To be published in IEEE Transactions on Information Theory

NOTE ON RECURRENT CODES*

[redacted]

Wyner and Ash[1] have given bounds on the minimum guard space necessary to correct all bursts of a specified maximum length with a recurrent code. Following the methods of a previous paper[2], NASA CR56894 this note gives an explicit construction for acheiving their lower bound for recurrent codes designed to correct error bursts of a known phase (called "Type B2 Codes"). This proves that the inequalities throughout section III of Wyner and Ash[1] may be replaced by equality to the lower value. The notation used here coincides with Wyner and Ash[1] and Berlekamp[2].

We assume a solution of the following form: Should say that this is a special case (b=6)

$$B = \begin{matrix} 100000 \\ 010000 \\ 001000 \\ 000100 \\ 000010 \\ 000001 \\ 000000 \\ 00000A \\ 0000CB \\ 000FED \\ 00JIHG \\ 0ØNMLK \end{matrix}$$

UNPUBLISHED PRELIMINARY DATA

N65-83837

Code none

NASA CR 56894

where A,B,C,... are binary variables whose values we shall subsequently choose. The parity check matrix of this code is given by:

[redacted]

100000					
010000	100000				
001000	010000	100000			
000100	001000	010000	100000		
000010	000100	001000	010000	100000	
000001	000010	000100	001000	010000	100000
000000	000001	000010	000100	001000	010000
00000A	000000	000001	000010	000100	001000
0000CB	00000A	000000	000001	000010	000100
000FED	0000CB	00000A	000000	000001	000010
00JIHG	000FED	0000CB	00000A	000000	000001
0ØNMLK	00JIHG	000FED	0000CB	00000A	000000

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*This work was supported in part by the U.S. Army, Navy, and Air Force under Contract DA36-039-AMC-03200(E); the National Science Foundation(Grant GP-2495); the National Institute of Health (Grant MH-04737-04); and the National Aeronautics and Space Administration (Grant NsG-496).

Adding in the necessary columns from the first block to eliminate all ones above the critical line in successive blocks gives:

0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0
0	0	0	0	A	0	0	0	0	A	0	1	0	0	A	0	1	0	0	A	0	1	0	0
0	0	0	0	B	A	0	0	0	B	0	0	0	0	B	0	0	1	0	B	0	0	1	0
0	0	F	E	(D+K)	B	0	F	E	D	0	A	F	E	D	0	0	0	E	D	0	0	0	1
0	J	I	(H+F)	(G+E)	D	J	I	H	G	0	B	I	H	G	0	0	A	H	G	0	0	0	0
0	N	(M+J)	(L+I)	(K+H)	G	N	M	L	(K+F)	E	D	M	L	K	0	0	B	L	K	0	0	0	A
M_1						M_2						M_3						M_4					

0	1	0	0	0	0
A	0	1	0	0	0
B	0	0	1	0	0
D	0	0	0	1	0
G	0	0	0	0	1
K	0	0	0	0	0
M_5					

The code is capable of type B2 error correction iff all the matrices M_1, M_2,... are simultaneously nonsingular. If some matrix M_i is singular, then some nonzero combination of its column vectors is zero, and the error pattern corresponding to this combination in the ith block is indistinguishable from an error pattern in the original ~~first~~ block.

We now show that it is possible to choose the binary variables A, B, ... in such a way as to make all the matrices M_i nonsingular. We choose the elements A, B,... successively in alphabetical order in such a way as to make nonsingular all the square submatrices whose upper right corner coincides with the upper right corner of one of the M_i. This is possible because each letter occurs on the minor diagonal of exactly one of the M_i. When it occurs in that position, all the other elements in the submatrix of which it is the lower left corner are ~~of lower~~ its alphabetical ~~order~~ predecessors (and hence previously determined). The cofactor of this lower left corner elem[illegible] is nongingular by a previous construction (or, initially, because it is an identity). Thus it is possible to choose the lower left corner element uniquely in such a way that the square submatrix containing it is nonsingular. (For further discussion of this procedure, see Berlekamp[2].)

[redacted]

For example, A occurs in the lower left corner of the submatrix $\begin{matrix}01\\A0\end{matrix}$, which is nonsingular iff we choose A to be 1. B occurs first in the lower left corner of the submatrix $\begin{matrix}010\\A01\\B00\end{matrix} = \begin{matrix}010\\101\\B00\end{matrix}$. To make this matrix nonsingular, we choose B to be 1. Continuing alphabetically, we choose the other letters. For any value of b, this method enables us to find a 2b x b matrix:

$$B = \begin{bmatrix} I \\ 00000000..00 \\ A \\ CB \\ FED \\ \ddots \end{bmatrix} \qquad \begin{matrix} A \\ CB \\ FED \\ JIHG \\ ØNMLK \\ UTSRQP \\ \ddots \end{matrix} = \begin{matrix} 1 \\ 11 \\ 111 \\ 1101 \\ 10001 \\ 100011 \\ \ddots \end{matrix}$$

The values of A, B, C,... in the triangular array are independent of b.

This method will also yield a 2b x b matrix having any specified elements (not necessarily zeros) on or above the minor diagonal in the bottom half. Modifications of the method can also be applied when the top half of B is an invertible bxb matrix other than the identity.

The search for a general B1 matrix satisfying the Wyner and Ash lower bound has not been so successful. An exhaustive search by hand reveals that of the 16 possible normalized (with check digit last) 4x2 matrices, only 3 are of type B1:

01	01	11
00	10	10
10	10	00
10	00	10

A computer search of the 4096 6x3 matrices revealed that 7 of them are of type B1:

011	001	001	111	101	011	101
010	110	110	100	100	110	100
110	010	100	110	100	100	010
000	100	010	100	010	110	000
100	010	100	100	000	000	010
100	010	100	010	010	010	110

This indicates that in general the identity matrix or any reflection or permutation of it is a poor choice for the top half of B.

Judging from the last matrix in each set, one is led to expect an 8x4 B1 matrix of the form

```
XYZ1
1000
0100
0010
0000
00A0
0CB0
FED0
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However, it can be proved that no such matrix exists, for any values of the letters. The existence of 2b x b matrices of type B1 for arbitrary b remains an open question.

(1) A. D. Wyner and R. B. Ash, "Analysis of Recurrent Codes", IEEE Transactions of Information Theory, vol. IT9, pp. 143-156; July, 1963.

(2) E. R. Berlekamp, "A Class of Convolution Codes", Information and Control, vol.6, pp 1-13; March, 1963.

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