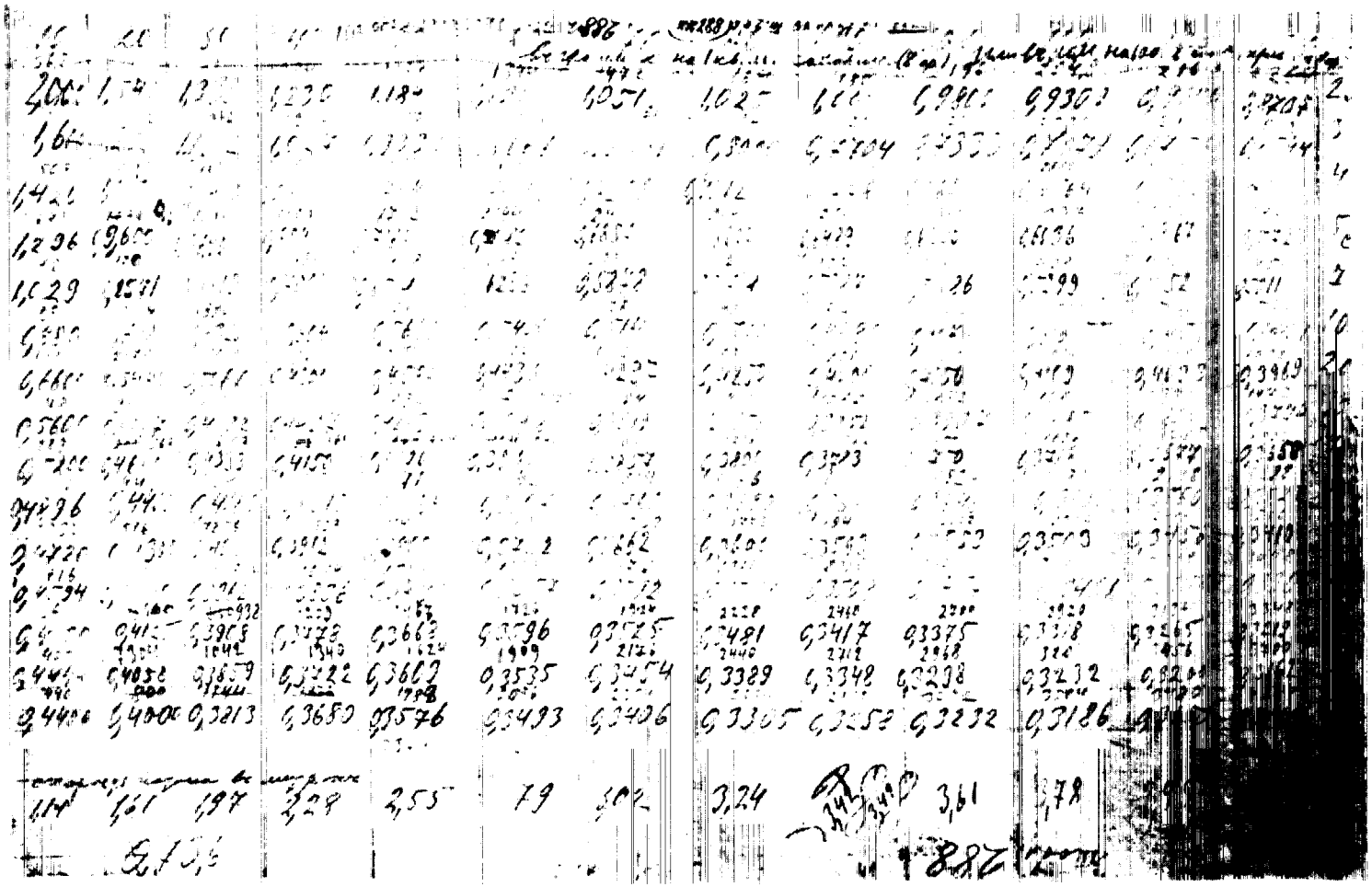


Fig. 119b.

No.	Length in direction of flow in cm	Width in cm	Aspect ratio	Perimeter - double width
1	28.1	28.1	1.000	56.4
2	40.6	25.7	1.580	51.6
3	34.5	14.0	2.464	28.2
4	37.1	12.5	2.968	25.2
5	56	15.7	3.567	31.6
6	70	15.9	4.402	32.0

Load (pounds)	10	20	30	40
1	18.9	32.27	49.82	64.18
2	22.20	37.40	54.07	68.41
3				58.51
4				66.27
5	23.73	43.67	62.66	79.11
6	30.00	56.56	80.00	99.10

No.	Aspect ratio	10	20	30	40 pounds
1	1.00	0.900	0.933	1.013	1.024
2	1.58	0.8476	0.8037	0.8223	0.8144
3	2.46				0.8070
4	2.97				0.7531
5	3.57	0.7084	0.7194	0.7320	0.7152
6	4.40	0.7463	0.7695	0.7692	0.7378

a constant double width of 100 cm and a variable length (in accordance with table 310).

Now, having compared the friction of a tapered surface with that of a rectangle of the same length and width, I got the following ratios (see table on p. 277).

314. From the table we see that, in general, this coefficient is less than unity, which is understandable since the area of the curvilinear figure is less than that of the rectangle. For a circle the conclusions are doubtful: the coefficient is too large, but this is due partly to the high drag of its 1-mm thick edges. The drag of these edges and their different inclination to the flow may perhaps also account for the small difference between the coefficients for the figures with different aspect ratios. Excluding the circle and taking their average value, these coefficients will be:

Aspect ratio:	1.58	2.46	2.97	3.57	4.40
Coefficient:	0.822	0.807	0.753	0.719	0.756

AIR RESISTANCE AND FLIGHT IN AIR*

In 1899 I requested funds from the Imperial Academy of Sciences in order to carry out experiments on air resistance. In this request I indicated my previous experiments, which had been described in Vestnik opytnoy fiziki (also available as a separate publication: "Pressure of Air....") and in Trudy Moskovskogo otdeleniya Obshchestva lyubiteleyyestestvoznaniya (Transactions of the Moscow Section, Society of Amateurs of Natural Science).

The Academy charged Academician M. A. Rykachev with consideration of my work. His favorable report and recommendation to the Academy led the latter to provide a grant (470 rubles) for new experiments, whose lines I had indicated to the Academy. This was in May 1900. I then acquired as equipment a large air blower (vaned air blower or fan, resembling a winnowing machine), which gave a flow of height and width one arshine (71.12 cm). Measuring instruments were also constructed. All of this equipment was taken down and reassembled no less than 6 times before a satisfactory air flow was obtained.

At the end of 1900 I could begin to make models and perform experiments. By the end of the following year (December 16, 1901) I had carried out some of the proposed program and submitted a progress report to the Academy. This report represented merely extracts from my manuscripts; it was 80 pages of text with 58 tables and drawings. Here I deal with the most important and practical conclusions from this work.

Neither descriptions or drawings of apparatus, nor descriptions of the experiments, are given -- there is no space for this here; for these details I refer where appropriate to my separate publication ("Pressure of Air...."), although the drawing of the principal machine given there is incorrect.

At the time of my report to the Academy I had not completed the program, and I was still far from exhausting or solving the problems discussed in it. There is still much to do before I will have solved the problems arising. This will probably require fresh work with better apparatus in order to obtain more accurate results in some cases.

Here it seems desirable to explain some of the terms I shall use most extensively in what follows.

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The elongation or aspect ratio of a body is the ratio of its length to the diameter of its largest cross-section. By diameter is meant here that of the circle having the area of the cross-section. The projection of a body is the area of the projection on a plane perpendicular to the air flow; its magnitude is, of course, dependent on the position of the body relative to the flow.

The form factor is a number indicating by how many times the force of wind pressure on the plane projection exceeds the resistance of the body itself; in relation to flight in air, this number actually represents the advantage of the body as regards ease of parting of the air. The reciprocal of this ratio is called the resistance coefficient.

All values and data relate to air of density 0.0012 of the density of water.

The quantity K in

$$P = \frac{K}{1000} \cdot S \cdot V^2,$$

is a coefficient giving the pressure force P in kg on any surface.

Here S is usually (not always) the projection of the body and is always expressed in square meters; V is the speed of the air flow and is always in meters per second; and K is a variable dependent on S and V, which has to be found by experiment and which is given in the tables below.

The aspect ratio of a rectangle is the ratio of its dimension along the flow to its dimension along a direction normal to the flow. Sometimes it is simply the ratio of the length to the width.

Plane Figures: Wind Normal to Figure

1. The wind forces on unelongated equal figures (ones having the same area) without holes are equal.

For instance, the forces from a wind of a given speed on a circle and on the square of the same area are equal.

2. The forces from a wind of a given speed on elongated figures of a given area (rectangle, ellipse) and on ones with holes

(ring, lattice) are larger than those on continuous and unelongated bodies (see 1). The difference in the forces for a given area and wind speed may be as much as 40%.

This explains the good effects of sails with holes.

3. The wind forces on geometrically similar figures are proportional to the area and to the square of the flow speed. This law is followed, for example, by squares, circles, ellipses of fixed axial ratio, and rectangles with a fixed ratio of the sides or a fixed elongation.

Aspect ratio of rectangle	K	Aspect ratio of rectangle	K	Aspect ratio of rectangle	K
1	73	8	90	16	95.7
1.5	74	9	91	17	96.1
2	76.5	10	92	18	96.5
3	80.5	11	92.8	19	96.9
4	83.5	12	93.5	20	97.2
5	85.5	13	94.1	30	98.4
6	87	14	94.7	40	99.4
7	89	15	95.2	50	100.1

4. A practical formula for the force on a figure subject to geometrically similar variation is

$$P = \frac{K}{1000} S \cdot V^2,$$

in which P is in kg, K is a constant to be determined from experiment, which is constant for a given shape and air density, S is area in square meters, and V is wind speed in meters per second.

5. For figures without holes and unelongated (see 1) I have taken K as 73 for air densities close to 0.0012 of the density of water, which is almost exactly Cailletet and Colardo's value.

6. The force on a rectangle of fixed area from a wind of fixed speed increases with the aspect ratio; K in the above formula thus increases with the aspect ratio. The above table gives the relation of K to aspect ratio as found by experiment.

This relation I have plotted as a curve, with K as ordinate and the aspect ratio as abscissa. This curve gives K for any aspect ratio.

The curve and table show that the force on a square is almost exactly that on a rectangle of small elongation (for a fixed area and wind speed) up to aspect ratio of 1.3; then the force increases with the aspect ratio, at first fairly rapidly, but then more slowly as the limit is approached, at which the force exceeds that on the square by 40%.

7. The formula of (4) may be used to determine the force on a rectangle by taking the coefficient corresponding to the aspect ratio from the table or the curve. I think that this formula is also applicable to an ellipse.

Normal Pressure on a Rectangular Plate Inclined to an Air Flow

8. The resultant of all the wind forces on an inclined plate is almost normal to the latter; this is the force I envisage,

and I therefore call it the normal-pressure force.

9. The force on a rectangle or other figure having a fixed position (perhaps inclined) relative to the flow is proportional to the square of the flow speed.

10. The wind force on a rectangle (including a square) having one of its sides (the long or the short one, not equivalent) perpendicular to the flow is given in the following table for unit area and for a flow speed of one meter per second. The numbers also represent the K of formula (4), which is thus suitable for the determination of the force on an inclined square or rectangle.

11. If the angle of inclination is 90°, the K to be used in (4) are best taken from (6).

12. With the angle as abscissa and K as ordinate, the force can be expressed in relation to the angle as a curve for any given elongation. The table gives the aspect ratio as negative when the short side is perpendicular to the flow, and as positive when the long side is normal to the flow. This curve enables one to find the pressure corresponding to any inclination.

13. Aspect ratios from -5 to +5 may be taken as abscissa to obtain curves each one representing the relation of aspect ratio to force for a fixed inclination.

14. These curves all represent sections of a surface whose equation is $z = F(x,y)$, which I cannot determine until more exact experiments are done with improved apparatus. This equation shows that the force z or coefficient K is a function of the inclination x and aspect ratio y of the plate.

15. The columns in the table show that K increases continuously for aspect ratios from -5 to +5 for any given inclination up to 25°; for instance, an aspect ratio of 5 together with an inclination of 10° would imply a force on one side of the plate with its long side perpendicular to the flow that is 5 times the force when the short side is perpendicular to it.

16. The table also shows that the difference of the K in such a case is the larger the smaller the inclination to the flow and the less the elongation of the plate.

17. Langley and others have shown that the force on a plate of fixed inclination is dependent on the plate's disposition in the

		Aspect ratio	Inclination to flow, degrees														
			5	10	15	20	25	30	35	40	45	50	55	60	70	80	90
25 × 5	Short side normal to flow	-5	3	77	13	21	30	40	50	59.2	69	79	85	86	84.5	84	85
20 × 5		-4	4	11	18	26	34	44	54	62.7	72	79	83	83.5	82.5	82.5	82
15 × 5		-3	5	12	20	29	40	51	60.5	68	75	78.6	79.5	79.5	79.5	79.7	80.0
10 × 5		-2	6	14	24	36	48	58.5	70	78	81	82	81.3	80.5	79	78	78
10 × 10 (square)		+1	9	21	32	44	54.5	63	67.7	70.5	72.5	73.6	74.8	75	74.8	73.7	73.6
5 × 10	Long side normal to flow	+2	12	27	37	49	83.3	54.5	57	61.3	66.2	71.2	74.5	76.5	77.3	76.5	76.8
15 × 15		+3	17	32	39.5	52	58	60.3	61	62.5	65	69.1	72.5	75	77.3	79.5	80
15 × 20		+4	17.5	35.5	42	55.4	59	61.2	63	66	69	73.8	77.5	80.1	83.2	83.6	82.4
15 × 25		+5	19	36	44	54.5	57	58	59.5	62	65.5	72	77.1	80.5	84.2	85.2	86

flow; but they have found no exact relationship and have not expressed it in tables or curves.

They also did not know the relation of K for an elongated plate to the aspect ratio, which is expressed by my table 6.

Usually (and perhaps even always, so far as I know), only the area of the plate and the inclination to the flow have been used in calculations; the significance of the elongation has been ignored as being an unknown but important factor [25].

18. The curves constructed from the table of (10) show that the force on a rectangular wing having its long side perpendicular to the flow direction is expressed by the simple formula

$$P = \frac{K}{1000} iSv^2,$$

provided that the inclination of the wing to the wind does not exceed 10-15°. Here i is the angle of inclination in degrees, P is the pressure in kilograms, S is the area in square meters, v is the flow velocity in meters per second, and K is a coefficient, whose value for rectangular plates of various aspect ratios is expressed by the following table.

19. The above shows why the flat wings of insects have their long sides perpendicular to the head-on air flow (or flight direction).

20. The last formula is applicable to calculations on the flight of insects of aeroplanes whose flat wings are inclined to the flow at not more than 15°.

By v we may understand not only the speed of the wind but also the speed of the plate in stationary air (more precisely, a stagnant atmosphere); it can mean both at once, namely the relative velocity between model and air. This remark applies also to all other cases of resistance.

21. The pressure force [26] on any plane figure inclined to a flow has so far usually been determined from some one formula, e.g., the ones given by Langley and Duchmin, Newton, Leslie, or Lord Rayleigh. I now consider that some of these formulas are quite

Aspect ratio, long side of rectangle perpendicular to flow	K, inclination not more than 15°	Aspect ratio, long side of rectangle perpendicular to flow	K, inclination not more than 15°
0.00	2.10	3.25	3.31
1.25	2.14	3.50	3.28
1.50	2.28	3.75	3.44
1.75	2.50	4.00	3.50
2.00	2.70	4.25	3.54
2.25	2.86	4.50	3.57
2.50	3.00	4.75	3.59
2.75	3.11	5.00	3.60
3.00	3.20		

unsatisfactory, while others are applicable only to a square or (with some error) to other shapes that are not elongated, e.g., circles. This is the case with Rayleigh's formula, which for angles of inclination up to 10° gives values close to the ones I have found, and Langley's formula, which agrees with my experiments for less acute angles and which in general does not conflict seriously with my experiments.

Resistance of a Right Circular Cylinder Having its Axis or Height Either Parallel to the Flow Direction or Perpendicular to it, or an Inclination

22. The following table gives the resistance of a circular cylinder whose axis is parallel to the flow direction.

The area S in formula (4) in this case denotes (in square meters) the area of the base of the circular cylinder or the area of the base of any right prism having a base of small elongation.

23. I made tests with cylinders of various sizes and found that the values in the table are only approximately correct if they are applied to cylinders of base diameter more than 10 cm and (especially) are less; the same is true if the wind speed deviates greatly from 1 meter per second.

24. The table shows that the resistance of a cylinder with its axis parallel to the flow at first decreases as the aspect ratio increases, but then it increases continuously, although slowly. Dubois and Duchmin reported an analogous effect for the resistance of bodies in water.

25. The following table gives the resistance coefficient K for the flow direction perpendicular to the axis:

The area S appearing in (4) is in this case the area of the longitudinal axial section of the cylinder. Here, as in the previous table, K decreases as the flow speed or base diameter increases (the latter above 10 cm); further, K will be larger if the diameter is less than 10 cm or if the speed V is less than 1 meter per second. But in the latter case (reduction of V and D) the error will be larger.

Aspect ratio of cylinder, or ratio of height to diameter of base	K of formula (4)	Aspect ratio of cylinder, or ratio of height to diameter of base	K of formula (4)	Aspect ratio of cylinder, or ratio of height to diameter of base	K of formula (4)
0.0	73.6	0.9	53.0	3.0	62.5
0.2	73.6	1.0	51.5	3.5	64.5
0.3	73.4	1.2	50.5	4.0	66.0
0.4	72.5	1.4	50.7	4.5	67.0

[Table cont'd. on next page]

[Table cont'd.]

Aspect ratio of cylinder, or ratio of height to diameter of base	K of formula (4)	Aspect ratio of cylinder, or ratio of height to diameter of base	K of formula (4)	Aspect ratio of cylinder, or ratio of height to diameter of base	K of formula (4)
0.5	68.0	1.6	51.8	5.0	68.0
0.6	62.5	1.8	54.1	5.5	68.5
0.7	57.5	2.0	56.5	6.0	69.0
0.8	54.6	2.5	60.0		

Aspect ratio, or ratio of height to diameter	K of formula (4)	Ratio of resistance to force on projection	Aspect ratio, or ratio of height to diameter	K of formula (4)	Ratio of resistance to force on projection	Aspect ratio, or ratio of height to diameter	K of formula (4)	Ratio of resistance to force on projection
0.2	33.8	0.460	2.0	40.6	0.552	4.4	46.4	-
0.3	34.2	-	2.2	41.1	0.558	4.6	46.7	-
0.4	34.7	-	2.4	41.6	-	4.8	46.9	-
0.5	35.0	0.475	2.6	42.3	-	5.0	47.1	0.640
0.6	35.4	-	2.8	42.8	-	5.2	47.3	-
0.7	35.7	-	3.0	43.3	0.588	5.4	47.4	-

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[Table cont'd.]

Aspect ratio, or ratio of height to diameter	K of formula (4)	Ratio of resistance to force on projection	Aspect ratio, or ratio of height to diameter	K of formula (4)	Ratio of resistance to force on projection	Aspect ratio, or ratio of height to diameter	K of formula (4)	Ratio of resistance to force on projection
0.8	36.1	-	3.2	44.0	-	5.6	47.5	-
1.0	37.1	0.504	3.4	44.5	-	5.8	47.6	-
1.2	37.7	-	3.6	45.0	-	6.0	47.7	0.648
1.4	38.5	0.523	3.8	45.4	0.617	6.2	47.7	-
1.6	39.8	0.534	4.0	45.8	0.622			
1.8	39.8	-	4.2	46.2	-			

26. It can be deduced from the table that the resistance per unit area of the longitudinal section increases continuously with the length but tends to a fixed limit. This is true for aspect ratios of 0.2 upward, at least; the effect is somewhat analogous to that observed for a rectangular plate (see 6).

27. The following table gives the resistance of a circular wire for various thicknesses.

For instance, for a wind speed close to 3 meters per second the resistance of a wire 1 mm thick, in kg, is $P = (59/1000) \times SV^2$, in which S is the area of the longitudinal axial section of the wire in square meters. The K for speeds greater than 3 meters per second will be somewhat less.

28. The resistance of a thin wire at a low speed may be greater than the force on a square plate having the area of the maximum longitudinal cross-section of the wire. In fact, K is 73 for a square plate (see 5), whereas for a wire it is in some cases somewhat larger (see table).

29. I have performed many other experiments with cylinders and have shown that the resistance is dependent on the diameter of the base and the wind speed (apart from other factors), as the last table shows.

In my experiments the resistance coefficient varied from 0.46 to 1.29; this variation would be even larger for larger differences in the diameters and speeds. It is clear that the resistance even of a simple body such as a right circular cylinder is a very complicated question; attempts to solve it must be abandoned until the laws of friction have been completely elucidated. The above also shows how unfounded are the usual theoretical calculations of the resistance coefficient, which give (no matter what the basic formula) a fixed value for the coefficient without relation to the flow speed, aspect ratio of the cylinder, and diameter of the base (see 67).

30. The resistance of a cylinder of infinite length whose axis is inclined to the flow direction is expressed by combining the table of (27) with the following one.

Then the components of the resistance of a wire inclined to the flow are given by

Thickness, mm	K of (4) for speeds of			Thickness, mm	K of (4) for speeds of		
	1 m/sec	2 m/sec	3 m/sec		1 m/sec	2 m/sec	3 m/sec
0.5	95	70	61.0	6	65	59.5	55.5
1	85	67.5	59	7	64.7	59.0	55.5
2	75	64	57.5	8	64.5	58	55.5
3	70.5	63	56.5	9	64	58	55.5
4	67.5	61.0	55.5	10	64	58	55.5
5	66	60.0	55.5				

Inclination of axis to flow, degrees	Flow horizontal		Inclination of axis to flow, degrees	Flow horizontal	
	k_1, coefficient for horizontal component of resistance	k_2, coefficient for vertical component of resistance		k_1, coefficient for horizontal component of resistance	k_2, coefficient for vertical component of resistance
0	0.06823	0.0000	50	0.5340	0.3361
10	0.09063	0.06103	60	0.7065	0.3263
20	0.1417	0.1296	70	0.8704	0.2717
30	0.2356	0.2034	80	0.9816	0.1609
40	0.3644	0.2863	90	1.0000	0.0000

$$P_1 = \frac{K}{1000} \cdot k_1 \cdot S \cdot v^2$$

and

$$P_2 = \frac{K}{1000} \cdot k_2 \cdot S \cdot v^2,$$

in which K is taken from the table of 27, and k_1 and k_2 are from the last table above. By S we must understand the area of the axial cross-section of the cylinder in square meters. These tables may be used to determine the wind force on a wire or cord retaining a kite (aeroplane) or balloon, and also to determine the curve it takes up, or the forces on any inclined cylindrical parts of any structure generally.

31. The resistance of the part of the lateral surface of a right circular cylinder as taken between two generators is given by the following table for increasing values of the length of the arc of the cross-section normal to the generators.

The quadrilateral composed of the generators and the chords of the bases was always a square in the tests.

A section of the surface at right angles to the generator is an arc of a circle. The first column gives the ratio of the sagitta of this arc to its chord.

The table shows that even considerable concavity (up to 0.3) increases the resistance only a little.

32. This indicates that the concavity of the wing of a bird in vertical motion in an atmosphere at rest is of little value to the bird. The significance of this concavity will be explained later on.

On the other hand, the resistance of a convex cylindrical surface falls rapidly as the convexity increases.

The last column shows that two represents the limit of the ratio of the resistances of a surface having its convex or concave side facing into the wind.

33. In formula 4 we must understand by S the area of the

Ratio of sagitta to chord	K for concavity to wind	K for convexity to wind	Ratio	Ratio of sagitta to chord	K for concavity to wind	K for convexity to wind	Ratio
0.0	74	74	1.00	0.3	76	49	1.55
0.1	74	60	1.23	0.4	82	44	1.78
0.2	74	55	1.35	0.5	84	42	2.00

projection of the cylinder on a plane perpendicular to the flow direction if we wish to determine the resistance of a cylindrical surface.

34. Experiment shows that the resistance of a semicylindrical surface presenting its side is only 1.7 times less than when the convex side faces the wind (axial section normal to the flow).

Square Plate Bent to Form Part of the Lateral Surface of a Right Circular Cylinder Between Two Generators and Inclined to the Flow Direction at Various Angles

35. The cylindrical surface was placed to bring its generators (or the axis of the cylinder) normal to the flow. I determined by experiment the component of the resistance along the wind direction and the component along a direction perpendicular to the wind and to the generator (or to the axis of the cylindrical surface). Consider a cylindrical wing having its generators horizontal; then the first force (that along the wind direction) may be called the drag (or resistance proper), while the second we call the lift force of the wing, because it tends to move the wing along the vertical. The next table gives these drag and lift forces, and also the ratio of the lift to the drag (or the lift per unit drag).

36. The section of the cylindrical surface by a plane normal to the axis of the cylinder is an arc of a circle. The ratio of the lengths of the sagitta and chord of this (the concavity) is given in the first column.

By inclination is meant that of a plane passing through the chord relative to the flow direction (line 1).

37. The table gives the K of formula 4, in which by S we must understand the area of the projection of the surface on the plane of the chord when the ratio of the sagitta to the chord is 1/2. S denotes the area of the curved surface itself for the other surfaces.

38. The table shows that the greatest lift in every case relates to an inclination close to 35°.

It is unexpected that the lift on any cylindrical wing having its concavity turned to face the wind persists even when the in-

clination is zero (no inclination). The curves of variation of the lift show that a ratio of sagitta to chord of 1/12 causes the lift to vanish only for a negative inclination of -4.5°. Lilienthal found for this same concavity of 1/12 that the effect occurred at -9°.

39. The lift of a wing with its convexity facing into the wind is extremely small (last line of table).

40. The following deductions I have drawn from calculations and construction of curves:

a) The best concavity of a cylindrical surface for aeroplane or bird is close to 1/15; Lilienthal gave 1/12.

This concavity facilitates flight (relative to plane wings) by a factor two. Taking into account also the gain from the elongation of the wings, we find that the two together facilitate flight by a factor 4.

In insects the weight of the wing is a negligible fraction of the total weight and does not load the insect. The subtlety of nature is restricted here to elongation of the wings. For the higher animals economy in wing weight is extremely important, because the wing weight in heavy birds is not a small fraction of the total weight; and here the subtlety of nature appears again in the use of concave wings, to reduce the weight and area.

41. b) The best inclination of a cylindrical wing is 12.5°; but the work required of an aeroplane changes by not more than 16% for angles of the aeroplane to the flow between limits of 7.5 and 20°. Here of course we assume the absence of any resistance other than that of the wings; if this is not so, the angle will be more than 12.5°.

42. c) The propelling screw, in order to work best in the air, should not have plane blades; this conflicts with Hiram Maxim's experiments with various kinds of screw.

43. d) The work done by an aeroplane with fixed wings of optimum inclination, elongation, and concavity (while still of simple cylindrical form, of course) is given by

$$T = 0.414P \sqrt{\frac{P}{S}},$$

	Concavity, or ratio of sagitta to chord	Angle, degrees	0	2.5	5	7.5	1.0	12.5
Plane	0	Lift	0	4.4	9.3	14.0	19.5	25.0
		Drag (resistance generally)	2	2.2	2.8	4.0	5.5	7.5
		Ratio of lift to drag	0	2	3.32	3.50	3.50	3.33
Concavity turned to wind	1/32	Lift	11	13.5	16.5	21.4	26.5	33.5
		Drag	4.00	4.5	5.0	6.0	7.5	9.5
		Ratio	2.75	3.00	3.30	3.57	3.51	3.53
	1/12	Lift	14.7	20.5	27.0	34.0	40.5	48.5
		Drag	7.0	7.0	7.5	8.5	10.0	12.5
		Ratio	2.10	2.93	3.60	4.00	4.05	3.88
Convexity turned to wind	1/8	Lift	11.0	18.5	25.5	32.0	39.0	46.0
		Drag	10.0	11.0	12.5	14.0	16.5	18.5
		Ratio	1.0	1.68	2.48	2.29	2.36	2.49
	1/7	Lift	10.0	18.0	24.5	31.0	37.5	4.40
		Drag	12.3	13.3	14.7	16.5	18.5	20.5
		Ratio	0.813	1.35	1.67	1.88	2.27	2.15
	1/2	Lift	12.5	12.3	12.5	13.5	15.6	18.5
		Drag	24.6	23.8	23.5	23.8	24.5	25.5
		Ratio	0.51	0.52	0.53	0.57	0.61	0.73
0.083 = 1/12		Lift	-14.8	-	+2	-	+12.5	-

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	Concavity, or ratio of sagitta to chord	Angle, degrees	15	17.5	20	25	30	35
Plane	0	Lift	30.0	35.5	40.5	49.5	55.0	56.0
		Drag (resistance generally)	10.0	13.0	16.0	23.0	30.0	38.0
		Ratio of lift to drag	3.00	2.73	2.53	2.15	1.80	1.47
Concavity turned to wind	1/32	Lift	39.5	46.5	53.0	59.0	72.0	7.35
		Drag	12.5	15.5	19.0	28.0	39.0	47.0
		Ratio	3.16	3.00	2.79	2.32	1.85	1.56
	1/12	Lift	55.5	63.5	70.3	31.0	86.5	84.7
		Drag	16.0	20.3	24.5	33.5	42.0	50.5
		Ratio	3.47	3.13	2.87	2.42	2.06	1.68
Convexity turned to wind	1/8	Lift	53.0	60.0	66.0	77.0	83.3	84.0
		Drag	21.0	24.0	27.0	35.0	44.0	52.5
		Ratio	2.52	2.50	2.44	2.20	1.89	1.60
	1/7	Lift	50.5	56.0	61.5	71.5	79.0	82.0
		Drag	22.8	26.0	28.6	30.7	45.4	54.0
		Ratio	2.21	2.15	2.15	2.33	1.74	1.52
	1/2	Lift	25.5	35.5	49.0	82.0	95.5	99.5
		Drag	27.5	30.5	34.0	46.5	59.5	68.3
		Ratio	0.93	1.164	1.440	17.64	1.605	1.458
0.083 = 1/12		Lift	+21.5	-	+28.8	34.0	32.5	39.2

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[Table cont'd.]

	Concavity, or ratio of sagitta to chord	Angle, degrees	40	50	60	70	80	90
Plane	0	Lift	54.5	47.0	36.6	25.5	13.0	0
		Drag (resistance generally)	45.6	56.5	65.0	70.0	72.5	73.5
		Ratio of lift to drag	1.19	-	-	-	-	0
Concavity turned to wind	1/32	Lift	70.5	57.5	44.0	30.0	15.0	0
		Drag	53.5	62.0	70.5	75.0	75.5	74.0
		Ratio	1.32	-	-	-	-	0
	1/12	Lift	84.5	60.0	44.5	30.0	15.5	0
		Drag	58.3	64.0	71.0	75.0	76.0	74.0
		Ratio	1.45	-	-	-	-	0
Convexity turned to wind	1/8	Lift	80.5	63.0	44.5	30.0	15.0	0
		Drag	60.5	64.5	69.5	74.0	76.0	75.0
		Ratio	1.33	-	-	-	-	-
	1/7	Lift	82.0	70.0	50.4	32.5	16.5	0
		Drag	62.0	61.5	69.5	74.5	76.3	76.0
		Ratio	1.32	-	-	-	-	-
	1/2	Lift	100.0	95.0	91.0	42.0	19.3	0
		Drag	74.5	82.5	85.7	87.0	87.5	86.0
		Ratio	1.342	1.151	-	-	-	-
0.083 = 1/12		Lift	39.8	37.5	31.8	23.5	13	0

in which T is the work done against drag in kg-m, neglecting any resistance apart from that of the wings (effects of screw and so on); P is the weight of the vehicle with all its contents (in kg), and S is the area of the wings in square meters.

The total minimum work of the aeroplane, with allowance for all resistance and loss on the basis of analogies with resistance in water (Froude), is found approximately as

$$T = 1.242P\sqrt{\frac{P}{S}}.$$

This formula almost exactly fits the data on the flying machines (aeroplanes) of Maxim, Langley, and Ader. The reason why Maxim's and Ader's machines did not fly may be that the wings were not in accordance with the most favorable conditions for the least work of flight.

Right Elliptic Cylinders

44. Tests were done only on right elliptic cylinders whose generators were perpendicular to the flow, one of the axes of the base lying along the flow. The ratio of this axis to the one normal to the flow I call the aspect ratio of the cylinder.

If the cylinder had the long axis of its base along the flow, the aspect ratio was greater than unity (less in the converse case). The aspect ratio of the cylinders I used ranged from 0.164 to 8.63, the heights of all being 10 cm and the minor axis of the ellipse (or width) being half this.

45. Here I give the K of formula 4 as calculated for flow speeds of 1, 2, and 3 m/sec.

46. The K of formula (4)

$$P = \frac{K}{1000} \cdot S \cdot V^2$$

Aspect ratio	K	K	K
	Flow speed, m/sec		
	1	2	3
0.2	70.0	Almost as for 1 m/sec	
0.3	65.5		
0.4	60.5		
0.5	55.5		
0.6	51.0		
0.7	46.5		
0.8	44.5		
0.9	43.5		
1.0	43.0		
1.1	42.5		

Aspect ratio	K	K	K
	Flow speed, m/sec		
	1	2	3
3.2	22.0	20.0	18.0
3.4	21.6	19.6	17.5
3.6	21.1	19.1	16.8
3.8	20.9	18.7	16.4
4.0	20.6	18.4	16.0
4.2	20.3	18.1	15.7
4.4	20.2	17.8	15.4
4.6	20.2	17.6	15.1
4.8	20.2	17.6	15.0
5.0	20.1	17.5	15.0

[Table cont'd. on next page]

[Table cont'd.]

Aspect ratio	K	K	K
	Flow speed, m/sec		
	1	2	3
1.2	41.0	Almost as for 1 m/sec	
1.3	35.0		
1.4	33.0	32.0	31.0
1.6	31.0	29.5	28.0
1.8	29.0	27.5	26.0
2.0	28.0	26.0	24.5
2.2	26.5	24.5	23.0
2.4	25.5	23.5	22.0
2.6	24.4	22.4	21.6
2.8	23.5	21.5	19.5
3.0	22.8	20.6	18.8

Aspect ratio	K	K	K
	Flow speed, m/sec		
	1	2	3
5.2	20.1	17.6	14.9
5.4	20.0	17.6	14.9
5.6	20.0	17.5	14.8
5.8	19.9	17.5	14.7
6.0	19.9	17.5	14.6
6.5	20.0	17.7	14.7
7.0	20.6	18.0	15.2
7.5	21.0	18.5	15.6
8.0	21.4	19.0	15.9
8.5	22.0	19.5	16.3
9.0	22.8	20.4	16.6

should be chosen in accordance with the aspect ratio of the cylinder and the flow speed.

K will be less than that given in the column headed 3 in the table for speeds greater than 3 m/sec.

On the other hand, for speeds less than 1 meter per second, K will be larger than that given in the column headed 1

By S is meant the area of the projection of the cylinder in square meters.

47. The formula is applicable to cylinders of about the sizes I have used; K will be smaller for larger sizes and conversely. The coefficient will also be slightly altered if the ratio of the height of the cylinder to the width is different.

48. The table shows that the resistance decreases as the aspect ratio increases up to a value of 6, after which the resistance starts to increase. This was so at all the flow speeds used.

49. It is also clear that K decreases as the flow speed increases, so the resistance does not increase as rapidly as the square of the flow speed.

An aspect ratio slightly larger or smaller than one (i.e., for an elliptic cylinder that is close to a right circular cylinder) gives hardly any change in K, as is best seen by constructing a curve of K as a function of the cylinder's aspect ratio.

50. The form factor is a number that indicates the factor by which the force on a square equal in area to the projection of the body exceeds the resistance of the body itself under the same conditions.

The form factor varied from 1 to 4.8 in these tests; the form factor for a cylinder increases with the aspect ratio (only up to an aspect ratio of 6), with the flow speed, with the absolute dimensions, and as the height is reduced in relation to the width.

Long Right Cylinders and Prisms of Various Shapes

51. The aspect ratio of each of the prisms and cylinders was close to 5.

Shape of base of cylinder or prism All bases symmetrical with respect to flow Curvilinear figures not having corners with side faces	Flow speed, m/sec	Coefficient in formula (4) most favorable position	Coefficient with model in reverse position	Form factor	The same, but for model in reverse position
1	2	3	4	5	6
Rectangle	2 m/sec	58.85	-	1.251	-
Rectangle having angles of 55° at both ends	"	29.20	-	2.52	-
The same, but with semicircle at one end. Semicircle to wind and reverse	"	28.05	27.95	2.62	2.63
The same, but with semicircles at both ends	"	26.12	-	2.82	-
Isosceles triangle (wedge), acute angle to wind and reverse	"	23.10	26.85	3.187	1.29

[Table cont'd. on next page]

[Table cont'd.]

1	2	3	4	5	6
Rhomb, elongated	2 m/sec	16.69	-	4.345	-
Symmetrical quadrilateral, leading angle 20°, trailing 30°	"	-	-	-	-
Large angle forward and the reverse	"	16.67	18.20	4.41	4.04
Ellipse	3 m/sec	15.24	-	4.83	-
Arcs of circles of various radii	"	12.51	-	5.89	-
Front part (turned to wind) elliptical, rear part bounded by arcs of circles	"	13.56	14.90	5.43	4.94
Front part bounded by arcs of smaller radii than rear	"	12.51	14.51	5.89	5.08
Front part an ellipse less elongated than rear one	"	14.21	16.29	5.18	4.52

[Table cont'd. on next page]

[Table cont'd.]

1	2	3	4	5	6
Imitations of birds and fishes; head sharp and sides somewhat sunken	3 m/sec	12.66	16.79	5.82	4.31
The same, but elliptical head	"	12.40	16.54	5.94	4.45
The same, but head sharp and bounded by arcs of circles	"	12.40	15.14	5.94	4.88

The ratio of the height to the greatest width was 2, as for the elliptic cylinders. The results are given the table above.

52. The table shows that some angular bodies have form factors not much less than that of an elliptic cylinder. In general, the latter is far from being the best shape as regards resistance.

The highest form factor at a flow speed of 3 m/sec was 5.94; that at 4 m/sec, 7. The highest form factors are often obtained for bodies whose front parts (those facing the flow) are blunter than the rear.

It is instructive to compare the present results with data on the resistance of the water to the motion of flat-bottomed river vessels, which may be taken as right cylinders with curvilinear bases, but relatively very low ones.

A river steamboat of good design is taken to have a resistance coefficient (see start of article for explanation of terms) of 0.16 to 0.20. Dubois and Duchmin's results would indicate that for a flat plate moving perpendicular to its plane we should take this coefficient as 1.43, in which case the form factor for the best river steamships would range from 7.15 to 9, the mean form factor being about 8.1, which is not far from the factor 5.94 found for the best of my shapes for a flow speed of 3 m/sec. The value is 7 for 4 m/sec, which is even closer to that of the steamships.

Speed, m/sec	Elliptic cylinder		Body of best shape	
	K	Form factor	K	Form factor
1	19.0	3.874	19.00	3.874
$\sqrt{2}$	18.0	4.089	17.25	4.267
$\sqrt{3}$	17.33		16.17	
2	16.75	4.394	15.12	4.866
$\sqrt{5}$	16.02		14.30	

[Table cont'd. on next page]

[Table cont'd.]

Speed, m/sec	Elliptic cylinder		Body of best shape	
	K	Form factor	K	Form factor
$\sqrt{6}$	15.75		13.58	
$\sqrt{7}$	15.36		13.07	
$\sqrt{8}$	15.00		12.51	
3	14.67	5.018	12.22	6.022
$\sqrt{10}$	14.44		11.90	
$\sqrt{11}$	14.14		11.59	
$\sqrt{12}$	13.88		11.33	
$\sqrt{13}$	13.64		11.12	
$\sqrt{14}$	13.38		10.89	
$\sqrt{15}$	13.17		10.70	
4	12.97	5.676	10.50	7.010

The form factor of a steamship is somewhat higher than that of my models not so much on account of the shape as on account of the size, which affects the resistance coefficient, as we have seen.

53. The above table gives the resistance of an elliptic cylinder of aspect ratio 5.7 and of the best bird-shaped (in cross section) model of aspect ratio 4.7.

We see from this that the resistance coefficients fall as the speed increases, while the form factor increases. The form factor is the same for both of these bodies at low speeds, but a difference

appears in favor of the best shape as the speed increases.

54. Here, as always, there is a certain analogy between resistance effects in water and in air, although it might seem that two media so different in nature as water and air should not give comparable effects.

Resistance to a Flow Directed Along the Axis of a Surface Generated by Rotating an Arc of a Circle Around its Chord

55. Tests with such surfaces show that the resistance does not increase as rapidly as the square of the speed, as I have already noted for elongated cylinders, although there it is not so pronounced as for bodies of rotation (continuous fall in K as the speed increases).

56. Tests were made with bodies having aspect ratios ranging from 2.3 to 7.4, the diameter of the mean cross section in all cases being close to 10 cm.

The least resistance (for any flow speed) occurred with the body of aspect ratio 3.33, which means that the resistance at a fixed flow speed falls rapidly as the aspect ratio increases (e.g., from one) to 3.33. Further increase in aspect ratio causes the resistance to rise again; this resistance apparently increases without limit with the length of the body. The best aspect ratio may be different for a body of another shape; for instance, it is larger for a bird, if we include the tail.

57. With v^2 (square of flow speed) as abscissa x and the resistance of a given shape as ordinate y, the relation is found to be an arc of a hyperbola. The origin lies on the curve at some distance from the vertex. The coordinate axes are parallel to the axes of the hyperbola.

58. This arc has a curvature that decreases as the aspect ratio increases (although the curvature is always variable in a hyperbola); that is, the origin lies further from the vertex of the hyperbola.

59. The equation of the curve is

$$y = \sqrt{Ax^2 + Bx + C - D}.$$

The constants A, B, C, and D have been determined for all aspect ratios from 2.3 to 7.4.

60. The form factor U_f is given by

$$U_f = \frac{9.2 \cdot x}{\sqrt{Ax^2 + Bx + C - D}} .$$

In the limit, when the speed increases indefinitely, x or v^2 also becomes infinitely large, and the formula becomes

$$U_f = \frac{9.2}{A}.$$

61. The results this gives for the limiting U_f for various aspect ratios are as follows:

Aspect ratio	2.3	3.3	4.1	5.0	6.3
U_f	30.50	31.40	19.33	13.45	12.83

i.e., the form factor is largest for the shape of aspect ratio 3.33.

62. The following table gives form factors for low wind speeds (1 to 10 m/sec).

63. The table shows that the form factor is largest for the shape of aspect ratio 3.3, which is found for any fixed flow speed. Further, the form factor varies in response to speed change

Aspect ratio \ Speed, m/sec	1	2	3	4	5	6	7	8	9	10
2.30	4.6	6.3	8.5	10.8	12.3	13.4	14.1	14.7	15.0	15.3
3.33	7.9	9.8	11.7	14.0	16.0	17.1	18.0	18.7	19.1	19.4
4.15	7.6	9.2	11.0	12.5	13.4	14.0	14.4	14.7	14.9	15.2
5.04	7.9	8.2	9.4	10.4	11.2	11.7	12.0	12.2	12.4	12.5
6.31	7.9	8.2	8.8	9.5	10.1	10.5	10.7	10.7	11.0	11.1
7.40	5.9	6.5	7.2	7.7	8.0	8.2	8.3	8.4	8.4	8.4

in a more pronounced fashion the blunter the body, i.e., the less the aspect ratio. This may apply only for a certain degree of bluntness, perhaps up to an aspect ratio not exceeding 2 (so far, I do not know whether this is so).

There is much that might be said about the curves that express the laws, but it is difficult to speak of this without considering the curves themselves.

Resistance of Similar Elongated Bodies (Bodies of Different Sizes But of the Same Shape)

64. The form factor increases with the size for fixed shape and elongation for bodies smooth in the lengthwise direction and having square cross-sections.

The following table gives results from tests on bodies of similar shape whose aspect ratio was about 4:

Speed, m/sec		$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	2	$2\sqrt{2}$	4
Area of projection, cm^2	17.6	4.7	4.4	4.9	6.9	7.4	7.9
	66	4.7	35.1	5.4	7.1	8.2	9.5
	263	5.4	6.3	6.9	8.0	8.8	9.9

The table shows that the form factor increases with the size as well as with the speed.

65. This result shows that the form factor found here for elongated bodies of rotation is not the greatest possible for the given speed and elongation; it should undoubtedly increase with the size of the body up to the dimensions of, say, an aerostat.

The results of 55 for elongated bodies of simple shape indicate that the resistance, or the form factor, or the coefficient K, is an unknown function of the wind speed v, of the aspect ratio, and of the absolute size of the body.

I reported an attempt to solve this problem to the Academy in the following year together with an account of new experiments constituting a continuation of the present work.

66. So far the resistance of any body in a uniform flow of air, and sometimes in water, has usually been found by calculation, and it has always been assumed that the resistance coefficient is dependent on the shape of the body, but not on its size or on the flow speed. In other words, it has been assumed that the resistance of geometrically similar bodies is proportional to the square of the flow speed and to the area of the projection on a plane perpendicular to the flow.

Now we can see how far these deductions of the generally accepted theory are from the true position.

67. The basis for determining the resistance of any shape has been the pressure on an inclined surface. The surface of any body may be represented as consisting of a multitude of plane faces. A double integration then gives the total force from the pressure on the body. The first error arising here is that friction is neglected, since it has always been assumed that the resistance of the medium decreases without limit as the aspect ratio is increased, which is never observed. We can go further. There are many formulas for the pressure on an inclined elementary area. If we adopt as basis Newton's formula (which is now rejected by all as conflicting with experiment), we obtain results that, while not correct, are sometimes not far from the true ones. For instance, experiment gives the resistance coefficient of a sphere as about 0.43, while theory gives 0.5. This may be why Newton's formula has been accepted for centuries as reasonably reliable; many students have written dissertations and obtained degrees on the subject of resistance. But now Newton's formula is thrown overboard, and preference is given to other formulas that give the pressure on an inclined unelongated plane fairly reliably; these have begun to be applied to determine the pressures on the surfaces of polyhedra and curves, but alas the results are cruder than before, for the deviation from experiment

Angle, degrees	0	10	20	30	40	50	60	70	80	90	100
Edge to wind	4	6	13	22	31	41.5	52.3	64.4	71.4	88	99.5
Rear to wind	4	11	24	40	58	74	90	105	115.5	129	136.5
Faces separated, mutual inclination unchanged	4	5.6	11	20	31	46	60	75.4	91	102	112.5

becomes 100 or 300% (see my "Pressure of Air....").

The consequences of using any of these formulas show that the generally accepted method is never satisfactory and constitutes a second error; we need new views on the resistance phenomenon and also experiments, the only way of testing the reliability of these views.

Wedge with Two Faces

68. This I made up from two equal squares, each of area 100 cm^2. The wedge was placed symmetrically in the flow, with the line of junction normal to the flow direction. The angle was varied from 0 to 100°, and the wedge was set with its line of junction facing into the flow or the other way round. The table above gives the results in the form of the K of formula 4. The last line gives the resistance of the faces of this wedge as placed separately some distance apart in the flow without change of angle of inclination.

This table differs slightly from that presented to the Academy of Sciences because in the latter the data were given in the form of curves, which I almost always have done for reports to the Academy, apart from this case.

The table shows that the resistance of the separated faces is at first somewhat less than that of the wedge with its edge facing into the wind, but then it becomes more, the difference being up to 28% at 80°.

The initial slight difference may be a result of experimental error.

The second and third lines of the table show that the resistance of the wedge with its edge facing into the wind is at first almost twice that for the reversed position, but the ratio of the resistances becomes less as the angle increases.

PRESSURE ON A PLANE SURFACE MOVING NORMALLY THROUGH AIR*

A Note from the Editor of Volume II of the Selected Works

This first part of the book "Pressure on a Plane Surface" is devoted to the thermodynamics of gases; the work required for compression and expansion in adiabatic and isothermal processes is derived. Tsiolkovskiy here derives, in place of the generally accepted adiabatic exponent $k = C_p/C_v$, where C_p and C_v are the specific heats at constant pressure and at constant pressure and at constant volume, respectively, another coefficient A (redesignated as B by the present

$V \cdot T^A = \text{const}$, where V is the specific volume and T is the absolute temperature.

Tsiolkovskiy did not use the gas constant R, in place of which he introduced the gas pressure at unit density and unit temperature, as well as the density at unit pressure and unit temperature. The first quantity is the gas constant, and the second is its reciprocal.

The remarks made by the editors on the relations linking these three variables and the generally employed parameters render Tsiolkovskiy's formulas readily applicable to any gas.

The forms of the tables compiled by Tsiolkovskiy for the compression and expansion of gases present some interest to the reader.

The second part of the book is devoted to a determination of the pressure of the oncoming stream. Tsiolkovskiy examines three degrees of accuracy in the determination of the pressure on a plane surface positioned at right angles to the direction of flow, namely:

a) the pressure exerted by the stream in accord with the

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*First printed in 1930 at Kaluga in a separate brochure published by the author. Reprinted in volume II of the Selected Works of K. E. Tsiolkovskiy, GONTI [State United Scientific and Tech. Press], 1932.

conventional formula for low flight speeds (31);

b) the pressure obtaining under the assumption that the density of the compressed air in front of the plate is proportional to the pressure of the air per unit surface area (32), (34);

c) the pressure obtaining under the assumption that the density of the compressed air in front of the plate corresponds to adiabatic compression (37).

These findings are only a rough approximation where high speeds are involved.

Towards the end of his book, Tsiolkovskiy cites, giving no special calculations, a table of allowable flight velocities for bodies presenting different aspect ratios.

In view of the fact that the literature contains extremely little data on motion at high velocities, this part of the book is of interest for the further development of the theory.

F. Tsander

NOMENCLATURE

Formulas (1) through (30). Work - L. Pressure per unit surface area - p. Volume - V. Absolute temperature - T. Specific weight by volume - γ. The same variables with subscripts indicate constants. The mechanical equivalent of heat A = 1/427. Specific heat or heat capacity at constant volume - C_v. Weight of the gas - G. Specific heat of hydrogen - C_{vH}. Molecular weight of the gas - μ. Same, for hydrogen - μ_H.

Formulas (31) through (45). p_a denotes the atmospheric pressure, and p_n the excess pressure on a plate moving normally to its plane. The velocity is c. The acceleration due to the earth's gravitational attraction is g.

VARIATION IN GAS VOLUME

We first turn to an explanation of the phenomena occurring when the volume of the gas undergoes a change. This is necessary in order to determine the resistance of the air to the motion of the flat plate.

THE WORK DONE BY THE GAS IN ADIABATIC COMPRESSION AND ADIABATIC EXPANSION

We have a certain volume V_1 of an ideal gas of specific weight γ_1 at a pressure p_1 per unit surface area and at an absolute temperature T_1. The gas was expanded or compressed with no loss of heat, and these variables changed in the process, to become:

$$V, \gamma, p, T.$$

The gas then did a certain amount of work L corresponding to the change in its temperature.

Using these arguments, we can set up the following quite familiar equations:

$$dL = pdV \tag{1}$$

(d is the differential symbol).

$$p_1 = p_{11}T_1\gamma_1, \tag{2}$$

where p_{11} denotes the gas pressure per unit specific weight and per unit temperature.

Consequently,

$$p_{11} = p_1 : (\gamma_1 T_1)^*, \tag{3}$$

$$\gamma : \gamma_1 = V_1 : V \tag{4}$$

(the mass of the gas remains unchanged, of course). Moreover,

$$dL = G \cdot C_v \frac{dT}{A} . \tag{5}$$

Here we have the notation: weight of the gas G; its specific heat (or heat capacity) at constant volume C_v; the mechanical e ui-valent of heat A.

For the weight of the gas we have

$$G = \gamma_1 \cdot V_1 . \tag{6}$$

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*This is the equation of state of the gas, and p_{11} constitutes the gas constant, usually denoted by the letter R.

--- Note by editor Tsander

Solving equations (1) to (6), we find

$$\ln\left(\frac{V}{V_1}\right) \quad \frac{C_v}{p_{11}A} \cdot \ln\left(\frac{T_1}{T}\right). \tag{7}$$

This is the absolute temperature dependence of the volume for the case of expansion. For compression we have

$$\ln\left(\frac{V_1}{V}\right) = \frac{C_v}{p_{11}A} \cdot \ln\left(\frac{T}{T_1}\right) \quad . \tag{8}$$

In the case of air, $C_v = 0.169$; $\gamma = 0.00129$; $T_1 = 273^\circ$ (or zero degrees Centrigade); $p_1 = 10.33 \frac{\text{tons}}{m^2}$; $\frac{1}{A} = 427$. This then means: $p_{11} = 29.26$; $1/p_{11} = \gamma_{11} = 0.0343$; $C_v/(p_{11} \cdot A) = 2.481$;

$$p_{11}A:C_v = 0.4032,$$

where γ_{11} is the specific weight of the gas at unit pressure and unit temperature.

Accordingly,

$$\ln(V_1:V) = 2.481 \cdot \ln(T:T_1), \tag{9}$$

$$\ln(T:T_1) = 0.4032 \cdot \ln(V_1:V). \qquad (10)$$

On the basis of this last formula, we compile the table:

$\frac{V_1}{V}$	2	3	4	5	6	7	8	9	10	11
$\frac{T_1}{T}$	1.32	1.56	1.75	1.91	2.06	2.19	2.31	2.42	2.53	2.63

Consequently, whenever the air is compressed 6 times, its absolute temperature will be increased more than twice. And conversely, the absolute temperature will be reduced to half whenever the gas is rarefied 6 times.

Since p_{11} is independent of the density, volume, and temperature of the air, then no matter what the volume of the air and no matter how rarefied or dense the air may be, the reduction in volume by a definite number of times (say by ten times) will always produce, in turn, a definite change in the absolute temperature by a definite number of times(by 2.53 times according to the table).

On the basis of the foregoing, we present the following approximate table on the next page.

Leaving aside the third row giving the centigrade temperature, we have exclusively absolute temperature entries here. Clearly, from the table, compression of ordinary air (at sea level) can easily lead to a temperature increase to 2000° - 3000°C. Given the rarefied gas at great heights, on the other hand, the temperature brought about by compression of the gas could climb to several tens of thousands of degrees, judging by the density and by the initial temperature of that

Rarefaction				Compression				
1296	216	36	6	1	6	36	216	1296
Cooling				Heating				
18.7	37.5	75	150	300	600	1200	2400	4800
-254.3	-235.5	-198	-123	+27	327	923	2123	4527
15.6	31.2	62.5	125	250	500	1000	2000	4000
12.5	25	50	100	200	400	800	1600	3200
9.3	18.7	37.5	75	150	300	600	1200	2400
6.2	12.5	25	50	100	200	400	800	1600
3.1	6.2	12.5	25	50	100	200	400	800
1.6	3.1	6.2	12.5	25	50	100	200	400

rarefied gas: for the lower the density, the higher the temperature may climb, since multiple condensation becomes possible. The rarefied air at great heights will be less likely to experience heat resulting from the compression if the air is very cold.

But, in the atmosphere we do not have air at temperatures below 80° freezing, which corresponds to the rather high absolute temperature of 193°.

For brevity, let us put (cf. 3):

$$C_v:(A \cdot p_{11}) = C_v\gamma_1 \cdot T_1:(p_1A) = B^*. \tag{11}$$

Then, on the basis of the fundamental equations (1) to (6), we find

$$V:V_1 = (T_1:T)^B, \tag{12}$$

$$T:T_1 = (V_1:V)^{1:B}, \tag{13}$$

$$p:p_1 = (T:T_1)^{B+1}, \tag{14}$$

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*The variable B is related to the adiabatic exponent $k = C_p/C_v$ where C_p and C_v are respectively the specific heat at constant pressure and the specific heat at constant volume, by the formula $B = \frac{1}{k-1}$

---Note by editor Tsander

$$T:T_1 = (p:p_1)^{1:(B+1)} \tag{15}$$

$$\gamma:\gamma_1 = (T:T_1)^{B}, \tag{16}$$

$$T:T_1 = (\gamma:\gamma_1)^{1:B}, \tag{17}$$

$$p:p_1 = (\gamma:\gamma_1)^{1+(1:B)} \tag{18}$$

$$\gamma:\gamma_1 = (p:p_1)^{B:(1+B)} \tag{19}$$

$$L = G \cdot C_v \cdot T \cdot \left(1 - \frac{T}{T_1}\right) : A, \tag{20}$$

$$L = B \cdot p_1 \cdot V_1 \left[1 - (V_1:V)^{1:B}\right]. \tag{21}$$

Formulas (12) and (13) show the change in volume as a function of the change in temperature, and vice versa: the change in the temperature as a function of the change in volume. Air must be compressed in the engines for the engines to deliver an intensified output even when the air is highly rarefied. That is precisely one of the applications of these formulas.

Formula (19) yields the change in density as a function of the change in pressure. This formula may be applied to a determination of the drag on the plate in rapid flow normal to the plate.

In compressing the gas, we are performing work. The amount of this work must be known. Formulas (20) and (21) serve this purpose.

Formulas (13), (14), and (16) may be stated in a single line:

$$T:T_1 = (V_1:V)^{1:B} = (p:p_1)^{1:(B+1)} = (\gamma:\gamma_1)^{1:B} . \qquad (22)$$

We have, of course, for all these "constant"gases:

$$C_v = C_{vH} \cdot \frac{\mu_H}{\mu}^{*} , \qquad (23)$$

$$\gamma = \gamma_H \cdot \frac{\mu}{\mu_H} . \qquad (24)$$

Here, the dependence of the specific heat C_v and of the specific weight of any gas γ on the specific heat and the specific weight of hydrogen C_{vH} and γ_H and the dependence of the molecular weights μ and μ_H, are duly expressed. We now find, from formulas (23), (24), (3), and (11):

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*This applies only to gases having the same number of atoms per molecule; for ideal monatomic gases the molecular specific heat at constant volume is 3, for diatomic gases, 5, and it is even higher for polyatomic gases.

--- Note by editor Tsander.

$$B = C_{vH} \cdot \gamma_H T_1 : (p_1 \cdot A) \ . \qquad (25)$$

But the product $C_{vH} \cdot \gamma_H$ is constant, so that B is also constant. Hence, all the formulas and laws derived therefrom are applicable to all constant gases.

In all these formulas (12) through (21), we see a dependence of the ratios on B. According to formula (25)*, B depends on the temperature T_1 in the case of all the constant gases.

In the case of sea-level air at zero Centigrade, B = 2.481; 1/B = 0.403; B + 1 = 3.48; 1 + 1/B = 1.403; 1/B(B + 1) = 0.287; B/(B + 1) = 0.713 (cf. section 11.25).

Now, in place of formula (22) we find

$$T:T_1 = (V_1:V)^{0.403} = (p:p_1)^{0.287} = (\gamma:\gamma_1)^{0.403} =$$

$$= \sqrt[2.48]{V_1:V} = \sqrt[3.48]{p:p_1} = \sqrt[2.48]{\gamma:\gamma_1} \ .$$

This means that for air the change in the absolute temperature will be inversely proportional to the root of power 2.48 of the change in volume, or directly proportional to the root of power 3.48 of the change in pressure, or directly proportional to the root of power 2.48 of the change in density.

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*This holds true because of the fact that the specific heat C_v increases with the temperature.

--- Note by editor Tsander

THE WORK IN AN ISOTHERMAL CHANGE OF STATE

Let us suppose that the gas has a constant temperature T, despite the change in volume. This may be the case in actual practice whenever the change in volume takes place slowly, so that the gas will have time to cool or to be heated by the external environment. This will even be the case when proper measures are taken to keep the temperature of the gas at a certain level.

Then $T = T$ in equations (1)-(14) and will be a constant quantity. From these equations we find

$$dL = p_{11} \cdot T_1 \cdot \gamma_1 \cdot V_1 \cdot \frac{dV}{V} , \qquad (26)$$

or, on the basis of formula (3):

$$dL = p_1 \cdot V_1 \cdot \frac{dV}{V} .$$

On integrating, we find

$$L = p_1 \cdot V_1 \cdot \ln V + \text{const.}$$

When $V = V_1$, we have $L = 0$; and hence

$$L = p_1 \cdot V_1 \cdot \ln \cdot \left(\frac{V}{V_1}\right) . \qquad (27)$$

When the gas expands ($V > V_1$), the work will be positive, i.e., work will be done by the gas. When the gas is compressed ($V < V_1$), work will be done on the gas, for external work will be required to compress the gas, so that the work will be negative.*

COMPARISON OF THE WORK DONE IN ADIABATIC AND IN ISOTHERMAL CHANGES OF STATE

If the pressure were kept constant, then the work done in expanding the gas from a volume V_1 to a volume V would be

$$L_1 = p_1(V - V_1) = p_1 \cdot V_1 \left(\frac{V}{V_1} - 1\right). \tag{28}$$

Dividing equation (27) term-by-term by equation (28), we obtain the relative work involved in the change of volume with the temperature kept constant

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*We learn from formula (11) that $B = C_v/(Ap_{11})$; but $p_{11} = R$ is inversely proportional to the molecular weight, so that R is proportional to the molecular specific heat $C_{v\mu}$ which, according to (23) for ideal gases having the same number of atoms per molecule, is a constant. Hence, B is also practically an approximately constant quantity for real gases.

--- Note by editor Tsander.

$$L:L_1 = \ln\left(\frac{V}{V_1}\right) : \left(\frac{V}{V_1} - 1\right). \qquad (29)$$

Proceeding in the same manner with formulas (21) and (28), we now obtain the relative work done in compression and in expansion, without losses and without the gas acquiring heat from the outside environment (adiabatic expansion or compression)

$$L:L_1 = B\left[1 - \left(\frac{V}{V_1}\right)^{1:B}\right] : \left(\frac{V}{V_1} - 1\right). \qquad (30)$$

Here we have $1/B = 0.403$.

Formulas (28), (29), and (30) enable us to compile a table of relative work figures responding to a change in volume:

1) at constant pressure, 2) at constant temperature, and

3) in an adiabatic process.

All of the work figures in the table are assumed to be positive. In order to obtain the true work figures, of course, we would have to multiply these figures by the work corresponding to formula (28). For example, for air under normal conditions we may assume:

$T_1 = +273^\circ$ (or $0^\circ C$); $\gamma_1 = 0.0013$; $p_1 = 10.3 \frac{\text{tons}}{m^2}$; $p_{11} = 29.26$.

We may now compute

$$L_1 = p_1 V \left(\frac{V}{V_1} - 1\right) = 10.3\ V \left(\frac{V}{V_1} - 1\right) \text{TM}.$$

For instance, in a triple expansion $L_1 = 20.6$ ton-meters (TM) or 20,600 kilogram-meters (kgm).

Let us analyze the table appearing below. It consists of two parts. The first refers to compression of the gas, and the second to its expansion. The first row indicates a slight change in volume, the second the relative difference change in the volume, and simultaneously the work done at constant pressure.

The unit of work used throughout the table is the work involved in changing the volume by one unit with the pressure kept constant ($10.3\ T/m^2$).

The third row refers to the work of compression with the temperature held constant, and the fifth row to the same with temperature variable.

Let us turn to the work done in compression. We see that, as a result of the continuously increasing pressure, the work done in compression is far greater than the work done under constant pressure.

Thus, at a compression of 1000 times, the work done increases by almost seven times. In response to the natural temperature increase (i.e., with no effort to eliminate the temperature increase), and a volume reduction of 1000 times, the work increases by 37 times, on the other hand. The less the compression, the less these deviations will be.

The seventh row indicates the number of times the work in compressing the gas will increase as a result of the increase in temperature resulting from compression of the gas. For instance, in response to a volume reduction of 1000 times, the work done will increase 5.4 times because of the heating of the gas.

We now turn to that portion of the table which refers to rarefaction of the gas. Here the work done by the gas is far less than the work done with the pressure held constant. The fourth row shows how many times less it will be when the temperature is kept constant (or when artificial heating of the gas from the external surroundings is resorted to). For instance, in a 10-fold expansion the work will be almost 4 times less than at constant pressure. If natural cooling is allowed to take place, however, the work will be six times less in response to the same volume reduction (see line six).

Nevertheless, at constant temperature the work of infinite expansion of the gas will increase without bounds, even though it increases slowly as the volume is increased [formula (27)]. On the other hand, the work done by the gas in expanding, and being allowed to cool naturally, is completely limited, despite the infinite increase in the volume of the gas [formulas (20) and (21)].

The eighth line indicates the effect of natural cooling on the work done by the expanding gas. This effect is a slight one in response to a slight expansion, but will take on ever greater proportions as the expansion becomes more pronounced. For instance, in response to a 1000-fold expansion the natural work done by the gas in expanding (with

TABLE

	Expansion						
1.	1000	100	10	5	4	3	2
2. $(V : V_1) - 1$	0.999	0.99	0.9	0.8	0.75	0.667	0.5
3. By formula (29)	6.90	4.65	2.62	2.01	1.84	1.65	1.39
4. Inverse ratio	-	-	-	-	-	-	-
5. By formula (30)	37.2	13.54	4.22	2.83	2.48	2.08	1.59
6. Inverse ratio	-	-	-	-	-	-	-
7. Ratio 30/29	5.4	2.9	1.6	1.4	1.3	1.25	1.2
8. Inverse ratio	-	-	-	-	-	-	-

TABLE

Compression							
1.	2	3	4	5	10	100	1000
2. $(V : V_1) - 1$	1	2	3	4	9	99	999
3. By formula (29)	0.693	0.549	0.400	0.402	0.256	0.046	0.0069
4. Inverse ratio	1.44	1.82	2.17	2.49	3.91	21.7	145.0
5. By formula (30)	0.600	0.459	0.355	0.295	0.166	0.0211	0.00233
6. Inverse ratio	1.67	1.82	2.82	3.39	6.02	47.4	429.0
7. Ratio 30/29	0.87	0.85	0.77	0.73	0.65	0.46	0.34
8. Inverse ratio	1.14	1.2	1.3	1.4	1.5	2.2	2.9

the temperature allowed to drop freely) will be almost 3 times less than when the temperature is kept fixed artificially.

It's hardly necessary to remind the reader that our table remains applicable to all ideal gases and to superheated nonconstant gases and vapors. It also applies to all densities and to all temperatures at which the gas remains a constant-parameter gas.

PRESSURE OF THE ONCOMING AIR. APPROXIMATE DETERMINATION OF AIRSTREAM PRESSURE

The aeroplane flies at a high speed through the rarefied layers of air. The air impinging on the nose of the aeroplane is therefore compressed and can thereby diminish the work that the compressor has to perform. The pressure of the oncoming flow is exerted on the air in the nose section of the airplane; this pressure is the same as that exerted on a flat plate placed normal to the flow. How great will this pressure be at different flight speeds (aeroplane speeds)?

The reader is cautioned that all of the subsequent calculations must not be taken as either exact or rigorously scientific. It is well enough in this context that the calculations yield so much as some idea of the amount of pressure exerted on the plane normal to the flow.

When the air in front of the plane is slightly compressed, we may make use of the familiar formula

$$p_n = \frac{c^2}{2g} \cdot \gamma_a, \tag{31}$$

where the speed should not exceed 100 meters/sec. This formula was also derived by me for the inertial resistance presented by the air.*

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*Cf. Trudy ob-va lyubiteleyyestestvoznaniya. Fiz. otd. (Proceedings of the Society of Natural Science Amateurs. Physics Section). 4, Moscow, 1891.

In this formula p_n denotes the excess pressure, g is the acceleration due to gravity, and γ_a is the specific weight (by volume) of the air.

When the speed exceeds 100 meters/sec, the air in front of the plane will be compressed by the strong pressure, and formula (31) will produce a large error. In that formula the specific weight γ_a must be replaced in accord with the formula

$$\gamma_a = \gamma_1 \left(\frac{p_n + p_a}{p_a} \right) \tag{32}$$

where γ_1 is the density of air at rest, and p_a is the atmospheric pressure.

From formulas (31) and (32), we obtain

$$p_n = \frac{c^2}{2g} \cdot \gamma_1 \cdot \frac{p_n + p_a}{p_a} . \tag{33}$$

Hence

$$p_n = \frac{c^2}{2g} \cdot \gamma_1 : \left(1 - \frac{\gamma_a}{p_a} \cdot \frac{c^2}{2g} \right) . \tag{34}$$

At low speed c, formula (31) is derived from the above formula.

Clearly, from formula (34), the speed cannot be very great; the constraint

$$c < \sqrt{2g \cdot p_a : \gamma_a} \cdot \qquad (35)$$

must be observed.

Suppose g = 10 square meters per sec; p_a = 10 tons/square meter; γ_a = 0.0013. Then, from formula (35), we compute c < 392 m/sec.

Clearly, at that velocity we seem to end up with an infinite condensation, so that no further increase in the velocity is possible. But this is an error, first because the gases are nonconstant gases, and because they cannot be condensed infinitely; secondly, because we failed to take into account the heating due to compression of the gas.

But for the time being let us return to our last formulas. The pressure, according to formula (34), will be greater than the pressure according to (31) by as many times as

$$p_{n(34)} : p_{n(31)} = 1 : \left(1 - \frac{\gamma_a}{p_a} \cdot \frac{c^2}{2g}\right). \qquad (36)$$

Equations (31), (34), and (36) yield the table shown on the following page.

The second line indicates the stream pressure in tons per square meter according to formula (31). The third line gives the same according to formula (34). The fourth line gives us the ratio of these pressures. According to formula (34), of course, the pressure is greater because of the compression of the air considered in the calculations. Thus, at a velocity of 50 meters a second error is 1.6%, while at 100 m/sec it is 7%, and at 250 m/sec the error attains a figure of 68%.

It is obvious that formula (31) may be resorted to only for speeds up to 100 m/sec. But formula (34) cannot be used for high speeds either. Actually, at a speed of 350 m/sec the air pressure becomes 3.9 ton per square meter, so that the air becomes heated, becomes less dense, and the pressure diminishes as a result. It is evident from the table that when the gas is compressed four-fold the absolute temperature will be increased by a factor of 1.75. The air

TABLE

Velocity, in m/sec	50	100	150	200	250	300	350	392
$p_{n(31)}$	0.162	0.65	1.46	2.60	4.06	5.85	7.96	10.0
$p_{n(34)}$	0.165	0.695	1.7	3.5	6.85	14	39	Infinite
Ratio $\frac{p_{n(34)}}{p_{n(31)}}$	1.016	1.070	1.171	1.35	1.68	2.41	4.90	Infinite

will become rarefied, and the pressure lessened, by the same factor.

PRESSURE ON A PLATE AS THE AIR DENSITY IS VARIED ADIABATICALLY

In order to obtain a more accurate formula for the pressure, we have to take the temperature increase due to compression of the air into account. Formula (19) gives

$$\gamma = \gamma_a \cdot (p : p_a)^{B : (1 + B)} .$$

But here

$$p = p_a + p_n ,$$

where p_a is the pressure in a quiet atmosphere, and p_n is the excess pressure. From these last formulas and formula (31), we derive the equation

$$p_n = \frac{c^2}{2g} \cdot \gamma_a \cdot \left(\frac{p_n + p_a}{p_a}\right)^{B : (1 + B)} . \qquad (37)$$

When p_n, or the excess pressure, is many times greater than the pressure p_a in the quiet air, then we may put

$$p_n = \frac{c^2}{2g} \cdot \gamma_a \cdot \left(\frac{p_n}{p_a}\right)^{B : (1 + B)} ; \qquad (38)$$

and hence, once the unknown p_n is determined, we find

$$p_n = \left(\frac{\gamma_a}{2g}\right)^{B+1} \cdot \left(\frac{c^{2\ B+2}}{p_a^B}\right) . \qquad (39)$$

Clearly, from the foregoing, B = 2.481; accordingly

$$p_n : p_a = K \cdot c^{6.96} , \qquad (40)$$

or, approximately:

$$p_n : p_a = K \cdot c^7 ;$$

where

$$K = [(\gamma_a : p_a) : 2g]^{B + 1} . \qquad (41)$$

At sea level and zero temperature we may put:

$\gamma_a = 0.0013$; $g = 10$; $p_a = 10$. We then find $K = 89 \cdot 10^{-20}$.

For example, putting c = 400 m/sec, we find

$$p_n : p_a = 1.46,$$

or

$$p_n = 14.6\ T/M^2.$$

According to formula (31) we find, for almost the same speed, $p_n = 10$ tons/m^2, while according to formula (34) we end up with infinite pressure.

Once the pressure ratio (40) is known, and with the assumption that the excess pressure p_n is so great that we may assume

$$p_n : p = 1,$$

where p is the absolute pressure of compressed air, we may use formulas (15) and (18) to compute the ratio of the absolute temperatures T/T_1 and the ratio of specific weights γ/γ_a.

At zero temperature and at sea level, we obtain

$$p_n : p_a = p : p_a = 89 \cdot 10^{-20} \cdot c^7.$$

Also, we have

$$T : T_a = (p : p_a)^{1:\ (B + 1)} = (p : p_a)^{0.287} = 6.61 \cdot 10^{-6} \cdot c^2.$$

And further

$$\gamma : \gamma_a = (p : p_a)^{B : (1+B)} = 1.35 \cdot 10^{-13} \cdot c^{5.01} ,$$

or approximately $1.35 \cdot 10^{-13} \cdot c^{5}$.

From these last two equalities, we see clearly that the absolute temperature will increase in proportion to the square of the speed, while the density will increase in proportion to the fifth power of the same speed.

All this now enables us to compile the following table:

Speed, in km/sec						
0.4	0.5	1	2	3	4	5
Pressure ratio (p/p_a) at sea level and zero temperature						
1.46	6.95	889.6	113 920	1 946 000	14 580 000	69 530 000
Absolute temperature ratio T/T_a						
1.0576	1.65	6.61	26.44	59.49	105.76	165.25
Corresponding absolute temperature T						
289	450	1804	7218	16 241	28 870	65 100
Density ratio γ/γ_a						
1.38	4.31	138	4420	33 530	141 400	431 200

From formulas (41) and from the sequel, it is clear that the pressure ratio, temperature ratio, and density ratio will not vary when the ratio γ_a/p_a is constant. This means that the table refers to all layers of the atmosphere equally, i.e., to any rarefaction or compression of the gas whatever, provided the initial temperature of the undisturbed air remains the same. The higher figures listed in this table may be applied to the rarefied layers of air, for instance when the air is rarefied thousands or millions of times, i.e., at very great heights.

At a speed of 400 m/sec, the pressure will attain a figure of almost 1.5 atm, the absolute temperature will rise to 289°, or 16°C, and the density will increase by a factor of 1.38. At a speed of 0.5 km/sec, the temperature is still not very high (177°) and the air is not yet capable of glowing. Here the air is condensed 4.3 times; at a speed of 1 km/sec the air would be capable of heating a body to produce a glow. The density of the air would be 138-fold at that point.

At a speed of 2 km/sec and higher, the computed condensation would not be justified at sea level in view of the nonconstant nature of the gas. But at great heights this could be partially justified if by nothing else at least in the ratio of the high temperatures. Assume, for example, that the air has undergone a thousand-fold rarefaction. Then it may be compressed 500 times without its ideal properties, inherent in a constant gas, being impaired (taking into account very high heating). This means that a speed of 5 km/sec and a density increase of 431,200 times is possible. The absolute temperature would reach 65,000° in the process.

Here we cannot help thinking of using the high temperature of compressed, greatly rarefied gases for different purposes. For example, would not diatomic gases decompose into monatomic gases? Would not radioactive phenomena ensue?

The tables and formulas could still be used at even higher speeds if the plate were moving through even more rarefied layers of air.

Moreover, we have to remember that the greater the speed at which the plate is moving and the greater the degree of condensation of the medium -- even in the rarefied layers of the atmosphere -- the more accurate will be the figures arrived at using our formulas. In fact, they would be true provided similitude is observed with respect to the densities. At low speeds, the density of the medium is almost constant throughout the entire range. It is precisely under these conditions that the formulas will be valid. At high speeds, however, the condensation will occur only in the immediate vicinity of the moving plate. The farther away from the plate, the less the density will be perturbed. The extreme lack of uniformity

of the densities constrains us to introduce a variable coefficient less than unity into our formulas, and this coefficient will be increasingly farther from unity as the speed of the plate increases. We leave it to the mathematicians to carry out more exact studies on the drag on the plate, in order to find the correct factor for these drag formulas.

Thus, the tabulated figures will appear slightly too high to the degree that the speed is greater.

From (37), for any speed and pressure, we obtain

$$\frac{p_n}{p_a} = c^2 \left(\frac{\gamma_a}{2g \cdot p_a} \right) \cdot \left(\frac{p_n}{p_a} + 1 \right)^{B:\,(1 + B)} . \quad (42)$$

This means

$$c = \sqrt{\frac{(p_n : p_a)\ 2g \cdot p_a}{\gamma_a \left(\frac{p_n}{p_a} + 1 \right)^{B:\,(B + 1)}}} . \quad (43)$$

For high pressures, however, we would have obtained from formula (39):

$$c^2 = (p : p_a)^{1:(B + 1)} \cdot (2g \cdot p_a : \gamma_a). \quad (44)$$

Let us now use formula (43) for any speed. But the p_n/p_a ratio for high pressures must be expressed, in this formula, in terms of the density ratio γ/γ_a by means of formula (18). We

TABLE

p_n/p_a	0.1	0.3	0.5	1.0
γ/γ_a	1.07	1.205	1.33	1.64
Speed, in m/sec	108.5	178	217	277
Rarefaction required	1.18	3.17	4.71	7.67
p_n/p_a	5	7	10	20
γ/γ_a	3.58	4.40	5.53	8.70
Speed, in m/sec	420	448	479	537
Rarefaction required	17.6	20.1	24.9	28.8
p_n/p_a	500	1000	5000	10 000
γ/γ_a	84.1	138	435	711
Speed in m/sec	871	955	1200	1340
Rarefaction required	75.9	91.2	144	180
p_n/p_a	1 000 000	5 000 000	10 000 000	50 000 000
γ/γ_a	19 000	59 900	98 800	309 000
Speed, in m/sec	590	3440	3630	4780
Rarefaction required	671	1183	1318	2285

[Table cont'd. next page]

[Table continued]

p_n/p_a	1.5	2	3
γ/γ_a	1.92	2.19	2.69
Speed, in m/sec	313	339	375
Rarefaction required	9.80	11.5	14.1
p_n/p_a	50	100	209
γ/γ_a	16.3	26.9	43.9
Speed, in m/sec	605	687	759
Rarefaction required	36.6	47.2	57.6
p_n/p_a	50 000	100 000	500 000
γ/γ_a	2240	3670	11 600
Speed, in m/sec	1780	1860	2470
Rarefaction required	317	346	610
p_n/p_a	100 000 000	500 000 000	
γ/γ_a	506 000	1 600 000	
Speed, in m/sec	5040	6640	
Rarefaction required	2540	4409	

then have

$$c^2 = \frac{2gp_a}{\gamma_a} \cdot \frac{(\gamma : \gamma_a)^{(B + 1) : B}}{[(\gamma : \gamma_a)^{(B + 1) : B} + 1]^{B : (2 + 2B)}}. \tag{45}$$

We put $g = 10$; $p_a = 10$; $\gamma_a = 0.0013$; $B = 2.48$; $B/(2 + 2B) = 3.356$; $B/(1 + B) = 0.713$; $(1 + B)/B = 1.403$.

Now we may use formulas (42) and (45) to compile our next table.

We see from the table that the condensation due to the oncoming airstream is insufficient to support the operation of the engines at speeds up to 955 m/sec. But beyond this point the condensation is even far greater than needed. For example, at a speed of 1200 m/sec the condensation will be 435, i.e., it will be triple that required, so that the engine performance will be tripled: all that is needed is the fuel and strong working cylinders. At a speed of 1860 m/sec, the same table indicates that the condensation would be 11 times more intense. This means that compressors would become superfluous at a speed of 1000 m/sec. Unfortunately, the temperature would reach 1500°C at that speed of 1 km/sec.

However, we already pointed out that the condensation and the temperature must be much less than that.

Let us say a few words on the formulas for the resistance of the air normal to the moving plate. Only at speeds up to 200 to 300 m/sec will these formulas be considered suitable [cf. the table on p. /245]. At higher speeds, though, these formulas will yield pressure, density, and temperature values which will be too high. But we shall have to settle for these formulas until better ones are derived.

Vehicles traveling through the atmosphere could hardly acquire a speed greater than 1 km/sec. And the table indicates that the density of the air bathing the plate would be increased 138-fold in that case.

Is travel economically feasible under those conditions?

The point is that the vehicles have no blunt ends, particularly in the front.

According to the table, a velocity of 300 m/sec would entail only a doubling of the density of the air in front of the plate. Accordingly, this velocity is entirely tolerable for spherical

bodies: the condensation of the medium in front of such bodies is virtually nil.

PERMISSIBLE FLIGHT SPEED FOR BODIES OF DIFFERENT ASPECT RATIO

For a smoothly streamlined body with double the oblong aspect of a sphere, a speed of 600 m/sec would be attainable with no appreciable condensation of the medium. We can compile a table on the basis of this reasoning.

Aspect ratio of bodies, or length/diameter ratio									
1	2	3	4	5	6	8	10	15	20
Permissible economic speed, in km/sec									
0.3	0.6	0.9	1.2	1.5	1.8	2.4	3	4.5	6

At these speeds, there will be no appreciable condensation of the medium, so that there will be no increased inordinate expenditure of energy in the flying of a bird-like flying machine.

At high altitudes the tail end may even be flat; but we have to fly downward as well. Therefore a flat tail would be unfeasible. It would also be impractical from the standpoint of design. But the portion of the tail from which the orifices of the conical tubes ejecting the combustion products protrude, would necessarily be flat.

FLIGHT OF METEORS. THEIR HEATING

Let us now turn to the table [page/245] with the object of demonstrating its plausibility, at least partially.

Meteors, or stones from outer space, when flying through the atmosphere heat to a glow and give off light. The reason for this is the delay imposed on their motion by the atmosphere, so that all the mineral in the meteor becomes heated as a result: the energy associated with the motion becomes converted to heat. Another reason is the condensation of the air ahead of the stone. Since the stone is of irregular shape, the condensation and the temperature of the air will be far less than indicated in the table. For example, at a speed of 5 km/sec the medium will be condensed 400,000 times, and the temperature of the medium will ascend to 65,000°. Bearing in mind the enormous speed of aerolites (up to 50 km/sec and faster), the condensation and the temperature should be justified.

But we must remember that this condensation and temperature do not appear all of a sudden, but gradually: time and a long path traversing the medium are necessary for the figures tabulated to be achieved. The atmosphere is bounded, however, and fireballs for the most part traverse a path not longer than 100 km through it. At a height of 100 km, the air is rarefied to an astonishing degree, now let us suppose a million times. If 100 km of this air were to be contracted to 1 m, we would end up with a medium of 0.1 the density of the sea-level atmosphere.

It is understandable that neither the condensation nor the temperature indicated in the table would be achieved. The resistance of the air would be so low that the meteor itself would not become heated: it would fly past unnoticed and dark.

That is why a certain height is needed for the meteor to glow and become visible. The boundary for these shooting stars is 100-200 km. They will not be visible higher up. The air higher up would be condensed so little by fireballs that it could not glow.

APPENDIX 1

Zhurnal Russkogo fiz.-khim. ob-va (Journal of the Russian Physico-Chemical Society), 1882, XIV, No. 8, Section 1, Physics Series, p. 480.

Extract from Official Record of Fortieth Meeting of the Physics Braanch of the Russian Physico-Chemical Society, 26 October 1882.

6. Professor P. P. Fan-der-Flit presented a paper on the theory of gases by Mr. Tsiolkovskiy, a teacher at the Borovsk district school, Kaluga Government. He reported that although the paper itself contained nothing new and the conclusions were not strictly correct, nevertheless, it revealed the unusual capabilities and keenness of the author, who had not received a regular education and had acquired his knowledge entirely on his own. The author's only sources for the present paper had been a few elementary textbooks on mechanics, Professor Petrushevskiy's course on experimental physics, and Professor Mendeleyev's "Foundations of Chemistry."

In view of this it was desirable to foster the author's further self-education.

The Society decided to petition the trustee of the Petersburg or Moscow District to transfer Mr. Tsiolkovskiy, if he so desires, to a city where he might have the advantage of greater scientific resources.

APPENDIX 2

Zhurnal Russkogo fiz.-khim. ob-va, 1897, XXIV, Physics Series, No. 9, Section 1, p. 350.

Official Record 168 of Two Hundred and Eighteenth Meeting of the Physics Branch of the Russian Physico-Chemical Society, 23 September 1897.

In the chair: F. F. Petrushevskiy

2. The clerk read a letter from Mr. Tsiolkovskiy of Kaluga containing a brief description of a series of experiments which Mr. Tsiolkovskiy intends to perform in order to determine the pressure of moving air on solids of revolution and the resistance offered to the motion of such bodies. Referring to all his previous work on the problem of the controlled aerostat or dirigible, Mr. Tsiolkovskiy begged the Branch to examine his proposals and, if it found them sufficiently important and interesting, to make him a grant of 200 rubles from the Branch funds to finance them.

It was resolved to refer Mr. Tsiolkovskiy's proposals to a review committee composed of D. K. Bobylev, V. V. Lermontov and I. V. Meshcherskiy and to request him to send the committee a more detailed description of his apparatus and his proposed experiments.

APPENDIX 3

Zhurnal Russkogo fiz.-khim. ob-va, 1898, XXX, Physics Series, No. 1, Section 1, p. 28.

Official Record 170 of the Two Hundred and Twentieth Meeting of the Physics Branch of the Russian Physico-Chemical Society, 11 November 1897.

In the chair: F. F. Petrushevskiy

3. The clerk, on behalf of Professor D. N. Bobylev, read the decision reached by the committee responsible for reviewing the program of experiments on the investigation of the resistance of air submitted by K. Tsiolkovskiy. The committee had come to the conclusion that, in the form in which Tsiolkovskiy proposed to perform them, the experiments could not lead to any results of practical importance. For this to be achieved, it would be necessary for the experiments to be conducted on a much larger scale, requiring considerable funds which the Society did not have currently available. Accordingly, the committee recommended that Mr. Tsiolkovskiy be informed that he could not expect material assistance from the Society in carrying out his experiments, the refusal being justified on the grounds above-mentioned.

After a brief discussion of the matter, it was decided to send Mr. Tsiolkovskiy a reply framed in the terms suggested.

APPENDIX 4

10 September 1899

To his excellency, the Vice-President of the Imperial Academy of Sciences, from Konstantin Tsiolkovskiy, author of works on aeronautics and physical astronomy (address: Kaluga, Georgiyevskaya ul., dom Speranskoy).

I beg your excellency to inform the Academy of Sciences of my recent experiments on the resistance of air described in "Vestnik opytnoy fiziki" (Odessa, Nos. 269-272, 1898-1899).

I have no offprints of the article and the journal is all sold out; accordingly, I much regret I can not send you an actual description of the experiments themselves.

Perhaps the competent members of the Academy would do me the honor of reading my work and estimating its value.

If their opinion is favorable, then perhaps I may hope for the material cooperation of the Academy in performing new experiments on the resistance of air. It is my intention that these will be more extensive, more numerous, more accurate, more varied and carried out with a greater flow of air and at greater velocities than the previous ones.

As early as eight years ago, in 1891, I published a description of some of my earlier experiments in Volume IV of the "Trudy Otd. fizich. nauk Imper. Moskovskogo Obshchestva lyubiteley yestyestvoznaniya." Since that time I have been considering the idea of carrying out much more accurate and extensive experiments on the resistance of air, and I flatter myself with the hope that the foremost scientific institution in Russia will not deny me its support.

The results of the experiments would be sent to the Academy and a description of them, together with the drawings and computations, could be published in the "Zapisky Akademii" (if they should so merit).

I remain, your most humble and respectful servant, Konstantin Tsiolkovskiy.

Photographs of all the apparatus and models (after completion of the new experiments) would be sent to the Academy.

Archives of the Academy of Sciences USSR

File 38, schedule 2, No. 633, items 1, 2.

APPENDIX 5

Official Record of Meeting of Academy of Sciences. Imperial Academy of Sciences, Physico-Mathematical Branch, No. X, 22 September 1899.

242. In a letter addressed to the Vice-President of the Academy and dated 10 September, Mr. Tsiolkovskiy, the author of works on aeronautics and physical astronomy, drew attention to his recent experiments on the resistance of air (see "Vestnik opytnoy fiziki," Odessa, Nos. 269-272, 1898-1899) and begged the Academy, in the event that they were found to be of sufficient scientific merit, to extend him material assistance in conducting further experiments.

It was decided to refer Mr. Tsiolkovskiy's letter to Academician M. A. Rykachev for review.

APPENDIX 6

Extract from Official Record of Meeting of the Imperial Academy of Sciences, Physico-Mathematical Branch, No. XI, 6 October 1899.

270. In accordance with para. 242 of the minutes of the meeting of 22 September, Academician M. A. Rykachev gave the following report on the note received from Mr. Tsiolkovskiy:

"The author begs the Academy to extend him material assistance in order to enable him to continue his experiments on the resistance of air described in his article 'Davleniye vozdukha na poverkhnosti, vvedennyye v iskusstvennyy vozdushnyy potok' (Air Pressure on Surfaces Introduced into an Artificial Flow of Air), published in Nos. 269-272 of 'Vestnik opytnoy fiziki i elementarnoy matematiki.'

These experiments deserve the full attention of the Academy, both as regards the idea behind them and their diversity. In spite of the primitive home-made apparatus available to the author, he has succeeded in obtaining some very interesting results.

In order to obtain a flow of air, Mr. Tsiolkovskiy uses a wind machine, rather like a winnowing machine, driven by weights of various sizes (1/2 lb., 1 lb., 2 lb., 4 lb., 8 lb., and 16 lb.).

In order to give the stream of air a single general direction, he has arranged in the mouth of the machine a box, open on two sides, fitted with thin horizontal and vertical plates, designed to direct the air in parallel jets, so that the air leaves the box in a single horizontal flow. The test model is placed in the center of this flow, a short distance from the mouth of the wind machine. The cross-sectional area of the flow is 1200 sq. cm and the cross-sectional area of the test body not more than 80 sq. cm. The model is placed on supports fastened to a tank that floats in water contained in a second vessel fixed rigidly to a table. To the same table there is fastened an upright supporting a thin, light needle which can pivot freely like a pendulum. A very light and mobile wire tie connects this needle with the supports on which the model stands. The slightest pressure on the model will cause it to move, together with its floating tank, thus deflecting the needle until the weight of the deflected needle balances the pressure. The amount of deflection, determined with the aid of a fixed horizontal millimeter scale mounted behind the needle, also gives the pressure on the model. Absolute values of the pressure, corresponding to a given deflection of the needle, are measured with the aid of weights suspended from a string attached to the floating tank and carried over a pulley block. In order to determine the flow velocity at the different loads used to drive the wind machine, the author measured the pressure produced by the flow on plates arranged normal to the flow and then computed the velocity from the formula proposed by Cailletet and Colardo, who have determined the relation between pressure and velocity for the rectilinear motion of a plate. Among other things, the author determined the air pressure on a plate at different angles of inclination, every 5 degrees, obtaining the unexpected result that for a plate moving at about 50 degrees to the direction of flow the normal pressure on the plate is greater than for a plate arranged perpendicular to the direction of flow. Experiments were performed on right prisms, cylindrical surfaces with various cross sections, polyhedra, a sphere, elongated solids of revolution, and so on. The author determined the resistance as a function of the flow velocity and the aspect ratio of the shape. Very interesting experiments were performed with the object of determining the effect of the rear part of the model. It was found that for an elongated body, one end of which was spherical and the other end a sharp cone (Schwarz's aerostat), the resistance is much less when the spherical, rather than the conical end is turned into the flow. The extent to which the author has succeeded in obtaining reliable results is clear from the

agreement observed in testing the pressure on a square plate at flow velocities obtained by driving the wheel of the wind machine with loads regularly increased by a factor of two. The pressure on the plate was then found to be: 4.5, 9, 18, 36.5, 73, and 145, i.e., the variation was almost exactly that which theory might lead one to expect. Some of the experiments were devoted to determining the air friction on a plate arranged parallel to the flow The experiments were performed with small models and at low flow velocities Moreover, the velocity in different sections of the flow was not determined. For all these reasons it would be highly desirable to perform experiments on a larger scale with more accurate instruments, and I beg to suggest that the Branch grant the author's request and extend him material assistance from the fund intended to meet scientific needs."

The Branch decided to ask Academician M. A. Rykachev to enter into communication with Mr. Tsiolkovskiy and request him to submit a program of proposed experiments and an estimate of expenses, warning him that this estimate must be prepared as economically as possible in view of the limited resources of the Academy.

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File 1, schedule 1a, No. 146.

APPENDIX 7

Extract from Official Record of Meeting of the Imperial Academy of Sciences, Physico-Mathematical Branch, No. XVI, 15 December 1899.

384. In accordance with an earlier Branch resolution (para. 270), Academician M. A. Rykachev presented Mr Tsiolkovskiy's detailed program of experiments on the resistance of air and his estimate of the corresponding expenses. According to this estimate, the completion of all the experiments described in the program would require an expenditure of approximately 1000 rubles, but Mr. Tsiolkovskiy was ready to accept, according to his statement, any material assistance that the Academy might find it possible to extend. In view of the interest which the Branch had shown in these experiments, Academician M. A. Rykachev requested that the Conference, when it distributed the funds for the scientific needs of the Academy in 1900, should determine how much assistance it could grant to Mr. Tsiolkovskiy.

It was decided to examine the matter further at a joint meeting of Branches I and II on 12 January 1900.

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File 1, schedule 1a, No. 146.

APPENDIX 8

Extract from Official Record of Meeting of the Imperial Academy of Sciences, Joint Meeting of Branches I and III, 12 January 1900.

3. Academician M. A. Rykachev: "In connection with the matter of cooperation between the Academy and Mr. Tsiolkovskiy in the conduct of his experiments on the resistance of air (see para. 384 of the minutes of the meeting of the Physico-Mathematical Branch of 15 December 1899). To carry out all the experiments described in Mr. Tsiolkovskiy's proposed program would cost 1000 rubles, but Mr. Tsiolkovskiy, according to his statement, is prepared to accept a lesser amount of aid. Academician M. A. Rykachev considers it desirable to grant Mr. Tsiolkovskiy a thousand rubles in the year 1900 in order to carry out all the experiments he has planned."

The joint meeting resolved:

3. To Academician M. A. Rykachev for the execution of Mr. Tsiolkovskiy's experiments on the resistance of air the sum of 470 rubles.

Archives of the Academy of Sciences USSR

File 1, schedule 1a, No. 147.

APPENDIX 9

Abridged report by Konstantin Tsiolkovskiy (address: Kaluga, Georgiyevskaya ul., dom. Pobedina) to the Imperial Academy of Sciences in connection with his experiments on the resistance of air performed under the auspices of the Academy between March 1900 and December 1901.

This report includes: 58 table-drawings (86 drawings), a summary of conclusions based on my notes and raw material -- the series of figures recorded by my assistant during the experiments and serving as the basis of my work (24 pages).

The drawings are copies of the original drawings which I have retained. These copies have been made more roughly, but are adapted for easy intelligibility and, if necessary, publication.

The summary of conclusions based on my notes is useful for a rapid review and evaluation of my work. The raw material, in the form of rows of figures, indicates the fluctuations of the needle during the observations.

I requested 1000 rubles from the Academy for these experiments. The Academy granted me 470. I accepted this money with much gratitude and pleasure and decided to use it to finance all my proposed experiments.

If God grants me health and strength, I shall do what I propose to do even if I have to draw upon my own rather limited means.

Whatever the position of the Academy, I shall never abandon my research.

At the same time, I am anxious to have the material as well as the moral support of the Academy, which will be possible only if, in the eyes of the Academy, my work deserves it.

Thus, I shall await the verdict of the Academy and assistance commensurate with this verdict. I shall accept the verdict uncomplainingly, but in no circumstances shall I cease my experiments, since it is permissible to hold one's own opinion, and in mine these experiments are of some value.

If the Academy does not reject them, I shall prepare a note on my further experiments for next December (December 1902).

If the Academy does me the honor of publishing my work, I humbly beg that instructions be given for me to be sent the galley proofs.

I also beg, most humbly, that after my work has been examined and evaluated I be informed of the opinion and decision of the Academy in the persons of its highly esteemed members.

This is of the greatest importance for me, since it will determine my future path in connection with the communication of my discoveries concerning the resistance of air.

With the most profound respect,

I remain,

K. Tsiolkovskiy.

Reprinted from an authentic letter of K. E. Tsiolkovskiy's, written on the first page of the manuscript which he submitted to the Academy of Sciences and returned to him together with the same. It is now preserved in the archives of the Moscow Branch of the Archives of the Academy of Sciences USSR, file No. 555.

APPENDIX 10

Letter Written by K. E. Tsiolkovskiy to Academician M. A. Rykachev Dated 1 September 1901.

Your excellency,

I have now been working hard on my experiments on the resistance of air for more than a year, but my task is still far from completed. The description and analysis of the experiments I have already performed take up more than 60 pages of the final report I am preparing for the Academy, and to this must be added 20 drawings (sketches of apparatus and graphs). In spite of this the proposed program of research is still not exhausted. I can not even predict with confidence when this will be. However, I do know that I have worked hard and accomplished something. Please advise me what to do now: continue my experiments or pause for three months in order to prepare a copy of my findings for presentation to the Imperial Academy of Sciences in December 1901.

I am inclined toward the latter course, since my health is not too good, and God only knows what next year may bring.

I would like to have the report published somewhere and would much appreciate your advice in the matter.

With the utmost respect, I remain,

Your

K. Tsiolkovskiy

A period of teaching activity has begun for me and this, though it takes up only a little of my time, takes up a great deal of my strength, since I am as interested in the business of teaching as in my experiments.

In spite of these and other unfavorable circumstances, I am hopeful of completing all my work by December 1902. Address: Kaluga, Georgiyevskaya ul., d. Pobedina. K. Tsiolkovskiy.

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File 38, schedule 2, No. 633, items 6, 7.

APPENDIX 11

Academician M. A. Rykachev's Reply to K. E. Tsiolkovskiy

Director of the Nikolayevskaya Main
Physical Observatory,
26 September 1901

Copy to:

Kaluga,
Georgiyevskaya ul., d. Pobedina,
to his excellency
K. Tsiolkovskiy

Dear Sir,

On returning from an official mission, I found your kind letter of 1 September 1901.

I hasten to reply that in my opinion it is, of course, preferable

to interrupt your experiments in time to prepare a report on what you have already accomplished. When the report is written, if you will kindly forward it to me, I am prepared to heed your request to consider where it might be published

With warm wishes first for your health and then for the success of your work, I remain,

Your devoted

M. Rykachev

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APPENDIX 12

Letter Written by K. E. Tsiolkovskiy to Academician M. A. Rykachev

Kaluga, 6 March 1902

Your excellency,

Already two and a half months have passed since I had the honor to present my findings on the resistance of air to the Imperial Academy of Sciences. I am now beginning to experience some disquiet about the fate of my report. Immediately after sending it off, I cheerfully began a series of new experiments However, as time passes, I become more and more concerned. Forgive me if I bother you prematurely, but please understand my feelings and my anxious state of mind, which is causing me to lose heart, and kindly let me know how my report was received or what recommendations you made concerning it, whether I may expect it to be published, and whether I may hope for material or at least moral support from the Imperial Academy of Sciences.

My apartment is crammed absolutely full with my apparatus. I dream of building an observatory for research into the resistance of air, where I could keep all my machines, instruments, and tools for making models.

It seems to me that with even a minimum of support from the Academy I could realize my dream (of course, not without my own small means).

In such an observatory it would be easy to repeat my experiments for any scientists anxious to verify them. It would also be easy to store models and conduct experiments for an indefinite number of years, since this is a big project, a very big project, as big as the ocean . . . .

It would be impossible to do anything without the help of His Imperial Highness, the President of the Academy. I believe that much depends on your opinion of my work. Be assured of my utmost respect.

K. Tsiolkovskiy

Address: Kaluga, Girls' Diocesan School, K. Tsiolkovskiy,

Physics Teacher.

Archives of the Academy of Sciences USSR

File 38, schedule 2, No. 633, Items 4, 5.

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2. Aeroplan, ili ptitsepodobnaya (aviatsionnaya) letatel'naya mashina (The Airplane or Bird-Like Flying Machine). Article in the journal "Nauka i zhizn" (Science and Life), Moscow, 1894 Nos. 43-46 (10 figs). Issued as a separate offprint in 1895.

3. Letter to the editor of the journal "Tekhnicheskiy sbornik" (Technical Collection), St. Petersburg, 1896, on mistakes made by Professor M. M. Pomortsev in calculations on the resistance of air in his work "Privyaznoy, svobodnyy i upravlyayemyy aerostaty. Mekhanicheskiye usloviya ikh ravnovesiya i dvizheniya" (Tethered, Free and Controlled Aerostats. The Mechanical Principles of Their Equilibrium and Motion). St Petersburg, 1895.

4. Samostoyatel'noye gorizontal'noye dvizheniye upravlyayemogo aerostata (novyye formuly soprotivleniya vozdukha i dvizheniya aerostata)(The Independent Horizontal Motion of a Controlled Aerostat (New Formulas for the Resistance of Air and the Motion of an Aerostat). Article in the journal "Vestnik opytnoy fiziki i elementarnoy matematiki" (Herald of Experimental Physics and Elementary Mathematics). Odessa, 1897, Nos. 258-259 (3 figs.). A separate offprint of this article was published in 1895 in the form of a brochure (pp. 3).

5. Davleniye vozdukha na poverkhnosti, vvedennyye v iskusstvennyy vozdushnyy potok (Air Pressure on Surfaces Introduced into an Artificial Flow of Air). Article in the journal "Vestnik opytnoy fiziki i elementarnoy matematiki", Odessa, 1898, Nos. 269-272. A separate offprint of this article was published in 1899 (pp. 48).

6. Voprosy vozdukhoplavaniya (Problems of Aeronautics) (in connection with the work on aeronautics of Division VII of the "Russkoye tekhnicheskoye obshchestvo" (Russian Technical Society) for the period 1895-1900). Article in the journal "nauchnoye obozreniye" (Science Review), St. Petersburg, 1900, No. 10. A separate offprint of this article was published in 1900 (pp. 16).

7. Review of the book by Dr. K. Ya. Danilevskiy "Upravlyayemyy letatel'nyy snaryad" (The Controlled Projectile), Kharkov, 1900, in the journal "Nauchnoye obozreniye," St. Petersburg, 1901, No. 5.

8. Review of the book by D. Chumakov "Osnovy k resheniyu zadachi vozdukhoplavaniya" (The Basis of the Solution of the Problem of Flight), Askhabad, 1901, in the journal "Nauchnoye obozreniye," 1901, No. 9.

9 Soprotivleniye vozdukha i vozdukhoplavaniye (The Resistance of Air and Aeronautics). Article in the journal "Nauchnoye obozreniye," St. Petersburg, 1902, No. 5. A separate offprint of this article was published in 1902 (pp. 20).

10. Chapter XIV "Soprotiveleniye vozdukha" (The Resistance of Air) in K. E. Tsiolkovskiy's large work "Aerostat i aeroplan" (The Aerostat and the Airplane), published in the journal "Vozdukhoplavatel" (Aeronaut), 1905-1908. Chapter XIV was published in No. 8, 1908.

11. Ustroystvo letatel'nogo apparata nasekomykh i ptits i sposoby ikh poleta (The Organization of the Flying Apparatus of Insects and Birds and Their Modes of Flight) (with 5 figs.). Article in the journal "Tekhnika vozdukhoplavaniya" (Aeronautical Engineering), St. Petersburg, organ of Division VII of the "Russkoye tekhnicheskoye obshchestvo," 1912, No. 3. A separate offprint of this article was published in the same year (1912).

12 Soprotivleniye vozdukha i skoryy poyezd (The Resistance of Air and the Fast Train). Kaluga, 1927, published by the author (pp. 72, 3 figs.).

13 Izdannyye trudy K. E Tsiolkovskogo (The Published Works of K. E. Tsiolkovskiy) (brochure). Kaluga, 1927, published by the author, pp. 24. In this brochure written by Tsiolkovskiy there are a number of notes on his work on the resistance of air.

14. Davleniye na ploskost' pri yeye normal'nom dvizhenii v

vozdukhe (The Pressure on a Plane in Normal Motion in Air). Kaluga, 1930, published by the author (pp. 48).

The same article was republished in Vol. II of "Izbrannykh trudov K. E. Tsiolkovskogo" (Selected Works of K. E. Tsiolkovskiy), ONTI, 1934, editor F. A. Tsander (pp. 112).

15. Planer i veter (Gliders and the Wind). Article in No. 3 of the journal "Tekhnika molodezhi" (Engineering for Young People), 1935.

NOTES*

"Davleniye...

1. This foreword was found in the manuscript. It appears in print for the first time. 21.

2. Tsiolkovskiy was responsible for the first theoretical and experimental investigation of the influence of the aspect ratio of a wing on the aerodynamic forces. It should be noted that this investigation was carried out as early as 1890-1891, i.e., long before the development of modern airfoil theory. 28.

"Ob'yasneniya...

3. This article is a supplement to Tsiolkovskiy's "Davleniye zhidkosti na ravnomerno dvizhushchuyusya v ney ploskost" (The Pressure ...). In this volume it appears in print for the first time. Here Tsiolkovskiy, developing the hypothesis on which the main article is based, arrives at the conclusion that in any flow around an obstacle there are really two opposing flows, which create the circular motion we actually observe. Thus, his starting hypothesis is again confirmed. The notion of a return flow beyond an obstacle, expounded by Tsiolkovskiy in this article of 1891, determined for many years the further development of aerodynamics. 30.

"Aeroplan ili...

4. It is worth noting Tsiolkovskiy's original formulation of the problem of the part played by translational motion in the flight of heavier-than-air machines. He shows, both theoretically and experimentally (with the aid of apparatus he himself constructed), that the work required to keep a flapping-wing machine airborne decreases with increase in the rate of translational motion. 41.

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*The figures that follow each note indicate the [Russian] page to which it refers.

5. Tsiolkovskiy was the first in the world to suggest the use of coaxial airscrews rotating in opposite directions. 44.

6. It is significant that, as the text makes clear, even as early as 1890-1894 Tsiolkovskiy emerges as a convinced advocate of the use of gasoline internal combustion engines in flying machines. His views were undoubtedly progressive at the time and long anticipated the actual installation of such engines in real aircraft. 44.

7. Here, for the first time, Tsiolkovskiy sketches (in text and drawings -- Figs. 2-4) a flying machine very similar to our modern concept of an airplane. Note the following basic features of his design, which are not found in any previous machine, either proposed or actually constructed: the fuselage is streamlined, the wings are unbraced, cantilevered, and curved in both longitudinal and transverse section, the coaxial airscrews rotate in opposite directions, and the engines are of the gasoline-powered internal combustion type. Before this, nobody had ever built or even suggested an airplane of this kind. Tsiolkovskiy not only put forward a novel idea, he tested it thoroughly from the point of view of both aerodynamics and structural strength. All this gives Tsiolkovskiy undoubted priority as the author of the modern cantilevered-wing monoplane. 44.

8. In these pages Tsiolkovskiy developed various elements of the aerodynamic design of aircraft for the first time in the literature of aviation. 45.

9. By P_2 the author understands the weight of the engines needed for motion in the absence of a lift force; P_1 is the additional weight of the engines connected with the supplementary drag that develops in the presence of a lift force. 48.

10. Figs. 3 and 4 are of considerable interest, since here for the first time Tsiolkovskiy sketches a monoplane with a cantilevered wing, the thickness of which decreases in the direction from root to tip (equal strength). Fig. 4 is also of interest in that here for the first time we find a sketch of an aircraft of the "gull-wing" type, which, as we know, possesses a number of aerodynamic advantages. Tsiolkovskiy put forward this idea in 1894, i.e., long before it was first applied in practice. 52.

11. Here, for the first time in the literature of aviation, Tsiolkovskiy suggested using gyroscopes in aircraft as a very simple automatic pilot. He also proposes a form of automatic electromagnetic

control system for the rudder. Moreover, it is known that he had previously (in "Aerostat, metallicheskiy, upravlyayemyy" (The Metal Dirigible) No. 2, Kaluga, 1893) worked out a scheme for the automatic electromagnetic control of dirigibles. These facts establish Tsiolkovskiy's priority as the founder of aviation electro-automatics. 73.

"Davleniye vozdukha na...

12. Tsiolkovskiy's bladed "wind machine" (1896-1897) was the first wind tunnel to be built in Russia.

Tsiolkovskiy also designed measuring apparatus for this machine and, in particular, an aerodynamic balance of the floating type characterized, as may be seen from the description, by high sensitivity and great accuracy. With the aid of his wind tunnel and measuring apparatus and using an independently developed experimental technique, Tsiolkovskiy carried out, for the first time in the history of science, a systematic experimental investigation of the drag and lift forces acting on bodies of different geometry, including models of wings and dirigibles. As a result of this investigation, Tsiolkovskiy established a number of very important laws of aerodynamics, relating to the influence of the geometrical parameters of a body and the criteria of dynamic similarity on the resistance of the medium. Thus, with perfect justice we may call Tsiolkovskiy the founder of modern experimental aerodynamics.

Tsiolkovskiy's experimental investigations in the region of aerodynamics, carried out with the aid of his home-made apparatus, are described in the present article "Davleniye vozdukha na ...) and in two later works also to be found in this volume ("Otchet ..." and "Soprotivleniye ..."). 91.

13. Here and in the rest of the article the author neglects the component of the drag acting along the plate. 97.

14. Here Tsiolkovskiy establishes for the first time the laws of friction for an air medium. Here he is the first to show experimentally that air friction is in no way proportional to the area of the surface exposed nor to the square of the flow velocity, as had previously been supposed. Tsiolkovskiy was able to determine, with surprising accuracy, the small force exerted by air friction on the surface of a model. Tsiolkovskiy's remarkable experimental skill and intuition are also evident in his analysis of the results. Having selected power laws for the dependence of the friction on the flow

velocity and the dimensions of the model, Tsiolkovskiy clearly established that in the equation for the coefficient of friction the velocity and the size (length) of the plate have the same exponent, as the theory of dynamic similarity predicts.

The numerical values of the coefficients of friction determined by Tsiolkovskiy are close to those found considerably later by boundary layer methods. In modern notation equation (89) is equivalent to the following formula:

$$c_f = \frac{T}{\rho \frac{V^2}{2} L} = \frac{0.61}{\left(\frac{VL}{\nu}\right)^{0.4}} = \frac{0.61}{R^{0.4}},$$

where $R = \frac{VL}{\nu}$; the results of computations based on this formula prove to be close to the results obtained from the known equation of boundary layer theory (for laminar flow in the layer):

$$c_f = \frac{1.328}{R^{0.5}}. \quad 101$$

"Otchet...

15. Tsiolkovskiy was thus the first to equalize the flow in a wind tunnel with the aid of a system of oriented baffle plates. 124.

16. Here Tsiolkovskiy establishes for the first time the fact that holes cut in a plate increase the drag. 137.

17. Sections 45-46, like a number of later sections, are missing from Tsiolkovskiy's manuscript. Apparently this is a mistake in numbering. 138.

18. Tsiolkovskiy, who had been preoccupied with balloons since

1885, was always interested in the drag of solids of revolution. His was the first experimental study of streamlined solids of revolution to establish the relation between the drag of such bodies and their aspect ratio, their size, the flow velocity, and other parameters. The results of this investigation were published partly in "Davleniye vozdukha na ..." but more fully in the present report.

It is interesting to note that the very term "prodolgovatost'" (oblongness) used by Tsiolkovskiy conveys the notion better than the modern term "udlineniye" (elongation, aspect ratio). 143.

19. Here the author neglects the component of the drag acting along the plate. 147.

20. To Tsiolkovskiy belongs the credit not only for the first theoretical investigation of the influence of the aspect ratio of a wing on the aerodynamic forces acting on it but also for the first thorough experimental investigation of the phenomenon.

In the text Tsiolkovskiy gives the principal results of his experiments to determine the pressure on plates with different aspect ratios. It should be noted that he varies the aspect ratio of the plates within very wide limits and, in particular, also experimented with plates having small aspect ratios. He was the first to point out the significance of wings with a low aspect ratio, which, however, were not introduced into practice until much later. 152.

21. Here the author neglects the air friction on the lateral surface of the cylinder. 158.

22. The explanation of this paradoxical effect was first given by Professor Zhukovskiy who pointed out that the reason for the higher resistance of a body in a moving medium is the eddies formed at the solid surfaces confining the fluid (for example, the walls of a channel). 160 and 137.

23. Here Tsiolkovskiy expounds for the first time certain elements of the aerodynamic design of aircraft. 184.

24. Tsiolkovskiy's experiments with long cylinders (up to 1 meter), arranged with the axis parallel to the flow, enabled him, as may be seen from the text, to make considerable improvements in the relations he had previously derived (in "Davleniye vozdukha na ...) and to establish the fundamental laws of turbulent friction. 198.

"Soprotivleniye vozdukha i vozdukhoplavaniye"

25. As already pointed out in the note on page 152, Tsiolkovskiy was the first to establish the relations between the aspect ratio of a wing and the forces acting on it. The text gives a compressed version of Tsiolkovskiy's main conclusions concerning this problem, reached as a result of his experiments with his second wind tunnel. 213.

26. Tsiolkovskiy also used the term "davleniye" (pressure) in the sense of "soprotivleniye" (resistance, drag) and "sila davleniya" (force). Here and throughout the article these three concepts are differentiated. 214.

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