SU-SEL-65-044

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April 1965

FACILITY FORM 602

N65-32199 (ACCESSION NUMBER)

(THRU)

33 (PAGES)

1 (CODE)

CR-64460 (NASA CR OR TMX OR AD NUMBER)

30 (CATEGORY)

Scientific Report No. 7
Prepared under
National Aeronautics and Space Administration
Research Grant no. NsG-377

Scientific Report No. 1
Prepared under
National Aeronautics and Space Administration
Research Grant no. NGR 05-020-065

GPO PRICE $

CSFTI PRICE(S) $

Hard copy (HC) 2.00

Microfiche (MF) .50

ff 653 July 65

RADIOSCIENCE LABORATORY

STANFORD ELECTRONICS LABORATORIES

STANFORD UNIVERSITY • STANFORD, CALIFORNIA

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Radioscience Laboratory
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Abstract: A method for the study of planetary ionospheres is presented which is based on the use of radio waves propagated between the earth and a spacecraft on an occulting trajectory beyond the planet. Phase path, group path, or amplitude measurements made during spacecraft immersion into, and emersion from, the occulted zone could be used to deduce vertical electron density profiles at the two limb positions probed by the waves. By using two or more harmonically related frequencies, the ionospheric measurements would be self-calibrating, thus avoiding the extreme measurement and computational precision that otherwise would be required. Furthermore, the use of more than one frequency makes it possible to separate dispersive ionospheric effects from the non-dispersive refractive effects of the neutral atmosphere, so that both neutral and ionized regions of the atmosphere could be studied in some detail. It is suggested that this simple technique would be particularly applicable for the initial exploration of planetary atmospheres.

author

Introduction: Not much is known about the ionized upper regions of the atmospheres of our neighboring planets and existing ionospheric models have a large degree of uncertainty. As an example, let us consider some ionospheric densities suggested for Mars. Chamberlain [1962] gives a peak electron density of about 10^{11} elec/m^3. Computations based on atmospheric models developed more recently, however, suggest a peak density of the order of 10^{12} elec/m^3. Norton [1964] finds that the peak density could be as high as 2×10^{13} elec/m^3, even for quiet solar conditions. The main reason for the large uncertainty in the electron density is that the ratio between the density of atomic oxygen and molecular nitrogen is not known.

Many of the questions concerning the parameters of the Martian atmosphere have now assumed considerable practical importance because of the interest in sending orbiters and landing probes to the planet. The design of such probes can be simplified and the probability of mission success improved if the atmospheric parameters can be specified with better accuracy.

To be discussed here is a bistatic radar occultation method for the study of planetary ionospheres. Bistatic radar is differentiated from monostatic radar in that the transmitter and receiver are at different locations in the former. For astronomical applications, we imply that one

is on the earth and the other on a space probe. If the trajectory is such that the space probe passes behind a planet as viewed from the earth, the radio ray path from transmitter to receiver will pass tangentially through the atmosphere and be occulted at the limb. Perturbations imposed on the radio waves by the planetary atmosphere provide a sensitive measure of atmospheric parameters. The basic quantity in such an experiment is the profile in height of the refractive index of the atmosphere. Both the neutral and the ionized regions of the atmosphere will in general contribute to this profile.

The main purpose of this paper is to discuss in some detail how the two-frequency, bistatic, radar-occultation method can be utilized for determination of the distribution in height of electron density in a planetary ionosphere. In a separate publication, emphasis was placed on the determination of the scale height and density of the lower neutral atmosphere, assuming that measurements at only one frequency were available [Fjeldbo and Eshleman, 1965].

A single-frequency, bistatic, radar-occultation experiment is to be conducted using the tracking and telemetry system of the Mariner spacecraft due to reach Mars in July, 1965. Measurements made at a single radio frequency do not allow experimental separation of dispersive ionospheric refraction effects from the non-dispersive

effects of the neutral atmosphere. The difference in height of the two regions may still make it possible to separate effects of the neutral atmosphere near the surface from higher ionospheric perturbations, if the electron density distribution in the ionosphere is sufficiently close to being spherically symmetrical. However, one can avoid these uncertainties and obtain a more accurate determination of the parameters of both the upper and lower regions of the atmosphere by using two (or more) frequencies in such an experiment.

General Discussion: The two-frequency, bistatic, radar-occultation method to be discussed here requires a spacecraft that moves along a trajectory which involves occultation by the planetary ionosphere as viewed from the earth. Two signals, one at high and one at low frequency, are used to probe the ionosphere during immersion and emersion. The frequencies should be chosen such that the highest frequency is relatively unaffected by the ionized medium and can serve as a basis of comparison for the lower frequency. Self-calibration of this type makes the ionospheric phase path measurement relatively insensitive to possible errors in the computed spacecraft trajectory. Phase-path measurements of the neutral atmosphere, and single-frequency, phase-path measurements of ionospheric effects require information on spacecraft motion to a very high degree of precision.

The phase of the radio waves will advance and their amplitude will vary as the ray paths penetrate the planetary ionosphere. The amplitude variations are produced by focussing of the waves, as the amount of refraction imposed by the ionosphere is a function of the depth of penetration of the propagation path. It will be shown that continuous measurements of amplitude or phase variations during the occultation may be used to determine the electron density profile in those regions of the atmosphere which are probed by the radio waves.

For the purpose of illustrating the problem, it is convenient to use a numerical example worked out on the basis of a particular model ionosphere. Chamberlain [1962] developed a model for the Martian atmosphere which has a pressure at the surface of about 85 mb. This model leads to an F_1 region at an altitude of about 320 km with a noon density of about 10^{11} elec/m^3. Chamberlain's model ionosphere also contains more detailed altitude variations, but for the purpose of illustrating the problem, it is sufficient to use one layer. Therefore, in the example to follow, we will use only a single Chapman layer to represent the Martian ionosphere:

$$N(h) = N_{max} \cdot \exp \frac{1}{2} \left\{ 1 + (h_o - h)/H - \exp[(h_o - h)/H] \right\} \quad (1)$$

For N_{max}, h_o, and H, we will use 10^{11} elec/m^3, 320 km, and 130 km, respectively.

The phase advance due to the ionosphere may conveniently be measured by transmitting harmonically related frequencies and comparing the phase of the two signals after they have propagated through the Martian ionosphere.

For the assumed model, one might use 50 Mc for the lower of the two frequencies. The power requirement at this frequency makes it advantageous to transmit from the earth and receive in the spacecraft. A second frequency could be provided by the tracking and telemetry system. The present Mariner-Mars mission uses a telemetry frequency of about 2300 Mc [Kliore et al, 1964]. For the purpose of illustrating the problem, it will in the following be assumed that the 50 and 2300 Mc signals are derived from the same frequency source and transmitted from the earth to the spacecraft. The beats between the 50 Mc and the 1/46 subharmonic of the 2300 Mc signals can be counted during occultation and telemetered back to the earth to measure ionospheric phase advance vs. time.

As an approximation to the radio phase effect, one can assume straight-line propagation through the ionosphere. This simplification gives a first approximation Φ_1 to the phase path increase

$$\Phi_1(\rho) = \frac{1}{\lambda} \int_{-\infty}^{+\infty} (\mu-1)\, dz \qquad (2)$$

where λ is the free space wavelength, and μ is the refractive index in the ionosphere. The geometrical quantities involved are illustrated in Fig. 1. The ionospheric phase path increase $\Phi_1(\rho)$ used in this publication is dimensionless since it is normalized to the free space wavelength λ.

The magnetoionic theory relates μ to the electron density N, and the radio frequency f by:

$$\mu = 1 - \frac{40.3}{f^2} N \tag{3}$$

where it is assumed that the radio frequency is much larger than the maximum plasma-, collision-, and gyro-frequencies of the ionosphere. All formulas given in this paper will be in MKS units. A combination of eqs. (2) and (3) gives

$$\Phi_1(\rho) = -40.3\ I(\rho)/cf \qquad \text{(cycles)} \tag{4}$$

where c is the free-space phase velocity and $I(\rho)$ is the electron content along the straight-line approximation to the propagation path. The function $I(\rho)$ is given by:

$$I(\rho) = \int_{-\infty}^{+\infty} N\, dz \qquad (\text{elec/m}^2) \tag{5}$$

Figure 2 shows $\Phi_1(\rho)$ or $I(\rho)$ for the ionospheric model adopted here. The ionospheric phase advance of the 50-Mc signal is seen to reach a maximum of more than 900 cycles.

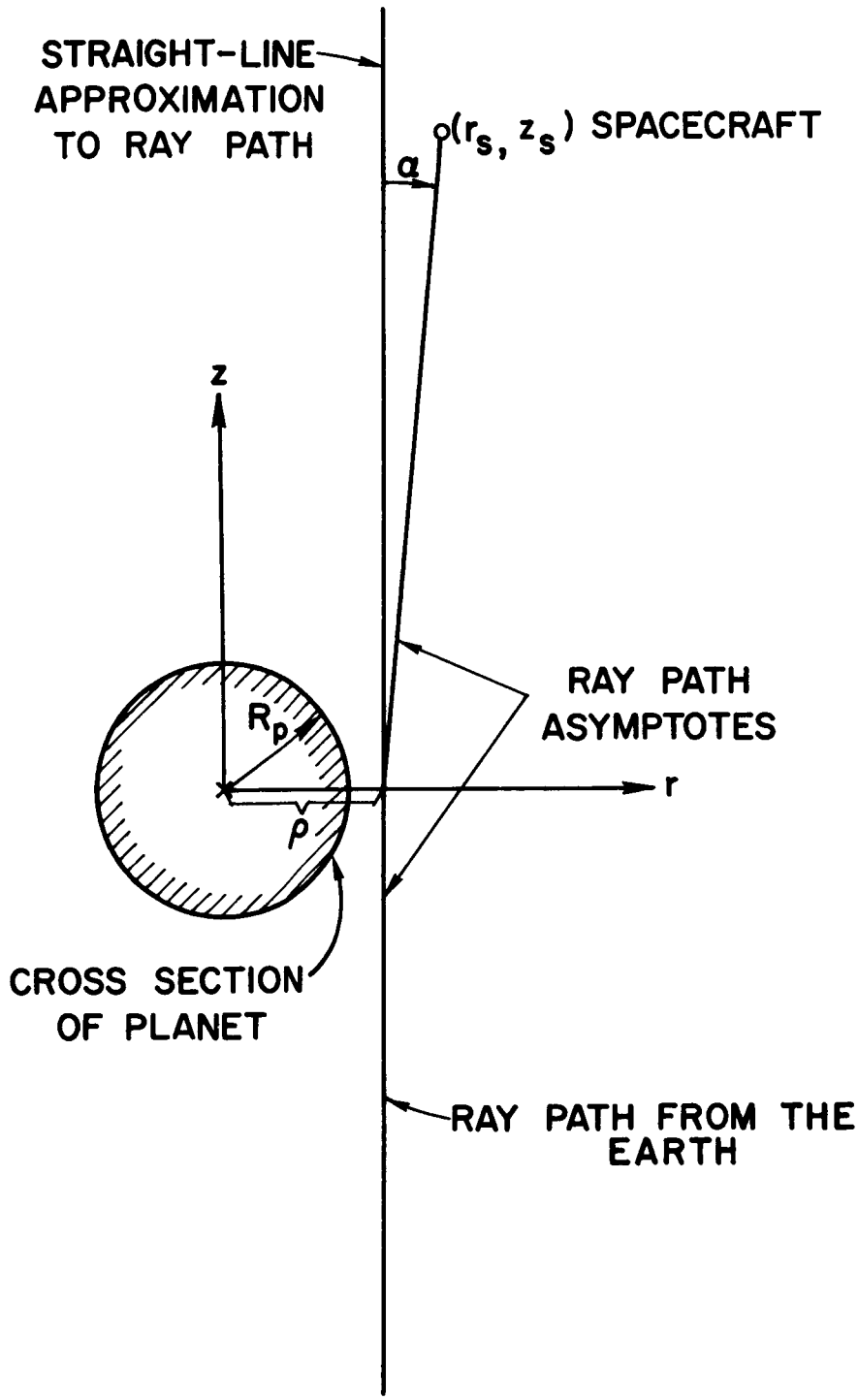

Fig. 1: RAY PATH GEOMETRY

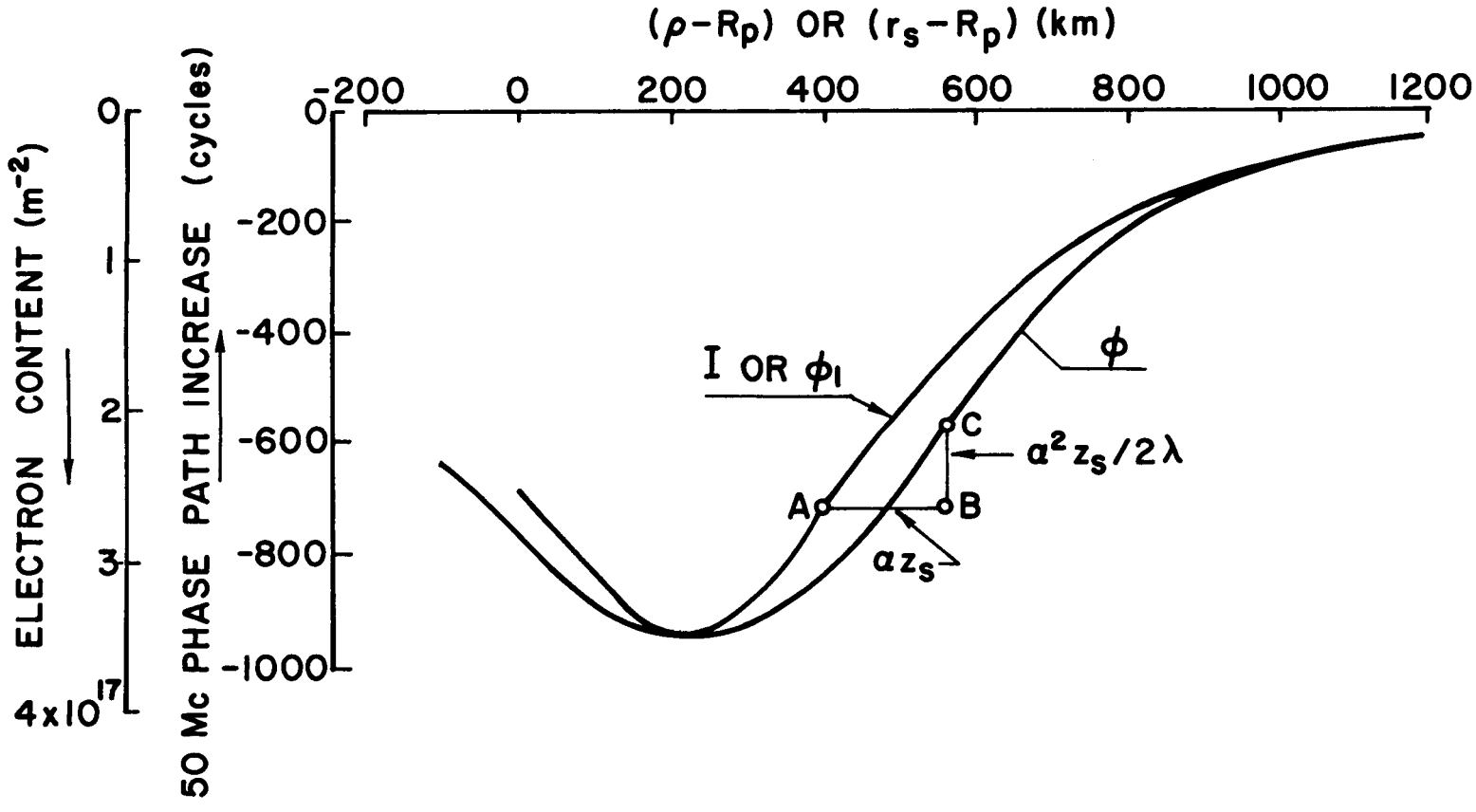

Fig. 2: ELECTRON CONTENT I AND 50 Mc IONOSPHERIC PHASE PATH INCREASE Φ. Occultation is assumed to occur 15,000 km behind Mars.

The phase path approximation obtained by assuming straight-line propagation is adequate for small angles of refraction, when the space receiver is close behind the planet.

The angle α by which the ray path is refracted can now be related to $\Phi_1(\rho)$. The wave fronts emerging from the planetary ionosphere have been perturbed a distance $[\lambda\Phi_1(\rho)]$ in the z-direction. The corresponding change in the wave normal direction gives

$$\alpha = \lambda \frac{d\Phi_1(\rho)}{d\rho} \tag{6}$$

or

$$\alpha = -\frac{40.3}{f^2} \frac{dI(\rho)}{d\rho} \tag{7}$$

which is valid for small angles of refraction.

Figure 3 shows the way in which the 50-Mc waves are refracted by the assumed model ionosphere. Note that the scale is different in the directions longitudinal and transverse to the incoming propagation path. The cross section of Mars, therefore, looks like an ellipsoid in this space.

The Chapman model has the property that it causes two caustics, each with two branches, to be formed behind the ionosphere. The signal is received simultaneously via

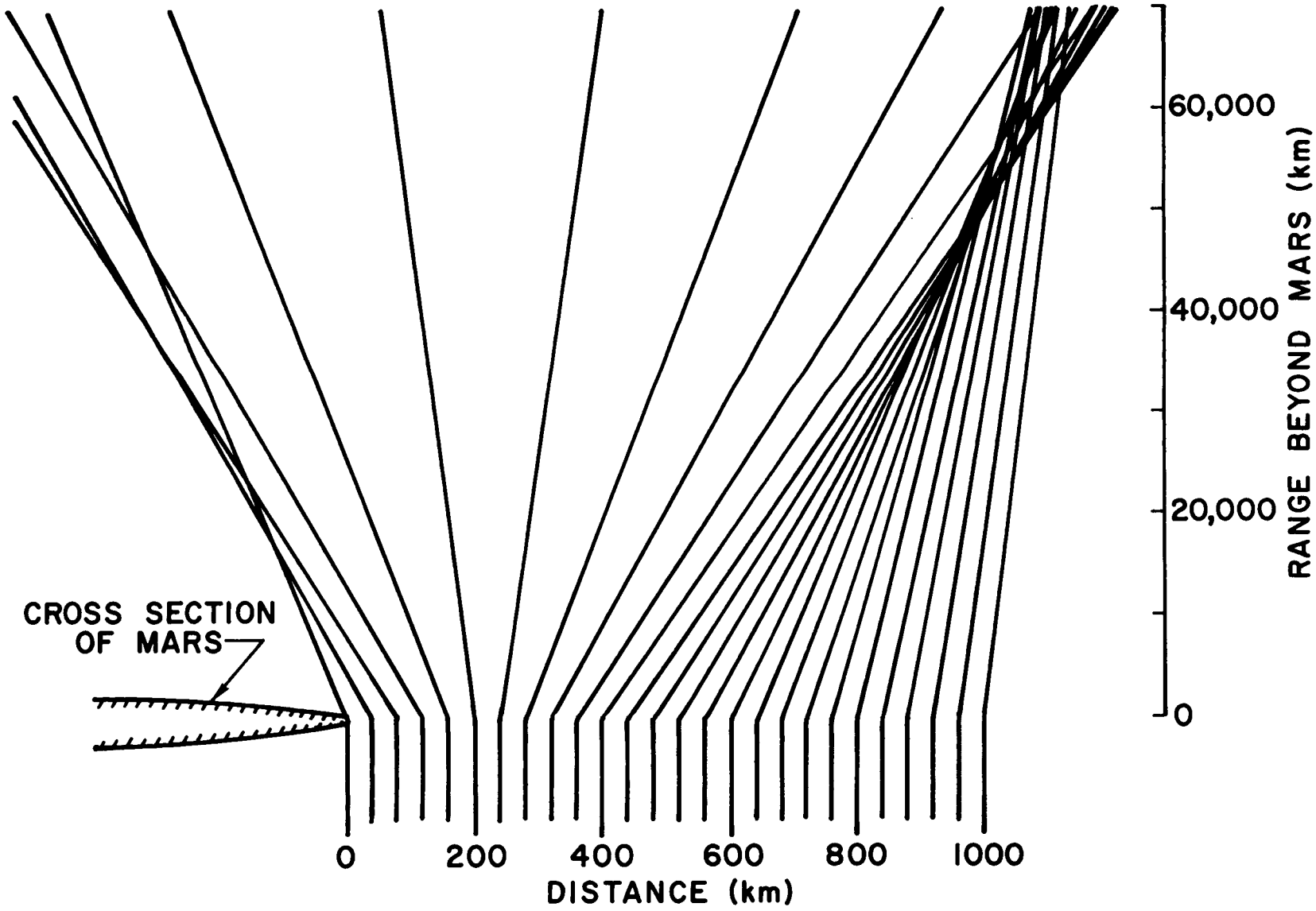

Fig. 3: REFRACTION OF THE 50 Mc SIGNAL BY THE MARTIAN IONOSPHERE

three different propagation paths in the regions beyond the caustics. This phenomenon is also illustrated in Fig. 3, although there the caustic closest to Mars is degenerated into a one-branch caustic due to shadowing by the limb. The spacecraft trajectory should avoid the caustic regions in order that the phase and amplitude measurements not be confused by multiple rays.

It is important to take into account the bending of the propagation path when the spacecraft is far behind the planet. Approximating the propagation path with two straight line segments along the ray path asymptotes gives the phase path increase Φ due to the planetary ionosphere:

$$\Phi(r_s) = \Phi_1(\rho) + \alpha^2 z_s/2\lambda \quad \text{(cycles)} \tag{8}$$

or

$$\Phi(r_s) = -40.3\, I(\rho)/cf + (812.0\, z_s/cf^3)[dI(\rho)/d\rho]^2 \tag{9}$$

where r_s and z_s are the coordinates of the spacecraft. An absolute phase path measurement yields Φ for different positions of the spacecraft along the trajectory, and we will therefore consider Φ a function of r_s.

The first term on the right-hand side of equation (9) is the familiar phase effect proportional to the electron content $I(\rho)$, and inversely proportional to the radio frequency f. The last term in equation (9) accounts for

the increase in phase path length caused by the bending of the propagation path. This effect is proportional to the square of the gradient in $I(\rho)$ and inversely proportional to f^3.

The $\Phi(r_s)$-curve can be obtained from $\Phi_1(\rho)$. This construction is indicated in Fig. 2. By moving from point A on the $\Phi_1(\rho)$-curve, a distance αz_s parallel to the abscissa and $\alpha^2 z_s/2\lambda$ cycles parallel to the ordinate gives the corresponding point C on the phase path curve $\Phi(r_s)$.

The frequency shift caused by the ionosphere is given by the negative of the product of $d\Phi(r_s)/dr_s$ and the radial spacecraft velocity dr_s/dt. This frequency is shown in Fig. 4. Assuming $dr_s/dt = -3.5$ km/sec, it is seen from Fig. 4 that the model ionosphere adopted here produces a maximum frequency shift of about 6.5 cps. This effect would be additive to the standard doppler shift caused by relative motion of the transmitter and receiver.

An absolute measurement of the phase path changes taking place during the occultation requires only a single frequency. This measurement yields $\Phi(r_s)$, from which the refractive index profile of the entire atmosphere may be calculated [Fjeldbo and Eshleman, 1965]. With two frequencies one can also make a self-calibrated measurement of the ionospheric phase path effects alone.

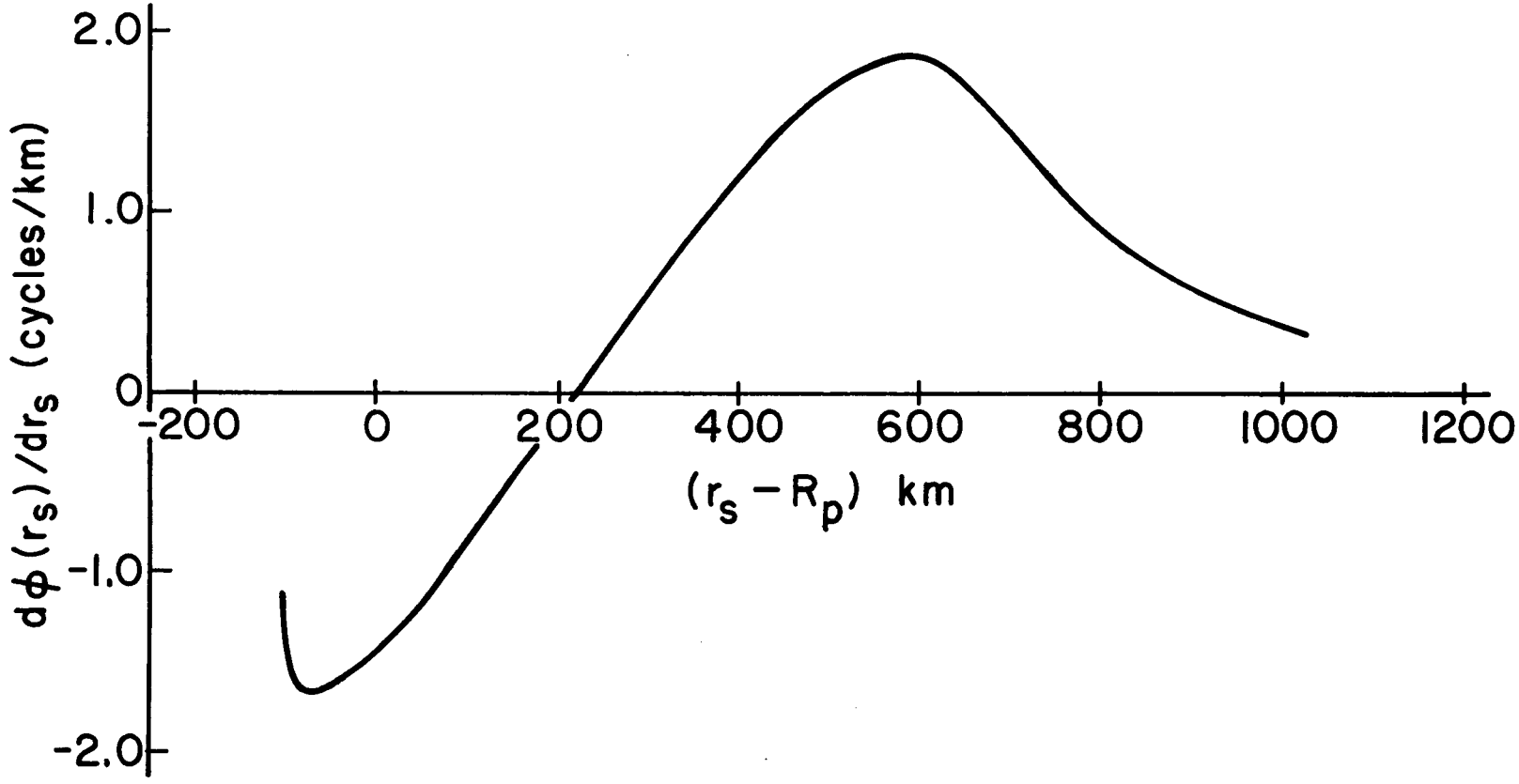

Fig. 4: RATE OF PHASE PATH CHANGE $d\Phi(r_s)/dr_s$ AT 50 Mc.
Occultation is assumed to occur 15,000 km behind Mars.

In the following it will be assumed that the two harmonically related frequencies f and mf are used to probe the ionosphere ($m > 1$). By counting the beats between f and the $1/m$-subharmonic of mf, one obtains a measure of differential phase path $[\Delta\phi(r_s)]$. From equation (9) it is seen that one can neglect the effect of ionospheric ray path bending at the highest frequency mf if m is sufficiently large. (This assumption is not strictly necessary.) For the differential phase path $[\Delta\phi(r_s)]$, one obtains:

$$\Delta\phi(r_s) = [\phi_1(\rho) + \alpha^2 z_s/2\lambda]_f - \frac{1}{m} [\phi_1(r_s)]_{mf}$$

where the first bracket on the right-hand side refers to the phase path at frequency f. The propagation path at this frequency has a radius of closest approach ρ. The second bracket refers to the phase path at frequency mf. This last propagation path is a straight line with distance r_s from the center of the planet.

The equation for the differential ionospheric phase path can be rewritten in the following form:

$$\Delta\phi(r_s) = \phi_1(\rho) + \alpha^2 z_s/2\lambda - \frac{1}{m^2} \phi_1(r_s) \tag{10}$$

where $\phi_1(\rho)$, $\phi_1(r_s)$, α, and λ in the last equation all refer to the lowest frequency f. Equation (10) can also be expressed in terms of $I(\rho)$ and that gives:

$$\Delta\phi(r_s) = -40.3\ I(\rho)/cf + (812.0\ z_s/cf^3)\ [dI(\rho)/d\rho]^2$$

$$+40.3\ I(r_s)/cf\ m^2 \qquad (11)$$

The refraction in the ionosphere causes focussing of the waves as Fig. 3 illustrates. Defining the refraction gain G_r as the change in amplitude due to differential refraction one obtains

$$G_r = -10 \log \left| 1 + z_s\ d\alpha/d\rho \right| \quad (db) \qquad (12)$$

which is valid when $|\alpha z_s| << r_s$. The refraction gain can also be related to the second derivative of the electron content:

$$G_r = -10 \log \left| 1 - (40.3\ z_s/f^2)\ d^2\ I(\rho)/d\rho^2 \right| \qquad (13)$$

Figure 5 shows how G_r would vary during occultation of the Chapman ionosphere adopted here.

The curves in Figs. 2, 4, and 5 are shown as a function of the straight-line miss distance $(r_s - R_p)$, where R_p is the radius of the planet. The occultation is assumed to take place 15,000 km behind Mars. Under these conditions the spacecraft will move with a velocity of the order of 3.5 km/sec in the r-direction. At this velocity the probing ray path sweeps through the ionosphere in about 7 minutes. The changes in the phase and amplitude

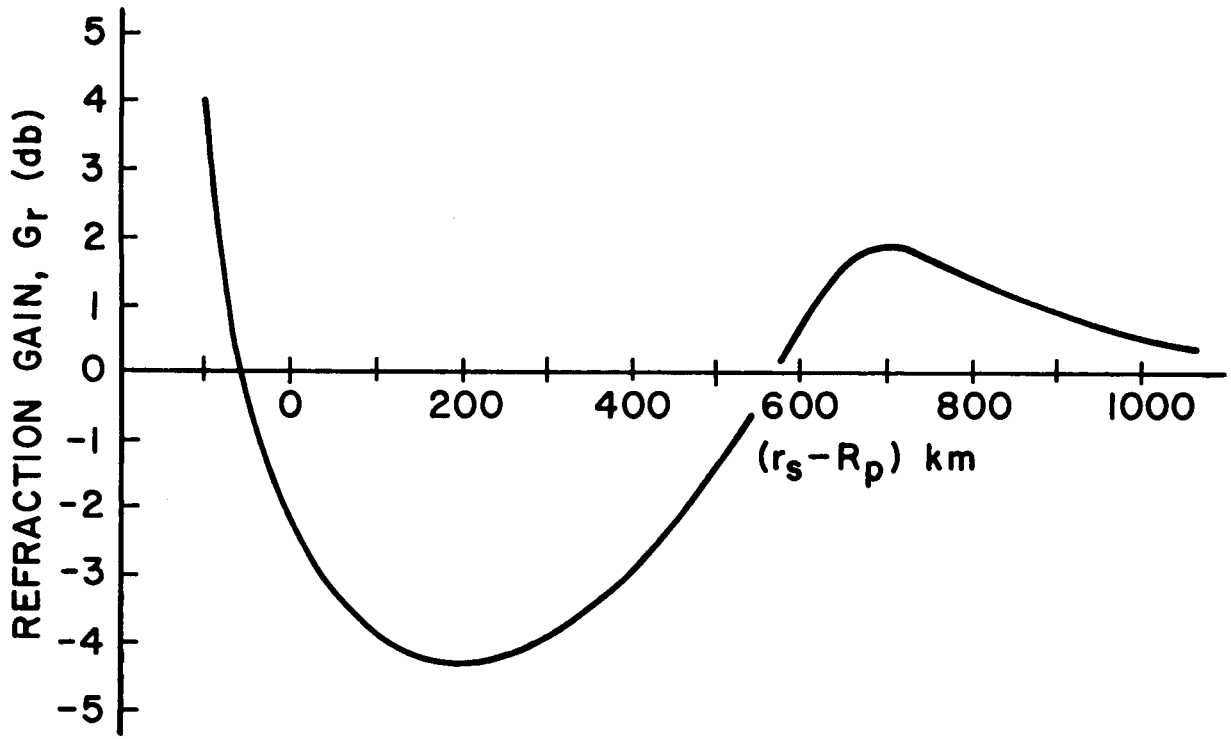

Fig. 5: REFRACTION GAIN G_r AT 50 Mc DUE TO THE MARTIAN IONOSPHERE
Occultation is assumed to take place 15,000 km behind the planet.

take place relatively slowly during the occultation of the ionosphere, and a low data sampling rate may be sufficient to recover most of the variations.

Equations (12) and (13) can be simplified further when

$$\left| z_s \, d\alpha/d\rho \right| \ll 1 \quad .$$

In terms of the second derivative of the electron content, one obtains

$$G_r = (175.0 \; z_s/f^2) \; d^2I(\rho)/d\rho^2 \tag{14}$$

A more complete treatment of the phase and amplitude variations caused by refraction in a planetary ionosphere has been given by Fjeldbo [1964].

It will now be outlined how the electron density profile can be determined from the radio occultation measurements. The solution to this problem can conveniently be divided into two steps:

1. First, one can determine the straight-line phase path $\Phi_1(\rho)$, or the electron content profile $I(\rho)$, from a measurement of the phase or amplitude variations that occur during immersion and emersion.
2. Second, one can use the electron content profile to calculate the electron density profile, assuming that the ionosphere may be considered spherically symmetrical in those regions probed by the radio signals.

The first step in this procedure is easily illustrated by assuming that the differential phase path $\Delta\phi(r_s)$ in equation (10) has been measured during immersion or emersion. When considering the slope of $\Delta\phi(r_s)$, one finds that:

$$\lambda d[\Delta\phi(r_s)]/dr_s = \lambda d[\phi_1(\rho)+\alpha^2 z_s/2\lambda]/d(\rho+\alpha z_s)-(\lambda/m^2)d\phi_1(r_s)/dr_s$$

$$= (1 - \frac{1}{m^2})\ \alpha$$

for small angles of refraction.

This derivation yields α as a function of the spacecraft coordinate r_s:

$$\alpha = [m^2\lambda/(m^2-1)]\ d[\Delta\phi(r_s)]/dr_s \tag{15}$$

In other words, the slope of $\phi_1(\rho)$ and the slope of $\Delta\phi(r_s)$ at $r_s = \rho + z_s\alpha$ both yield the angle of refraction α for the propagation path with radius of closest approach ρ.

For the regions where $z_s\lambda \left\{d[\Delta\phi(r_s)]/dr_s\right\}^2/2$ is negligible, one obtains from equation (10):

$$\phi_1(\rho) = m^2\ \Delta\phi(r_s)/(m^2 - 1) \tag{16}$$

where $\rho = r_s$, since the propagation follows essentially straight lines.

Next, it will be shown how $\phi_1(\rho)$ may be obtained from $\Delta\phi(r_s)$ in regions where $d[\Delta\phi(r_s)]/dr_s > 0$. From equation (10) it is seen that knowing $\phi_1(r_s)$ permits calculation of $\phi_1(r_s - \alpha z_s)$ when $\Delta\phi(r_s)$ has been obtained from the phase path measurements. This procedure allows a stepwise determination of $\phi_1(\rho)$ starting at large and moving towards small values of ρ.

A similar procedure can be followed in regions where $d[\Delta\phi(r_s)]/dr_s < 0$. When $\phi_1(\rho)$ is known, one can find $\phi_1(\rho + \alpha z_s)$ from equation (10). In this way one can calculate $\phi_1(\rho)$ by starting at large and successively moving towards smaller values of ρ. Having found $\phi_1(\rho)$, we also know the electron content $I(\rho)$ since the two only differ by a constant factor.

The procedure discussed here can also be used to calculate $\phi_1(\rho)$ from an absolute measurement of the ionospheric phase path increase $\phi(r_s)$. This mainly amounts to setting $1/m^2$ equal to zero in the above expression.

Note that the differential phase path method does not yield $\phi_1(\rho)$ for those rays that are bent further behind the planet than the calibration signal. This limitation can be minimized by choosing the lower frequency sufficiently high such that the bending of the propagation paths around the planetary limb is negligible.

It can also be shown that group path or amplitude measurements may be used to find the electron content $I(\rho)$ [Fjeldbo, 1964]. The last method utilizes equation (13). In practice it would be desirable to measure phase path, group path, amplitude, and polarization during immersion and emersion to obtain both redundant and complementary information on the profiles of electron density, absorption or scattering loss, and the planetary magnetic field.

The final step in calculating the electron density profile from the phase path, group path, or amplitude measurements will now be outlined. Equation (5) relates the electron density N to the electron content $I(\rho)$. Assuming spherical symmetry one can show that this equation is a special case of Abel's integral equation. Solving for $N(\rho)$ gives:

$$N(\rho) = \frac{1}{\pi} \int_{\xi=\rho}^{\infty} [I(\rho) - I(\xi)]\xi \, (\xi^2-\rho^2)^{-3/2} \, d\xi \qquad (17)$$

where ξ is a dummy variable of integration.

The pole of $(\xi^2-\rho^2)^{-3/2}$ may cause inaccuracies if equation (17) is used in the calculation of $N(\rho)$. These errors may be avoided by expanding the integrand in a Taylor series the interval from ρ to $\rho+\Delta\rho$ and evaluating this portion of the integral analytically. In this way one obtains:

$$N(\rho) = -\frac{1}{\pi}(\Delta\rho/2\rho)^{1/2}\, dI(\rho)/d\rho + \frac{1}{\pi}\int_{\rho+\Delta\rho}^{\rho_i} [I(\rho)-I(\xi)]\xi\,(\xi^2-\rho^2)^{-3/2} d\xi$$

$$+ \frac{1}{\pi} I(\rho)\,(\rho_i{}^2-\rho^2)^{-1/2} \qquad (18)$$

where $\Delta\rho$ should be chosen small enough such that $I(\rho)$ can be approximated by its tangent in this interval. In deriving equation (18), it was also assumed that $I(\xi)$ is negligible for ξ larger than the outer radius ρ_i of the ionosphere. Numerical calculations of $N(\rho)$ using equation (18) have the advantage that integration is required only over a finite interval in ξ, and that the integrand contains no pole in this region.

Measurements made during immersion and emersion give the electron density profile at two different local times, latitudes, and seasons. These profiles are furthermore related to the neutral constituents in the atmosphere. A measurement of electron concentration plus knowledge of recombination and attachment coefficients permit the production rate to be determined for steady state conditions. This, in turn, permits the calculation of the concentration of the ionizable constituents, since the intensity of the ionizing agents are relatively well known from rocket and satellite measurements.

The lower frequency signals may suffer some absorption as the propagation path passes through the lower ionospheric regions. In that case it would not be possible to use the amplitude measurements at the lower frequency to calculate the electron density profile unless a separation of refraction gain and absorption can be made using their different dependence on z_s. On the other hand, these amplitude measurements may be used to estimate the vertical collision frequency profile, assuming that the electron density profile has been found from the phase-path or group-path measurements. The collision frequency is again related to the temperature and density of the neutral gas.

A non-spherical distribution of the electron density will cause some ambiguity since it is necessary to know the form of the horizontal variations in order to determine the vertical density profile from the content profile. However, examples taking into account the solar zenith angle dependence in the Chapman layer, which is omitted in equation (1), show that the average electron density profile can be obtained with good accuracy when the horizontal variations in N are neglected in the regions probed by the radio signals [Fjeldbo, 1964].

Changes in the earth's ionosphere and in the interplanetary medium may also affect the measurement. For instance, the maximum rate of change of ionospheric electron content in the afternoon and evening (the time of Martian encounter for the 1964-65 mission) is approximately 10^{13} elec/m^2/sec. From ionosonde measurements it is conservatively estimated that 80 percent of this variation may be predicted, leaving an uncertainty of only $\pm 2 \times 10^{12}$ elec/m^2/sec. This rate of change corresponds to a wavelength uncertainty at 50 Mc of only 0.005 wavelengths/sec, or about one wavelength every three minutes. In the period of approximately 7 minutes during which the minimum ray path altitude varies from about 1500 km to the Martian surface, the earth's ionosphere would contribute an uncertainty of only about two wavelengths to the total of approximately 600 wavelengths. Measurements of the electron content near the earth with moon echoes, or by using signals from satellites, might further reduce this uncertainty.

There does not appear to be sufficient data at present to make a good estimate of the possible interplanetary effect. One might expect that if a major solar stream crosses a large fraction of the ray path during the encounter measurements, it could be difficult to separate the effects. With a steady solar wind, there may be fluctuations due to temporal and spatial irregularities, but we know of no good

measures of this effect. The moon echo measurements being conducted at Stanford could perhaps set an upper limit [Howard et al, 1964]. It is found that there are RMS fluctuations in the frequency of the reflected echoes of about 0.1 cps at 25 Mc for a 2.5 second pulse. No doubt most or all of this uncertainty is due to ionospheric irregularities, libration of the rough lunar surface, and measurement limitations. However, if it is pessimistically assumed that all of it is due to irregularities in the interplanetary medium, and that the space and time variations can be treated as independent and random, one finds an 11 cycle uncertainty in a 7 minute measurement of phase path from the earth to the Mars probe. This very pessimistic assumption for an upper limit of the uncertainty due to the interplanetary medium during quiet solar conditions thus leads to an uncertainty in phase path which is much less than the representative Martian ionospheric effect illustrated in Fig. 2.

The radio waves are also refracted by the lower neutral atmosphere and diffracted at the planetary limb. These effects provide a sensitive measure of scale height and density in the neutral atmosphere [Fjeldbo and Eshleman, 1965], especially if the experiment provides separation of dispersive ionospheric refraction effects from the non-dispersive effects of the neutral atmosphere.

In order also to illustrate the advantage of using two frequencies in the determination of the refractive index profile of the lower neutral atmosphere, it will in the following be assumed that $\phi_1(\rho)$ has been calculated from measurements at two frequencies f and mf. Taking into account the effect of both the ionized and the neutral part of the atmosphere, one can rewrite equation (2) in the following form:

$$\phi_1(\rho,f) = \frac{1}{\lambda}\int_{-\infty}^{+\infty}(\mu_n - 1)\,dz - \frac{40.3}{cf}\int_{-\infty}^{+\infty} N dz \qquad (19)$$

where $\phi_1(\rho,f)$ denotes the straight-line phase path at frequency f, λ the free space wavelength corresponding to the frequency f, and μ_n the refractive index profile of the neutral atmosphere. Similarly, one can write for the straight-line phase path at frequency mf:

$$\phi_1(\rho,mf) = \frac{m}{\lambda}\int_{-\infty}^{+\infty}(\mu_n - 1)\,dz - \frac{40.3}{cfm}\int_{-\infty}^{+\infty} N dz \qquad (20)$$

From equations (19) and (20), one can show that:

$$\frac{1}{\lambda}\int_{-\infty}^{+\infty}(\mu_n - 1)\,dz = \frac{m\phi_1(\rho,mf) - \phi_1(\rho,f)}{m^2 - 1} \qquad (21)$$

from which the vertical refractive index profile of the lower neutral atmosphere may be determined. Equation (21)

shows that it is not necessary to assume a spherically symmetric ionosphere in order to determine the refractive index profile of the neutral atmosphere, when two frequencies are used in the occultation experiment. This result is very important in cases where very large ionospheric effects might otherwise mask the effect of a neutral atmosphere of low density.

The best choice of frequencies for the radio occultation experiment depends on the electron density profile of the ionosphere to be explored, and on the distance between the spacecraft and the planet during the occultation. There is a lower limit to the frequency that can be used, if the caustic region is to be avoided. This minimum usable frequency f_{min} is given by:

$$f_{min} = \sqrt{40.3 z_s \left[\frac{d^2 I(\rho)}{d\rho^2}\right]_{max}} \tag{22}$$

Equation (22) can be obtained from equation (13) by making use of the fact that the refraction gain G_r approaches infinity at the caustic.

It is seen from equation (22) that the minimum usable frequency f_{min} is proportional to the square root of the spacecraft-planet separation distance z_s and the maximum of the second derivative of the electron content $I(\rho)$. For a single Chapman-layer model, one finds that f_{min} is proportional to the maximum plasma frequency in the ionosphere [Fjeldbo, 1964].

A 50-Mc signal has been used in the numerical examples in this publication. This choice was based on the 10^{11} elec/m^3 peak density obtained by Chamberlain [1962]. Atmospheric models developed more recently suggest that the peak electron density might be as high as 10^{13} elec/m^3 [Norton, 1964], so that a frequency around 500 Mc may, therefore, be a better choice.

An even higher minimum usable frequency is obtained if one considers the possibility of patches of dense sporadic-E ionization similar to those observed in the earth's ionosphere. Very large density gradients may be encountered here, so that f_{min} estimated from equation (22) might be many thousands of megacycles. This emphasizes the importance of using frequencies considerably higher than the maximum plasma frequency, although increasing the frequency decreases measurement precision for other regions of the ionosphere. Possible signal reception over several different propagation paths due to sporadic-E would not affect the quality of the data for the rest of the ionosphere. However, ionization patches of this type would tend to reduce the accuracy with which the refractive index profile of the lower neutral atmosphere could be determined. This last situation may be avoided by increasing the lower radio frequency or decreasing the planet spacecraft separation distance. Under certain conditions of ionospheric layer density, sporadic-E gradients, and neutral atmospheric

density, it may be highly advantageous to use at least three frequencies if both ionospheric and atmospheric data are to be obtained with good accuracy.

To be discussed next is the power and antenna gain required to conduct the ionospheric radio occultation experiment. Cosmic noise is the limiting factor in signal reception at 50 Mc. Assuming a signal-to-noise ratio at the space receiver of about 25 db in a 30 cps bandwidth, and an earth-Mars distance of 1.5 Astronomical Units, one finds that it is necessary to radiate about 7×10^6 watts/steradian during the occultation. This requirement can be satisfied with a 75 foot diameter dish and a 1200 kw CW transmitter. A 150 foot dish requires a 300 kw transmitter. These numbers all refer to a 50 Mc signal. The power requirement at 500 Mc is down by about a factor of ten, mainly due to a lower noise temperature, and 30 kw should be sufficient if a 150 foot dish is used for transmission from the earth.

It is seen from these power estimates that it would not be feasible at present to transmit a signal in the VHF range from the spacecraft and receive it on the earth. At these frequencies, one has to make the transmission from the ground and telemeter measurements of the changes in phase and amplitude back to the earth.

Conclusions: It is necessary to have fairly good atmospheric models before more sophisticated missions can be designed to explore the planets in our solar system. Entry probes can also be used to study these atmospheres [Seiff and Reese, 1965]. However, the radio occultation method is superior from the point of view of simplicity. In addition, this method permits diurnal and seasonal changes in different regions of the atmosphere to be studied from a single orbiting satellite. On the basis of these considerations, it is suggested that the two frequency, bistatic, radar-occultation experiment be used, particularly for the initial exploration of planetary atmospheres.

If an orbiting satellite is used, one might also employ the radio echo reflected from the planet to study the planetary surface. A separate publication is being prepared to discuss this subject. It should, furthermore, be mentioned that the spacecraft equipment required for the two-frequency occultation experiment can very well be used to monitor the average interplanetary electron density while the space probe is in transit.

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