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ANALYSIS OF THE HYDROSTATIC JOURNAL AND THRUST GAS BEARING FOR THE NASA AB-5 GYRO GIMBAL BEARING

By

J. W. Lund
R. J. Wernick
S. B. Malanoski

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ABSTRACT

This report presents a summary of the work performed by Mechanical Technology Incorporated under Contract NAS8-2588 (Subcontract 213-03-72120-6244 with the General Electric Company), entitled "Gas Lubricated Gimbal Bearing Investigation," with the National Aeronautics and Space Administration, Hunstville, Alabama.

The investigation is an analytical study of the static and dynamic load carrying capacity and the flow for the externally pressurized, gas lubricated journal and thrust bearing with orifice restricted feeding.

The investigation comprises the following analyses:

1. An approximate analysis of the static load and the flow for the journal and thrust bearing, restricted to small values of the eccentricity ratio. The number of orifice restricted feeding holes is assumed to be large such that the feeding planes become line sources. The solution is a first order perturbation calculation on the eccentricity ratio.
2. An analysis of the static load and the flow for the thrust bearing where the orifices are represented by point sources.
3. An approximate analysis of the static load and the flow for the journal bearing with the orifices as point sources. The analysis is a first order perturbation calculation on the eccentricity ratio ϵ, i.e., the results are valid for small ϵ only.
4. An approximate analysis of the static load for the journal bearing, valid for all eccentricity ratios. The feeding planes are taken as line sources and the bearing is assumed to be infinitely short.
5. An approximate analysis of the dynamic stiffness for the journal and the thrust bearing. The feeding planes are represented by line sources and the solution is a first order perturbation calculation on

the amplitude, i.e., the results are limited to motions with small amplitude. The motion is a harmonic vibration around the concentric position and is either a pure translation or a pure transverse rotation.

All five analyses have been programmed for the IBM 704 computer. Numerical results are obtained and compared to test data.

INTRODUCTION

The externally pressurized, gas lubricated gimbal axis bearing for a gyroscope in a missile must meet a series of severe specifications. As specifically related to the pressurized gas bearing the most important of these specifications are

1. The bearing must be capable of withstanding high static and dynamic loads imposed by acceleration and vibrations during launching and space flight conditions.
2. The bearing and its auxiliary equipment should have minimum weight, volume and power consumption.
3. The error drift rate caused by turbine torque should be as low as possible.
4. The bearing must be stable.

The design problems arising from these specifications require for their solution a development of manufacturing techniques, experimental methods and theoretical analysis. It is the purpose of this report to describe the analytical methods which have been developed to assist in the design of the gimbal gas bearing. The present analysis has three objectives:

a) to furnish methods for optimization of the design,
b) to calculate the load carrying capacity and the flow of the bearing,
c) to calculate the dynamic bearing stiffness for use in a resonant frequency calculation.

In formulating the theoretical analysis the two basic equations are:

1. Reynolds equation which represents the bearing gas film (see eq. (A.5) and (A.17), Appendix A). This means that gas inertia, turbulence and shock conditions are not considered. Furthermore, the gas film is assumed to be isothermal.

2. The thermodynamic, one-dimensional orifice flow equation (eq. (A.22), Appendix A). The equation contains a vena contracta coefficient to define the effective flow area through the orifice. This coefficient is determined experimentally.

Hence the basic requirement of the analysis is to find methods for solving Reynolds equation and the orifice equation simultaneously. In general, these equations are too complex to permit an exact solution so that instead various approximate solutions have been obtained, each serving its particular purpose.

The first solution assumes that the number of feeding holes in a feeding plane is sufficiently large to let the feeding planes become line sources. This simplification allows a direct solution of the thrust bearing whereas the journal bearing with its circumferentially varying film thickness is solved by a first order perturbation on the eccentricity ratio ϵ (i.e., the results are valid for small ϵ values only). The advantage of the solution is that the results for the load and the flow are functions of only three parameters: the L/D ratio, the pressure ratio P_s/P_a and the feeding parameter Λ_t (see Figs. 3-6). Hence the analysis provides an easily interpreted, yet comprehensive picture of the bearing performance so that the effect of a change in any design parameter is readily evaluated.

The second solution uses a more accurate representation of the orifices by making them discreet point sources. For the thrust bearing, Reynolds equation reduces to Laplace's equation and an exact solution can be obtained for all eccentricity ratios (i.e., for all values of the film thickness). The journal bearing, however, does not have a uniform film thickness and therefore an exact solution cannot be obtained. Restricting the analysis to small eccentricity ratios a first order perturbation calculation is performed which gives an accurate evaluation of the bearing stiffness at $\epsilon = 0$. To find the journal bearing load vs. eccentricity ratio relationship for large values of ϵ an

approximate analysis is made. The analysis assumes the orifice feeding to be a line source and in addition that the bearing is infinitely short (i.e., that the axial flow gradient is much larger than the circumferential flow gradient). This results in a load vs. ϵ curve which may be improved by normalizing it with respect to the accurately calculated slope (stiffness) at $\epsilon = 0$.

The last part of the analysis is concerned with the dynamic stiffness of the bearings. In this case Reynolds equation becomes non-linear. Limiting the analysis to harmonic vibrations with small amplitude around the concentric journal position, a numerical calculation is carried out by means of a first order perturbation solution. The analysis assumes two vibratory motions: either a pure translation or a pure transverse rotation. Hence the increase in bearing stiffness caused by the squeeze film effect can be estimated (see Figs. 26-33) and included in a resonant frequency calculation.

All the above analyses have been programmed for the IBM 704 computer. Numerical results have been obtained and compared to experimental data (see Figs. 7-14 and Figs. 20-25).

RESULTS

The scope of the investigation is outlined in the preceeding section. The general theoretical background and all the analyses are given in detail in the appendices. This section is concerned with the numerical results of the investigation as they are given in Figs. 1-33. The results are of two categories: a) general dimensionless data from which the effect of a change in design parameters may be evaluated, and b) specific numerical data for comparison with test data.

1. Orifice and Millipore Characteristics

The thermodynamic, one dimensional orifice flow equation is given by eq. (A.22). To account for the effective orifice area the equation contains a vena contracta coefficient γ which is evaluated by fitting the theoretical equation to NASA test data 2588-1 to -4 giving the orifice flow as a function of downstream pressure for a fixed supply pressure. The results for the sleeve and the endplate are given in Fig. 2 and serve as a basis for the subsequent calculations. In addition to using orifice restricted feeding, tests have also been conducted with millipore restriction. The millipore restrictor is a filter of five layers: 2 screens, 2 cement wafers and a membrane filter. An empirical expression is used to describe the characteristics of the millipore, eq. (A.29), which contains five experimentally evaluated coefficients. From NASA test data 2588-21 to -27 of the millipore flow as a function of supply pressure for a fixed downstream pressure these coefficients are found to be:

	Sleeve No. 4	Endplate No. 2 and No. 4
a_o	= 3.892	3.859
B	= 1.406	1.267
$C(psi^{-1})$	= .101	.108
D(psi)	= -.335	-.40
$a^2(in^2)$	= $1.399.10^{-6}$	$1.348.10^{-6}$

2. Bearing Dimensions and Air Data

The general bearing geometry is shown in Fig. 1. The actual bearing dimensions are summarized below as taken from NASA drawings:

orifice bearing: D10581725, C10582503, F10582504, F10582710

millipore bearing: D10582619, FX1783550, BX1796031, BX1796032, FX1796040

			Orifice Bearing Sleeve No. 5256 Endplate No. E339,E445	Millipore Bearing Sleeve No. 4 Endplate No. 2&4
Journal Bearing	Length outside feeding planes	: L	.724	.724
	Length between feeding planes	: L_1	1.2532	1.2532
	Bearing dia.	: D	2.1622	2.1625
	Radial clearance	: C	.00055,.000925	.0005,.00074,.00095
	Orifice radius	: a	.0025	-
	Feeding hole diameter	: d	.028	.028
	Number of feeding holes	: N	32	48
Thrust Bearing	Outer radius	: R_1	1.057	1.0676
	Inner radius	: R_2	.650	.650
	Orifice circle radius	: R_c	.835	.835
	Axial clearance	: C	.00085	.00055,.00075,.001
	Orifice radius	: a	.0035	-
	Feeding hole diameter	: d	.018	.028
	Number of feeding holes	: N	12	18
	Depth of recess	: h_r	.0002	.0002
	Radius of recess	: R_r	.109	.109

(all dimensions in inches)

The air is at 75°F with:

Viscosity : $\mu = 2.83.10^{-9}$ lbs.sec/in^2

(Gas constant).(Total temp.) : RT = 342,500 in

Four ratios of supply pressure to ambient pressure have been tested:

Ambient Pressure P_a psia	Supply Pressure ΔP_s psig	P_s psia	Pressure Ratio $V = P_s/P_a$
14.7	12.8	27.5	1.87075
14.7	1.25	15.95	1.08503
7.0	1.25	8.25	1.17857
1.0	1.25	2.25	2.25

3. Bearing Stiffness and Flow at Eccentricity = 0

The first part of the investigation is based on an analysis which is a first order perturbation solution on the eccentricity ratio ϵ around the concentric journal position (Appendix B and C). Hence the analysis gives results for the stiffness and the flow at $\epsilon = 0$ for the journal bearing and the double acting thrust bearing. In dimensionless form the results are given in Figs. 3-6 as function of the feeding parameter Λ_t. This parameter may be visualized as expressing the ratio between the bearing flow resistance and the orifice flow resistance. When the bearing resistance is very high (f. inst. for a very small clearance) or the orifice resistance is low, then there is no pressure drop through the orifice. Hence, the pressure is uniform around the circumference of the journal bearing and there is no load carrying capacity. In other words, as $\Lambda_t \rightarrow \infty$, the load $\rightarrow 0$. Conversely, when the bearing resistance is small (f. inst for a very large clearance) or the orifice resistance is high, the pressure downstream from the orifice reduces to ambient pressure and again there is no load carrying capacity, i.e., as $\Lambda_t \rightarrow 0$, the load $\rightarrow 0$. Therefore, the load has a maximum value for a given bearing geometry and a given pressure ratio, corresponding to a particular value of Λ_t. This is the case for both the journal bearing and the double acting thrust bearing as shown by Figs. 3 and 5. Thus, in general the bearing clearance and the orifice

area should be chosen to give a Λ_t which corresponds to the maximum load. This is illustrated in Figs. 7, 9, 11 and 13 where the orifice area is held fixed while varying the clearance. It should furthermore be observed that for a fixed Λ_t and a given ambient pressure, the flow increases with the cube of the bearing clearance. Hence, from the point of view of minimizing the flow, and still have a maximum load, i.e., Λ_t unchanged, the clearance should be as small as consistent with manufacturing considerations. However, the bearings capacity for shock absorbtion is diminished by reducing the clearance so that a need for a compromise is indicated.

In applying the above approximate line source analysis to an actual bearing design it is apparent that there is a need for introducing a correction to account for the difference between the idealized model and the actual bearing. Experience has shown that the analysis is able to predict the bearing flow rather accurately, from which it can be deduced that the analysis also calculates the downstream orifice pressure correctly. However, the analysis ignores the reduction of pressure between the orifices so that the calculated load tends to be too large. The load is generated by the circumferential pressure variation set up by the eccentric journal, or expressed in terms of the analysis, by the pressure perturbation on the symmetrical, concentric-position pressure. This variation (or perturbation) is governed by the slope of the orifice flow vs. downstream pressure relationship at the operating concentric journal position (see. Fig. 4, Appendix B). Therefore, it seems reasonable to attempt to apply an empirical correction factor to the slope while recognising that a formal justification is lacking. A correction factor of 2.0 has been used in calculating Figs. 3, 5, 7, 9, 11 and 13, i.e., m_o' in Appendix B has been multiplied by 2.0. The

analysis is programmed for the IBM 704 computer.

Figs. 7-14 show a comparison between NASA test data (2588-5 to 20, -38 to 50 and data of June 26, June 28 and July 10, 1962) and theoretical results obtained as explained above. The agreement is quite satisfactory.

The same analysis is used to study the effect of the axial position of the feeding planes in the journal bearing. The results are shown in Fig. 15 where the abscissa is the ratio of the distance between the feeding planes and the total bearing length. The curves give the maximum obtainable dimensionless load together with the corresponding values of the dimensionless flow and the feeding parameter Λ_t. Within a practical range for the feeding plane distance the load does not exhibit a maximum as long as Λ_t is changed according to the given curves. If Λ_t is kept constant, though, while the distance is changed, an optimum feeding plane position can be determined. The ratio of load to flow is maximum with one feeding plane in the middle of the bearing. It should be noted that these results are concerned with the load for an aligned journal only, or in approximation with the stiffness for translatory journal motion. If instead the stiffness for journal rotation around a transverse axis was considered, it would be expected that an optimum feeding plane position existed. This calculation has not been performed

4. <u>Bearing Load vs. Eccentricity Ratio</u>

Since the preceeding line source analysis is restricted to small values of the eccentricity ratio ϵ and predicts a linear relationship between load and ϵ, additional analyses are required to determine the load for large values of ϵ.

The thrust bearing presents the simplest case because the gas film thickness is uniform. Even the line source analysis need not be limited to small ϵ although this was done above in order to simplify the results for the double acting thrust bearing. For the present analysis a more accurate representation of the orifices is introduced by making them point sources. This is a logical method since Reynolds equation for the thrust bearing reduces to Laplace's equation in the square of the pressure. The orifices are taken as logarithmic singularities and the physical boundary conditions are satisfied by means of image sources and sinks (Appendix E). The general dimensionless results are given in Figs. 16 and 17 for the thrust bearing with a recess around the feeding hole and in Figs. 18 and 19 for the bearing without a recess. Figs. 16 and 18 give the dimensionless orifice downstream pressure P_i as a function of the feeding hole radius r_o and the orifice source strength C. The effect of recess depth h_r or number or feeding holes N is also shown. Combining this data with the orifice flow equation (see eq. (E.6), Appendix E), the actual downstream pressure and source strength can be determined for any particular case with a given film thickness and bearing geometry. Knowing the source strength the bearing load is found directly from Fig. 17 or 19. Such a calculation has been carried out for four specific test conditions and the results are compared to the experimental data in Figs. 20, 21 and 22. The agreement is good. The pressure distribution has also been computed and compared to measurements in a few cases. The correlation was very satisfactory, but no results are given here. It should be noted that in performing the calculations, the effective feeding hole radius r_o was taken as .007 inch instead of the actual dimension of .009 inch to account for the orifice jet not filling out the entire feeding hole

area. This value of the effective radius is in agreement with the measurements of the velocity head of the orifice jet as it enters the bearing.

The corresponding analysis for the journal bearing is vastly more difficult since the film thickness varies circumferentially when the journal is in an eccentric position. Instead of attempting a complete solution the analysis has been limited to finding the bearing stiffness (i.e., the slope of the load curve) at $\epsilon = 0$. This is done by perturbing the pressure and the orifice source strength with respect to ϵ around the concentric position. The orifices are point sources expressed mathematically by the impulse function (the Dirac delta function). Performing a first order perturbation, Reynolds equation reduces to two equations of the Laplacian type, the first equation being for the pressure in the concentric portion and the second equation being for the pressure perturbation which determines the load. The equations are solved in terms of infinite series (see Appendix C and D) and calculated on the IBM 704 computer. Due to the many parameters and the lengthy computation time no general results have been given. However, the program is used to obtain the stiffness for four particular test conditions (Figs. 23-25) in connection with the subsequent analysis.

In order to extend the analysis of the journal bearing to larger values of the eccentricity ratio an approximate analysis has been set up, see Appendix F. The orifice feeding is again represented by line sources and in addition the bearing is considered infinitely short (or in other words the axial flow gradient >> the circumferential flow gradient). A similar approach has been successfully employed for the hydrodynamic bearing (Ref. 6). Solving the equations results in the load as a

function of ϵ. This curve is scaled down so that its slope at $\epsilon = 0$ becomes equal to the previously calculated, accurate slope (stiffness). The results are shown and compared to NASA test data in Figs. 23-25. The agreement is reasonably good.

5. Dynamic Bearing Stiffness

If the gyroscope is subjected to time changing accelerations the journal will perform vibratory motions. The velocity of the motion will "squeeze" the gas film thereby increasing the film pressure and hence increase the bearing load and the stiffness. Since the velocity is proportional to the vibratory frequency the dynamic stiffness is frequency dependent. This effect is important in a resonant frequency calculation.

The dynamic Reynolds equation (eq. (A.6) and (A.17), Appendix A) is non-linear and only an approximate solution is attempted. The equation is linearized by assuming the journal motion to be harmonic with small amplitude around the concentric position. Two vibrating modes are studied for both the journal and the thrust bearing: a pure translation with amplitude $\epsilon.\cos\omega t$ and a pure rotation around a transverse symmetry axis with angular amplitude $\alpha.\cos\omega t$, where ω is the frequency. Assuming ϵ and α small, a first order perturbation calculation is performed. In addition, the feeding planes are treated as line sources. The resulting equations are rewritten in terms of finite difference equations and integrated numerically on the computer. Results for the dimensionless translatory stiffness, the dimensionless rotational stiffness and the phase angle (i.e., the angle by which the stiffness lags the amplitude) are given in Figs. 26-33 as function of frequency. As would be expected, the stiffness increases with increasing frequency

but the effect is first noticeable at high frequencies, f. inst. for an ambient pressure P_a = 1 psia at 3 - 40 cps and for P_a = 14.7 at 200 to 400 cps. At low frequencies the dynamic stiffness is equal to the static stiffness.

Three distinct resonant frequencies may be visualized: a) a pure translatory mode governed by the journal bearing stiffness, b) a pure translatory mode governed by the thrust bearing stiffness, and c) a pure rotational mode (i.e., a rotation around an axis through the center of gravity of the journal perpendicular to the journal axis) governed by the combined journal and thrust bearing rotational stiffness. Denoting the translatory stiffness: K (lbs/in) and the rotational stiffness: m (lbs.in/rad) the resonant frequencies ω are given by:

translatory $\omega = \sqrt{\frac{K}{M}} \ \frac{rad}{sec}$

rotational $\omega = \sqrt{\frac{m}{I}} \ \frac{rad}{sec}$

where M is the journal mass in lbs.sec^2/in and I is the transverse mass moment of inertia in lbs.in.sec^2. Hence:

$$K = M\omega^2$$

$$m = I\omega^2$$

where K and m are found from Figs. 26, 30, 28 and 32 by:

$$K = \frac{W_D}{C\epsilon}$$

$$m = \frac{M_D}{C}$$

Thus K and m may be plotted as functions of frequency ω for a given bearing and a given operating condition. On the same graphs the parabolic curves $M\omega^2$ and $I\omega^2$ can be plotted and their intersection with the corresponding stiffness curves define the resonant frequencies.

The actual calculations have not been carried out. It should be noted that the dynamic stiffness actually is a combination of a stiffness term and a damping term. Let it be assumed that the journal is in static equilibrium at an eccentricity ratio ϵ_o. If the journal is given a small displacement X and a corresponding velocity $\dot{X}$ the restoring force becomes:

$$F = \frac{1}{C}\left(\frac{\partial F}{\partial \varepsilon}\right)_{\varepsilon_o} x + \frac{1}{C}\left(\frac{\partial F}{\partial \dot{\varepsilon}}\right)_{\varepsilon_o} \dot{x}$$

For a translatory journal motion with amplitude $C\cos\omega t$ the restoring force is from the analysis (see Eq. (G.62), Appendix G)

$$F = W_D = P_a D(L+L_1) W_D' \, \varepsilon \cos(\omega t - \varphi_w) = \frac{P_a D(L+L_1) W_D'}{C}\left(\cos\varphi_w x - \frac{\sin\varphi_w}{\omega}\right)$$

where $W_D' = \frac{W_D}{P_a D(L+L_1)\varepsilon}$ is given in Fig. 26 and φ_w is given in Fig. 27.

Hence we may introduce:

$$\text{the stiffness}: \quad K_D = \frac{1}{C}\left(\frac{\partial F}{\partial \varepsilon}\right)_{\varepsilon_o} = \frac{P_a D(L+L_1) W_D'}{C} \cos\varphi_w$$

$$\text{the damping}: \quad C_D = \frac{1}{C}\left(\frac{\partial F}{\partial \dot{\varepsilon}}\right)_{\varepsilon_o} = -\frac{P_a D(L+L_1) W_D'}{\omega C} \sin\varphi_w$$

where K_D is the actual dynamic stiffness. Both K_D and C_D are functions of frequency because W' and φ_w are. If the sleeve is subjected to a forced vibration with amplitude $b\cos\omega t$, then the journal will vibrate with an amplitude y relative to the sleeve, where:

$$y = \frac{M\omega^2 b}{\sqrt{(K_D - M\omega^2)^2 + (\omega C_D)^2}} \cos(\omega t - \psi_w) \qquad \psi_w = \tan^{-1}\left(\frac{\omega C_D}{K_D - M\omega^2}\right)$$

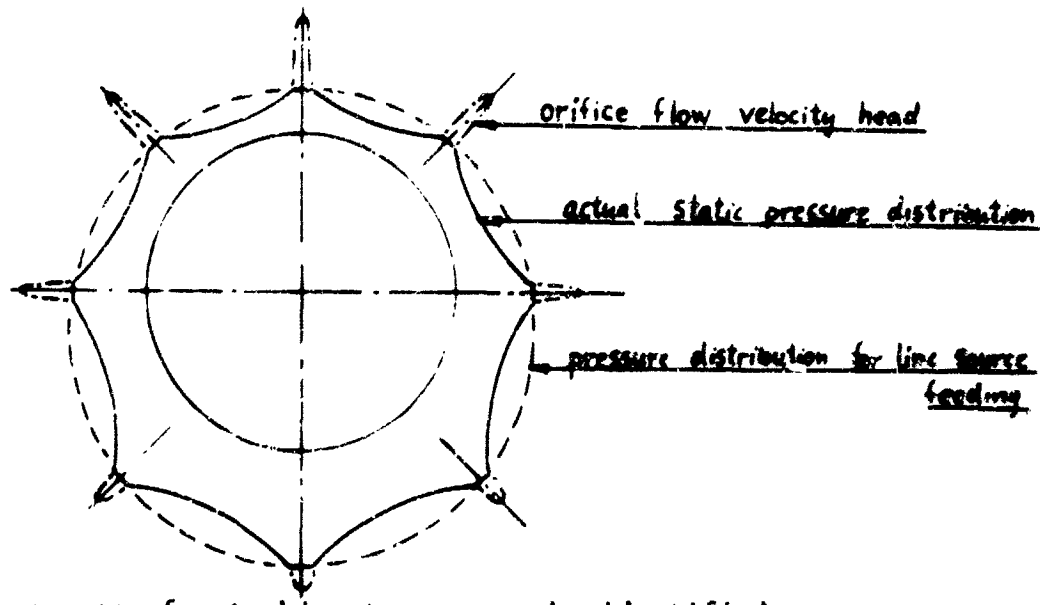
Thus for a given value of the frequency ω, the amplitude of the journal motion can be determined.

6. <u>Turbine Torque</u>

The actual bearing and orifice dimensions will always deviate slightly from the ideal drawing dimensions. Thus there will always be dissymmetries in the flow through the bearing and the circumferential component of those dissymmetries will exert a net torque on the journal.

For the function of the gyroscope this torque is highly undesirable and must be kept to a minimum. The torque depends on the eccentricity ratio and on the journal motion and is not constant over the operating range of the bearing. This makes it impossible to eliminate the torque by any simple compensation.

In the following a few simple calculations are made to bring out some of the important factors contributing to turbine torque. Limiting the discussion to the journal bearing the circumferential pressure distribution is in principle as shown below:



Three possible sources for turbine torque may be identified:

a) The orifice jet impinging on the journal, i.e., the orifice flow velocity head above.

b) The overall circumferential flow from the bottom towards the top caused by the eccentric journal. This flow may be considered to come from the distribution labeled: "pressure distribution for line source feeding" above, since this distribution represents the orifice downstream pressure.

c) The local circumferential flow variation between the orifices caused by the above "scalloped" pressure distribution.

Let the total bearing flow over all N orifices be Q_T in^3/sec at ambient pressure P_a. The orifice downstream pressure is P_o and the orifice jet is assumed to have the same radius, a, as the orifice. If all orifice centerlines pass the journal center by the distance δ inch, then the total torque caused by orifice jet impingement becomes:

$$T_1 = \frac{1}{2} \frac{P_a^2}{P_o(RT)} \frac{Q_T^2}{N\pi a^2} \delta$$

where RT is the (gas constant).(total temperature) in in^2/sec^2. To convert the torque to dyn.cm, multiply by $1.130.10^6$. For the particular test condition of a supply pressure = 12.8 psig and an ambient pressure of 14.6 psia the flow is 7.63 in^3/sec at an orifice downstream pressure P_o = 21.4 psia. If furthermore the orifice centerline eccentricity δ due to both tolerances and journal eccentricity is set to δ = .001 in. for all orifices (conservative) the torque becomes:

$$T_1 = 3.94 \quad dyn \cdot cm$$

Let the gas film pressure be P so that the circumferential shear stress on the journal at an angle Θ from the top of the bearing is:

$$\tau = \frac{1}{2} \frac{C}{R} \frac{\partial P}{\partial \Theta} (1+\varepsilon \cos\Theta) \qquad \frac{lbs}{in^2}$$

where C is the radial clearance and R is the journal radius. The total torque on one bearing half becomes:

$$T_2 = RC \int_0^{\frac{L}{2}} \int_0^{\pi} \frac{\partial P}{\partial \Theta} (1+\varepsilon \cos\Theta) \, d\Theta \, dz$$

When the feeding planes are line sources and the eccentricity ratio ϵ is small, a first order perturbation calculation shows that the pressure may be expressed by:

$$P = P_0(z) + \varepsilon P_1(z) \cos\Theta$$

Hence the torque becomes:

$$T_2 = -2\varepsilon RC \int_0^{\frac{L}{2}} P_1(z) \, dz$$

The bearing load W lbs is:

$$W = -2\int_0^{\frac{L}{2}}\int_0^{2\pi} P\cos\theta \, R\,d\theta\, dz = -2\pi\epsilon R\int_0^{\frac{L}{2}} P_1(z)\,dz$$

so that the shear torque is given by:

$$T_2 = \frac{1}{\pi} C W$$

For the same test condition as used in arriving at the above value for the orifice jet torque T_1, the shear torque becomes:

$$T_2 = 1380 \text{ dyn·cm}$$

at an eccentricity ratio ϵ =.1, a load of 4.16 lbs and a radial clearance C = .000925 in. The net torque on the journal is the balance between the torques on the two bearing halves, i.e.,

$$T_{2,net} = \text{error in } (1380 - 1380) \quad \text{dyn·cm}$$

On the other hand the net orifice jet torque is

$$T_{1,net} = \text{error in } (4) \quad \text{dyn·cm}$$

Hence it may be concluded that the jet impingement effect is small compared to the circumferential flow effect.

The third contribution to the turbine torque from the pressure variation between the orifices can be shown to be of the same order of magnitude as the shear torque T_2. Thus this effect is very important but no numerical calculations will be given.

7. Summary of Results

Briefly summarized, the results of the investigation are:

a) A simplified analysis of the bearing load and the flow for use in selecting the design parameters. The results are dimensionless and may readily be presented in convenient graphs as shown in Figs. 3-6. The validity of the results is established in Figs. 7-14 by a comparison with test data.

b) A calculation method for determining the load as a function

of journal eccentricity for both the journal and the thrust bearing. Comparison with test data is shown in Figs. 20-25. The form of the general theoretical results, from which the numerical data is calculated, is illustrated by Figs. 16-19.

c) A simplified analysis for the dynamic stiffness. The results may be given in dimensionless charts as in Figs. 26-33 and are used for calculating the resonant frequencies of the bearing and the amplitude.

d) Computer programs have been written for all the analyses.

DISCUSSION

Having presented the scope and the results of the analysis it remains to discuss the implications of the results and the limitations of the analysis.

One of the basic design requirements of the gimbal gas bearing is to obtain maximum stiffness and load with a minimum of flow and power. Since the stiffness is not constant over the operating range the design specifications must give information leading to a definition of where the stiffness should be optimized. No such explicit specifications exist at the present time (i.e., for steady state acceleration, vibratory acceleration, frequency spectrum, impulse loading etc.), but it is the current feeling that the stiffness should be optimized at a large eccentricity ratio (approximately $\epsilon = .9$) to ensure that the bearings load carrying capacity may be utilized to its limit. This is clearly not sufficient since the load curve may be such that the stiffness is large at $\epsilon = .9$ and relatively small at $\epsilon = 0$, the latter case being a possible space flight condition at zero g. Furthermore the area under the load curve cannot be neglected since it is a measure of the bearings capacity for shock absorbtion. Also, the gas film damping is important in establishing the vibratory amplitude at resonance where the cylinder must not touch the bearing sleeve. However, in any case an analysis is needed to give a rather accurate calculation of the load curve and the present investigation has been partially successful in doing so. It is apparent, though, that the journal bearing analysis need to be refined. In addition it seems clear that a more detailed and more physically correct orifice representation would improve the analysis. It is hardly to be expected that a one dimensional orifice model can account accurately for the complex three dimensional flow existing around the feeding hole entrance to the bearing. It is essential for future progress to explore this area.

Another problem is the dynamics of the bearing. The investigation has analyzed the dynamic stiffness but only for the concentric journal position and

based on a simplified orifice feeding representation. A more extensive analysis is needed which includes the effect of eccentricity ratio and, if at all possible, allows for the influence of the bearing on the orifice under dynamic conditions. This is an extremely difficult problem since it involves both a method for solving the non-linear Reynolds equation and in addition a method for solving the time dependent orifice characteristic, a problem which has not yet been well defined. A study of this type is necessary not only for a more accurate frequency response calculation but it is also an essential prerequisite for a stability analysis.

The investigation has ignored the possible existence of shock conditions in the bearing. Such conditions may occur around the feeding hole entrance and in the gas film when the ambient pressure is low. Since the analysis assumes isothermal conditions it is not suited to investigate shock. Therefore, a new analysis is needed if this problem becomes important.

The analysis is limited to the bearing geometry of Fig. 1, whereas an actual design analysis should consider all possible configurations. Thus it would seem natural to attempt a more uniformly distributed gas feeding to reduce turbine torque, f. inst. by a porous sleeve wall or by suitable grooving of the bearing surface. Obviously this requires a new analysis.

CONCLUSIONS

1. The investigation has demonstrated that analytical methods can be used in selecting the optimum bearing dimensions and design parameters.
2. A method has been developed to calculate the load as a function of journal displacement for both the thrust and the journal bearing. This is needed for evaluating the bearing performance over the entire operating range.
3. An analysis has been set up for calculating the dynamic stiffness of both the thrust and the journal bearing and, hence, for calculating the resonant frequencies of the bearing. The analysis provides for determining the vibration amplitude caused by a forced vibration of the bearing.
4. The investigation has demonstrated the need for a detailed study of the flow through the orifice and around the feeding hole entrance.
5. Calculations have been made to show that the orifice jet impingement plays a minor role in producing turbine torque. The circumferential flow, caused by the eccentric journal and by the pressure drop between the orifices, is a major factor.

RECOMMENDATIONS

1. A study of the pressure and velocity distribution around the orifice and the feeding hole should be undertaken. Preferably the study should include time dependent effects. Such an analysis is necessary for obtaining closer agreement between experimental and theoretical results and also for setting up a realistic stability analysis.
2. A more comprehensive analysis of the dynamic behavior of the bearing is desirable both for calculating the vibratory response and the stability. It may be necessary to resort to a numerical solution based on finite difference equations. Since the simultaneous solution of the Reynolds equation with the orifice equation is a highly non-linear problem, a separate mathematical study of related numerical techniques is indicated.
3. Other promising bearing configurations should be analyzed similarly to the present investigation. Thus f.inst. the bearing, where the gas feeding takes place through narrow slits or grooves, may have a reduced turbine torque and the same may be true for the porous wall bearing.
4. A study should be undertaken to establish the operating specifications for the bearing, i.e., specifications for steady acceleration, dynamic acceleration, frequency spectrum, impulse loading, allowable turbine torque, etc.

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Dr. C. H. T. Pan has served as advisor on several parts of the theoretical analysis, Mr. F. Walsh has assisted in obtaining the numerical calculations and the graphs and Mr. R. Geguzys has written the computer program for the journal bearing with source feeding. Their contributions are appreciated by the authors.

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APPENDIX A

General Equations

A. Journal Bearing

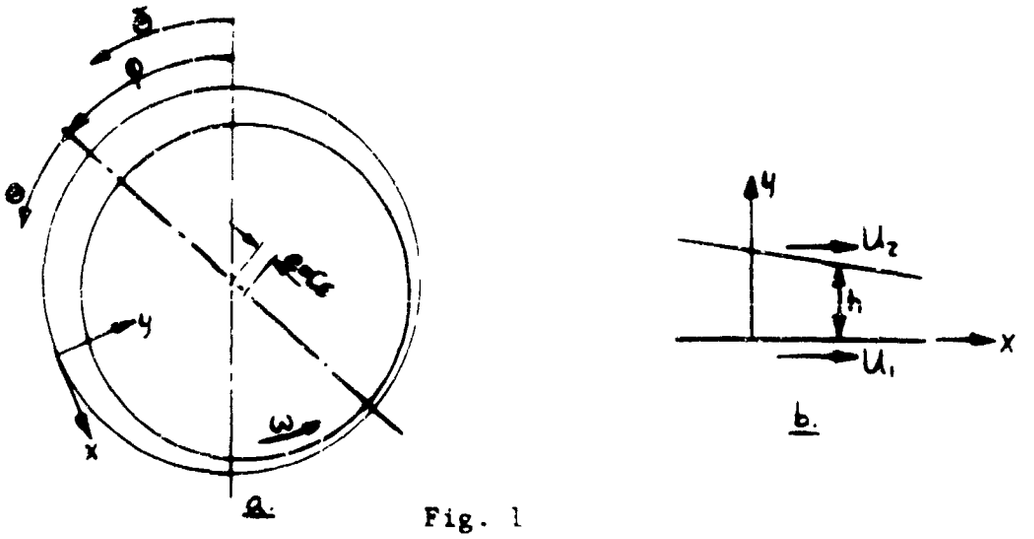

Fig. 1

If fluid film inertia is neglected and if the gas film is considered so thin that pressure, density and viscosity may be considered constant across the film and velocity gradients across the film much greater than gradients along the film, Navier-Stokes equations reduce to:

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial z}$$

where u and w are the velocities in the x- and z-direction, μ is the viscosity and P is the pressure. Integrating twice making use of Fig. 1b:

$$u = \frac{1}{2\mu}\frac{\partial P}{\partial x}\, y(y-h) + \frac{h-y}{h}U_1 + \frac{y}{h}U_2 \tag{A.1}$$

$$w = \frac{1}{2\mu}\frac{\partial P}{\partial z}\, y(y-h) \tag{A.2}$$

Hence the mass flow per inch becomes:

$$Q_x = \int_0^h \rho u\, dy = -\frac{h^3}{12\mu}\rho\frac{\partial P}{\partial x} + \frac{1}{2}\rho h(U_1+U_2) \tag{A.3}$$

(A.4) $$Q_z = \int_0^h \rho w \, dy = -\frac{h^3}{12\mu} \rho \frac{\partial P}{\partial z}$$

These equations are used to establish the flow balance for an infinitesimal prism $h \cdot dx \cdot dz$ where external gas feeding is introduced as the mass flow M over the area A. Hence we get:

$$\frac{\partial Q_x}{\partial x} \cdot dx\,dz + \frac{\partial Q_z}{\partial z} dz\,dx + \frac{\partial(\rho h)}{\partial t} dx\,dz = \frac{M}{A} dx\,dz$$

Introducing eq. (A.3)-(A.4) we obtain Reynolds equation:

(A.5) $$\frac{\partial}{\partial x}\left[\frac{h^3}{12\mu} \rho \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{h^3}{12\mu} \rho \frac{\partial P}{\partial z}\right] = \frac{1}{2} \frac{\partial}{\partial x}\left[\rho h (U_1 + U_2)\right] + \frac{\partial(\rho h)}{\partial t} - \frac{M}{A}$$

This equation is made dimensionless by setting:

$$\bar{P} = \frac{P}{P_a} \qquad \bar{h} = \frac{h}{C} = 1 + \varepsilon \cos\theta \qquad \varepsilon = \frac{e}{C}$$, see fig. 1a

$$\theta = \frac{x}{R} \qquad \zeta = \frac{z}{R} \qquad \tau = \omega t$$

$$U_2 = R\omega \qquad U_1 = 0$$

Assuming isothermal conditions such that μ is constant and $\rho = \frac{P}{RT}$, eq. (5) becomes upon neglecting the bar notation:

(A.6) $$\frac{\partial}{\partial \theta}\left[h^3 P \frac{\partial P}{\partial \theta}\right] + \frac{\partial}{\partial \zeta}\left[h^3 P \frac{\partial P}{\partial \zeta}\right] = \Lambda\left[\frac{\partial(Ph)}{\partial \theta} + 2\frac{\partial(Ph)}{\partial \tau}\right] - 2\Lambda_t V m \frac{\pi R^2}{NA}$$

where:

(A.7) $$\Lambda = \frac{6\mu\omega}{P_a}\left(\frac{R}{C}\right)^2$$

(A.8) $$\Lambda_t = \frac{6\mu N \alpha a^2 \sqrt{RT}}{P_a C^3}$$ α : orifice coefficient

(A.9) $$V = \frac{P_s}{P_a}$$

(A.10) N : number of feeding holes

(A.11) $$m = \frac{M\sqrt{RT}}{\alpha \pi a^2 P_s}$$

Since $\theta = \bar{\theta} - \phi$, where ϕ may vary with time, we can write:

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial \tau}\Big|_{\theta} - \dot{\phi}\frac{\partial}{\partial \theta} \qquad \text{where} \qquad \dot{\phi} = \frac{\partial \phi}{\partial \tau} = \frac{1}{\omega}\frac{\partial \phi}{\partial t}$$

Hence, if $\frac{\partial}{\partial \tau}$ refers to the time derivative only, Reynolds equation becomes:

$$\text{(A 12)} \quad \frac{\partial}{\partial \theta}\left[h^3 P \frac{\partial P}{\partial \theta}\right] + \frac{\partial}{\partial \zeta}\left[h^3 P \frac{\partial P}{\partial \zeta}\right] = \Lambda\left[(1-2\dot{\phi})\frac{\partial (Ph)}{\partial \theta} + 2\frac{\partial (Ph)}{\partial \tau}\right] - 2\Lambda_t V_m \frac{\pi R^2}{NA}$$

For the hydrostatic bearing there is no journal rotation. However, the journal center may still have motion and assuming this motion to be periodic with frequency ω eq. (A.12) becomes:

$$\text{(A.13)} \quad \frac{\partial}{\partial \theta}\left[h^3 \frac{\partial P^2}{\partial \theta}\right] + \frac{\partial}{\partial \zeta}\left[h^3 \frac{\partial P^2}{\partial \zeta}\right] = 4\Lambda\left[-\dot{\phi}\frac{\partial (Ph)}{\partial \theta} + \frac{\partial (Ph)}{\partial \tau}\right] - 4\Lambda_t V_m \frac{\pi R^2}{NA}$$

which is the general Reynolds equation for the hydrostatic journal bearing. For steady state operation it reduces to:

$$\text{(A.14)} \quad \frac{\partial}{\partial \theta}\left[h^3 \frac{\partial P^2}{\partial \theta}\right] + \frac{\partial}{\partial \zeta}\left[h^3 \frac{\partial P^2}{\partial \zeta}\right] = -4\Lambda_t V_m \frac{\pi R^2}{NA}$$

B. <u>Thrust Bearing</u>

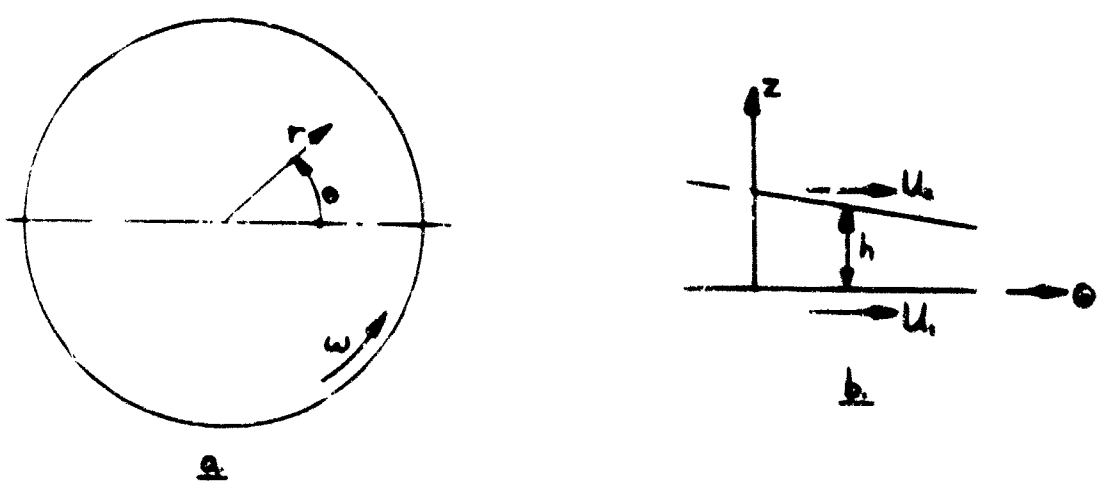

Fig. 2

Under the previous assumptions Navier-Stokes equations reduce to:

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial P}{\partial r}$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{\mu}\frac{\partial P}{r\partial\theta}$$

where u and v are the velocities in the radial and tangential direction. The mass flow per inch becomes:

$$Q_r = -\frac{h^3}{12\mu}\rho\frac{\partial P}{\partial r} \tag{A.15}$$

$$Q_\theta = -\frac{h^3}{12\mu}\rho\frac{\partial P}{r\partial\theta} + \frac{1}{2}\rho h(U_1+U_2) \tag{A.16}$$

A mass flow balance gives:

$$\frac{\partial(rQ_r)}{\partial r}dr\,d\theta + \frac{\partial Q_\theta}{r\partial\theta}r\,d\theta\,dr + \frac{\partial(\rho h)}{\partial t}r\,d\theta\,dr = \frac{M}{A}r\,d\theta\,dr$$

where M is the external gas feeding over the area A. Introducing eq. (A.15)-(A.16) we obtain Reynolds equation in polar coordinates:

$$\frac{\partial}{\partial r}\left[r\frac{h^3}{12\mu}\rho\frac{\partial P}{\partial r}\right] + \frac{\partial}{r\partial\theta}\left[\frac{h^3}{12\mu}\rho\frac{\partial P}{r\partial\theta}\right] = \frac{1}{2}\frac{\partial}{r\partial\theta}\left[\rho h(U_2+U_1)\right] + \frac{\partial(\rho h)}{\partial t} - \frac{M}{A} \tag{A.17}$$

In this equation the local film thickness h changes with time not only due to a normal velocity difference between the two plates but also by the rotation of the upper plate in Fig. 2b. For this reason Reynolds equation is normally given in coordinates stationary with respect to the lower plate. Thus the new θ-coordinate becomes:

$$\bar{\theta} = \theta - \frac{U_1}{r}t$$

Therefore:

$$\frac{\partial(\rho h)}{\partial t} = \frac{\partial(\rho h)}{\partial t}\Big|_{\bar{\theta}} + \frac{\partial(\rho h)}{\partial\bar{\theta}}\frac{\partial\bar{\theta}}{\partial t} = \frac{\partial(\rho h)}{\partial t}\Big|_{\bar{\theta}} - U_1\frac{\partial(\rho h)}{r\partial\bar{\theta}}$$

Dropping the bar notation eq. (A.17) becomes:

$$\text{(A.18)}\quad \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{h^3}{12\mu}\rho\frac{\partial P}{\partial r}\right]+\frac{\partial}{r\partial\theta}\left[\frac{h^3}{12\mu}\rho\frac{\partial P}{r\partial\theta}\right]=\frac{1}{2}(U_2-U_1)\frac{\partial(\rho h)}{r\partial\theta}+\frac{\partial(\rho h)}{\partial t}-\frac{M}{A}$$

To make dimensionless set:

$$\bar{P}=\frac{P}{P_a} \qquad \bar{h}=\frac{h}{C}$$

$$\bar{r}=\frac{r}{R} \qquad \tau=\omega t$$

$$U_2=r\omega \qquad U_1=0$$

Assuming isothermal conditions and a perfect gas with $\rho=\frac{P}{RT}$ and dropping the bar notation eq. (A.18) becomes:

$$\text{(A.19)}\quad \frac{1}{r}\frac{\partial}{\partial r}\left[rh^3P\frac{\partial P}{\partial r}\right]+\frac{\partial}{r\partial\theta}\left[h^3P\frac{\partial P}{r\partial\theta}\right]=\Lambda\frac{\partial(Ph)}{\partial\theta}+2\Lambda\frac{\partial(Ph)}{\partial\tau}-2\Lambda_t V_m\frac{\pi R^2}{NA}$$

where the dimensionless parameters are defined in eq. (A.7)-(A.11). For the purely hydrostatic bearing there is no rotation. Assuming the upper plate to have motion with frequency ω eq. (A.19) reduces to:

$$\text{(A.20)}\quad \frac{1}{r}\frac{\partial}{\partial r}\left[rh^3\frac{\partial P^2}{\partial r}\right]+\frac{\partial}{r\partial\theta}\left[h^3\frac{\partial P^2}{r\partial\theta}\right]=2\Lambda\left[\dot{\varphi}\frac{\partial(Ph)}{\partial\theta}+2\frac{\partial(Ph)}{\partial\tau}\right]-4\Lambda_t V_m\frac{\pi R^2}{NA}$$

where $\dot{\varphi}=\frac{\partial\varphi}{\partial\tau}=\frac{1}{\omega}\frac{\partial\varphi}{\partial t}$ and φ is the angular motion of the top plate.

For steady state operation eq. (A.20) becomes:

$$\text{(A.21)}\quad \frac{1}{r}\frac{\partial}{\partial r}\left[rh^3\frac{\partial P^2}{\partial r}\right]+\frac{\partial}{r\partial\theta}\left[h^3\frac{\partial P^2}{r\partial\theta}\right]=-4\Lambda_t V_m\frac{\pi R^2}{NA}$$

C. External Gas Feeding

In the above equations the external gas feeding is introduced directly into the Reynolds equation. In general gas is only fed to the bearing at discrete points. Hence, it is frequently convenient to solve Reynolds equation

without feeding and introduce the feeding through the boundary conditions as will be shown in later appendices. In accordance with eq. (A.11) we define the dimensionless mass flow from the external feeding as

$$m = \frac{M\sqrt{RT}}{\alpha \pi a^2 P_s}$$

where M is the mass flow, lbs.sec^2/in; α is a flow coefficient; πa^2 is the restrictor area, P_s is the supply pressure and RT is the gas constant·total temperature, in^2/sec^2. The dimensionless mass flow m is in general a function of the downstream pressure P_i from the feeding hole, i.e.

$$m = m\left(\frac{P_i}{V}\right)$$

In the present analysis two kinds of feeding restrictors are used: the orifice restricted and the millipore restricted feeding.

1. <u>Orifice Restriction</u>

The mass flow through an orifice with radius a is given by:

$$M = \nu \pi a^2 \sqrt{\eta \frac{2K}{K-1}} \frac{P_s}{\sqrt{RT}} \left(\frac{P_i}{V}\right)^{1/K} \sqrt{1 - \left(\frac{P_i}{V}\right)^{\frac{K-1}{K}}}$$

Setting the orifice coefficient $\alpha = \sqrt{\eta}$ the dimensionless mass flow becomes:

$$\text{(A.22)} \quad m = \nu \sqrt{\frac{2K}{K-1}} \left(\frac{P_i}{V}\right)^{1/K} \sqrt{1 - \left(\frac{P_i}{V}\right)^{\frac{K-1}{K}}}$$

where K is the adiabatic gas exponent, ν is the vena contracta coefficient and η is the adiabatic efficiency. For later use:

$$\text{(A.23)} \quad m' = \frac{\partial m}{\partial\left(\frac{P_i}{V}\right)} = \frac{1}{V}\frac{\partial \nu}{\partial\left(\frac{P_i}{V}\right)} m - \frac{2\nu^2}{K-1} \frac{\left(\frac{P_i}{V}\right)^{1/K}\left[\frac{K+1}{2} - \left(\frac{P_i}{V}\right)^{\frac{1-K}{K}}\right]}{m}$$

Eq. (A.22) and (A.23) apply to the unchoked orifice. When the orifice is choked, i.e,

(A.24) $$\left(\frac{P_i}{V}\right) \leq \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$$

then eq. (A.22) and (A.23) become:

(A.25) $$m = \nu \sqrt{\frac{2K}{K+1}} \left(\frac{2}{K+1}\right)^{\frac{1}{K-1}}$$

(A.26) $$m' = \frac{\partial m}{\partial\left(\frac{P_i}{V}\right)} = \frac{1}{\nu} \frac{\partial \nu}{\partial\left(\frac{P_i}{V}\right)} m$$

Because of the discontinuity in the above equations at the critical pressure ratio, it is sometimes convenient to replace the exact orifice equation by an approximate expression. This can be done by setting:

$$M = \frac{\alpha \pi a^2}{\sqrt{RT}} P_s \sqrt{1 - \left(\frac{P_i}{V}\right)^2}$$

so that

(A.27) $$m = \sqrt{1 - \left(\frac{P_i}{V}\right)}$$

(A.28) $$m' = \frac{\partial m}{\partial\left(\frac{P_i}{V}\right)} = \frac{-P_i}{\sqrt{V^2 - P_i^2}}$$

2. <u>Millipore Restriction</u>

The millipore restrictor is a filter consisting of 5 layers: 2 screens, 2 cement wafers and a membrane filter. No theoretical expression is available for the flow through this filter and instead the following empirical formula is used:

(A.29) $$M = \frac{\pi a^2 P_s}{\sqrt{RT}} \frac{1}{V}\left[C P_i^2 + P_i + D\right] \frac{(V - P_i)(V - P_i + B)}{V - P_i + a_0}$$

where the coefficients a_0, B, C and D are determined experimentally.

In dimensionless form:

$$(A.30)\quad m = \frac{1}{V}\left[C P_i^2 + P_i + D\right]\frac{(V-P_i)(V-P_i+B)}{V-P_i+a_0}$$

$$(A.31)\quad m' = \frac{\partial m}{\partial\left(\frac{P_i}{V}\right)} = -\left[C P_i^2+P_i+D\right]\left[1+\frac{a_0(B-a_0)}{(V-P_i+a_0)^2}\right] + \frac{(V-P_i)(V-P_i+B)}{V-P_i+a_0}\left[2CP_i+1\right]$$

3. <u>Orifice Restriction with Inherent Compensation</u>

If the diameter d of the feeding hole from the orifice to the bearing is small, the annular restrictor area between the journal and the rim of the feeding hole becomes important.

For two orifices in series the mass flow equation becomes:

$$(A.32)\quad \frac{M\sqrt{RT}}{A_1\sqrt{\eta_1}\,\frac{2K}{K-1}P_s\left(\frac{P_m}{P_s}\right)} = \frac{\left(\frac{P_m}{P_s}\right)^{1/K}\sqrt{1-\left(\frac{P_m}{P_s}\right)^{\frac{K-1}{K}}}}{\left(\frac{P_m}{P_s}\right)} = \frac{A_2\sqrt{\eta_2}}{A_1\sqrt{\eta_1}}\left(\frac{P_i}{P_m}\right)^{1/K}\sqrt{1-\left(\frac{P_i}{P_m}\right)^{\frac{K-1}{K}}}$$

where A_1 is the area of the first orifice, A_2 of the second orifice, η is the adiabatic efficiency, P_s is the pressure before the first orifice, P_m is the pressure between the orifices and P_i is the pressure after the second orifice.

When $\frac{A_2\sqrt{\eta_2}}{A_1\sqrt{\eta_1}}$ is known this equation may be solved numerically to give $m = \frac{M\sqrt{RT}}{A_1 P_s}$ as a function of $\frac{P_i}{P_s}$. If the orifice radius is a, the feeding hole diameter is d and the film thickness is h we get:

$$\frac{A_2\sqrt{\eta_2}}{A_1\sqrt{\eta_1}} = \frac{\nu_2\sqrt{\eta_2}\,d\,h}{\nu_1\sqrt{\eta_1}\,a^2}$$

where ν is the vena contracta coefficient. Since ν is a function of the orifice pressure ratio and h varies around the circumference in the journal bearing an exact solution is cumbersome and an approximate expression is more convenient. If it is assumed that h does not vary too much (i.e., for small

eccentricity ratios in the journal bearing) and if $\alpha = \nu\sqrt{\eta}$ may be considered the same for the two restrictors we get:

$$\frac{A_2\sqrt{\eta_2}}{A_1\sqrt{\eta_1}} = \frac{dC(1+\varepsilon\cos\theta)}{a^2} = \frac{1+\varepsilon\cos\theta}{\delta}$$

(A.33) $$\delta = \frac{a^2}{dC}$$

where C is the radial clearance. Then a reasonable approximation to eq. (A.32) is:

(A.34) $$m = \frac{M\sqrt{RT}}{\alpha\pi a^2 P_s} = \frac{1}{\sqrt{1+\frac{\delta^4}{(1+\varepsilon\cos\theta)^2}}}\sqrt{1-\left(\frac{P_i}{V}\right)^2}$$

where α depends on δ .

When δ is small the bearing is predominantly orifice restricted whereas a large δ indicates a predominantly inherent compensated bearing.

APPENDIX B

The Static Load Carrying Capacity of Hydrostatic Bearings With Orifice Restricted Line Feeding

Frequently the number of orifices in a feeding plane is sufficiently large to permit representing the orifices by a line feed. This considerably simplifies the analysis, since the gas feeding does not enter Reynolds equation directly but instead becomes one of the boundary conditions.

A. Journal Bearing

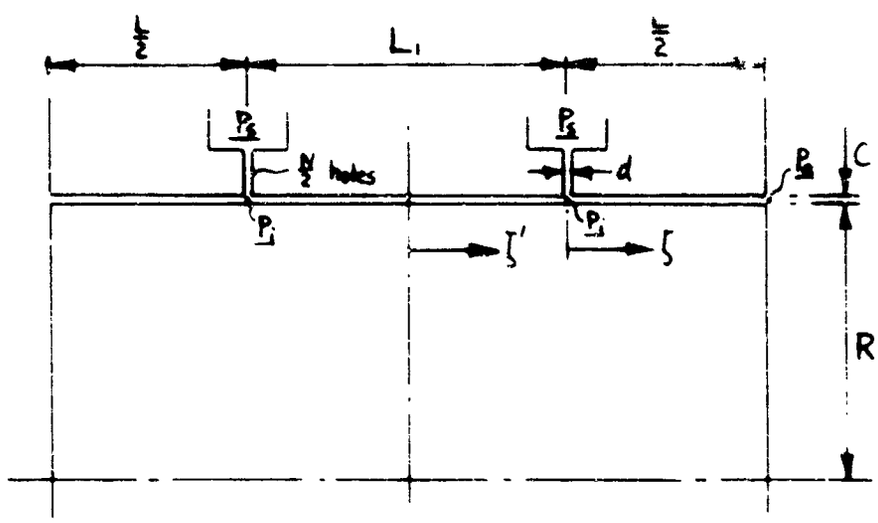

Fig. 3

The journal bearing is fed from two feeding planes, each with $\frac{1}{2}$ N equidistant orifice restricted feeding holes around the circumference. The supply pressure is P_s, the downstream orifice pressure is P_i and the ambient pressure at the ends of the bearing is P_a. The dimensionless axial coordinate is ζ, defined as $\zeta = \frac{z}{R}$ in Appendix A, and the circumferential

coordinate is Θ. The ga film between and outside the feeding planes is described by Reynolds equation which in dimensionless form is (see eq. A 14):

(B.1) $$\frac{\partial}{\partial\Theta}\left[h^3\frac{\partial P^2}{\partial\Theta}\right]+\frac{\partial}{\partial\zeta}\left[h^3\frac{\partial P^2}{\partial\zeta}\right]=0$$

where P is the dimensionless pressure with respect to ambient pressure and $h=1+\varepsilon\cos\Theta$ is the dimensionless film thickness. To solve the equation set:

(B.2) $$P=P_0+\varepsilon P_1+\varepsilon^2 P_2+\cdots$$
$$P^2=P_0^2+2\varepsilon P_0P_1+\cdots$$

Substituting eq. (B.2) into eq. (B.1) and setting the coefficients to $\varepsilon^0,\varepsilon,\varepsilon^2,\cdots$ equal to zero yields an infinite number of equations of which the first two become:

(B.3) $$\frac{\partial^2 P_0^2}{\partial\Theta^2}+\frac{\partial^2 P_0^2}{\partial\zeta^2}=0$$

(B.4) $$\frac{\partial^2(P_0P_1)}{\partial\Theta^2}+\frac{\partial^2(P_0P_1)}{\partial\zeta^2}=0$$

Thus the analysis is limited to small values of ε. Since P_0 is the pressure when the journal is concentric in the bearing, P_0 is independent of Θ and eq. (B.3) gives:

(B.5) $\underline{0\le\zeta\le\xi}$ $\qquad P_0^2=1+q(\xi-\zeta)$

(B.6) $\underline{0\le\zeta'\le\eta}$ $\qquad P_0^2=1+q\xi$

where

(B.7) $\xi=\frac{L}{D}$

see fig. 3

(B.8) $\eta=\frac{L_1}{D}$

and q will be determined later.

Eq. (B.5) satisfies the boundary condition of ambient pressure at the end of the bearing, i.e., where $P=1$. At the feeding plane ($\zeta=0$, $\zeta'=\eta$) the flow entering the bearing from the orifices must equal the flow into the bearing film. The orifice flow per inch circumference is $\frac{\frac{1}{2}NM}{2\pi R}$ whereas the flow into the film is given by eq. (A.4). Equating the two flows:

$$-\frac{h^3}{12\mu}\rho\left.\frac{\partial P}{\partial z}\right|_i = \frac{NM}{4\pi R}$$

Make this equation dimensionless as defined in Appendix A:

$$\text{(B.9)}\qquad (1+\varepsilon\cos\theta)^3\left[-\frac{\partial P^2}{\partial\zeta}+\frac{\partial P^2}{\partial\zeta'}\right]_i = \Lambda_t V m$$

where Λ_t, V and m are defined in eq. (A.8), (A.9) and (A.11). Expand m in a Taylor series around the operating point for concentric journal:

$$\text{(B.10)}\qquad m = m_0 + \left.\frac{\partial m}{\partial(\frac{P_i}{V})}\right|_0\left(\frac{P_i}{V}-\frac{P_0}{V}\right)+\cdots\cdots$$

Substitute eq. (B.2) into the equations and collect coefficients according to powers of ε:

$$\text{(B.11)}\qquad \left(\frac{\partial P_0^2}{\partial\zeta}-\frac{\partial P_0^2}{\partial\zeta'}\right)_i = -\Lambda_t V m_0$$

$$\text{(B.12)}\qquad \left(\frac{\partial(P_0P_1)}{\partial\zeta}-\frac{\partial(P_0P_1)}{\partial\zeta'}\right)_i = \frac{3}{2}\cos\theta\,\Lambda_t V m_0 - \frac{1}{2}\Lambda_t m_0' P_{1,i}$$

where

$$m_0' = \left.\frac{\partial m}{\partial(\frac{P_i}{V})}\right|_0$$

Introducing eq. (B.5)-(B.6) into eq. (B.11) gives

$$\text{(B.13)}\qquad q = \Lambda_t V m_0$$

Eq. (B.12) serves as a boundary condition to eq. (B.4). The remaining boundary conditions are:

$$P_1(\theta,\zeta) = P_1(\theta+2\pi,\zeta)$$

$$P_1(\theta,\xi) = 0$$

$$\frac{\partial P_1}{\partial \zeta'}(\theta,0) = 0$$

Setting

(B.14) $$P_0 P_1 = H(\zeta)\cos\theta$$

and substituting into eq. (B.4) gives:

$$\frac{d^2H}{d\zeta^2} - H = 0$$

with the solution:

$$0 \le \zeta \le \xi \qquad H = C_1 \sinh(\xi-\zeta)$$

$$0 \le \zeta' \le \eta \qquad H = C_2 \cosh\zeta'$$

where C_1 and C_2 are integration constants to be determined from the boundary conditions.

Setting $H_{\zeta=0} = H_{\zeta'=\eta}$ and using eq. (B.12) yields:

(B.15) $\underline{0 \le \zeta \le \xi}$ $$P_1 = \frac{H\cos\theta}{P_0} = \frac{-\frac{3}{2}q\cos\theta}{\left[\cosh\xi + \left(\tanh\eta - \frac{\Lambda_2 \eta'}{2\sqrt{1+q\xi}}\right)\sinh\xi\right]} \frac{\sinh(\xi-\zeta)}{\sqrt{1+q(\xi-\zeta)}}$$

(B.16) $\underline{0 \le \zeta' \le \eta}$ $$P_1 = \frac{H\cos\theta}{P_0} = \frac{-\frac{3}{2}q\sinh\xi\cos\theta}{\left[\cosh\xi + \left(\tanh\eta - \frac{\Lambda_2 \eta'}{2\sqrt{1+q\xi}}\right)\sinh\xi\right]\sqrt{1+q\xi}} \frac{\cosh\zeta'}{\cosh\eta}$$

The load carrying capacity is given by:

$\underline{0 \le \zeta \le \xi}$ $$W_2 = -2P_a R^2 \int_0^{\xi}\int_0^{2\pi} P\cos\theta \, d\theta \, d\zeta$$

$$0 \le \zeta' \le \eta \qquad W_1 = -2\Gamma_a R^2 \int_0^{\eta}\int_0^{2\pi} P\cos\theta \, d\theta \, d\zeta'$$

Substituting for P from eq. (B.2) gives

(B.17) $\underline{0 \le \zeta \le \xi}$

$$\frac{4W_2}{\pi D L P_a \varepsilon} = \frac{3q}{\xi\left[\cosh\xi + \left(\tanh\eta - \frac{\Lambda_2 m_0'}{2\sqrt{1+q\xi}}\right)\sinh\xi\right]} \int_0^{\xi} \frac{\sinh(\xi-\zeta)\,d\zeta}{\sqrt{1+q(\xi-\zeta)}}$$

(B.18) $\underline{0 \le \zeta' \le \eta}$

$$\frac{4W_1}{\pi D L_1 P_a \varepsilon} = \frac{3q \cdot \sinh\xi}{\left[\cosh\xi + \left(\tanh\eta - \frac{\Lambda_2 m_0'}{2\sqrt{1+q\xi}}\right)\sinh\xi\right]\sqrt{1+q\xi}} \; \frac{\tanh\eta}{\eta}$$

The integral in eq. (B.17) may be solved by the substitution:

$$t = \sqrt{\frac{1}{q} + (\xi - \zeta)}$$

so that:

(B.19)
$$\int_0^{\xi} \frac{\sinh(\xi-\zeta)\,d\zeta}{\sqrt{1+q(\xi-\zeta)}} = \frac{e^{-\frac{1}{q}}}{\sqrt{q}}\left[\psi\left(\sqrt{\tfrac{1}{q}+\xi}\right) - \psi\left(\sqrt{\tfrac{1}{q}}\right)\right] - \frac{\sqrt{\pi}}{2}\frac{e^{\frac{1}{q}}}{\sqrt{q}}\left[\mathrm{erf}\left(\sqrt{\tfrac{1}{q}+\xi}\right) - \mathrm{erf}\left(\sqrt{\tfrac{1}{q}}\right)\right]$$

where:

(B.20)
$$\psi(x) = \int_0^x e^{t^2}\,dt$$
$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

$\psi(x)$ is tabulated in Jahnke and Emde: Table of Functions. However, in most cases it is more convenient to evaluate the integrals by numerical integration. Since the axial flow per inch circumference is given by eq. (A.4) the total flow from both ends of the bearing is given by:

$$M_T = -2\int_0^{2\pi} \frac{h^3}{12\mu}\rho\frac{\partial P}{\partial z} R \, d\theta$$

Substituting dimensionless expressions from Appendix A and making use of

eq. (B.2), (B.5) and (B.15) gives:

$$M_T = \frac{C^3 P_a^2}{12\mu RT}\int_0^{2\pi}\left[-\frac{\partial P_0^2}{\partial \zeta} - 2\varepsilon\frac{\partial(P_0 P_1)}{\partial \zeta}\right](1-3\varepsilon\cos\theta)\,d\theta = \frac{\pi C^3 P_a^2}{6\mu RT}\cdot q \tag{B.21}$$

Hence q may be denoted the dimensionless bearing flow. To determine q use is made of eq. (B.13). This shall be illustrated for the case of the orifice bearing. The solution for the millipore bearing is analogous.

For the orifice bearing m_0 is given by eq. (A.22) and (A.25):

$$\left(\frac{2}{K+1}\right)^{\frac{K}{K-1}} \le \left(\frac{P_{0i}}{V}\right) \le 1 \qquad m_0 = \nu\sqrt{\frac{2K}{K-1}}\left(\frac{P_{0i}}{V}\right)^{1/K}\sqrt{1-\left(\frac{P_{0i}}{V}\right)^{\frac{K-1}{K}}}$$

(B.22)

$$\frac{1}{V} \le \left(\frac{P_{0i}}{V}\right) \le \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}} \qquad m_0 = \nu\sqrt{\frac{2K}{K+1}}\left(\frac{2}{K+1}\right)^{\frac{1}{K-1}}$$

From eq. (B.5) or (B.6) we get at $\zeta = 0$:

$$P_{0i}^2 = 1 + q\xi$$

i.e

$$q = \frac{V^2}{\xi}\left[\left(\frac{P_{0i}}{V}\right)^2 - \frac{1}{V^2}\right] \tag{B.23}$$

Thus eq. (B.13) becomes:

$$\frac{V}{\xi\Lambda_t}\left[\left(\frac{P_{0i}}{V}\right)^2 - \frac{1}{V^2}\right] = m_0 \tag{B.24}$$

This relationship may be shown graphically:

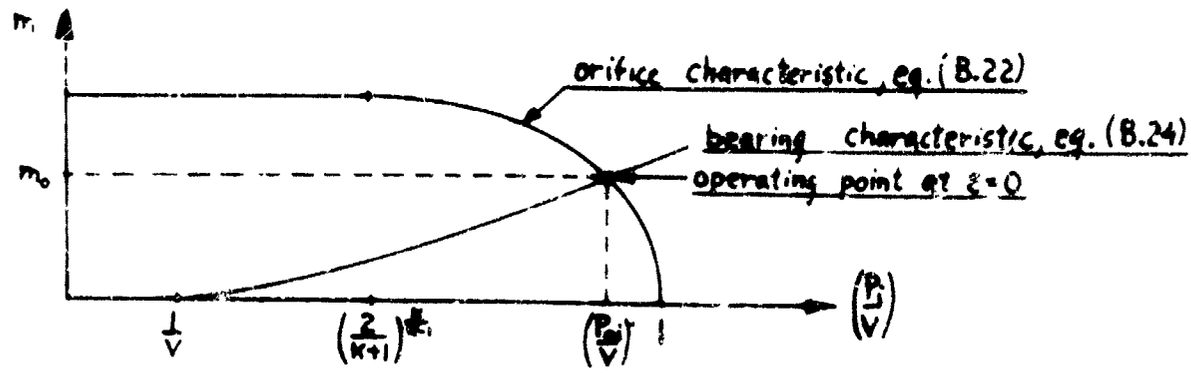

Fig. 4

Hence eq. (B.24) can be solved graphically or by iterations when using a computer.

If the approximate orifice formula is used(eq. (A.27)) a closed form solution can be obtained as follows:

$$q = \Lambda_t V m_o = \Lambda_t V \sqrt{1-\left(\frac{P_{ci}}{V}\right)^2} = \Lambda_t \sqrt{V^2-1-q\xi}$$

or

(B.25) $$q = \frac{1}{2}\xi \Lambda_t^2 \left[-1 + \sqrt{1 + \frac{4(V^2-1)}{\xi^2 \Lambda_t^2}} \right]$$

and from eq. (A.28):

(B.26 $$-\frac{\Lambda_t m_o'}{2\sqrt{1+q\xi}} = \frac{\Lambda_t^2}{2q}$$

for use in the calculation of the load.

When the feeding hole diameter is small, we may use the approximate eq. (A.34) to estimate the effect of inherent compensation. Substituting eq. (A.34) into eq. (B.9), then eq. (B.11) and (B.12) becomes:

$$\left(\frac{\partial P_o^2}{\partial \zeta} - \frac{\partial P_i^2}{\partial \zeta'}\right) = -\frac{\Lambda_t}{\sqrt{1+\sigma^2}} \sqrt{V^2-1-q\xi}$$

$$\left(\frac{\partial (P_o P_i)}{\partial \zeta} - \frac{\partial (P_o P_i)}{\partial \zeta'}\right)_i = \frac{3}{2} q \cos\theta \, \frac{1+\frac{2}{3}\sigma^2}{1+\sigma^2} + \frac{1}{2q}\left(\frac{\Lambda_t}{\sqrt{1+\sigma^2}}\right)^2 (P_o P_i)_i$$

where $$\sigma = \frac{\xi^2}{\alpha C}$$

Hence when inherent compensation is important, i.e, $\sigma > .4 - .5$, the journal bearing load becomes:

(B.27) $$\frac{4 W_\sigma}{\pi P_a D L C} = \frac{1+\frac{2}{3}\sigma^2}{1+\sigma^2} \, \frac{4 W_{\sigma=0}}{\pi P_a D L C}$$

where $W_{\delta=0}$ is given by eq. (B.17) and (B.18). In calculating $W_{\delta=0}$, set

(B.28) $$\Lambda_{t,\delta=0} = \frac{\Lambda_t}{\sqrt{1+\sigma^2}}$$

where Λ_t is as previously defined.

B. Single Acting Thrust Bearing

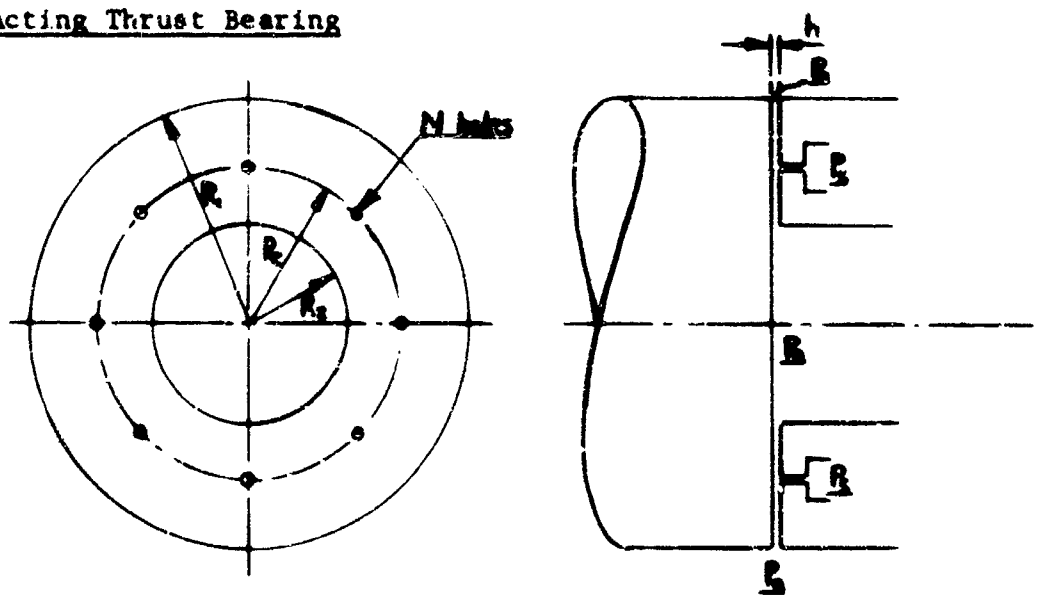

Fig. 5

The thrust bearing consists of an annular flat plate with inner and outer radius R_2 and R_1 . It is fed through N orifice restricted feeding holes located at radius R_c .

For static load and uniform film thickness the dimensionless Reynolds equation on both side of the feeding plane is obtained from eq. (A.21):

(B.29) $$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial P^2}{\partial r}\right] + \frac{\partial^2 P^2}{r^2 \partial \theta^2} = 0$$

where the second term is zero due to axial symmetry. Hence the solution for the dimensionless pressure becomes:

$$P^2 = 1 + C_1 \ln r$$

where C_1 is an integration constant. Introducing the dimensionless bearing dimensions:

(B.30) $$\eta = \frac{R_1}{R_c}$$

(B.31) $$\gamma = \frac{R_{i_1}}{R_2}$$

(B.32) $$\beta = \frac{\ln \eta}{\ln \gamma}$$

C_1 can be evaluated to give the dimensionless pressure:

(B.33) $1 \le r \le \eta$ $$P^2 = 1 + q_T \ln\left(\frac{\eta}{r}\right)$$

(B.34) $\frac{1}{\gamma} \le r \le 1$ $$P^2 = 1 + \beta q_T \ln(\gamma r)$$

where r is the radius made dimensionless by dividing by R_c. q_T is determined by equating the orifice and the bearing flow analogous to the analysis for the journal bearing. Thus:

$$\left(-\frac{\partial P^2}{\partial r}\bigg|_{r \ge 1} + \frac{\partial P^2}{\partial r}\bigg|_{r \le 1} \right)_{r=1} = \frac{12 \mu N \alpha a^2 \sqrt{RT}}{P_a h^3} V m$$

Inserting eq. (B.33) we get:

(B.35) $$q_T = \Lambda_T V m$$

where

(B.36) $$\Lambda_T = \frac{12 \mu N \alpha a^2 \sqrt{RT}}{P_a h^3 (1+\beta)}$$

To determine q_T substitute eq. (B.33) into eq. (B.35), setting $r=1$:

(B.37) $$\frac{V}{\Lambda_T \ln \eta}\left[\left(\frac{P_i}{V}\right)^2 - \frac{1}{V^2} \right] = m$$

This equation is analogous to eq. (B.24) and is solved in the same way. If the approximate orifice formula (eq. (A.27)) is used we get:

(B.38) $$q_T = \frac{1}{2}\ln\eta\ \Lambda_T^2\left[-1+\sqrt{1+\frac{4(V^2-1)}{\ln^2\eta\cdot\Lambda_T^2}}\right]$$

The load is found by integrating the pressure:

$$W = P_a R_c^2 \int_0^{2\pi}\int_{\frac{1}{\gamma}}^{\eta} r(P-1)\,dr\,d\theta$$

i.e.

(B.39) $$\frac{W}{\pi R_1^2 P_a} = \frac{2\sqrt{1+q_T\ln\eta}}{\eta^2}\int_1^{\eta} r\sqrt{1-\frac{q_T}{1+q_T\ln\eta}\ln r}\,dr + \frac{2}{(\gamma\eta)^2}\int_1^{\gamma} r'\sqrt{1+\beta q_T\ln r'}\,dr' + \frac{1}{(\gamma\eta)^2} - 1$$

The two integrals may be solved in closed form by the substitutions:

$$\dot{t} = \sqrt{2\left(\frac{1}{q_T}+\ln\eta-\ln r\right)}$$

$$t = \sqrt{2\left(\frac{1}{q_T\beta}+\ln r'\right)}$$

to get:

(B.40) $$\frac{W}{\pi R_1^2 P_a} = \frac{\sqrt{\pi}}{2}\sqrt{\frac{q_T}{2}}\,e^{\frac{2}{q_T}}\left[\mathrm{erf}\left(\sqrt{2\left(\frac{1}{q_T}+\ln\eta\right)}\right)-\mathrm{erf}\left(\sqrt{\frac{2}{q_T}}\right)\right]-\frac{1}{(\gamma\eta)^2}e^{-\frac{2}{\beta q_T}}\sqrt{\frac{\beta q_T}{2}}\left[\psi\left(\sqrt{2\left(\frac{1}{\beta q_T}+\ln\gamma\right)}\right)-\psi\left(\sqrt{\frac{2}{\beta q_T}}\right)\right]$$

where $\mathrm{erf}(x)$ and $\psi(x)$ are defined in eq. (B.20). Frequently, it is more convenient to evaluate the integrals in eq. (B.39) by numerical integration. The total mass flow from the thrust bearing is determined by:

$$M_T = \frac{C^3 P_a^2}{24\mu RT}\int_0^{2\pi}\left[-\eta\left.\frac{\partial P^2}{\partial r}\right|_{r=\eta} + \frac{1}{\gamma}\left.\frac{\partial P^2}{\partial r}\right|_{r=\frac{1}{\gamma}}\right]d\theta$$

or

(B.41) $$M_T = \frac{\pi C^3 P_a^2}{12\mu RT}(1+\beta)\,q_T$$

so that q_T denotes the dimensionless flow.

If inherent compensation is important Λ_T should be divided by $\sqrt{1+\sigma^2}$ where $\sigma = \frac{a^2}{dh}$, see. eq. (A.34).

C. Double Acting Thrust Bearing

The double acting thrust bearing consists of two thrust bearings as shown on Fig. 5, one bearing at each end of the cylinder. From eq. (B.29) Reynolds equation is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r h^3 \frac{\partial P^2}{\partial r}\right] = 0 \tag{B.42}$$

In the center position of the cylinder the clearance in both bearings is C. If the cylinder is displaced a small distance εC the dimensionless film thickness in the bearings becomes $h = 1 \pm \varepsilon$. Limiting the analysis to $\varepsilon \ll 1$ we may express the pressure as:

$$\begin{aligned} P &= P_0 \pm \varepsilon P_1 \\ P^2 &= P_0^2 \pm 2\varepsilon P_0 P_1 \end{aligned} \tag{B.43}$$

Substituting eq. (B.43) into eq. (B.42) and collecting terms according to powers of ε we get:

$$\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial P_0^2}{\partial r}\right] = 0 \\ &\frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial (P_0 P_1)}{\partial r}\right] = 0 \end{aligned} \tag{B.44}$$

with the solutions (see eq. (B.33)-(B.34)):

$$1 \le r \le \eta \qquad P_0^2 = 1 + q_T \ln\left(\frac{\eta}{r}\right) \tag{B.45}$$

$$\frac{1}{\gamma} \le r \le 1 \qquad P_0^2 = 1 + \beta q_T \ln(\gamma r) \tag{B.46}$$

$$1 \le r \le \eta \qquad P_0 P_1 = C_1 \ln\left(\frac{\eta}{r}\right) \tag{B.47}$$

$$\frac{1}{\gamma} \le r \le 1 \qquad P_0 P_1 = \beta C_1 \ln(\gamma r) \tag{B.48}$$

where C_1 is an integration constant. q_T and C_1 are determined by equating the orifice flow with the flow through the bearing:

$$\frac{C^3 P_a^2 (1 \pm \varepsilon)^3}{24 \mu R T R_c}\left[-\frac{\partial P^2}{\partial r}\bigg|_{r \ge 1} + \frac{\partial P^2}{\partial r}\bigg|_{r \le 1}\right]_{r=1} = \frac{N M}{2\pi R_c}$$

Substituting from eq. (B.43) and using eq. (A.8), (A.9) and (A.11), we get:

$$\left(-\frac{\partial P_0^2}{\partial r}\Big|_{r\pm 1} + \frac{\partial P_0^2}{\partial r}\Big|_{r\pm 1}\right)_{r=1} = 2\Lambda_t V m_0$$

$$\left(\frac{\partial (P_0 P_1)}{\partial r}\Big|_{r\pm 1} - \frac{\partial (P_0 P_1)}{\partial r}\Big|_{r\pm 1}\right)_{r=1} = 3\Lambda_t V m_0 - \frac{\Lambda_t m_0'}{P_{0,r=1}} (P_0 P_1)_{r=1}$$

Inserting eq. (B.45)-(B.48) we get:

(B.49) $$q_T = \Lambda_T V m_0$$

(B.50) $$C_1 = \frac{-\frac{3}{2} q_T}{1 - \frac{\Lambda_T m_0'}{2\sqrt{1+q_T \ln\eta}} \ln\eta}$$

where $\Lambda_T = \frac{2\Lambda_t}{1+\beta}$ is defined in eq. (B.36) by setting $h = C$. Thus in total:

(B.51) $$1 \le r \le \eta \qquad P^2 = 1 + q_T\left[1 \mp \varepsilon \frac{3}{1 - \frac{\Lambda_T m_0'}{2\sqrt{1+q_T \ln\eta}} \ln\eta}\right] \ln\left(\frac{\eta}{r}\right)$$

(B.52) $$\frac{1}{\gamma} \le r \le 1 \qquad P^2 = 1 + \beta q_T\left[1 \mp \varepsilon \frac{3}{1 - \frac{\Lambda_T m_0'}{2\sqrt{1+q_T \ln\eta}} \ln\eta}\right] \ln(\gamma r)$$

The combined load for the double acting bearing is:

$$W = P_a R_c^2 \int_0^{2\pi} \int_{\frac{1}{\gamma}}^{\eta} r\left[P(-\varepsilon) - P(+\varepsilon)\right] dr\, d\theta$$

or

(B.53) $$\frac{W}{\pi R_1^2 P_a \varepsilon} = \frac{6 q_T}{1 - \frac{\Lambda_T m_0'}{2\sqrt{1+q_T \ln\eta}} \ln\eta} \left\{ \frac{\beta}{(\gamma\eta)^2} \int_1^{\gamma} \frac{r' \ln r'\, dr'}{\sqrt{1+\beta q_T \ln r'}} - \int_{\frac{1}{\eta}}^{1} \frac{r' \ln r'\, dr'}{\sqrt{1 - q_T \ln r'}} \right\}$$

The two integrals on the right hand side may be evaluated by means of the substitutions:

$$t = \sqrt{2\left(\frac{1}{\beta q_T} + \ln r'\right)}$$

$$\dot{t} = \sqrt{2\left(\frac{1}{q_T} - \ln r'\right)}$$

to get:

$$\text{(B.54)} \quad \frac{W}{\pi R_1^2 P_a \varepsilon} = \frac{6q_T}{1-\frac{\Lambda_T m_s'}{2\sqrt{1+q_T \ln\eta}}\ln\eta}\left\{\frac{1}{2q_T}\left(1-\frac{1}{(\gamma\eta)^2}\right)+\frac{\sqrt{\pi}}{2}e^{\frac{2}{q_T}}\sqrt{\frac{2}{q_T}}\left(\frac{1}{4}-\frac{1}{q_T}\right)\left[\mathrm{erf}\left(\sqrt{2\left(\frac{1}{q_T}+\ln\eta\right)}\right)-\mathrm{erf}\left(\sqrt{\frac{2}{q_T}}\right)\right]\right.$$
$$\left.-e^{-\frac{2}{\beta q_T}}\sqrt{\frac{2}{\beta q_T}}\,\frac{\beta}{(\gamma\eta)^2}\left(\frac{1}{4}+\frac{1}{\beta q_T}\right)\left[\psi\left(\sqrt{\frac{2}{\beta}\left(\frac{1}{q_T}+\ln\eta\right)}\right)-\psi\left(\sqrt{\frac{2}{\beta q_T}}\right)\right]\right\}$$

where $\psi(x)$ and $\mathrm{erf}(x)$ are given by eq. (B.20). The mass flow from each thrust bearing is given by eq. (B.41).

If inherent compensation is important we may use the approximate equation (A.34) to get

$$\frac{W_\delta}{\pi R_1^2 P_a \varepsilon} = \frac{1+\frac{2}{3}\delta^2}{1+\delta^2}\,\frac{W_{\delta=0}}{\pi R_1^2 P_a \varepsilon}$$

where $W_{\delta=0}$ is given by eq. (B.53) and

$$\Lambda_{T,\delta} = \frac{\Lambda_{T,\delta=0}}{\sqrt{1+\delta^2}}$$

Appendix C

Static Load Carrying Capacity of Hydrostatic Journal Bearing with Orifice Source Feeding

The dimensionless steady state Reynolds equation with feeding is derived in Eq. (14), Appendix A. In this derivation it is assumed that the gas feeding takes place over a finite area A. However, the present analysis assumes that the orifice is represented by a point source. Hence, the feeding term is replaced by the source strength C_j and Eq. (A.14) becomes:

$$\text{(C.1)} \qquad \frac{\partial}{\partial\theta}\left[h^3 \frac{\partial(p^2)}{\partial\theta}\right] + \frac{\partial}{\partial\zeta}\left[h^3 \frac{\partial(p^2)}{\partial\zeta}\right] = -\,\delta(\zeta-\zeta_0)\sum_{j=1}^{N} C_j\,\delta(\theta-\varphi_j),$$

where

$\delta(x)$ = the Dirac delta function,

ζ_0 = the ζ - coordinate of the point sources,

φ_j = the angular position of the jth source,

N = the number of sources on each side of the ζ - origin (See Figure 6 below),

C_j = the strength of the jth point source.

The solution of the problem is then that of finding a Green's function for the equation

$$\frac{\partial}{\partial\theta}\left[h^3 \frac{\partial(p^2)}{\partial\theta}\right] + \frac{\partial}{\partial\zeta}\left[h^3 \frac{\partial(p^2)}{\partial\zeta}\right] = 0,$$

subject to certain boundary conditions

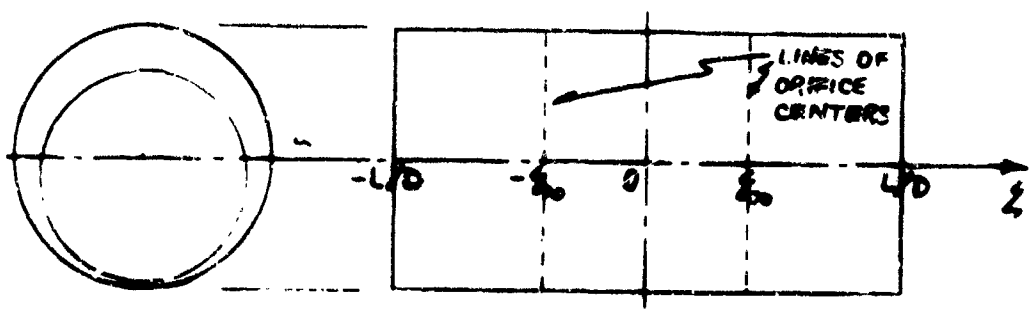

Figure 6

Since the orifices are assumed uniformly spaced with respect to θ, then

$$\varphi_j = (2j-1)\pi/N, \qquad j = 1, 2, \ldots N.$$

Moreover, since the problem is symmetric in ζ, we need consider only that portion of the bearing for which $0 \le \zeta \le \frac{L}{D}$. Figure 7 shows the bearing unfolded and the boundary conditions.

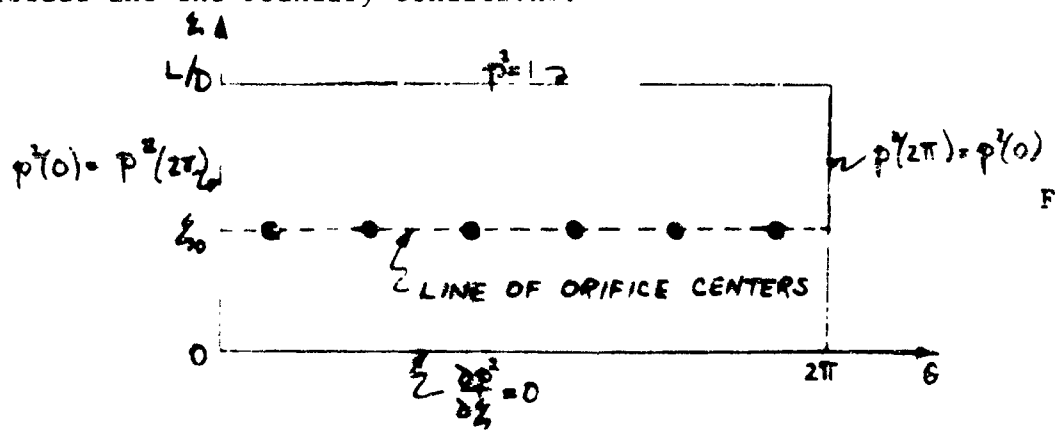

Figure 7

We note that $h = 1 + \epsilon \cos\theta$. If we expand p and C_j in perturbation series in powers of ϵ and retain only first degree terms, we have

$$\text{(C.2)} \qquad \begin{aligned} p^2 &= p_0^2 + 2\epsilon p_0 p_1 , \\ h^3 &= 1 + 3\epsilon \cos\theta , \\ C_j &= C_j^{(0)} + \epsilon C_j^{(1)} . \end{aligned}$$

Substitution of Eq. (C.2) in (C.1), retaining again only terms of first degree in , yields

$$\nabla^2(p_0^2) = -\delta(\zeta - \zeta_0)\sum_{j=1}^{N} C_j^{(0)} \delta(\theta - \varphi_j),$$

as the equation of zeroth degree in ϵ. Since this corresponds to the symmetric case, we note that all values of $C_j^{(0)}$ are identical ($=C^{(0)}$) for a uniform source pressure; therefore, we need only consider the sector

$$\frac{2(j-1)\pi}{N} \le \theta \le \frac{2j\pi}{N},$$

and the above equation reduces to

$$\text{(C.3a)} \qquad \nabla^2(p_0^2) = \frac{\partial^2}{\partial \zeta^2}(p_0^2) + \frac{\partial^2}{\partial \theta^2}(p_0^2) = -C^{(0)}\,\delta(\zeta-\zeta_0)\,\delta(\theta-\varphi_j),$$

with the boundary conditions

$$\text{(C.3b)} \begin{cases} p_0^2 = 1 \quad \text{at } \zeta = L/D\,; \quad \frac{\partial}{\partial \zeta}(p_0^2) = 0 \quad \text{at } \zeta = 0\,; \\ p_0^2\big|_{\theta_{j-1}} = p_0^2\big|_{\theta_j}\,; \qquad \theta_j = \frac{2j\pi}{N}\,; \quad j = 1, 2, \ldots, N. \end{cases}$$

The first degree terms in ϵ yield the equation

$$\text{(C.4a)} \qquad \nabla^2(p_0 p_1) = \nabla^2(U_1 + U_2 + U_3),$$

where

$$\nabla^2 U_1 = \frac{3}{2}\sin\theta\,\frac{\partial}{\partial\theta}(p_0^2),$$

$$\nabla^2 U_2 = \frac{3}{2}C^{(0)}\cos\theta\;\delta(\zeta-\zeta_0)\sum_{j=1}^{N}\delta(\theta-\varphi_j),$$

$$\nabla^2 U_3 = -\frac{1}{2}\,\delta(\zeta-\zeta_0)\sum_{j=1}^{N} C_j^{(0)}\,\delta(\theta-\varphi_j).$$

The boundary conditions on $p_0 p_1$ (and U_1, U_2, U_3) are:

$$\text{(C.4b)} \begin{cases} p_0 p_1 = 0 \quad \text{at } \zeta = L/D, \quad \frac{\partial}{\partial \zeta}(p_0 p_1) = 0 \quad \text{at } \zeta = 0\,; \\ p_0 p_1\big|_{\theta=0} = p_0 p_1\big|_{\theta=2\pi}. \end{cases}$$

Solution of the Zeroth Degree Equation

We write (C.3a) as

$$(C.5) \qquad p_o^{\,2} = 1 + U_o.$$

We then have the equation

$$(C.6) \qquad \nabla^2 U_o = -C^{(0)}\,\delta(\zeta-\zeta_0)\,\delta(\theta-\varphi_j),$$

with the boundary conditions as in Eq. (C.3b), except that at $\zeta = L/D$, $U_o=0$. Eq. (C.6) is then the equation for a Green's function:

$$U_o = U_o(\zeta,\theta;\zeta_0,\varphi_j)$$

It is found by separation of variables in a direct product function space (See Reference 3.)

We write U_o as a Fourier series:

$$(C.7) \qquad U_o(\zeta,\theta) = \frac{N}{\pi}\left\{\frac{1}{2}\gamma_0(\zeta) + \sum_{n=1}^{\infty}\gamma_n(\zeta)\cos nN\theta\right\},$$

where

$$\gamma_n(\zeta) = \int_{2\pi(j-1)/N}^{2\pi j/N} U_o(\zeta,\theta)\cos nN\theta\, d\theta\,; \qquad n=0,1,2,\ldots$$

Eq. (C.7) satisfies the boundary conditions in the θ direction. We now multiply Eq. (C.6) by $\cos nN\theta$ and integrate with respect to θ from $\frac{2\pi(j-1)}{N}$ to $\frac{2\pi j}{N}$. The result is

$$(C.8) \qquad \frac{d^2}{d\zeta^2}\gamma_0(\zeta) = -C^{(0)}\,\delta(\zeta-\zeta_0)\,, \qquad n=0,$$

$$(C.9) \qquad \frac{d^2}{d\zeta^2}\gamma_n(\zeta) - (nN)^2\gamma_n(\zeta) = -C^{(0)}\delta(\zeta-\zeta_0)\cos nN\varphi_j\,, \qquad n=1,2,\ldots$$

The solutions to both (C.8) and (C.9) must obey the remaining boundary conditions, must be continuous across $\xi = \xi_0$, and must have the proper jump in their first derivative at that point. The solution of the homogeneous part of (C.8) is $\gamma_0 = A\xi + \text{const.}$ Thus,

$$\gamma_0(\xi) = \begin{cases} A_0\left(\frac{L}{D} - \xi_0\right), & \xi < \xi_0 \\ A_0\left(\frac{L}{D} - \xi\right), & \xi > \xi_0 \end{cases}$$

The jump in $\gamma_0'(\xi)$ at ξ_0 is $-C^{(0)}$; thus

$$\frac{d}{d\xi}\left[A_0\left(\frac{L}{D} - \xi\right)\right]\Bigg|_{\xi=\xi_0} - \cancel{\frac{d}{d\xi}\left[A_0\left(\frac{L}{D} - \xi_0\right)\right]\Bigg|_{\xi=\xi_0}}^{\,0} = -C^{(0)}$$

and

$$A_0 = C^{(0)}$$

Hence, if we write

$\xi_< =$ minimum of (ξ, ξ_0),

$\xi_> =$ maximum of (ξ, ξ_0),

then we obtain

(C.10) $$\gamma_0(\xi) = C^{(0)}\left(\frac{L}{D} - \xi_>\right).$$

The solution of the homogeneous part of (C.9) is given by hyperbolic functions. To satisfy the boundary conditions, our solution (continuous at $\xi = \xi_0$) is given by

$$\gamma_n(\xi) = \begin{cases} A_n \cosh nN\xi \sinh nN\left(\frac{L}{D} - \xi_0\right), & \xi < \xi_0, \\ A_n \cosh nN\xi_0 \sinh nN\left(\frac{L}{D} - \xi\right), & \xi > \xi_0. \end{cases}$$

The constant A_n is chosen to satisfy the jump condition across $\xi = \xi_0$:

$$-nN A_n\left[\cosh nN\left(\frac{L}{D} - \xi_0\right)\cosh nN\xi_0 + \sinh nN\xi_0 \sinh nN\left(\frac{L}{D} - \xi_0\right)\right] =$$

$$= -nN A_n \cosh \frac{nNL}{D} = -C^{(0)} \cos nN\varphi_1$$

Therefore,

$$A_n = \frac{C^{(0)} \cos nN\varphi_j}{nN \cosh(nNL/D)} .$$

But $\varphi_j = (2j-1)\pi/N$, and $\cos nN\varphi_j = \cos n\pi(2j-1) = (-1)^n$. Hence, A_n is independent of j, and is given by

$$A_n = (-1)^n C^{(0)} \Big/ \left(nN \cosh \frac{nNL}{D}\right), \qquad n = 1, 2, \ldots$$

Thus,

$$Y_n(\xi) = \frac{(-1)^n C^{(0)} \cosh nN\xi_< \sinh nN\left(\frac{L}{D} - \xi_>\right)}{nN \cosh(nNL/D)} , \quad n = 1, 2, \ldots$$

The solution for U_o is then

$$\text{(C.11)} \quad U_0(\xi, \theta; \xi_0) = \frac{C^{(0)}}{\pi} \left[\frac{N}{2}\left(\frac{L}{D} - \xi_>\right) + \sum_{n=1}^{\infty} (-1)^n \frac{\cos nN\theta \cosh nN\xi_< \sinh nN\left(\frac{L}{D} - \xi_>\right)}{m \cosh(mNL/D)} \right],$$

and p_o^2 is obtained by adding 1 to Eq. (C.1).

The method for determining $C^{(o)}$ will be described later.

<u>Solution of the First Degree Equations</u>

To obtain the functions U_1, U_2, and U_3 whose sum is the solution of Eq. (C.4a), we must first obtain the Green's function $G(\xi, \theta; \xi, \psi)$ for the Laplacean with boundary conditions (C.4b). From the fact that

$$\text{(C.12)} \qquad \nabla^2_{\xi,\theta} G(\xi, \theta; \xi, \psi) = -\delta(\xi - \xi)\, \delta(\theta - \psi),$$

(where the subscripts on the Laplacean operator indicate the variables with respect to which we differentiate), we find that

$$(C.13)\quad U_1 = -\frac{3}{2}\int_0^{L/D}\int_0^{2\pi} G(\xi,\theta;\xi,\psi)\frac{\partial}{\partial\psi}\left[p_0^2(\xi,\psi;\xi_0)\right]\sin\psi \; d\psi \, d\xi \, .$$

Moreover, since U_2 and U_3 are also Green's functions satisfying the same boundary conditions as G, we may obtain U_2 and U_3 by a comparatively simple modification of G.

To obtain G, then, we follow the same procedure as in the solution for p_o^2, expanding this time in Fourier series in integers. Thus,

$$(C.14)\quad G(\xi,\theta;\xi,\psi) = \frac{1}{\pi}\left\{\frac{1}{2}\left(\frac{L}{D}-y\right) + \sum_{n=1}^{\infty}\left[\frac{\cos n\psi \cos n\theta}{n} \cdot \frac{\cosh nx \sinh n\left(\frac{L}{D}-y\right)}{\cosh(nL/D)}\right]\right\},$$

where

$y =$ max (ξ, ξ),

$x =$ min (ξ, ξ),

If we exclude the source points we may show uniform convergence of both the series p_o^2 and its term-by-term derivative with respect to θ. Therefore, from Eqs. (C.11), (C.13), and (C.14)

$$U_1(\xi,\theta;\xi_0) = \frac{3NC^{(0)}}{2\pi^2}\int_0^{L/D}\int_0^{2\pi}\left\{\frac{1}{2}\left(\frac{L}{D}-y\right) + \sum_{n=1}^{\infty}\left[\frac{\cos n\psi \cos n\theta}{n}\,\frac{\cosh nx \sinh n\left(\frac{L}{D}-y\right)}{\cosh(nL/D)}\right]\right\} \times$$

$$\times\left\{\sum_{m=1}^{\infty}(-1)^m \frac{\sin mN\psi \cosh mN\bar{x} \sinh mN\left(\frac{L}{D}-\bar{y}\right)}{\cosh(mNL/D)}\right\}\sin\psi \, d\psi \, d\xi$$

where

$\bar{y} =$ max (ξ_0, ξ),

$\bar{x} =$ min (ξ_0, ξ).

(<u>NOTE</u>: U_1 takes on a slightly different form in the case N = 1 (i.e., a single orifice). The analysis given herein includes this case, but the computer program assumes N > 1.)

Integration of the above gives

$$(C.15)\quad U_1(\xi,\theta;\xi_0)=\frac{3NC^{(0)}}{4\pi}\sum_{m=1}^{\infty}(-1)^m\left\{\frac{\cos\alpha_m\theta}{\alpha_m(2mN-1)}\left[\frac{mN\cosh\alpha_m\xi_<\sinh\alpha_m(\frac{L}{D}-\xi_>)}{\cosh(\alpha_m L/D)}-\frac{\alpha_m\cosh mN\xi_<\sinh mN(\frac{L}{D}-\xi_>)}{\cosh(mNL/D)}\right]\right.$$

$$\left.+\frac{\cos\beta_m\theta}{\beta_m(2mN+1)}\left[\frac{mN\cosh\beta_m\xi_<\sinh\beta_m(\frac{L}{D}-\xi_>)}{\cosh(\beta_m L/D)}-\frac{\beta_m\cosh mN\xi_<\sinh mN(\frac{L}{D}-\xi_>)}{\cosh(mNL/D)}\right]\right\},$$

(For the special case N = 1,

$$U_1(\xi,\theta;\xi_0)=\frac{3C^{(0)}}{4\pi}\left\{\left[\frac{\cosh\xi_<\sinh(\frac{L}{D}-\xi_>)}{\cosh(L/D)}-(\frac{L}{D}-\xi_>)\right]\right.$$

$$+\sum_{m=2}^{\infty}(-1)^m\left\{\frac{\cos\alpha_m\theta}{\alpha_m(2m-1)}\left[\frac{m\cosh\alpha_m\xi_<\sinh\alpha_m(\frac{L}{D}-\xi_>)}{\cosh(\alpha_m L/D)}\right.\right.$$

$$\left.-\frac{\alpha_m\cosh m\xi_<\sinh m(\frac{L}{D}-\xi_>)}{\cosh(mL/D)}\right]$$

$$+\frac{\cos\beta_m\theta}{\beta_m(2m+1)}\left[\frac{m\cosh\beta_m\xi_<\sinh\beta_m(\frac{L}{D}-\xi_>)}{\cosh(\beta_m L/D)}\right.$$

$$\left.\left.\left.-\frac{\beta_m\cosh m\xi_<\sinh m(\frac{L}{D}-\xi_>)}{\cosh(mL/D)}\right]\right\}.)$$

where

$$\alpha_m = mN-1$$

$$\beta_m = mN+1$$

As noted above, the function U_2 is also a Green's function satisfying the same boundary conditions as G, but with a different jump in the first derivative:

$$\text{(C.16)} \qquad U_2 = -\frac{3C^{(0)}}{2\pi}\sum_{j=1}^{N}\cos\varphi_j\left\{\frac{1}{2}\left(\frac{L}{D}-\xi_>\right) + \sum_{n=1}^{\infty}\left[\frac{\cos n\varphi_j \cos n\theta \cosh n\xi_< \sinh n\left(\frac{L}{D}-\xi_>\right)}{n\cosh(nL/D)}\right]\right\}.$$

(For the special case N=1, where $\phi = \pi$, we obtain

$$U_2 = \frac{3C^{(0)}}{4\pi}\left[\left(\frac{L}{D}-\xi_>\right) - \sum_{m=1}^{\infty}(-1)^m \frac{\cos m\theta \cosh m\xi_< \sinh m\left(\frac{L}{D}-\xi_>\right)}{m\cosh(mL/D)}\right].\ )$$

We note that $\phi_j = (2j-1)\pi/N$. Interchanging the order of summation, we make use of the orthogonality of trigonometric functions under summation to obtain

$$\text{(C.17a)} \qquad \sum_{j=1}^{N}\cos\frac{(2j-1)\pi}{N} = 0,$$

$$\text{(C.17b)} \qquad \sum_{j=1}^{N}\cos\frac{(2j-1)\pi}{N}\cos\frac{n(2j-1)\pi}{N} = \begin{cases} 0, & \frac{n}{2} \neq mN\pm 1 \\ (-1)^m\frac{N}{2}, & n = mN\pm 1 \end{cases}$$

Hence,

$$(C.18)\qquad U_2(\xi,\theta;\xi_0) = -\frac{3NC^{(o)}}{4\pi}\sum_{m=1}^{\infty}(-1)^m\left[\frac{\cos\alpha_m\theta\,\cosh\alpha_m\xi_<\,\sinh\alpha_m(\frac{L}{D}-\xi_>)}{\alpha_m\cosh(\alpha_m L/D)} + \frac{\cos\beta_m\theta\,\cosh\beta_m\xi_<\,\sinh\beta_m(\frac{L}{D}-\xi_>)}{\beta_m\cosh(\beta_m L/D)}\right].$$

Combination of Eqs. (C.15) and (C.18) results in

$$(C.19)\qquad \left[U_1+U_2\right](\xi,\theta;\xi_0) = -\frac{3NC^{(o)}}{4\pi}\sum_{m=1}^{\infty}(-1)^m\left\{\frac{\cos\alpha_m\theta}{(2mN-1)}\left[\frac{\cosh\alpha_m\xi_<\,\sinh\alpha_m(\frac{L}{D}-\xi_>)}{\cosh(\alpha_m L/D)} - \frac{\cosh mN\xi_<\,\sinh mN(\frac{L}{D}-\xi_>)}{\cosh(mNL/D)}\right] + \frac{\cos\beta_m\theta}{(2mN+1)}\left[\frac{\cosh\beta_m\xi_<\,\sinh\beta_m(\frac{L}{D}-\xi_>)}{\cosh(\beta_m L/D)} + \frac{\cosh mN\xi_<\,\sinh mN(\frac{L}{D}-\xi_>)}{\cosh(mNL/D)}\right]\right\}.$$

In a similar fashion we may write

$$(C.20)\qquad U_3(\xi,\theta;\xi_0) = \frac{1}{2\pi}\sum_{j=1}^{N}C_j^{(1)}\left[\frac{1}{2}\left(\frac{L}{D}-\xi_>\right)+\sum_{n=1}^{\infty}\frac{\cos n\varphi_j\cos n\theta\,\cosh n\xi_<\,\sinh n(\frac{L}{D}-\xi_>)}{n\cosh(nL/D)}\right].$$

(For the special case N=1, Eq. (C.20) becomes

$$U_3(\xi,\theta;\xi_0) = \frac{C^{(1)}}{2\pi}\left[\frac{1}{2}\left(\frac{L}{D}-\xi_>\right)+\sum_{m=1}^{\infty}(-1)^m\frac{\cos m\theta\,\cosh m\xi_<\,\sinh m(\frac{L}{D}-\xi_>)}{m\cosh(mL/D)}\right].\ )$$

The solution for $p_o p_1$ is given by the sum of U_1, U_2, and U_3, i.e., by the sum of Eqs. (C.19) and (C.20).

This result depends on the symmetric source strength $C^{(o)}$ and the perturbed source strengths $C_j^{(1)}$ (j=1,2...,N). In the next section we describe the procedure for their calculation.

Calculations of Source Strengths from the Equations of Mass Flow

From Eqs. (A.3) and (A.4), we may obtain the mass flow at the jth source as

(C.21a) $$M = -\frac{p_a^2 c^3}{24\mu RT} \oint_j h^3 \frac{\partial}{\partial n}(p^2)\, ds ,$$

where

$\oint_j [\quad] ds$ denotes the line integral about the jth source,

$\frac{\partial}{\partial n}$ = the outward normal derivative.

If we use the perturbation series in Eq. (C.21), we obtain, at the jth orifice

(C.21b) $$M = M_0 + \varepsilon M_1 = -\frac{p_a^2 c^3}{24\mu RT} \oint_j \left\{ \frac{\partial}{\partial n}(p_0^2) + \varepsilon \left[2 \frac{\partial}{\partial n}(p_0 p_1) + 3 \cos\theta \frac{\partial}{\partial n}(p_0^2) \right] \right\} ds .$$

From Eq. (A.22), we obtain the mass flow at the jth orifice to be

(C.22) $$M = \frac{\pi a^2 \eta^{1/2} \sqrt{\frac{2k}{k-1}}\, p_a V \cdot [\nu(p/V)]}{\sqrt{RT}} \left(\frac{p}{V}\right)^{1/k} \sqrt{1 - \left(\frac{p}{V}\right)^{\frac{k-1}{k}}}$$

Expanding Eq. (C 22) in a Taylor's series about $p=p_o$ and retaining only the first two terms yields

$$M = M(p_o) + (p-p_o) \left.\frac{dM}{dp}\right|_{p=p_o} .$$

But,

$$p-p_o = \epsilon p_1 ;$$

Therefore,

(C.23) $$M = M(p_o) + \epsilon p_1 \left.\frac{dM}{dp}\right|_{p=p_o} .$$

We note that Eq. (C.23) holds at the jth orifice. Equating the coefficients like powers of ϵ in Eqs. (C.21b) and (C.23) at the jth orifice gives

(C.24a) $$M_0 = M(p_0) = -\frac{p_a^2 c^3}{24\mu RT} \oint_j \frac{\partial}{\partial n}(p_0^2)\, ds ,$$

(C.24b) $$M_1 = M'(p_0)\cdot[p_1]_j = -\frac{p_a^2 c^3}{24\mu RT} \oint_j \left[2\frac{\partial}{\partial n}(p_0 p_1) + 3\cos\theta \frac{\partial}{\partial n}(p_0^2)\right] ds ,$$

where $M(p_o)$ and $M'(p_o)$ are obtained from Eq. (C.22).

Consider first the integral on the right hand side of Eq. (C.24a). By Green's theorem and Eq. (C.3a),

(C.25) $$\oint_j \frac{\partial}{\partial n}(p_0^2)\, ds = \iint \nabla^2(p_0^2)\, d\theta\, d\xi = -C^{(0)} .$$

Applying Green's theorem and Eq. (C.4a) to the integral in Eq. (C.24b) we find

(C.26) $$\oint_j \left[2\frac{\partial}{\partial n}(p_0 p_1) + 3\cos\theta \frac{\partial}{\partial n}(p_0^2)\right] ds = -C_j^{(1)}$$

We now define the following quantities:

(C.27) $$\begin{cases} x = p_0/V , \\ \Lambda_t = 6\mu\eta^{1/2} a^2 \sqrt{RT}\, N / p_a c^3 , \\ f(x) = \sqrt{\frac{2k}{k-1}}\, [\nu(x)]\, x^{1/k} \sqrt{1 - x^{\frac{k-1}{k}}} . \end{cases}$$

To solve for $C^{(o)}$, we consider Eq. (C.24a). Using Eqs. (C.22), (C.25), and (C.27), Eq. (C.24a) becomes

(C.28) $$\Lambda_t V f(x) = \frac{N}{4}\left[\frac{C^{(0)}}{\pi}\right] .$$

At the jth orifice, $\theta=(2j-1)\pi/N$. Substituting this value in Eq. (C.11), we have

$$p_0^2\Big|_{\theta=\varphi_j} = 1+\frac{C^{(0)}}{\pi}\left[\frac{N}{2}\left(\frac{L}{D}-\xi_>\right)+\sum_{m=1}^{\infty}\frac{\cosh mN\xi_< \sinh mN(\xi-\xi_>)}{m\cosh(mNL/D)}\right],$$

$$= 1+\frac{C^{(0)}}{\pi}S_0(\xi,\xi_0).$$

To represent S_0 at the orifice we choose

$$\bar{S}_0 = \frac{1}{2}\left[S_0(\xi_0+a,\xi_0)+S_0(\xi_0-a,\xi_0)\right].$$

(In Appendix D, a transformation is made in the series S_0 to obtain quicker convergence.) Hence, at the jth orifice we have

$$\frac{C^{(0)}}{\pi} = \frac{p_0^2-1}{\bar{S}_0} = \frac{V^2}{\bar{S}_0}\left[x^2-\frac{1}{V^2}\right].$$

Eq. (C.28) becomes

$$\text{(C.29)} \qquad \Lambda_t f(x) = \frac{NV}{4\bar{S}_0}\left[x^2-\frac{1}{V^2}\right].$$

The above equation has one and only one root for positive $x \leq 1$. Calling this root x^*, we then obtain

$$C^{(0)} = \frac{\pi V^2}{\bar{S}_0}\left[(x^*)^2-\frac{1}{V^2}\right].$$

The root x^* also defines the value of $\nu(x)$ and $\nu'(x)$ to be used subsequently.

We next solve for $C_j^{(1)}$. From Eq. (C.22),

$$\text{(C.30)} \qquad M_0' = M_0\Psi/V,$$

where

$$\Psi = \frac{\nu'(x^*)}{\nu(x^*)} + \frac{1}{k}\left\{\frac{1}{x^*} - \frac{k-1}{2(x^*)^{1/k}\left[1-(x^*)^{(k-1)/k}\right]}\right\} .$$

We multiply and divide the left hand side of (C.24b) by $(p_o)_j$, and use Eqs. (C.27) and (C.30) to obtain

$$f(x)\frac{\Lambda_t \Psi}{V\ x^*}\left[p_0 p_1\right]_j = \frac{N}{4}\left[\frac{C_j^{(1)}}{\pi}\right] .$$

Clearing of fractions gives

(C.31) $$\left[p_0 p_1\right]_j = \frac{NV\,x^*}{4\pi\Lambda_t \Psi f(x)} C_j^{(1)}, \qquad j = 1,2,\ldots,N.$$

The above equation is a linear system wherein the unknowns are the source strengths $C_j^{(1)}$. Therefore, it can be put in the equivalent form

(C.32) $$[A]\ \vec{C}^{(1)} = \vec{B} ,$$

where

$[A]$ is a matrix of order N x N whose elements are A_{jk},

$\vec{C}^{(1)}$ is a column vector whose N elements are the unknowns $C_j^{(1)}$,

$\vec{B}$ is a column vector containing N elements.

In the following, it is assumed that all expressions are evaluated first for $\xi = \xi_o - a$, then for $\xi = \xi_o + a$, and an average taken.

We recall first that

$$r_o p_1 = U_1 + U_2 + U_3.$$

Moreover, at the jth orifice, the angular coordinate is given by

$$(2j-1)\pi/N.$$

From Eq. (C.19), at the jth orifice

$$\text{(C.33a)} \qquad (U_1 + U_2)_j = (\bar{S}_1)_j, \qquad j = 1,2,\ldots,N$$

where

$(\bar{S}_1)_j$ is the right hand side of (C.19) with $\theta = (2j-1)\pi/N$.

Also, from Eq. (C.20), at the jth orifice

$$\text{(C.33b)} \qquad (U_3)_j = \sum_{k=1}^{N} C_k^{(1)} T_{jk}, \qquad j = 1,2,\ldots,N,$$

where

$$T_{jk} = \frac{1}{2\pi}\left[\frac{1}{2}\left(\frac{L}{D} - \xi_>\right) + \sum_{n=1}^{\infty} \frac{\cos\frac{n\pi(2k-1)}{N}\cos\frac{n\pi(2j-1)}{N}\cosh n\xi_< \sinh n\left(\frac{L}{D}-\xi_>\right)}{n\cosh(nL/D)}\right].$$

(This series is also put into more easily calculable form by the transformation described in Appendix D.) Therefore, the elements of the matrix $[A]$ and the vector $\vec{B}$ of Eq. (C.31) are given by

$$\text{(C.34)} \quad \begin{cases} A_{jk} = T_{jk} - \left[\dfrac{NV^2 x^*}{4\pi\Lambda_t \Psi}\right]\delta_{jk} = A_{kj}, \\[2ex] B_j = -(\bar{S}_1)_j. \end{cases}$$

where

$$\delta_{jk} = \begin{cases} 0, & j \neq k \\ 1, & j = k \end{cases}$$

The $C^{(1)}$ vector is then found by inverting $[A]$ and multiplying $\vec{B}$ by $[A]^{-1}$.

Appendix D

Reformulation of an Infinite Series Encountered in the Analysis Performed in Appendix C

Several expressions in the previous Appendix (e.g., Eqs. (C.11), (C.20)) involve infinite series containing several parameters, of the form

$$S = \sum_{n=1}^{\infty} \frac{\cos n\alpha \cos n\beta \cosh nx \sinh n(R-y)}{n \cosh nR}, \tag{D.1}$$

where

$$R > y > x \geq 0.$$

The series is absolutely convergent.

We make the following substitutions:

$$\begin{aligned} a_1 &= \alpha+\beta \\ a_2 &= \alpha-\beta \\ \delta &= y-x \\ \sigma &= y+x \end{aligned} \tag{D.2}$$

after which Eq. (D.1) becomes

$$S = \frac{1}{4} \sum_{n=1}^{\infty} \frac{[\cos na_1 + \cos na_2][\sinh n(R-\delta) + \sinh n(R-\sigma)]}{n \cosh nR}, \tag{D.3}$$

noting that

$$2R > \sigma > \delta > 0.$$

We represent a typical part of the series as

$$T = \sum_{n=1}^{\infty} \frac{\cos n\varphi \sinh n(R-\xi)}{n \cosh nR}, \tag{D.4}$$

where

$$2R > \xi > 0.$$

Then, writing the trigonometric and hyperbolic functions as sums of exponentials, we have

$$T = \frac{1}{2}\sum_{n=1}^{\infty} \frac{\left[e^{-n(\xi+i\varphi)} + e^{-n(\xi-i\varphi)} - e^{-n(2R-\xi+i\varphi)} - e^{-n(2R-\xi-i\varphi)}\right]}{n\left(1+e^{-2nR}\right)}.$$

But,

$$\left(1+e^{-2nR}\right)^{-1} = \sum_{k=0}^{\infty} (-1)^k e^{-2knR} \qquad (nR > 0);$$

hence,

$$\text{(D.5)} \qquad T = \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}\sum_{k=0}^{\infty}(-1)^k\Big[e^{-n(2kR+\xi+i\varphi)} + e^{-n(2kR+\xi-i\varphi)} - e^{-n(2kR+2R-\xi+i\varphi)} - e^{-n(2kR+2R-\xi-i\varphi)}\Big].$$

Interchanging the order of summation yields

$$T = \frac{1}{2}\sum_{k=0}^{\infty}(-1)^k\sum_{n=1}^{\infty}\frac{1}{n}\Big[e^{-n(2kR+\xi+i\varphi)} + e^{-n(2kR+\xi-i\varphi)} - e^{-n(2kR+2R-\xi+i\varphi)} - e^{-n(2kR+2R-\xi-i\varphi)}\Big].$$

Since

$$\sum_{n=1}^{\infty}\frac{z^n}{n} = -\log(1-z) \qquad (|z|^2 < 1),$$

we then have

$$T = \frac{1}{2}\sum_{k=0}^{\infty}(-1)^k \log\left\{\frac{\left[1-e^{-(2kR+2R-\xi+i\varphi)}\right]\left[1-e^{-(2kR+2R-\xi-i\varphi)}\right]}{\left[1-e^{-(2kR+\xi+i\varphi)}\right]\left[1-e^{-(2kR+\xi-i\varphi)}\right]}\right\},$$

which is still absolutely convergent. We now convert the expressions within the curly brackets back to trigonometric and hyperbolic functions:

$$T = \frac{1}{2}\sum_{k=0}^{\infty}(-1)^k \log\left\{e^{-2(R-\xi)}\left[\frac{\sinh^2(kR+R-\frac{\xi}{2})\cos^2\frac{\varphi}{2} + \cosh^2(kR+R-\frac{\xi}{2})\sin^2\frac{\varphi}{2}}{\sinh^2(kR+\frac{\xi}{2})\cos^2\frac{\varphi}{2} + \cosh^2(kR+\frac{\xi}{2})\sin^2\frac{\varphi}{2}}\right]\right\}$$

$$= \frac{1}{2}\sum_{k=0}^{\infty}(-1)^k \log\left\{e^{-2(R-\xi)}\left[\frac{\cosh^2(kR+R-\frac{\xi}{2}) - \cos^2\frac{\varphi}{2}}{\cosh^2(kR+\frac{\xi}{2}) - \cos^2\frac{\varphi}{2}}\right]\right\}$$

Finally,

$$\text{(D.6)} \quad T = \frac{1}{2}\sum_{k=0}^{\infty}(-1)^k \log\left\{\frac{e^{-(2R-\xi)}\left[\cosh(2R-\xi) + \tanh 2kR \sinh(2R-\xi) - \frac{\cos\varphi}{\cosh 2kR}\right]}{e^{-\xi}\left[\cosh\xi + \tanh 2kR \sinh\xi - \frac{\cos\varphi}{\cosh 2kR}\right]}\right\}.$$

Eq. (D.6) is then substituted in Eq. (D.3), for $\xi = \sigma$, $\xi = \delta$, $\varphi = a_1$, $\varphi = a_2$.

The computational advantages of the new form Eq. (D.6) are as follows:

1) In the initial formulation Eq. (D.1), two trigonometric and three hyperbolic functions are required for each value of the summation index n. Even if these are calculated from recurrence formulas, the form (D.6) is more efficient, since after an initial calculation of entry values, only two hyperbolic functions are necessary at each step.

2) The new formulation is alternating, and the rate of convergence can be increased by means of Euler's method.

APPENDIX E

Static Load Carrying Capacity of Hydrostatic Thrust Bearing with Orifice Source Feeding

Reynolds equation for the hydrostatic annular thrust bearing with feeding is given in eq. (A.21). If the feeding is represented by a point source instead of a flow over a finite area, eq. (A.21) reduces to Laplaces equation with singularities at the orifices. Assuming that the orifice may be described by a simple source (a logarithmic singularity) use can be made of the methods of conformal mapping. (See f. inst. Ref. 4 and 5) Thus in the present case it is most convenient first to consider a rectangular pad with a source in the center, ambient pressure at two sides and no flow from the other two sides. This pad is equivalent to a radial sector of the annular thrust bearing.

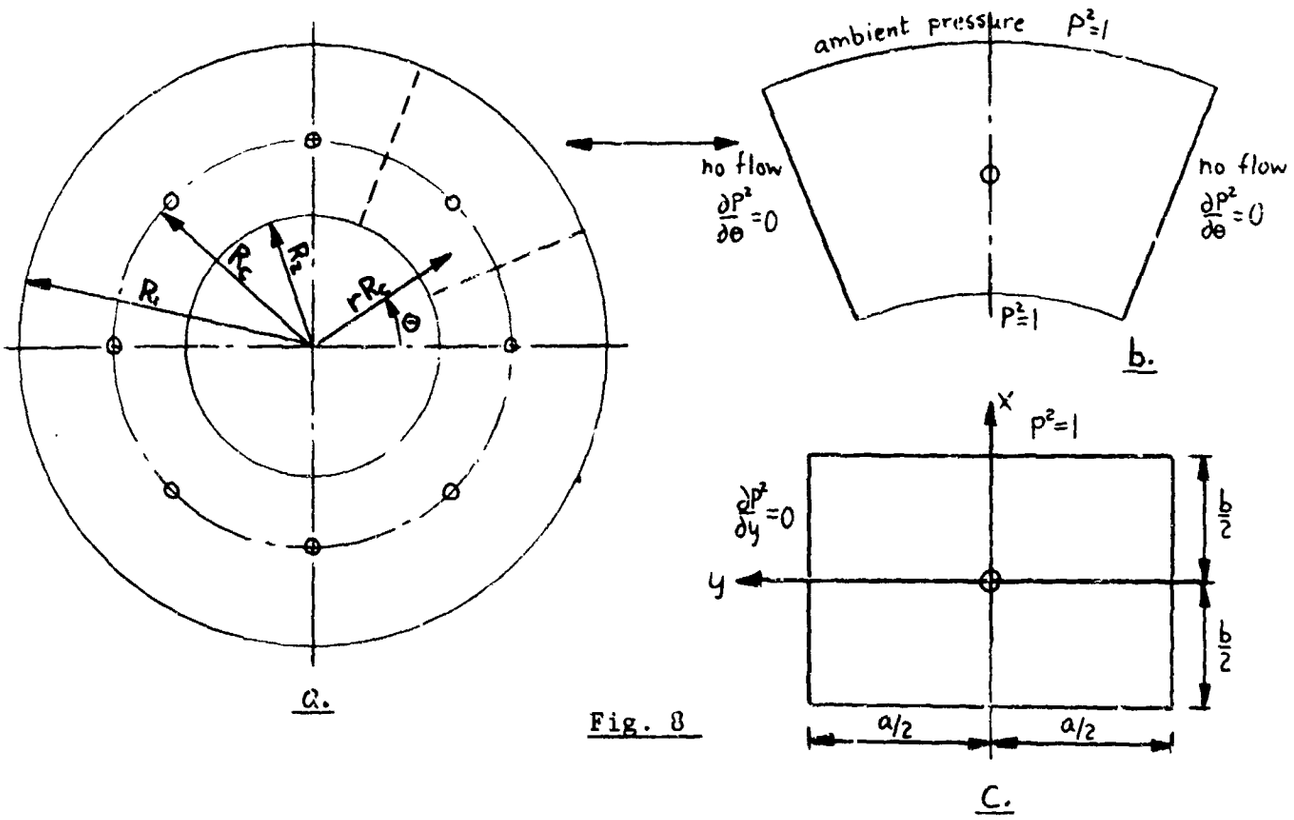

Fig. 8

The boundary condition of zero flow at $y = \pm \frac{a}{2}$ is satisfied if we along the y-axis take a infinite number of equidistant sources of equal strength. Similarly we need an infinite number of alternating sources and sinks in the x-direction to satisfy the ambient pressure condition. Such an array is represented by

$$P^2 = 1 + Re\left\{ -C \sum_{n=-\infty}^{\infty} (-1)^n \cdot \ln\left[\frac{\sinh \pi(\bar{z} + n\bar{b})}{K_n} \right] \right\} \tag{E.1}$$

where P is the dimensionless pressure (with respect to ambient), C is the source strength, $\bar{b} = \frac{b}{a}$, b is the length of the rectangle in the x-direction, a is the length in the y-direction, K_n is a constant and $\bar{z} = \frac{1}{a}(x+iy)$.

Taking the real part as indicated gives:

$$P^2 = 1 - \frac{1}{2} C \sum_{n=-\infty}^{\infty} (-1)^n \cdot \ln\left[\frac{\cosh 2\pi(\frac{x}{a} + n\bar{b}) - \cos(2\pi \frac{y}{a})}{\cosh \pi\bar{b}(2n+1) - 1} \right] \tag{E.2}$$

which satisfies the boundary conditions:

$$x = \pm \frac{b}{2} : \qquad P^2 = 1$$

$$y = \pm \frac{a}{2} : \qquad \frac{\partial P^2}{\partial y} = 0$$

From eq. (E.2) we get:

$$\frac{\partial P^2}{\partial(\frac{x}{a})} = -\pi C \sum_{n=-\infty}^{\infty} (-1)^n \frac{\sinh 2\pi(\frac{x}{a} + n\bar{b})}{\cosh 2\pi(\frac{x}{a} + n\bar{b}) - \cos(2\pi \frac{y}{a})} \tag{E.3}$$

To determine the source strength C the bearing flow must be equated to the orifice mass flow M:

$$M = -\frac{P_a^2 h^3}{24 \mu RT} \oint \frac{\partial P^2}{\partial n} ds \tag{E.4}$$

where $\oint ds$ is a closed line integral around the source and n is the outward directed normal.

Using the boundaries of the rectangle as the integration path, where $\frac{\partial P^2}{\partial y} = 0$ along $y = \pm \frac{a}{2}$, eq. (E.4) reduces to:

$$M = -\frac{P_a^2 h^3}{12\mu RT}\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\partial P^2}{\partial(\frac{x}{a})}\bigg|_{x=\frac{b}{2}} d\left(\frac{y}{a}\right)$$

Introducing eq. (E.3) gives:

(E.5) $$M = \frac{\pi P_a^2 h^3}{12\mu RT}\cdot C$$

Substituting the dimensionless mass flow m from eq. (A.11) and the feeding parameter Λ_t from eq. (A.8) (setting $h = C(1+\varepsilon)$) gives:

(E.6) $$\frac{CN(1+\varepsilon)^3}{2\Lambda_t V} = m$$

This equation may be solved graphically or numerically as shown in Appendix B, Fig. 4, where C is known as a function of the downstream orifice pressure P_i. This relationship is calculated from eq. (E.2) based on a known feeding hole diameter (i.e. known values of x and y).

The rectangular pad is transformed into the radial sector shown in Fig. 8b by the transformation:

(E.7) $$\bar{z} = \frac{1}{a}(x+iy) = \frac{N}{2\pi}\ln(re^{i\theta})$$

i.e.

(E.8) $$\frac{x}{a} = \frac{N}{2\pi}\cdot\ln r$$

(E.9) $$\frac{y}{a} = \frac{N}{2\pi}\theta$$

where N is the number of orifices, r is the dimensionless radius with respect to the orifice circle radius R_c and θ is the angular coordinate. Substituting eq. (E.8) and (E.9) into eq. (E.2) gives:

(E.10) $$P^2 = 1 - \frac{1}{2}C\sum_{n=-\infty}^{\infty}(-1)^n \ln\left[\frac{\cosh N(\ln r + n\ln(\frac{R_1}{R_2})) - \cos N\theta}{\cosh(N(n+\frac{1}{2})\cdot\ln(\frac{R_1}{R_2})) - 1}\right]$$

where C still is given by eq. (E.5) or (E.6). Thus C is determined as the intersection between the orifice flow curve and the C vs. $\left(\frac{P_i}{V}\right)$ relationship

determined from eq. (E.10) for a given feeding hole diameter.

It should be noted that since the isobars are not circles this relationship is determined by taking the average pressure along the rim of the feeding hole. However, this is not a serious assumption since the isobars tend to circle in the limit of approaching the singularity. The load carrying capacity is calculated by integrating the pressure:

$$W = 2N P_a R_c^2 \int_0^{\frac{\pi}{N}} \int_{\frac{R_2}{R_c}}^{\frac{R_1}{R_c}} r(P-1)\, dr\, d\theta \tag{E.11}$$

Since the orifice was located in the center of the rectangular pad, the transformation given by eq. (E.7) results in:

$$R_c = \sqrt{R_1 R_2} \tag{E.12}$$

i.e., the analysis is only valid for this particular value of R_c. Hence eq. (E.11) becomes:

$$\frac{W}{\pi R_1^2 P_a} = \frac{2N}{\pi} \cdot \frac{R_2}{R_1} \int_0^{\frac{\pi}{N}} \int_{\sqrt{\frac{R_2}{R_1}}}^{\sqrt{\frac{R_1}{R_2}}} r \cdot (P-1)\, dr\, d\theta \tag{E.13}$$

where P is given by eq. (E.10). The integration is performed numerically. Rearranging eq. (E.10) we get:

$$P^2 = 1 - \frac{1}{2} C \left\{ \ln\left[\frac{\cosh(N \ln r) - \cos N\theta}{\cosh(\frac{N}{2}\ln(\frac{R_1}{R_2})) - 1}\right] + \sum_{n=1}^{\infty} (-1)^n \ln\left[\frac{\cosh^2(nN\ln(\frac{R_1}{R_2})) - 2\cosh(nN\ln(\frac{R_1}{R_2}))\cosh(N\ln r)\cos N\theta + \cosh^2(N\ln r) + \cos^2 N\theta}{\left[\cosh(nN\ln(\frac{R_1}{R_2})) - \cosh(\frac{N}{2}\ln(\frac{R_1}{R_2}))\right]^2}\right]\right\} \tag{E.14}$$

It can be shown that this series is absolute convergent except at the source. Since the series is alternating, the truncation error after n terms is less than the absolute value of the $(n+1)$'st term. The series converges very fast so that only the two first terms are necessary for an evaluation.

If the bearing has a circular recess around the orifice with depth h_r and radius R_r eq. (E.10) and (E.14) still describes the pressure outside the

recess. Inside the recess the pressure becomes:

$$\text{(E.15)} \quad P_r^2 = 1 + C_2 - \frac{1}{2}\left(\frac{1}{1+\frac{h_r}{h}}\right)^3 C \sum_{n=-\infty}^{\infty} (-1)^n \ln\left[\frac{\cosh N\left(\ln r + n \ln\left(\frac{R_1}{R_2}\right)\right) - \cos N\theta}{\cosh\left(N\left(n+\frac{1}{2}\right)\ln\left(\frac{R_1}{R_2}\right)\right) - 1}\right]$$

where C_2 is determined from the condition that $P = P_r$ along the circumference of the recess. This solution is only approximate since the isobars are not exactly circular. An average value is used.

When the orifices are not located as given by eq. (E.12) the above analysis is no longer valid. Instead we may use the method developed in Appendix C. Let us again start with a rectangular pad where the location of the orifice in this case is not restricted to the center of the pad.

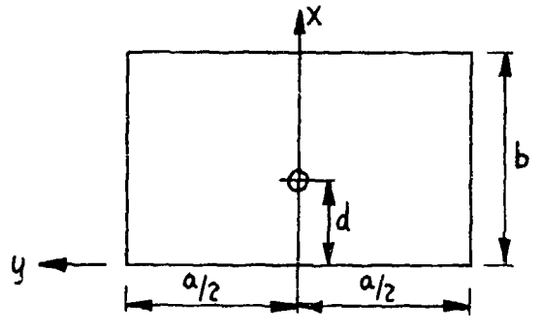

Fig. 9

If the gas feeding is represented by a point source of strength C it can be written $C \cdot \delta(x-d) \cdot \delta(y)$ where δ is the Dirac delta function. Thus Reynolds equation, eq. (A.14) becomes, in dimensionless form:

$$\text{(E.16)} \quad \frac{\partial^2 P^2}{\partial \zeta^2} + \frac{\partial^2 P^2}{\partial \varphi^2} = -C \cdot \delta(\zeta - 2\pi\varkappa)\,\delta(\varphi)$$

where:

$$\text{(E.17)} \quad \zeta = 2\pi\frac{x}{a} \qquad \varphi = 2\pi\frac{y}{a} \qquad \xi = \frac{b}{a} \qquad \varkappa = \frac{d}{a}$$

The boundary conditions for eq. (E.16) are:

(E.18) $\varphi = \pm\pi$: $\frac{\partial P^2}{\partial \varphi} = 0$

(E.19) $\zeta = 0$ and $\zeta = 2\pi\xi$: $P^2 = 1$

To make the boundary conditions homogenous set:

(E.20) $$P^2 = 1 + U(\zeta, \varphi)$$

and express U in a Fourier expansion:

(E.21) $$U(\zeta, \varphi) = \frac{1}{2\pi}\gamma_0 + \frac{1}{\pi}\sum_{n=1}^{\infty}\gamma_n(\zeta)\cos n\varphi$$

where the coefficients $\gamma_n(\zeta)$ are determined by the inverse Fourier transform:

(E.22) $$\gamma_n(\zeta) = \int_{-\pi}^{\pi} U(\zeta, \varphi)\cdot\cos n\varphi \, d\varphi$$

Substituting eq. (E.20) into eq. (E.16) gives:

$$\frac{\partial^2 U}{\partial \zeta^2} + \frac{\partial^2 U}{\partial \varphi^2} = -C\,\delta(\zeta - 2\pi\varkappa)\cdot\delta(\varphi)$$

Then

(E.23) $$\int_{-\pi}^{\pi}\left[\frac{\partial^2 U}{\partial \zeta^2} + \frac{\partial^2 U}{\partial \varphi^2}\right]\cos n\varphi \, d\varphi = -C\,\delta(\zeta - 2\pi\varkappa)\int_{-\pi}^{\pi}\delta(\varphi)\cos n\varphi \, d\varphi = -C\,\delta(\zeta - 2\pi\varkappa)$$

The left hand side may be evaluated by:

$$\int_{-\pi}^{\pi}\frac{\partial^2 U}{\partial \zeta^2}\cos n\varphi \, d\varphi = \frac{\partial^2}{\partial \zeta^2}\left[\int_{-\pi}^{\pi} U\cos n\varphi \, d\varphi\right] = \frac{\partial^2 \gamma_n}{\partial \zeta^2}$$

$$\int_{-\pi}^{\pi}\frac{\partial^2 U}{\partial \varphi^2}\cos n\varphi \, d\varphi = \frac{\partial U}{\partial \varphi}\cos n\varphi\Big|_{-\pi}^{\pi} + n\int_{-\pi}^{\pi}\frac{\partial U}{\partial \varphi}\sin n\varphi \, d\varphi = nU\sin n\varphi\Big|_{-\pi}^{\pi} - n^2\int_{-\pi}^{\pi} U\cos n\varphi \, d\varphi = -n^2\gamma_n$$

making use of eq. (E.18). Therefore eq. (E.23) becomes:

(E.24) $$\frac{d^2\gamma_n}{d\zeta^2} - n^2\gamma_n = -C\cdot\delta(\zeta - 2\pi\varkappa)$$

Integrating:

(E.25) $$\left.\frac{d\gamma_n}{d\zeta}\right|_{2\pi\varkappa+0} - \left.\frac{d\gamma_n}{d\zeta}\right|_{2\pi\varkappa-0} = -C$$

The general solution of eq. (E.24) is:

(E.26) $$\gamma_0 = A_0 + B_0\zeta$$
$$\gamma_n = A_n \cosh(n\zeta) + B_n \sinh(n\zeta)$$

Introducing the boundary conditions eq. (E.18) and (E.19) together with eq. (E.20) and using eq. (E.25) yields the values of the coefficients A_n and B_n . Substituting into eq. (E.21) and (E.20) gives:

(E.27) $\underline{0 \leq \zeta \leq 2\pi\varkappa}$ $$P^2 = 1 + \frac{C}{\pi}\left\{\frac{\xi-\varkappa}{2\xi}\zeta + \sum_{n=1}^{\infty}\frac{1}{n}\frac{\sinh(2\pi n(\xi-\varkappa))}{\sinh(2\pi n\xi)}\sinh(n\zeta)\cos(n\varphi)\right\}$$

(E.28) $\underline{2\pi\varkappa \leq \zeta \leq 2\pi\xi}$ $$P^2 = 1 + \frac{C}{\pi}\left\{\frac{\varkappa}{2\xi}(2\pi\xi-\zeta) + \sum_{n=1}^{\infty}\frac{1}{n}\frac{\sinh(2\pi n\varkappa)}{\sinh(2\pi n\xi)}\sinh(n(2\pi\xi-\zeta))\cos(n\varphi)\right\}$$

To find the flow we must take the derivative of the above equations:

$\underline{0 \leq \zeta \leq 2\pi\varkappa}$ $$\frac{\partial P^2}{\partial\zeta} = \frac{C}{\pi}\left\{\frac{\xi-\varkappa}{2\xi} + \sum_{n=1}^{\infty}\frac{\sinh(2\pi n(\xi-\varkappa))}{\sinh(2\pi n\xi)}\cosh(n\zeta)\cos(n\varphi)\right\}$$

$\underline{2\pi\varkappa \leq \zeta \leq 2\pi\xi}$ $$\frac{\partial P^2}{\partial\zeta} = \frac{C}{\pi}\left\{-\frac{\varkappa}{2\xi} + \sum_{n=1}^{\infty}-\frac{\sinh(2\pi n\varkappa)}{\sinh(2\pi n\xi)}\cosh(n(2\pi\xi-\zeta))\cos(n\varphi)\right\}$$

The mass flow M is given by eq. (E.4). Let the integration path be around the boundaries of the rectangle so that:

$$M = -\frac{P_a^2 h^3}{24\mu RT}\int_{-\pi}^{\pi}\left[-\left.\frac{\partial P^2}{\partial\zeta}\right|_{\zeta=2\pi\xi} + \left.\frac{\partial P^2}{\partial\zeta}\right|_{\zeta=0}\right]d\varphi$$

Thus:

(E.29) $$M = \frac{P_a^2 h^3}{24\mu RT}C$$

Analogous to eq. (E.6) we get:

(E.30) $$\frac{C N (1+\varepsilon)^3}{4\pi \Lambda_t V} = m$$

The transformation from rectangular to polar coordinates is:

(E.31) $$\zeta + i\varphi = N \cdot \ln (r' e^{i\theta})$$

i.e.

(E.32) $$\zeta = N \cdot \ln r'$$

(E.33) $$\varphi = N\theta$$

where r' is the dimensionless radius with respect to the inner radius R_2 .

Therefore

(E.34) $$\xi = \frac{N}{2\pi} \cdot \ln\left(\frac{R_1}{R_2}\right)$$

(E.35) $$\varkappa = \frac{N}{2\pi} \cdot \ln\left(\frac{R_c}{R_2}\right)$$

Substituting eq. (E.32)-(E.35) into eq. (E.27)-(E.28) and introducing the previous nomenclature

(E.36) $$\eta = \frac{R_1}{R_c} \qquad \gamma = \frac{R_c}{R_2} \qquad \beta = \frac{\ln \eta}{\ln \gamma}$$

yields:

(E.37) $\underline{1 \leq r' \leq \gamma}$ $$p^2 = 1 + \frac{C}{\pi}\left\{\frac{N}{2}\frac{\beta}{1+\beta}\ln r' + \sum_{n=1}^{\infty}\frac{1}{n}\frac{\sinh(nN\ln\eta)}{\sinh(nN\ln\eta\gamma)}\sinh(nN\ln r')\cos(nN\theta)\right\}$$

(E.38) $\underline{\gamma \leq r' \leq \eta\gamma}$ $$p^2 = 1 + \frac{C}{\pi}\left\{\frac{N}{2}\frac{1}{1+\beta}\ln\left(\frac{\eta\gamma}{r'}\right) + \sum_{n=1}^{\infty}\frac{1}{n}\frac{\sinh(nN\ln\gamma)}{\sinh(nN\ln\eta\gamma)}\sinh\left(nN\ln\left(\frac{\eta\gamma}{r'}\right)\right)\cos(nN\theta)\right\}$$

The load is determined by eq. (E.11) and is evaluated numerically.

Since it is more convenient to have the above series in the same general form as the series in eq. (E.14), eq. (E.37)-(E.38) shall be rearranged. These series have the form:

(E.39) $$T = \sum_{n=1}^{\infty} \frac{1}{n} \frac{\sinh(na)}{\sinh(nb)} \sinh(nc) \cos(nd)$$

Rewrite to get:

(E.40) $$T = \frac{1}{4} \sum_{n=1}^{\infty} \frac{e^{-n(b-a-c)}}{n} \cdot \left(1-e^{-2nb}\right)^{-1} \left[e^{ind} + e^{-ind} - e^{-n(2a-id)} - e^{-n(2a+id)} - e^{-n(2c-id)} - e^{-n(2c+id)} + e^{-n(2a+2c-id)} + e^{-n(2a+2c+id)} \right]$$

From the binomial expansion:

$$(1-x)^{-1} = \sum_{k=0}^{\infty} x^k \qquad |x^2| < 1$$

we get

(E.41) $$\left(1-e^{-2nb}\right)^{-1} = \sum_{k=0}^{\infty} e^{-2knb} \qquad b>0$$

Substituting eq. (E.41) into eq. (E.40) and interchange order of summation gives:

(E.42) $$T = \frac{1}{4} \sum_{k=0}^{\infty} \sum_{n=1}^{\infty} \frac{1}{n} \Big[e^{-n(2kb+b-a-c-id)} + e^{-n(2kb+b-a-c+id)} - e^{-n(2kb+b+a-c-id)} - e^{-n(2kb+b+a-c+id)}$$
$$- e^{-n(2kb+b-a+c-id)} - e^{-n(2kb+b-a+c+id)} + e^{-n(2kb+b+a+c-id)} + e^{-n(2kb+b+a+c+id)} \Big]$$

Introduce the expansion:

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x) \qquad |x^2| < 1$$

Since $b > a+c$ and a, b and c are positive all the exponentials in eq. (E.42) are less than 1. Therefore eq. (E.42) becomes

(E.43) $$T = \frac{1}{4} \sum_{k=0}^{\infty} \ln \left[\frac{\sinh\frac{1}{2}(2kb+b-a-c-id) \cdot \sinh\frac{1}{2}(2kb+b+a-c+id) \cdot \sinh\frac{1}{2}(2kb+b-a+c-id) \cdot \sinh\frac{1}{2}(2kb+b-a+c+id)}{\sinh\frac{1}{2}(2kb+b+a+c-id) \cdot \sinh\frac{1}{2}(2kb+b+a+c+id) \cdot \sinh\frac{1}{2}(2kb+b-a-c-id) \sinh\frac{1}{2}(2kb+b-a-c+id)} \right]$$

From the relationships:

$$\sinh\tfrac{1}{2}(x+y)\cdot\sinh\tfrac{1}{2}(x-y) = \tfrac{1}{2}\left[\cosh x - \cosh y\right]$$

$$\cosh(x \pm iy) = \cosh x\cdot\cos y \pm i\,\sinh x\cdot\sin y$$

eq. (E.43) reduces to:

$$\text{(E.44)}\quad T = \frac{1}{4}\sum_{k=0}^{\infty}\ln\left[\frac{\cosh^2(2k+1)b - 2\cosh(2k+1)b\cdot\cosh(a-c)\cdot\cos d + \cosh^2(a-c) + \cos^2 d - 1}{\cosh^2(2k+1)b - 2\cosh(2k+1)b\cdot\cosh(a+c)\cdot\cos d + \cosh^2(a+c) + \cos^2 d - 1}\right]$$

Comparing eq. (E.37) and (E.38) with eq. (E.39) we get:

$$\text{(E.45)}\quad 1 \le r' \le \gamma \qquad \frac{1}{\gamma} \le r \le 1 \qquad a-c = N\cdot\ln\left(\frac{\eta}{\gamma r}\right) \qquad a+c = N\cdot\ln(\eta\gamma r) \qquad b = N\cdot\ln(\eta\gamma) \qquad d = N\theta$$

$$\text{(E.46)}\quad \gamma \le r' \le \eta\gamma \qquad 1 \le r \le \eta \qquad a-c = N\cdot\ln\left(\frac{\gamma r}{\eta}\right) \qquad a+c = N\cdot\ln\left(\frac{\eta\gamma}{r}\right) \qquad b = N\cdot\ln(\eta\gamma) \qquad d = N\theta$$

where r is the dimensionless radius with respect to R_c whereas r' is dimensionless with respect to R_2, i.e, $r' = \gamma r$. Thus eq. (E.37)-(E.38) become:

$$\text{(E.47)}\quad \underline{\frac{1}{\gamma} \le r \le 1} \qquad P^2 = 1 + \frac{C}{4\pi}\left\{2N\frac{\beta}{1+\beta}\ln(\gamma r) + 4T\right\}$$

$$\text{(E.48)}\quad \underline{1 \le r \le \eta} \qquad P^2 = 1 + \frac{C}{4\pi}\left\{2N\frac{1}{1+\beta}\ln\left(\frac{\eta}{r}\right) + 4T\right\}$$

where T is determined from eq. (E.44) in combination with eq. (E.45) and (E.46).

APPENDIX F

Effect of Eccentricity Ratio on Load Carrying Capacity of Hydrostatic Journal Bearing

The analyses of the hydrostatic journal bearing in Appendix B and C are limited to small values of the eccentricity ratio and predict a linear relationship between load and eccentricity ratio. The present appendix is concerned with an approximate analysis of the journal bearing where the eccentricity ratio is not small. If it is assumed that the orifice feeding may be represented by line feeding as in Appendix B, the Reynolds equation outside the feeding planes is given by eq. (A.14) without local feeding:

(F.1) $$\frac{\partial}{\partial\theta}\left[h^3\frac{\partial P^2}{\partial\theta}\right]+\frac{\partial}{\partial\zeta}\left[h^3\frac{\partial P^2}{\partial\zeta}\right]=0$$

where $h=1+\varepsilon\cos\theta$. If it is assumed that the bearing is short so that $\frac{\partial P^2}{\partial\zeta}\gg\frac{\partial P^2}{\partial\theta}$, eq. (F.1) can be approximated by:

(F.2) $$\frac{\partial}{\partial\zeta}\left[(1+\varepsilon\cos\theta)^3\frac{\partial P^2}{\partial\zeta}\right]=0$$

with the solution (see Fig. 3, Appendix B, and eq. (B.7) and (B.8)):

(F.3) $\underline{0\le\zeta\le\xi}$ $$P^2=1+f(\theta)\cdot\left(1-\frac{\zeta}{\xi}\right)$$

(F.4) $\underline{0\le\zeta'\le\eta}$ $$P^2=1+f(\theta)$$

where $f(\theta)$ is a function of θ only. A similar approximation has been employed successfully for the hydrodynamic bearing, see Ref. 6.

Eq. (F.3) satisfies the boundary condition of ambient pressure at the end of the bearing, i.e., $P=1$ for $\zeta=\xi$. At $\zeta=0$ the bearing flow must equal the orifice flow as given by eq. (B.9):

(F.5) $$(1+\varepsilon\cos\theta)^3\left[-\frac{\partial P^2}{\partial\zeta}\Big|_{\zeta=0}+\frac{\partial P^2}{\partial\zeta'}\Big|_{\zeta'=\eta}\right]=\Lambda_t V m$$

Substitute eq. (F.3) and (F.4) into eq. (F.5):

(F.6) $$(1+\varepsilon\cos\theta)^3 \cdot f = \xi \Lambda_t V m$$

If m is replaced by the approximate orifice flow equation eq. (A.27):

$$m = \sqrt{1-\left(\frac{P_i}{V}\right)^2}$$

where in the present case $P_i^2 = 1 + f(\theta)$ from eq. (F.3) we get:

$$(1+\varepsilon\cos\theta)^3 \cdot f = \xi \Lambda_t \sqrt{V^2-1-f}$$

or

(F.7) $$f(\theta) = \frac{1}{2}\frac{\xi^2\Lambda_t^2}{(1+\varepsilon\cos\theta)^6}\left[-1+\sqrt{1+\frac{4(V^2-1)}{\xi^2\Lambda_t^2}(1+\varepsilon\cos\theta)^6}\right]$$

The load carrying capacity W is found by integrating the pressure:

$$W = -4P_a R^2 \int_0^{\pi}\left[\int_0^{\xi} P\,d\zeta + \int_0^{\eta} P\,d\zeta'\right]\cos\theta\,d\theta$$

Using eq. (F.3) and (F.4) we get:

(F.8) $$\frac{W}{P_a D(L+L_1)} = -\frac{\xi}{\xi+\eta}\cdot\frac{2}{3}\int_0^{\pi_1}\frac{(1+f)^{3/2}-1}{f}\cos\theta\,d\theta - \frac{\eta}{\xi+\eta}\int_0^{\pi}\sqrt{1+f}\,\cos\theta\,d\theta$$

The integration is carried out numerically.

The total mass flow is given by:

(F.9) $$M_T = \frac{P_a^2 C^3}{12\mu RT}\,\frac{1}{\xi}\int_0^{\pi} f\cdot(1+\varepsilon\cos\theta)^3\,d\theta$$

APPENDIX G

The Dynamic Load Carrying Capacity of the Hydrostatic Journal Bearing with Orifice Restricted Line Feeding

When the journal performs harmonic vibrations, pressure will be generated in the gas film due to the compressibility of the gas as indicated in Appendix A. The present appendix is concerned with an analysis of this dynamic load when the eccentricity ratio and the vibration amplitude is small.

The governing equation is Reynolds equation (A.13) and the bearing configuration is shown in Fig. 3, Appendix B. If it is assumed that the journal perform harmonic vibrations with frequency ω such that the motion is either purely translatory or a rotation around a transverse axis, then outside and between the feeding planes eq. (A.13) becomes:

(G.1) $$\frac{\partial}{\partial\theta}\left[h^3\frac{\partial P^2}{\partial\theta}\right]+\frac{\partial}{\partial\zeta}\left[h^3\frac{\partial P^2}{\partial\zeta}\right]=2\sigma\frac{\partial(Ph)}{\partial\tau}$$

where σ is the frequency number:

(G.2) $$\sigma=\frac{12\mu\omega}{P_a}\left(\frac{R}{C}\right)^2$$

When the journal performs translatory motion with amplitude $Re\{C\varepsilon e^{i\omega t}\}$ around the concentric position the dimensionless film thickness is:

(G.3) $$h=Re\{1+\varepsilon e^{i\tau}\cos\theta\}$$

Expanding the pressure in a power series of the amplitude and assuming $\varepsilon \ll 1$ so that only the two first terms are retained, we get:

(G.4) $$P=P_0+Re\{\varepsilon e^{i\tau}P_1\}$$

(G.5) $$P^2=P_0^2+Re\{2\varepsilon e^{i\tau}P_0P_1\}$$

Also:

(G.6) $$h^3=1+Re\{3\varepsilon e^{i\tau}\cos\theta\}$$

Set:

(G.7) $$P_1 = H(\zeta)\cos\theta$$

and substitute eq (G.4)-(G.6) into eq. (G.1) to get:

(G.8) $$\frac{\partial P_0^2}{\partial \theta^2} + \frac{\partial P_0^2}{\partial \zeta^2} = 0$$

where $\frac{\partial P_0}{\partial \theta} = 0$ because of symmetry

(G.9) $$\frac{d^2H}{d\zeta^2} - \left(1 + \frac{i\sigma}{P_0}\right)H = i\sigma P_0$$

Referring to Fig. 3, Appendix B, the solution of eq. (G.8) is:

(G.10) $\underline{0 \le \zeta \le \xi}$ $$P_0^2 = 1 + q(\xi + \zeta)$$

(G.11) $\underline{0 \le \zeta' \le \eta}$ $$P_0^2 = 1 + q\xi$$

At $\zeta = 0$ the bearing flow must equal the orifice flow as expressed by eq. (B.9):

(G.12) $$\left(1 + \varepsilon e^{i\tau}\cos\theta\right)^3 \left[-\frac{\partial P^2}{\partial \zeta}\bigg|_{\zeta=0} + \frac{\partial P^2}{\partial \zeta'}\bigg|_{\zeta'=\eta} \right] = \Lambda_t V m$$

Insert eq. (G.4)-(G.7) and expend m as in eq. (B.10) to get:

(G.13) $$-\frac{\partial P_0^2}{\partial \zeta}\bigg|_{\zeta=0} + \frac{\partial P_0^2}{\partial \zeta'}\bigg|_{\zeta'=\eta} = \Lambda_t V m_0$$

(G.14) $$\frac{dH}{d\zeta}\bigg|_{\zeta=0} - \frac{dH}{d\zeta'}\bigg|_{\zeta'=\eta} = \frac{3}{2}\Lambda_t V m_0 - \frac{\Lambda_t m_0'}{2P_{0i}} H_i$$

Eq. (G.13) gives:

(G.15) $$q = \Lambda_t V m_0$$

For $0 \le \zeta' \le \eta$ eq. (G.9) may be solved since P_0 is constant, see eq. (G.11). From symmetry $\frac{dH}{d\zeta'}\Big|_{\zeta'=0} = 0$ and therefore the solution of eq. (G.9) is:

(G.16) $\underline{0 \le \zeta' \le \eta}$ $$H = \left[H_i + \frac{i\sigma P_0}{1 + \frac{i\sigma}{P_0}}\right] \frac{\cosh \zeta'(\alpha + i\beta)}{\cosh \eta(\alpha + i\beta)} - \frac{i\sigma P_0}{1 + \frac{i\sigma}{P_0}}$$

where $H_i = H_{\zeta'=\eta}$ and:

(G.17) $$\alpha + i\beta = \sqrt{1 + \frac{i\sigma}{P_{0i}}}$$

i.e.

(G.18) $$\alpha = \sqrt{\frac{1}{2}\left[1+\sqrt{1+\left(\frac{\sigma}{P_{oi}}\right)^2}\right]}$$

(G.19) $$\beta = \sqrt{\frac{1}{2}\left[-1+\sqrt{1+\left(\frac{\sigma}{P_{oi}}\right)^2}\right]}$$

Substituting eq. (G.16) and (G.15) into eq. (G.14) gives:

(G.20) $$\left.\frac{dH}{d\zeta}\right|_{\zeta=0} = \frac{3}{2}q + \frac{i\sigma P_{oi}}{1+\frac{i\sigma}{P_{oi}}}(\alpha+i\beta)\tanh\eta(\alpha+i\beta) + \left[(\alpha+i\beta)\tanh\eta(\alpha+i\beta) - \frac{\Lambda_t m_o'}{2P_{oi}}\right]H_i$$

where:

(G.21) $$P_{oi} = \sqrt{1+q\xi}$$

Thus eq. (G.9) should be solved in $0 \leq \zeta \leq \xi$ with eq. (G.20) as one boundary condition and $H(\xi)=0$ as the other boundary condition. The solution is obtained numerically as shown later, but first the equations for rotational vibrations will be derived.

When the journal performs rotational motion with the angular displacement $Re\{\alpha e^{i\omega t}\}$ around a transverse axis perpendicular to the journal axis at $\zeta'=0$ (see Fig. 3, Appendix B) the dimensionless film thickness becomes:

(G.22) $$h = 1+\left(\frac{R}{C}\right)\zeta'\alpha e^{i\tau}\cos\theta$$

Proceeding as in the translatory case we get:

(G.23) $$P = P_0 + Re\{\alpha e^{i\tau}P_1\}$$

(G.24) $$P^2 = P_0^2 + Re\{2\alpha e^{i\tau}P_0P_1\}$$

(G.25) $$h^3 = 1 + Re\left\{3\left(\frac{R}{C}\right)\alpha e^{i\tau}\zeta'\cos\theta\right\}$$

Set:

(G.26) $$P_0P_1 = \left(\frac{R}{C}\right)H(\zeta)\cdot\cos\theta$$

so that eq. (G.1) becomes in addition to eq. (G.8):

(G.27) $$\frac{d^2H}{d\zeta^2} - \left(1+\frac{i\sigma}{P_0}\right)H = -\frac{3}{2}\frac{\partial P_0^2}{\partial\zeta} + i\sigma P_0\zeta'$$

Since $\frac{\partial P_0^2}{\partial\theta}=0$, P_0 is given by eq. (G.10) and (G.11). As P_0 is

constant for $0 \leq \zeta' \leq \eta$ eq. (G.27) may be solved between the feeding planes. From symmetry $H_{\zeta'=0} = 0$ so that eq. (G.27) gives:

(G.28) $\underline{0 \leq \zeta' \leq \eta}$ $$H = \left[H_i + \frac{i\sigma P_0 \eta}{1+\frac{i\sigma}{P_0}}\right] \frac{\sinh \zeta'(\alpha+i\beta)}{\sinh \eta(\alpha+i\beta)} - \frac{i\sigma P_0 \zeta'}{1+\frac{i\sigma}{P_0}}$$

where α and β are given by eq. (G.18) and (G.19). The boundary condition at $\zeta = 0$ is given analogous to eq. (G.12) by:

(G.29) $$\left(1+3\left(\frac{R}{C}\right)\eta\alpha e^{i\tau}\cos\theta\right)\left(-\frac{\partial P^2}{\partial \zeta}\bigg|_{\zeta=0} + \frac{\partial P^2}{\partial \zeta'}\bigg|_{\zeta'=\eta}\right) = \Lambda_t\left(V m_0 + \left(\frac{R}{C}\right)\alpha e^{i\tau}\frac{m_0'}{P_{0i}} H_i \cos\theta\right)$$

Introducing eq. (G.28) we get in addition to eq. (G.13) and (G.15):

(G.30) $$\frac{dH}{d\zeta}\bigg|_{\zeta=0} = \frac{3}{2} q\eta + \frac{i\sigma P_{0i}}{1+\frac{i\sigma}{P_{0i}}}\left[\eta(\alpha+i\beta)\coth\eta(\alpha+i\beta) - 1\right] + \left[(\alpha+i\beta)\coth\eta(\alpha+i\beta) - \frac{\Lambda_t m_0'}{2P_{0i}}\right]H_i$$

Hence eq. (G.27) should be solved in $0 \leq \zeta \leq \xi$ where eq. (G.30) serves as one boundary condition and $H(\xi)=0$ as the other condition.

Eq. (G.9) and (G.27) has the general form:

(G.31) $\underline{0 \leq \zeta \leq \xi}$ $$\frac{d^2H}{d\zeta^2} - \left(1+\frac{i\sigma}{P_0}\right)H = F = \begin{cases} i\sigma P_0 & \underline{\text{translatory}} \\ \frac{3}{2}q + i\sigma P_0(\zeta+\eta) & \underline{\text{rotational}} \end{cases}$$

where P_0 is given by eq. (G.10) and H and F are complex:

(G.32) $$H = h_r + i h_i$$

(G.33) $$F = f_r + i f_i$$

The boundary conditions are:

(G.34) $$H(\xi) = 0$$

(G.35) $$H_0' = \frac{dH}{d\zeta}\bigg|_{\zeta=0} = a + ib + (c+id)H_0$$

Eq. (G.31) is solved numerically. To get it in finite difference form, integrate

twice:

(G.36) $$H = \int_0^{\zeta} \left(\int_0^{x} \left[\left(1 + \frac{i\sigma}{P_0}\right) H + F \right] dy \right) dx + \zeta H_0' + H_0$$

Set:

(G.37) $$G = \left(1 + \frac{i\sigma}{P_0}\right) H + F$$

and integrate the double integral by parts to get:

(G.38) $$H = \int_0^{\zeta} (\zeta - x)\, G\, dx + \zeta H_0' + H_0$$

To write this equation in finite difference form subdivide the interval $0 \to \xi$ in m increments of length $\Delta\zeta = \frac{\xi}{m}$ and integrate by the trapezoidal rule:

(G.39) $0 \leq n \leq m-1$ $$H_{n+1} = H_n + (\Delta\zeta)^2 \left[\tfrac{1}{2} G_0 + G_1 + G_2 + \cdots + G_n \right] + \Delta\zeta \cdot H_0'$$

For computer calculations it is convenient to set:

(G.40) $$H_n = s_{rn} + i s_{in} + (t_{rn} + i t_{in}) H_0$$

(G.41) $$(\Delta\zeta)^2 \left[\tfrac{1}{2} G_0 + G_1 + G_2 + \cdots + G_n \right] = p_{rn} + i p_{in} + (q_{rn} + i q_{in}) H_0$$

Then from eq. (G.39), (G.35), (G.37) and (G.33):

(G.42) $$P_{on} = \sqrt{1 + q(m-n)\Delta\zeta}$$

(G.43) $$p_{rn} = p_{r,n-1} + (\Delta\zeta)^2 \left[s_{rn} - \frac{\sigma}{P_{on}} s_{in} + f_{rn} \right]$$

(G.44) $$p_{in} = p_{i,n-1} + (\Delta\zeta)^2 \left[s_{in} + \frac{\sigma}{P_{on}} \cdot s_{rn} + f_{in} \right]$$

(G.45) $$q_{rn} = q_{r,n-1} + (\Delta\zeta)^2 \left[t_{rn} - \frac{\sigma}{P_{on}} \cdot t_{in} \right]$$

(G.46) $$q_{in} = q_{i,n-1} + (\Delta\zeta)^2 \left[t_{in} + \frac{\sigma}{P_{on}} t_{rn} \right]$$

(G.47) $$s_{r,n+1} = s_{rn} + p_{rn} + a \cdot \Delta\zeta$$

(G.48) $$s_{i,n+1} = s_{in} + p_{in} + b \cdot \Delta\zeta$$

(G.49) $$t_{r,n+1} = t_{rn} + q_{rn} + c \cdot \Delta\zeta$$

(G.50) $$t_{i,n+1} = t_{in} + q_{in} + d \cdot \Delta\zeta$$

To initiate calculations set

(G.51) $$p_{ro} = \frac{1}{2}(\Delta\zeta)^2 f_{ro}$$

(G.52) $$p_{io} = \frac{1}{2}(\Delta\zeta)^2 f_{io}$$

(G.53) $$q_{ro} = \frac{1}{2}(\Delta\zeta)^2$$

(G.54) $$q_{io} = \frac{1}{2}(\Delta\zeta)^2 \frac{\alpha}{P_{oo}}$$

(G.55) $$s_{ro} = s_{io} = t_{io} = 0$$

(G.56) $$t_{ro} = 1$$

Thus the method of calculation is: 1) preset $p_{ro}, p_{io}, q_{ro}, \cdots, t_{ro}$ from eq. (G.51)-(G.56); 2) set $n=0$ and calculate eq. (G.47)-(G.50); 3) increase n by 1 and repeat the cycle eq. (G.42)-(G.50) to $n=m-1$, where the boundary condition eq. (G.34) gives:

$$H_m = s_{rm} + i s_{im} + (t_{rm} + i t_{im}) H_o = 0$$

or:

(G.57) $$H_o = -\frac{s_{rm} + i s_{im}}{t_{rm} + i t_{im}} = h_{ro} + i h_{io}$$

Having determined H_o we may find $H_n = h_{rn} + i h_{in}$ from eq. (G.40). The pressure is calculated from eq. (G.7) and (G.4) or eq. (G.26) and (G.23):

(G.58) <u>translatory</u> $$P_n = P_{on} + \frac{\varepsilon}{P_{on}}(h_{rn}\cos\omega t - h_{in}\sin\omega t)\cos\theta$$

(G.59) <u>rotational</u> $$P_n = P_{on} + \left(\frac{R}{C}\right)\frac{\alpha}{P_{on}}(h_{rn}\cos\omega t - h_{in}\sin\omega t)\cos\theta$$

The dynamic load carrying capacity W_D and the restoring moment M_D are obtained by integration:

(G.60) <u>translatory</u> $$W_D = -2P_a R^2 \int_0^{2\pi}\left[\int_0^{\xi}\left(P_o + \varepsilon e^{i\tau}\frac{H}{P_o}\cos\theta\right)d\zeta + \int_0^{\eta}\left(P_o + \varepsilon e^{i\tau}\frac{H}{P_o}\cos\theta\right)d\zeta'\right]\cos\theta\, d\theta$$

(G.61) <u>rotational</u> $$M_D = -2P_a R^3 \int_0^{2\pi}\left[\int_0^{\xi}(\zeta+\eta)\left(P_o + \left(\frac{R}{C}\right)\alpha e^{i\tau}\frac{H}{P_o}\cos\theta\right)d\zeta + \int_0^{\eta}\zeta'\left(P_o + \left(\frac{R}{C}\right)\alpha e^{i\tau}\frac{H}{P_o}\cos\theta\right)d\zeta'\right]\cos\theta\, d\theta$$

In dimensionles form:

$$\text{(G.62)}\quad \frac{W_D}{P_a D(L+L_1)\varepsilon} = \frac{\pi}{2(\xi+\eta)}\left\{-\cos\omega t\left[\int_0^{\xi}\frac{h_{rn}}{P_{on}}d\zeta + \int_0^{\eta}\frac{h_r}{P_o}d\zeta'\right] + \sin\omega t\left[\int_0^{\xi}\frac{h_{in}}{P_{on}}d\zeta + \int_0^{\eta}\frac{h_i}{P_o}d\zeta'\right]\right\} = W_D'\cos(\omega t-\varphi_W)$$

$$\text{(G.63)}\quad \frac{M_D}{P_a D^2(L+L_1)(\frac{R}{c})\alpha} = \frac{\pi}{4(\xi+\eta)}\left\{-\cos\omega t\left[\int_0^{\xi}(\zeta+\eta)\frac{h_{rn}}{P_{on}}d\zeta + \int_0^{\eta}\zeta'\frac{h_r}{P_o}d\zeta'\right] + \sin\omega t\left[\int_0^{\xi}(\zeta+\eta)\frac{h_{in}}{P_{on}}d\zeta + \int_0^{\eta}\zeta'\frac{h_i}{P_o}d\zeta'\right]\right\} = M_D'\cos(\omega t-\varphi_M)$$

The integrals $\int_0^{\xi} d\zeta$ are calculated numerically from the above obtained values of H_n. The integrals $\int_0^{\eta} d\zeta'$ can be evaluated from eq. (G.16) and (C.28):

$$\text{(G.64)}\quad \underline{\text{translatory}}\qquad -\frac{e^{i\omega t}}{P_o}\int_0^{\eta} H\,d\zeta' = -\frac{\cos\omega t + i\sin\omega t}{P_o + i\sigma}\left\{\left(H_o + \frac{i\sigma P_o}{1+\frac{i\sigma}{P_o}}\right)(\alpha+i\beta)\tanh\eta(\alpha+i\beta) - i\sigma P_o\eta\right\}$$

$$\text{(G.65)}\quad \underline{\text{rotational}}\qquad -\frac{e^{i\omega t}}{P_o}\int_0^{\eta}\zeta' H\,d\zeta' = -\frac{\cos\omega t + i\sin\omega t}{P_o + i\sigma}\left\{\left(H_o + \frac{i\sigma P_o\eta}{1+\frac{i\sigma}{P_o}}\right)\left(\eta(\alpha+i\beta)\coth\eta(\alpha+i\beta) - 1\right) - i\tfrac{1}{3}\sigma P_o\eta^3\right\}$$

where:

$$\text{(G.66)}\quad (\alpha+i\beta)^2 = 1 + \frac{i\sigma}{P_{oi}} \qquad \text{(see eq.(G.17))}$$

$$\text{(G.67)}\quad (\alpha+i\beta)\tanh\eta(\alpha+i\beta) = \frac{\alpha\cdot\sinh(2\eta\alpha) - \beta\cdot\sin(2\eta\beta) + i(\alpha\sin(2\eta\beta) + \beta\sinh(2\eta\alpha))}{\cosh(2\eta\alpha) + \cos(2\eta\beta)}$$

$$\text{(G.68)}\quad (\alpha+i\beta)\coth\eta(\alpha+i\beta) = \frac{\alpha\sinh(2\eta\alpha) + \beta\sin(2\eta\beta) - i(\alpha\sin(2\eta\beta) - \beta\sinh(2\eta\alpha))}{\cosh(2\eta\alpha) - \cos(2\eta\beta)}$$

$$\text{(G.69)}\quad \frac{i\sigma P_{oi}}{1+\frac{i\sigma}{P_{oi}}} = \frac{\sigma^2 + i\sigma P_{oi}}{1+\left(\frac{\sigma}{P_{oi}}\right)^2}$$

APPENDIX H

The Dynamic Load Carrying Capacity of the Hydrostatic Thrust Bearing with Orifice Restricted Line Feeding

When one face of the thrust bearing performs harmonic vibrations, pressure will be generated in the gas film due to the compressibility of the gas. The present appendix is concerned with an analysis of this dynamic load when the vibration amplitude is small.

The governing equation is Reynolds equation (A.20) and the bearing geometry is shown in Fig. 5, Appendix B. If it is assumed that the vibrational motion with frequency ω is either purely translatory or a rotation around a diameter, then outside the feeding planes eq. (A.20) becomes:

(H.1) $$\frac{1}{r}\frac{\partial}{\partial r}\left[rh^3\frac{\partial P^2}{\partial r}\right]+\frac{\partial}{r\partial\theta}\left[h^3\frac{\partial P^2}{r\partial\theta}\right]=2\sigma\frac{\partial(Ph)}{\partial\tau}$$

where σ is the frequency number:

$$\sigma=\frac{12\mu\omega}{P_a}\left(\frac{R_c}{C}\right)^2$$

and R_c is the radius of the orifice circle.

Let the vibration be purely translatory with amplitude $Re\{C\varepsilon e^{i\omega t}\}$ where C denotes the equilibrium film thickness. Then the dimensionless film thickness is:

(H.2) $$h=1+Re\{\varepsilon e^{i\tau}\}$$

Performing a first order pertubation in ε we get:

(H.3) $$P=P_0+Re\{\varepsilon e^{i\tau}P_1\}$$

(H.4) $$P^2=P_0^2+Re\{2\varepsilon e^{i\tau}P_0P_1\}$$

(H.5) $$h^3=1+3\varepsilon e^{i\tau}$$

Set:

(H.6) $$P_0P_1=H(r)$$

and substitute eq. (H.2)-(H.6) into eq. (H.1) to get:

(H.7) $$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial P_0^2}{\partial r}\right] + \frac{\partial^2 P_0^2}{r^2 \partial\theta^2} = 0$$

(H.8) $$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial H}{\partial r}\right] - \frac{i\sigma}{P_0} H = i\kappa P_0$$

Since $\frac{\partial P_0^2}{\partial\theta} = 0$ because of symmetry eq. (H.7) gives:

(H.9) $\underline{1 \le r \le \eta}$ $\quad P_0^2 = 1 + q_T \ln\left(\frac{\eta}{r}\right)$

(H.10) $\underline{\frac{1}{\gamma} \le r \le 1}$ $\quad P_0^2 = 1 + \beta q_T \ln(\gamma r)$

where $r = \frac{R}{R_i}$ and η, γ and β are defined in eq. (B.30)-(B.32). At $r=1$ the bearing flow must equal the orifice flow:

(H.11) $$(1+3\varepsilon e^{i\tau})\left(-\frac{\partial P^2}{\partial r}\Big|_{r=1+0} + \frac{\partial P^2}{\partial r}\Big|_{r=1-0}\right) = (1+\beta)\Lambda_T V m = (1+\beta)\Lambda_T\left(V m_0 + \frac{m_0'}{P_{0i}}\varepsilon e^{i\tau} H_{r=1}\right)$$

where Λ_T is as defined in eq. (B.36):

(H.12) $$\Lambda_T = \frac{12\mu N \alpha a^2 \sqrt{RT}}{P_a C^3 (1+\beta)}$$

Hence:

(H.13) $$q_T = \Lambda_T V m_0$$

(H.14) $$\frac{\partial H}{\partial r}\Big|_{r=1+0} - \frac{\partial H}{\partial r}\Big|_{r=1-0} = (1+\beta)\left(\frac{3}{2} q_T - \frac{\Lambda_T m_0'}{2 P_{0i}} H_i\right)$$

Thus eq. (H.8) is to be solved with eq. (H.14) as one boundary condition and

(H.15) $$H\left(\frac{1}{\gamma}\right) = H(\eta) = 0$$

as the other boundary condition. This will be done numerically as shown later. If the vibration is pure rotational with angular displacement $Re\{\alpha e^{i\omega t}\}$ around a diameter, the dimensionless film thickness becomes:

(H.16) $$h = 1 + Re\left\{\left(\frac{R_i}{C}\right)\alpha e^{i\tau} r \sin\theta\right\}$$

A first order pertubation gives:

(H.17) $$P = P_0 + Re\{\alpha e^{i\tau} P_1\}$$

(H.18) $$P^2 = P_0^2 + Re\{2\alpha e^{i\tau} P_0 P_1\}$$

(H.19) $$h^3 = 1 + 3\left(\frac{R_c}{C}\right)\alpha e^{i\tau} r \sin\theta$$

Set:

(H.20) $$P_0 P_1 = \left(\frac{R_c}{C}\right) H(r) \sin\theta$$

and substitute eq. (H.16)-(H.20) into eq. (H.1) to get in addition to eq. (H.7):

(H.21) $$\frac{1}{r}\frac{d}{dr}\left[r\frac{dH}{dr}\right] - \left(\frac{1}{r^2} + \frac{i\sigma}{P_0}\right)H = -\frac{3}{2}\frac{\partial P_0^2}{\partial r} + i\sigma P_0 r$$

where P_0 is given by eq. (H.9)-(H.10). At $r=1$ the boundary condition is given by eq. (H.14) and the other boundary condition is eq. (H.15). Eq. (H.8) and (H.21) are solved numerically. They have the general form:

(H.22) $$\frac{1}{r}\frac{d}{dr}\left[r\frac{dH}{dr}\right] - EH = F$$

where:

(H.23) $$E = e_r + ie_i = \begin{cases} \frac{i\sigma}{P_0} \\ \frac{1}{r^2} + \frac{i\sigma}{P_0} \end{cases}$$

(H.24) $$F = f_r + if_i = \begin{cases} i\sigma P_0 \\ -\frac{3}{2}\frac{\partial P_0^2}{\partial r} + i\sigma P_0 r \end{cases}$$

(H.25) $$H = h_r + ih_i$$

The boundary conditions are:

$$H(\eta) = H\left(\frac{1}{\gamma}\right) = 0$$

(H.26) $$\left.\frac{dH}{dr}\right|_{r=1+0} - \left.\frac{dH}{dr}\right|_{r=1-0} = a + bH_i$$

where:

(H.27) $$a = \frac{3}{2}(1+\beta)q_T$$

(H.28) $$b = -(1+\beta)\frac{\Lambda_T m_0'}{2P_{0i}}$$

To get eq. (H.22) into finite difference form, integrate twice:

$$H = \int_1^r \frac{1}{x}\left(\int_1^x y(EH+F)\,dy\right)dx + \ln r \cdot H_i' + H_i$$

where $H_i' = \frac{dH}{dr}\Big|_{r=1}$ and $H_i = H_{r=1}$. Set:

$$G = EH + F$$

and integrate the double integral by parts to get:

(H.29) $$H = \int_1^r x \cdot \ln\left(\frac{r}{x}\right) G\,dx + \ln r \cdot H_i' + H_i$$

Set

(H.30) $$H_0' = \frac{dH}{dr}\Big|_{r=1+0}$$

(H.31) $$H_0 = H_i = H_{r=1}$$

Then eq. (H.29) by use of eq. (H.26) becomes:

(H.32) $\underline{1 \le r \le \eta}$ $$H = \int_1^r x \ln\left(\frac{r}{x}\right) \cdot G\,dx + \ln r\, H_0' + H_0$$

(H.33) $\underline{\frac{1}{\gamma} \le r \le 1}$ $$H = \int_1^r x \ln\left(\frac{r}{x}\right) G\,dx - a \cdot \ln r + \ln r \cdot H_0' + (1 - b \cdot \ln r) H_0$$

Subdivide the integration interval in m increments of length

(H.34) $$\Delta r = \begin{cases} \frac{\eta - 1}{m} & \underline{1 \le r \le \eta} \\ \frac{1 - \frac{1}{\gamma}}{m} & \underline{\frac{1}{\gamma} \le r \le 1} \end{cases}$$

and integrate eq. (H.32)-(H.33) numerically by the trapezoidal rule to obtain their finite difference form:

(H.35) $\underline{1 \le r \le \eta}$ $$H_{n+1} = H_n + \ln\left(\frac{1+(n+1)\Delta r}{1+n\Delta r}\right) \Delta r\left[\tfrac{1}{2}G_0 + (1+\Delta r)G_1 + \cdots + (1+n\Delta r)G_n\right] + H_0' \ln\left(\frac{1+(n+1)\Delta r}{1+n\Delta r}\right)$$

(H.36) $\underline{1 \ge r \ge \frac{1}{\gamma}}$ $$H_{n+1} = H_n + \ln\left(\frac{1-(n+1)\Delta r}{1+n\Delta r}\right) \Delta r\left[\tfrac{1}{2}G_0 + (1-\Delta r)G_1 + \cdots + (1-n\Delta r)G_n\right]$$
$$+ (-a + H_0' - b H_0) \cdot \ln\left(\frac{1-(n+1)\Delta r}{1-n\Delta r}\right)$$

To calculate eq. (H.35) on the computer set:

(H.37) $$H_n = s_{rn} + i s_{in} + (t_{rn} + i t_{in}) H_0' + (v_{rn} + i v_{in}) H_0$$

(H.38) $$\Delta r\left[\tfrac{1}{2}G_0 + (1+\Delta r)G_1 + \text{----} + (1+n\Delta r)G_n\right] = p_{rn} + i p_{in} + (q_{rn} + i q_{in}) H_0' + (w_{rn} + i w_{in}) H_0$$

(H.39) $$G_n = E_n H_n + F_n = e_{rn} s_{rn} - e_{in} s_{in} + f_{rn} + i(e_{rn} s_{in} + e_{in} s_{rn} + f_{in}) + \left[e_{rn} t_{rn} - e_{in} t_{in} + i(e_{rn} t_{in} + e_{in} t_{rn})\right] H_0' + \left[e_{rn} v_{rn} - e_{in} v_{in} + i(e_{rn} v_{in} + e_{in} v_{rn})\right] H_0$$

Therefore:

(H.40 $$P_{on} = \sqrt{1 + q_r \ln\left(\frac{\eta}{1+n\Delta r}\right)}$$

(H.41) $e_{rn} = 0$ (translatory) $\left(\frac{1}{1+n\Delta r}\right)^2$ (rotational)

(H.42) $e_{in} = \frac{\sigma}{P_{on}}$ (——"——) $\frac{\sigma}{P_{on}}$ (——"——)

(H.43) $f_{rn} = 0$ (——"——) $\frac{3}{2}\frac{q_r}{(1+n\Delta r)}$ (——"——)

(H.44) $f_{in} = \sigma P_{on}$ (——"——) $\sigma P_{on}(1+n\Delta r)$ (——"——)

(H.45) $$p_{rn} = p_{r,n-1} + \Delta r(1+n\Delta r)(e_{rn} s_{rn} - e_{in} s_{in} + f_{rn})$$

(H.46) $$p_{in} = p_{i,n-1} + \Delta r(1+n\Delta r)(e_{rn} s_{in} + e_{in} s_{rn} + f_{in})$$

(H.47) $$q_{rn} = q_{r,n-1} + \Delta r(1+n\Delta r)(e_{rn} t_{rn} - e_{in} t_{in})$$

(H.48) $$q_{in} = q_{i,n-1} + \Delta r(1+n\Delta r)(e_{rn} t_{in} + e_{in} t_{rn})$$

(H.49) $$w_{rn} = w_{r,n-1} + \Delta r(1+n\Delta r)(e_{rn} v_{rn} - e_{in} v_{in})$$

(H.50) $$w_{in} = w_{i,n-1} + \Delta r(1+n\Delta r)(e_{rn} v_{in} + e_{in} v_{rn})$$

(H.51) $$s_{r,n+1} = s_{rn} + \ln\left(\frac{1+(n+1)\Delta r}{1+n\Delta r}\right) p_{rn}$$

(H.52) $$s_{i,n+1} = s_{in} + \ln\left(\frac{1+(n+1)\Delta r}{1+n\Delta r}\right) p_{in}$$

(H.53) $$t_{r,n+1} = t_{rn} + \ln\left(\frac{1+(n+1)\Delta r}{1+n\Delta r}\right)(1+q_{rn})$$

(H.54) $$t_{i,n+1} = t_{in} + \ln\left(\frac{1+(n+1)\Delta r}{1+n\Delta r}\right) q_{in}$$

(H.55) $$v_{r,n+1} = v_{rn} + \ln\left(\frac{1+(n+1)\Delta r}{1+n\Delta r}\right) w_{rn}$$

(H.56) $$v_{i,n+1} = v_{in} + \ln\left(\frac{1+(n+1)\Delta r}{1+n\Delta r}\right) w_{in}$$

To initiate the calculations we have:

(H.57) $$s_{ro} = s_{io} = t_{ro} = t_{io} = v_{io} = 0$$

(H.58) $$v_{ro} = 1$$

(H.59) $$p_{ro} = \frac{1}{2}\Delta r f_{ro}$$

(H.60) $$p_{io} = \frac{1}{2}\Delta r f_{io}$$

(H.61) $$q_{ro} = q_{io} = 0$$

(H.62) $$w_{ro} = \frac{1}{2}\Delta r e_{ro}$$

(H.63) $$w_{io} = \frac{1}{2}\Delta r e_{io}$$

The calculations proceed as follows: 1) preset $s_{ro}, s_{io}, t_{ro}, \text{----} w_{io}$ from eq. (H.57)-(H.63); 2) set $n=0$ and calculate eq. (H.51)-(H.56); 3) advance n by 1 and repeat the cycle eq. (H.40)-(H.56) till $n=m-1$. The test calculation results in:

(H.64) $$H_\eta = H_m = s_{r\eta} + i s_{i\eta} + (t_{r\eta} + i t_{i\eta}) H_0' + (v_{r\eta} + i v_{i\eta}) H_0 = 0$$

A similar calculation is performed for $\frac{1}{\gamma} \leq r \leq 1$ from eq. (H.36). The previous eq. (H.37)-(H.56) are used by replacing n by $-n$ and $(n+1)$ by

$-(n+1)$ and making the following changes:

(H.40a) $P_{on} = \sqrt{1+\beta q_T \ln(\gamma(1-n\Delta r))}$

(H.43a) $f_r = 0$ (translatory) $\quad -\frac{3}{2}\frac{\beta q_T}{1-n\Delta r}$ (rotational)

(H.51a) $s_{r,n+1} = s_{rn} + \ln\left(\frac{1-(n+1)\Delta r}{1-n\Delta r}\right)(p_{rn}-a)$

(H.55a) $v_{r,n+1} = v_{rn} + \ln\left(\frac{1-(n+1)\Delta r}{1-n\Delta r}\right)(w_{rn}-b)$

Proceeding as above we get:

(H.65) $$H_\gamma = H_m = s_{r\gamma} + i s_{i\gamma} + (t_{r\gamma} + i t_{i\gamma}) H_0' + (v_{r\gamma} + i v_{i\gamma}) H_0$$

Solve eq. (H.64) and (H.65) together:

(H.66) $$H_0' = -\frac{(s_{r\eta}+i s_{i\eta})(v_{r\gamma}+i v_{i\gamma}) - (s_{r\gamma}+i s_{i\gamma})(v_{r\eta}+i v_{i\eta})}{(t_{r\eta}+i t_{i\eta})(v_{r\gamma}+i v_{i\gamma}) - (t_{r\gamma}+i t_{i\gamma})(v_{r\eta}+i v_{i\eta})} = h_{ro}' + i h_{io}'$$

(H.67) $$H_0 = -\frac{(t_{r\eta}+i t_{i\eta})(s_{r\gamma}+i s_{i\gamma}) - (t_{r\gamma}+i t_{i\gamma})(s_{r\eta}+i s_{i\eta})}{(t_{r\eta}+i t_{i\eta})(v_{r\gamma}+i v_{i\gamma}) - (t_{r\gamma}+i t_{i\gamma})(v_{r\eta}+i v_{i\eta})} = h_{ro} + i h_{io}$$

Having determined H_0' and H_0, we can calculate $H_n = h_{rn} + i h_{in}$ from eq. (H.37). The pressure is obtained from eq. (H.6) and (H.3) or eq. (H.20) and (H.17):

(H.68) <u>translatory</u> $\quad P_n = P_{on} + \frac{\varepsilon}{P_{on}}(h_{rn}\cdot\cos\omega t - h_{in}\cdot\sin\omega t)$

(H.69) <u>rotational</u> $\quad P_n = P_{on} + \left(\frac{R_c}{C}\right)\frac{\alpha}{P_{on}}(h_{rn}\cos\omega t - h_{in}\sin\omega t)\sin\theta$

The dynamic load carrying capacity W_D and the restoring moment M_D are obtained by integration over the thrust bearing:

(H.70) $$W_D = -P_a R_c^2 \int_0^{2\pi}\int_{\frac{1}{\gamma}}^{\eta} r P\, dr\, d\theta$$

(H.71) $$M_D = -P_a R_c^3 \int_0^{2\pi}\int_{\frac{1}{\gamma}}^{\eta} r^2 P \sin\theta\, dr\, d\theta$$

If, in calculating W_D , the static load is disregarded we get:

$$\text{(H.72)} \qquad \frac{W_D}{\pi R_1^2 P_a \varepsilon} = \frac{-2}{\eta^2}\left[\cos\omega t \int_{\frac{1}{\gamma}}^{\eta} r \frac{h_r}{P_0}\, dr - \sin\omega t \int_{\frac{1}{\gamma}}^{\eta} r \frac{h_i}{P_0}\, dr\right] = W_D' \cos(\omega t - \varphi_W)$$

$$\text{(H.73)} \qquad \frac{M_D}{\pi R_1^2 P_a R_c \left(\frac{R_c}{C}\right) \alpha} = \frac{-1}{\eta^2}\left[\cos\omega t \int_{\frac{1}{\gamma}}^{\eta} r^2 \frac{h_r}{P_0}\, dr - \sin\omega t \int_{\frac{1}{\gamma}}^{\eta} r^2 \frac{h_i}{P_0}\, dr\right] = M_D' \cos(\omega t - \varphi_M)$$

The integration is carried out numerically.

COMPUTER PROGRAM PN0020

Static Load and Flow for Journal and
Thrust Bearing with Line Source Feeding

```
C     MECHANICAL TECHNOLOGY,INC.  JORGEN W.LUND
C     PN0020=HYDROSTATIC JOURNAL AND THRUST BEARING
      DIMENSION FLAM(100),AREA(100),VLST(100),PSL(100),PAL(100),EPS(50),
     1EPS3(50),PZ(21),P1(21),GAML(100),DGAL(100),QST(50),PRST(50),
     2PR2ST(50),DUMMY(500)
  200 READ 100
      READ 101
      WRITE OUTPUT TAPE 3,102
      WRITE OUTPUT TAPE 3,103
      WRITE OUTPUT TAPE 3,100
      WRITE OUTPUT TAPE 3,101
      READ 104,LMB,NV,NJT,MIL,MDIM,NDIAG,INP,NPRT
      READ 107,PCFC
      WRITE OUTPUT TAPE 3,105
      WRITE OUTPUT TAPE 3,106,LMB,NV,NJT,MIL,MDIM,NDIAG,INP,NPRT
      WRITE OUTPUT TAPE 3,136
      WRITE OUTPUT TAPE 3,137,PCFC
      IF(MDIM) 204,201,204
  201 READ 107,(FLAM(I),I=1,LMB)
      READ 107,(VLST(I),I=1,NV)
      IF(NJT) 203,202,203
  202 READ 107,FLD,FLID
      GO TO 207
  203 READ 107,ETA,GAM
      IF(NJT) 207,202,245
  245 READ 107,(EPS(I),I=1,NJT)
      GO TO 207
  204 READ 107,VISC,RT,HN,CLRZ
      READ 107,(PSL(I),PAL(I),I=1,NV)
      READ 107,(AREA(I),I=1,LMB)
      IF(NJT) 206,205,206
  205 READ 107,BD,BL,BL1
      GO TO 207
  206 READ 107,R1,R2,RC
      IF(NJT) 207,205,245
  207 IF(MIL) 220,209,208
  208 READ 107,CB,SA,CAC,CAD
      GO TO 209
  220 IF(MIL+1) 214,247,209
  247 READ 139,ADC,LVC
      IF(LVC) 254,255,254
  254 READ 107,(GAML(I),I=2,26)
      GAMZ=1.0
      DGAMZ=0.0
      GAML(1)=1.0
      WRITE OUTPUT TAPE 3,138
      WRITE OUTPUT TAPE 3,112,(GAML(I),I=1,25)
      GO TO 256
  255 READ 107,(GAML(I),DGAL(I),I=1,NV)
  256 EX6=2.0/(ADC+1.0)
      EX1=1.0/(ADC-1.0)
      EX7=(EX6**EX1)*SQRTF(EX6*ADC)
      EX4=2.0*EX1
      EX1=ADC*EX1
      CRP=EX6**EX1
      EX3=SQRTF(2.0*EX1)
      EX2=1.0/EX1
      EX5=-EX2
      EX6=1.0/EX6
      EX1=1.0/ADC
      DIT=(1.0-CRP)/50.0
      CX=CRP
      DO 258 I=1,50
```

```
      CX=CX+DIT
      C5=CX**FX1
      DST(I)=FX3*C5*SQRTF(1.0-CX/C5)
      PR2ST(I)=CX*CX
  258 PRSI(I)=CX
C     INITIAL CONDITIONS
  209 IF(MDIM) 210,214,210
  210 DO 211 I=1,NV
  211 VLST(I)=PSL(I)/PAL(I)
      IF(NJT) 213,212,213
  212 BET1=2.0
      FLD=BL/BD
      FL1D=BL1/BD
      GO TO 217
  213 ETA=R1/RC
      GAM=RC/R2
  214 IF(NJT) 215,217,215
  215 FLETA=LOGF(ETA)
      C1=LOGF(GAM)
      BETA=FLETA/C1
      BET1=1.0+BETA
      IF(NJT) 249,217,248
  248 DO 216 I=1,NJT
      C1=1.0+EPS(I)
  216 EPS3(I)=C1**3
      IF(MDIM) 218,300,218
  217 C1=EXPF(FLD)
      C2=1.0/C1
      SHKS=(C1-C2)/2.0
      CHKS=(C1+C2)/2.0
      IF(FL1D) 276,277,276
  276 C1=-2.0*FL1D
      C1=EXPF(C1)
      TAH=(1.0-C1)/(1.0+C1)
      THET=TAH/FL1D
      GO TO 249
  277 TAH=0.0
      THET=1.0
  249 EPS3(1)=1.0
      EPS(1)=0.0
      IF(MDIM) 218,300,218
  218 C1=12.0*VISC
      C1=EFT1/C1
      C2=SQRTF(386.069*RT)
      CLAM=HN*C2/C1
      CFLWZ=3.1415927*C1/RT
C     INITIATE LAMBDA AND V
  300 DO 431 I=1,LMB
      IF(MIL+1) 251,252,252
  251 PL=FLAM(I)
      FLM=PL
      IF(NJT) 354,371,354
  252 IF(MDIM) 275,301,274
  301 FLM=FLAM(I)
      GO TO 303
  274 AR=AREA(I)
      CLR=CLRZ
      GO TO 302
  275 AR=CLRZ
      CLR=AREA(I)
  302 FLMT=CLR**3
      CFLW=FLMT*CFLWZ
      FLM=CLAM*AR/FLMT
```

```
303 DO 431 J=1,NV
    V=VLST(J)
    V2=V*V
    VM1=V2-1.0
    IF(MDIM) 304,305,304
304 PA=PAL(J)
    PA2=PA*PA
    FLM=FLMT/PA
    FLFCT=CFLW*PA2
    CACP=CAC*PA
    CADP=CAD/PA
    GO TO 241
305 FLM=FLMT
241 IF(MIL+1) 306,242,306
242 IF(LVC) 306,257,306
257 GAMZ=GAML(J)
    DGAMZ=DGAL(J)
    IF(MDIM) 243,306,243
243 FLM=FLM*GAMZ
C   P2 CALCULATION
306 K=1
307 FLME=FLM/EPS3(K)
    FLFE=FLFCT*EPS3(K)
    IF(NJT) 309,308,309
308 P2LD=FLD
    GO TO 310
309 P2LD=FLETA
310 IF(MIL) 221,311,312
311 C1=FLME/2.0*P2LD
    C2=C1*C1
    C1=C1*FLME
    P2=C1*(-1.0+SQRTF(1.0+VM1/C2))
    C1=3.0*P2
    C2=FLME/2.0*FLME/P2
    GO TO 325
312 C1=FLME*P2LD
    C2=VM1
    MTST=0
    C3=V-1.0
    C4=0.01*C3
    CX=C4
    IF(NPRT) 329,313,329
329 WRITE OUTPUT TAPE 3,108
313 C5=V-CX
    C5=C5*C5-1.0-C1*CX/(CX+SA)*(CX+CB)*(CACP*C5*C5+C5+CADP)
    IF(NPRT) 314,315,314
314 WRITE OUTPUT TAPE 3,109,CX,C5
315 IF(MTST) 316,316,319
316 C6=C5*C2
    IF(C6) 318,330,317
317 CX=CX+C4
    C2=C5
    IF(CX-C3) 313,313,440
318 MTST=1
    C7=C5
    CX=CX-0.5*C4
    GO TO 313
319 C1=C4*C4
    C1=2.0*(C2+C7-2.0*C5)/C1
    C3=(C7-C2)/C4
    IF(C1) 321,320,321
320 C6=-C5/C3
    GO TO 324
```

```
321 C4=0.5*C3/C1
    C6=SQRTF(C4*C4-C5/C1)
    IF(C4) 322,323,323
322 C6=-C6
323 C6=C6-C4
324 CX=CX+C6
330 C6=V-CX
    P2=(C6*C6-1.0)/P2LD
    C5=CX+SA
    C2=CX*(CX+CB)/C5
    C3=CACP*C6*C6+C6+CADP
    C1=3.0*FLME*C2*C3
    C2=0.5*FLME/C6*((1.0+SA/C5*(CB-SA)/C5)*C3-C2*(2.0*CACP*C6+1.0))
    GO TO 325
221 IF(MIL+1) 253,259,311
259 C1=FLME*P2LD*V
    CX=SQRTF(1.0+C1*EX7)/V
    IF(CRP-CX) 223,222,222
222 P2=FLME*V*EX7
    C1=3.0*P2
    C2=-0.5*FLME/V*EX7/CX*DGAMZ/GAM7
    GO TO 244
223 C2=V2*CRP*CRP-1.0-C1*EX7
    MTST=0
    L=2
    C4=2.0*DIT
    IF(NPRT) 224,260,224
224 WRITE OUTPUT TAPE 3,132
260 IF(LVC) 261,225,261
225 C5=V2*PR2ST(L)-1.0-C1*QST(L)
    CX=PRST(L)
    IF(NPRT) 226,227,226
226 WRITE OUTPUT TAPE 3,109,CX,C5
227 IF(MTST) 228,228,231
228 C6=C5*C2
    IF(C6) 230,237,229
229 L=L+2
    C2=C5
    IF(L-50) 225,225,441
230 MTST=1
    C7=C5
    L=L-1
    GO TO 225
261 M=2
262 VC=GAML(M)
263 C5=V2*PR2ST(L)-1.0-C1*VC*QST(L)
    CX=PRST(L)
    IF(NPRT) 264,265,264
264 WRITE OUTPUT TAPE 3,109,CX,C5
265 IF(MTST) 266,266,231
266 C6=C5*C2
    IF(C6) 268,237,267
267 L=L+2
    M=M+1
    C2=C5
    IF(L-50) 262,262,441
268 MTST=1
    C7=C5
    L=L-1
    VCM=GAML(M-1)
    VCS=VC
    DVC=(VC-VCM)/C4
    VC=(VC+VCM)/2.0
```

```
      GO TO 263
  231 C1=C4*C4
      C1=2.0(C2+C7-2.0*C5)/C1
      C3=(C7-C2)/C4
      IF(C1) 233,232,233
  232 C6=-C5/C3
      GO TO 236
  233 C4=0.5*C3/C1
      C6=SQRTF(C4*C4-C5/C1)
      IF(C4) 234,235,235
  234 C6=-C6
  235 C6=C6-C4
  236 CX=CX+C6
  237 P2=(V2*CX*CX-1.0)/P2LD
      EX2=CX**EX1
      C1=3.0*P2
      IF(LVC) 270,269,270
  269 C4=P2/FLME/V
      C5=-DGAMZ/GAMZ
      C6=0.5
      GO TO 271
  270 VC=VC+C6*DVC
      C4=VC*FLME*V
      C4=P2/C4
      C5=-DVC/VC
      C6=0.5*VC
  271 C2=C6/V*FLME/CX*(C5*C4+EX4*EX2/C4*(EX6-EX2/CX))
  244 IF(NPRT) 238,325,238
  238 WRITE OUTPUT TAPE 3,109,CX,C1
      GO TO 325
  253 C1=3.0*P2
      C2=SQRTF(1.0+P2*P2LD)
      C2=0.5*V/C2
  325 IF(NJT) 350,326,340
C     JOURNAL BEARING FORCE CALCULATION
  326 SLP=C2
      C2=C2+TAH
      C5=FLD*(CHKS+C2*PCFC*SHKS)
      PFCT=C1/C5
      C5=FLD*(CHKS+C2*SHKS)
      P1FC=C1/C5
      IF(MIL+1) 372,371,371
  372 FRC=PFCT*FINT1
      FRCM=FLD*PFCT*P1(1)*THET
      GO TO 400
C     THRUST BEARING FORCE CALCULATION
  340 C1=ETA*ETA
      C2=1.0/GAM/GAM/C1
      C3=1.0+P2*FLETA
      C4=SQRTF(C3)/C1
      C3=P2/C3
      C5=BETA*P2
      C8=(ETA-1.0)/20.0
      CX=1.0
      DO 341 L=1,21
      C7=LOGF(CX)
      C7=1.0-C3*C7
      PZ(L)=CX*SQRTF(C7)
  341 CX=CX+C8
      C9=(GAM-1.0)/20.0
      CX=1.0
      DO 342 L=1,21
      C7=LOGF(CX)
```

```
      C7=1.0+C5*C7
      P1(L)=CX*SQRTF(C7)
  342 CX=CX+C9
      C6=PZ(1)-PZ(21)
      C7=P1(1)-P1(21)
      DO 343 L=2,20,2
      C6=C6+4.0*PZ(L)+2.0*PZ(L+1)
  343 C7=C7+4.0*P1(L)+2.0*P1(L+1)
      FINT1=C6*C8/3.0
      FINT2=C7*C9/3.0
      FRC=(C4*FINT1+C2*FINT2)*2.0+C2-1.0
      FRCM=P1(21)/GAM
      GO TO 400
  350 PFCT=1.0+C2*PCFC*FLETA
      PFCT=2.0*C1/PFCT
      P1FC=1.0+C2*FLETA
      P1FC=0.5*C1/P1FC
      SLP=C2
      IF(MIL+1) 355,354,354
  355 FRC=PFCT*(FINT1+FINT2)
      FRCM=P1FC*PZ(1)*ETA
C     OUTPUT
  400 DPW=P2*LOGF(V)
      IF(NJT) 402,401,402
  401 WRITE OUTPUT TAPE 3,110,FLD,FL1D
      GO TO 403
  402 WRITE OUTPUT TAPE 3,111
      WRITE OUTPUT TAPE 3,115
      WRITE OUTPUT TAPE 3,112,EPS(K),ETA,GAM,FLETA,BETA
  403 WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,113,FLME,V,P2,FRC,FRCM,DPW
      IF(MIL) 240,405,404
  240 IF(MIL+1) 405,246,405
  246 WRITE OUTPUT TAPE 3,133
      IF(LVC) 272,273,272
  272 WRITE OUTPUT TAPE 3,122,ADC,VC,DVC
      GO TO 405
  273 WRITE OUTPUT TAPE 3,122,ADC,GAMZ,DGAMZ
      GO TO 405
  404 WRITE OUTPUT TAPE 3,116
      WRITE OUTPUT TAPE 3,117,SA,CB,CAC,CAD
  405 IF(MDIM) 406,415,406
  406 WRITE OUTPUT TAPE 3,118
      WRITE OUTPUT TAPE 3,113,PSL(J),PA,VISC,RT,HN,AR
      FLW-FLFE*P2
      PW=RT*FLFE/6600.0*DPW
      IF(NJT) 411,407,411
  407 WRITE OUTPUT TAPE 3,120
      WRITE OUTPUT TAPE 3,119,BD,BL,BL1,CLR
      CX=0.78539816*BD*PA
      C7=CX*BL1
      CX=CX*BL
      FRC=FRC*CX
      FRCM=FRCM*C7
      TFR=FRC+FRCM
      STF=TFR/CLR
      WRITE OUTPUT TAPE 3,121
      WRITE OUTPUT TAPE 3,113,FLW,PW,FRC,FRCM,TFR,STF
      CX=PA*FLD/2.0*P1FC
      C6=PA*PZ(1)
      C7=CX*P1(1)
      SLP=-2.0*PZ(1)/FLME*SLP
      CRT=PZ(1)/V
```

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      WRITE OUTPUT TAPE 3,123
      WRITE OUTPUT TAPE 3,112,C6,C7,FINT1,SLP,CRT
      IF(NDIAG) 408,410,408
  408 WRITE OUTPUT TAPE 3,124
      C6=BL/40.0
      C7=0.0
      DO 409 L=1,21
      PZ(L)=PA*PZ(L)
      P1(L)=CX*P1(L)
      WRITE OUTPUT TAPE 3,125,C7,PZ(L),P1(L)
  409 C7=C7+C6
  410 GO TO 430
  411 CX=(1.0+EPS(K))*CLR
      WRITE OUTPUT TAPE 3,126
      WRITE OUTPUT TAPE 3,112,R1,R2,RC,CLR,CX
      CX=3.1415927*PA*R1*R1
      FRC=CX*FRC
      FRCM=PA*FRCM
      IF(NJT) 424,430,423
  423 WRITE OUTPUT TAPE 3,127
      WRITE OUTPUT TAPE 3,119,FLW,PW,FRC,FRCM
      IF(NDIAG) 412,414,412
  412 WRITE OUTPUT TAPE 3,129
      C1=C4*C1*PA*RC
      C4=PA*R2
  426 C2=(R1-RC)/20.0
      C3=RC
      C5=(RC-R2)/20.0
      C6=R2
      DO 413 L=1,21
      PZ(L)=PZ(L)*C1/C3
      P1(L)=P1(L)*C4/C6
      WRITE OUTPUT TAPE 3,128,C6,P1(L),C3,PZ(L)
      C3=C3+C2
  413 C6=C6+C5
  414 GO TO 430
  424 STF=FRC/CLR
      C3=PA*SQRTF(1.0+P2*FLETA)
      WRITE OUTPUT TAPE 3,134
      WRITE OUTPUT TAPE 3,113,FLW,PW,FRC,STF,C3,FRCM
      IF(NDIAG) 425,430,425
  425 WRITE OUTPUT TAPE 3,129
      C1=P1FC*PA
      C4=R2*BETA*C1
      C1=C1*R1
      GO TO 426
  415 IF(NJT) 427,416,420
  416 WRITE OUTPUT TAPE 3,123
      CX=FLD/2.0*P1FC
      C6=P1(1)*CX
      SLP=-2.0*PZ(1)/FLME*SLP
      CRT=PZ(1)/V
      WRITE OUTPUT TAPE 3,112,PZ(1),C6,FINT1,SLP,CRT
      IF(NDIAG) 417,419,417
  417 WRITE OUTPUT TAPE 3,124
      C5=FLD/20.0
      C6=0.0
      DO 418 L=1,21
      C1=P1(L)*CX
      WRITE OUTPUT TAPE 3,125,C6,PZ(L),C1
  418 C6=C6+C5
  419 GO TO 430
  420 IF(NDIAG) 421,430,421
```

```
421 WRITE OUTPUT TAPE 3,129
    C1=C1*C4
    C2=(FTA-1.0)/20.0
    C3=1.0
    C4=(GAM-1.0)/20.0
    C5=1.0
    DO 422 L=1,21
    C6=C1*PZ(L)/C3
    C7=P1(L)/C5
    WRITE OUTPUT TAPE 3,128,C5,C7,C3,C6
    C3=C3+C2
422 C5=C5+C4
    GO TO 430
427 WRITE OUTPUT TAPE 3,135
    C4=SQRTF(1.0+P2*FLETA)
    SLP=-2.0*C4/FLME*SLP
    C5=C4/V
    WRITE OUTPUT TAPE 3,113,C4,FRCM,FINT1,FINT2,SLP,C5
    IF(NDIAG) 428,430,428
428 C1=P1FC*BETA
    C2=(FTA-1.0)/20.0
    C3=1.0
    C4=(GAM-1.0)/20.0
    C5=1.0
    WRITE OUTPUT TAPE 3,129
    DO 429 L=1,21
    C6=C3*PZ(L)*P1FC
    C7=C1/C5*P1(L)
    WRITE OUTPUT TAPE 3,128,C5,C7,C3,C6
    C3=C3+C2
429 C5=C5+C4
C   START A NEW LOOP
430 K=K+1
    IF(K-NJT) 307,307,431
431 CONTINUE
    IF(INP) 432,200,432
432 END FILE 3
    STOP 77777
440 WRITE OUTPUT TAPE 3,130
    GO TO 430
441 WRITE OUTPUT TAPE 3,131
    GO TO 430
371 CX=FLD
    C3=FLD/20.0
    DO 327 L=1,21
    C4=EXPF(CX)
    C6=0.5*(C4-1.0/C4)
    C4=SQRTF(1.0+P2*CX)
    PZ(L)=C4
    CX=CX-C3
327 P1(L)=C6/C4
    C7=P1(1)
    DO 328 L=2,20,2
328 C7=C7+4.0*P1(L)+2.0*P1(L+1)
    FINT1=C7*C3/3.0
    IF(MIL+1) 303,372,372
354 C1=GAM*ETA
    C1=BETA/C1/C1
    CX=1.0/ETA
    C8=(1.0-CX)/20.0
    C5=BETA*P2
    DO 351 L=1,21
    C7=-LOGF(CX)
```

```
      PZ(L)=CX*C7/SQRTF(1.0+P2*C7)
351   CX=CX+C8
      C9=(GAM-1.0)/20.0
      CX=1.0
      DO 352 L=1,21
      C7=LOGF(CX)
      P1(L)=CX*C7/SQRTF(1.0+C5*C7)
352   CX=CX+C9
      C6=PZ(1)-PZ(21)
      C7=P1(1)-P1(21)
      DO 353 L=2,20,2
      C6=C6+4.0*PZ(L)+2.0*PZ(L+1)
353   C7=C7+4.0*P1(L)+2.0*P1(L+1)
      FINT1=C6*C8/3.0
      FINT2=C1*C7/3.0*C9
      IF(MIL+1) 303,355,355
100   FORMAT(72H0
     1                    )
101   FORMAT(72H
     1                    )
102   FORMAT(84H1                                        MECHANICAL TECHNO
     1LOGY,INC.    JORGEN W.LUND    )
103   FORMAT(84H                                      PN0020   HYDROSTATI
     1C JOURNAL AND THRUST BEARING)
104   FORMAT(8I5)
105   FORMAT(120H0  NO.LAMBDA     NO.PRESS.RAT.  JOURN/THRUST  ORIF/MILL
     1IPOR  DIMENSIONLESS  PRESS.OUTPUT    LAST INPUT    ITERAT.PRINT  )
106   FORMAT(I8,7I15)
107   FORMAT(1P4E15.7)
108   FORMAT(30H0       V-P0            F(V-P0) )
109   FORMAT(1P2E16.7)
110   FORMAT(56H0                                      JOURNAL BEARING,L/D=
     11PE15.7)
111   FORMAT(66H0                                                THRUST BE
     1ARING      )
112   FORMAT(1P5E18.7)
113   FORMAT(1P6E18.7)
114   FORMAT(108H0        LAMBDA           PRESS.RATIO          DIM.FLOW
     1       DIM.FORCE       DIM.ORIF.PRESS.       DIM.POWER  )
115   FORMAT(90H0       FCC.RATIO        OUT.R/ORIF.R      ORIF.R/INNER R
     1    LOG(R1/RC)            BETA      )
116   FORMAT(36H0     MILLIPORE FLOW COEFFICIENTS     )
117   FORMAT(13H      COEFF.A=1PE14.7,12H    COEFF.B=1PE14.7,12H    COFFF
     1.C=1PE14.7,12H    COEFF.D=1PE14.7)
118   FORMAT(108H0    SUPPLY PRESS.       AMB.PRESSURE        VISCOSITY
     1   GAS CONST*TEMP.    NO.FEED.HOLES       FEEDER AREA )
119   FORMAT(1P4E18.7)
120   FORMAT(72H0     BRG.DIAMETER       BRG.LENGTH        FEED.PL.DIST.
     1      RAD.CLEAR. )
121   FORMAT(108H0         FLOW                POWER             END FORCE
     1    CENTER FORCE        TOTAL FORCE         STIFFNESS  )
122   FORMAT(1P3E18.7)
123   FORMAT(90H0    P0 AT ORIFICE      P1 AT ORIFICE         INTEGRAL
     1        SLOPE          ORIF.PR.RATIO)
124   FORMAT(48H0   DISTANCE           P0                  P1        )
125   FORMAT(F12.6,1P2E17.7)
126   FORMAT(90H0     OUTER RADIUS       INNER RADIUS       ORIF.RADIUS
     1      CLEARANCE         FILM THICKN.)
127   FORMAT(72H0          FLOW                POWER               FORCE
     1   ORIF.PRESSURE)
128   FORMAT(F12.6,1PE15.7,0PF12.6,1PE15.7)
129   FORMAT(54H0    RADIUS     PRESSURE         RADIUS     PRESSURE    )
130   FORMAT(42H0NO ROOT FOUND IN MILLIPORE CALCULATION    )
```

```
131 FORMAT(36H0NO ROOT FOUND IN ORIFICE ITERATION )
132 FORMAT(30H0        P0/V              F(P0/V) )
133 FORMAT(54H0      ADIAB.EXP.          AREA COEFF.      DERIV.AREA CF.)
134 FORMAT(108H0          FLOW                 POWER                FORCE
   1       STIFFNESS          P0 AT ORIFICE      P1 AT ORIFICE)
135 FORMAT(108H0     P0 AT ORIFICE      P1 AT ORIFICE      OUT.INTEGRAL
   1     INN.INTEGRAL            SLOPE            ORIF.PR.RATIO)
136 FORMAT(18H0     PR.CORR.FACT.)
137 FORMAT(1PE18.7)
138 FORMAT(32H0     VENA CONTRACTA COEFFICIENTS)
139 FORMAT(1PE15.7,I5)
    END(0,1,0,1,1)
```

COMPUTER PROGRAM PN0031

Static Load for Thrust Bearing with Point Source Feeding

```
C     CALCULATION OF PRESSURE DISTRIBUTION FOR GAS LUBRICATED GIMBEL
C     BEARING RD 3  BOB BURGESS FOR  STAN MALANOSKI
      DIMENSION  P(10,10),PSQ(10,10),A(10,10),THETA(10),RHO(10)
   10 FORMAT(3F6.3,2I2)
   20 FORMAT (8H1  C(1)=F6.3,8H   R(1)=F6.3,8H   R(2)=F6.3,
     1   5H   N=I2,8H   R(I)=F6.3,8H   WBAR=F9.5,8H   C(2)=F6.3,
     2   8H   C(3)=F7.4)
   30 FORMAT (39H0   RHO(L)          THETA(K)          P(K,L))
   40 FORMAT(F10.5,F16.5,F14.5/(9H          F16.5,F14.5))
   50 FORMAT (3I2,F5.2)
   55 FORMAT  (56H1PRESSURE DISTRIBUTION FOR GAS LUBRICATED GIMBAL BEARI
     1NG)
   60 FORMAT  (45H0       MECHANICAL TECHNOLOGY INC.,LATHAM,N.Y.)
      WRITE OUTPUT TAPE 3,55
      WRITE OUTPUT TAPE 3,60
   80 READ 10,CONE,RONE,RTWO,N,I
      IF (6-I) 160,300,85
   85 AN=N
      DTHETA = 3.14159265/(10.0*AN)
      RI=SQRTF(RONE*RTWO)
      DRHO1= .22222222*(RI-RTWO)/RI
      DRHO2= .18181818*(RONE-RI)/RI
      THETA(1)= DTHETA/2.0
      DO 100 K=2,10
  100 THETA(K)= THETA(K-1)+ DTHETA
      RHO(1)= RTWO/RI + .5*DRHO1
      DO 110 L=2,5
  110 RHO(L)= RHO(L-1)+ DRHO1
      RHO(6)= 1.0 + DRHO2
      DO 120 L=7,10
  120 RHO(L) = RHO(L-1) + DRHO2
      Z=AN * LOGF(RONE/RTWO)
      COSHZ =COSH(Z)
      COSHZ2 = COSH(.5*Z)
      E = COSHZ2 -1.0
      G = (COSHZ - COSHZ2)**2
      H= G/E
      DO 130 L=1,10
      X= AN * LOGF(RHO(L))
      COSHX = COSH(X)
      COSH2X = COSH(2.0*X)
      DO 130 K=1,10
      Y= AN * THETA(K)
      COSY = COSF(Y)
      COS2Y = COSF(2.0*Y)
      D = COSHX - COSY
      F = COSHZ**2 -2.0*COSHZ * COSHX * COSY + .5 *(COSH2X + COS2Y)
  130 A(K,L) = LOGF(H * (D/F))
      DO 140 L= 1,10
      DO 140 K= 1,10
      PSQ(K,L) = 1.0 - .5 * CONE * A(K,L)
  140 P(K,L) = SQRTF(PSQ(K,L))
      J=1
      GO TO 541
C     DO RECESS CALCULATION IF I=2,QUIT IF I=1
  150 GO TO (80,155),I
  155 READ 50,LONE,LTWO,KONE,CTWO
      IF (LTWO-LONE) 160,160,165
  160 STOP
  165 AM=LTWO-LONE-1+2*KONE
      PAVG1=0.0
      KMAX=KONE-1
      DO 170 K=1,KMAX
```

```
170 PAVG1=PAVG1+PSQ(K,LONE)+PSQ(K,LTWO)
    DO 180 L=LONE,LTWO
180 PAVG1=PAVG1+PSQ(KONE,L)
    PAVG1=PAVG1/AM
    DO 190 K=1,KMAX
    PSQ(K,LONE)=1.0-.5*CTWO*A(K,LONE)
190 PSQ(K,LTWO)=1.0-.5*CTWO*A(K,LTWO)
    DO 200   L=LONE,LTWO
200 PSQ(KONE,L)=1.0-.5*CTWO*A(KONE,L)
    PAVG2=0.0
    DO 210 K=1,KMAX
210 PAVG2=PAVG2+PSQ(K,LONE)+PSQ(K,LTWO)
    DO 220 L=LONE,LTWO
220 PAVG2=PAVG2+PSQ(KONE,L)
    PAVG2=PAVG2/AM
    CTHREE=PAVG1-PAVG2
    DO 230  L=LONE,LTWO
    DO 230  K=1,KONE
    PSQ(K,L)=1.0-.5*CTWO*A(K,L)+CTHREE
230 P(K,L)=SQRTF(PSQ(K,L))
    J=2
    GO TO 541
300 STOP
541 WBAR=0.0
    DO 550 L=1,5
    T=0.0
    DO 545 K=1,10
545 T=T+P(K,L)
550 WBAR=WBAR+RHO(L)*(T-10.0)
    WBAR=WBAR*DRHO1
    TT=0.0
    DO 570 L=6,10
    T=0.0
    DO 560 K=1,10
560 T=T+P(K,L)
570 TT=TT+RHO(L)*(T-10.0)
    WBAR=(WBAR+TT*DRHO2)/5.0
    WRITE OUTPUT TAPE 3,20,CONE,RONE,RTWO,N,RI,WBAR,CTWO,CTHREE
    WRITE OUTPUT TAPE 3,30
    DO 580 L=1,10
580 WRITE OUTPUT TAPE 3,40,RHO(L),(THETA(K),P(K,L),K=1,10)
    GO TO (150,80),J
    END (0,1,0,1,1)
    FUNCTION COSH(X)
    EX=EXPF(X)
    COSH=.5*(EX+1.0/EX)
    RETURN
    END (0,1,0,1,1)
```

COMPUTER PROGRAM PN0040

Static Load for Journal Bearing with Point Source Feeding

```
C       ANALYSIS  OF  NASA  AIR  BEARING        (MAIN  PROG)    BN0040
C       PROGRAMMED FOR  R.WERNICK  BY R.GEGUZYS     MTI ST5-5886   7-26-62
        DIMENSION  ALFA(51),ALOD(10),AMDAL(100),V(100),F(51),X(51),T(12),
       1Q2(51,61)  ,DUMMY(100),C1(200),DAM(61)  ,ZN(3),XN(4),XZ(5),CH(3),SH
       2H(3),CHP(3),SHP(3),CHM(3),SHM(3),CHN(3),SHN(3),          RVEC(50)
       3, XCOS(200),COS1(50,50),COS2(50,50),ERASE(50)  , AMAT(50,50),CCOS(
       4,50),COS1(61,3),CSIN(50),SINT(61,3),TEMP(51,61)
        COMMON PA,C,A,ZTA,AK,RT12,N,NA,NLD,NAA,NV,ALFA,ALOD,AMDAL,V,F,X,M
       1DIAG,MORCS,RS,XS,YS,COSA1,COSA2,CS,SO,T,SINHK,SINHDK,COSHK,COSHDK,
       2Z,NTH,PO2,AAR,B,CB,D,ICOUNT,E,EN,ALODI,WK,TAR,ZTAN,ALFP,ALFSP,DFL
       3,CO,   DELTH,C1,DIN,NDIN,CN,SN,S1,IS1,ZH,ZL,      RVEC,AMAT,COS1,C
       4S2,ERASE,XCOS,ZTAP,ZTAM,CCOS,COST,CSIN,SINT,TEMP
  500  READ 150, PA,C,A,ZTA,AK,RT12,DIN
C       READ  AND  PRINT INPUT
        WRITE OUTPUT TAPE 3,151
        WRITE OUTPUT TAPE 3,152,PA,C,A,ZTA,AK,RT12,DIN
        READ 153,N,NA,NLD,NAA,NV,MDIAG,MORCS,NZ,NTH,NDIN,NP
        READ 150, (ALFA(I),I=1,NA)
        WRITE OUTPUT TAPE 3,154,N,NA,NLD,NAA,NV,NZ,NTH
        WRITE OUTPUT TAPE 3,155,(ALFA(I),I=1,NA)
        READ 150, (ALOD(I),I=1,NLD)
        READ 150, (AMDAL(I),I=1,NAA)
        WRITE OUTPUT TAPE 3,156,(ALOD(I),I=1,NLD)
        WRITE OUTPUT TAPE 3,157,(AMDAL(I),I=1,NAA)
        READ 150,(V(I),I=1,NV)
        WRITE OUTPUT TAPE 3,158,(V(I),I=1,NV)
        WRITE OUTPUT TAPE 3,166,MDIAG,MORCS,NDIN,NP
        WRITE OUTPUT TAPE 3,159
C       INITIAL CALCULATIONS -- PO/V  AND F TABLE
        EX1 =AK/(AK-1.0)
        EX2 = 1.0/EX1
        EX3 = 1.0/AK
        EX4= SQRTF((AK+AK)/(AK-1.))
        ZTAP= ZTA+A
        ZTAM= ZTA-A
        EN =N
        ZTAN = ZTA*EN
        ENA = NA
        POVCR = (2.0/(AK+1.0))**EX1
        DELX  (1.0-POVCR)/ (ENA-1.0)
        NAM = NA-1
        F(1)=((2./(AK+1.))**(1./(AK-1.)))*SQRTF((AK+AK)/(AK+1.))
        X(1)= POVCR
        DO1 I=2,NAM
        X(I)=X(I-1) +DELX
        VAR = SQRTF( 1.0 -X(I)**EX2)
      1 F(I)= ALFA(I)* VAR *(X(I)**EX3)*EX4
        X(NA)= 1.0
        F(NA) =0.0
        ENZ =NZ
        ENTH=NTH
        DELTH= 6.2831853/(ENTH-1.0)
C       HERE WE SET UP THE COSINE MATRICES WHICH WILL BE USED IN THE
C       CALCULATION OF THE C1 VECTOR AND P1 MATRIX
        TAR= 0.0
        ICOUNT = 2
        E= 3.1415927/EN
        AAR= SINF(E)
        CB=AAR
        B= COSF(E)
        D=0
        NAM=4*N
        C1(1)= CB
```

```
      XCOS(1)=D
      DO 7I=2,NAM
      CALL SINCOS
      C1(I)=CB
    7 XCOS(I)=D
      DO 120 I=1,N
      DO 120 J=1,N
      K1=(I+J-1)*2
      K2=XABSF((I-J)*2)
      IF(K2)122,121,122
  121 COS2(I,J)=1.
      GO TO 120
  122 COS2(I,J)=XCOS(K2)
  120 COS1(I,J)=XCOS(K1)
C                 THE ARRAYS COS1, AND COS2, CONTAIN
C                            COS(2(I+J-1)PI/N)      AND
C                            COS(2(I-J)PI/N)   RESPECTIVELY
C
      DO 603 I= 1,N
      K2=2*I-1
      CSIN(I)= C1(K2)
  603 CCOS(I) = XCOS(K2)
C           THE ARRAYS CCOS, CSIN, CONTAIN COS, SIN OF (2I-1)*PI/( N).
      ICOUNT = 1
      TAR =0.0
      E= DELTH
      AAR = SINF(E)
      B= COSF(E)
      CB = 0.0
      D= 1.0
      COST(1,2)= D
      COST(1,3)= D
      SINT(1,2)=0.0
      SINT(1,3)=0.0
      DO 601 I=2,NTH
      CALL SINCOS
      COST( I,2)= D
  601 SINT(I,2)= CB
      TAR = 0.0
      E= FN*DELTH
      AAR= SINF(E)
      B= COSF(E)
      CB= 0.0
      D= 1.0
      ICOUNT = 1
      DO 604 I=2,NTH
      CALL SINCOS
      COST(I,3)=D
  604 SINT(I,3)= CB
C           THE ARRAYS SINT(I,J), COST(I,J) CONTAIN SIN , COS RSPCTVLY OF
C                 0.5*THETA(I) FOR J= 1
C                 THETA(I)     FOR J=2
C                 N*THETA(I)   FORJ=3 .
      IF (MDIAG) 102,101,102
  102 WRITE OUTPUT TAPE 4, 151
      WRITE OUTPUT TAPE 4,161
      WRITE OUTPUT TAPE 4,162,EX1,EX2,FX3,ZTAN,POVCR,DELX,DELTH,FX4,ZTAP
     1,ZTAM
      WRITE OUTPUT TAPE 4,163,(X(I),F(I),I=1,NA)
      WRITE OUTPUT TAPE 4,162,(RCOS(I) ,I=1,N)
C     THE L/D LOOP BEGINS HERE  -- SO IS CALCULATED IN  SUBROUTINE SOR
  101 DO 2 I=1,NLD
      RS = FN*ALOD(I)
      YS = FN*ZTA
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```
      XS = FN*(ZTA-A)
      CS = 1.0
      COSA1=CS
      COSA2=COSA1
      ALODI=ALOD(I)
      DELZ = ALODI  /(FN2-1.0)
      CALL SOR
      VAR = SO
      XS =YS
      YS = FN*(ZTA +A)
      CALL SOR
      SO =(SO +VAR)*0.5
      TESO=SO
C     SO IS GIVEN BY THE AVERAGE OF SOI AND SOII
C     THE LAMBDA(T)/ALPHA  LOOP BEGINS HERE   ENDS WITH STATEMENT 3
      DO 3 J=1,NAA
      A1= 4.0*AMDAL(J)/FN
      CRMO = A1 * F(1)
C     THE V LOOP BEGINS HERE
      DO 4 K=1,NV
      XSTR=SQRTF((TESO*CRMO+1.0/V(K))/V(K))
      IF(XSTR-POVCR) 5,5,6
C     IF XSTAR LARGER THAN PO/V CRIT GO TO 6 IF NOT GO TO 5
    5 PO = V(K)*XSTR
      ALFP =0.0
      ALFSR=1.0
      GO TO 75
C     LOOP6 USED TO FIND ROOT XSTR BETWEEN X(I) AND X(I-1)
    6 VSO = V(K)/TESO
      RV2 = 1.0/V(K)**2
      Y1 = CRMO -VSO*(X(1)**2-RV2)
      DO 8 L=2,NA
      L=L
      Y3 = A1*F(L)-VSO*(X(L)**2-RV2)
C     IF  Y3=0  PO = V*X(L),IF PLUS CONTINUE, IF MINUS PUT QUADRATIC
C     THRU Y1,Y2,Y3
      IF (Y3) 9,10,11
   11 Y1=Y3
      GO TO 8
   10 PO =V(K)*X(L)
      ALFSR=ALFA(L)
      IF (L-NA)  76,77,77
   77 ALFP =(ALFA(L)-ALFA(L-1))/DELX
      GO TO 75
   76 ALFP=0.5*(ALFA(L+1)-ALFA(L-1))/DELX
      GO TO 75
    9 VAR = (X(L)+X(L-1))**2/2.-RV2
      Y2 = A1*(F(I)+F(L-1))/2.0 - VSO*VAR
      AA=X(L-1)*Y3
      BB=X(L)*Y1
      CC=0.25*(X(L-1)*Y1+X(L)*Y3)
      AAA=0.5*(Y1+Y3)-Y2
      BBB=Y2*(X(L-1)+X(L))-CC-0.75*(AA+BB)
      CCC =(AA+BB)*(X(L-1)+X(L))/4.-Y2*X(L)  *X(L-1)
      VAR = BBB **2 -4.0*AAA*CCC
      IF(VAR) 12,13,13
   12 WRITE OUTPUT TAPE 3,160
      GO TO 99
   13 ROOT1=(-BBB +SQRTF(VAR))/2.0/AAA
      ROOT2=(-BBB -SQRTF(VAR))/2.0/AAA
      IF(ROOT1-X(L-1)) 14,14,15
   15 IF(ROOT1-X(L))16,14,14
   14 IF(ROOT2-X(L-1))17,17,18
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```
   17 WRITE OUTPUT TAPE 3,160
      GO TO 99
   18 IF(ROOT2-X(L))19,17,17
   16 XSTR =ROOT1
      GO TO 20
   19 XSTR =ROOT2
      GO TO 20
    8 CONTINUE
      GO TO 100
   20 PO= V(K)*XSTR
      ALFP =(ALFA(L)-ALFA(L-1))/DELX
      ALFSR =(XSTR-X(L))/DELX*(ALFA(L)-ALFA(L-1))+ ALFA(L)
      GO TO 75
C     THESE CALCULATIONS ARE MADE ON THE ASSUMPTION THAT THERE IS ONLY
C     ONE ROOT BETWEEN X(I-1) AND X(I) THE FUNCTION IS ALSO ASSUMED TO
C     BE QUADRATIC. LINEAR INTERPOLATION IS USED TO DETERMINE ALPHA STAR
C     IF PROGRAM GOES TO N99 IT WILL INCUR AN ERROR STOP. THIS MEANS A
C     COMPLEX ROOT HAS BEEN FOUND OR THE REAL ROOTS ARE NOT IN THE
C     PROPER RANGE   N100 WILL BE REACHED ON AN INPUT ERROR
   99 STOP 77772
  100 STOP 77773
   75 CO = (PO**2-1.0)*3.1415927/TESO
      IF (MDIAG) 103,104,103
  103 WRITE OUTPUT TAPE 4,162,RS,XS,YS,CS,COSA1,COSA2,SO,A1,CRMO,XSTR,PO
     1,VSO,RV2,Y1,Y2,Y3,VAR,ALFP,ALFSR,AA,BB,CC,AAA,BBB,CCC,ROOT1,ROOT2
  104 WRITE OUTPUT TAPE 3,165, ALODI,AMDAL(J),V(K)
      WRITE OUTPUT TAPE 3,164,CO,ALFSR,ALFP,XSTR
C     COMPUTATION OF PO    AT EACH MESH POINT
      RS=FN*ALODI
      CS= 1.0
      K1=(NTH+1)/2
      DO 78 M=1,K1
      K2= NTH+1 -M
      COSA1=-CO P(M,3)
   82 COSA2=COSA1
      Z=0.0
      YS =ZTAN
      DO 78 L=1,NZ
      IF (L-NZ) 80,81,81
   81 PO2(L,M)=1.0
      PO2(L,K2)= 1.0
      GO TO 78
   80 ZN = FN* Z
      IF (Z -ZTAM)  84,84,85
   84 XS =ZN
      GO TO 86
   85 IF(Z-ZTAP) 126,127,127
  126 YS=ZTAN
      XS=FN*ZTAM
      CALL SOR
      PO2(L,M)=SO
      YS= FN*ZTAP
      XS =ZTAN
      CALL SOR
      SO= 0.5*(SO+PO2(L,M))
      GO TO 128
  127 YS =ZN
      XS =ZTAN
   86 CALL SOR
  128 PO2(L,M) = SQRTF(1.0 + (  3.1415927*SO)
      PO2(L,K2) PO2(L,M)
      IF(MDIAG) 105,178,105
```

```
  178 Z = Z +DELZ
   78 CONTINUE
      NGO=1
      IF(NP) 106,107,106
  106 WRITE OUTPUT TAPE 3,167
      WRITE OUTPUT TAPE 3,177,DELTH,ALODI,DELZ
  137 NAM=NTH-10
      LL=-9
      L=0
      GO TO 1088
  108 WRITE OUTPUT TAPE 3, 178
 1088 IF(NAM) 109,110,110
  109 L=NTH
      LL=10+LL
      M11=2
      GO TO 111
  110 L=10+L
      LL=10+LL
      NAM=NAM-10
      M11=1
      GO TO 111
  111 DO 112M=1,NZ
  112 WRITE OUTPUT TAPE 3,168,(PO2(M,JJ),JJ=LL,L)
      GO TO( 108,107),M11
  107 GO TO (134,135     ),NGO
  134 VAR=1.0-XSTR**EX2
      XSTRR= XSTR**EX1
      PHI= ALFP/ALFSR+EX3/XSTR-((0.5/XSTRR)*(AK-1.)/VAR)*EX3
      CAPK=((6.2831853/PHI)*PO/CO)*V(K)
C     HERE WE BEGIN THE CALCULATION OF THE C1 VECTOR BY SETTING UP
C     WHAT SHALL BE CALLED THE B VECTOR AND THE A MATRIX
      RS= ALODI
      CS= 1.
      IS1=1
      ZH= ZTA
      ZL=ZTAM
      CALL S1
      VAR= S1
      ZH= ZTAP
      ZL= ZTA
      CALL S1
      S1=(S1+VAR)/2.0
      TEM = CO*FN*S1*1.5
      YS=ZTA
      XS=ZTAM
      DO 123 L=1,N
      BVEC(L)= CCOS(L)*TEM
      DO 123 M=L,N
      COSA1=COS1(L,M)
      COSA2=COS2(L,M)
      CALL SOR
  123 AMAT(L,M)= SO+0.5*(RS-YS)
      XS=YS
      YS= ZTAP
      IF (MDIAG) 132,133,132
  132 WRITE OUTPUT TAPE 4,171, (BVEC(L),L=1,N)
      WRITE OUTPUT TAPE 4,172,((AMAT(L,M), L=1,N),M=1,N)
      WRITE OUTPUT TAPE 4,162, VAR,XSTRR,PHI,CAPK,ZH,ZL,S1,TEM ,XS,YS
  133 DO 124 L=1,N
      DO 124 M=L,N
      COSA1= COS1(L,M)
      COSA2= COS2(L,M)
      CALL SOR
```

```
      AMAT(L,M)=(AMAT(L,M)+SQ+0.5*(RS-YS))*0.5
  124 AMAT(M,L)= AMAT(L,M)
      DO 125 L=1,N
  125 AMAT(L,L)= AMAT(L,L)- CAPK
C      SUBROUTINE XSIMEQ IS USED TO SOLVE AX=B WHERE X IS THE C1 VECTOR
      SCALE= 1.
      IERR= XSIMEQE(50,N,1,AMAT,BVEC,SCALE,ERASE)
      IF (IERR-2)129,130,131
C     IERR IS AN ASSIGNED FIXED POINT VARIABLE
C                1  IF SOLUTION SUCESSFUL    --    CONTINUE
C                2  IF UNDERFLOW OR OVERFLOW     -- PRINT ERROR PROCEED
C                3  IF AMAT IS SINGULAR          -- PRINT ERROR PROCEED
  130 WRITE OUTPUT TAPE 3,169
      GO TO 4
  131 WRITE OUTPUT TAPE 3,170
      GO TO 4
  129 WRITE OUTPUT TAPE 3, 173,  (AMAT(L,1),L=1,N)
      DO 140 L=1,N
  140 C1(L)=AMAT(L,1)
      CALL SUBP1
      IF(NP) 138,135,138
  138 WRITE OUTPUT TAPE 3,174
      WRITE OUTPUT TAPE 3,177,DELTH,ALODI,DELZ
      NGO =2
      GO TO 137
  135 CALL LCALC
      WK=WK+WK
      WRITE OUTPUT TAPE 3,175,WK
      IF (NDIN) 136,4,136
  136 DWK= WK*PA*ALODI*DIN*DIN
       WRITE OUTPUT TAPE 3,176, DWK
    4 CONTINUE
    3 CONTINUE
    2 CONTINUE
      IF (MORCS)  500,501,500
  501 STOP
C                FORMAT STATEMENTS
  150 FORMAT (6E12.5)
  151 FORMAT( 30H1 ANALYSIS OF NASA AIR BEARING/ 39H  R.WERNICK  MTI LAT
     1HAM,NY   ST 5-5886 )
  152 FORMAT( 7H0 INPUT/117H     P(A)                C                A
     1            ZETA              K              SQRT(RT)            DIA
     2 METER /1P7E17.7 )
  154 FORMAT( 87H0  N    NO OF ALPHAS    NO OF L/D    NO OF LAMBDA(T)
     1  NO OF V  NO OF Z  NO OF THETA  / 1I5,1I10,3I15,2I10)
  155 FORMAT(23H0 TABLE OF ALPHA VALUES/(1P7E17.7))
  156 FORMAT(21H0 TABLE OF L/D VALUES/(1P7E17.7))
  157 FORMAT(33H0 TABLE OF LAMBDA(T) VALUES      /(1P7E17.7))
  158 FORMAT(19H0 TABLE OF V VALUES/(1P7E17.7))
  159 FORMAT( 8H0 OUTPUT)
  160 FORMAT(29H0   OOPS  SOMETHING IS WRONG )
  161 FORMAT(22H0 DIAGNOSTIC PRINTOUT )
  162 FORMAT(1P7E17.7)
  163 FORMAT(24H0    X(1)          F(I) /(1P2E15.7))
  164 FORMAT(90H0    CO             ALFA STAR       ALFA STR PRIM
     1PO/V                              /1P4F17.7)
  165 FORMAT(44H0  L/D(I)          LAMBDA(T)           V(I) / 1P3F17.7)
  153 FORMAT(11I5)
  166 FORMAT( 29H0 (1) MEANS YES, (0) MEANS NO/59H  PRINT DIAG   MORE
     1CASES  DIMENS. LOAD   PRESS, PRINTOUT / 1I7,3I15)
  167 FORMAT (61H0  DIMENSIONLESS ZEROTH ORDER PRESSURE PROFILE PO(Z,THE
     1ETA).        )
  168 FORMAT (1P10E12.4)
```

```
169 FORMAT( 38H0  UNDERFLOW OR OVERFLOW IN XSIMEQ     )
170 FORMAT( 24H0  A MATRIX IS SINGULAR )
171 FORMAT( 13H0  B VECTOR  /( 1P7E17.7))
172 FORMAT( 13H0  A MATRIX  /(1P7E17.7))
173 FORMAT( 13H0  C1 VECTOR /(1P7E17.7))
174 FORMAT( 61H0  DIMENSIONLESS FIRST ORDER PRESSURE PROFILE  P1(Z,1H
   1ETA).          )
175 FORMAT      (36H0 FULL BEARING DIMENSIONLESS LOAD    /1P1E17.7)
176 FORMAT(31H0 FULL BEARING LOAD IN POUNDS  /1P1E17.7)
177 FORMAT(104H EACH COLUMN BELOW GIVES PRESSURE FOR A GIVEN VALUE OF
   1THETA(THETA GOES FROM 0 TO 2*PI IN INCREMENTS OF 1P1E14.5,2H)./
   234H IN EACH COLUMN, Z GOES FROM 0 TO 1P1E14.5, 21H  , IN INCREMENT
   3S OF 1P1E14.5,  3H  .)
178 FORMAT(31H0CONTINUATION OF PRESSURE FIELD)
    END( 0,1,0,1,1)
```

```
      SUBROUTINE SOR
      DIMENSION  ALFA(51),ALOD(10),AMDAL(100),V(100),F(51),X(51),T(12),P
     1O2(51,61) ,DUMMY(100),C1(200),DAM(61) ,ZN(3),XN(4),XZ(5),CH(3),SH(
     13),CHP(3),SHP(3),CHM(3),SHM(3),CHN(3),SHN(3),          BVEC(50)
     2, XCOS(200),COS1(50,50),COS2(50,50),ERASE(50) , AMAT(50,50),CCOS(5
     30),COST(61,3),CSIN(50),SINT(61,3),TEMP(51,61)
      COMMON PA,C,A,ZTA,AK,RT12,N,NA,NLD,NAA,NV,ALFA,ALOD,AMDAL,V,F,X,MD
     1IAG,MORCS,RS,XS,YS,COSA1,COSA2,CS,SO,T,SINHK,SINHDK,COSHK,COSHDK,N
     2Z,NTH,PO2,AAB,B,CB,D,ICOUNT,E,FN,ALODI,WK,TAR,ZTAN,ALFP,ALFSR,DFLZ
     3,CO,   DELTH,C1,DIN,NDIN,CN,SN,S1,IS1,ZH,ZL,     BVEC,AMAT,COS1,CO
     4S2,ERASE,XCOS,ZTAP,ZTAM,CCOS,COST,CSIN,SINT,TEMP
      DELTA = YS-XS
      SIGMA = XS+YS
      TR = 2.0*RS
      IF( TR  ) 14,15,15
   15 IF(SIGMA) 14,16,16
   16 IF(DELTA) 14,17,17
   17 IF(TR-SIGMA)14,18,18
   18 IF(SIGMA-DELTA)14,19,19
C     IF PROGRAM GOES TO N14 AN ERROR HAS OCCURRED. EITHER 2R SIGMA OR
C     DELTA ARE MINUS
   14 STOP 77771
   19 EX2R = EXPF(-TR)
      EXSG = EXPF(-SIGMA)
      EXDT = EXPF(-DELTA)
      EX2RS = EX2R/EXSG
      EX2RD = EX2R/EXDT
      SINHK = 0.0
      COSHK = 1.0
      TRMSG = TR-SIGMA
      TRMDT=TR-DELTA
      COHRS = 0.5*( EX2RS+ 1.0/EX2RS)
      SIHRS = 0.5*(-EX2RS+ 1.0/EX2RS)
      COHRD = 0.5*( EX2RD+ 1.0/EX2RD)
      SIHRD = 0.5*(-EX2RD+ 1.0/EX2RD)
      COHSG = 0.5*( EXSG + 1.0/EXSG)
      SIHSG = 0.5*(-EXSG + 1.0/EXSG)
      COHDT = 0.5*( EXDT + 1.0/EXDT)
      SIHDT = 0.5*( EXDT + 1.0/EXDT)
      COSHDK = 0.5*(EX2R + 1.0/EX2R)
      SINHDK = 0.5*(-EX2R+ 1.0/EX2R)
C     IF THE ARGUMENT (EXCLUDING THOSE CONTAINING THE INCREMENT K ) OF
C     ANY HYPERBOLIC FUNCTION  EXCEEDS 5.0 AN ASYMTOTIC APPROXIMATION
C     IS USED TO COMPUTE T(K)
      IF (COSA1-COSA2) 63,64,63
   64 M11 = 2
      GO TO 163
   63 M11 = 1
  163 DO 50 K=1,12
      IF (K-1) 51,51,52
   51 T21N =0.0
      T22N =0.0
      T21D =0.0
      T22D =0.0
      TM31=COSA1
      TM32=COSA2
      TANHK= 0.0
      GO TO 53
   52 AKK=K-1
      IF (TR*AKK -60.0) 54,54,55
   55 T(K)=T(K-1)
      GO TO 50
   54 CALL SINH
```

```
      TANHK = SINHK/COSHK
      T21N = TANHK * SIHRS
      T22N = TANHK * SIHRD
      T21D = TANHK* SIHSG
      T22D = TANHK *SIHDT
      IF QUOTIENT OVERFLOW 56,56
   56 TM31 = COSA1/COSHK
      IF QUOTIENT OVERFLOW 57,58
   57 TM31=0.0
      GO TO (58,53),M11
   58 TM32 = COSA2/COSHK
      IF QUOTIENT OVERFLOW 59,53
   59 TM32 =0.0
C     IF ZRK IS GREATER THAN 60.0 WE ASSUME THE SERIES HAS CONVERGED
C     SINCE REMAINING FORWARD DIFFERENCES ARE ESSENTIALLY ZERO
   53 IF (TRMSG -5.0) 20,21,21
   20 T1N = ( COHRS + T21N - TM31) * EX2RS
      GO TO(65,22), M11
   65 T3N = ( COHRS + T21N - TM32) * EX2RS
      GO TO 22
   21 IF ACCUMULATOR OVERFLOW  23,23
   23 TM1 =  EX2RS * TM31
      IF ACCUMULATOR OVERFLOW 24,255
   24 TM1 = 0.0
  255 GO TO (25,27), M11
   25 TM2 = EX2RS *  TM32
      IF ACCUMULATOR OVERFLOW 26,27
   26 TM2 =0.0
   27 T1N = 0.5*(1.0+TANHK)-TM1
      GO TO (66,22),M11
   66 T3N = 0.5*(1.0+TANHK)-TM2
   22 IF(TRMDT-5.0) 28,29,29
   28 T2N =(COHRD+ T22N-TM31)* EX2RD
      GO TO (67,49),M11
   67 T4N = (COHRD +T22N-TM32)*EX2RD
      GO TO 49
   29 TM1 = EX2RD *TM31
      IF ACCUMULATOR OVERFLOW 30,311
   30 TM1 = 0.0
  311 GO TO (31,33),M11
   31 TM2 = EX2RD * TM32
      IF ACCUMULATOR OVERFLOW 32,33
   32 TM2 =0.0
   33 T2N =0.5*(1.0+TANHK)-TM1
      GO TO (68,49),M11
   68 T4N = 0.5*(1.0+TANHK)-TM2
   49 IF (SIGMA -5.0) 34,35,35
   34 T1D =(COHSG +T21D -TM31)*EXSG
      GO TO (69,37),M11
   69 T3D = (COHSG + T21D -TM32)*EXSG
      GO TO 37
   35 IF ACCUMULATOR OVERFLOW 36,36
   36 TM1 = EXSG * TM31
      IF ACCUMULATOR OVERFLOW 38,399
   38 TM1 =0.0
  399 GO TO (39,41),M11
   39 TM2=EXSG*TM32
      IF ACCUMULATOR OVERFLOW 40,41
   40 TM2 = 0.0
   41 T1D = 0.5*(1.0+TANHK)- TM1
      GO TO (70,37),M11
   70 T3D = 0.5 *(1.0+TANHK) -TM2
   37 IF (DELTA -5.0) 42,43,43
```

```
 42 T2D = (COHDT + T22D - TM31)* FXDT
    GO TO (71,72),M11
 71 T4D = (COHDT + T22D - TM32)* FXDT
    GO TO 60
 43 IF ACCUMULATOR OVERFLOW 44,44
 44 TM1 = FXDT * TM31
    IF ACCUMULATOR OVERFLOW 45,466
 45 TM1 =0.0
466 GO TO (46,48),M11
 46 TM2 = EXDT * TM32
    IF ACCUMULATOR OVERFLOW 47,48
 47 TM2 =0.0
 48 T2D =0.5*(1.0+TANHK) -TM1
    GO TO (73,72),M11
 73 T4D = 0.5*(1.0+TANHK)-TM2
    GO TO 60
 72 VAR =(T1N/T1D *T2N/T2D)**2
    GO TO 74
 60 VAR = T1N/T1D*T2N/T2D*T3N/T3D*T4N/T4D
 74 T(K) =LOGF(VAR)
    IF (MDIAG) 113,50,113
113 WRITE OUTPUT TAPE 4,169, DELTA,SIGMA,TR,COHRS,COHRD,SIHSG,SIHDT,CO
   1SHDK,T21N,T21D,TANHK,TM1,T1N,T2N,T3N,T4N,T1D,T2D,T3D,T4D,T(K),VAR
169 FORMAT(16H0 SO SUBROUTINE /(1P7E16.7))
 50 CONTINUE
    VAR = T(1)-T(2)+T(3)-T(4)+T(5)/2.0
    OM= 1.0
    DIV= 2.0
    DO 61 J=1,7
    OM=-OM
    DIV=DIV*2.0
    M=8 -J
    DO 62 K=1,M
 62 T(K+4)=T(K+4)-T(K+5)
    VAR= VAR +OM*T(5)/DIV
 61 CONTINUE
    SO = 0.5*(RS-YS  +(CS *VAR)/4.)
    RETURN
    END(0,1,0,1,0)
```

```
      SUBROUTINE S1
      DIMENSION  ALFA(51),ALOD(10),AMDAL(100),V(100),F(51),X(51),T(12),P
     1O2(51,61) ,DUMMY(100),C1(200),DAM(61) ,ZN(3),XN(4),XZ(5),CH(3),SH(
     13),CHP(3),SHP(3),CHM(3),SHM(3),CHN(3),SHN(3),          BVEC(50)
     2, XCOS(200),COS1(50,50),COS2(50,50),ERASE(50) , AMAT(50,50),CCOS(5
     30),COST(61,3),CSIN(50),SINT(61,3),TEMP(51,61)
      COMMON PA,C,A,ZTA,AK,RT12,N,NA,NLD,NAA,NV,ALFA,ALOD,AMDAL,V,F,X,MD
     1IAG,MORCS,RS,XS,YS,COSA1,COSA2,CS,SO,T,SINHK,SINHDK,COSHK,COSHDK,N
     2Z,NTH,PO2,AAB,B,CB,D,ICOUNT,E,FN,ALODI,WK,TAR,ZTAN,ALFP,ALFSR,DFLZ
     3,CO,   DFLTH,C1,DIN,NDIN,CN,SN,S1,IS1,ZH,ZL,      BVEC,AMAT,COS1,CO
     4S2,ERASE,XCOS,ZTAP,ZTAM,CCOS,COST,CSIN,SINT,TEMP
C          ENTER WITH
C               FN= NO. ORIFICES, ZH= MAX(Z,ZETA), ZL= MIN(Z,ZETA),
C               ALODI= CURRENT (L/D).
C               IS1= 1 FOR C1 CALCULATION,
C                  = 2 REQUIRES SINE, COSINE N*THETA,THETA.
C               E=N THETA              ICOUNT=1
C               B=COS(E)               TAR= 0
C               AAB=SIN(E)             CN=COS(THETA)
C               CB= 0.                 SN= SIN(THETA)
C               D= 1.
C          FIRST COMPUTE ARGUMENTS AND INITIAL VALUES.
    1 ZN(1)=FN*ZL
      ZN(2)=FN*(ALODI- ZH)
      ZN(3)=FN*ALODI
      ENN=  FN+ FN
      EMNZ1= ZN(1)
      EMNZ2= FN*ZH
      ENZH=EMNZ2
      EMNZ3= ZN(3)
      BOTM= ENN- 1.
      BOTP= ENN+ 1.
C          COMPUTE EXPONENTIAL FUNCTIONS.
    3 DO  4  IQ=1,3
    4 XN(IQ)= EXPF(ZN(IQ))
      XZ(1)= EXPF(ZL)
      XZ(2)= EXPF(ALODI- ZH)
      XZ(3)= EXPF(ALODI)
      XZ(5)= XZ(2)/XZ(3)
      XN(4)= XN(2)*XN(1)/XN(3)
      XZ(4)= XZ(5)*XZ(1)
C          COMPUTE HYPERBOLIC FUNCTIONS
    5 DO 6 IQ=1,3
      VA= 1./XN(IQ)
      VB= 1./XZ(IQ)
      CHN(IQ)= (XN(IQ) + VA)/2.
      SHN(IQ)= (XN(IQ) - VA)/2.
      CH (IQ)= (XZ(IQ) + VB)/2.
    6 SH (IQ)= (XZ(IQ) - VB)/2.
C          STORE HYPERBOLIC FUNCTIONS FOR M= 0.
      DO 7 IQ=1,3
      CHP(IQ)= 1.
    7 SHP(IQ)= 0.
      OM = -1.0
      STS= 0.
C          NOW COMPUTE COSH, SINH OF-  MN*ZL, MN*(L/D-ZH), MN*(L/D)
      ASSIGN 8 TO NHA
      GO TO 500
C          RETURN WITH HYPERBOLIC FUNCTS IN CHM, SHM ARRAYS. NEXT COMPUTE
C               UAB1, UAB2, UAB3 USING HYPERBLC FUNCTS.
    8 ASSIGN 9 TO NUABA
      GO TO 600
C          RETURN WITH UAB,S.
```

```
    9 S1= UAB1+ UAB3+ UAB2/BOTM+ UAB2/BOTP
      SPR= S1
      GO TO (12,10), IS1
C           FOLLOWING COMPUTES SUM WHEN THETA TERM ENTERS- CASE 2.
   10 ASSIGN 12 TO  NIS12
      CALL SINCOS
      SMN= D*CN
      SMP=CB*SN
      CI=SMN+SMP
      C2=SMN-SMP
   11 STS= OM*(CI*(UAB1 + UAB2/BOTM) +C2*(UAB3 + UAB2/BOTP)) + STS
      OM= -OM
      GO TO NIS12, (12,26)
   12 FMNZ3= FMNZ3 + ZN(3)
      EMNZ2= FMNZ2 + FNZH
      FMNZ1= EMNZ1 + ZN(1)
      BOTM = BOTM + FNN
      BOTP = BOTP + FNN
C           ABOVE STATEMENTS ADVANCE SUMMATION INDEX M BY 1.
   13 IF (FMNZ3- 8.0) 14,14,17
C           FOLLOWING COMPUTE UAB1-3 BY HYPERBOLIC FUNCTIONS.
   14 ASSIGN 16 TO NHA
      GO TO 500
   16 ASSIGN 24 TO NUABA
      GO TO 600
C           FOLLOWING FOR MN*(L/D) GREATER 8. TEST MN*ZL.
   17 IF (FMNZ1- 8.0) 18,18,23
   18 IF (FMNZ3- 50.0) 21,19,19
   19 CHM(1)= CHN(1)*CHP(1)+ SHN(1)*SHP(1)
      SHM(1)= SHN(1)*CHP(1)+ CHN(1)*SHP(1)
      CHP(1)= CHM(1)
      SHP(1)= SHM(1)
      XS= EXPF(FMNZ2)
      UAB2= CHM(1)/XS
      T(1)= CHM(1)*CH(1)
      T(2)= SHM(1)*SH(1)
   20 UAB1= (T(1)- T(2))/(XS* X7(5)*BOTM)
      UAB3=((T(1)+ T(2))/XS)*X7(5)/BOTP
      GO TO 24
C           FOLLOWING COMPUTE UAB1-3 BY HYPERBOLIC FUNCTIONS.
   21 ASSIGN 22 TO NHA
      GO TO 500
   22 ASSIGN 24 TO NUABA
      GO TO 600
C           FOLLOWING EVALUATE UAB1-3 BY ASYMPTOTIC FORMULAS.
   23 UAB2= 0.5*EXPF(FMNZ1-FMNZ2)
      UAB1= UAB2/(X7(4)*BOTM)
      UAB3= UAB2*X7(4)/BOTP
C           FOLLOWING CALCULATE SUMS AND CHECK CONVERGERGENCE.
   24 S1= S1+ UAB1 + UAB3 + UAB2/BOTM + UAB2/BOTP
      GO TO (26,25),IS1
   25 ASSIGN 26 TO NIS12
      GO TO 11
   26 IF (SPR/S1- 0.9999)27,28,28
   27 SPR= S1
      GO TO 12
   28 GO TO (30,29),IS1
   29 S1= STS
   30 RETURN
C           CALCULATION OF HYPERBOLIC FUNCTS OF MN*ARGUMENT.
  500 DO 501 IQ= 1,3
      CHM(IQ)= CHN(IQ)*CHP(IQ)+ SHN(IQ)*SHP(IQ)
      SHM(IQ)= SHN(IQ)*CHP(IQ)+ CHN(IQ)*SHP(IQ)
```

```
      CHP(IQ)= CHM(IQ)
  501 SHP(IQ)= SHM(IQ)
      GO TO NHA, (8,16,22)
C          CALCULATION OF UAB1-3 USING HYPERBOLIC FUNCTIONS.
  600 UAB2 = CHM(1)*SHM(2)/CHM(3)
      T(1)= CHM(1)*CH(1)
      T(2)= SHM(1)*SH(1)
      T(3)= SHM(2)*CH(2)
      T(4)= CHM(2)*SH(2)
      T(5)= CHM(3)*CH(3)
      T(6)= SHM(3)*SH(3)
  601 UAB1= ((T(1)-T(2))/(T(5)-T(6)))*(T(3)-T(4))/BOTM
      UAB3= ((T(1)+T(2))/(T(5)+T(6)))*(T(3)+T(4))/BOTP
      GO TO NUABA,(9,24)
      END(0,1,0,1,0)
```

```
      SUBROUTINE SURP1
C               THIS SUBROUTINE COMPUTES THE FIRST ORDER PRESSURE PROFILE
C               WHICH UPON EXIT FROM THE ROUTINE IS STORED IN PO2. THE
C               S1 AND SOR SUBPROGRAMS ARE USED. POP1= (U1+U2)+U3  ,
C               POP1/(PO2) = P1
C               U1+U2 INVOLVES USE OF S1
      DIMENSION  ALFA(51),ALOD(10),AMDAL(100),V(100),F(51),X(51),T(12),P
     1O2(51,61) ,DUMMY(100),C1(200),DAM(61) ,ZN(3),XN(4),XZ(5),CH(3),SH(
     13),CHP(3),SHP(3),CHM(3),SHM(3),CHN(3),SHN(3),          BVEC(50)
     2, XCOS(200),COS1(50,50),COS2(50,50),ERASE(50) , AMAT(50,50),CCOS(5
     30),COST(61,3),CSIN(50),SINT(61,3),TEMP(51,61)
      COMMON PA,C,A,ZTA,AK,RT12,N,NA,NLD,NAA,NV,ALFA,ALOD,AMDAL,V,F,X,MD
     1IAG,MORCS,RS,XS,YS,COSA1,COSA2,CS,SO,T,SINHK,SINHDK,COSHK,COSHDK,N
     2Z,NTH,PO2,AAB,B,CB,D,ICOUNT,E,FN,ALODI,WK,TAR,ZTAN,ALFP,ALFSR,DFL7
     3,CO,   DELTH,C1,DIN,NDIN,CN,SN,S1,IS1,ZH,ZL,      BVEC,AMAT,COS1,CO
     4S2,ERASE,XCOS,ZTAP,ZTAM,CCOS,COST,CSIN,SINT,TEMP
      VAR = FN*DELTH
      E= -VAR
      IS1=2
      FMPT= -CO*FN/3.1415927*0.75
      DO 2 J=1,NTH
      E=E+VAR
      Z= 0.0
      CN=COST(J,2)
      SN=SINT(J,2)
      ZH= ZTA
      DO 2 I=1,NZ
      CB=0.0
      D=1.0
      ICOUNT = 1
      TAR =0.
      B= COST(J,3)
      AAB= SINT(J,3)
      IF(I-NZ) 3,2,2
    3 IF(Z-ZTAM) 5,5,6
    5 ZL=Z
      GO TO 7
    6 IF(Z-ZTAP) 8,9,9
    8 ZH= ZTA
      ZL=ZTAM
      CALL S1
      TEMP(I,J)= S1
      ZH= ZTAP
      ZL= ZTA
      CB= 0.
      D= 1.
      ICOUNT = 1
      TAR= 0.0
      B= COST(J,3)
      AAB= SINT(J,3)
      CALL S1
      S1= 0.5*(S1+TEMP(I,J))
      GO TO 10
    9 ZH= Z
      ZL= ZTA
    7 CALL S1
   10 TEMP(I,J)= S1        *FMPT
      Z=Z+DELZ
    2 CONTINUE
      RS= ALODI
      DO 11J=1,NTH
      Z=0.0
      YS=ZTA
```

```
      DO 11 I=1,NZ
      IF(I-NZ) 12,91,91
   12 IF (Z-ZTAM) 13,13,14
   13 XS=Z
      M11=1
      GO TO 15
   14 IF(Z-ZTAP) 16,17,17
   16 YS= ZTA
      XS= ZTAM
      M11=2
      GO TO 15
   17 YS= Z
      XS= ZTA
      M11=1
   15 DO 21 K=1,N
      A1= COST(J,2)*CCOS(K)
      A2= SINT(J,2)*CSIN(K)
      COSA1= A1-A2
      COSA2= A1+A2
      GO TO 22
   18 VAR= SO+ 0.5*(RS-YS)
      YS= ZTAP
      XS= ZTA
      M11=3
   22 CALL SOR
      GO TO ( 19,18,20),M11
   20 SO=(VAR+SO+(RS-YS)*0.5 )*0.5
      YS= ZTA
      XS= ZTAM
      M11=2
      GO TO 21
   19 SO= SO+0.5*(RS-YS)
   21 TEMP(I,J)= SO* C1(K)/6.2831853 + TEMP(I,J)
      PO2(I,J)= TEMP(I,J)/PO2(I,J)
      GO TO 11
   91  PO2(I,J) = PO2(I,1)
   11 CONTINUE
      RETURN
      END (0,1,0,1,0)
```

```
      SUBROUTINE SINH
      DIMENSION  ALFA(51),ALOD(10),AMDAL(100),V(100),F(51),X(51),T(12),P
     1O2(51,61) ,DUMMY(100),C1(200),DAM(61) ,ZN(3),XN(4),XZ(5),CH(3),SH(
     13),CHP(3),SHP(3),CHM(3),SHM(3),CHN(3),SHN(3),          BVEC(50)
     2, XCOS(200),COS1(50,50),COS2(50,50),ERASE(50) , AMAT(50,50),CCOS(5
     30),COST(61,3),CSIN(50),SINT(61,3),TEMP(51,61)
      COMMON PA,C,A,ZTA,AK,RT12,N,NA,NLD,NAA,NV,ALFA,ALOD,AMDAL,V,F,X,MD
     1IAG,MORCS,RS,XS,YS,COSA1,COSA2,CS,SO,T,SINHK,SINHDK,COSHK,COSHDK,N
     2Z,NTH,PO2,AAB,B,CB,D,ICOUNT,E,FN,ALODI,WK,TAR,ZTAN,ALFP,ALFSR,DFLZ
     3,CO,   DELTH,C1,DIN,NDIN,CN,SN,S1,IS1,ZH,ZL,      BVEC,AMAT,COS1,CO
     4S2,ERASE,XCOS,ZTAP,ZTAM,CCOS,COST,CSIN,SINT,TEMP
      SINHK1= SINHK*COSHDK + SINHDK*COSHK
      COSHK = SINHDK*SINHK + COSHDK*COSHK
      SINHK =SINHK1
      RETURN
      END(0,1,0,1,0)
```

```
C     SINE  COS SUBROUTINE -- DUE TO POSSIBLE ROUNDOFF ERROR IN THIS
C     ROUTINE  EVERY TENTH    SET OF VALUES  IS COMPUTED USING A SERIES
C     OR EXPONENTIAL APPROXIMATION ROUTINE AAB=SIN(DARG),B=COS(DARG)
C     CB=SIN(ARG), D=COS(ARG)
      SUBROUTINE SINCOS
      DIMENSION  ALFA(51),ALOD(10),AMDAL(100),V(100),F(51),X(51),T(12),P
     1O2(51,61) ,DUMMY(100),C1(200),DAM(61) ,ZN(3),XN(4),XZ(5),CH(3),SH(
     13),CHP(3),SHP(3),CHM(3),SHM(3),CHN(3),SHN(3),          BVEC(50)
     2, XCOS(200),COS1(50,50),COS2(50,50),ERASE(50) , AMAT(50,50),CCOS(5
     30),COST(61,3),CSIN(50),SINT(61,3),TEMP(51,61)
      COMMON PA,C,A,ZTA,AK,RT12,N,NA,NLD,NAA,NV,ALFA,ALOD,AMDAL,V,F,X,MD
     1IAG,MORCS,RS,XS,YS,COSA1,COSA2,CS,SO,T,SINHK,SINHDK,COSHK,COSHDK,N
     2Z,NTH,PO2,AAB,B,CB,D,ICOUNT,E,FN,ALODI,WK,TAR,ZTAN,ALFP,ALFSR,DFLZ
     3,CO,   DELTH,C1,DIN,NDIN,CN,SN,S1,IS1,ZH,ZL,      BVEC,AMAT,COS1,CO
     4S2,ERASE,XCOS,ZTAP,ZTAM,CCOS,COST,CSIN,SINT,TEMP
      IF (ICOUNT-10  ) 1,2,2
    2 FIC = ICOUNT
      TAR = 1.0+TAR
      ICOUNT =1
      CB = SINF(FIC   *TAR*E)
      D= COSF(FIC*TAR*E)
      GO TO 3
    1 ICOUNT =ICOUNT +1
      FIC =  B*CB + AAB*D
      D = -AAB*CB + B*D
      CB =FIC
    3 RETURN
      END(0,1,0,1,0)
```

```
C     DOUBLE INTEGRATION OF  WK =// PK COS(TH) DTH DZ . THE  THETA RANGE
C     IS APPROXIMATED BY TRAPEZOIDAL RULE. THE Z RANGE IS APPROXIMATED
C     BY SIMPSONS RULE.
      SUBROUTINE LCALC
      DIMENSION  ALFA(51),ALOD(10),AMDAL(100),V(100),F(51),X(51),T(12),P
     1O2(51,61) ,DUMMY(100),C1(200),DAM(61) ,ZN(3),XN(4),XZ(5),CH(3),SH(
     13),CHP(3),SHP(3),CHM(3),SHM(3),CHN(3),SHN(3),          BVEC(50)
     2, XCOS(200),COS1(50,50),COS2(50,50),ERASE(50) , AMAT(50,50),CCOS(5
     30),COST(61,3),CSIN(50),SINT(61,3),TEMP(51,61)
      COMMON PA,C,A,ZTA,AK,RT12,N,NA,NLD,NAA,NV,ALFA,ALOD,AMDAL,V,F,X,MD
     1IAG,MORCS,RS,XS,YS,COSA1,COSA2,CS,SO,T,SINHK,SINHDK,COSHK,COSHDK,N
     2Z,NTH,PO2,AAB,B,CB,D,ICOUNT,E,EN,ALODI,WK,TAR,ZTAN,ALFP,ALFSR,DELZ
     3,CO,   DELTH,C1,DIN,NDIN,CN,SN,S1,IS1,ZH,ZL,      BVEC,AMAT,COS1,CO
     4S2,ERASE,XCOS,ZTAP,ZTAM,CCOS,COST,CSIN,SINT,TEMP
      COSTH =1.0
      DZ3 = DELZ/3.0
      TDZ3= DZ3+DZ3
      FDZ3=TDZ3+TDZ3
      NX =NZ-2
      WK=0.0
      DO 1 I=1,NTH
      VAR=0.0
      VAR1= PO2(I,2)
      COSTH= COST(I,2)
    3 DAM(I)    = DZ3*(PO2(I,1))
      DO 2 J=3,NX,2
      VAR       = PO2(I,J) + VAR
    2 VAR1      = PO2(I,J+1) + VAR1
      DAM(I)= DAM(I)+TDZ3*VAR+FDZ3*VAR1
      DAM(I)=DAM(I)*COSTH
    1 WK=WK+DAM(I)
      WK= (WK-(DAM(1)+DAM(NTH))*0.5)*DELTH
      RETURN
      END(0,1,0,1,0)
```

COMPUTER PROGRAM PN0045

Static Load for Infinitely Short Journal Bearing with Line Source Feeding

```
C     MECHANICAL TECNOLOGY,INC.   JORGEN W.LUND   12-8-1962
C     PN0045=INFINITELY SHORT HYDROSTATIC JOURNAL BEARING
      DIMENSION VLST(200),FLST(200),EPL(200),CS(200),DW(200),DUM(500),
     1DW1(200)
  200 READ INPUT TAPE 2,100
      READ INPUT TAPE 2,101
      READ INPUT TAPE 2,105,NV,NLL,NEP,NTH,NDIA,INP
      READ INPUT TAPE 2,106,(VLST(I),I=1,NV),(FLST(I),I=1,NLL)
     1,(EPL(I),I=1,NEP)
      WRITE OUTPUT TAPE 3,102
      WRITE OUTPUT TAPE 3,100
      WRITE OUTPUT TAPE 3,101
      WRITE OUTPUT TAPE 3,103
      WRITE OUTPUT TAPE 3,104,NV,NLL,NEP,NTH,NDIA,INP
      WRITE OUTPUT TAPE 3,107
      WRITE OUTPUT TAPE 3,108,(VLST(I),I=1,NV)
      WRITE OUTPUT TAPE 3,109
      WRITE OUTPUT TAPE 3,108,(FLST(I),I=1,NLL)
      WRITE OUTPUT TAPE 3,110
      WRITE OUTPUT TAPE 3,108,(EPL(I),I=1,NEP)
      DTH=NTH
      DTH=3.1415927/DTH
      NTHE=NTH+1
      TH=0.0
      DO 201 I=1,NTHE
      CS(I)=COSF(TH)
  201 TH=TH+DTH
      DO 205 J=1,NV
      V=VLST(J)
      V2=(V*V-1.0)*4.0
      DO 205 K=1,NLL
      FL=FLST(K)
      A=FL*FL
      A2=0.5*A
      DO 205 L=1,NEP
      EPS=EPL(L)
      W=0.0
      W2=0.0
      DO 202 I=1,NTHE
      CTH=CS(I)
      H6=(1.0+EPS*CTH)**6.0
      TH=V2/A*H6
      IF(TH-.0001) 207,208,208
  207 Q=V2/4.0
      GO TO 209
  208 Q=A2*(-1.0+SQRTF(1.0+TH))/H6
  209 Q1=Q+1.0
      QSR=SQRTF(Q1)
      DW(I)=(Q1*QSR-1.0)/Q*CTH
      DW1(I)=QSR*CTH
      W2=W2+DW1(I)
  202 W=W+DW(I)
      W=DTH*(DW(1)+DW(NTHE)-2.0*W)/3.0
      W2=DTH*(0.5*DW1(1)+0.5*DW1(NTHE)-W2)
      W1=1.2732395*W/EPS
      IF(NDIA) 203,204,203
  203 WRITE OUTPUT TAPE 3,111
      WRITE OUTPUT TAPE 3,112,V,FL,EPS,V2,A,A2,DTH,TH,CTH,H6,Q,Q1,QSR,
     1W,W1
      WRITE OUTPUT TAPE 3,113,(CS(I),DW(I),I=1,NTHE)
  204 WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,115,V,FL,EPS,W,W1,W2
  205 CONTINUE
```

```
    IF(INP) 206,200,206
206 END FILE 3
    STOP 77777
100 FORMAT(72H0
   1                    )
101 FORMAT(72H
   1                    )
102 FORMAT(84H1  PN0045=INFINITELY SHORT HYDROSTATIC JOURNAL BEARING
   1   JORGEN W.LUND  12-8-1962 )
103 FORMAT(90H0 NO.PR.RATIO   NO.L/D*LAMBDA   NO.EPSILON      NO.THETA
   1      DIAGNOSTIC         INPUT      )
104 FORMAT(I8,5I15)
105 FORMAT(6I5)
106 FORMAT(1P4E15.7)
107 FORMAT(18H0  PRESSURE RATIOS)
108 FORMAT(1P7E16.7)
109 FORMAT(18H0  L/D*LAMBDA-T    )
110 FORMAT(12H0  EPSILONS )
111 FORMAT(18H0  DIAGNOSTIC      )
112 FORMAT(1P6E16.7)
113 FORMAT(1P2E16.7)
114 FORMAT(108H0   PRESSURE RATIO      L/D*LAMBDA-T        EPSILON
   1     DIMENS.LOAD     4/PI*LOAD/EPS.      CENTER LOAD )
115 FORMAT(1P6E18.7)
    END (0,0,1)
```

COMPUTER PROGRAM PN0047

Dynamic Load for Journal and Thrust
Bearing with Line Source Feeding

```
C     MECHANICAL TECHNOLOGY,INC.    JORGEN W.LUND    9-2-62
C     PN0047 DYNAMIC LOAD FOR HYDROSTATIC JOURNAL AND THRUST BEARING
      DIMENSION FVEE(100),FLAM(100),FSIG(100),FALFA(26),FXI(50),FGAM(50)
     1,QST(50),PR2ST(50),PRST(50),G(101),P0(101),PZG(101),PR1(101),
     2PI1(101),PR2(101),PI2(101),SR(101),SI(101),TR(101),TI(101),VR(101)
     3,VI(101),TSR(101),TSI(101),TTR(101),TTI(101),TVR(101),TVI(101),
     4DUMMY(1000)
```



```
  200 READ 100
      READ 101
      READ 102,NSIG,NLAM,NVEE,KOP1,NX,NGAM,MT,NDIAG,NPR,FKA
      READ 103,(FXI(I),I=1,NX)
      READ 103,(FGAM(I), I=1,NGAM)
      READ 103,(FVEE(I),I=1,NVEE)
      READ 103,(FLAM(I),I=1,NLAM)
      READ 103,(FSIG(I),I=1,NSIG)
      READ 103,(FALFA(I),I=2,26)
      WRITE OUTPUT TAPE 3,115
      WRITE OUTPUT TAPE 3,120
      WRITE OUTPUT TAPE 3,100
      WRITE OUTPUT TAPE 3,101
      WRITE OUTPUT TAPE 3,136
      WRITE OUTPUT TAPE 3,137,NX,NGAM,NVEE,NLAM,NSIG,KOP1,MT,NPR,NDIAG,
     1FKA
      WRITE OUTPUT TAPE 3,131
      WRITE OUTPUT TAPE 3,124,(FXI(I),I=1,NX)
      WRITE OUTPUT TAPE 3,132
      WRITE OUTPUT TAPE 3,124,(FGAM(I),I=1,NGAM)
      WRITE OUTPUT TAPE 3,125
      WRITE OUTPUT TAPE 3,124,(FVEE(I),I=1,NVEE)
      WRITE OUTPUT TAPE 3,123
      WRITE OUTPUT TAPE 3,124,(FLAM(I),I=1,NLAM)
      WRITE OUTPUT TAPE 3,126
      WRITE OUTPUT TAPE 3,124,(FSIG(I),I=1,NSIG)
      WRITE OUTPUT TAPE 3,127
      WRITE OUTPUT TAPE 3,124,(FALFA(I),I=2,26)
C     CALCULATION OF ORIFICE FLOW TABLE
      FALFA(1)=1.0
      EX6=2.0/(FKA+1.0)
      EX1=1.0/(FKA-1.0)
      EX7=(EX6**EX1)*SQRTF(EX6*FKA)
      EX4 = 2.0 * EX1
      EX1=FKA*EX1
      CRP = EX6 ** EX1
      EX3= SQRTF(2.0*EX1)
      EX2= 1.0/EX1
      EX5 = -EX2
      EX6 = 1.0/EX6
      EX1=1.0/FKA
      DIT =(1.0 -CRP)/50.0
      CX=CRP
      DO 258 I=1,50
      CX = CX+DIT
      C5 = CX**EX1
      QST(I)=EX3*C5*SQRTF(1.0-CX/C5)
      PR2ST(I)=CX*CX
  258 PRST(I)= CX
      AME=MT
C     INITIATE L/D LOOP
      DO 242 IA=1,NX
      FLD=FXI(IA)
      DELTA=FLD/AME
      DELTAS=DELTA*DELTA
      IF(KOP1) 202,201,202
```

```
  201 P2LD=FLD
      GO TO 203
  202 P2LD=LOGF(FLD)
C     INITIATE L1/D-LOOP
  203 DO 242 IB=1,NGAM
      FL1D=FGAM(IB)
      IF(KOP1) 204,205,204
  204 FLG=LOGF(FL1D)
      BETAT=P2LD/FLG
      BP1=BETAT+1.0
      GMNV=1.0/FL1D
      FRFT=1.0/(FLD*FLD)
      DELTA=(FLD-1.0)/AMF
      DELG=(1.0-GMNV)/AMF
C     INITIATE PRESSURE RATIO LOOP
  205 DO 242 IC=1,NVEE
      V=FVEE(IC)
      V2=V*V
      VNV=1.0/V2
C     INITIATE LAMBDA-T LOOP
      DO 242 ID=1,NLAM
      FLT=FLAM(ID)
C     CALCULATE DIMENSIONLESS FLOW AND ORIFICE PRESSURE
  259 C1=FLT*P2LD*V
      CX=SQRTF(1.0+C1*EX7)/V
      IF(CRP-CX) 223,222,222
  222 P2=FLT*V*EX7
      C1=1.5*P2
      C2=0.0
      P00=CX*V
      SLP=0.0
      VC=1.0
      GO TO 244
  223 C2=V2*CRP*CRP-1.0-C1*EX7
      MTST =0
      L=2
      C4=2.0*DIT
  261 M=2
  262 VC= FALFA(M)
  263 C5=V2 * PR2ST(L)-1.0-C1*VC* QST(L)
      CX=PRST(L)
  265 IF (MTST) 266,266,231
  266 C6=C5* C2
      IF(C6) 268,268,267
  267 L=L+2
      M=M+1
      C2=C5
      IF(L-50) 262,262,275
  268 MTST =1
      C7=C5
      L=L-1
      VCM = FALFA(M-1)
      VCS=VC
      DVC=( VC- VCM)/C4
      VC = (VC+VCM)/2.0
      GO TO 263
  231 C1 = C4 * C4
      C1 = 2.0*(C2+C7-2.0*C5)/C1
      C3 =(C7-C2)/C4
      IF (C1) 233,232,233
  232 C6= -C5/C3
      GO TO 236
  233 C4= 0.5* C3/C1
```

```
      C6= SQRTF(C4*C4-C5/C1)
      IF(C4) 234,235,235
  234 C6= -C6
  235 C6= C6-C4
  236 C6= CX + C6
  237 P2=(V2*CX*CX-1.0)/P2LD
      P00=CX*V
      EX2=CX**EX1
      C1=1.5*P2
      VC=VC+C6*DVC
      C4=VC*FLT*V
      C4=P2/C4
      C5=-DVC/VC
      C6=0.5*VC
      C2=C6*FLT/P00*(C5*C4+EX4*EX2/C4*(EX6-EX2/CX))
      SLP=-2.0*C2/FLT*P00
  244 IF(KOP1) 245,271,245
C          JOURNAL BEARING CALCULATIONS
  271 P0(1)=P00
      MTST=1+MT
      C4=P2*DELTA
      C6=P2*FLD-C4
      DO 213 J=2,MTST
      P0(J)=SQRTF(1.0+C6)
  213 C6=C6-C4
C     INITIATE FREQUENCY NO. LOOP
      DO 246 J=1,NSIG
      SIG=FSIG(J)
C     CALCULATE A,B,C AND D AND CENTER LOAD CONSTANTS
      SGPZ=SIG*P00
      FFCI=SIG/P00
      FUNK=1.0+FFCI*FFCI
      AL =   SQRTF( FUNK)
      BETAJ = AL - 1.0
      BETAJ = SQRTF(0.5 * BETAJ)
      AL = AL +1.0
      AL = SQRTF(0.5 * AL)
      FFCR=1.0/(FUNK*P00)
      FFCI=-FFCI/(FUNK*P00)
      FSPR=SIG/FUNK*SIG
      FSPI=SIG/FUNK*P00
      SH2A=2.0*FL1D*AL
      SH2A=EXPF(SH2A)
      SN2B=1.0/SH2A
      CH2A=(SH2A+SN2B)*0.5
      SH2A=(SH2A-SN2B)*0.5
      SN2B=2.0*FL1D*BETAJ
      CS2B=COSF(SN2B)
      SN2B=SINF(SN2B)
      CTR=CH2A+CS2B
      THR=(AL*SH2A-BETAJ*SN2B)/CTR
      THI=(AL*SN2B+BETAJ*SH2A)/CTR
      IF(FL1D) 211,219,211
  21 CTR=0.0
      CTI=0.0
      FSPR=0.0
      FSPI=0.0
      GO TO 212
  211 CTI=CH2A-CS2B
      CTR=(AL*SH2A+BETAJ*SN2B)/CTI
      CTI=(BETAJ*SH2A-AL*SN2B)/CTI
  212 A=C1+THR*FSPR-THI*FSPI
      B=TH *FSPI+THI*FSPR
```

```
      C=THR+C2
      D=THI
C     FINITE DIFFERENCE EQUATIONS
      FF=0.0
      K=1
      M=MT+1
      DO 207 N=1,M
  207 G(N)=SIG*PO(N)
  218 QR=0.5*DELTAS
      QI=QR/POO*SIG
      PR=QR*FF
      PI=QR*G(1)
      SR(1)=0.0
      SI(1)=0.0
      TI(1)=0.0
      TR(1)=1.0
      DO 209 N=2,M
      SR(N)=SR(N-1)+PR+DELTA*A
      SI(N)=SI(N-1)+PI+DELTA*B
      TR(N)=TR(N-1)+QR+DELTA*C
      TI(N)=TI(N-1)+QI+DELTA*D
      IF(N-MT) 208,208,210
  208 C6=SIG/PO(N)
      PR=PR+DELTAS*(SR(N)-C6*SI(N)+FF)
      PI=PI+DELTAS*(SI(N)+C6*SR(N)+G(N))
      QR=QR+DELTAS*(TR(N)-C6*TI(N))
  209 QI=QI+DELTAS*(TI(N)+C6*TR(N))
  210 DHRI=TR(M)*TR(M)+TI(M)*TI(M)
      HIO=(TI(M)*SR(M)-TR(M)*SI(M))/DHRI
      HRO=-(TI(M)*SI(M)+TR(M)*SR(M))/DHRI
      GO TO (600,601),K
C     JOURNAL BEARING LOAD,TRANSLATION
  600 REL=0.0
      EEL=0.0
      DO 302 N=1,M
      PR1(N)=(SR(N)+TR(N)*HRO-TI(N)*HIO)/PO(N)
      PI1(N)=(SI(N)+TR(N)*HIO+TI(N)*HRO)/PO(N)
      REL=REL+PR1(N)
  302 EEL=EEL+PI1(N)
      REL=DELTA*(0.5*PR1(1)-REL)
      EEL=DELTA*(EEL-0.5*PI1(1))
      IF(FL1D) 214,215,214
  215 RCL=0.0
      ECL=0.0
      GO TO 216
  214 RF=HRO+FSPR
      EF=HIO+FSPI
      C6=RF*THR-EF*THI
      EF=RF*THI+EF*THR-SGPZ*FL1D
      RCL=-FFCR*C6+FFCI*EF
      ECL=FFCR*EF+FFCI*C6
  216 C6=REL+RCL
      C4= -(EEL+ECL)
      W=1.5707963/(FLD+FL1D)*SQRTF(C6*C6+C4*C4)
      THETA=ANG(C6,C4)
C     JOURNAL BEARING MOMENT,ROTATION
      FF=C1
      K=2
      C4=FL1D
      DO 217 N=1,M
      G(N)=G(N)*C4
  217 C4=C4+DELTA
      A=FL1D*(C1+FSPR*CTR-FSPI*CTI)-FSPR
```

```
      B=FL1D*(FSPR*CTI+FSPI*CTR)-FSPI
      C=CTR+C2
      D=CTI
      HTRZ=HR0
      HTEZ=HI0
      GO TO 218
  601 REM=0.0
      EEM=0.0
      C4=FL1D
      DO 220 N=1,M
      PR2(N)=(SR(N)+TR(N)*HR0-TI(N)*HI0)/P0(N)
      PI2(N)=(SI(N)+TR(N)*HI0+TI(N)*HR0)/P0(N)
      REM=REM+C4*PR2(N)
      EEM=EEM+C4*PI2(N)
  220 C4=C4+DELTA
      REM=DELTA*(0.5*PR2(1)*FL1D-REM)
      EEM=DELTA*(EEM-0.5*PI2(1)*FL1D)
      IF(FL1D) 224,221,224
  221 RCM=0.0
      ECM=0.0
      GO TO 225
  224 RF=HR0+FL1D*FSPR
      EF=HI0+FL1D*FSPI
      C4=FL1D*CTR-1.0
      C6=FL1D*CTI
      ECM=RF*C4-EF*C6
      C4=RF*C6+EF*C4-SGPZ/3.0*(FL1D**3)
      RCM=FFCI*C4-FFCR*ECM
      ECM=FFCR*C4+FFCI*ECM
  225 C6=REM+RCM
      C4=-(EEM+ECM)
      RMT=0.78539816/(FLD+FL1D)*SQRTF(C6*C6+C4*C4)
      THMT=ANG(C6,C4)
C     JOURNAL BEARING OUTPUT
      WRITE OUTPUT TAPE 3,106
      WRITE OUTPUT TAPE 3,107,FLD,FL1D,V,FLT,SIG,P2,CX
      WRITE OUTPUT TAPE 3,108
      WRITE OUTPUT TAPE 3,107,P00,VC,SLP,HTRZ,HTEZ,HR0,HI0
      WRITE OUTPUT TAPE 3,109
      WRITE OUTPUT TAPE 3,110,W,THETA,REL,EEL,RCL,ECL
      WRITE OUTPUT TAPE 3,111
      WRITE OUTPUT TAPE 3,110,RMT,THMT,REM,EEM,RCM,ECM
      IF(NPR) 240,246,240
  240 WRITE OUTPUT TAPE 3,112
      WRITE OUTPUT TAPE 3,113
      C4=FL1D
      DO 241 N=1,M
      WRITE OUTPUT TAPE 3,110,C4,P0(N),PR1(N),PI1(N),PR2(N),PI2(N)
  241 C4=C4+DELTA
  246 CONTINUE
      GO TO 242
C     THRUST BEARING CALCULATION
  245 A=BP1*C1
      B=BP1*C2
      M=1+MT
      C3=BETAT*P2
      C4=1.0
      C5=1.0
      C6=1.0+P2*P2LD
      DO 247 N=1,M
      C=LOGF(C4)
      D=LOGF(C5)
      P0(N)=SQRTF(C6-P2*C)
```

```
      PZG(N)=SQRTF(C6+C3*D)
      C4=C4+DELTA
  247 C5=C5-DELG
C     INITIATE FREQUENCY NO. LOOP
      DO 290 J=1,NSIG
      SIG=FSIG(J)
C     FINITE DIFFERENCE EQUATIONS
      K=1
      ERC=0.0
      FRC=0.0
      ER=0.0
      FR=0.0
  248 ADR=DELTA
      BDR=DELTA
      C=0.0
      D=0.0
      DO 249 N=1,M
  249 G(N)=PO(N)
  250 C3=1.0
      C4=1.0+BDR
      SR(1)=0.0
      SI(1)=0.0
      TR(1)=0.0
      TI(1)=0.0
      VI(1)=0.0
      VR(1)=1.0
      C6=0.5*ADR
      PR=C6*FRC
      PI=C6*SIG*P00
      WR=C6*ERC
      WI=C6/P00*SIG
      QR=0.0
      QI=0.0
      DO 254 N=2,M
      C5=LOGF(C4/C3)
      SR(N)=SR(N-1)+C5*(PR-C)
      SI(N)=SI(N-1)+C5*PI
      TR(N)=TR(N-1)+C5*(1.0+QR)
      TI(N)=TI(N-1)+C5*QI
      VR(N)=VR(N-1)+C5*(WR-D)
      VI(N)=VI(N-1)+C5*WI
      IF(N-M) 251,255,255
  251 C3=C3+BDR
      C4=C4+BDR
      FE=G(N)
      EE=SIG/FE
      FE=SIG*FE
      IF(K-2) 253,253,252
  252 FE=FE*C3
      FR=FRC/C3
      ER=1.0/C3
      ER=ER*ER
  253 C5=ADR*C3
      PR=PR+C5*(ER*SR(N)-FE*SI(N)+FR)
      PI=PI+C5*(ER*SI(N)+FE*SR(N)+FE)
      QR=QR+C5*(ER*TR(N)-EE*TI(N))
      QI=QI+C5*(ER*TI(N)+FE*TR(N))
      WR=WR+C5*(ER*VR(N)-FE*VI(N))
  254 WI=WI+C5*(ER*VI(N)+FE*VR(N))
  255 GO TO (256,278,270,278),K
  270 FRC=-C1*BETAT
  256 ADR=DELG
      BDR=-DELG
```

```
      C=A
      D=B
      DO 257 N=1,M
      G(N)=PZG(N)
      TSR(N)=SR(N)
      TSI(N)=SI(N)
      TTR(N)=TR(N)
      TTI(N)=TI(N)
      TVR(N)=VR(N)
  257 TVI(N)=VI(N)
      K=K+1
      GO TO 250
  278 D1=TSR(M)
      D2=TSI(M)
      D3=TTR(M)
      D4=TTI(M)
      D5=TVR(M)
      D6=TVI(M)
      G1=SR(M)
      G2=SI(M)
      G3=TR(M)
      G4=TI(M)
      G5=VR(M)
      G6=VI(M)
      C3=D4*G6+G3*D5-D3*G5-G4*D6
      C4=D3*G6+D4*G5-G3*D6-G4*D5
      C5=D1*G5-D2*G6-G1*D5+G2*D6
      C6=D1*G6+D2*G5-G1*D6-G2*D5
      C7=C3*C3+C4*C4
      HPR=(C5*C3-C6*C4)/C7
      HPE=(C5*C4+C6*C3)/C7
      C5=D3*G1-D4*G2-G3*D1+G4*D2
      C6=D3*G2+D4*G1-G3*D2-G4*D1
      HRO=(C5*C3-C6*C4)/C7
      HIO=(C5*C4+C6*C3)/C7
      IF(K-2) 279,279,264
C     THRUST BEARING LOAD,TRANSLATION
  279 ERC=1.0
      FRC=C1
      K=3
      STW=-0.5*(POO+FLD)
      STC=-0.5*(POO+GMNV)
      REL=0.0
      EEL=0.0
      RCL=0.0
      ECL=0.0
      C3=1.0
      C4=1.0
      DO 260 N=1,M
      PR1(N)=(TSR(N)+TTR(N)*HPR-TTI(N)*HPE+TVR(N)*HRO-TVI(N)*HIO)/PO(N)
      PI1(N)=(TSI(N)+TTR(N)*HPE+TTI(N)*HPR+TVR(N)*HIO+TVI(N)*HRO)/PO(N)
      PR2(N)=(SR(N)+TR(N)*HPR-TI(N)*HPE+VR(N)*HRO-VI(N)*HIO)/PZG(N)
      PI2(N)=(SI(N)+TR(N)*HPE+TI(N)*HPR+VR(N)*HIO+VI(N)*HRO)/PZG(N)
      REL=REL+C3*PR1(N)
      EEL=EEL+C3*PI1(N)
      RCL=RCL+C4*PR2(N)
      ECL=ECL+C4*PI2(N)
      STW=STW+C3*PO(N)
      STC=STC+C4*PZG(N)
      C3=C3+DELTA
  260 C4=C4-DELG
      REL=DELTA*(REL-0.5*PR1(1))
      EEL=DELTA*(EEL-0.5*PI1(1))
```

```
      RCL=DELG*(RCL-0.5*PR2(1))
      ECL=DELG*(ECL-0.5*PI2(1))
      STW=DELTA*STW*2.0*FRFT
      STC=DELG*STC*2.0*FRFT
      WST=STW+STC
      C6=-(REL+RCL)
      C4=-EEL-ECL
      W=2.0*FRFT*SQRTF(C6*C6+C4*C4)
      THETA=ANG(C6,C4)
      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,107,FLD,FLID,V,FLT,SIG,P2,CX
      WRITE OUTPUT TAPE 3,116
      WRITE OUTPUT TAPE 3,110,P00,VC,SLP,STW,STC,WST
      WRITE OUTPUT TAPE 3,117
      WRITE OUTPUT TAPE 3,110,W,THETA,REL,EEL,RCL,ECL
  286 WRITE OUTPUT TAPE 3,118
      WRITE OUTPUT TAPE 3,119,HR0,HI0,HPR,HPE
      IF(NPR) 281,283,281
  281 WRITE OUTPUT TAPE 3,128
      C3=1.0
      C4=1.0
      DO 282 N=1,M
      WRITE OUTPUT TAPE 3,121,C3,P0(N),PR1(N),PI1(N),C4,PZG(N),PR2(N)
     1PI2(N)
      C3=C3+DELTA
  282 C4=C4-DELG
  283 IF(K-3) 248,248,290
C     THRUST BEARING MOMENT,ROTATION
  264 REM=0.0
      EEM=0.0
      RCM=0.0
      ECM=0.0
      C3=1.0
      C4=1.0
      DO 285 N=1,M
      PR1(N)=(TSR(N)+TTR(N)*HPR-TTI(N)*HPE+TVR(N)*HR0-TVI(N)*HI0)/P0(
      PI1(N)=(TSI(N)+TTR(N)*HPE+TTI(N)*HPR+TVR(N)*HI0+TVI(N)*HR0)/P0(N
      PR2(N)=(SR(N)+TR(N)*HPR-TI(N)*HPE+VR(N)*HR0-VI(N)*HI0)/PZG(N)
      PI2(N)=(SI(N)+TR(N)*HPE+TI(N)*HPR+VR(N)*HI0+VI(N)*HR0)/PZG(N)
      C5=C3*C3
      C6=C4*C4
      REM=REM+C5*PR1(N)
      EEM=EEM+C5*PI1(N)
      RCM=RCM+C6*PR2(N)
      ECM=ECM+C6*PI2(N)
      C3=C3+DELTA
  285 C4=C4-DELG
      REM=DELTA*(REM-0.5*PR1(1))
      EEM=DELTA*(EEM-0.5*PI1(1))
      RCM=DELG*(RCM-0.5*PR2(1))
      ECM=DELG*(ECM-0.5*PI2(1))
      C6=-(REM+RCM)
      C4=-EEM-ECM
      RMT=FRFT*SQRTF(C6*C6+C4*C4)
      THMT=ANG(C6,C4)
      WRITE OUTPUT TAPE 3,122
      WRITE OUTPUT TAPE 3,110,RMT,THMT,REM,EEM,RCM,ECM
      GO TO 286
  290 CONTINUE
  242 CONTINUE
      IF(NDIAG) 243,200,243
  243 END FILE 3
      STOP 77777
```

```
275 WRITE OUTPUT TAPE 3,129
    GO TO 242
100 FORMAT(72H0
   1                  )
101 FORMAT(72H
   1                  )
102 FORMAT(9I5,1PE15.7)
103 FORMAT(1P4E15.7)
106 FORMAT(////120H          L/D                L1/D          PRESS.RATIO
   1        LAMBDA-T       FREQUENCY NO.        DIM.FLOW       ORIF.PR.RAT
   2IO )
107 FORMAT(1P7E17.7)
108 FORMAT(120H0     ORIF.PRESS.      VENA CONT.CF.       DM/D(P/V)
   1TRANS.RE(H0)      TRANS.IM(H0)      ROTAT.RE(H0)      ROTAT.IM(H0)   )
109 FORMAT(102H0          LOAD          PHASE ANGLE      RE(END LOAD)
   1IM(END LOAD)      RE(CTR.LOAD)      IM(CTR.LOAD) )
110 FORMAT(1P6E17.7)
111 FORMAT(102H0         MOMENT         PHASE ANGLE      RE(END MOMT)
   1IM(END MOMT)      RE(CTR.MOMT)      IM(CTR.MOMT) )
112 FORMAT(96H0                                                  TRANSLATO
   1RY                          ROTATIONAL      )
113 FORMAT(102H   DIST.FR.FD.PL.          P0              RE(P1)
   1   IM(P1)            RE(P1)            IM(P1)     )
114 FORMAT(////120H    OUT.R/ORIF.R.    ORIF.R/INN.R.      PRESS.RATIO
   1        LAMBDA-T       FREQUENCY NO.        DIM.FLOW       ORIF.PR.RAT
   2IO )
115 FORMAT(84H1                                          MECHANICAL TECHNO
   1LOGY,INC.   JORGEN W.LUND    )
116 FORMAT(102H0     ORIF.PRESS.      VENA CONT.CF.       DM/D(P/V)
   1OUT.STAT.LOAD     INN.STAT.LOAD      STATIC LOAD )
117 FORMAT(102H0     DYNAMIC LOAD      PHASE ANGLE      RE(OUT.LOAD)
   1IM(OUT.LOAD)      RE(INN.LOAD)      IM(INN.LOAD) )
118 FORMAT(66H0       RE(H0)            IM(H0)            RE(HPR0)
   1 IM(HPR0) )
119 FORMAT(1P4E17.7)
120 FORMAT(108H                  PN0047  DYNAMIC LOAD FOR HYDROSTATIC JO
   1URNAL AND THRUST BEARING-FIRST ORDER PERTURBATION     )
121 FORMAT(1PE14.6,1P3E15.6,1PE16.6,1P3E15.6)
122 FORMAT(102H0       MOMENT          PHASE ANGLE      RE(OUT.MOMT)
   1IM(OUT.MOMT)      RE(INN.MOMT)      IM(INN.MOMT) )
123 FORMAT(23H0    FEEDING PARAMETERS)
124 FORMAT(1P7E17.7)
125 FORMAT(18H0    PRESS. RATIOS)
126 FORMAT(22H0    FREQUENCY NUMBERS)
127 FORMAT(32H0    VENA CONTRACTA COEFFICIENTS)
128 FORMAT(120H0RADIUS/ORIF.R          P0              RE(P1)          IM(P1
   1)       RADIUS/ORIF.R        P0              RE(P1)          IM(P1)    )
129 FORMAT(36H0NO ROOT IN ORIFICE FLOW CALCULATION)
131 FORMAT(18H0    L/D-RATIO     )
132 FORMAT(18H0    L1/D-RATIO    )
136 FORMAT(120H0  NO.L/D       NO.L1/D   NO.PR.RATIO  NO.LAMBDA   NO.FR
   1EQ.   JOURN/THR.   NO.INCR.   PRESS.OUTP.   INPUT    ADIAB.EXP.   )
137 FORMAT(I7,8I12,1PE16.7)
    END(0,1,0,1,1)
```

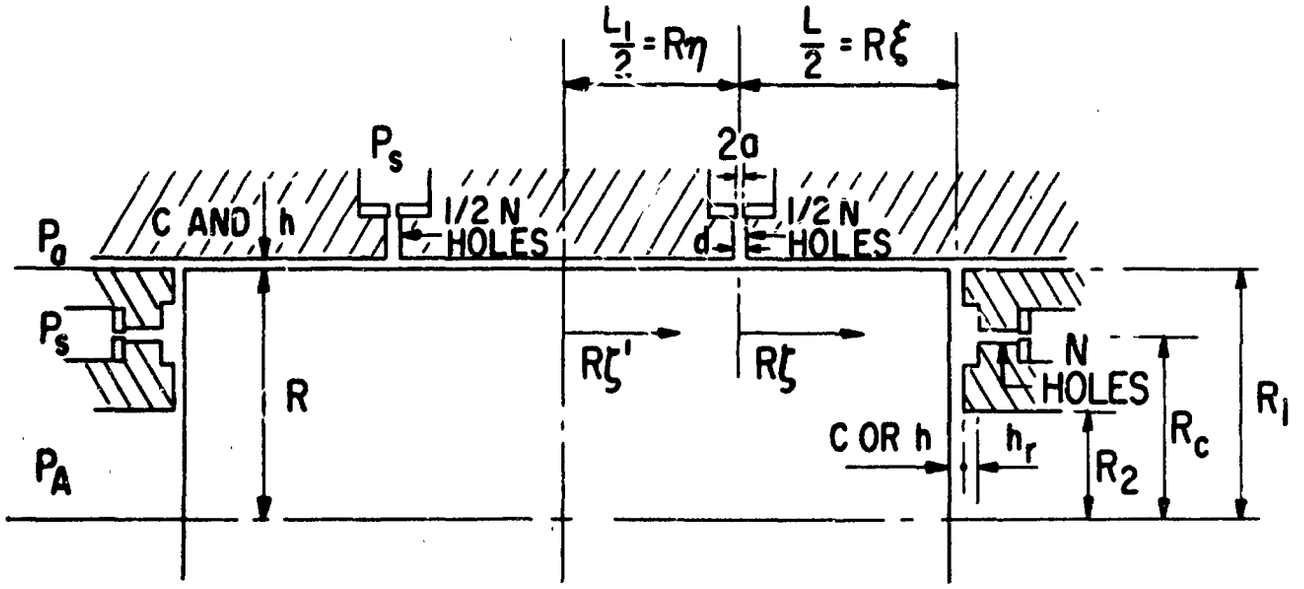


FIG. 1 GEOMETRY OF GIMBAL AIR BEARING

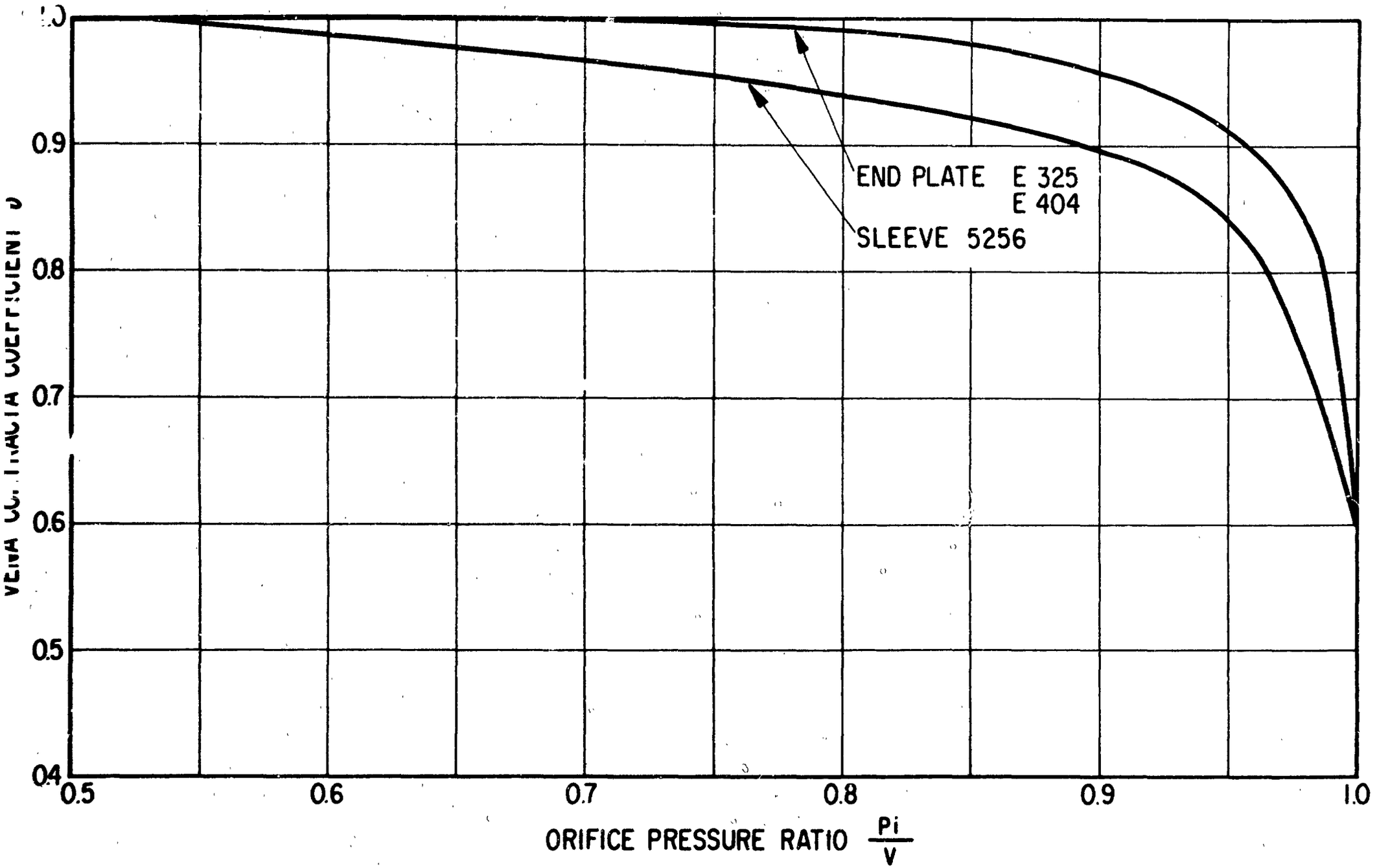


FIG. 2 VENA CONTRACTA COEFFICIENT

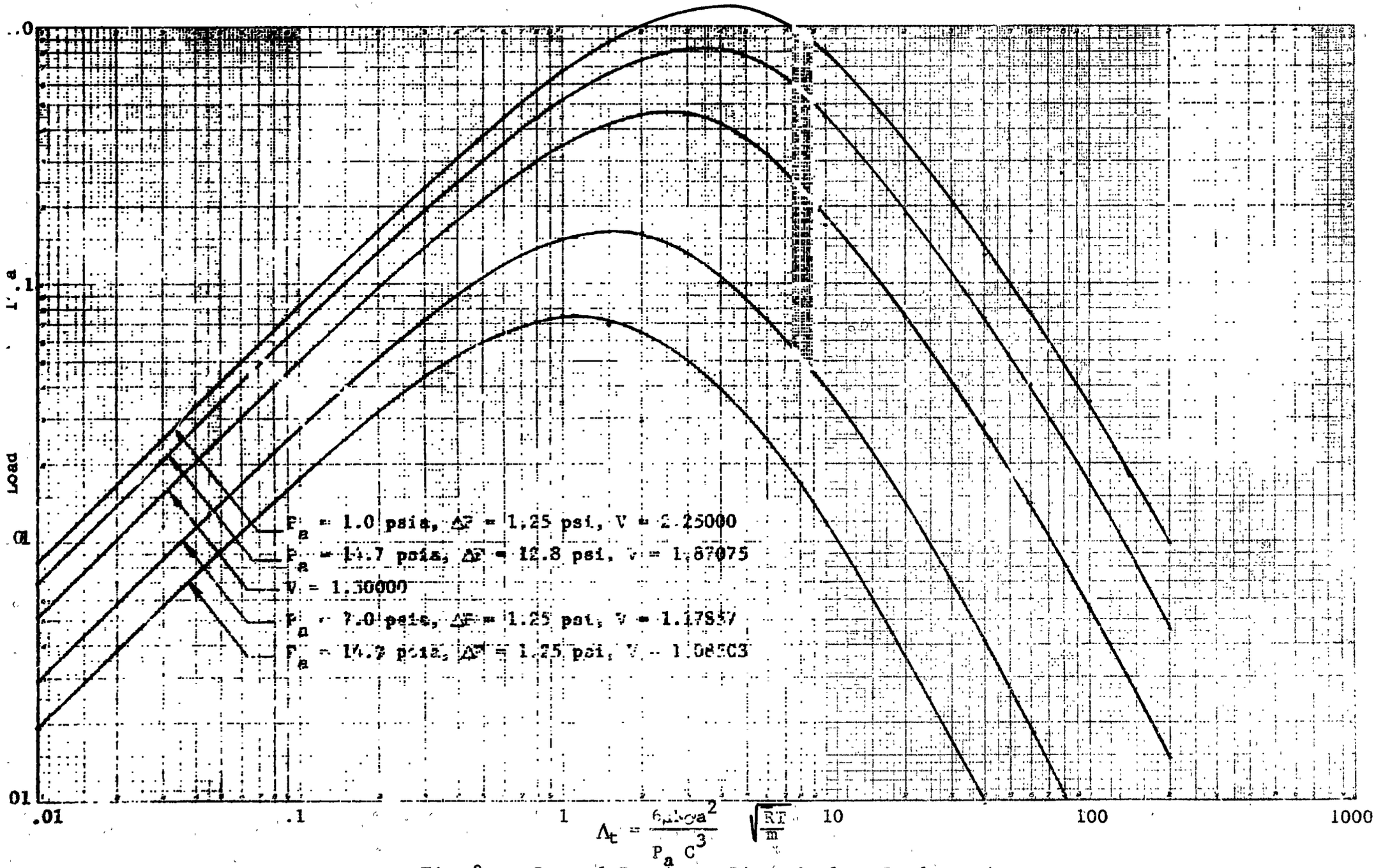


Fig. 3 Journal Bearing - Dimensionless Load vs. Λ_t

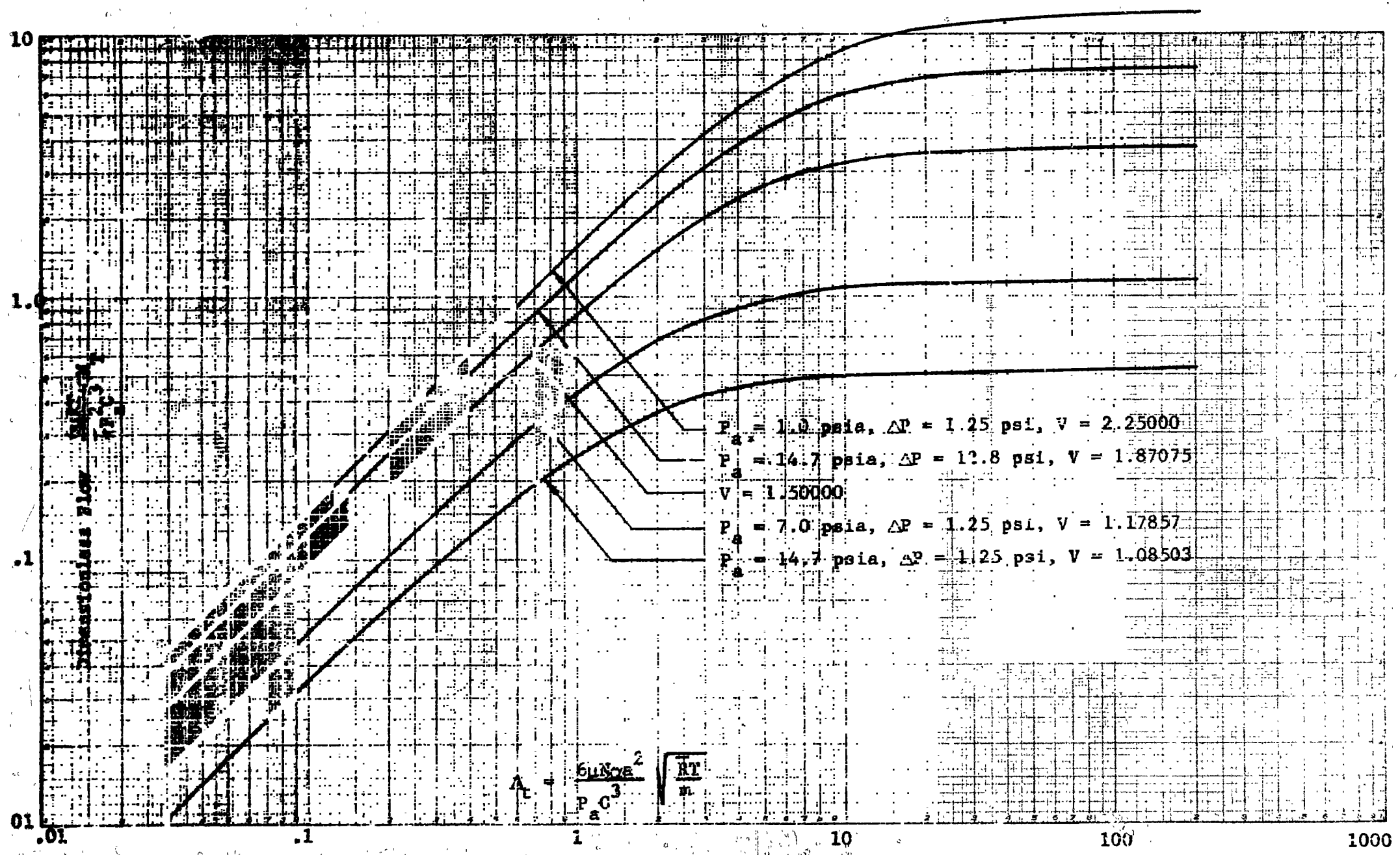


Fig. 4 Journal Bearing - Dimensionless Flow vs. Λ_t

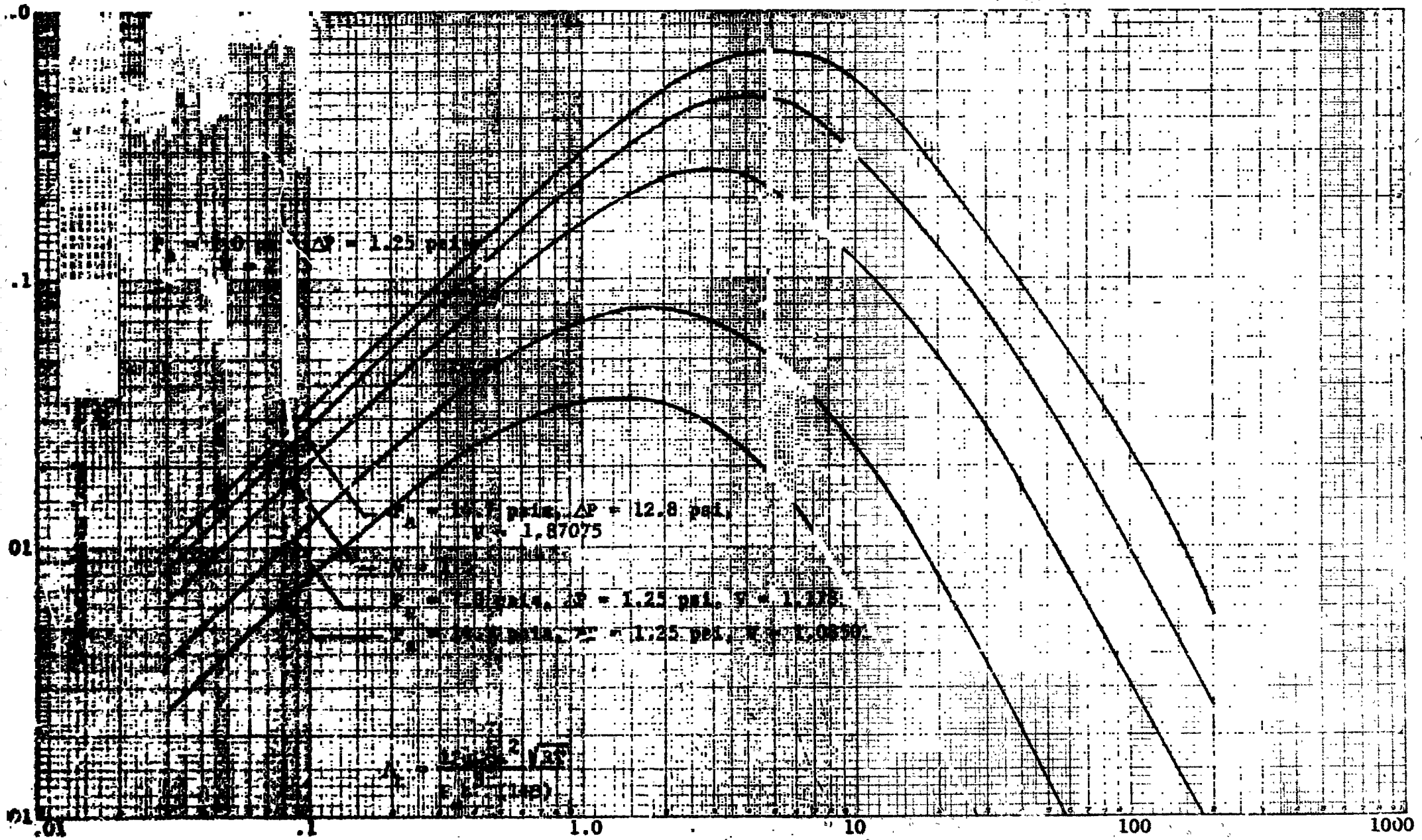


Fig. 5 Thrust Bearing - Dimensionless Load vs. Λ_t

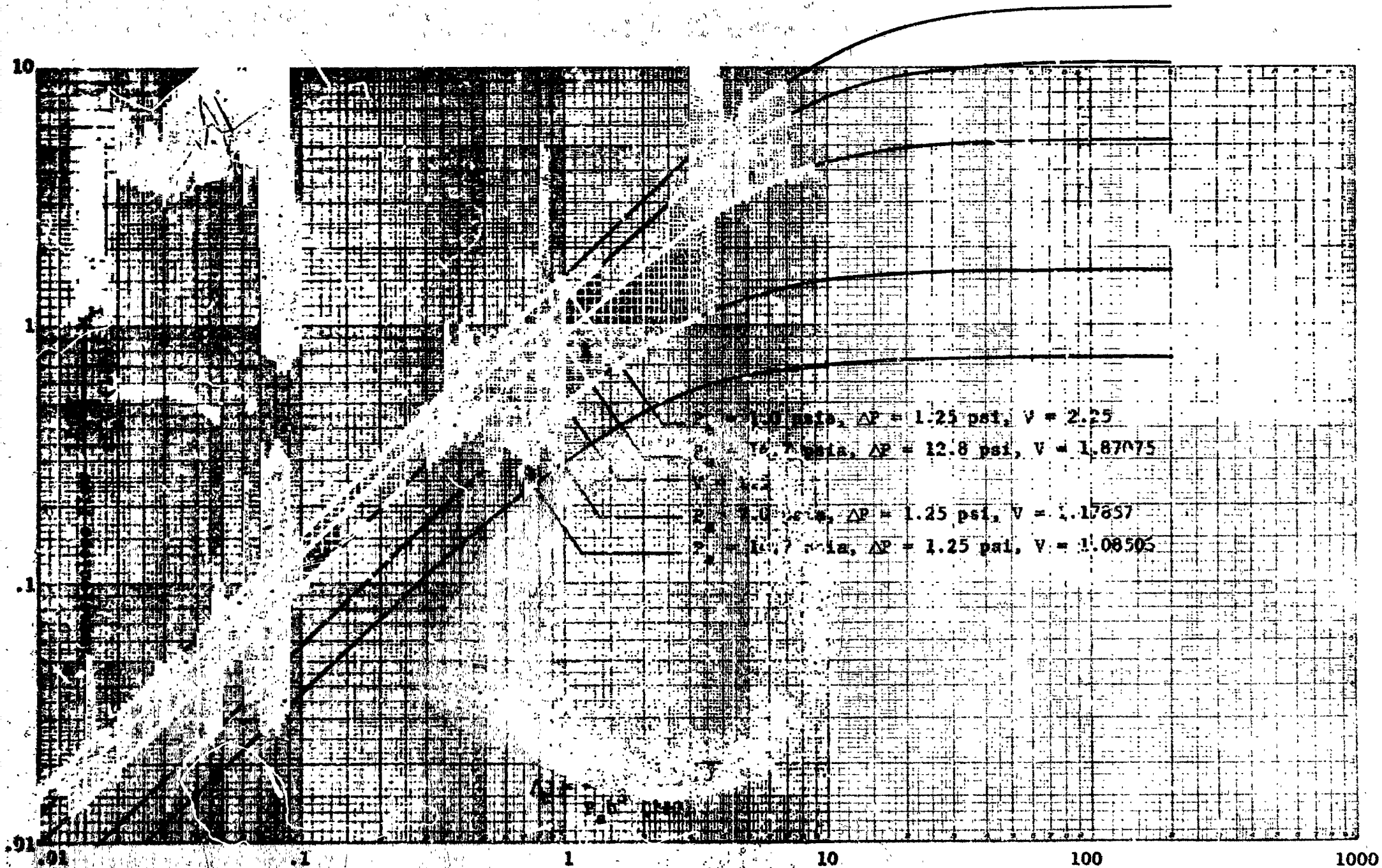


Fig. 6 Thrust Bearing - Dimensionless Flow vs. Λ_t

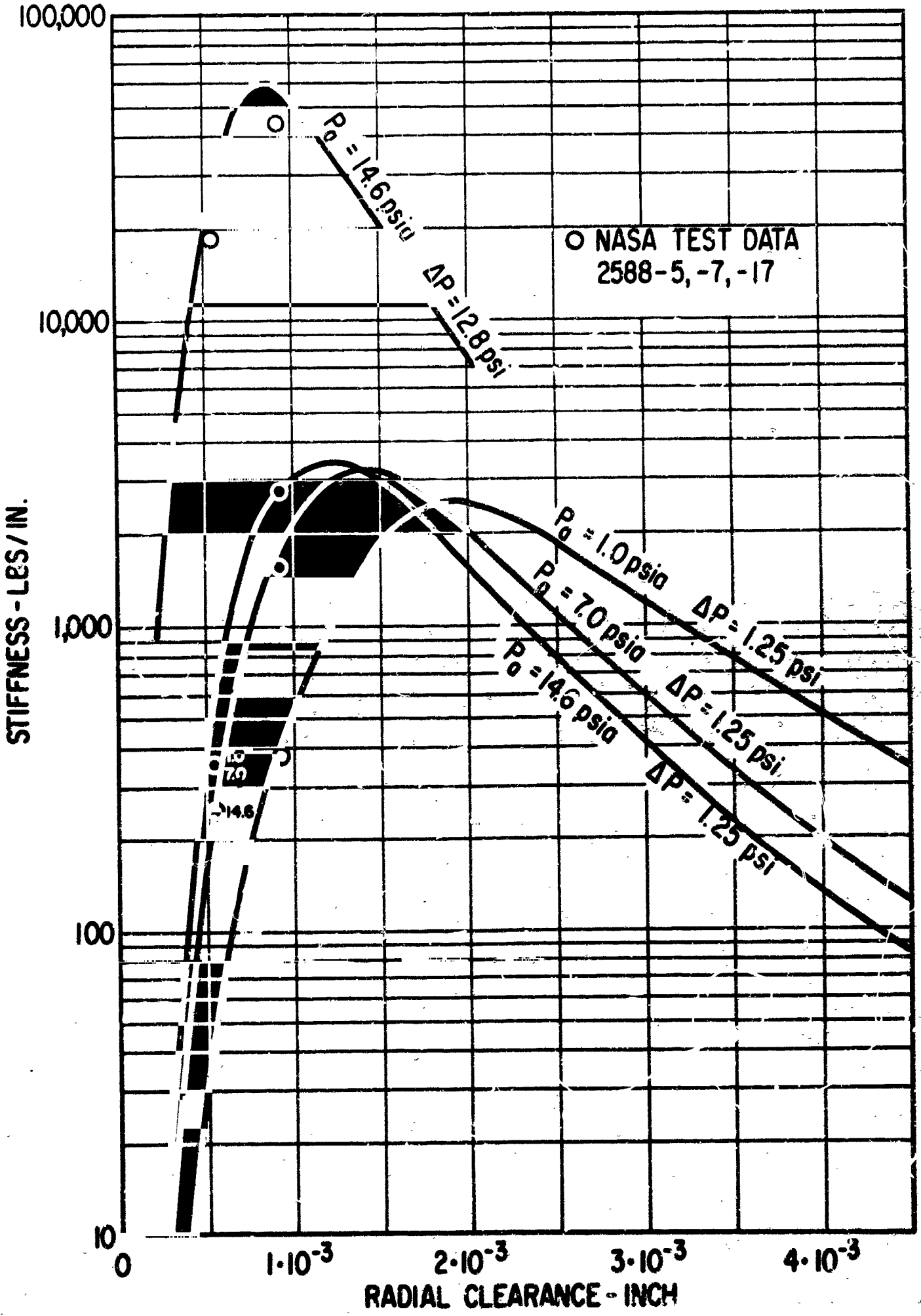


FIG. 7 AB-5 PIERCED HOLES AIRBEARING RADIAL STIFFNESS vs CLEARANCE

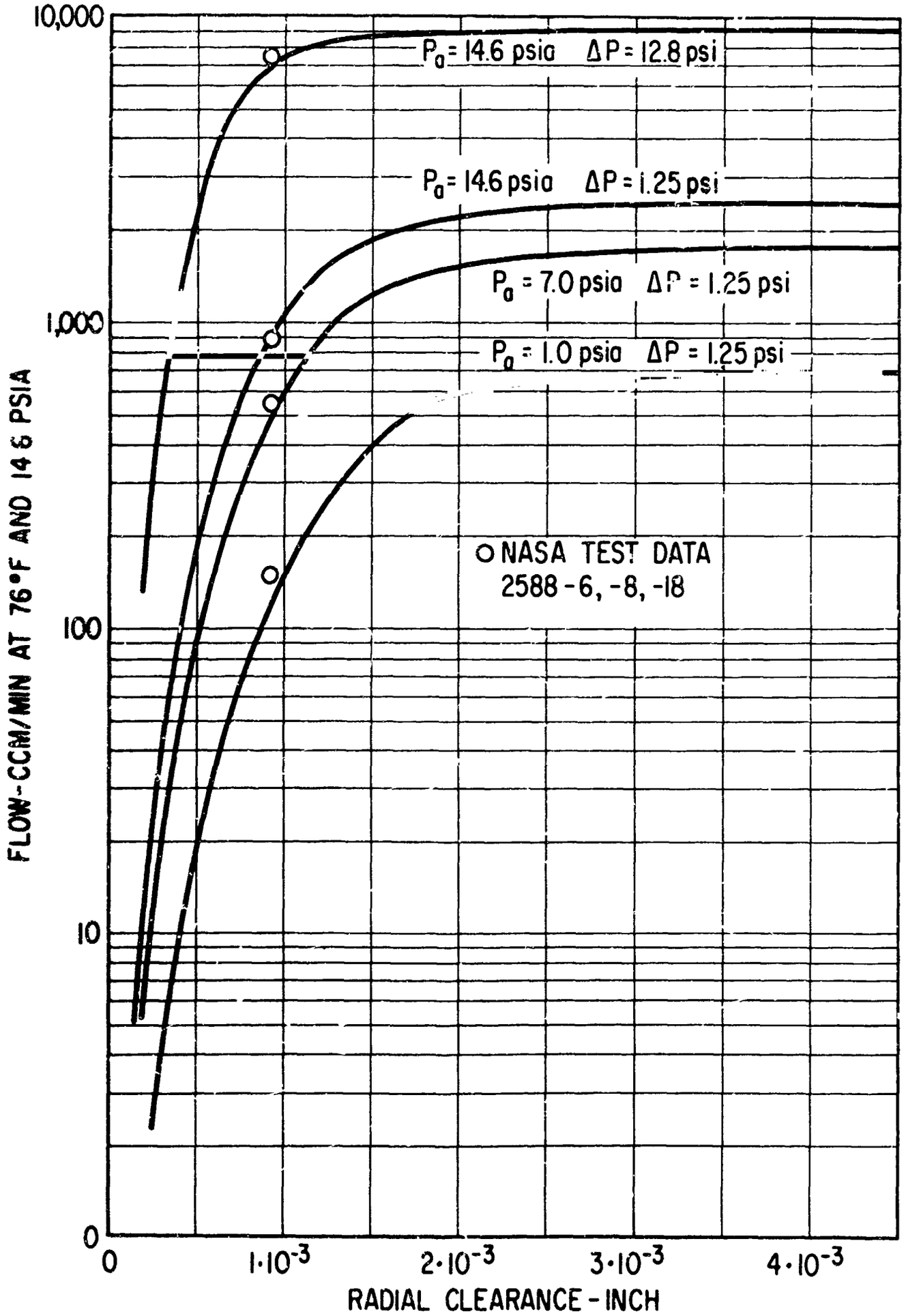


FIG. 8 AB-5 PIERCED HOLES AIRBEARING RADIAL LOADING - SLEEVE FLOW vs CLEARANCE

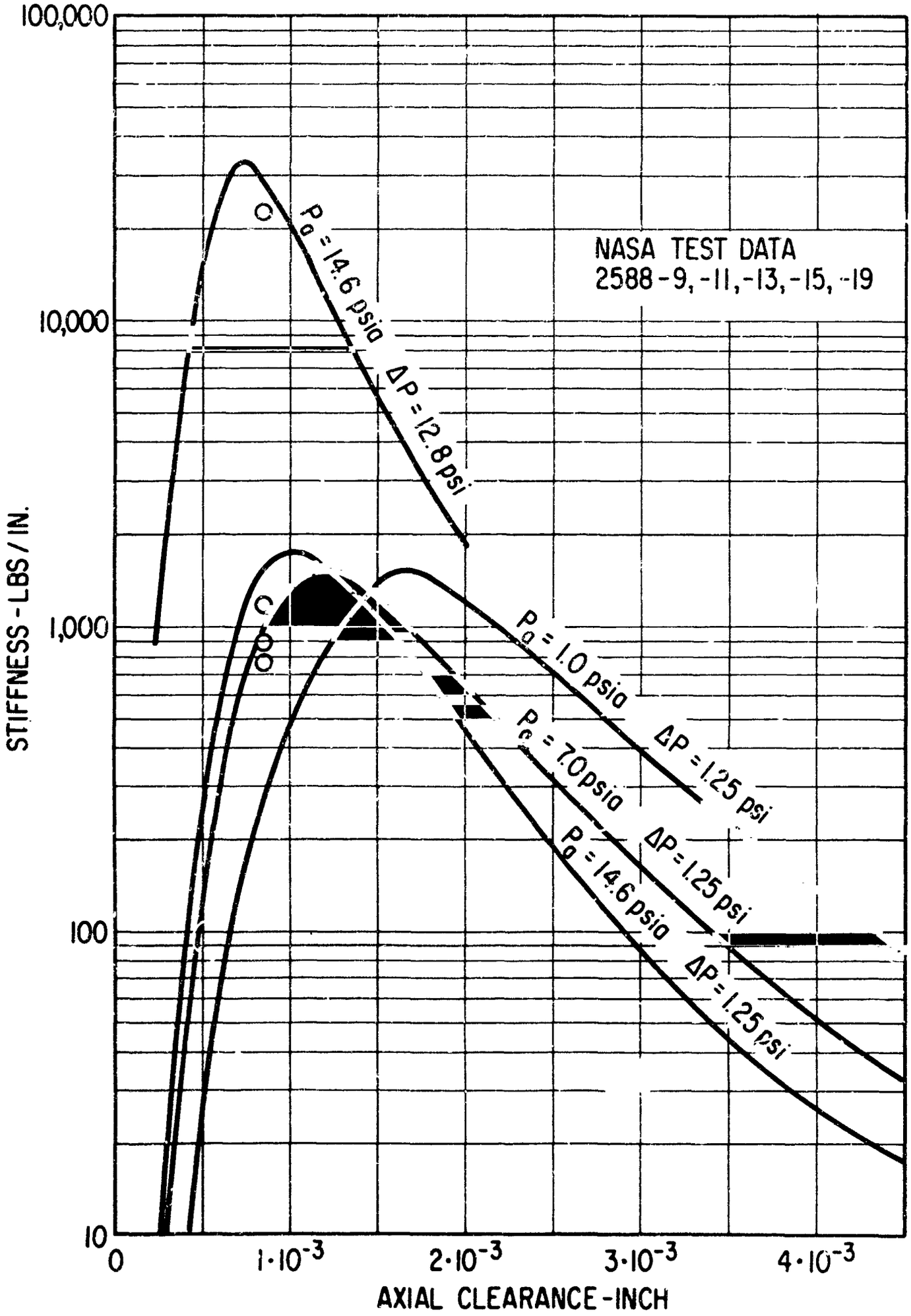


FIG. 9 AB-5 PIERCED HOLES AIRBEARING AXIAL STIFFNESS vs CLEARANCE

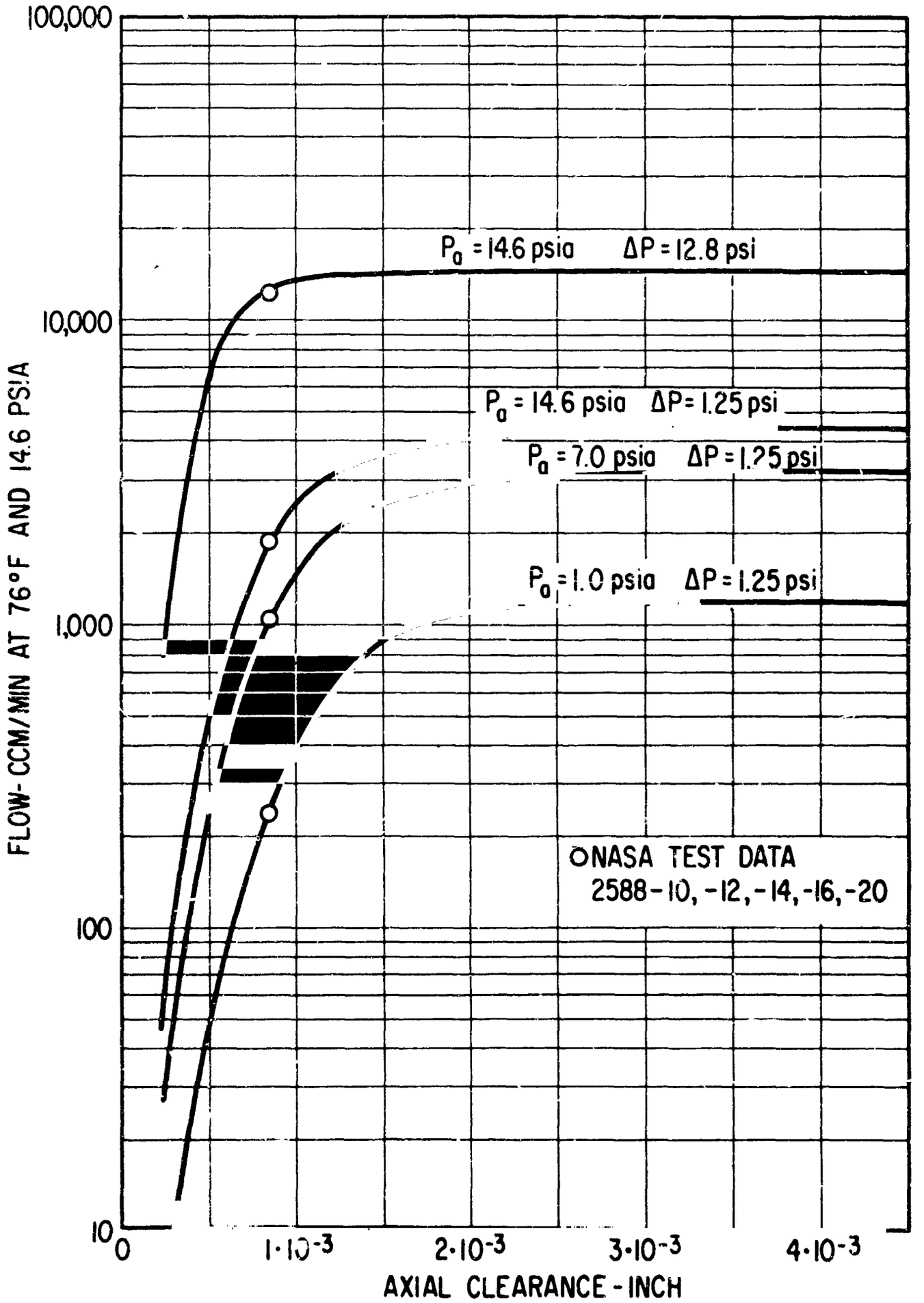


FIG. 10 AB-5 PIERCED HOLES AIRBEARING AXIAL LOADING - ENDPLATE FLOW vs CLEARANCE

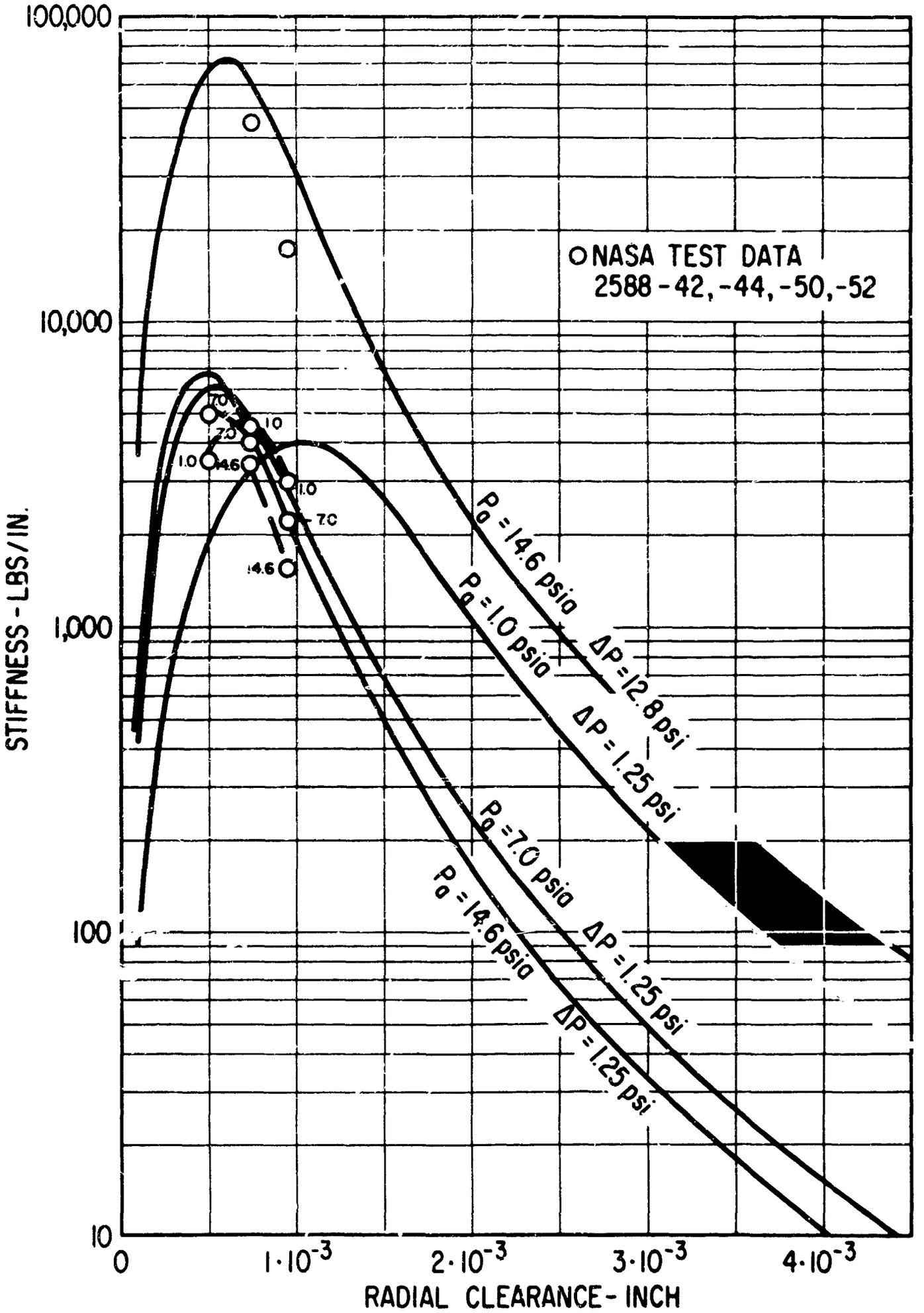


FIG. 11 AB-5 MILLIPORE AIRBEARING
RADIAL STIFFNESS vs CLEARANCE

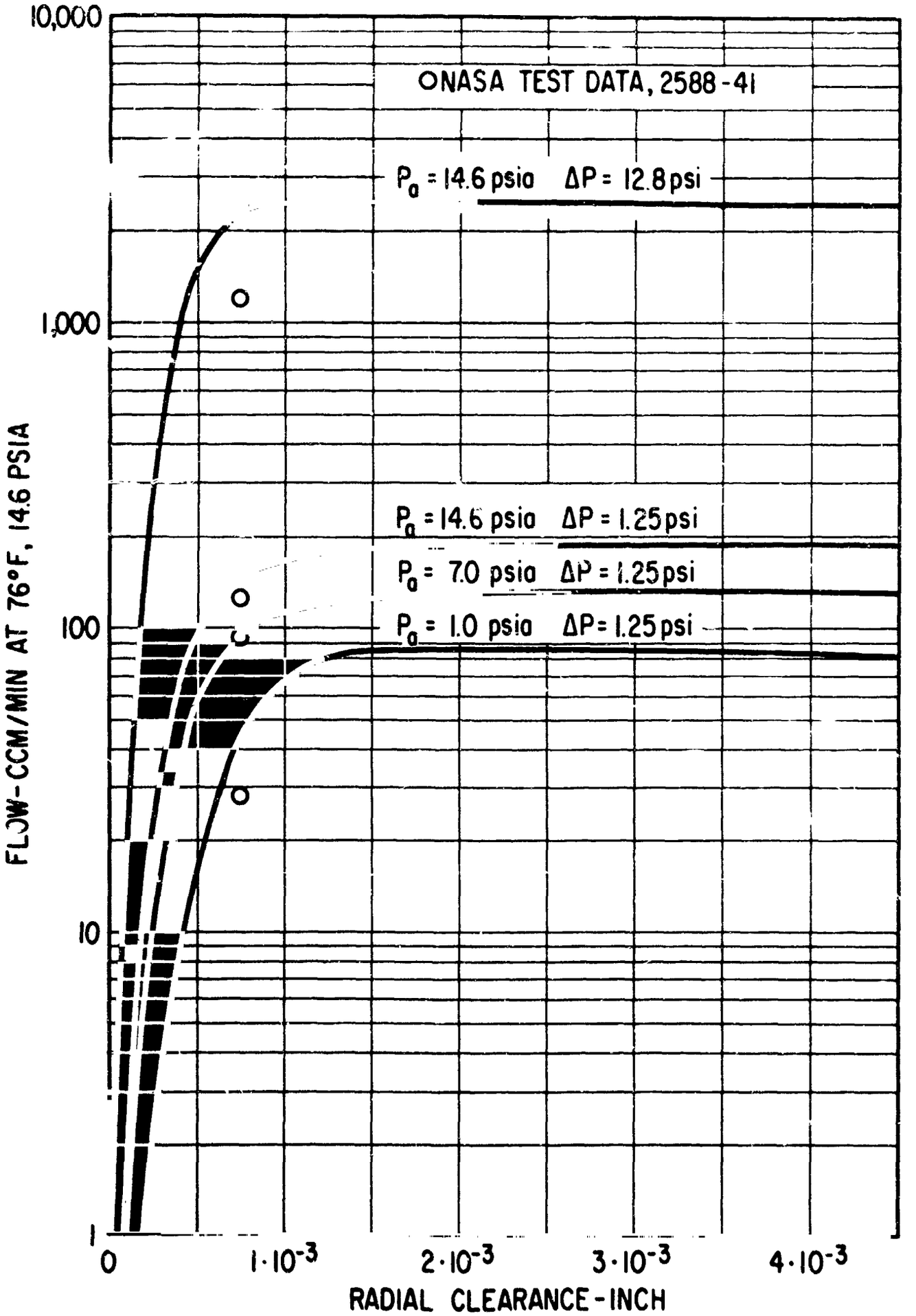


FIG. 12 AB-5 MILLIPORE AIRBEARING
RADIAL LOADING - SLEEVE
FLOW vs CLEARANCE

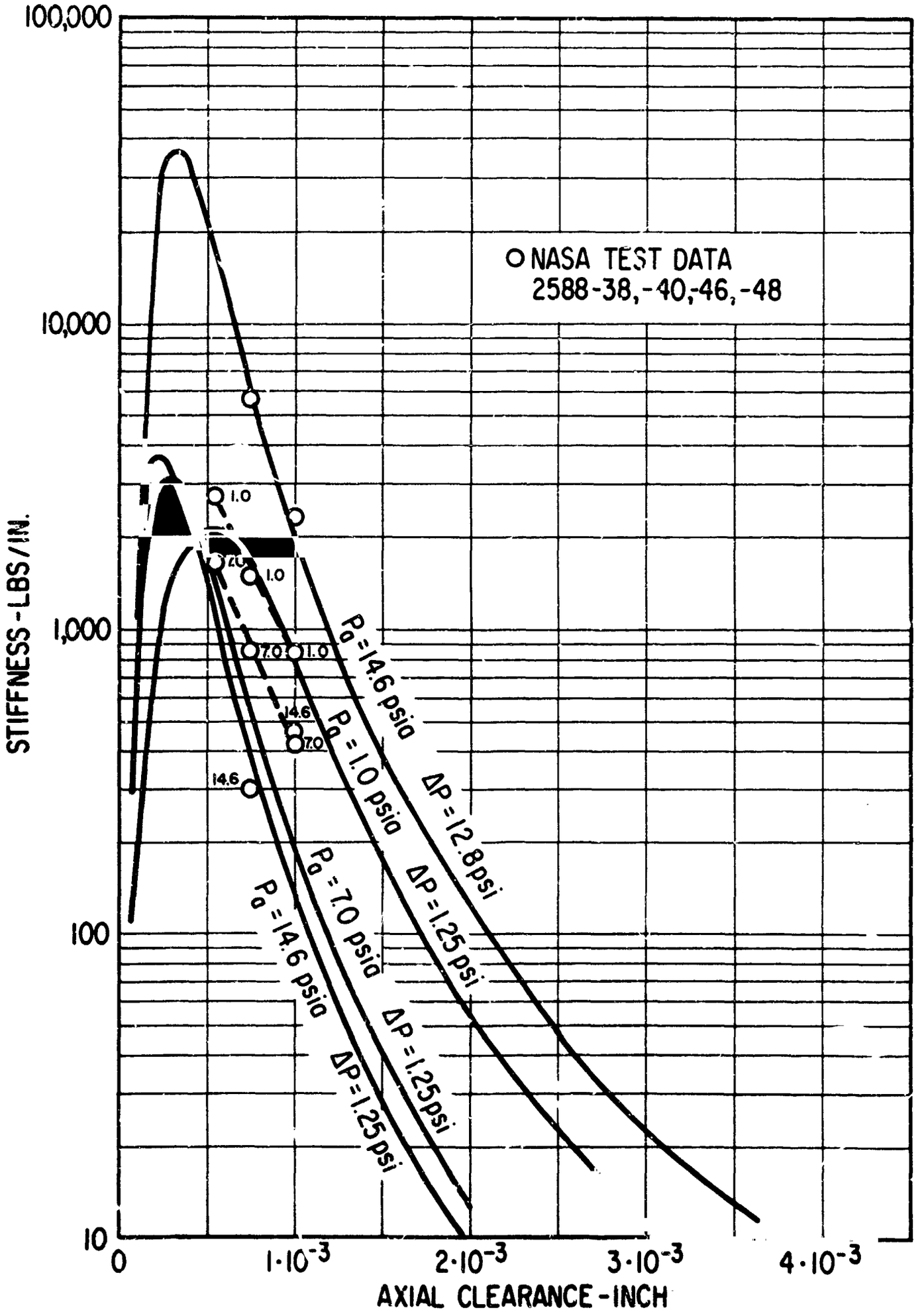


FIG. 13 AB-5 MILLIPORE AIRBEARING
AXIAL STIFFNESS vs CLEARANCE

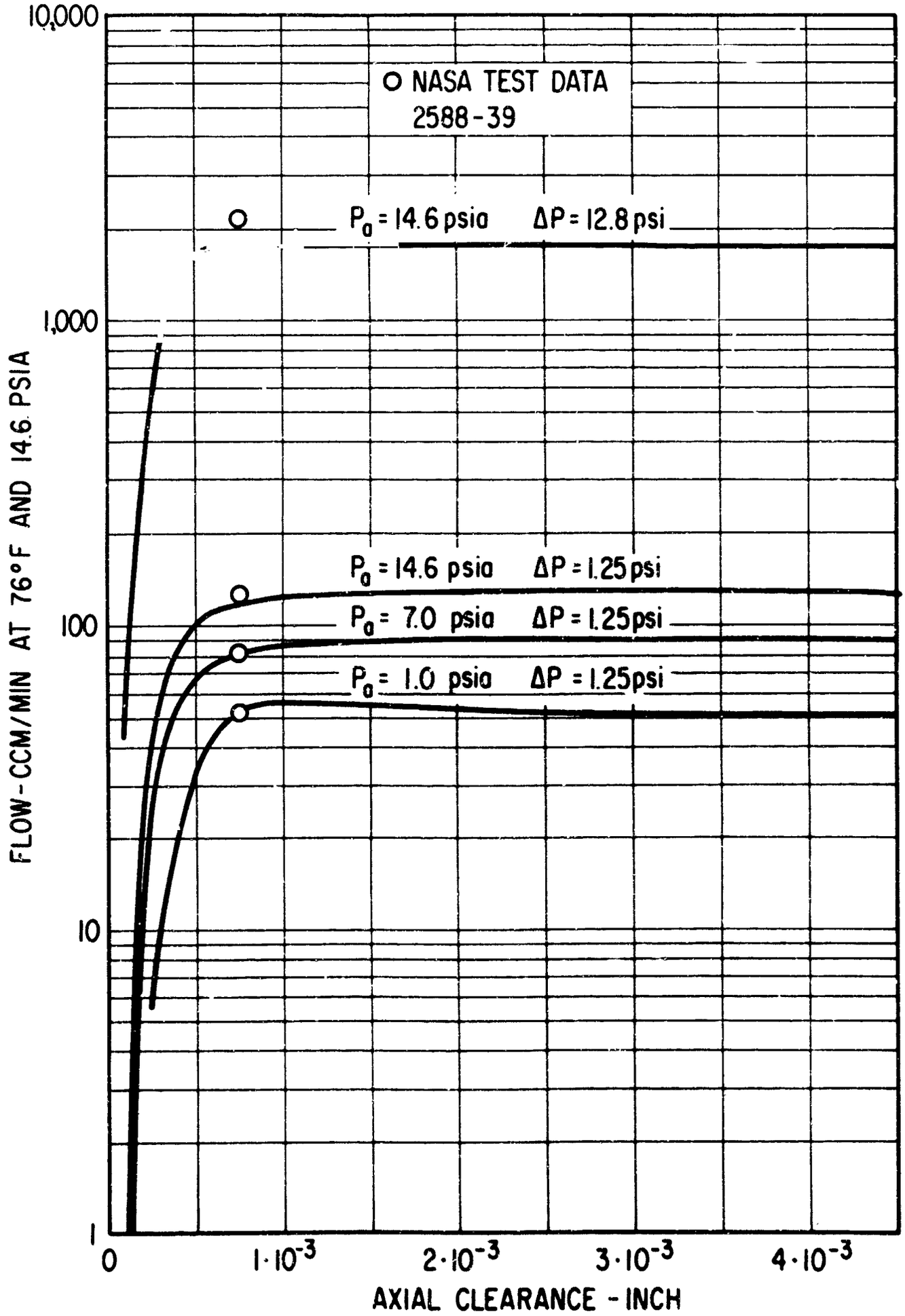


FIG. 14 AB-5 MILLIPORE AIRBEARING
AXIAL LOADING - ENDPLATE
FLOW vs CLEARANCE

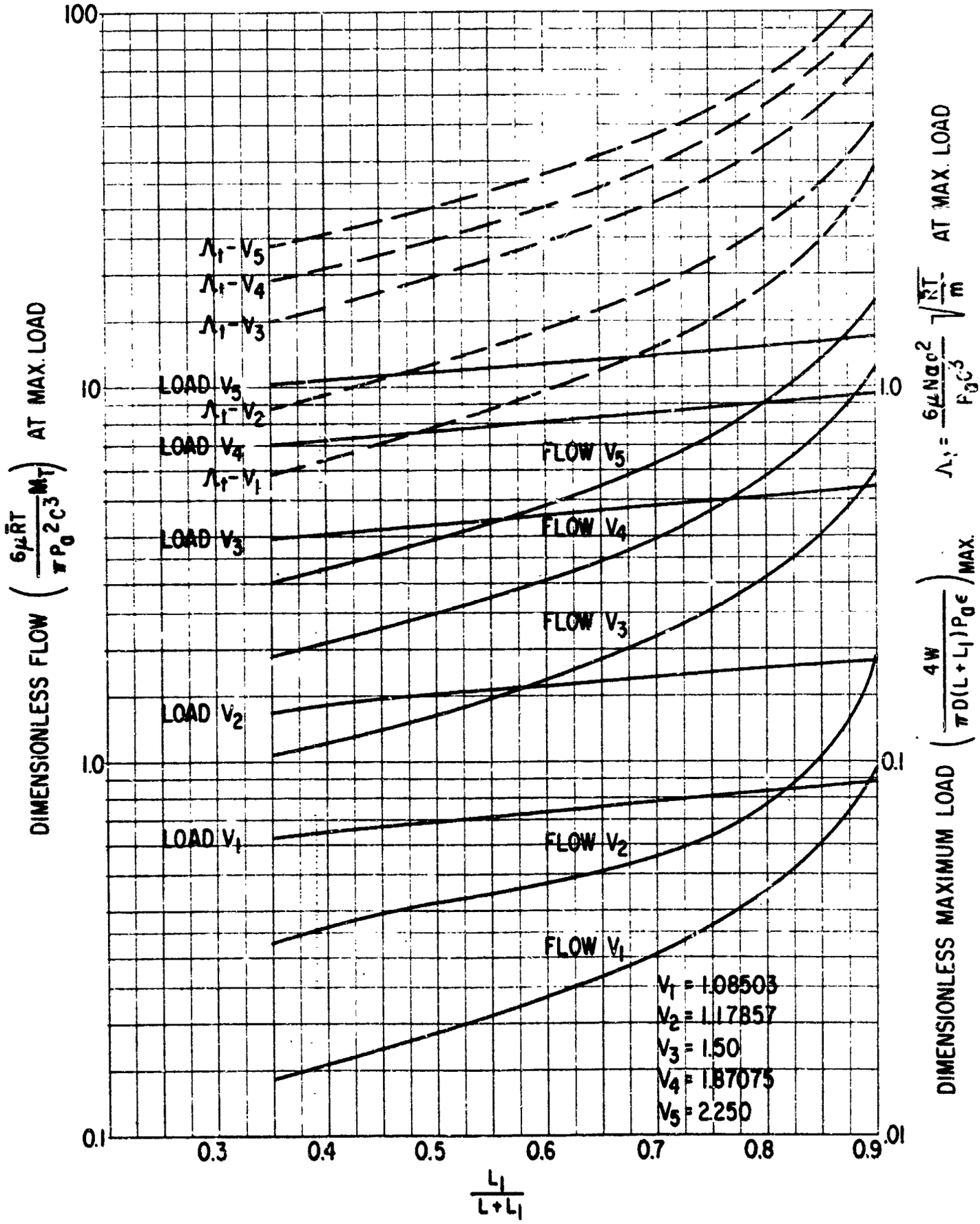
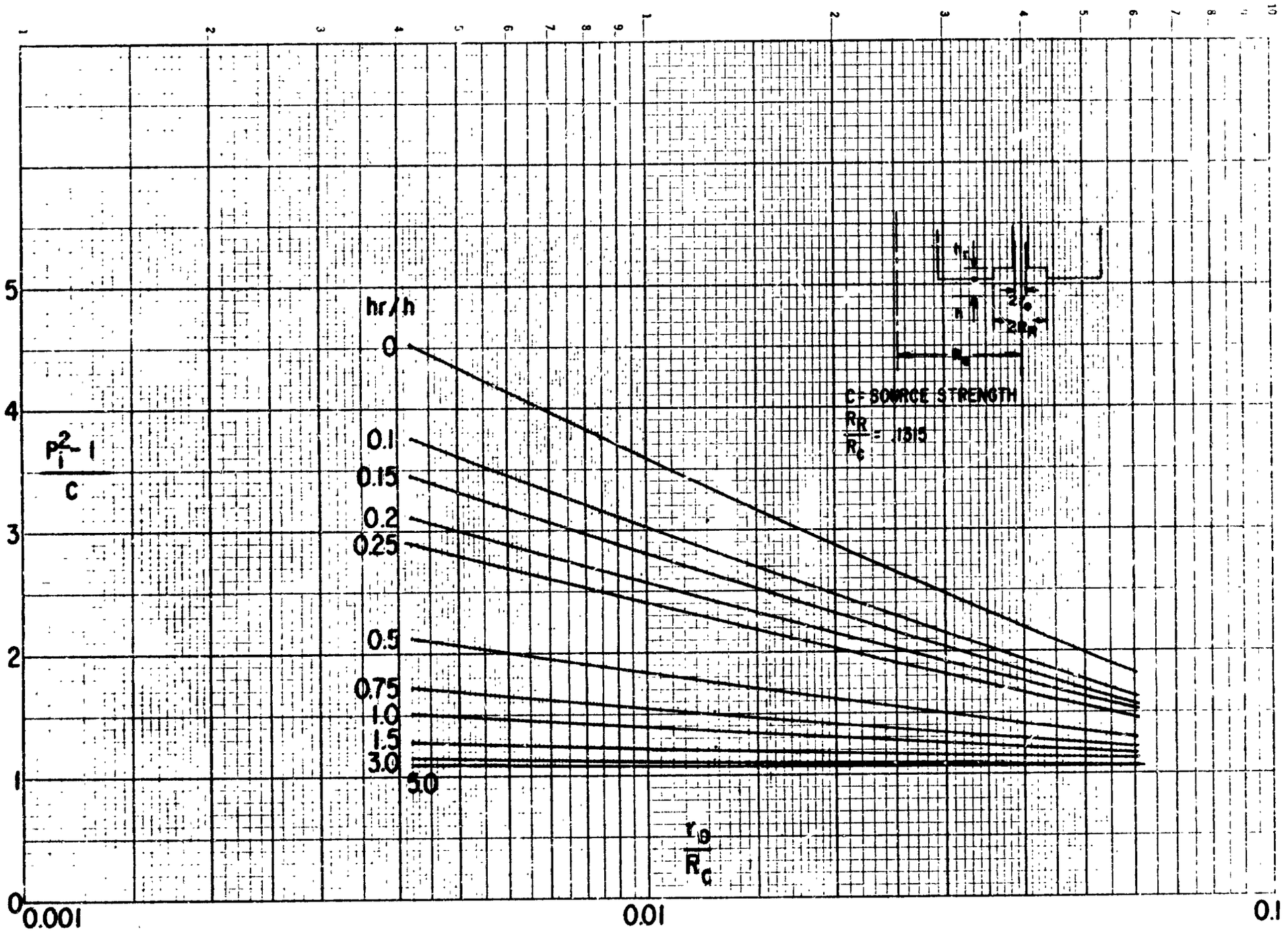


FIG. 15 JOURNAL BEARING-LOAD AND FLOW OPTIMIZATION

2 CYCLES X 70 DIVISIONS



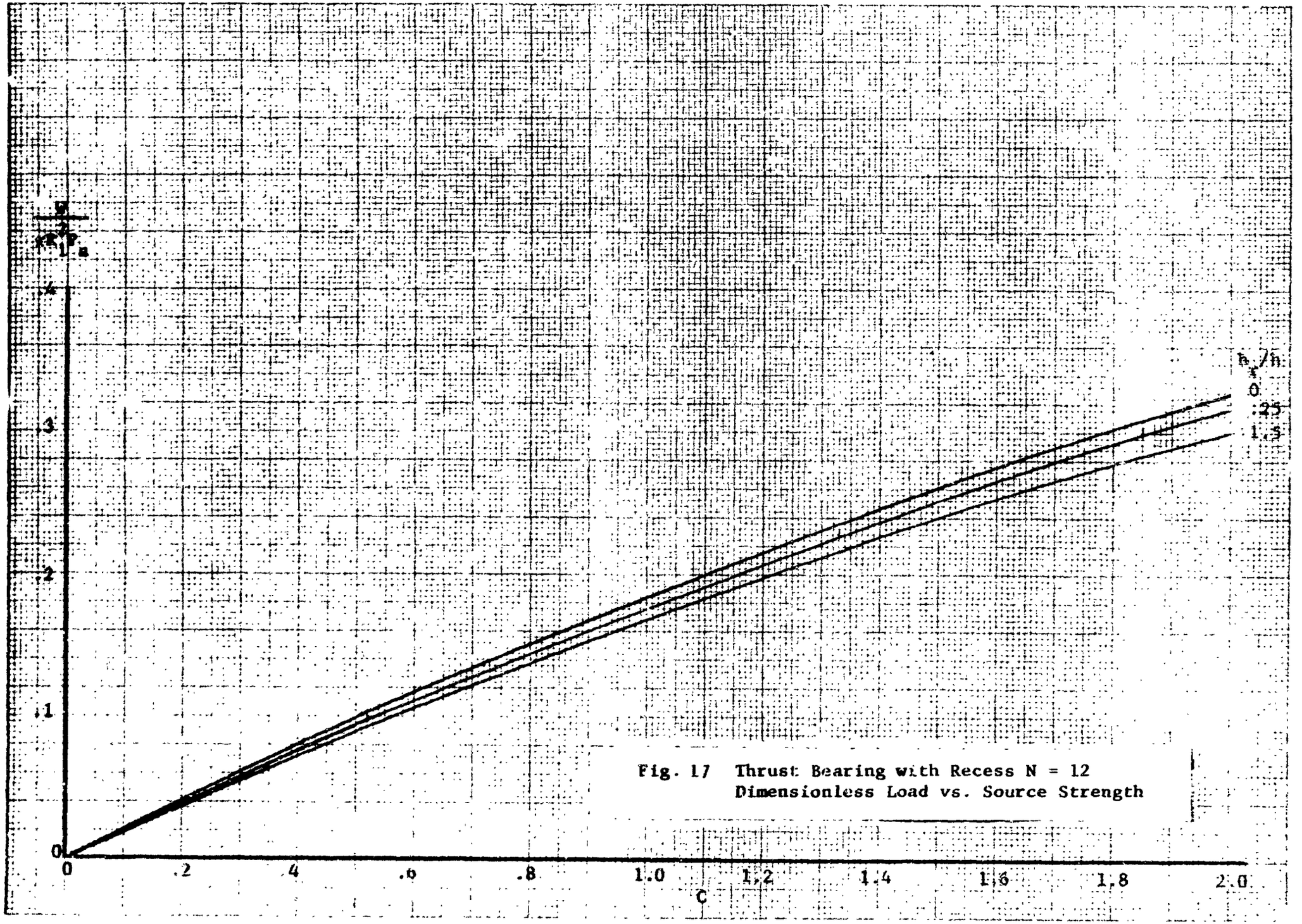


Fig. 17 Thrust Bearing with Recess N = 12
Dimensionless Load vs. Source Strength

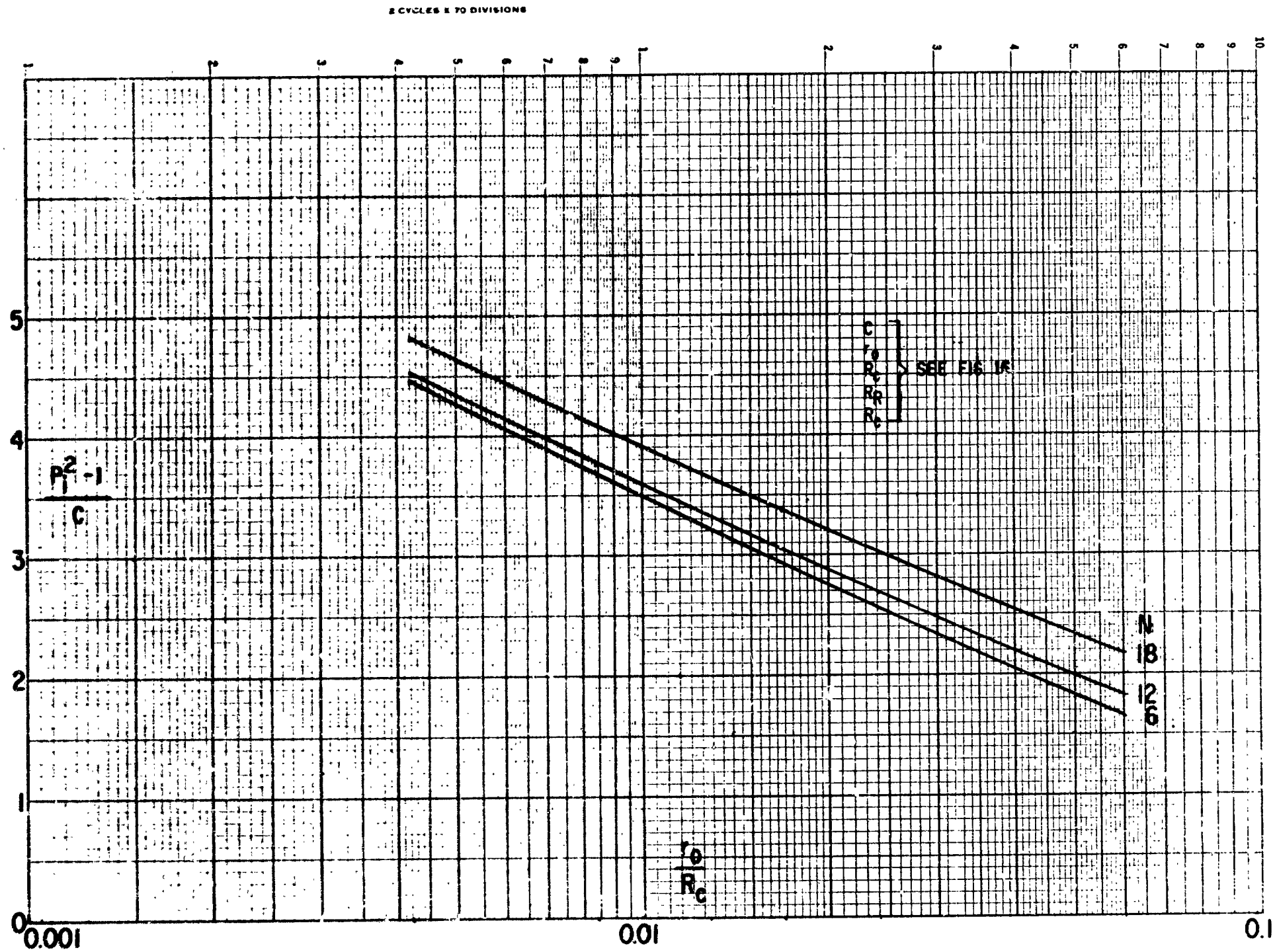


FIG. 18 THRUST BEARING WITHOUT RECESS N = 6, 12, 18

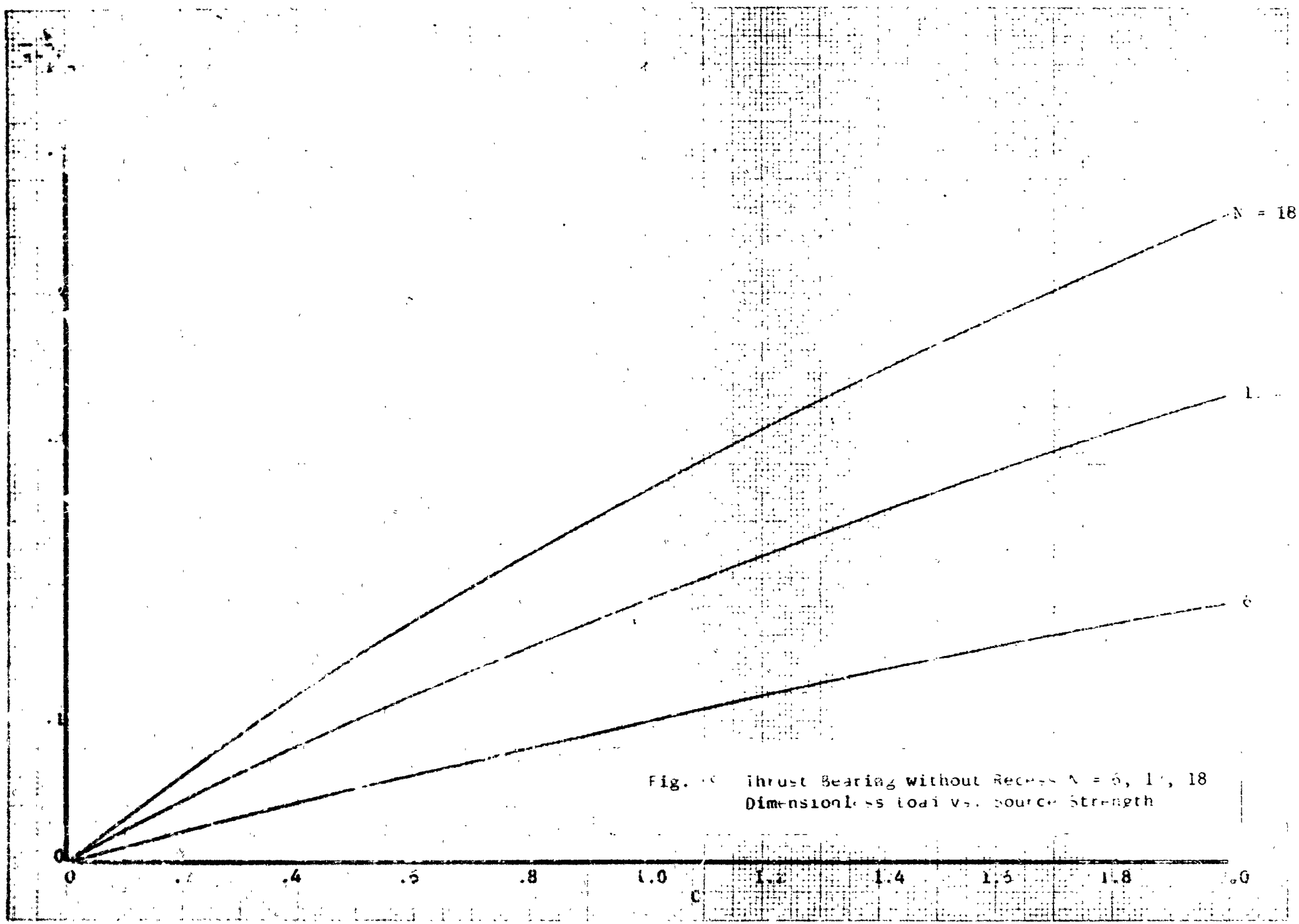


Fig. [illegible] Thrust Bearing Without Recess N = 6, 1[illegible], 18
Dimensionless Load Vs. Source Strength

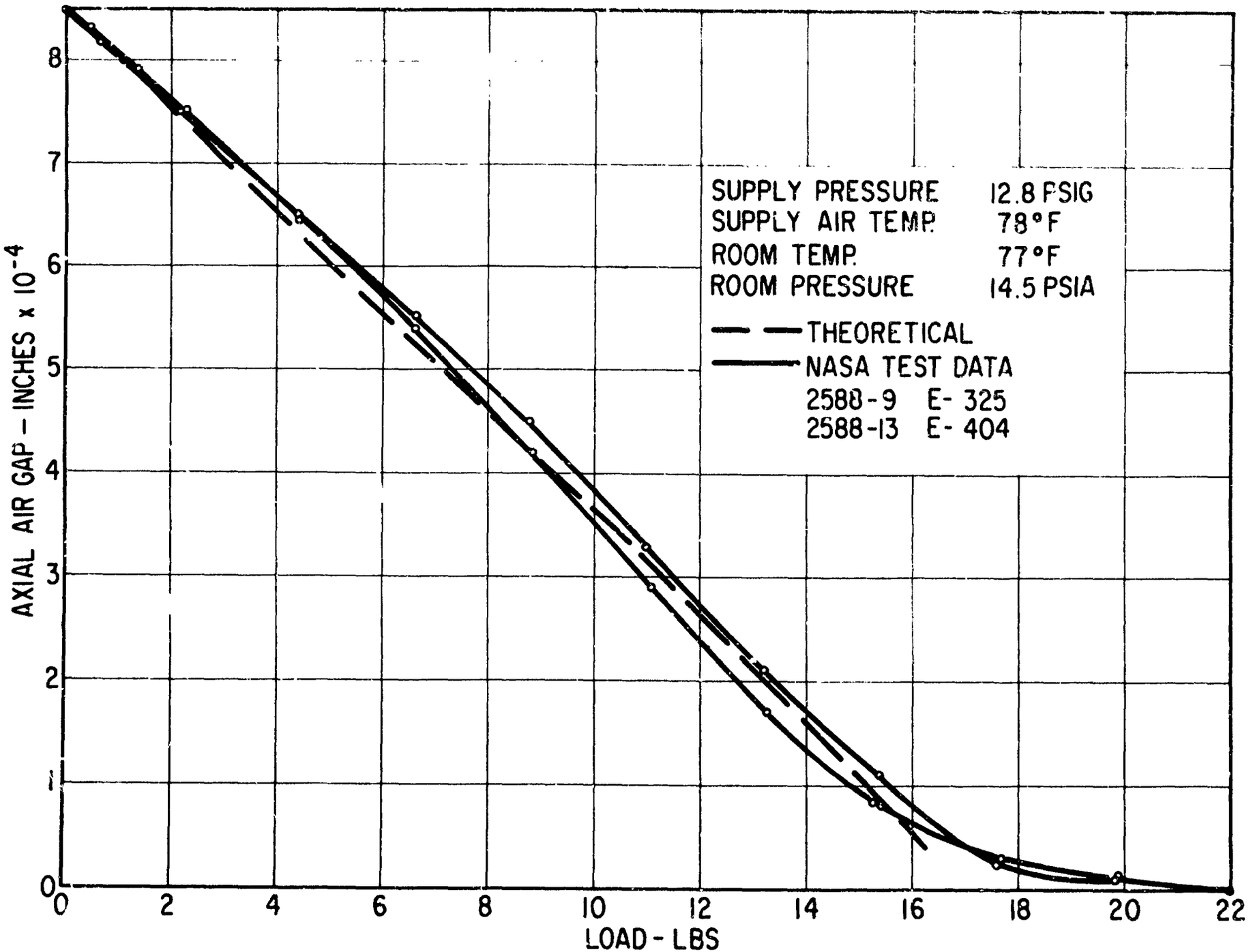


FIG 20 AXIAL LOADING - AIR GAP vs LOAD

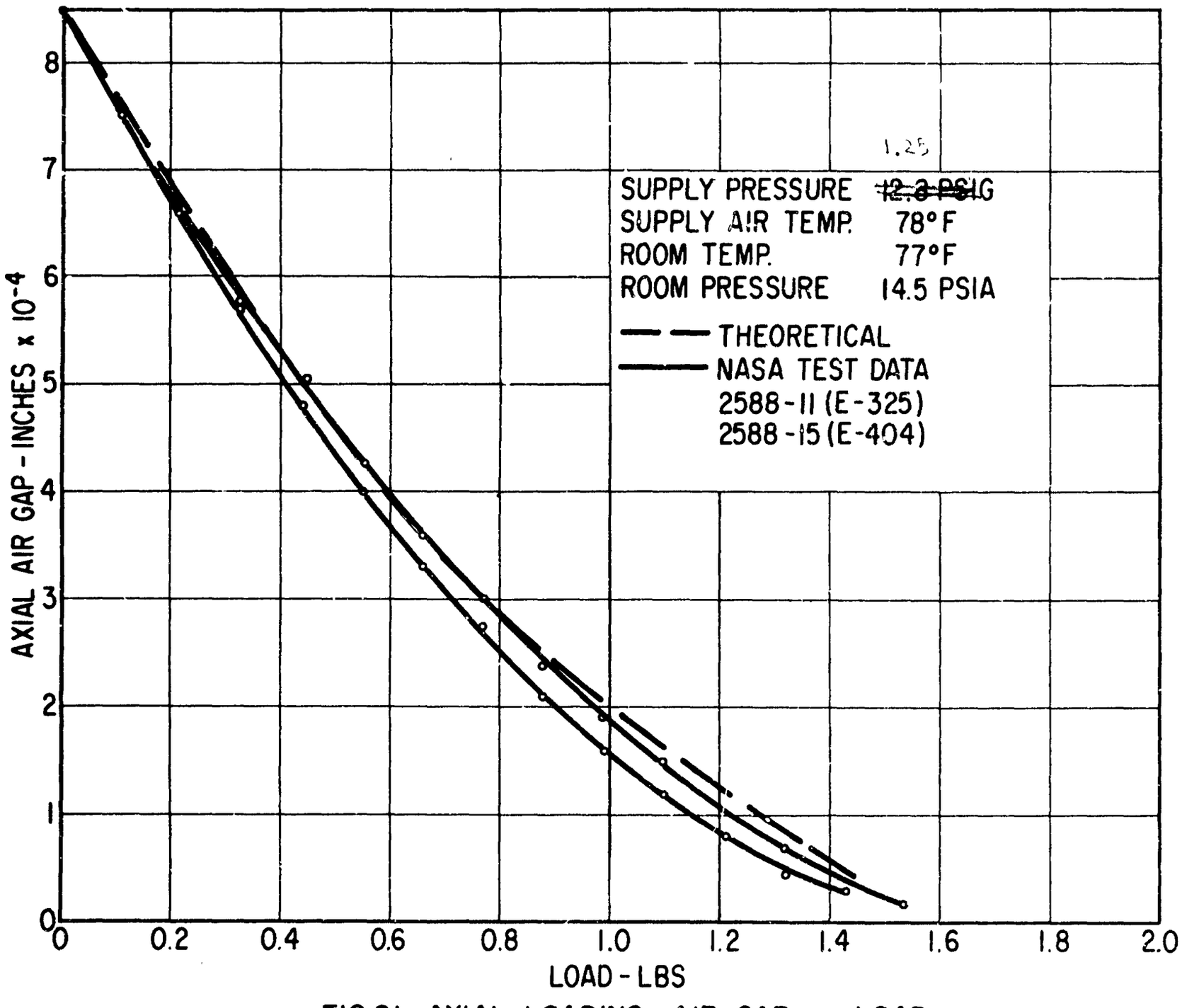


FIG. 21 AXIAL LOADING - AIR GAP vs LOAD

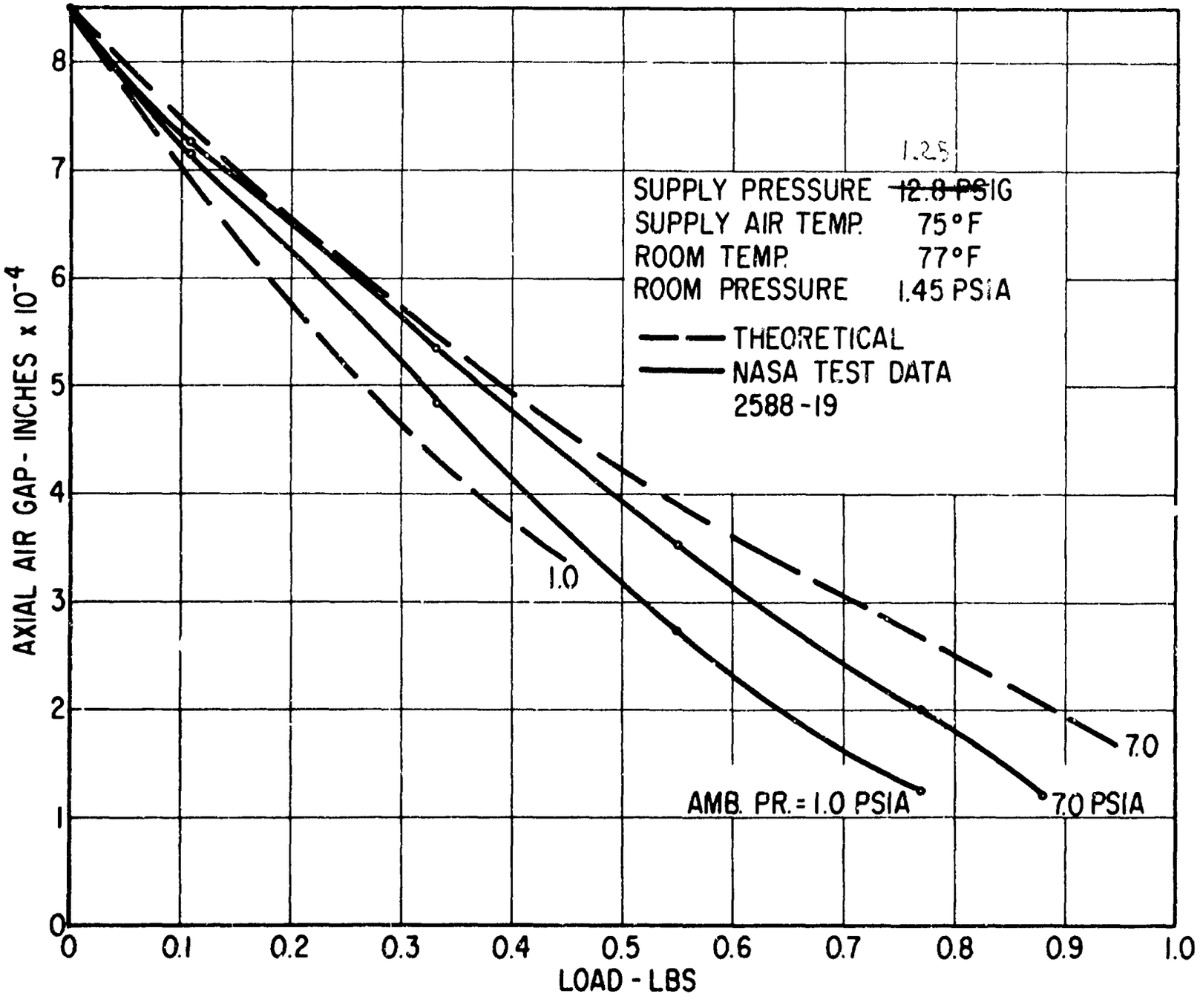


FIG.22 AXIAL LOADING-LOW AMBIENT PRESSURE
AIR GAP vs LOAD

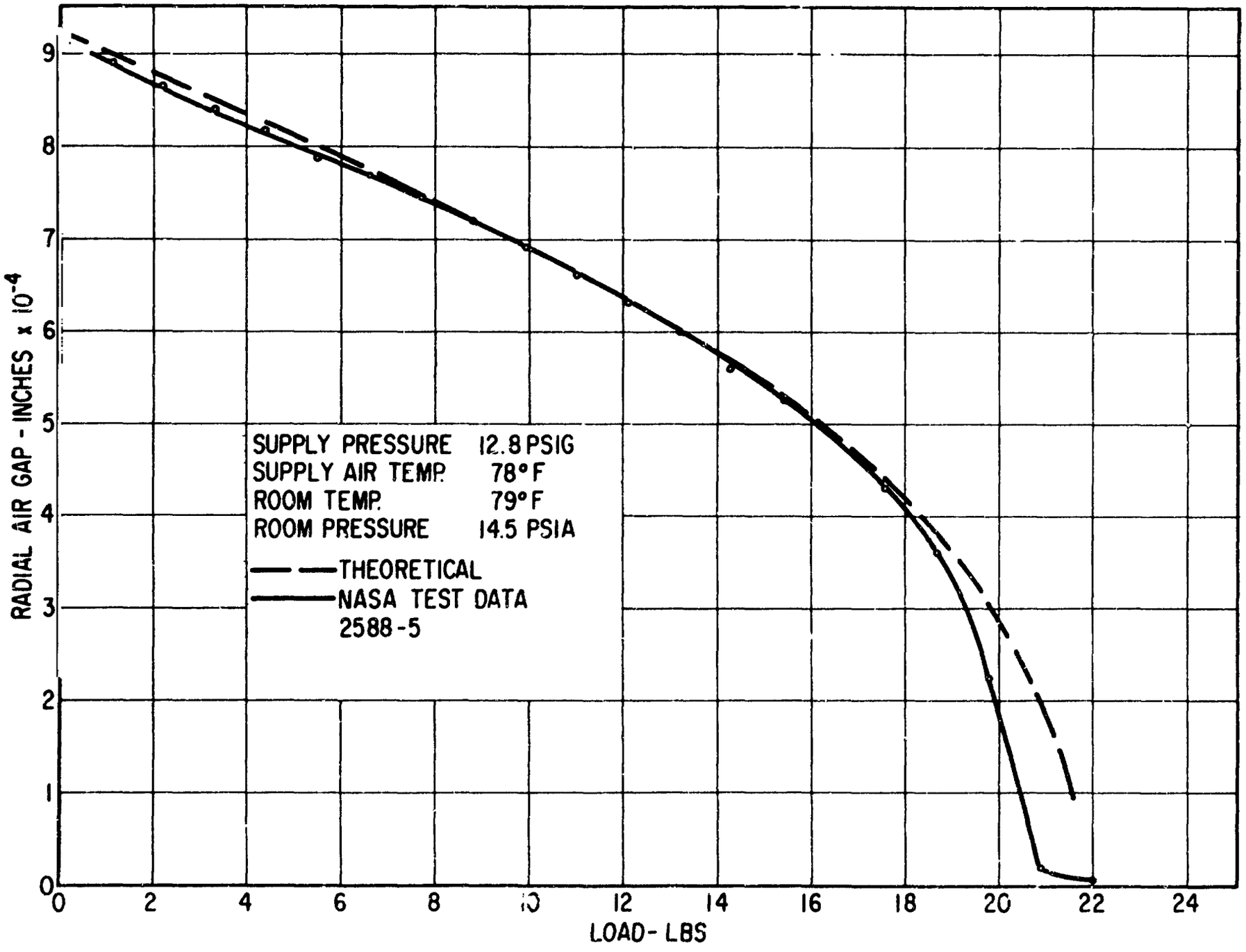


FIG.23 RADIAL LOADING - AIR GAP vs LOAD

SLEEVE NO. 5256

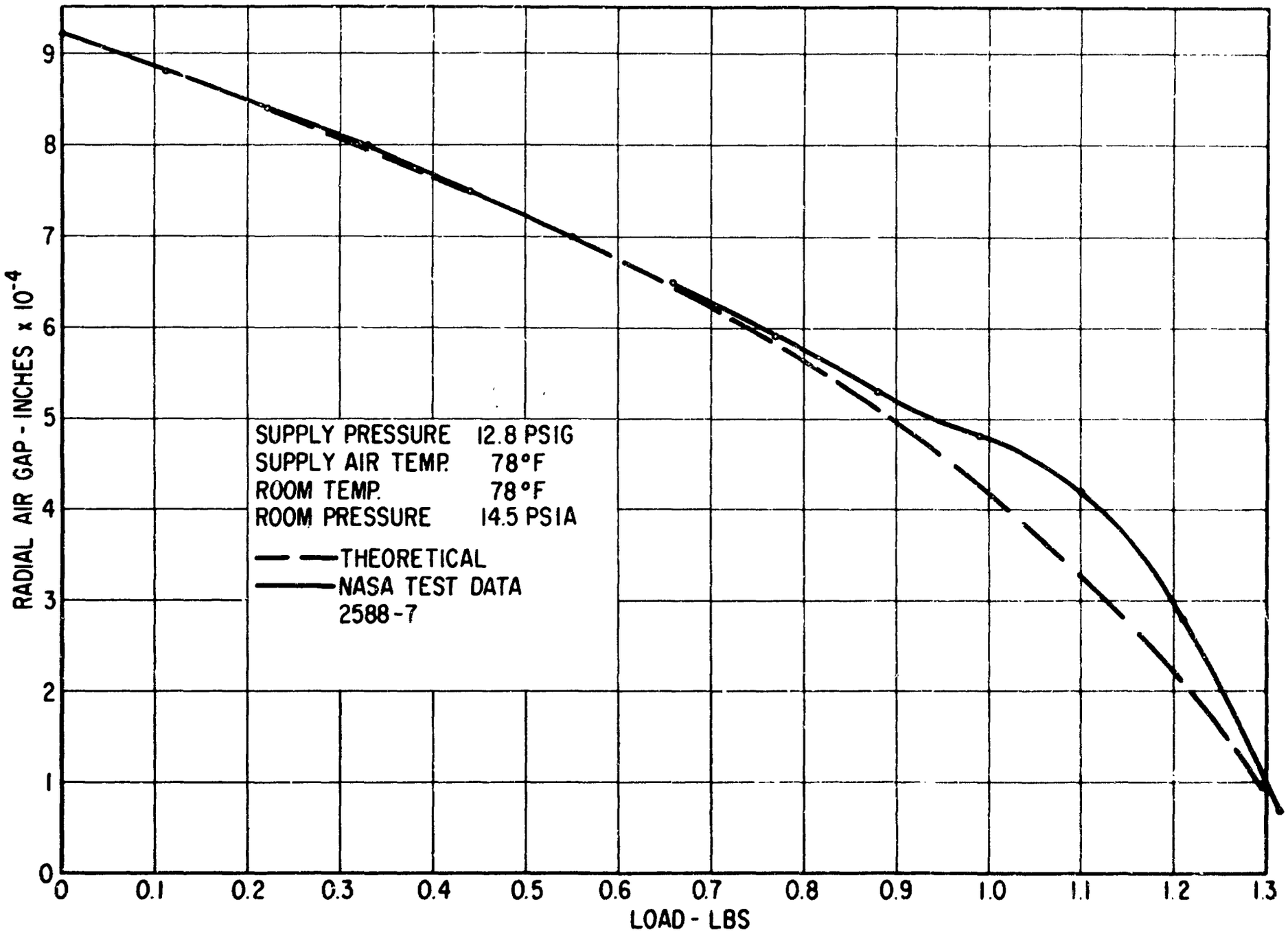


FIG.24 RADIAL LOADING - AIR GAP vs LOAD
SLEEVE NO. 5256

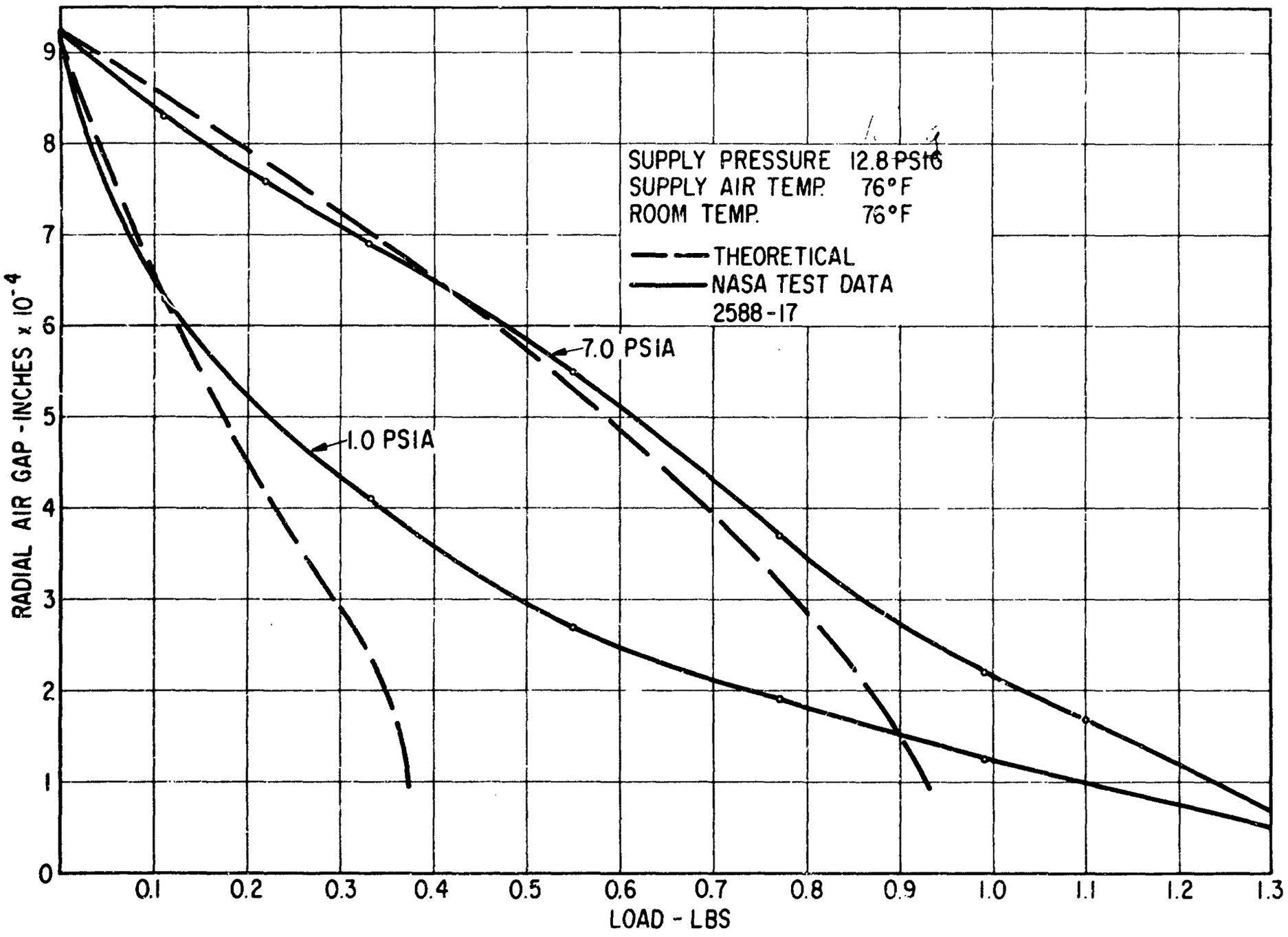


FIG.25 RADIAL LOADING - LOW AMBIENT PRESSURE
AIR GAP vs LOAD - SLEEVE NUMBER 5256

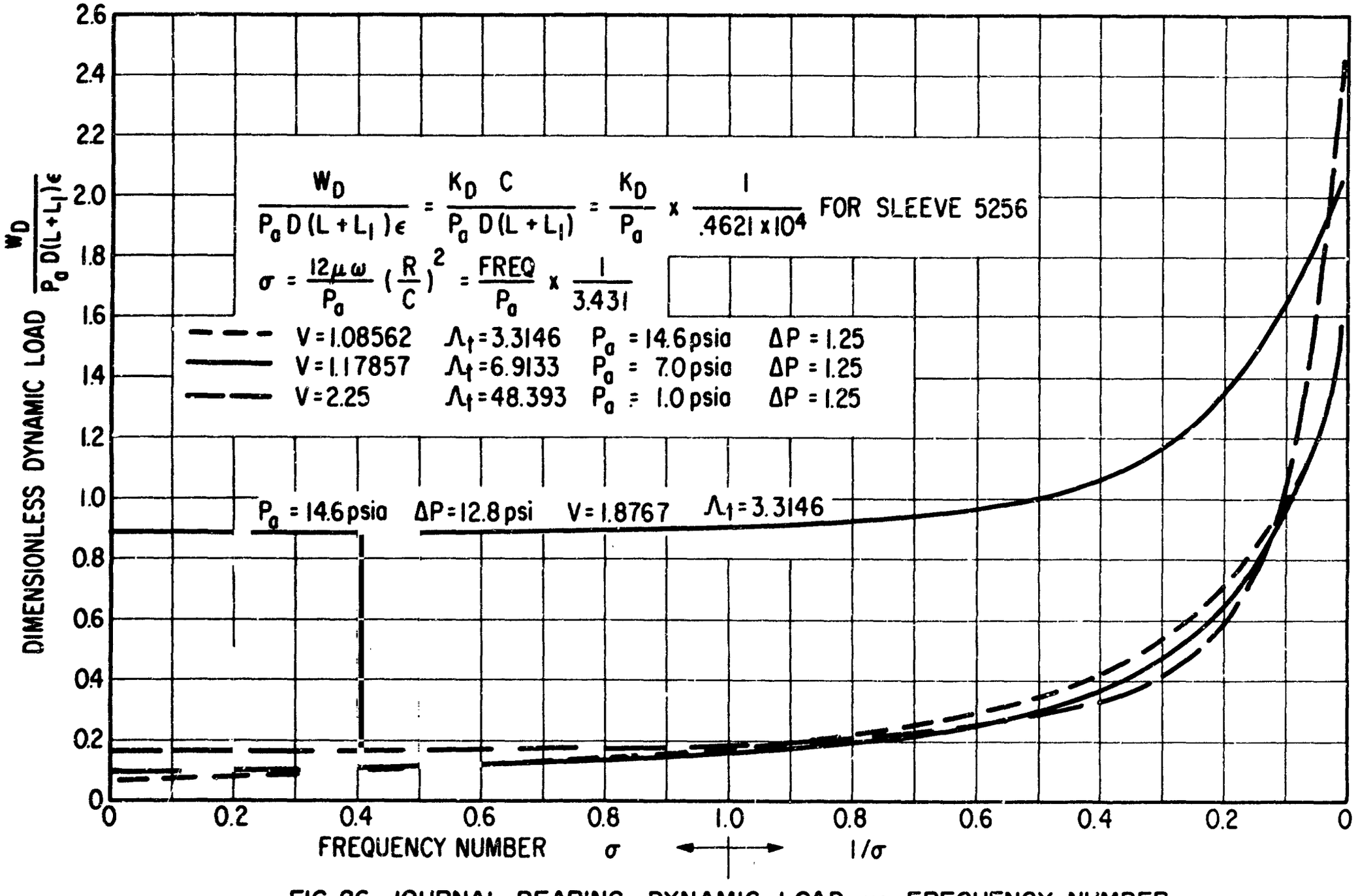


FIG. 26 JOURNAL BEARING - DYNAMIC LOAD vs FREQUENCY NUMBER

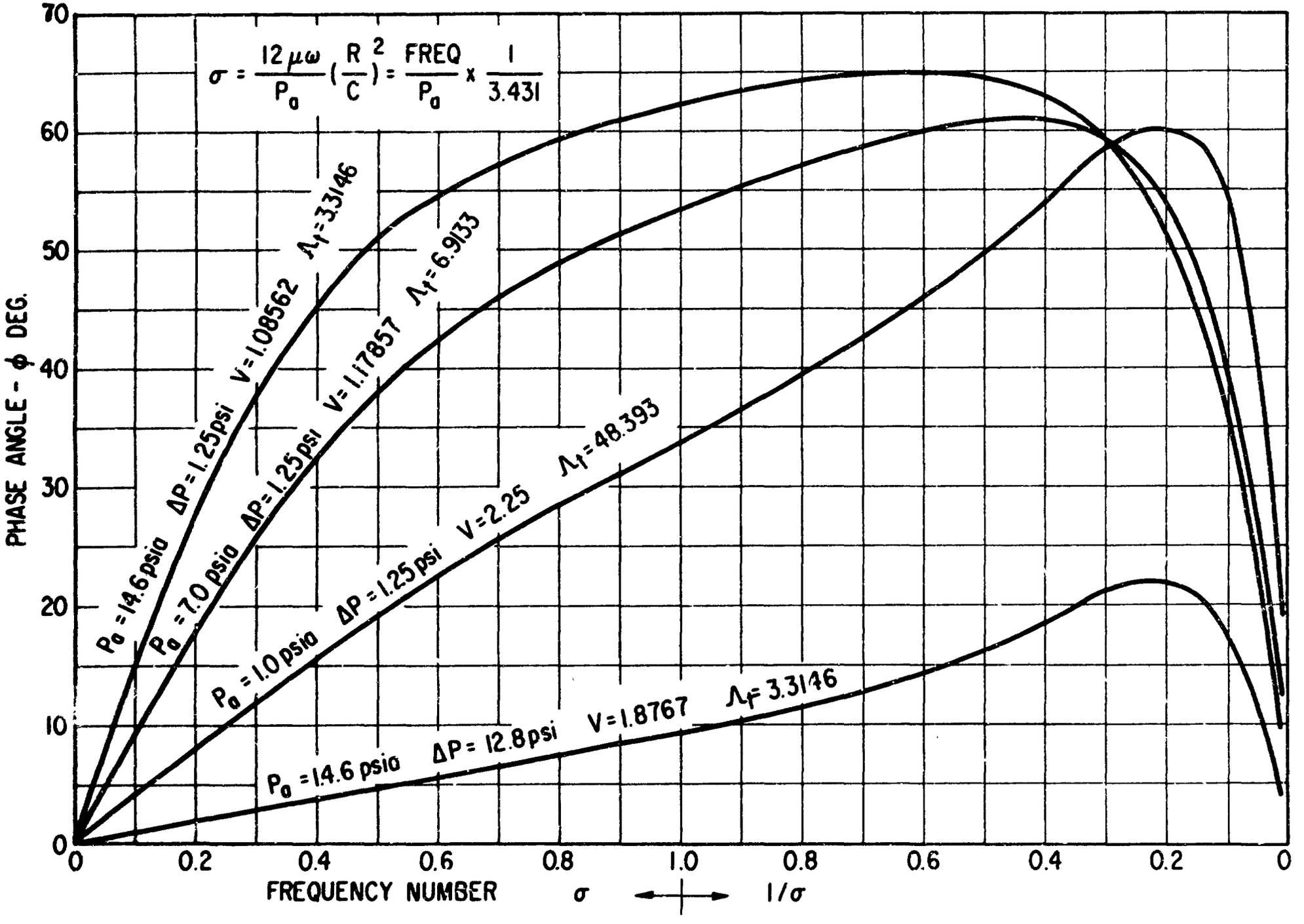


FIG. 27 JOURNAL BEARING-LOAD PHASE vs FREQUENCY NUMBER

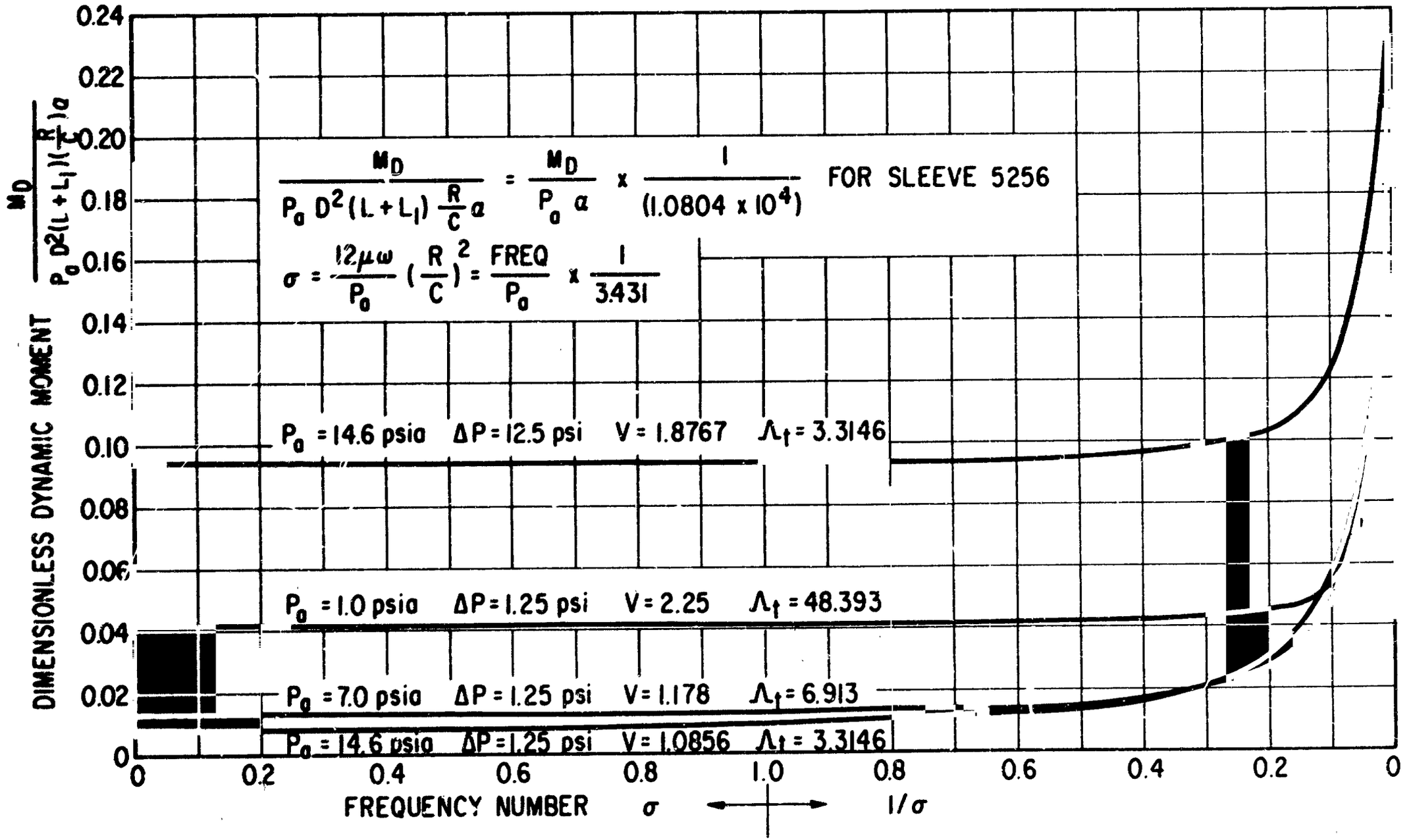
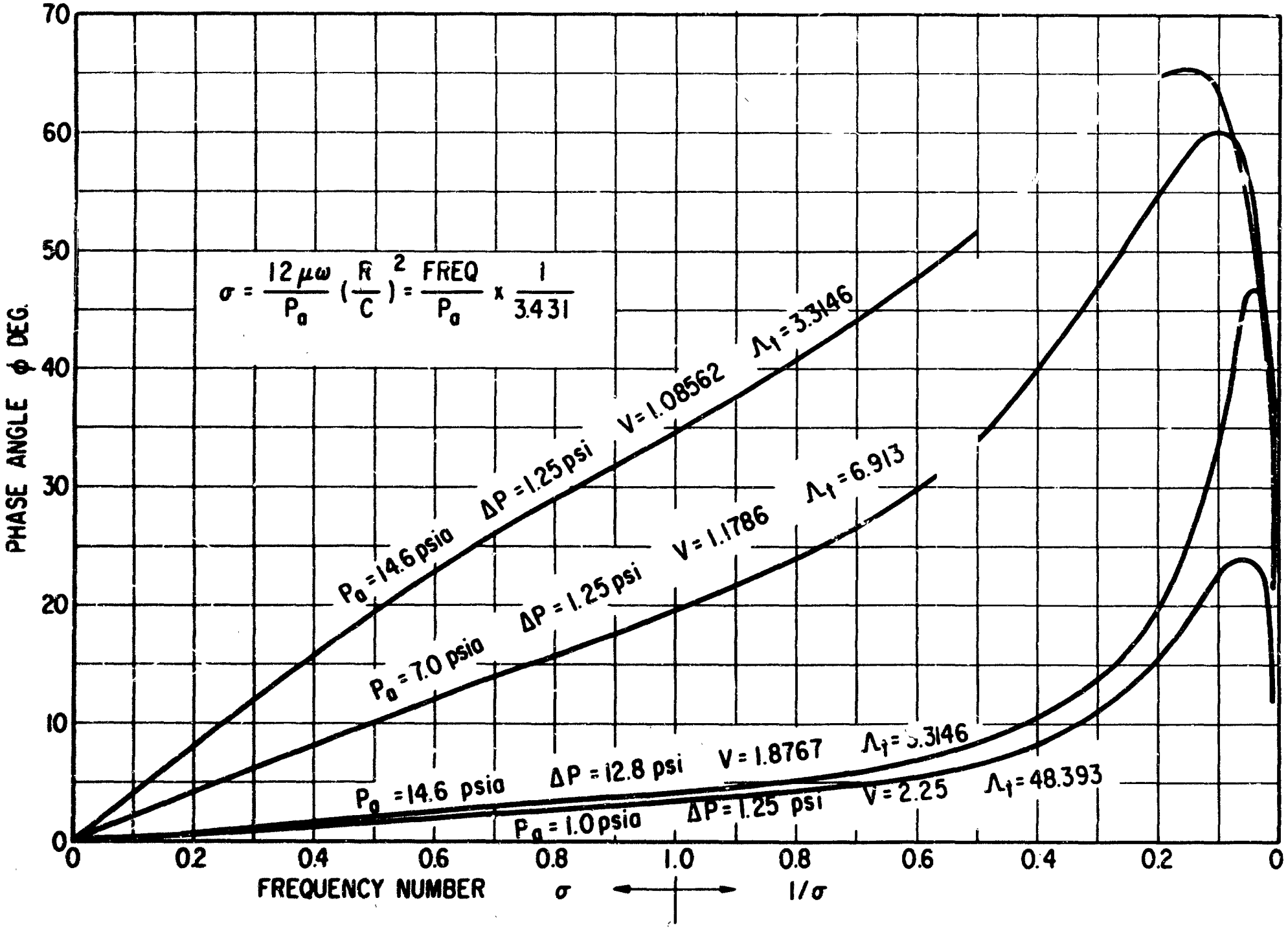


FIG. 28 JOURNAL BEARING - DYNAMIC MOMENT vs FREQUENCY NUMBER


$\sigma = \frac{12\mu\omega}{P_a}\left(\frac{R}{C}\right)^2 = \frac{FREQ}{P_a} \times \frac{1}{3.431}$
PHASE ANGLE ϕ DEG.
FREQUENCY NUMBER σ ← → $1/\sigma$
P_a = 14.6 psia ΔP = 1.25 psi V = 1.08562 Λ_1 = 3.3146
P_a = 7.0 psia ΔP = 1.25 psi V = 1.1786 Λ_1 = 6.913
P_a = 14.6 psia ΔP = 12.8 psi V = 1.8767 Λ_1 = 3.3146
P_a = 1.0 psia ΔP = 1.25 psi V = 2.25 Λ_1 = 48.393

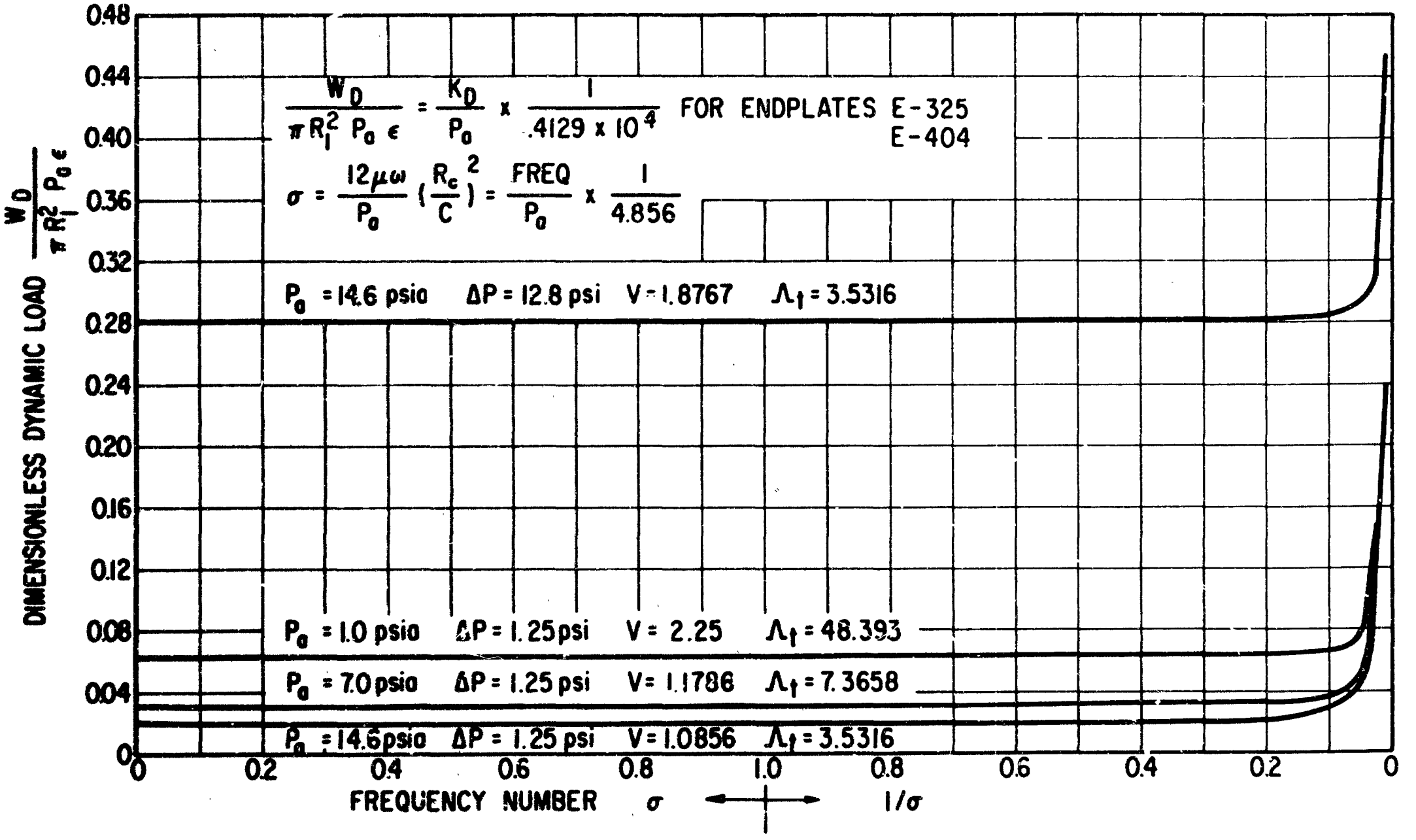


FIG. 30 THRUST BEARING - DYNAMIC LOAD vs FREQUENCY NUMBER

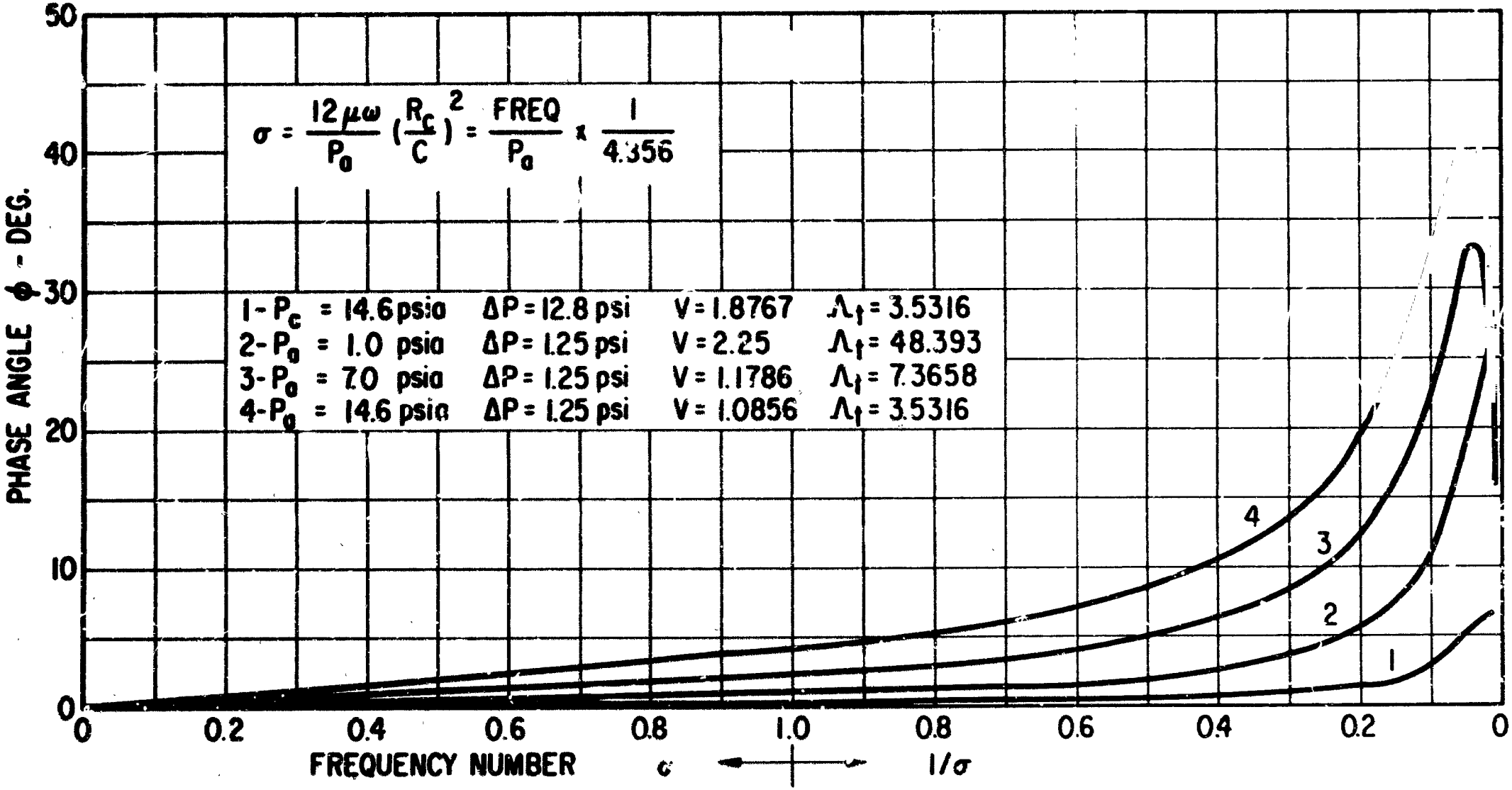


FIG. 31 THRUST BEARING - LOAD PHASE vs FREQUENCY NUMBER

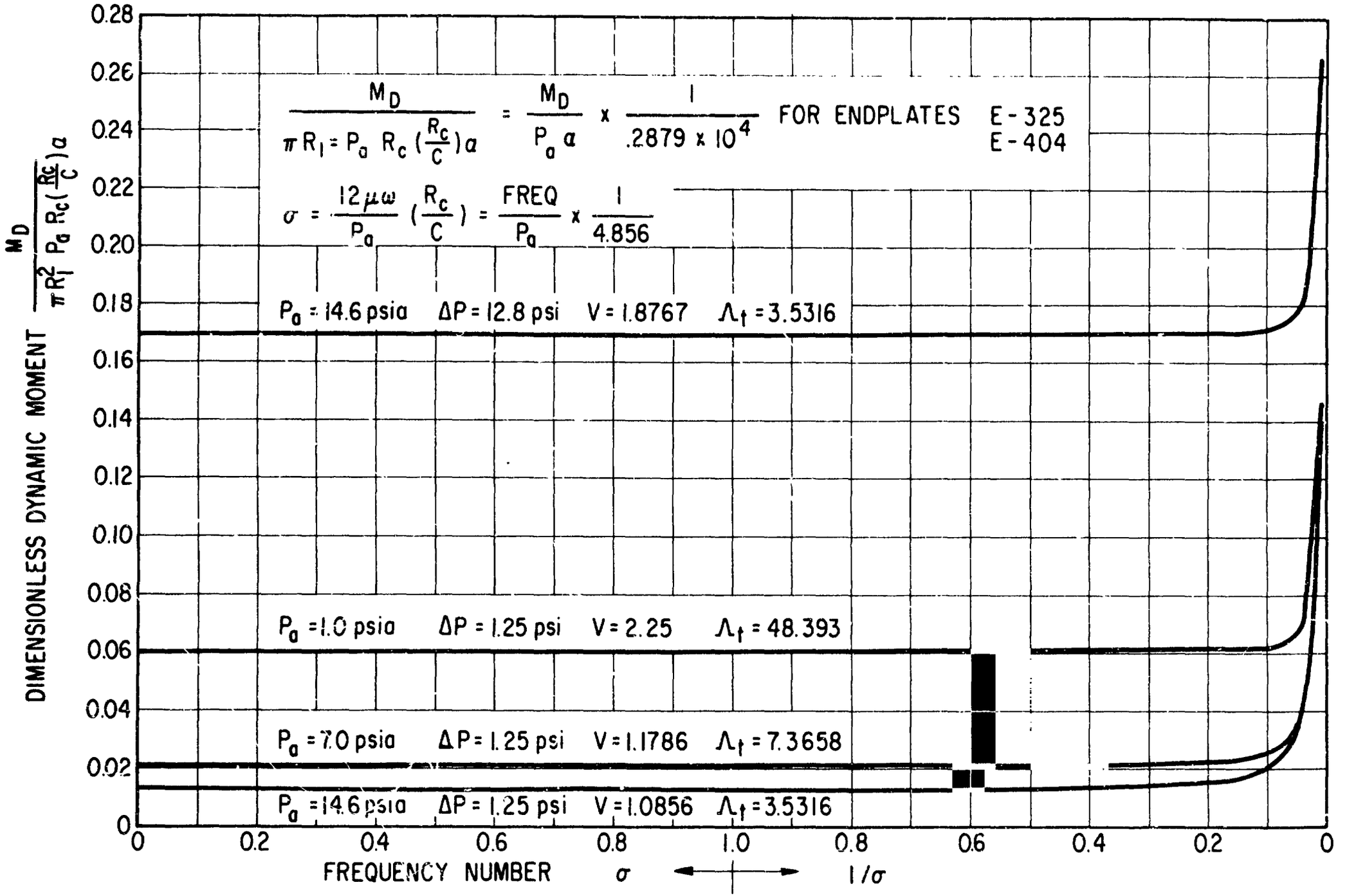


FIG. 32 THRUST BEARING - DYNAMIC MOMENT vs FREQUENCY NUMBER

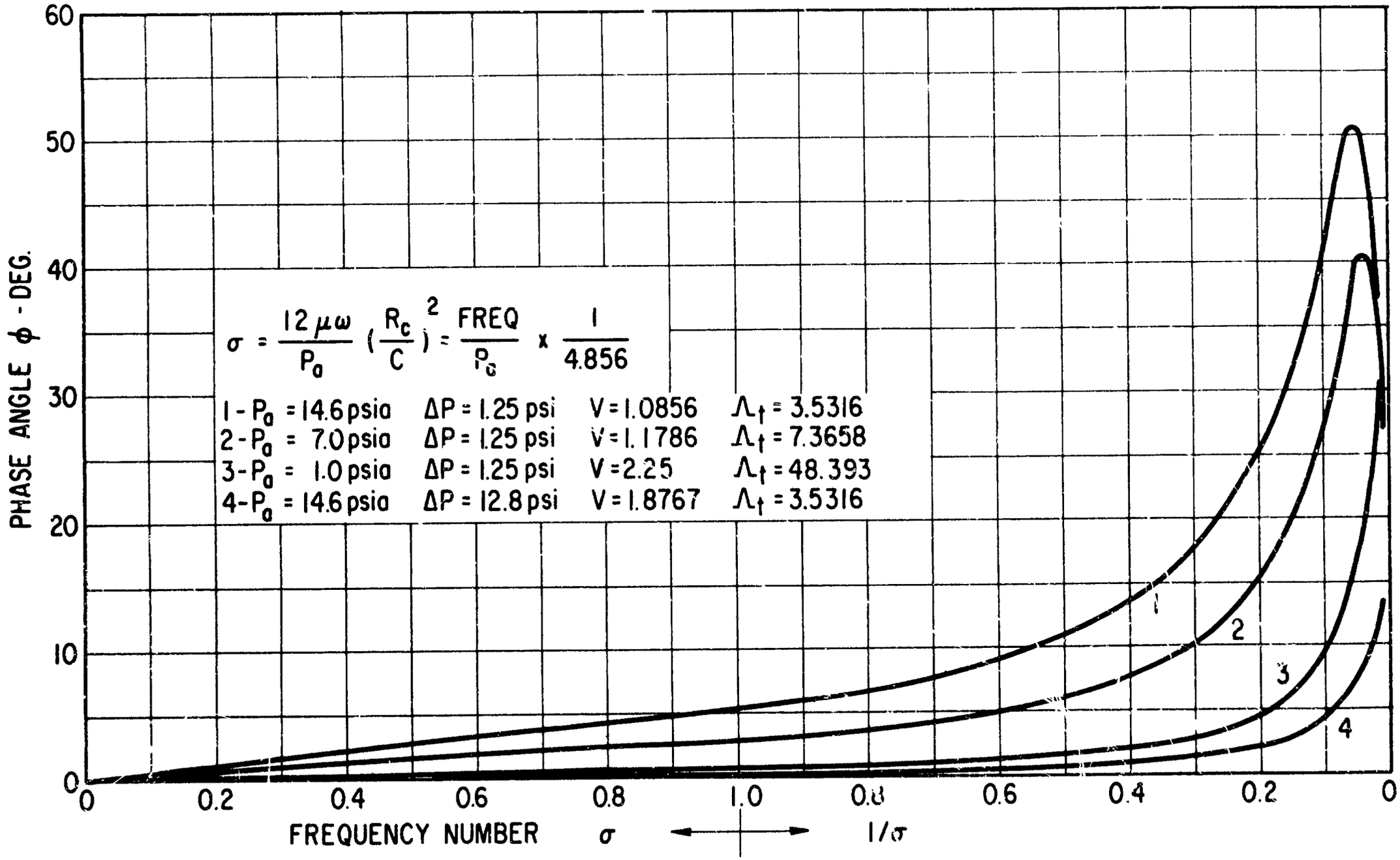


FIG. 33 THRUST BEARING - MOMENT PHASE vs FREQUENCY NUMBER

NOMENCLATURE

A	Orifice flow area - inch^2
a, B, C, D	Millipore flow coefficients
a	Orifice radius - inch
a, b	Dimensions of rectangular pad, representing thrust segment, figs. 8 and 9 - inch
C	Radial clearance in journal bearing, axial clearance in thrust bearing - inch
C	Source strength
D	Journal bearing diameter - inch
d	Source location, see fig. 9 - inch
d	Feeding hole diameter - inch
E, F, G	Functions for use in finite difference equations, see eqs. (H.23), (H.24), (H.30), (G.31), (G.37)
e	Journal bearing eccentricity - inch
f	Pressure function, short journal bearing, see eq. (F.7)
H	Pressure variable, representing $(P_0 P_1)$, see eqs. (B.14), (G.7), (G.26), (H.6), (H.20)
H_i, H_o	Value of H at downstream orifice
H_o'	$\frac{dH}{d\zeta}$ or $\frac{dH}{dr}$ at feeding plane
h	Bearing film thickness, dimensionless: $1+\varepsilon\cos\theta$, or in inch
k	Adiabatic gas exponent
L	Journal bearing length, from feeding plane to bearing end, see fig. 3 - inch
L_1	Journal bearing length between feeding planes, see fig. 3 - inch
M	Mass flow through one orifice - lbs.sec/in
M_T	Total bearing mass flow - lbs.sec/in
M_D	Dynamic bearing moment - lbs.inch
M_D'	Dimensionless dynamic moment, see eqs. (G.63) and (H.73)
m	Dimensionless mass flow through one orifice, see eq. (A.11)

NOMENCLATURE (cont'd)

m'	$= dm/d\left(\frac{P_i}{P}\right)$
m_o	Dimensionless orifice mass flow when $\varepsilon=0$
m_o'	$= \left[dm/d\left(\frac{P_i}{P}\right)\right]_{\varepsilon=0}$
m	Number of finite difference increments
N	Number of feeding holes in bearing
n	Running index
P	Gas film pressure, dimensionless: $\frac{P(psia)}{P_a}$, or in psia
P_a	Ambient pressure at end of bearing - psia
P_s	Supply pressure - psia
P_i	Orifice downstream pressure, dimensionless: $\frac{P_i(psia)}{P_a}$, or in psia
P_r	Dimensionless pressure in thrust bearing recess
P_o	Dimensionless pressure when $\varepsilon=0$
P_1	Dimensionless first order pressure coefficient, see eq. (B.2)
p,q,s,t,v,w	Functions for use in finite difference equations, see eqs. (G.40), (G.41), (H.37), (H.38)
Q_x,Q_z,Q_r,Q_θ	Mass flow per inch - lbs.sec/in^2
q	$= \Lambda_t V m_o$, dimensionless mass flow in journal bearing
q_T	$= \Lambda_T V m_o$, dimensionless mass flow in thrust bearing
R	Journal bearing radius - inch
R_1	Outer radius of thrust bearing- inch
R_2	Inner radius of thrust bearing - inch
R_c	Radius of feeding plane circle in thrust bearing - inch
RT	(Gas constant).(total temperature) - in^2/sec^2
r	Radial coordinate in thrust bearing, dimensionless: $\frac{r(inch)}{R_c}$, or in inch
r'	$= \frac{r(inch)}{R_2}$ dimensionless radial thrust bearing coordinate
T	Infinite series given by eq. (E.39)
t	Time - seconds

NOMENCLATURE (cont'd)

U_1, U_2	Bearing surface velocities, see figs. 1 and 2 - in/sec
U	Dimensionless pressure function, see eq. (E.20)
u, w	Circumferential and axial gas velocity in journal bearing - in/sec
u, v	Radial and tangential gas velocity in thrust bearing - in/sec
V	$= P_s/P_a$, pressure ratio
W	Load of journal and thrust bearing - lbs
W_2	Load of journal bearing outside feeding planes - lbs
W_1	Load of journal bearing between feeding planes - lbs
W_σ	Load including effect of inherent compensation- lbs
W_D	Dynamic bearing load - lbs
W_D'	Dimensionless dynamic load, see eqs. (G.62) and (H.72)
x, y, z	Circumferential, radial and axial coordinates for journal bearing, see fig. 1 - inch
α	Orifice flow coefficient
α	Angular vibration amplitude - radius
α, β	$\alpha + i\beta = \sqrt{1 + \frac{i\sigma}{P_a}}$, see eqs. (G.18) and (G.19)
β	$= \ln\eta/\ln\gamma$, geometrical thrust bearing parameter
γ	$= R_c/R_2$, dimensionless inner radius of thrust bearing
γ_n	Fourier coefficients for U , see eq. (E.21)
ε	$= e/C$, eccentricity ratio
ε	Dimensionless vibration amplitude
ζ, ζ'	$= z/R$, dimensionless axial coordinates for journal bearing, see fig. 3
ζ	$= 2\pi \frac{x}{a}$, dimensionless x-coordinate for rectangular pad, appendix E
η	$= L_1/D$, dimensionless length between feeding planes in journal bearing
η	$= R_1/R_c$, dimensionless outer radius of thrust bearing
η	Adiabatic efficiency of orifice

NOMENCLATURE (cont'd)

$\theta, \bar{\theta}$	Circumferential coordinate in journal bearing, see fig. 1 - radians
θ	Tangential coordinate in thrust bearing - radians
$\varkappa$	$= d/a$, dimensionless coordinate for orifice in rectangular pad, appendix E
μ	Gas viscosity - lbs.sec/in^2
ν	Vena contracta coefficient of orifice
ξ	$= L/D$, dimensionless length outside feeding planes in journal bearing, see fig. 3
ξ	$= b/a$, aspect ratio of rectangular pad, appendix E, see figs. 8 and 9
ρ	Mass density - lbs.sec^2/in^4
σ	$= \frac{12\mu\omega}{P_a}\left(\frac{R}{C}\right)^2$, frequency number
σ	$= a^2/dC$, inherent compensation factor
τ	$=\omega t$, dimensionless time
φ	Journal bearing attitude angle, see fig. 1 - radians
$\dot{\varphi}$	Angular velocity of journal center - rad/sec
φ	$=2\pi\, y/a$, dimensionless y-coordinate for rectangular pad, appendix E
φ_w	Phase angle for translatory vibration, see eq. (G.62) and (H.72) - radians
φ_M	Phase angle for rotational vibration, see eqs. (G.63) and (H.73) - radians
$\psi(x)$	$= \int_0^x e^{t^2} dt$, related error function
ω	Angular speed - rad/sec
ω	Frequency - rad/sec
$\Delta\zeta$	Dimensionless axial increment in finite difference equation for journal bearing
Δr	Dimensionless radial increment in finite difference equation for thrust bearing
Λ	$= \frac{6\mu\omega}{P_a}\left(\frac{R}{C}\right)^2$, bearing number
Λ_t	$= \frac{6\mu N\alpha a^2\sqrt{RT}}{P_a C^3}$, feeding parameter for journal bearing
Λ_T	$= \frac{12\mu N\alpha a^2\sqrt{RT}}{P_a C^3(1+\beta)}$, feeding parameter for thrust bearing