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[illegible] ANALYSIS

## I. INTRODUCTION

The purpose of this analysis is to determine, for a representative nominal probe trajectory, whether an on-board navigation system and a separation thrust control system, of attainable accuracies, will permit a lander to enter the Mars atmosphere within a narrow band of entry angles, to a high degree of probability, and to land on a designated target site area with high probability. Conversely, extrapolation is used to determine navigation system requirements.

The analysis presented herein is an extension and amplification of work done previously by Mr. G. Reehl, as well as unpublished work done by the writer. Several significant features should be pointed out, which distinguish the present material from Mr. Reehl's:

1) Equations for error coefficients are presented in greatly simplified form, as a result of use of bus and lander trajectory parameters.

2) The derivation of expressions for certain error coefficients, based upon geometry considerations, is clarified.

3) Equations are delineated (in an appendix) for the determination of lander nominal trajectory parameters, such that this trajectory meets the constraints of being in the bus trajectory plane, intercepting the Mars atmosphere at a specified entry angle, and landing at the designated target site.

4) Navigation uncertainties are computed in a bus trajectory system, rather than as Earth-centered equatorial system. The transformations to convert these uncertainties, at separation, to lander uncertainties, is presented in detail.

The nominal probe heliocentric trajectory was selected on the basis of meeting an Earth visibility criterion, at Martian atmospheric entry. This trajectory is considered representative for the 1971 launch contour, constrained by the aforesaid criterion, since the probe asymptotic approach velocity does not significantly change in this contour.

Numerical results, derived in this analysis, indicate that a landing, such that allowable entry angle and surface dispersion criteria are met, can be accomplished with (what is believed to be attainable) thrust control and navigation system tolerances, provided that on-board navigation measurements are made on the order of not less than 4 hour intervals.

## II. NUMERICAL RESULTS

### A. Assumptions

The nominal heliocentric trajectory chosen was the Type I, Class I of May 19, 1971 launch date, and time of flight of 200 days. The corresponding approach

[illegible] was assumed to be applied in such [illegible] that the [illegible] of the lander was the same as that of the bus; i.e., no speeding of lander and lander and bus approach trajectory planes coincident. The desired [illegible] (which affects the orientation of the approach plane) was taken to be [illegible]; an additional constraint was that entry to Mars atmosphere should [illegible] be at a 30° path angle (measured down from local horizontal).

It was assumed that an on-board navigation system would measure two star-planet angles simultaneously, with this pair of measurements made every 8 hours and starting when the bus would be 2 million miles from Mars. Orbit determination and prediction was assumed to take place concurrently with measurements, these measurements continuing, at the sampling rate, down to the nominal separation distance. The total number of sample pairs was 37, and the 1 σ value of error in star-planet angle was taken as 1 milliradian. It was assumed that orbit determination would be accomplished by the maximum likelihood technique, using a priori statistics from Earth - based measurements prior to the initiation of the on-board measurements. A navigation error program was run to obtain the covariance matrix of bus position and velocity component uncertainties at the separation point; A.U. uncertainty effects were included. The resulting matrix is shown, Table I.

It should be noted that Mars atmosphere uncertainties are not considered in this analysis. Based upon present estimates of atmosphere extremes the error effect should be small compared to navigation error effects.

A. Entry Angle Dispersion

The required velocity separation increment for the stated conditions, is 92.8 f.p.s. A 1 σ error of 1% was assumed in the magnitude of the velocity increment; 1 σ errors, both in-plane and out-of-plane, of from 0° to 3° were taken for the direction of the velocity increment. Even up to 3°, the angle error effect is negligible compared to the [illegible] magnitude error effect. Furthermore, as shown in Table II, the execution error effect is only [illegible] as large as the r.m.s. of the execution and navigation error effects.

TABLE II

| $\sigma(\delta\beta)$ | $\sigma(\delta\gamma_2)$ | $\sigma(\delta\gamma_2)_N$ | $\sigma(\delta\gamma_2)_T$ |
|---|---|---|---|
| 0° | [illegible] | 2.[illegible]° | 2.43° |
| 1° | [illegible] | | |
| 2° | [illegible] | | |
| 3° | [illegible] | | |

TABLE I

Covariance Matrix of Uncertainties @ $r_s$ = 150,000 n.miles;

IF Tracking Angle Error = 1 milliradian;

Sampling rate; 1 pair every 3 hrs.

| | $Y^2$ | $Z^1$ | $\dot{X}^1$ | $\dot{Y}^1$ | $\dot{Z}^1$ |
|---|---|---|---|---|---|
| [illegible] $\times 10^4$ | $-.037 \times 10^4$ | $2.844 \times 10^4$ | $1.703 \times 10^{-3}$ | $-1.852 \times 10^{-3}$ | $9.089 \times 10^{-3}$ |
| | $1.523 \times 10^4$ | $-1.551 \times 10^4$ | $-2.162 \times 10^{-3}$ | $-2.277 \times 10^{-3}$ | $-4.060 \times 10^{-3}$ |
| | | $7.703 \times 10^4$ | $7.131 \times 10^{-3}$ | $2.088 \times 10^{-3}$ | $22.169 \times 10^{-3}$ |
| | | | $6.783 \times 10^{-10}$ | $6.514 \times 10^{-10}$ | $10.094 \times 10^{-10}$ |
| | | | | $11.570 \times 10^{-10}$ | $2.099 \times 10^{-10}$ |
| | | | | | $64.973 \times 10^{-10}$ |

[illegible] variances in $km^2$

[illegible] variances $km^2/sec.^2$

[illegible] velocity covariances in $km^2/sec.$

If the nominal entry angle were set at 27.5°, assuming little change in entry angle dispersion, the entry angle would lie between 20° and 35° with 99.7% probability.

On the other hand, if no more than a 3σ value of 5° in $\sigma(\delta\gamma_E)_T$ is to be permitted, and assuming $\sigma(\delta\beta) = 3°$ and $\sigma(\delta\Delta v_s) = 1\%\,\Delta v_s$, then an allowable navigation error effect is obtained as follows:

$$\sigma(\delta\gamma_E)_N\Big]_{\text{allowable}} = \sqrt{\left(\frac{5}{3}\right)^2 - (.44)^2}$$

$$= 1.6°$$

To achieve this, it would be necessary to increase the number of samples, or to decrease the inaccuracy in the sampled data. Approximately, the required increase in number of samples is given by

$$\frac{\sigma(\delta\gamma_E)_N\Big]_{\text{all}}}{2.39} = \frac{1}{\sqrt{n}}$$

$$n = 2.2$$

$$\text{no. of samples req'd (pairs)} = 2.2 \times 37 \approx 81$$

This can certainly be accomplished by reducing the sampling interval from 8 hours down to say 3½ hours. Alternatively, the allowable entry angle dispersion can be achieved, retaining the 8 hour sampling rate, by reducing the measurement error from 1 milliradian to 2/3 milliradian.

8. Surface Dispersion

The surface miss, due to both navigation uncertainties and execution error, is resolved into a down-range (in-plane) component $\delta\theta_s$ and a cross-range (out-of-plane) component $\delta\theta_c$. Thus, the resultant is

$$\delta\theta_s = \sqrt{(\delta\theta_s)^2 + (\delta\theta_c)^2}$$

Let subscript T denote total error, due to both sources:

$$(\delta\theta_s)_T = (\delta\theta_s)_N + (\delta\theta_s)_T$$

$$(\delta\theta_c)_T = (\delta\theta_c)_N + (\delta\theta_c)_T$$

Whence

$$\sigma(\delta\theta_s)_T = \sqrt{\sigma(\delta\theta_s)_N^2 + \sigma(\delta\theta_s)_T^2}$$

The [illegible]

The variances as well as covariance of $(\delta\theta_R)_T$ and $(\delta\theta_c)_T$ determine the shape and orientation of the surface dispersion ellipse.

In order to properly determine the probability of landing anywhere within the target area (and therefore assess the permissibility of execution tolerances and navigation instrument inaccuracy) it would be necessary to integrate the joint probability density function of $(\delta\theta_R)_T$ and $(\delta\theta_c)_T$ over the target area. If, however, the 3 r.s.s.value of the total components is less than the smallest dimension of the target area then certainly the probability of impact will be better than 99%. The extent of Pandora Return is about $10^\circ \times 40^\circ$. If the execution magnitude $1\sigma$ error is 1% of velocity increment, and the execution direction $1\sigma$ errors (both in-plane and out-of-plane) are each $3^\circ$, then from Table III, the criterion that $\sqrt{(\delta\theta_R)^2_T} = 5/3^\circ$ cannot be met, if the $1\sigma$, 1 milliradian navigation instrument inaccuracy and 8 hour sampling interval is kept. The allowable instrument inaccuracy is found as follows:

$$\left.\sqrt{\overline{(\delta\theta_R)^2_N}}\right]_{\text{allowable}} = \sqrt{\left(\frac{5}{3}\right)^2 - (1.442)^2}$$

$$= .835^\circ$$

$$\text{instrument error allowable} = \frac{.835}{3.36} = .25 \text{ milliradians, } 1\sigma$$

Doubling the sampling frequency would permit an instrument inaccuracy of .35 milliradians, $1\sigma$, which, it is believed, is obtainable.

Numerical value of error coefficients are listed in an appendix, for comparative purposes.

## III. NOMENCLATURE

$v$ - velocity

$\Delta v$ - velocity increment

$r$ - distance from center of planet

$\eta$ - true anomaly

$\gamma$ - path angle, measured downward from local horizontal when considering hyperbolic path

$p$ - semi-latus rectum

$\varepsilon$ - eccentricity of hyperbola

$\rho$ - angle between periapsis line and asymptotes approach vector

$a$ - semi-major distance

$n$ - mean motion

$t$ - time

$i$ - inclination

TABLE III NOTE: $\sigma(\delta\Delta\nu_s) = .01\,\Delta\nu_s$

| $\sigma(\delta\theta)$ | $\sigma(\delta\theta_R)_E$ | $\sigma(\delta\theta_R)_N$ | $\sigma(\delta\theta_R)_T$ | $\sigma(\delta\theta_c)_E$ | $\sigma(\delta\theta_c)_N$ | $\sigma(\delta\theta_c)_T$ | $\sqrt{(\delta\theta_R)^2_E}$ | $\sqrt{(\delta\theta_R)^2_N}$ | $\sqrt{(\delta\theta_R)^2_T}$ |
|---|---|---|---|---|---|---|---|---|---|
| | .5304° | 3.1434° | 3.188° | .0518° | 1.199° | 1.201° | .532° | 3.36° | 3.407° |
| | .5769° | ↓ | 3.196° | 1.319° | ↓ | 1.783° | 1.442° | ↓ | 3.360° |

$\Omega$ - longitude of ascending node

$\phi_M$ - celestial latitude of Mars

$\lambda_M$ - celestial longitude of Mars

$L$ - longitude in Mars equatorial system; $\xi$ - latitude in Mars equatorial system

$h$ - areocentric altitude from Mars surface

$u_4$ - central angle, measured in probe heliocentric orbit, locating Mars at encounter

$\sigma(\ )$ - std. deviation of ( ) error

$\beta$ - thrust angle of [illegible], measured from bus velocity direction, at separation

SUBSCRIPTS

$S$ - "at separation"

$B$ - "of bus"

$C$ - "of capsule (lander)"

$t$ - "due to time error"

$\infty$ - "at infinity"

$P$ - "at periapsis"; also, "of probe"

$N$ - "due to navigation uncertainties"

$E$ - "due to execution errors at separation"; also, "at entry"

$I$ - "at impact"

$M$ - "referred to Mars or Mars orbit"

CONSTANTS:

$\mu_M$ = 4.2906 x [illegible]

$\eta$ = [illegible]

$\Omega_4$ = Mars [illegible]

= [illegible]

[illegible]

REFERENCES:

Tech. Memo. 9752-004 - "Voyager Lander Error Analysis", G. Roehl

PIR 9751-033 - "Separation Velocity Increment Requirement", M. Levinson

PIR 9751-036 - "Preliminary Data, Earth Visibility", M. Levinson

Tech. Memo 9751-126 - "Earth Line-of-Sight Limitation"., M. Levinson

## IV. ANALYSIS

### A. Sources of Errors

In Fig. 1, unprimed quantities represent nominally desired values; primed quantities represent actual trajectory values. It is assumed that the approach guidance system is such that the proper separation point (i.e., separation distance $r_s$) and proper thrust angle $\beta$ is commanded, for the nominal separation velocity increment $\Delta v_s$ to be imparted, and for the nominally determined bus trajectory, so that lander capsule entry will be made at the prescribed entry angle $\gamma_E$ and landing will occur at a designated landing site. Because of noise errors in the navigation instruments which establish the bus trajectory (and therefore, the required $\beta$ and $r_s$) the actual bus trajectory will differ from the computed (nominal) trajectory; in other words, there will be uncertainties in the position and velocity of the bus, at separation. Furthermore, the thrust angle may deviate from the commanded value and the velocity increment will have some error tolerance around the design value. The former type are termed navigation errors, the latter type are termed execution errors. The net effect of these errors is to cause the lander to have an entry angle deviation and to miss the target site.

Additional sources of error are the design tolerances in lander $W/C_D A$, and uncertainties in Martian atmosphere parameters (essentially, density at surface and scale height). The effects of these on landing error are small for the expected variation (see reference memo 1) and are not numerically assessed in this memorandum.

Specification of the bus approach velocity ($v_{B-\infty}$) and periapsis distance ($r_{B-P}$) specifies the bus in-plane parameters ($a_B$, $e_B$). The specification of the orientation of the approach plane say, with respect to Mars equatorial plane and an inertial line in this latter plane depends upon the constraints of target site latitude ($\delta_T$), bus approach asymptote angular coordinates ($\delta_P$, $\gamma_P$), and lander entry angle ($\gamma_E$). The separation velocity increment ($\Delta v_s$) nominally required, is a function not only of $v_{B-\infty}$ and $r_{B-P}$ but of thrust angle ($\beta$), entry angle ($\gamma_E$) and separation distance ($r_s$). The relations required for delineation of nominal conditions are given in the appendix (see also references 2, 3 and 4).

Only the error analysis for in-plane targeting will be considered in this memorandum. That is, the nominal situation is one where the lander trajectory plane is coincident with bus trajectory plane; $\overline{\Delta v_s}$ has no out-of-plane component.

### B. In-Plane Error Coefficients

#### 1) Sensitivity of entry conditions to lander conditions after separation

It is desired to find the partials of entry conditions $v_E$, $\gamma_E$, $\theta_E$ with respect to capsule separation conditions $v_{c-s}$, $\gamma_{c-s}$, $r_s$, $\theta_s$. From conservation of energy and momentum considerations,

$$v_E^2 = v_{c-s}^2 + 2\mu_M\left(\frac{1}{r_E} - \frac{1}{r_s}\right) \tag{1}$$

$$\cos\gamma_E = \frac{r_s v_{c-s}\cos\gamma_{c-s}}{r_E v_E} \tag{2}$$

in which $r_E$ is a constant.

Differentiation yields

$$\frac{\partial v_E}{\partial v_{c-s}} = \frac{v_{c-s}}{v_E} \tag{3}$$

$$\frac{\partial v_E}{\partial r_s} = \frac{\mu_M}{r_s^2 v_E} \tag{4}$$

$$\frac{\partial v_E}{\partial \gamma_{c-s}} = 0 \; ; \; \frac{\partial v_E}{\partial \theta_s} = 0 \; ; \; \frac{\partial \gamma_E}{\partial \theta_s} = 0$$

$$\frac{\partial \gamma_E}{\partial v_{c-s}} = \left(\frac{v_{c-s}}{v_E^2} - \frac{1}{v_{c-s}}\right)\operatorname{ctn}\gamma_E \tag{5}$$

$$\frac{\partial \gamma_E}{\partial r_s} = \frac{1}{r_s}\left(\frac{\mu_M}{r_s v_E^2} - 1\right)\operatorname{ctn}\gamma_E \tag{6}$$

$$\frac{\partial \gamma_E}{\partial \gamma_{c-s}} = \frac{\tan\gamma_{c-s}}{\tan\gamma_E} \; ; \; \gamma_E \neq 90^\circ \tag{7}$$

From Fig. 1, a change in $\theta_s$ only will simply rotate the lander trajectory about the planet center, whence

$$\frac{\partial \theta_E}{\partial \theta_s} = 1 \tag{8}$$

Also, from Fig. 1,

$$(\theta' - \theta) = (\theta' - \theta) + (\theta' - \theta) - (\theta' - \theta)$$

Thus

$$\frac{\partial \theta_E}{\partial v_{c-s}} = \frac{\partial(\eta_{c-s} - \eta_{c-E})}{\partial v_{c-s}} \tag{9}$$

$$\frac{\partial \theta_E}{\partial r_s} = \frac{\partial(\eta_{c-s} - \eta_{c-E})}{\partial r_s} \tag{10}$$

$$\frac{\partial \theta_E}{\partial \gamma_{c-s}} = \frac{\partial(\eta_{c-s} - \eta_{c-E})}{\partial \gamma_{c-s}} \tag{11}$$

Now

$$\tan \eta_{c-s} = \tan \gamma_{c-s} \Big/ \left(1 - \frac{r_s}{p_c}\right) \tag{12}$$

$$\cos \eta_{c-E} = \frac{1}{\ell_c}\left(\frac{p_c}{r_E} - 1\right) \tag{13}$$

where

$$p_c = \frac{\left(r_s v_{c-s} \cos \gamma_{c-s}\right)^2}{\mu_M} \tag{14}$$

$$\ell_c^2 = \frac{p_c}{\mu_M}\left[v_{c-s}^2 - \frac{2\mu_M}{r_s}\right] + 1 \tag{15}$$

Differentiation of eq's, 12 thru 15 and subsequent substitution into eq's 9 thru 11 yields

$$\frac{\partial \theta_E}{\partial v_{c-s}} = \frac{-2 p_c}{\ell_c^2 r_s v_{c-s}} \tan \gamma_{c-s} + \frac{2 \operatorname{ctn} \gamma_E}{v_{c-s}} \cdot \left[1 - \frac{(p_c - r_E)}{\ell_c^2}\left(\frac{1}{a_c} + \frac{1}{r_s}\right)\right] \tag{16}$$

$$\frac{\partial \theta_E}{\partial r_s} = \frac{-p_c}{\ell_c^2 r_s} \tan \gamma_{c-s} + \frac{\operatorname{ctn} \gamma_E}{r_s}\left[2 - \frac{(p_c - r_E)}{\ell_c^2}\left(\frac{1}{a_c} + \frac{1}{r_s}\right)\right] \tag{17}$$

$$\frac{\partial \theta_c}{\partial \gamma_{c\text{-}s}} = \frac{p_c}{e_c^2 r_s}\left[\frac{p_c}{r_s} - \cos 2\gamma_{c\text{-}s}\right]\sec^2 \gamma_{c\text{-}s} - \left[2 - \frac{(p_c - r_E)}{a_c e_c^2}\right]\operatorname{ctn}\gamma_E \tan\gamma_{c\text{-}s} \tag{18}$$

wherein

$$a_c = p_c/(e_c^2 - 1) \tag{19}$$

Variations in conditions after separation affect not only position and velocity components at entry, but also the time of flight from separation to entry. This in turn has an effect on surface dispersion, i.e., the amount by which the lander misses the desired target site. Kepler's equation is written

$$n_c t_{s\text{-}E} = e_c\left[\sinh F_{c\text{-}s} - \sinh F_{c\text{-}E}\right] - \left[F_{c\text{-}s} - F_{c\text{-}E}\right] \tag{20}$$

where

$$n_c = \frac{\left[v_{c\text{-}s}^2 - \frac{2\mu_M}{r_s}\right]^{3/2}}{\mu_M} \tag{21}$$

$$\cosh F_{c\text{-}s} = \frac{e_c + \cos \eta_{c\text{-}s}}{1 + e_c \cos \eta_{c\text{-}s}} \tag{22}$$

$$\cosh F_{c\text{-}E} = \frac{e_c + \cos \eta_{c\text{-}E}}{1 + e_c \cos \eta_{c\text{-}E}} \tag{23}$$

Differentiation of eq'. 20 yields, for example,

$$n_c \frac{\partial t_{s\text{-}E}}{\partial r_s} = \left[\sinh F_{c\text{-}s} - \sinh F_{c\text{-}E}\right]\frac{\partial e_c}{\partial r_s} + \frac{1}{a_c}\left[r_s \frac{\partial F_{c\text{-}s}}{\partial r_s} - r_E \frac{\partial F_{c\text{-}E}}{\partial r_s}\right] - t_{s\text{-}E}\frac{\partial n_c}{\partial r_s} \tag{24}$$

Differentiation of eq's. 12, 13, 15, 21, 22, 23 and subsequent substitution in eq. 24 yields

$$\frac{\partial t_{s\text{-}E}}{\partial r_s} = -\frac{3 v_{c\text{-}s} t_{s\text{-}E}}{r_s^2 n_c} + \frac{1}{n_c e_c a_c}\sqrt{\frac{p_c}{a_c}}\left\{\frac{a_c Q}{r_s}\left[\frac{1}{a_c} + \frac{1}{r_s}\right] + \left[\frac{2 r_E}{r_s}\csc \eta_{c\text{-}E} - \frac{1}{e_c}\tan\gamma_{c\text{-}s}\right]\right\} \tag{25}$$

in which

$$Q = r_s\left(1+\frac{r_s}{p_c}\right)\sin\gamma_{c-s} - r_E\left(1+\frac{r_E}{p_c}\right)\sin\gamma_{c-E} - \frac{r_E^2}{a_c e_c}\cos\gamma_{c-E} \tag{26}$$

In similar fashion, we obtain

$$\frac{\partial t_{s-E}}{\partial v_{c-s}} = \frac{-3 v_{c-s} v_{c-s} t_{s-E}}{n_c \mu_M} + \frac{2}{n_c e_c a_c v_{c-s}}\sqrt{\frac{p_c}{a_c}}\left\{ a_c Q\left[\frac{1}{a_c}+\frac{1}{r_s}\right] + \left[r_E\cos\gamma_{c-s} - \frac{r_E}{a_c}\tan\gamma_{c-s}\right]\right\} \tag{27}$$

and

$$\frac{\partial t_{s-E}}{\partial \gamma_{c-s}} = \frac{1}{n_c e_c a_c}\sqrt{\frac{p_c}{a_c}}\tan\gamma_{c-s}\left\{\frac{2}{e_c\sin 2\gamma_{c-s}}\left(p_c - r_s\cos 2\gamma_{c-s}\right) - Q - \frac{2 e_c r_E^2}{p_c\tan\gamma_E}\right\} \tag{28}$$

note that

$$\frac{\partial t_{s-E}}{\partial \theta_s} = 0$$

2) Sensitivity of landing conditions to entry conditions

From Fig. 2, the central angle deviation at impact is given by

$$\theta_I' - \theta_I = (\theta_E' - \theta_E) + (\theta_{E-I}' - \theta_{E-I})$$

or

$$\delta\theta_I = \delta\theta_E + d\theta_{E-I} \tag{29}$$

where $\theta_{E-I}$ is the down-range angle traveled by the lander as it descends through the (Martian) atmosphere. The path actually bends and is almost vertical at impact, as the entry body is slowed down. $\theta_{E-I}$ is particularly affected by $\gamma_E$ and to a lesser

extent by $v_E$. For an assumed atmosphere (density-altitude relation) and specified $W/C_D A$ of the entry body, $\theta_{E-I}$ (as well as $t_{E-I}$, the time of descent) can be determined by means of a three-degree of freedom, point-mass computer program. Sensitivities to changes in $\gamma_E$ and $v_E$ can then be obtained. From eq. 29

$$\delta\theta_I = \frac{\partial\theta_I}{\partial\theta_E}\delta\theta_E + \frac{\partial\theta_I}{\partial v_E}\delta v_E + \frac{\partial\theta_I}{\partial\gamma_E}\delta\gamma_E \tag{30}$$

where

$$\frac{\partial\theta_I}{\partial\theta_E} = 1\,;\quad \frac{\partial\theta_I}{\partial v_E} = \frac{\partial\theta_{E-I}}{\partial v_E}\,;\quad \frac{\partial\theta_I}{\partial\gamma_E} = \frac{\partial\theta_{E-I}}{\partial\gamma_E}$$

the value of $\frac{\partial\theta_{E-I}}{\partial v_E}$ and $\frac{\partial\theta_{E-I}}{\partial\gamma_E}$ being obtained from plots of tabulated data.

Let $t_{S-I}$ denote time of flight of the lander, from separation to impact. Thus

$$t_{S-I} = t_{S-E} + t_{E-I}$$

$$\delta t_{S-I} = \delta t_{S-E} + \delta t_{E-I}$$

$$= \frac{\partial t_{S-E}}{\partial v_{C-S}}\delta v_{C-S} + \frac{\partial t_{S-E}}{\partial\gamma_{C-S}}\delta\gamma_{C-S} + \frac{\partial t_{S-E}}{\partial h_S}\delta h_S$$

$$+ \frac{\partial t_{E-I}}{\partial v_E}\delta v_E + \frac{\partial t_{E-I}}{\partial\gamma_E}\delta\gamma_E \tag{31}$$

where

$$\delta v_E = \frac{\partial v_E}{\partial v_{C-S}}\delta v_{C-S} + \frac{\partial v_E}{\partial h_S}\delta h_S \tag{32}$$

$$\delta\gamma_E = \frac{\partial\gamma_E}{\partial v_{C-S}}\delta v_{C-S} + \frac{\partial\gamma_E}{\partial h_S}\delta h_S + \frac{\partial\gamma_E}{\partial\gamma_{C-S}}\delta\gamma_{C-S} \tag{33}$$

Thus

$$\delta t_{S-I} = \frac{\partial t_{S-I}}{\partial v_{C-S}}\delta v_{C-S} + \frac{\partial t_{S-I}}{\partial h_S}\delta h_S + \frac{\partial t_{S-I}}{\partial\gamma_{C-S}}\delta\gamma_{C-S} \tag{34}$$

where

$$\frac{\partial t_{S-I}}{\partial v_{C-S}} = \frac{\partial t_{E-I}}{\partial v_E}\frac{\partial v_E}{\partial v_{C-S}} + \frac{\partial t_{E-I}}{\partial\gamma_E}\frac{\partial\gamma_E}{\partial v_{C-S}} + \frac{\partial t_{S-E}}{\partial v_{C-S}} \tag{35}$$

$$\frac{\partial t_{S-I}}{\partial r_S} = \frac{\partial t_{E-I}}{\partial v_E}\frac{\partial v_E}{\partial r_S} + \frac{\partial t_{E-I}}{\partial \gamma_E}\frac{\partial \gamma_E}{\partial r_S} + \frac{\partial t_{S-E}}{\partial r_S} \tag{36}$$

$$\frac{\partial t_{S-I}}{\partial \gamma_{C-S}} = \frac{\partial t_{E-I}}{\partial \gamma_E}\frac{\partial \gamma_E}{\partial \gamma_{C-S}} + \frac{\partial t_{S-E}}{\partial \gamma_{C-S}} \tag{37}$$

The value of $\frac{\partial t_{E-I}}{\partial v_E}$ and $\frac{\partial t_{E-I}}{\partial \gamma_E}$ are obtained from the aforementioned plots; i.e., partials are evaluated at the nominal values of $v_E$, $\gamma_E$. Note that eq. 34 implies

$$\frac{\partial t_{E-I}}{\partial \theta_S} = 0$$

C. Out-of-Plane Error Coefficients

In Fig. 3, define a coordinate system such that $\bar{x}$ is along $-\bar{r}_S$, $y$ is normal to the nominal lander approach plane, and $\bar{z}$ forms a right-handed system. The lander would nominally traverse a central angle of

$$\Delta\eta = \eta_{C-S} - \eta_{C-E} + \theta_{E-I} \tag{38}$$

to arrive at the target position $\bar{r}_I$. If there is a velocity error $\delta\dot{y}$ normal to the nominal plane, the actual trajectory plane will be at an angle $\theta_p$ to the nominal plane. Neglecting variation in time of flight due to $\delta\dot{y}$, the cross-range error will be $\delta\theta_C$. We have

$$\delta\theta_C \approx \tan\delta\theta_C = \tan\theta_p \sin\Delta\eta \tag{39}$$

But

$$\bar{v}'_{C-S} = v_{C-S}\left[\sin\gamma_{C-S}\,\bar{x} + \cos\gamma_{C-S}\,\bar{y}\right] + \delta\dot{y}\,\bar{z} \tag{40}$$

$$\bar{r}_S = -r_S\,\bar{x} \tag{41}$$

$$\bar{r}_S \times \bar{v}'_{C-S} = r_S\left[v_{C-S}\cos\gamma_{C-S}\,\bar{z} - \delta\dot{y}\,\bar{y}\right] \tag{42}$$

whence

$$\tan\theta_p = \frac{-\bar{y}\cdot(\bar{r}_S \times \bar{v}'_{C-S})}{\bar{z}\cdot(\bar{r}_S \times \bar{v}'_{C-S})}$$

$$= \delta\dot{y}\,/\,v_{C-S}\cos\gamma_{C-S}$$

Substituting eq. 43 into eq. 39,

$$\frac{\partial \theta_c}{\partial \dot{y}} \approx \frac{\delta \theta_c}{\delta \dot{y}} = \frac{\sec \gamma_{c-s} \sin \Delta\eta}{v_{c-s}} \tag{44}$$

If there is a normal position error $\delta y$ only, then $\bar{v}'_{c-s}$ is parallel to $\bar{v}_{c-s}$ and the deviated plane (the $\bar{r}'_s - \bar{v}'_{c-s}$ plane) intersects the nominal plane in the line $\bar{\lambda}_p$, situated $90° - \gamma_{c-s}$ from $\bar{r}_s$. We have (see Fig. 4)

$$\bar{v}'_{c-s} = v_{c-s}\left[\sin \gamma_{c-s}\, \bar{x} + \cos \gamma_{c-s}\, \bar{y}\right] \tag{45}$$

$$\bar{r}'_s = -r_s \bar{x} + \delta y\, \bar{y} \tag{46}$$

$$\bar{r}'_s \times \bar{v}'_{c-s} = v_{c-s}\left[r_s \cos \gamma_{c-s}\, \bar{y} + \delta y \left(\cos \gamma_{c-s}\, \bar{x} - \sin \gamma_{c-s}\, \bar{y}\right)\right] \tag{47}$$

$$\bar{\lambda}_p = \cos \gamma_{c-s}\, \bar{x} - \sin \gamma_{c-s}\, \bar{y} \tag{48}$$

Thus

$$\tan \Theta_p = \frac{\bar{\lambda}_p \cdot \left(\bar{r}'_s \times \bar{v}'_{c-s}\right)}{\bar{y} \cdot \left(\bar{r}'_s \times \bar{v}'_{c-s}\right)} = \frac{\delta y}{r_s} \sec \gamma_{c-s} \tag{49}$$

But

$$\delta \theta_c \approx \tan \delta \theta_c = \tan \Theta_p \cos\left(\Delta\eta - \gamma_{c-s}\right) \tag{50}$$

Substituting eq. 49 into eq. 50,

$$\frac{\partial \theta_c}{\partial \dot{y}} \approx \frac{\delta \theta_c}{\delta y} = \frac{1}{r_s}\left[\cos \Delta\eta + \sin \Delta\eta \tan \gamma_{c-s}\right] \tag{51}$$

D) Separation Error Effects

1) In-Plane Errors

From Fig. 1, considering the separation geometry, we have nominally,

$$v_{c-s}^2 = \left(\Delta v_s\right)^2 + v_{B-s}^2 + 2\left(\Delta v_s\right) v_{B-s} \cos \beta \tag{52}$$

Differentiation with respect to , and yields

$$\frac{\partial v_{c-s}}{\partial} = -\Delta v_s \left(v_{B-s}\right) \sin \beta \tag{53}$$

$$\frac{\partial v_{C-S}}{\partial \Delta v_S} = \left(\frac{\Delta v_S}{v_{C-S}}\right) + \left(\frac{v_{B-S}}{v_{C-S}}\right)\cos\beta \tag{54}$$

Also, from Fig. 1,

$$\tan\left(\gamma_{C-S} - \gamma_{B-S}\right) = \frac{\Delta v_S \sin\beta}{\left(\Delta v_S \cos\beta + v_{B-S}\right)} \tag{55}$$

Differentiation yields

$$\frac{\partial \gamma_{C-S}}{\partial \beta} = \frac{\operatorname{ctn}\beta \sin 2\phi}{2} + \sin^2\phi \tag{56}$$

$$\frac{\partial \gamma_{C-S}}{\partial \Delta v_S} = \frac{1}{\Delta v_S}\left[\frac{\sin 2\phi}{2} - \operatorname{ctn}\beta \sin^2\phi\right] \tag{57}$$

where $\phi \triangleq \gamma_{C-S} - \gamma_{B-S}$. As a good approximation to the situation of desired thrust normal to $v_{B-S}$ we may take $\beta$ such that $v_{C-S} = v_{B-S}$, that is, no change in specific total energy, so that $v_{E-B} = v_{E-C}$.

This **considerably** simplifies computation of normal conditions. In that case,

$$\left.\begin{aligned} \beta &= 90^\circ + \frac{\phi}{2} \\ \frac{\Delta v_S}{v_{C-S}} &= -2\cos\beta \end{aligned}\right\} \tag{58}$$

and the sensitivities to thrust (magnitude and direction) variations become

$$\frac{\partial v_{C-S}}{\partial \beta} = -\Delta v_S \sin\beta \tag{59}$$

$$\frac{\partial v_{C-S}}{\partial \Delta v_S} = -\cos\beta \tag{60}$$

$$\frac{\partial \gamma_{C-S}}{\partial \beta} = 2\cos^2\beta \tag{61}$$

$$\frac{\partial \gamma_{C-S}}{\partial \Delta v_S} = \frac{-\sin 2\beta}{\Delta v_S} \tag{62}$$

2) Out-of-plane error

In Fig. 5, let $\Delta v_s$ be rotated to $\Delta v_s'$, where this rotation occurs about an axis perpendicular to $\Delta v_s$, and lying in the $\bar{r}_s - \bar{v}_{c-s}$ plane. Evidently, $\bar{v}_{c-s}$ has the component

$$\delta y \approx \Delta v_s \cdot \delta\theta \tag{63}$$

Then from eq. 44,

$$\frac{\partial \theta_c}{\partial \theta} = \left(\frac{\Delta v_s}{v_{c-s}}\right) \sec \gamma_{c-s} \cdot \sin \Delta\eta \tag{64}$$

For nominal thrust such that $v_{c-s} = v_{B-s}$,

$$\frac{\partial \theta_c}{\partial \theta} = -2 \cos \beta \cdot \sec \gamma_{c-s} \cdot \sin \Delta\eta \tag{65}$$

It should be observed that effects of $\delta\theta$ on $v_{c-s}$ and $\gamma_{c-s}$ are second order and can be neglected. That is,

$$\frac{\partial v_{c-s}}{\partial \theta} \approx 0 \quad ; \quad \frac{\partial \gamma_{c-s}}{\partial \theta} \approx 0$$

E. Navigation Error Effects

Prior to separation of the capsule from the bus, either an Earth-bound or on-board navigation system may be used to determine the (bus) trajectory and to predict the bus position and velocity coordinates, in some coordinate system, at separation. For the nominally coincident plane case, evidently uncertainties in position are the same for bus and lander, at separation; the normal component of velocity uncertainty is also the same. However, uncertainties in $v_{B-s}$ and $\gamma_{B-s}$ must be related to uncertainties in $v_{c-s}$ and $\gamma_{c-s}$. Differentiation of eq's. 52 and 55 gives

$$\frac{\partial v_{c-s}}{\partial v_{B-s}} = \frac{1}{v_{c-s}} \left[ v_{B-s} + \Delta v_s \cos \beta \right] \tag{66}$$

$$\frac{\partial \gamma_{c-s}}{\partial v_{B-s}} = -\frac{1}{\Delta v_s} \frac{\sin^2 \phi}{\sin \beta} \tag{67}$$

$$\frac{\partial \gamma_{c-s}}{\partial \gamma_{B-s}} = 1 \tag{68}$$

Note that

$$\frac{\partial v_{c-s}}{\partial \gamma_{B-s}} = 0$$

For the special thrusting case such that $v_{c-s} = v_{B-s}$,

$$\frac{\partial v_{c-s}}{\partial v_{B-s}} = -\cos 2\beta \tag{69}$$

$$\frac{\partial \gamma_{c-s}}{\partial v_{B-s}} = \frac{\sin 2\beta}{v_{c-s}} \tag{70}$$

Now, assume that an on-board navigation system enables determination of bus position and velocity coordinates in the $\bar{i}_\infty - \bar{j}_\infty - \bar{k}_\infty$ system defined by $\bar{i}_\infty$ along $\bar{v}_{B-\infty}$, $\bar{j}_\infty = \bar{\eta}$ normal to bus approach plane, and $\bar{k}_\infty$ along the impact parameter $\bar{B}$. (For ease of notation, the $\bar{x} - \bar{y} - \bar{z}$ system will now be referred to as the $\bar{i}_A - \bar{j}_A - \bar{k}_A$ system, see Fig. 6). Then, measuring inertial angle $\theta_s$ from $-\bar{i}_\infty$,

$$\bar{r}_s = r_s\left[-\cos\theta_s\,\bar{i}_\infty + \sin\theta_s\,\bar{k}_\infty\right] \tag{71}$$

The actual position vector is

$$\bar{r}_s' = (r_s + \delta r_s)\left[-\cos\theta_s'\cos\varepsilon\,\bar{i}_\infty + \cos\varepsilon\sin\theta_s'\,\bar{k}_\infty + \sin\varepsilon\,\bar{j}_\infty\right] \tag{72}$$

Now

$$\begin{aligned}\bar{r}_s' - \bar{r}_s = \overline{\Delta r_s} &= \delta x\,\bar{i}_A + \delta y\,\bar{j}_A + \delta z\,\bar{k}_A \\ &= \delta X\,\bar{i}_\infty + \delta Y\,\bar{j}_\infty + \delta Z\,\bar{k}_\infty\end{aligned} \tag{73}$$

(Note that $|\overline{\Delta r_s}| \neq \delta r_s$ ). Observing that $\theta_s' = \theta_s + \delta\theta_s$, we have, after neglecting second order effects,

$$\begin{aligned}\delta X &= \overline{\Delta r_s}\cdot\bar{i}_\infty \\ &\simeq r_s\cdot\delta\theta_s\sin\theta_s - \delta r_s\cos\theta_s\end{aligned} \tag{74}$$

$$\begin{aligned}\delta Y &= \overline{\Delta r_s}\cdot\bar{j}_\infty = \delta y \\ &\simeq r_s\,\varepsilon\end{aligned} \tag{75}$$

$$\begin{aligned}\delta Z &= \overline{\Delta r_s}\cdot\bar{k}_\infty \\ &\simeq r_s\cdot\delta\theta_s\cos\theta_s + \delta r_s\sin\theta_s\end{aligned} \tag{76}$$

From eq's. 74 and 76 we obtain,

$$\delta\theta_s = \frac{\delta X}{r_s}\sin\theta_s + \frac{\delta Z}{r_s}\cos\theta_s \tag{77}$$

$$\delta r_s = -\delta X\cos\theta_s + \delta Z\sin\theta_s \tag{78}$$

From Fig. 6,

$$\begin{bmatrix} \bar{i}_r \\ \bar{j}_r \\ \bar{k}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_s & 0 & -\sin\theta_s \\ 0 & 1 & 0 \\ \sin\theta_s & 0 & \cos\theta_s \end{bmatrix} \begin{bmatrix} \bar{i}_o \\ \bar{j}_o \\ \bar{k}_o \end{bmatrix}$$

and from Fig. 7,

$$\begin{bmatrix} \bar{i}_r' \\ \bar{j}_r' \\ \bar{k}_r' \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & -\delta\theta_s \\ 0 & 1 & 0 \\ \delta\theta_s & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{i}_r \\ \bar{j}_r \\ \bar{k}_r \end{bmatrix}$$

Define

$$v_{B-S}' = v_{B-S} + \delta v_{B-S} \; ; \; \gamma_{B-S}' = \gamma_{B-S} + \delta\gamma_{B-S}$$

Then, from Fig. 8,

$$\bar{v}_{B-S} = v_{B-S}\left[\sin\gamma_{B-S}\,\bar{i}_r + \cos\gamma_{B-S}\,\bar{k}_r\right] \tag{79}$$

$$\bar{v}_{B-S}' = v_{B-S}'\left[\cos\varepsilon_v \sin\gamma_{B-S}'\,\bar{i}_r' + \sin\varepsilon_v\,\bar{j}_r' + \cos\varepsilon_v\cos\gamma_{B-S}'\,\bar{k}_r'\right] \tag{80}$$

But

$$\frac{d\,\overline{\Delta r}_s}{dt} = \delta\dot{X}\,\bar{i}_o + \delta\dot{Y}\,\bar{j}_o + \delta\dot{Z}\,\bar{k}_o = \bar{v}_{B-S}' - \bar{v}_{B-S} \tag{81}$$

Substituting eq's. 79 and 80, and making use of the above transformation matrices,

$$\delta\dot{y} = \frac{d\,\Delta\bar{r}_s}{dt} = v_{B-S}'\cos\varepsilon_v\sin\gamma_{B-S}'\cos(\theta_s + \gamma_{B-S}) + v_{B-S}'\sin\varepsilon_v\sin(\theta_s + \gamma_{B-S}) \tag{82}$$

$$\delta \dot{Y} = \frac{d\,\Delta \bar{r}_s}{dt} \cdot \bar{j}_* \approx v_{B-S}\,\varepsilon_v = \delta \dot{y} \tag{83}$$

$$\delta \dot{Z} = \frac{d\,\Delta \bar{r}_s}{dt} \cdot \bar{k}_\infty \approx -v_{B-S}\,\delta \gamma'_{B-S} \sin(\theta_s + \gamma_{B-S}) + \delta v_{B-S} \cos(\theta_s + \gamma_{B-S}) \tag{84}$$

**wherein** $\delta \gamma'_{B-S} \triangleq \delta \gamma_{B-S} + \delta \theta_s$

**Solving for** $\delta \gamma_{B-S}$ **and** $\delta v_{B-S}$ **from eq's. 83 and 85,**

$$\delta \gamma_{B-S} = \frac{\delta \dot{X}}{v_{B-S}} \cos(\theta_s + \gamma_{B-S}) - \frac{\delta \dot{Z}}{v_{B-S}} \sin(\theta_s + \gamma_{B-S}) - \delta \theta_s \tag{85}$$

$$\delta v_{B-S} = \delta \dot{X} \sin(\theta_s + \gamma_{B-S}) + \delta \dot{Z} \cos(\theta_s + \gamma_{B-S}) \tag{86}$$

**Now**

$$\delta \gamma_{C-S} = \frac{\partial \gamma_{C-S}}{\partial v_{B-S}} \delta v_{B-S} + \frac{\partial \gamma_{C-S}}{\partial \gamma_{B-S}} \delta \gamma_{B-S} \tag{87}$$

$$\delta v_{C-S} = \frac{\partial v_{C-S}}{\partial v_{B-S}} \delta v_{B-S} + \frac{\partial v_{C-S}}{\partial \gamma_{B-S}} \delta \gamma_{B-S} \tag{88}$$

**Substituting eq's. 68, 69, 76, 77, 85 and 86 into eq's. 87 and 88 and noting that** $\frac{\partial v_{C-S}}{\partial \gamma_{B-S}} = 0$ **, we have (only for nominal thrusting such that** $v_{C-S} = v_{B-S}$**, and** $\Delta v_S$ **in the bus approach plane)**

$$\begin{aligned}\delta \gamma_{C-S} = & -\frac{\sin \theta_s}{r_s} \delta \dot{X} - \frac{\cos \theta_s}{r_s} \delta \dot{Z} \\ & + \frac{\delta \dot{X}}{v_{C-S}} \left[ \sin 2\beta \sin(\theta_s + \gamma_{B-S}) + \cos(\theta_s + \gamma_{B-S}) \right] \\ & + \frac{\delta \dot{Z}}{v_{C-S}} \left[ \sin 2\beta \cos(\theta_s + \gamma_{B-S}) - \sin(\theta_s + \gamma_{B-S}) \right]\end{aligned} \tag{89}$$

$$\begin{aligned}\delta v_{C-S} = & -\delta \dot{X} \cos 2\beta \sin(\theta_s + \gamma_{B-S}) \\ & - \delta \dot{Z} \cos 2\beta \cos(\theta_s + \gamma_{B-S})\end{aligned} \tag{90}$$

Eq's. 75, 77, 78, 83, 89 and 90 relate the bus navigation uncertainties to capsule (lander) navigation uncertainties. In numerically assessing the moment matrix of bus uncertainties, a differently defined bus coordinate system $\bar{i}_\infty' - \bar{j}_\infty' - \bar{k}_\infty'$ was used, where $\bar{i}_\infty' = \bar{k}_\infty$, $\bar{j}_\infty' = -\bar{j}_\infty$, $\bar{k}_\infty' = \bar{i}_\infty$, necessitating the use of an additional transformation:

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \\ \delta \dot{X} \\ \delta \dot{Y} \\ \delta \dot{Z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & & & \\ 0 & -1 & 0 & & (0) & \\ 1 & 0 & 0 & & & \\ & & & 0 & 0 & 1 \\ & (0) & & 0 & -1 & 0 \\ & & & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta X' \\ \delta Y' \\ \delta Z' \\ \delta \dot{X}' \\ \delta \dot{Y}' \\ \delta \dot{Z}' \end{bmatrix}$$

F. <u>Entry Angle Dispersion</u>

The entry angle error due to execution errors at separation is

$$\begin{aligned} \delta \gamma_E \Big]_E &= \frac{\partial \gamma_E}{\partial v_{c-s}} \delta v_{c-s} \Big]_E + \frac{\partial \gamma_E}{\partial \gamma_{c-s}} \delta \gamma_{c-s} \Big]_E \\ &= \left[ \frac{\partial \gamma_E}{\partial v_{c-s}} \frac{\partial v_{c-s}}{\partial \beta} + \frac{\partial \gamma_E}{\partial \gamma_{c-s}} \frac{\partial \gamma_{c-s}}{\partial \beta} \right] \delta \beta \\ &\quad + \left[ \frac{\partial \gamma_E}{\partial v_{c-s}} \frac{\partial v_{c-s}}{\partial \Delta v_s} + \frac{\partial \gamma_E}{\partial \gamma_{c-s}} \frac{\partial \gamma_{c-s}}{\partial \Delta v_s} \right] \delta \Delta v_s \\ &= \frac{\partial \gamma_E}{\partial \beta} \delta \beta + \frac{\partial \gamma_E}{\partial \Delta v_s} \delta \Delta v_s \end{aligned} \tag{91}$$

The standard deviation of this (random) error is

$$\sigma(\delta \gamma_E)_E = \sqrt{\overline{\delta \gamma_E^2}\Big]_E}$$

where, since $\delta \beta$ and $\delta \Delta v_s$ are independent,

$$\overline{\delta \gamma_E^2}\Big]_E = \left(\frac{\partial \gamma_E}{\partial \beta}\right)^2 \sigma^2(\delta \beta) + \left(\frac{\partial \gamma_E}{\partial \Delta v_s}\right)^2 \sigma^2(\delta \Delta v_s) \tag{92}$$

The entry angle error due to navigation error is

$$\delta \gamma_E \Big]_N = \frac{\partial \gamma_E}{\partial h} \delta h \Big]_N + \frac{\partial \gamma_E}{\partial \dot{h}} \delta \dot{h} \Big]_N + \frac{\partial \gamma_E}{\partial \Delta v_s} \delta \Delta v_s \Big]_N$$

$$= \frac{\partial \gamma_E}{\partial X'} \delta X' + \frac{\partial \gamma_E}{\partial Z'} \delta Z' + \frac{\partial \gamma_E}{\partial \dot{X}'} \delta \dot{X}' + \frac{\partial \gamma_E}{\partial \dot{Z}'} \delta \dot{Z}' \tag{93}$$

Let $\widehat{\delta X}$ represent the matrix of navigation uncertainties:

$$\widehat{\delta X} \triangleq \begin{bmatrix} \delta X' \\ \delta Y' \\ \delta Z' \\ \delta \dot{X}' \\ \delta \dot{Y}' \\ \delta \dot{Z}' \end{bmatrix}$$

Then

$$\begin{bmatrix} \delta r_s \\ \delta \theta_s \\ \delta v_{c-s} \\ \delta \gamma_{c-s} \end{bmatrix} = \widehat{C}_A \, \widehat{\delta X} \tag{94}$$

$\widehat{C}_A$ is, from the discussion in the previous section, a 4 x 6 matrix, of which the non-zero-elements are given by

$$\frac{\partial r_s}{\partial X'} \triangleq C_{11} = \sin \theta_s \quad ; \quad \frac{\partial r_s}{\partial Z'} \triangleq C_{13} = -\cos \theta_s$$

$$\frac{\partial \theta_s}{\partial X'} \triangleq C_{21} = \frac{\cos \theta_s}{r_s} \quad ; \quad \frac{\partial \theta_s}{\partial Z'} \triangleq C_{23} = \frac{\sin \theta_s}{r_s}$$

$$\frac{\partial v_{c-s}}{\partial \dot{X}'} \triangleq C_{34} = -\cos 2\beta \cos(\theta_s + \gamma_{c-s}) \; ; \; \frac{\partial v_{c-s}}{\partial \dot{Z}'} \triangleq C_{36} = -\cos 2\beta \sin(\theta_s + \gamma_{c-s})$$

$$\frac{\partial \gamma_{c-s}}{\partial X'} \triangleq C_{41} = -\frac{\cos \theta_s}{r_s} \quad ; \quad \frac{\partial \gamma_{c-s}}{\partial Z'} \triangleq C_{43} = -\frac{\sin \theta_s}{r_s}$$

$$\frac{\partial \gamma_{c-s}}{\partial \dot{X}'} \triangleq C_{44} = \frac{1}{v_{c-s}} \left[ \sin 2\beta \cos(\theta_s + \gamma_{c-s}) - \sin(\theta_s + \gamma_{c-s}) \right]$$

$$\frac{\partial \gamma_{c-s}}{\partial \dot{Z}'} \triangleq C_{46} = \frac{1}{v_{c-s}} \left[ \sin 2\beta \sin(\theta_s + \gamma_{c-s}) + \cos(\theta_s + \gamma_{c-s}) \right]$$

Then

$$\operatorname{var}(\delta r_s) = \left[ \quad \right]$$

where

$$\overline{\delta\gamma_E^2}\Big]_N = \left[\frac{\partial\gamma_E}{\partial h_s} \quad 0 \quad \frac{\partial\gamma_E}{\partial v_{c-s}} \quad \frac{\partial\gamma_E}{\partial\gamma_{c-s}}\right] \tilde{C}_h \, \overline{\delta\bar{X}\,\delta\bar{X}^T} \, \tilde{C}_h' \begin{bmatrix} \frac{\partial\gamma_E}{\partial h_s} \\ 0 \\ \frac{\partial\gamma_E}{\partial v_{c-s}} \\ \frac{\partial\gamma_E}{\partial\gamma_{c-s}} \end{bmatrix} \tag{95}$$

$\overline{\delta\bar{X}\,\delta\bar{X}^T}$ is the moment matrix of navigation uncertainties, evaluated at separation point. From eq. 95, the following relations may be deduced:

$$\frac{\partial\gamma_E}{\partial x'} = \frac{\partial\gamma_E}{\partial h_s}\frac{\partial h_s}{\partial x'} + \frac{\partial\gamma_E}{\partial\gamma_{c-s}}\frac{\partial\gamma_{c-s}}{\partial x'}$$

$$\frac{\partial\gamma_E}{\partial z'} = \frac{\partial\gamma_E}{\partial h_s}\frac{\partial h_s}{\partial z'} + \frac{\partial\gamma_E}{\partial\gamma_{c-s}}\frac{\partial\gamma_{c-s}}{\partial z'}$$

$$\frac{\partial\gamma_E}{\partial \dot{x}'} = \frac{\partial\gamma_E}{\partial v_{c-s}}\frac{\partial v_{c-s}}{\partial \dot{x}'} + \frac{\partial\gamma_E}{\partial\gamma_{c-s}}\frac{\partial\gamma_{c-s}}{\partial \dot{x}'}$$

$$\frac{\partial\gamma_E}{\partial \dot{z}'} = \frac{\partial\gamma_E}{\partial v_{c-s}}\frac{\partial v_{c-s}}{\partial \dot{z}'} + \frac{\partial\gamma_E}{\partial\gamma_{c-s}}\frac{\partial\gamma_{c-s}}{\partial \dot{z}'}$$

$$\frac{\partial\gamma_E}{\partial y'} = 0 \quad ; \quad \frac{\partial\gamma_E}{\partial \dot{y}'} = 0$$

Since the execution and navigation errors are independent, the standard deviation of the total entry angle error is

$$\sigma(\delta\gamma_E)_T = \sqrt{\overline{\delta\gamma_E^2}\Big]_E + \overline{\delta\gamma_E^2}\Big]_N} \tag{96}$$

If the error sources are all Gaussian distributed, then $(\delta\gamma_E)_T$ will be Gaussian and $\pm 3\sigma(\delta\gamma_E)_T$ represents the 99.7% probability limits.

Error coefficients were evaluated numerically and are listed in the appendix, for the stated nominal conditions. It should be noted that the contribution to $\sigma(\delta\gamma_E)_E$ of the direction execution error is quite small. On a percentage basis,

Error effect on $\gamma_E$ / % error in $\beta$ $= \sigma\left(\frac{\partial\gamma_E}{\beta}\right)$

Error effect on $\gamma_E$ / % error in $\gamma_{c-s}$ $= \sigma\left(\frac{\partial\gamma_E}{\gamma_{c-s}}\right)$

This ratio, for the stated nominal conditions, is only .032. This fact explains the relative insensitivity of $\sigma(\delta\gamma_E)_E$ to change in $\sigma(\delta\beta)$, as shown in Table II.

G. Surface Dispersion

1) Time-of-flight error effects

In Fig. 9, suppose an error in $t_{s-r}$ only, so that the lander arrives at point T'' instead of point T; the target, during $\delta t_{s-r}$, will travel to point T'. It is desired to determine the down-range and cross-range errors due to $\delta t_{s-r}$. Denote the great circle arc through points T and T' as $\Delta$. Then

$$\sin\frac{\Delta}{2} = \cos\xi_T \sin\left(\frac{\omega_M \delta t_{s-r}}{2}\right)$$

or

$$\Delta \approx \omega_M \delta t_{s-r} \cdot \cos\xi_T \tag{97}$$

where $\omega_M$ = Mars rotational rate = 14.621 deg's/hr.

$\xi_T$ = Target latitude, with respect to Mars equatorial plane

From the T-T'-T'' triangle,

$$\tan \delta\theta_r\Big]_t = \tan\Delta \cdot \cos(A'-A) \tag{98}$$

but

$$\cos A = \sin i_p \cos(L_T - \Omega_p) \tag{99}$$

$$\sin A = \cos i_p \sec\xi_T \tag{100}$$

$$\cos A' = \tan\xi_T \tan\left(\frac{\Delta}{2}\right) \tag{101}$$

$$\sin A' = \cos\left(\frac{\omega_M \delta t_{s-r}}{2}\right) \sec\left(\frac{\Delta}{2}\right) \tag{102}$$

Substituting eq's. 99 through 102 and eq. 97 into eq. 98, and neglecting second order effects,

$$\delta\theta_r\Big]_t = \omega_M \delta t_{s-r} \cdot \cos i_p \tag{103}$$

Since this reduces the down-range error due to other sources, a negative sign should be appended; thus

$$\frac{\partial\theta_r}{\partial t_{s-r}} = -\omega_M \cos i_p \tag{104}$$

Again from the T-T'-T'' triangle,

$$\sin \delta\lambda_r\Big] = \sin\Delta \sin(A'-A) \tag{105}$$

[illegible]

$$\delta\theta_c\Big|_t = \omega_M\,\delta t_{S-I}\cos\xi_T\,\sin i_p\,\cos(L_T-\Omega_p) \qquad (106)$$

Proper consideration must be given to the sign of $\xi_T$ in accounting for the effect on $\delta\theta_c$ due to other error sources. Thus,

$$\frac{\partial\theta_c}{\partial t_{S-I}} = g\,\omega_M\cos\xi_T\,\sin i_p\,\cos(L_T-\Omega_p) \qquad (107)$$

$$g = \begin{cases} +1, & \text{for } \xi_T > 0 \\ -1, & \text{for } \xi_T < 0 \end{cases}$$

It should be noted that $i_p$ and $\Omega_p$ depend on $\xi_T$ as well as on bus and lander trajectory parameters. It can be shown that

$$\cos\xi_T\,\sin i_p\,\cos(L_T-\Omega_p) = \frac{\sin\xi_\infty + \sin\xi_T\cos\chi'}{\sin\chi'} \qquad (108)$$

Equations for $\xi_\infty$ and $\chi'$ are given in the appendix.

2) Navigation uncertainty effects

The down-range error, measured in the nominal plane, is

$$\delta\theta_r = \delta\theta_I + \frac{\partial\theta_r}{\partial t_{S-I}}\,\delta t_{S-I} \qquad (109)$$

where $\delta\theta_I$ is given by eq. 30 and $\delta t_{S-I}$ is given by eq. 34. Now note that

$$\delta\theta_E = \frac{\partial\theta_E}{\partial\theta_S}\delta\theta_S + \frac{\partial\theta_E}{\partial v_{C-S}}\delta v_{C-S} + \frac{\partial\theta_E}{\partial\gamma_{C-S}}\delta\gamma_{C-S} + \frac{\partial\theta_E}{\partial r_S}\delta r_S \qquad (110)$$

Substituting eq's. 32, 33 and 110 into eq. 30 and substituting the resulting equation and eq. 34 into eq. 109, the following is obtained:

$$\delta\theta_r = \frac{\partial\theta_r}{\partial r_S}\delta r_S + \frac{\partial\theta_r}{\partial\theta_S}\delta\theta_S + \frac{\partial\theta_r}{\partial v_{C-S}}\delta v_{C-S} + \frac{\partial\theta_r}{\partial\gamma_{C-S}}\delta\gamma_{C-S} \qquad (111)$$

where

$$\frac{\partial\theta_r}{\partial r_S} = \frac{\partial\theta_E}{\partial r_S} + \frac{\partial\theta_{E-I}}{\partial v_E}\frac{\partial v_E}{\partial r_S} + \frac{\partial\theta_{E-I}}{\partial\gamma_E}\frac{\partial\gamma_E}{\partial r_S} + \frac{\partial\theta_r}{\partial t_{S-I}}\frac{\partial t_{S-I}}{\partial r_S} \qquad (112)$$

$$\frac{\partial\theta_r}{\partial\theta_S} = 1 \qquad (113)$$

$$\frac{\partial\theta_r}{\partial v_{C-S}} = \frac{\partial\theta_E}{\partial v_{C-S}} + \frac{\partial\theta_{E-I}}{\partial v_E}\frac{\partial v_E}{\partial v_{C-S}} + \frac{\partial\theta_{E-I}}{\partial\gamma_E}\frac{\partial\gamma_E}{\partial v_{C-S}} + \frac{\partial\theta_r}{\partial t_{S-I}}\frac{\partial t_{S-I}}{\partial v_{C-S}} \qquad (114)$$

$$\frac{\partial\theta_r}{\partial\gamma_{C-S}} = \frac{\partial\theta_E}{\partial\gamma_{C-S}} + \frac{\partial\theta_{E-I}}{\partial v_E}\frac{\partial v_E}{\partial\gamma_{C-S}} + \frac{\partial\theta_{E-I}}{\partial\gamma_E}\frac{\partial\gamma_E}{\partial\gamma_{C-S}} + \frac{\partial\theta_r}{\partial t_{S-I}}\frac{\partial t_{S-I}}{\partial\gamma_{C-S}} \qquad (115)$$

The cross-range error, measured in a plane perpendicular to the nominal plane, is given by

$$\delta\theta_2 = \frac{\partial\theta_c}{\partial y}\,\delta y + \frac{\partial\theta_c}{\partial \dot{y}}\,\delta\dot{y} + \frac{\partial\theta_c}{\partial t_{s-r}}\,\delta t_{s-r}$$

$$= \frac{\partial\theta_c}{\partial y}\,\delta y + \frac{\partial\theta_c}{\partial \dot{y}}\,\delta\dot{y} + \frac{\partial\theta_c}{\partial h_s}\,\delta h_s + \frac{\partial\theta_c}{\partial v_{c_s}}\,\delta v_{c_s} + \frac{\partial\theta_c}{\partial \gamma_{c_s}}\,\delta\gamma_{c_s} \tag{116}$$

where

$$\frac{\partial\theta_c}{\partial h_s} = \frac{\partial\theta_c}{\partial t_{s-r}}\,\frac{\partial t_{s-r}}{\partial h_s} \tag{117}$$

$$\frac{\partial\theta_c}{\partial v_{c_s}} = \frac{\partial\theta_c}{\partial t_{s-r}}\,\frac{\partial t_{s-r}}{\partial v_{c_s}} \tag{118}$$

$$\frac{\partial\theta_c}{\partial \gamma_{c_s}} = \frac{\partial\theta_c}{\partial t_{s-r}}\,\frac{\partial t_{s-r}}{\partial \gamma_{c_s}} \tag{119}$$

It is seen that $\delta\theta_1$ and $\delta\theta_2$ are **unrelated** through $\delta t_{s-r}$. Eq's. 111 and 116 are generally applicable, whether considering execution error sources or navigation uncertainties.

To relate $\delta\theta_1$ and $\delta\theta_2$ to navigation uncertainties, note first that eq. 109 can be written in matrix form as follows:

$$\delta\theta_1 = \bar{A}_h \begin{bmatrix} \delta\theta_t \\ \delta v_t \\ \delta\gamma_t \\ \delta t_{s-r} \end{bmatrix} \tag{120}$$

where

$$\bar{A}_h = \begin{bmatrix} 1 & \frac{\partial\theta_{t-r}}{\partial v_t} & \frac{\partial\theta_{t-r}}{\partial\gamma_t} & \frac{\partial\theta_{t-r}}{\partial t_{s-r}} \end{bmatrix} \tag{121}$$

Also, eq's. 32, 33, 34 and 110 can be combined in matrix form:

$$\begin{bmatrix} \delta\theta_r \\ \delta v_r \\ \delta\gamma_r \\ \delta t_{s-r} \end{bmatrix} = \bar{B}_h \begin{bmatrix} \delta h_s \\ \delta\theta_s \\ \delta v_{c_s} \\ \delta\gamma_{c_s} \end{bmatrix} \tag{122}$$

where

$$\hat{B}_{\lambda} = \begin{bmatrix} \frac{\partial \theta_E}{\partial \lambda_s} & 1 & \frac{\partial \theta_E}{\partial v_{c-s}} & \frac{\partial \theta_E}{\partial \gamma_{c-s}} \\ \frac{\partial v_E}{\partial \lambda_s} & 0 & \frac{\partial v_E}{\partial v_{c-s}} & 0 \\ \frac{\partial \gamma_E}{\partial \lambda_s} & 0 & \frac{\partial \gamma_E}{\partial v_{c-s}} & \frac{\partial \gamma_E}{\partial \gamma_{c-s}} \\ \frac{\partial t_{s-I}}{\partial \lambda_s} & 0 & \frac{\partial t_{s-I}}{\partial v_{c-s}} & \frac{\partial t_{s-I}}{\partial \gamma_{c-s}} \end{bmatrix} \tag{123}$$

Thus, the derived coefficients can be obtained by matrix multiplication:

$$\hat{A}_{\lambda} \hat{B}_{\lambda} = \begin{bmatrix} \frac{\partial \theta_\lambda}{\partial \lambda_s} & \frac{\partial \theta_\lambda}{\partial \theta_s} & \frac{\partial \theta_\lambda}{\partial v_{c-s}} & \frac{\partial \theta_\lambda}{\partial \gamma_{c-s}} \end{bmatrix} \tag{124}$$

Finally, from eq. 94, we have

$$\delta \theta_\lambda \Big]_N = \hat{A}_\lambda \hat{B}_\lambda \hat{C}_\lambda \, \delta \hat{X} \tag{125}$$

where the elements of $\hat{A}_\lambda \hat{B}_\lambda \hat{C}_\lambda$ are the derived sensitivities of $\delta\theta_\lambda$ to navigation uncertainties:

$$\hat{A}_\lambda \hat{B}_\lambda \hat{C}_\lambda = \begin{bmatrix} \frac{\partial \theta_\lambda}{\partial x'} & 0 & \frac{\partial \theta_\lambda}{\partial z'} & \frac{\partial \theta_\lambda}{\partial \dot{x}'} & 0 & \frac{\partial \theta_\lambda}{\partial \dot{z}'} \end{bmatrix} \tag{126}$$

where

$$\frac{\partial \theta_\lambda}{\partial x'} = \frac{\partial \theta_\lambda}{\partial \lambda_s} \frac{\partial \lambda_s}{\partial x'} + \frac{\partial \theta_s}{\partial x'} + \frac{\partial \theta_\lambda}{\partial \gamma_{c-s}} \frac{\partial \gamma_{c-s}}{\partial x'} \tag{127}$$

$$\frac{\partial \theta_\lambda}{\partial z'} = \frac{\partial \theta_\lambda}{\partial \lambda_s} \frac{\partial \lambda_s}{\partial z'} + \frac{\partial \theta_s}{\partial z'} + \frac{\partial \theta_\lambda}{\partial \gamma_{c-s}} \frac{\partial \gamma_{c-s}}{\partial z'} \tag{128}$$

$$\frac{\partial \theta_\lambda}{\partial \dot{x}'} = \frac{\partial \theta_\lambda}{\partial v_{c-s}} \frac{\partial v_{c-s}}{\partial \dot{x}'} + \frac{\partial \theta_\lambda}{\partial \gamma_{c-s}} \frac{\partial \gamma_{c-s}}{\partial \dot{x}'} \tag{129}$$

$$\frac{\partial \theta_\lambda}{\partial \dot{z}'} = \frac{\partial \theta_\lambda}{\partial v_{c-s}} \frac{\partial v_{c-s}}{\partial \dot{z}'} + \frac{\partial \theta_\lambda}{\partial \gamma_{c-s}} \frac{\partial \gamma_{c-s}}{\partial \dot{z}'} \tag{130}$$

The mean square down-range error is given by

$$\overline{\delta \theta_\lambda^2} \Big]_N = \left[ \hat{A}_\lambda \hat{B}_\lambda \hat{C}_\lambda \right] \overline{\delta \hat{X} \, \delta \hat{X}'} \left[ \hat{A}_\lambda \hat{B}_\lambda \hat{C}_\lambda \right]' \tag{131}$$

whence

In [illegible] eq. [illegible] n [illegible]

$$\delta\theta_c = \bar{A}_c \bar{B}_c \begin{bmatrix} \delta\psi \\ \delta r_s \\ \delta\theta_s \\ \delta v_{c-s} \\ \delta\gamma_{c-s} \end{bmatrix} \tag{132}$$

where

$$\bar{A}_c = \begin{bmatrix} \frac{\partial\theta_c}{\partial x_{c-s}} & \frac{\partial\theta_c}{\partial y} & \frac{\partial\theta_c}{\partial \dot{y}} \end{bmatrix} \tag{133}$$

$$B_c = \begin{bmatrix} 0 & 0 & \frac{\partial x_{c-s}}{\partial r_s} & 0 & \frac{\partial x_{c-s}}{\partial v_{c-s}} & \frac{\partial x_{c-s}}{\partial \gamma_{c-s}} \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{134}$$

For the navigation error effects,

$$\delta\theta_c\Big]_N = \bar{A}_c \bar{B}_c \bar{C}_c \overline{\delta X} \tag{135}$$

where

$$\bar{C}_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ & & \bar{C}_n & & & \end{bmatrix} \tag{136}$$

Thus

$$\overline{\delta\theta_c^2}\Big]_N = \left[\bar{A}_c \bar{B}_c \bar{C}_c\right] \overline{\delta X \delta X^T} \left[\bar{A}_c \bar{B}_c \bar{C}_c\right]^T \tag{137}$$

and

$$\sigma(\delta\theta_c)_N = \sqrt{\overline{\delta\theta_c^2}\Big]_N}$$

Note that eq. 135 defines the cross-range sensitivities to navigation errors:

$$\frac{\partial\theta_c}{\partial x'} = \frac{\partial\theta_c}{\partial r_s}\frac{\partial r_s}{\partial x'} + \frac{\partial\theta_c}{\partial \gamma_{c-s}}\frac{\partial \gamma_{c-s}}{\partial x'} \tag{138}$$

$$\frac{\partial\theta_c}{\partial y'} = -\frac{\partial\theta_c}{\partial y} \tag{139}$$

$$\frac{\partial\theta_c}{\partial z'} = \frac{\partial\theta_c}{\partial r_s}\frac{\partial r_s}{\partial z'} + \frac{\partial\theta_c}{\partial \gamma_{c-s}}\frac{\partial \gamma_{c-s}}{\partial z'} \tag{140}$$

$$\frac{\partial \theta_c}{\partial y'} = \frac{\partial \theta_c}{\partial \tau_{c_s}} \frac{\partial \tau_{c_s}}{\partial y'} + \frac{\partial \theta_c}{\partial \tau_{c_s}} \frac{\partial \tau_{c_s}}{\partial y'} \tag{141}$$

$$\frac{\partial \theta_c}{\partial y'} = - \frac{\partial \theta_c}{\partial y} \tag{142}$$

$$\frac{\partial \theta_c}{\partial z'} = \frac{\partial \theta_c}{\partial \tau_{c_s}} \frac{\partial \tau_{c_s}}{\partial z'} + \frac{\partial \theta_c}{\partial \tau_{c_s}} \frac{\partial \tau_{c_s}}{\partial z'} \tag{143}$$

The resultant of down-range and cross-range errors is

$$\delta \theta_R \Big]_N = \sqrt{(\delta \theta_A)_N^2 + (\delta \theta_c)_N^2} \tag{144}$$

Since this is always positive, its probability density function is one-sided, and it has a non-zero mean value. Thus

$$\sigma(\delta \theta_R)_N \neq \sqrt{\overline{\delta \theta_R^2}\Big]_N}$$

However, note that the r.m.s. value is given simply by

$$\sqrt{\overline{\delta \theta_R^2}\Big]_N} = \sqrt{\sigma^2(\delta \theta_A)_N + \sigma^2(\delta \theta_c)_N} \tag{145}$$

For adequately large ratio of the semi-major axis to semi-minor axis of the dispersion ellipse, the 99% probability value of $\delta \theta_R$ is approximately 3 times the r.m.s. value.

3) Execution error effects

The down-range error due to execution errors is given by

$$\delta \theta_A \Big]_E = \frac{\partial \theta_A}{\partial \tau_{c_s}} \delta \tau_{c_s} \Big]_E + \frac{\partial \theta_A}{\partial \tau_{c_s}} \delta \tau_{c_s} \Big]_E$$

$$= \frac{\partial \theta_A}{\partial \Delta v_s} \delta \Delta v_s + \frac{\partial \theta_A}{\partial \beta} \delta \beta \tag{146}$$

where

$$\frac{\partial \theta_A}{\partial \Delta v_s} = \frac{\partial \theta_A}{\partial \tau_{c_s}} \frac{\partial \tau_{c_s}}{\partial \Delta v_s} + \frac{\partial \theta_A}{\partial \tau_{c_s}} \frac{\partial \tau_{c_s}}{\partial \Delta v_s} \tag{147}$$

$$\frac{\partial \theta_A}{\partial \beta} = \frac{\partial \theta_A}{\partial \tau_{c_s}} \frac{\partial \tau_{c_s}}{\partial \beta} + \frac{\partial \theta_A}{\partial \tau_{c_s}} \frac{\partial \tau_{c_s}}{\partial \beta} \tag{148}$$

Thus

$$\overline{\delta \theta_A^2}\Big]_E = \left(\frac{\partial \theta_A}{\partial \Delta v_s}\right)^2 \sigma^2(\delta \Delta v_s) + \left(\frac{\partial \theta_A}{\partial \beta}\right)^2 \sigma^2(\delta \beta) \tag{149}$$

and

In similar fashion,

$$\overline{\delta\theta_c^2}\Big]_E = \left(\frac{\partial\theta_c}{\partial\Delta v_s}\right)^2 \sigma^2(\Delta v_s) + \left(\frac{\partial\theta_c}{\partial\beta}\right)^2 \sigma^2(\beta) + \left(\frac{\partial\theta_c}{\partial\theta}\right)^2 \sigma^2(\theta) \tag{150}$$

where

$$\frac{\partial\theta_c}{\partial\Delta v_s} = \frac{\partial\theta_c}{\partial h_{c-s}}\left[\frac{\partial h_{c-s}}{\partial v_{c-s}}\frac{\partial v_{c-s}}{\partial\Delta v_s} + \frac{\partial h_{c-s}}{\partial h_{c-s}}\frac{\partial h_{c-s}}{\partial\Delta v_s}\right] \tag{151}$$

$$\frac{\partial\theta_c}{\partial\beta} = \frac{\partial\theta_c}{\partial h_{c-s}}\left[\frac{\partial h_{c-s}}{\partial v_{c-s}}\frac{\partial v_{c-s}}{\partial\beta} + \frac{\partial h_{c-s}}{\partial h_{c-s}}\frac{\partial h_{c-s}}{\partial\beta}\right] \tag{152}$$

and $\frac{\partial\theta_c}{\partial\theta}$ is given by eq. 65. Then

$$\sigma(\delta\theta_c)_E = \sqrt{\overline{\delta\theta_c^2}\Big]_E}$$

The r.m.s. value of the resultant error is

$$\sqrt{\overline{\delta\theta_R^2}\Big]_E} = \sqrt{\sigma^2(\delta\theta_c)_P + \sigma^2(\delta\theta_c)_E} \tag{153}$$

It is to be observed that the predominant contributing factor in eq. 149 is the velocity magnitude error; the predominant contribution in eq. 150 is the out-of-plane thrust direction error.

4) Total effects

Summing up the effects, from both error sources, in either of the two orthogonal directions, we may write

$$(\delta\theta_R)_P = (\delta\theta_c)_P + (\delta\theta_c)_E \tag{154}$$

$$(\delta\theta_R)_N = (\delta\theta_c)_E + (\delta\theta_c)_N \tag{155}$$

whence

$$\sigma(\delta\theta_R)_P = \sqrt{\overline{(\delta\theta_c)_P^2} + \overline{(\delta\theta_c)_E^2}} \tag{156}$$

$$\sigma(\delta\theta_R)_N = \sqrt{\overline{(\delta\theta_c)_E^2} + \overline{(\delta\theta_c)_N^2}} \tag{157}$$

By definition, the resultant of the total components is

$$\left(\delta\theta_R\right)_T = \sqrt{\left(\delta\theta_1\right)_T^2 + \left(\delta\theta_2\right)_T^2} \tag{158}$$

whence

$$\sqrt{\overline{\delta\theta_R^2}}_T = \sqrt{\overline{\delta\theta_1^2}_T + \overline{\delta\theta_2^2}_T}$$

$$= \sqrt{\overline{\delta\theta_1^2}_T + \overline{\delta\theta_2^2}_T} \tag{159}$$

Reference to Fig. 10 will somewhat clarify the terminology and concepts.

## APPENDIX I

### Nominal Conditions - Equations & Numerical Values

A. Heliocentric Data

For May 19, 1971 launch date, $T_f$ = 200 days, the Julian day of encounter is 2441290.5; from the Planetary Ephemerides, at encounter,

$$\lambda_M = 29.481^\circ, \quad \phi_M = -.6224^\circ$$

Also, for 1972 encounter (see Fig. I-1)

$$I_M = 1.85^\circ, \quad \Omega_M = 49.34^\circ, \quad \lambda_M = 25.2^\circ, \quad \Omega_M' = 41.0^\circ$$

Then

$$\sin \mu_f = \frac{\sin \phi_M}{\sin I_M} \qquad \text{(I-1)}$$

$$\cos \mu_f = \cos \phi_M \cos(\lambda_M - \Omega_M) \qquad \text{(I-2)}$$

$$\mu_f = 340.3^\circ$$

B. Bus Aerocentric Data

From J.P.L. trajectory data the orientation of $\vec{V}_{B\infty}$ with respect to Mars orbit plane is given by the pair

$$\zeta_p = 94^\circ \; ; \quad \gamma_p = -4.6^\circ$$

Then, from Fig. I-1,

$$\cos \sigma = \frac{-\cos \zeta_p}{\cos \gamma_p} \qquad \text{(I-3)}$$

$$-180^\circ \leq \sigma \leq 0^\circ$$

whence $\sigma$ = [illegible]

Also, from Fig. I-2,

$$\sin \zeta_\infty = -\cos \gamma_0 \sin \lambda_M \sin(\mu_f + \sigma - \Omega_M) + \sin \gamma_p \cos \lambda_M \qquad \text{(I-4)}$$

$$-180^\circ \leq \zeta_\infty \leq 90^\circ$$

whence $\zeta_\infty$ = [illegible]

It can be shown (see reference 6) that $L_\infty$ is given by

$$\tan L_\infty = \frac{\cos i_\infty \sin(\alpha_f + \gamma - \Omega_\infty) + \tan \gamma_f \sin i_\infty}{\cos(\alpha_f + \gamma - \Omega_\infty)} \qquad \text{(I-5)}$$

whence $L_\infty = 212.[illegible]$

From the J.P.L. data, $\gamma_{[illegible]} = [illegible]$ km/sec Given

$\alpha_{[illegible]} = [illegible]$ [illegible] = [illegible] km

$\gamma_{[illegible]} = [illegible]$ km

then $\gamma_{[illegible]} = [illegible]$ [illegible] = [illegible] km

**Given**

$\mu_{[illegible]} = [illegible] \times 10^{[illegible]}$ km.$^2$/sec.$^2$

**then**

$$[illegible] = [illegible] \qquad \text{(I-6)}$$

$$\Delta_3 = [illegible]$$

**Also,**

$$[illegible] \qquad \text{(I-7)}$$

$$[illegible]$$

**and**

$$[illegible] \qquad \text{(I-8)}$$

**Given** $\mu_{[illegible]} = [illegible]$ [illegible]

**then**

$$\cos \Delta_3 = [illegible] \qquad \text{(I-9)}$$

**Also**

[illegible]

[illegible]

[illegible]

$$\rho_{\varepsilon} = 160 \; [illegible] \qquad (I\text{-}12)$$

$$\rho_{\varepsilon} = 1.6[illegible]$$

[illegible] Lander Characteristic Data

Given $\gamma_{\varepsilon} = 30°$, $h_{\varepsilon} = 1 \times 10^{6}$ ft. = 304.[illegible] km.; given that $v_{\varepsilon}$ is to equal $v_{\varepsilon\text{-}s}$ and that bus and lander planes are coincident. Then $v_{\varepsilon\infty} = v_{\varepsilon\text{-}s}$, and

$$A_{\varepsilon} = R_{\varepsilon_{\gamma_{\varepsilon}}} + h_{\varepsilon} = [illegible]95 \text{ km}$$

$$v_{\varepsilon} = \sqrt{v_{\varepsilon\text{-}s}^{2} + \frac{2\mu_{\varepsilon}}{r_{\varepsilon}}} \qquad (I\text{-}13)$$

$$v_{\varepsilon} = 5.673[illegible] \text{ km/sec.}$$

$$p = \frac{\left(r_{\varepsilon} v_{\varepsilon} \cos \frac{\gamma}{\varepsilon}\right)^{2}}{\mu_{\varepsilon}} \qquad (I\text{-}14)$$

$$p_{\varepsilon} = 7.4349 \times 1[illegible] \text{ km.}$$

$$a_{\varepsilon} = \sqrt{\mu_{\varepsilon}/v_{\varepsilon}^{2}} \qquad (I\text{-}15)$$

$$= [illegible] \text{ km}$$

$$e_{\varepsilon} = \sqrt{1 + \sqrt{2/a_{\varepsilon}}} \qquad (I\text{-}16)$$

$$= [illegible]$$

$$h_{\varepsilon} = [illegible] \qquad (I\text{-}17)$$

$$= [illegible]$$

$$\beta = \beta_{\varepsilon} = \frac{[illegible]}{\gamma_{\varepsilon} \; [illegible]} \qquad (I\text{-}18)$$

$$\gamma_{\varepsilon\text{-}s} = 88.767°$$

$$\tan \eta_{c-s} = \frac{\tan \tau_{c-s}}{\left(1 - \frac{\eta_{c-s}}{\eta_{c-s}}\right)} \; ; \; 0 \le \eta_{c-s} \le 180^\circ \qquad (2\text{-}19)$$

[illegible]

[illegible] (2-20)

[illegible]

[illegible] (2-21)

[illegible] 0.0[illegible] lb/sec

[illegible] (2-22)

[illegible] 100°

[illegible]

$$k_{s\,e} = \frac{1}{\eta_{c-s}} \left\{ [illegible] \right\} \qquad (2\text{-}23)$$

where $\sinh F_{c-s} = [illegible]$ (2-24)

$\sinh [illegible] = [illegible]$ (2-25)

then

[illegible] = [illegible] hrs

Also, for $p/k_{s\,e}$ = [illegible] and 100 atmospheres [illegible] = 9.[illegible], whence

$$\Delta\eta = [illegible] \qquad (2\text{-}26)$$

$= [illegible]$

$$( = [illegible] \qquad (2\text{-}27)$$

$= [illegible]$

D. Approach Plane Orientation

From Fig. I-3, or from reference 4, for sun-lit landing, and for $\xi_T = -24^\circ$ (latitude of Pandorac Fratum)

$$\cos(L_\infty - L_T) = -\tan\xi_T \tan\xi_\infty - \sec\xi_T \sec\xi_\infty \cos\psi \qquad \text{(I-28)}$$

$$0^\circ \leq (L_\infty - L_T) \leq 180^\circ$$

$$L_\infty - L_T = 84.81^\circ$$

whence $$L_T = L_\infty - (L_\infty - L_T) = 127.6^\circ$$

Also, from Fig. I-3,

$$\cos i_p = \frac{\sin(L_\infty - L_T)\cos\xi_T \cos\xi_\infty}{\sin\psi} \qquad \text{(I-29)}$$

$$0^\circ \leq i_p \leq 180^\circ$$

$$i_p = 26.0^\circ$$

Also,

$$\tan(L_T - \Omega_p) = \frac{\sin\xi_T \sin(L_\infty - L_T)}{\tan\xi_\infty \cos\xi_T - \sin\xi_T \cos(L_\infty - L_T)} \qquad \text{(I-30)}$$

$$(L_T - \Omega_p) = -60.0^\circ$$

whence $\Omega_p = 113.6^\circ$

## APPENDIX II

### Error Co-efficients - Numerical Values

$\frac{\partial \theta_{t-r}}{\partial \theta_{r}} = .20$ degs./(km/sec) ; $\frac{\partial \theta_{r-r}}{\partial \theta_{r}} = -.15$ deg's/deg.

$\frac{\partial \theta}{\partial \theta_{r}} = 13.15$ degs./hr.

---

$\frac{\partial \theta_{E}}{\partial \theta_{s}} = 5.0 \times 10^{-4}$ deg's/km. ; $\frac{\partial \theta_{E}}{\partial \theta_{s}} = 26.66$ degs./(km/sec.)

$\frac{\partial \theta_{E}}{\partial \theta_{s}} = -112.5$ deg's./deg.

---

$\frac{\partial \theta_{E}}{\partial \theta_{s}} = 9.8 \times 10^{-8}$ (km/sec./km ; $\frac{\partial \theta_{E}}{\partial \theta_{s}} = .516 \frac{(km/sec.)}{(km/sec.)}$

---

$\frac{\partial \theta_{E}}{\partial \theta_{s}} = -3.55 \times 10^{-4}$ deg's/km. ; $\frac{\partial \theta_{E}}{\partial \theta_{s}} = -21.1$ deg's./(km/sec.)

$\frac{\partial \theta_{E}}{\partial \theta_{s}} = 80.5$ deg's/deg.

---

$\frac{\partial t_{t-r}}{\partial h_{s}} = 9.10 \times 10^{-5}$ hrs/km. ; $\frac{\partial t_{t-r}}{\partial h_{s}} = 6.17$ hrs./(km/sec.)

$\frac{\partial t_{t-r}}{\partial h_{s}} = -3.73$ hrs./deg.

---

$\frac{\partial t_{t-r}}{\partial \theta_{E}} = -.005$ hrs./(km/sec.) ; $\frac{\partial t_{t-r}}{\partial \theta_{E}} = -.002$ hrs./deg.

---

$\frac{\partial t_{t-r}}{\partial h_{s}} = 9.18 \times 10^{-5}$ hrs./km. ; $\frac{\partial t_{t-r}}{\partial h_{s}} = 6.12$ hrs./(km/sec.)

$\frac{\partial t_{t-r}}{\partial h_{s}} = [illegible]$ hrs./deg.

---

$C_{11} = \frac{\partial \lambda_s}{\partial x'} = .0316$ km/km. ; $C_{13} = \frac{\partial \lambda_s}{\partial z'} = -.999$ km/km.

$C_{21} = \frac{\partial \theta_s}{\partial x'} = 2.06 \times 10^{-4}$ deg's/km. ; $C_{23} = \frac{\partial \theta_s}{\partial z'} = .75 \times 10^{-5}$ deg's/km.

$C_{34} = \frac{\partial v_{c-s}}{\partial \dot{x}'} = -5.1 \times 10^{-4} \frac{\text{km/sec.}}{\text{km/sec.}}$ ; $C_{36} = \frac{\partial v_{c-s}}{\partial \dot{z}'} = 1.0 \frac{\text{km/sec.}}{\text{km/sec.}}$

$C_{41} = \frac{\partial \gamma_{c-s}}{\partial x'} = -2.06 \times 10^{-4}$ deg's/km. ; $C_{43} = \frac{\partial \gamma_{c-s}}{\partial z'} = -.75 \times 10^{-5} \frac{\text{degs.}}{\text{km.}}$

$C_{44} = \frac{\partial \gamma_{c-s}}{\partial \dot{x}'} = -19.2$ degs/(km/sec.) ; $C_{46} = \frac{\partial \gamma_{c-s}}{\partial \dot{z}'} = -.194 \frac{\text{degs.}}{\text{(km/sec.)}}$

---

$\frac{\partial \gamma_E}{\partial x'} = -.0106$ degs/km. ; $\frac{\partial \gamma_E}{\partial z'} = -9.6 \times 10^{-4}$ degs./km.

$\frac{\partial \gamma_E}{\partial \dot{x}'} = -154.6$ deg's/(km/sec.) ; $\frac{\partial \gamma_E}{\partial \dot{z}'} = -39.7$ deg's/(km/sec.)

---

$\frac{\partial v_{c-s}}{\partial \Delta v_s} = .00473 \frac{\text{km/sec.}}{\text{km/sec.}}$ ; $\frac{\partial v_{c-s}}{\partial \beta} = -4.93 \times 10^{-4} \frac{\text{(km/sec.)}}{\text{deg.}}$

$\frac{\partial \gamma_{c-s}}{\partial \Delta v_s} = 19.2$ deg's/(km/sec.) ; $\frac{\partial \gamma_{c-s}}{\partial \beta} = 4.47 \times 10^{-5}$ deg's/deg.

---

$\frac{\partial \gamma_E}{\partial \Delta v_s} = 1548$ deg's/(km/sec.) ; $\frac{\partial \gamma_E}{\partial \beta} = .0155$ deg's/deg.

---

$\frac{\partial \theta_c}{\partial \lambda_s} = .00958$ deg's/km. ; $\frac{\partial \theta_c}{\partial \dot{\lambda}_s} = [illegible].3$ deg's/(km/sec.)

$\frac{\partial \theta_c}{\partial \lambda_s} = -2.4[illegible]$ deg's/km.

---

$\frac{\partial \theta_c}{\partial \lambda_s} = [illegible].4 \times 10^{-5}$ deg's/km. ; $\frac{\partial \theta_c}{\partial \dot{\lambda}_s} = [illegible]$ deg's/(km/sec.)

$\frac{\partial \theta_c}{\partial \dot{\lambda}_s} = [illegible]$ deg's/(km/sec.)

---

$\frac{\partial \theta_c}{\partial \Delta v_3}$ = 183.2 deg's/(km/sec.) ; $\frac{\partial \theta_c}{\partial \beta}$ = .600 deg's/deg.

$\frac{\partial \theta_c}{\partial b}$ = .439 deg's/deg.

---

$\frac{\partial \theta_c}{\partial x'}$ = -2.67 x $10^{-4}$ deg's/km. ; $\frac{\partial \theta_c}{\partial z'}$ = 1.53 x $10^{-4}$ deg's/km.

$\frac{\partial \theta_c}{\partial \dot{x}'}$ = -182.6 deg's/(km/sec.) ; $\frac{\partial \theta_c}{\partial \dot{z}'}$ = 18.0 deg's/(km/sec.)

---

$\frac{\partial \theta_h}{\partial x_s}$ = -5.46 x $10^{-4}$ deg's/km. ; $\frac{\partial \theta_h}{\partial v_{z_s}}$ = 144.4 deg's/(km/sec.)

$\frac{\partial \theta_h}{\partial \theta_s}$ = 1 deg/deg. ; $\frac{\partial \theta_h}{\partial \theta_z}$ = -97.6 deg's/deg.

---

$\frac{\partial \theta_h}{\partial \Delta v_3}$ = -1878 deg's/(km/sec.) ; $\frac{\partial \theta_h}{\partial \beta}$ = .067 deg's/deg.

---

$\frac{\partial \theta_h}{\partial x'}$ = .0203 deg's/km. ; $\frac{\partial \theta_h}{\partial z'}$ = .00129 deg's/km.

$\frac{\partial \theta_h}{\partial \dot{x}'}$ = 1870 deg's/(km/sec.) ; $\frac{\partial \theta_h}{\partial \dot{z}'}$ = 103.3 deg's/(km/sec.)

---

TABLE III NOTE: $\sigma(\delta \Delta v_s) = .01\, \Delta v_s$

| [illegible] | [illegible] | $\sigma(\delta\theta_a)_F$ | $\sigma(\delta\theta_a)_N$ | $\sigma(\delta\theta_a)_T$ | $\sqrt{(\delta\theta_K)^2_F}$ | $\sqrt{(\delta\theta_K)^2_N}$ | $\sqrt{(\delta\theta_K)^2_T}$ |
|---|---|---|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 3.36° | 3.407° |
| [illegible] | [illegible] | [illegible] | ↓ | [illegible] | [illegible] | ↓ | [illegible] |

$\Omega$ – longitude of ascending node

$\phi_M$ – celestial latitude of Mars

$\lambda_M$ – celestial longitude of Mars

[illegible] – longitude in Mars equatorial system, $\xi$ – latitude in Mars equatorial system

$h$ – areocentric altitude above Mars surface

$\theta_M$ – central angle, measured in probe heliocentric orbit, locating Mars at encounter

$\sigma(\ )$ – std. deviation of ( ) error

$\beta$ – thrust angle of [illegible] measured from the velocity direction, at [illegible]

SUBSCRIPTS

[illegible] – "at departure [illegible]"

[illegible] – "at Mars"

[illegible] – "at departure [illegible]"

[illegible] – "due to [illegible]"

$\infty$ – "at infinity"

[illegible] – [illegible] perihelion [illegible]

N – "due to [illegible]"

[illegible] – "due to [illegible] [illegible]"

[illegible] – "at [illegible]"

[illegible] – [illegible]

[illegible]

[illegible]

REFERENCES:

Tech. Memo. 9752-004 - "Voyager Lander Error Analysis", G. Reehl

PIR 9751-033 - "Separation Velocity Increment Requirement", M. Levinson

PIR 9751-036 - "Preliminary Data, Earth Visibility", M. Levinson

Tech. Memo 9751-126 - "Earth Line-of-Sight Limitation", M. Levinson

## IV. ANALYSIS

### A. Sources of Errors

In Fig. 1, unprimed quantities represent nominally desired values; primed quantities represent actual trajectory values. It is assumed that the approach guidance system is such that the proper separation point (i.e., separation distance $R_s$) and proper thrust angle $\beta$ is commanded, for the nominal separation velocity increment $\Delta v_s$ to be imparted, and for the nominally determined bus trajectory, so that lander capsule entry will be made at the prescribed entry angle $\gamma_E$ and landing will occur at a designated landing site. Because of noise errors in the navigation instruments which establish the bus trajectory (and therefore, the required $\beta$ and $R_s$) the actual bus trajectory will differ from the computed (nominal) trajectory; in other words, there will be uncertainties in the position and velocity of the bus, at separation. Furthermore, the thrust angle may deviate from the commanded value and the velocity increment will have some error tolerance around the design value. The former type are termed navigation errors, the latter type are termed execution errors. The net effect of these errors is to cause the lander to have an entry angle deviation and to miss the target site.

Additional sources of error are the design tolerances in lander $W/C_D A$, and uncertainties in Martian atmosphere parameters (essentially, density at surface and scale height). The effects of these on landing error are small for the expected variation (see reference memo 1) and are not numerically assessed in this memorandum.

Specification of the bus approach velocity ($V_{B-\infty}$) and periapsis distance ($R_{B-P}$) specifies the bus in-plane parameters ($\alpha_B$, $R_B$). The specification of the orientation of the approach plane say, with respect to Mars equatorial plane and an inertial line in this latter plane depends upon the constraints of target site latitude ($\xi_T$), bus approach asymptote angular coordinates ($\xi_P$, $\gamma_P$), and lander entry angle ($\gamma_E$). The separation velocity increment ($\Delta v_s$) nominally required, is a function not only of $V_{B-\infty}$ and $\alpha_B$ but of thrust angle ($\beta$), entry angle ($\gamma_E$) and separation distance ($R_s$). The relations required for delineation of nominal conditions are given in the appendix (see also references 2, 3 and 4).

Only the error analysis for in-plane targeting will be considered in this memorandum. That is, the nominal situation is one where the lander trajectory plane is coincident with bus trajectory plane. $\Delta v_s$ has no out-of-plane component.

### B. In-Plane Error Coefficients

#### 1. Sensitivity of entry conditions to lander conditions after separation

It is desired to find the partials of entry conditions $v_E$, $\gamma_E$, $\theta_E$ with respect to capsule separation conditions $v_{c-s}$, $\gamma_{c-s}$, $R_s$, $\theta_s$. From conservation of energy and momentum considerations,

$$v_E^2 = v_{E-S}^2 + 2\mu_M\left(\frac{1}{r_E} - \frac{1}{r_S}\right) \qquad (1)$$

$$\cos\gamma_E = \frac{r_S\, v_{E-S} \cos\gamma_{E-S}}{r_E\, v_E} \qquad (2)$$

in which $r_E$ is a constant.

Differentiation yields

$$\frac{\partial v_E}{\partial v_{E-S}} = \frac{v_{E-S}}{v_E} \qquad (3)$$

$$\frac{\partial r_E}{\partial r_S} = \frac{\mu_M}{r_S\, v_E} \qquad (4)$$

$$\frac{\partial v_E}{\partial \gamma_{E-S}} = \frac{\partial v_E}{\partial \gamma_{E-S}} + \frac{\partial v_E}{\partial \gamma_{E-S}} = 0$$

$$\frac{\partial \gamma_E}{\partial \gamma_{E-S}} = \frac{\partial \gamma_E}{\partial \gamma_{E-S}} + \frac{\partial \gamma_E}{\partial \gamma_{E-S}} \qquad (5)$$

$$\frac{\partial \gamma_E}{\partial r_S} = \frac{1}{r_S} + \frac{\partial \gamma_E}{\partial r_S} - \frac{\partial \gamma_E}{\partial r_S} \qquad (6)$$

$$\frac{\partial \gamma_E}{\partial \gamma_{E-S}} = \frac{\partial \gamma_E}{\partial \gamma_{E-S}} \qquad (7)$$

From Fig. 1, a change in [illegible] trajectory about the planet center, whence

[illegible]

Thus

$$\frac{\partial \theta_E}{\partial v_{c-s}} = \frac{\partial(\eta_{c-s} - \eta_{c-E})}{\partial v_{c-s}} \tag{9}$$

$$\frac{\partial \theta_E}{\partial \lambda_s} = \frac{\partial(\eta_{c-s} - \eta_{c-E})}{\partial \lambda_s} \tag{10}$$

$$\frac{\partial \theta_E}{\partial \gamma_{c-s}} = \frac{\partial(\eta_{c-s} - \eta_{c-E})}{\partial \gamma_{c-s}} \tag{11}$$

Now

$$\tan \eta_{c-s} = \tan \gamma_{c-s} \Big/ \left(1 - \frac{\lambda_s}{p_c}\right) \tag{12}$$

$$\cos \eta_{c-E} = \frac{1}{e_c}\left(\frac{p_c}{\lambda_E} - 1\right) \tag{13}$$

where

$$p_c = \frac{(\lambda_s v_{c-s} \cos \gamma_{c-s})^2}{\mu_M} \tag{14}$$

$$e_c^2 = \frac{p_c}{\mu_M}\left[v_{c-s}^2 - \frac{2\mu_M}{\lambda_s}\right] + 1 \tag{15}$$

Differentiation of eq's. 12 thru 15 and subsequent substitution into eq's 9 thru 11 yields

$$\frac{\partial \theta_E}{\partial v_{c-s}} = \frac{-2p_c}{e_c^2 \lambda_s v_{c-s}} \tan \gamma_{c-s}$$

$$+ \frac{2 \cot \gamma_E}{v_{c-s}} \left[ - \frac{(p_c - \lambda_s)}{\lambda_s^2}\left(\frac{1}{e_c} + \frac{1}{\lambda_s}\right)\right] \tag{16}$$

$$\frac{\partial \theta_E}{\partial \lambda_s} = \frac{p_c}{e_c^2 \lambda_s} \tan \gamma_{c-s}$$

$$\frac{\cot \gamma_E}{\lambda_s^2}\left[ \frac{(p_c - \lambda_s)}{\lambda_s^2}\left(\frac{1}{e_c} + \frac{1}{\lambda_s}\right)\right] \tag{17}$$

$$\frac{\partial p_c}{\partial \gamma_{c\text{-}s}} = \frac{p_c}{e_c r_s}\left[\frac{p_c}{r_s} - \cos 2\gamma_{c\text{-}s}\right]\sec^2\gamma_{c\text{-}s} - \left[2 - \frac{(p_c - r_E)}{a_c e_c^2}\right]\operatorname{ctn}\gamma_E \tan\gamma_{c\text{-}s} \tag{18}$$

wherein

$$a_c = p_c/(e_c^2 - 1) \tag{19}$$

Variations in conditions after separation affect not only position and velocity components at entry, but also the time of flight from separation to entry. This in turn has an effect on surface dispersion, i.e., the amount by which the lander misses the desired target site. Kepler's equation is written

$$n_c t_{s\text{-}E} = e_c\left[\sinh F_{c\text{-}s} - \sinh F_{c\text{-}E}\right] - \left[F_{c\text{-}s} - F_{c\text{-}E}\right] \tag{20}$$

where

$$n_c = \frac{\left[v_{c\text{-}s}^2 - \frac{2\mu_M}{r_s}\right]^{3/2}}{\mu_M} \tag{21}$$

$$\cosh F_{c\text{-}s} = \frac{e_c + \cos\gamma_{c\text{-}s}}{1 + e_c\cos\gamma_{c\text{-}s}} \tag{22}$$

$$\cosh F_{c\text{-}E} = \frac{e_c + \cos\gamma_{c\text{-}E}}{1 + e_c\cos\gamma_{c\text{-}E}} \tag{23}$$

Differentiation of eq'. 20 yields, for example,

$$n_c\frac{\partial t_{s\text{-}E}}{\partial r_s} = \left[\sinh F_{c\text{-}s} - \sinh F_{c\text{-}E}\right]\frac{\partial e_c}{\partial r_s} + \frac{1}{e_c}\left[r_s\frac{\partial F_{c\text{-}s}}{\partial r_s} - r_E\frac{\partial F_{c\text{-}s}}{\partial r_s}\right] - t_{s\text{-}E}\frac{\partial n_c}{\partial r_s} \tag{24}$$

Differentiation of eq's. 12, 13, 15, 21, 22, 23 and subsequent substitution in eq. 24 yields

$$\frac{\partial t_{s\text{-}E}}{\partial r_s} = -\frac{3 n_c t_{s\text{-}E}}{r_s^2 a_c} + \frac{1}{n_c e_c a_c}\sqrt{\frac{p_c}{a_c}}\left\{\frac{e_c a_c}{r_s}\left[\frac{1}{a_c} + \frac{1}{r_s}\right] + \left[\frac{2 r_E}{r_s}\csc\gamma_{c\text{-}E} - \frac{1}{e_c}\tan\gamma_{c\text{-}s}\right]\right\} \tag{25}$$

in which

$$Q = \ldots \left( \ldots \right) \ldots \left( \ldots \right) \ldots$$

$$\ldots \qquad (26)$$

In similar fashion, we obtain

$$\frac{\partial t_{s-2}}{\partial w_{c-1}} = \frac{\ldots}{\ldots}$$

$$\ldots \left\{ \begin{array}{l} \ldots Q \left[ \frac{1}{\ldots} + \frac{1}{\ldots} \right] \\ \ldots \left[ \ldots \tan \ldots \right] \end{array} \right\} \qquad (27)$$

and

$$\frac{\partial t_{s-2}}{\partial \ldots} = \frac{\ldots}{\ldots} \sqrt{\frac{\ldots}{\ldots}} \tan \ldots \left\{ \begin{array}{l} \ldots \left( \ldots \tan 2\chi_{-1} \right) \\ \ldots \end{array} \right\} \qquad (28)$$

note that

$$\frac{\partial t_{s-2}}{\partial \ldots} = 0$$

2. Sensitivity of landing predictions to entry conditions

From Fig. 2, the normal angle deviation at impact is given by

$$\ldots$$

or

$$\ldots \qquad (29)$$

where $\theta_{s-1}$ is the down-range [illegible] through the (Martian) atmosphere. The [illegible] at impact, as the entry body is closed down [illegible]

extent by $\gamma_E$. For an assumed atmosphere (density-altitude relation) and specified $W/C_D A$ of the entry body, $\theta_{E-I}$ (as well as $t_{E-I}$, the time of descent) can be determined by means of a three-degree of freedom, point-mass computer program. Sensitivities to changes in $\gamma_E$ and $v_E$ can then be obtained. From eq. 29

$$\delta\theta_I = \frac{\partial\theta_I}{\partial\theta_E}\,\delta\theta_E + \frac{\partial\theta_I}{\partial v_E}\,\delta v_E + \frac{\partial\theta_I}{\partial\gamma_E}\,\delta\gamma_E \tag{30}$$

where

$$\frac{\partial\theta_I}{\partial\theta_E} = 1, \quad \frac{\partial\theta_I}{\partial v_E} = \frac{\partial\theta_{E-I}}{\partial v_E}, \quad \frac{\partial\theta_I}{\partial\gamma_E} = \frac{\partial\theta_{E-I}}{\partial\gamma_E}$$

the value of $\frac{\partial\theta_{E-I}}{\partial v_E}$ and $\frac{\partial\theta_{E-I}}{\partial\gamma_E}$ being obtained from plots of tabulated data.

Let $t_{S-I}$ denote time of flight of the lander, from separation to impact. Thus

$$t_{S-I} = t_{S-E} + t_{E-I}$$

$$\delta t_{S-I} = \delta t_{S-E} + \delta t_{E-I}$$

$$= \frac{\partial t_{S-E}}{\partial v_{E-S}}\,\delta v_{E-S} + \frac{\partial t_{S-E}}{\partial\theta_{E-S}}\,\delta\theta_{E-S} + \frac{\partial t_{S-E}}{\partial\lambda_S}\,\delta\lambda_S$$

$$+ \frac{\partial t_{E-I}}{\partial v_E}\,\delta v_E + \frac{\partial t_{E-I}}{\partial\gamma_E}\,\delta\gamma_E \tag{31}$$

where

$$\delta v_E = \frac{\partial v_E}{\partial v_{E-S}}\,\delta v_{E-S} + \frac{\partial v_E}{\partial\lambda_S}\,\delta\lambda_S \tag{32}$$

$$\delta\gamma_E = \frac{\partial\gamma_E}{\partial v_{E-S}}\,\delta v_{E-S} + \frac{\partial\gamma_E}{\partial\lambda_S}\,\delta\lambda_S + \frac{\partial\gamma_E}{\partial\theta_{E-S}}\,\delta\theta_{E-S} \tag{33}$$

Thus

$$\delta t_{S-I} = \frac{\partial t_{S-I}}{\partial v_{E-S}}\,\delta v_{E-S} + \frac{\partial t_{S-I}}{\partial\lambda_S}\,\delta\lambda_S + \frac{\partial t_{S-I}}{\partial\theta_{E-S}}\,\delta\theta_{E-S} \tag{34}$$

where

$$\frac{\partial t_{S-I}}{\partial v_{E-S}} = \frac{\partial t_{S-E}}{\partial v_{E-S}} + \frac{\partial t_{E-I}}{\partial v_E}\,\frac{\partial v_E}{\partial v_{E-S}} + \frac{\partial t_{E-I}}{\partial\gamma_E}\,\frac{\partial\gamma_E}{\partial v_{E-S}} \tag{35}$$

$$\frac{\partial t_{S-I}}{\partial r_S} = \frac{\partial t_{E-I}}{\partial v_E}\frac{\partial v_E}{\partial r_S} + \frac{\partial t_{E-I}}{\partial \gamma_E}\frac{\partial \gamma_E}{\partial r_S} + \frac{\partial t_{S-E}}{\partial r_S} \tag{36}$$

$$\frac{\partial t_{E-I}}{\partial \gamma_{E-S}} = \frac{\partial t_{E-I}}{\partial \gamma_E}\frac{\partial \gamma_E}{\partial \gamma_{E-S}} + \frac{\partial t_{E-I}}{\partial \gamma_{E-S}} \tag{37}$$

The value of $\frac{\partial t_{E-I}}{\partial v_E}$ and $\frac{\partial t_{E-I}}{\partial \gamma_E}$ are obtained from the aforementioned plots; i.e., partials are evaluated at the nominal values of $v_E$, $\gamma_E$. Note that eq. 34 implies

$$\frac{\partial t_{E-I}}{\partial \theta_S} = 0$$

C. Out-of-Plane Error Coefficients

In Fig. 3, define a coordinate system such that $\bar{x}$ is along $-\bar{r}_S$, y is normal to the nominal lander approach plane, and $\bar{z}$ forms a right-handed system. The lander would nominally traverse a central angle of

$$\Delta\eta = \theta_{C-S} - \theta_{E-S} + \theta_{C-E} \tag{38}$$

to arrive at the target position $\bar{r}_T$. If there is a velocity error $\delta\bar{y}$ normal to the nominal plane, the actual trajectory plane will be at an angle $\theta_T$ to the nominal plane. Neglecting variation in time of flight for $\delta\bar{y}$, the cross-range error will be $\delta\theta_C$. We have

$$\delta\theta_C = \tan\delta\theta_C = \tan\delta\theta_C = \frac{\delta\theta_C}{\delta\theta_C} \tag{39}$$

But

$$\delta\theta_C = \tan\delta\theta_C = \tan\delta\theta_C = \frac{\delta\theta_C}{\delta\theta_C} \tag{40}$$

$$\delta\theta_C = \tan\delta\theta_C = \tan\delta\theta_C = \frac{\delta\theta_C}{\delta\theta_C} \tag{41}$$

$$\delta\theta_C = \tan\delta\theta_C = \tan\delta\theta_C = \frac{\delta\theta_C}{\delta\theta_C} \tag{42}$$

Then

Substituting eq. 43 into eq. 39,

$$\frac{\partial \theta_c}{\partial \dot{y}} = \frac{\delta \theta_c}{\delta \dot{y}} = \frac{\sin \gamma_{c-s} \sin \Delta\eta}{\bar{v}_{c-s}} \tag{44}$$

If there is a normal position [illegible] only, then $\bar{v}'_{c-s}$ is parallel to $\bar{v}_{c-s}$ and the deviated plane (the $\bar{r}_s' \times \bar{v}_{c-s}'$ plane) intersects the nominal plane in the line $\bar{i}_q$, situated 90° $\gamma_{c-s}$ from $\bar{r}_s$. We have (see Fig. 4)

$$\bar{v}'_{c-s} = v_{c-s}\left[\sin \gamma_{c-s}\, \bar{x} + \cos \gamma_{c-s}\, \bar{z}\right] \tag{45}$$

$$\bar{r}_s' = -r_s\, \bar{x} + dy\, \bar{y} \tag{46}$$

$$\bar{r}_s' \times \bar{v}'_{c-s} = v_{c-s}\left[r_s \cos \gamma_{c-s}\, \bar{y} + dy\left(\cos \gamma_{c-s}\, \bar{x} - \sin \gamma_{c-s}\, \bar{z}\right)\right] \tag{47}$$

$$\bar{i}_q = \cos \gamma_{c-s}\, \bar{x} - \sin \gamma_{c-s}\, \bar{z} \tag{48}$$

Thus

$$\tan \theta_p = \frac{\bar{r}_s \cdot (\bar{r}_s' \times \bar{v}'_{c-s})}{\bar{y} \cdot (\bar{r}_s' \times \bar{v}'_{c-s})} = \frac{r_s}{r_s} \sin \gamma_{c-s} \tag{49}$$

and

$$\delta\theta_p = [illegible] \tag{50}$$

Substituting eq. [illegible] into eq. 50,

$$\left[\frac{\partial \theta_p}{\partial \dot{y}} = \frac{\delta \theta_p}{\delta \dot{y}} = [illegible] \sin \gamma_{c-s}\right] \tag{51}$$

[illegible]

[illegible]

[illegible] to have nominally,

$$[illegible] \tag{52}$$

Differentiation with respect to , and yields

$$\frac{\partial v_{c-1}}{\partial \Delta v_s} = \left(\frac{\Delta v_s}{v_{c-1}}\right) + \left(\frac{v_s}{v_{c-1}}\right) \cos\beta \tag{54}$$

Also, from Fig. 1,

$$\tan\left(\gamma_{c-1} - \gamma_{g-1}\right) = \frac{\Delta v_s \sin\beta}{\left(\Delta v_s \cos\beta + v_{g-1}\right)} \tag{55}$$

Differentiation yields

$$\frac{\partial \gamma_{c-1}}{\partial \beta} = \frac{\sin\beta \cos\beta}{2} + \sin^2\phi \tag{56}$$

$$\frac{\partial \gamma_{c-1}}{\partial \Delta v_s} = \frac{1}{\Delta v_s}\left[\frac{\sin 2\phi}{2} - \cos\beta \sin^2\phi\right] \tag{57}$$

where $\phi \triangleq \gamma_{c-1} - \gamma_{g-1}$. As a good approximation to the situation of desired thrust normal to $v_{g-1}$ we may take $\beta$ such that $v_{c-1} = v_{g-1}$, that is, no change in specific total energy, so that $v_{c-1} = v_{g-1}$.

This considerably simplifies computation of normal conditions. In that case,

$$\beta = \gamma_{g-1} + \frac{\phi}{2}$$

$$\frac{\Delta v_s}{v_{c-1}} = \frac{\Delta v_s}{v_{g-1}} \tag{58}$$

and the sensitivities to thrust (magnitude and direction) variations become

$$\frac{\partial \gamma_{c-1}}{\partial \beta} = \frac{\sin\beta \cos\beta}{2} + \sin^2\phi \tag{59}$$

$$\frac{\partial \gamma_{c-1}}{\partial \Delta v_s} = \frac{1}{\Delta v_s}\left[\frac{\sin 2\phi}{2} - \cos\beta \sin^2\phi\right] \tag{60}$$

$$\frac{\partial \gamma_{c-1}}{\partial \Delta v_s} = \frac{1}{\Delta v_s}\left[\frac{\sin 2\phi}{2} - \cos\beta \sin^2\phi\right] \tag{61}$$

2) [illegible]

In Fig. 5 let $\overline{\delta v_s}$ be rotated to $\overline{\delta v_s}$, where this rotation occurs about an axis perpendicular to $\overline{\delta v_s}$, and lying in the $\overline{\delta v_s} - \overline{v_{c-s}}$ plane. Evidently, $\overline{v_{c-s}}$ has the component

$$\delta v_s = \Delta v_s \cdot \delta\theta \tag{63}$$

Then from eq. 44,

$$\frac{\partial \theta_c}{\partial \theta} = \left(\frac{\Delta v_s}{v_{c-s}}\right) \sec \gamma_{c-s} \sin \Delta\eta \tag{64}$$

For nominal thrust such that $v_{c-s} = v_{s-s}$

$$\frac{\partial \theta_c}{\partial \theta} = -2 \cos \beta \cdot \sec \gamma_{c-s} \sin \Delta\eta \tag{65}$$

It should be observed that effects of $\delta\theta$ on $v_{c-s}$ and $\gamma_{c-s}$ are second order and can be neglected. That is,

$$\frac{\partial v_{c-s}}{\partial \theta} \approx 0 \quad ; \quad \frac{\partial \gamma_{c-s}}{\partial \theta} \approx 0$$

E Navigation Error Effects

Prior to separation of the capsule from the bus, either an Earth-bound or on-board navigation system may be used to determine the (bus) trajectory and to predict the bus position and velocity coordinates, in some coordinate system, at separation. For the nominally coincident plane case, evidently uncertainties in position are the same for bus and tender, at separation; the normal component of velocity uncertainty is also the same. However uncertainties in $\gamma_{c-s}$ and $\gamma_{s-s}$ must be related to uncertainties in $v_{c-s}$ and $\gamma_{c-s}$. Differentiation of eq's. 52 and 55 gives

$$\frac{\partial v_{c-s}}{\partial \theta} = \left[ v_{s-s} - \Delta v_s \cos \beta \right] \tag{66}$$

$$\frac{\partial \gamma_{c-s}}{\partial \theta} = \frac{\sin \beta}{\cos \beta} \tag{67}$$

$$\tag{68}$$

$\partial \gamma_{s-s}$

For the special thrusting case such that $v_{c-s} = v_{B-s}$,

$$\frac{\partial v_{c-s}}{\partial v_{B-s}} = -\cos 2\beta \tag{69}$$

$$\frac{\partial \gamma_{c-s}}{\partial v_{B-s}} = \frac{\sin 2\beta}{v_{c-s}} \tag{70}$$

Now, assume that an on-board navigation system enables determination of bus position and velocity coordinates in the $\bar{i}_\infty - \bar{j}_\infty - \bar{k}_\infty$ system defined by $\bar{i}_\infty$ along $\bar{v}_{B-s}$, $\bar{j}_\infty = \bar{j}$ normal to bus approach plane, and $\bar{k}_\infty$ along the impact parameter $\bar{B}$. (For ease of notation, the $\bar{i} - \bar{j} - \bar{j}$ system will now be referred to as the $\bar{i}_s - \bar{j}_s - \bar{k}_s$ system, see Fig. 6). Then, measuring inertial angle $\theta_s$ from $-\bar{i}_\infty$,

$$\bar{r}_s = r_s\left[-\cos\theta_s\,\bar{i}_\infty + \sin\theta_s\,\bar{k}_\infty\right] \tag{71}$$

The actual position vector is

$$\bar{r}_s' = (r_s + \delta r_s)\left[-\cos\theta_s'\cos\varepsilon\,\bar{i}_\infty + \cos\varepsilon\sin\theta_s'\,\bar{k}_\infty + \sin\varepsilon\,\bar{j}_\infty\right] \tag{72}$$

Now

$$\begin{aligned}\bar{r}_s' - \bar{r}_s = \overline{\delta r_s} &= \delta x\,\bar{i}_s + \delta y\,\bar{j}_s + \delta z\,\bar{k}_s \\ &= (\delta x\,\bar{i}_\infty - \delta y\,\bar{j}_\infty + \delta z\,\bar{k}_\infty)\end{aligned} \tag{73}$$

(Note that $|\overline{\delta r_s}| \ll \delta r_s$). Observing that $\theta_s' = \theta_s + \delta\theta_s$, we have, after neglecting second order effects,

$$\begin{aligned}\delta x &= \overline{\delta r_s}\cdot\bar{i}_\infty \\ &= r_s\,\delta\theta_s\sin\theta_s - \delta r_s\cos\theta_s\end{aligned} \tag{74}$$

$$\begin{aligned}\delta y &= \overline{\delta r_s}\cdot\bar{j}_\infty = \delta y \\ &= r_s\,\delta\varepsilon\end{aligned} \tag{75}$$

(76)

From eq's. 74 and 75 we obtain,

$$\delta\theta_s = \frac{\delta X}{r_s}\sin\theta_s + \frac{\delta Z}{r_s}\cos\theta_s \tag{77}$$

$$\delta r_s = -\delta X\cos\theta_s + \delta Z\sin\theta_s \tag{78}$$

From Fig. 6,

$$\begin{bmatrix} \bar{i}_r \\ \bar{j}_r \\ \bar{k}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_s & 0 & -\sin\theta_s \\ 0 & 1 & 0 \\ \sin\theta_s & 0 & \cos\theta_s \end{bmatrix} \begin{bmatrix} \bar{i}_0 \\ \bar{j}_0 \\ \bar{k}_0 \end{bmatrix}$$

and from Fig. 7,

$$\begin{bmatrix} \bar{i}_r' \\ \bar{j}_r' \\ \bar{k}_r' \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & -\delta\theta_s \\ 0 & 1 & 0 \\ \delta\theta_s & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{i}_r \\ \bar{j}_r \\ \bar{k}_r \end{bmatrix}$$

Define

$$v_{B-s}' = v_{B-s} + \delta v_{B-s} \; ; \; \gamma_{B-s}' = \gamma_{B-s} + \delta\gamma_{B-s}$$

Then, from Fig. 8,

$$\bar{v}_{B-s} = v_{B-s}\left[\sin\gamma_{B-s}\,\bar{i}_r + \cos\gamma_{B-s}\,\bar{k}_r\right] \tag{79}$$

$$\bar{v}_{B-s}' = v_{B-s}'\left[\sin\gamma_{B-s}'\,\bar{i}_r' + \cos\gamma_{B-s}'\,\bar{j}_r' + \cos\gamma_{B-s}'\cos\gamma_{B-s}'\,\bar{k}_r'\right] \tag{80}$$

But

$$\frac{d\,\bar{X}^B}{dt} = \bar{i}_r + \delta\bar{X}_{B-s} + \delta\bar{k}_r + \ldots \tag{81}$$

[illegible]

[illegible]

$$\delta \dot{Y} = \frac{d\,\delta Y}{dt} \approx \bar{v}_{B-S}\,\delta\gamma = \delta\dot{y} \tag{83}$$

$$\delta \dot{\Phi} = \frac{d\,\delta\Phi}{dt} = \bar{\Phi}_{\infty} = -\bar{v}_{B-S}\,\delta\gamma_{B-S}\,\sin(\theta_S + \gamma_{B-S}) + \delta v_{B-S}\cos(\theta_S + \gamma_{B-S}) \tag{84}$$

wherein $\delta\gamma_{B-S} \equiv \delta\gamma_B - \delta\gamma_S$

Solving for $\delta\gamma_{B-S}$ and $\delta v_{B-S}$ from eq's. 83 and 84,

$$\delta\gamma_{B-S} = \frac{\delta\dot{y}}{\bar{v}_{B-S}}\cos(\theta_S + \gamma_{B-S}) - \frac{\delta\dot{\Phi}}{\bar{v}_{B-S}}\sin(\theta_S + \gamma_{B-S}) - \delta\theta_S \tag{85}$$

$$\delta v_{B-S} = \delta\dot{y}\,\sin(\theta_S + \gamma_{B-S}) + \delta\dot{\Phi}\cos(\theta_S + \gamma_{B-S}) \tag{86}$$

Now

$$\delta\gamma_{B-S} = \frac{\partial\gamma_{B-S}}{\partial v_{B-S}}\,\delta v_{B-S} + \frac{\partial\gamma_{B-S}}{\partial\gamma_{B-S}}\,\delta\gamma_{B-S} \tag{87}$$

$$\delta v_{B-S} = \frac{\partial v_{B-S}}{\partial v_{B-S}}\,\delta v_{B-S} + \frac{\partial v_{B-S}}{\partial\gamma_{B-S}}\,\delta\gamma_{B-S} \tag{88}$$

Substituting eq's. 85, 86, 87, 88 and 81 into eq's. 87 and 88 and noting that $\frac{\partial\gamma_{B-S}}{\partial v_{B-S}} = 0$, we have (and for reasons discussing such (but $v_{B-S}$, $\gamma_{B-S}$ and $\Delta t_S$ in the 2nd approach [Line])

$$\delta v_{B-S} = -\frac{1}{\bar{v}_{B-S}}\left[\delta\dot{y}\,\sin(\theta_S + \gamma_{B-S}) + \delta\dot{\Phi}\cos(\theta_S + \gamma_{B-S})\right]$$

$$+ \frac{\Delta t_S}{\bar{v}_{B-S}}\left[\delta\dot{y}\,\sin(\theta_S + \gamma_{B-S}) + \delta\dot{\Phi}\cos(\theta_S + \gamma_{B-S})\right] \tag{89}$$

Eq's. 75, 77, 78, 83, 89 and 90 relate the bus navigation uncertainties to capsule (leader) navigation uncertainties. In numerically assessing the moment matrix of bus uncertainties, a differently defined bus coordinate system $\bar{x}_c - \bar{y}_c - \bar{z}_c$ was used, where $\bar{x}' = \bar{x}_\infty$, $\bar{y}' = -\bar{y}_\infty$, $\bar{z}' = \bar{z}_\infty$, necessitating the use of an additional transformation:

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \\ \delta \dot{X} \\ \delta \dot{Y} \\ \delta \dot{Z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & & & \\ 0 & -1 & 0 & & (0) & \\ 1 & 0 & 0 & & & \\ & & & 0 & 0 & 1 \\ & (0) & & 0 & -1 & 0 \\ & & & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta X' \\ \delta Y' \\ \delta Z' \\ \delta \dot{X}' \\ \delta \dot{Y}' \\ \delta \dot{Z}' \end{bmatrix}$$

F. Entry Angle Dispersion

The entry angle error due to errors at separation is

$$\delta \gamma_E \Big|_{t_E} = \frac{\partial \gamma_E}{\partial \bar{r}_{c_0}} \delta \bar{r}_{c_0} \Big|_{t_E} + \frac{\partial \gamma_E}{\partial \bar{v}_{c_0}} \delta \bar{v}_{c_0} \Big|_{t_E}$$

$$+ \left[ \frac{\partial \gamma_E}{\partial \alpha_0} \quad \frac{\partial \gamma_E}{\partial \beta_0} \quad \frac{\partial \gamma_E}{\partial \gamma_0} \quad \frac{\partial \gamma_E}{\partial \phi_0} \right] \delta \bar{\gamma}$$

$$+ \left[ \frac{\partial \gamma_E}{\partial \alpha_s} \quad \frac{\partial \gamma_E}{\partial \beta_s} \quad \frac{\partial \gamma_E}{\partial \gamma_s} \quad \frac{\partial \gamma_E}{\partial \phi_s} \right] \delta \bar{\gamma}_s$$

$$+ \frac{\partial \gamma_E}{\partial \bar{r}_{c_s}} \delta \bar{r}_{c_s} + \frac{\partial \gamma_E}{\partial \bar{v}_{c_s}} \delta \bar{v}_{c_s} \tag{91}$$

The standard deviation of the entry angle is

$$\sigma_{\gamma_E} = \sqrt{\sum_{i=1}^{n} \sigma_{\gamma_{E_i}}^2}$$

where, since [illegible]

$$\sigma_{\gamma_{E_i}} = \left[ \frac{\partial \gamma_E}{\partial \bar{r}_{c_0}} \quad \frac{\partial \gamma_E}{\partial \bar{v}_{c_0}} \quad \frac{\partial \gamma_E}{\partial \bar{\gamma}} \quad \frac{\partial \gamma_E}{\partial \bar{\gamma}_s} \quad \frac{\partial \gamma_E}{\partial \bar{r}_{c_s}} \quad \frac{\partial \gamma_E}{\partial \bar{v}_{c_s}} \right] \sigma_{\bar{\gamma}_i} \tag{92}$$

[illegible]

$$= \frac{\partial \gamma_E}{\partial x'} \delta x' + \frac{\partial \gamma_E}{\partial z'} \delta z' + \frac{\partial \gamma_E}{\partial \dot{x}'} \delta \dot{x}' + \frac{\partial \gamma_E}{\partial \dot{z}'} \delta \dot{z}' \tag{93}$$

Let $\widehat{\delta X}$ represent the matrix of navigation uncertainties:

$$\widehat{\delta X} \triangleq \begin{bmatrix} \delta x' \\ \delta y' \\ \delta z' \\ \delta \dot{x}' \\ \delta \dot{y}' \\ \delta \dot{z}' \end{bmatrix}$$

Then

$$\begin{bmatrix} \delta r_s \\ \delta \theta_s \\ \delta v_{E-s} \\ \delta \gamma_{E-s} \end{bmatrix} = \widehat{C}_n \, \widehat{\delta X} \tag{94}$$

$\widehat{C}_n$ is, from the discussion in the previous section, a 4 x 6 matrix, of which the non-zero elements are given by

$$\frac{\partial r_s}{\partial x'} \triangleq C_{11} = \sin\theta_s \quad \frac{\partial r_s}{\partial z'} \triangleq C_{13} = -\cos\theta_s$$

$$\frac{\partial \theta_s}{\partial x'} \triangleq C_{21} = \frac{\cos\theta_s}{r_s} \quad \frac{\partial \theta_s}{\partial z'} \triangleq C_{23} = \frac{\sin\theta_s}{r_s}$$

$$\frac{\partial \gamma_{E-s}}{\partial x'} \triangleq C_{24} = \cos\theta_s \quad \frac{\partial \gamma_{E-s}}{\partial z'} \triangleq C_{26} = -\cos 2\theta \, \sin(\theta_s + \gamma_{E-s})$$

$$\frac{\partial \gamma_{E-s}}{\partial x'} \triangleq C_{27} = \frac{\cos\theta_s}{r_s} \quad \frac{\partial \gamma_{E-s}}{\partial z'} \triangleq C_{29} = \frac{-\sin\theta_s}{r_s}$$

where

$$\overline{\delta x_E}\Big|_{s} = \left[\frac{\partial x_E}{\partial x_s} \;\; \frac{\partial x_E}{\partial x_{c_1}} \;\; \frac{\partial x_E}{\partial x_{c_2}}\right]_{s} \overline{\delta x\, \delta x^T}\, C^T \begin{bmatrix} \frac{\partial x_E}{\partial x_s} \\ I \\ \frac{\partial x_E}{\partial x_{c_1}} \\ \frac{\partial x_E}{\partial x_{c_2}} \end{bmatrix} \tag{95}$$

$\overline{\delta x\, \delta x^T}$ is the moment matrix of navigation uncertainties, evaluated at separation point. From eq. 95, the following relations may be deduced:

$$\frac{\partial \sigma_{x_E}^2}{\partial x^T} = \frac{\partial x_E}{\partial x_s}\frac{\partial x_s}{\partial x^T} + \frac{\partial x_E}{\partial x_{c_1}}\frac{\partial x_{c_1}}{\partial x^T}$$

$$\frac{\partial \sigma_{x_E}^2}{\partial t} = \frac{\partial x_E}{\partial x_s}\frac{\partial x_s}{\partial t} + \frac{\partial x_E}{\partial x_{c_1}}\frac{\partial x_{c_1}}{\partial t}$$

$$\frac{\partial \sigma_{x_E}^2}{\partial x^T} = \frac{\partial x_E}{\partial x_{c_1}}\frac{\partial x_{c_1}}{\partial x^T} + \frac{\partial x_E}{\partial x_{c_2}}\frac{\partial x_{c_2}}{\partial x^T}$$

$$\frac{\partial \sigma_{x_E}^2}{\partial t} = \frac{\partial x_E}{\partial x_{c_1}}\frac{\partial x_{c_1}}{\partial t} + \frac{\partial x_E}{\partial x_{c_2}}\frac{\partial x_{c_2}}{\partial t}$$

$$\frac{\partial x_E}{\partial x^T} = 0 \;, \quad \frac{\partial \sigma_{x_E}^2}{\partial t} = 0$$

Since the observation and navigation errors are independent, the standard deviation of the total entry angle error is

$$\sigma_{\theta_E} = \left[\sigma_{x_E}^2 + \sigma_{x_E}^2\right]^{1/2} \tag{96}$$

If the 3 root mean sum at $\theta_{E_{max}}$ is discriminated, then $\sigma_{\theta_E}$ will be Gaussian and $3\sigma_{\theta_E}$ expresses the 99.7% [illegible] error.

[illegible] listed in the appendix, [illegible]

This ratio, for the stated nominal conditions, is only .032. This fact explains the relative insensitivity of $\sigma(\delta\xi_T)_S$ to change in $\sigma(\delta\beta)$, as shown in Table II.

C. Surface Dispersion

1) Time-of-flight error effects

In Fig. 9, suppose an error in $t_{s-r}$ only, so that the lander arrives at point T' instead of point T; the target, during $\delta t_{s-r}$, will travel to point T'. It is desired to determine the down-range and cross-range errors due to $\delta t_{s-r}$. Denote the great circle arc through points T and T' as $\Delta$. Then

$$\sin\frac{\Delta}{2} = \cos\xi_T \sin\left(\frac{\omega_p\,\delta t_{s-r}}{2}\right)$$

or

$$\Delta \approx \omega_p\,\delta t_{s-r} \cdot \cos\xi_T \qquad (97)$$

where $\omega_p$ = Mars rotational rate = 14.631 deg's/hr

$\xi_T$ = Target latitude, with respect to Mars equatorial plane

From the T-T'-T'' triangle,

$$\tan\delta\theta_A\Big]_{\xi} = \tan\Delta \cdot \cos(A'-A) \qquad (98)$$

but

$$\cos A = \sin\delta_p \cos(L_T - \alpha_p) \qquad (99)$$

$$\sin A = \cos\delta_p \sec\xi_T \qquad (100)$$

$$\cos A' = \tan\delta_p \tan\left(\frac{\Delta}{2}\right) \qquad (101)$$

$$\sin A' = \cos\left(\delta_p - \frac{\Delta}{2}\right)\sec\left(\frac{\Delta}{2}\right) \qquad (102)$$

Substituting eq's [illegible] through [illegible] and [illegible] eq. 98, and neglecting second order effects,

$$\tan\delta\theta_A\Big]_{\xi} = \omega_p\,\delta t_{s-r} \cdot \cos\delta_p \qquad (103)$$

Since this reduces the down range error due to other sources, a negative sign should be appended, thus

$$\delta\theta_A = \qquad (104)$$

Again

[illegible] (106)

Proper consideration must be given to the [illegible] in accounting for the effect on [illegible] due to [illegible] on [illegible]. Thus

$$\frac{\partial \Delta \alpha}{\partial \zeta_T} = \frac{\partial}{\partial \zeta_T} \left[ \frac{W_T}{[illegible]} \cos \zeta_T \sin(\zeta_T - \zeta_p) \right] \tag{107}$$

$$\gamma = \begin{cases} +1, & \text{for } \zeta_T > \zeta_c \\ -1, & \text{for } \zeta_T < \zeta_c \end{cases}$$

It should be noted that [illegible] and [illegible] depend on [illegible] as well as on lag and lander trajectory parameters. It can be shown that

$$\zeta_c = \zeta_p + \sin^{-1}\left( \frac{\sin \zeta_p \cos \gamma}{\sin \gamma} \right) \tag{108}$$

Expressions for [illegible] and [illegible] are given in the appendix.

2. Deviations from nominal orbits

The long-range impact, measured in the nominal plane, is

$$\delta \xi_p = \delta \xi_1 + \frac{\partial \xi_p}{\partial \Omega_p} \delta \Omega_p \tag{109}$$

where $\delta \xi_1$ is given by eq. 30 and $\delta \Omega_p$ is given by eq. 3[illegible]. Recall that

$$\delta \Omega_p = \frac{\partial \Omega_p}{\partial \Omega_1} \delta \Omega_1 + \frac{\partial \Omega_p}{\partial \lambda_1} \delta \lambda_1 + \frac{\partial \Omega_p}{\partial \zeta_1} \delta \zeta_1 + \frac{\partial \Omega_p}{\partial \Delta \alpha} \delta \Delta \alpha \tag{110}$$

Substituting eq's. 32, 33 and 110 into eq. 3[illegible] and substituting the resulting equation and eq. 34 from eq. 109, the entire lag is obtained

$$\delta \xi = \frac{\partial \xi_p}{\partial \Omega_1} \delta \Omega_1 + \frac{\partial \xi_p}{\partial \lambda_1} \delta \lambda_1 + \frac{\partial \xi_p}{\partial \zeta_1} \delta \zeta_1 + \frac{\partial \xi_p}{\partial \Delta \alpha} \delta \Delta \alpha \tag{111}$$

where

$$\frac{\partial \xi_p}{\partial \Omega_1} = \frac{\partial \xi_p}{\partial \Omega_p} \frac{\partial \Omega_p}{\partial \Omega_1} \tag{112}$$

$$\frac{\partial \xi_p}{\partial \lambda_1} = \frac{\partial \xi_p}{\partial \Omega_p} \frac{\partial \Omega_p}{\partial \lambda_1} \tag{113}$$

$$\frac{\partial \xi_p}{\partial \zeta_1} = \frac{\partial \xi_p}{\partial \Omega_p} \frac{\partial \Omega_p}{\partial \zeta_1} \tag{114}$$

$$\frac{\partial \xi_p}{\partial \Delta \alpha} = \frac{\partial \xi_p}{\partial \Omega_p} \frac{\partial \Omega_p}{\partial \Delta \alpha} + \frac{\partial \xi_{s-T}}{\partial \Omega_p} \frac{\partial \Omega_p}{\partial \Delta \alpha} + \frac{\partial \Omega_A}{\partial \Omega_p} \frac{\partial \xi_{s-T}}{\partial \Omega_A} \tag{115}$$

The cross-range error, measured in a plane perpendicular to the nominal plane, is given by

$$\delta\theta_c = \frac{\partial\theta_c}{\partial\theta_1}\delta\theta_1 + \frac{\partial\theta_c}{\partial\theta_2}\delta\theta_2 + \frac{\partial\theta_c}{\partial\theta_3}\delta\theta_3 + \frac{\partial\theta_c}{\partial\theta_4}\delta\theta_4 + \frac{\partial\theta_c}{\partial\theta_5}\delta\theta_5 + \frac{\partial\theta_c}{\partial\theta_6}\delta\theta_6 \tag{116}$$

where

$$\frac{\partial\theta_c}{\partial\theta_1} = \frac{\partial\theta_c}{\partial\theta_4} = \frac{\partial\theta_c}{\partial\theta_5} \tag{117}$$

$$\frac{\partial\theta_c}{\partial\theta_2} = \frac{\partial\theta_c}{\partial\theta_3} = \frac{\partial\theta_c}{\partial\theta_6} \tag{118}$$

$$\frac{\partial\theta_c}{\partial\theta_3} = \frac{\partial\theta_c}{\partial\theta_2} = \frac{\partial\theta_c}{\partial\theta_6} \tag{119}$$

It is seen that $\delta\theta_1$ and $\delta\theta_4$ are combined through $\delta\theta_c$. Eq's 111 and 116 are generally applicable, whether considering antenna error angles or navigation uncertainties.

In order to $\delta\theta_2$ and $\delta\theta_3$ or navigation error similar, note first that eq. 109 can be written in matrix form as follows:

$$\delta\theta_c = A_n \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \\ \delta\theta_3 \\ \delta\theta_4 \\ \delta\theta_5 \\ \delta\theta_6 \end{bmatrix} \tag{120}$$

where

$$A_n = \begin{bmatrix} \frac{\partial\theta_c}{\partial\theta_1} & \frac{\partial\theta_c}{\partial\theta_2} & \frac{\partial\theta_c}{\partial\theta_3} & \frac{\partial\theta_c}{\partial\theta_4} & \frac{\partial\theta_c}{\partial\theta_5} & \frac{\partial\theta_c}{\partial\theta_6} \end{bmatrix} \tag{121}$$

Also, with $\delta\theta_2$ and $\delta\theta_3$ treated as independent variables, we have from

$$\delta\theta_c = \frac{\partial\theta_c}{\partial\theta_2}\delta\theta_2 + \frac{\partial\theta_c}{\partial\theta_3}\delta\theta_3 \tag{122}$$

where

$$\hat{B}_k = \begin{bmatrix} \frac{\partial \theta_E}{\partial \lambda_E} & \frac{\partial \theta_E}{\partial \gamma_{E_1}} & \frac{\partial \theta_E}{\partial \gamma_{E_2}} \\ \frac{\partial \phi_E}{\partial \lambda_E} & \frac{\partial \phi_E}{\partial \gamma_{E_1}} & 0 \\ \frac{\partial \theta_S}{\partial \lambda_E} & \frac{\partial \theta_S}{\partial \gamma_{E_1}} & \frac{\partial \theta_S}{\partial \gamma_{E_2}} \\ \frac{\partial \phi_S}{\partial \lambda_E} & \frac{\partial \phi_S}{\partial \gamma_{E_1}} & \frac{\partial \phi_S}{\partial \gamma_{E_2}} \end{bmatrix} \tag{123}$$

Then, the derived coefficients can be obtained by matrix multiplication:

$$\hat{A}_k \hat{B}_k = \begin{bmatrix} \frac{\partial \theta_S}{\partial \lambda_E} & \frac{\partial \theta_S}{\partial \theta_E} & \frac{\partial \theta_S}{\partial \gamma_{E_1}} & \frac{\partial \theta_S}{\partial \gamma_{E_2}} \end{bmatrix} \tag{124}$$

Finally, from eq. 94, we have

$$\delta \hat{\theta}_S \Big|_{\hat{\theta}_E} = \hat{A}_k \hat{B}_k \hat{C}_k \hat{\Gamma}_k \tag{125}$$

where the elements of $\hat{A}_k \hat{B}_k \hat{C}_k$ are the derived coefficients of $\delta \hat{\theta}_S$ to navigation uncertainties:

$$\hat{A}_k \hat{B}_k \hat{C}_k = \begin{bmatrix} \frac{\partial \theta_S}{\partial \lambda_E} & \frac{\partial \theta_S}{\partial \theta_E} & \frac{\partial \theta_S}{\partial \gamma_{E_1}} & \frac{\partial \theta_S}{\partial \gamma_{E_2}} \end{bmatrix} \tag{126}$$

where

$$\frac{\partial \theta_S}{\partial \lambda_E} = \frac{\partial \theta_S}{\partial \lambda_S} \frac{\partial \lambda_S}{\partial \lambda_E} + \frac{\partial \theta_S}{\partial \phi_S} \frac{\partial \phi_S}{\partial \lambda_E} \tag{127}$$

$$\frac{\partial \theta_S}{\partial \theta_E} = \frac{\partial \theta_S}{\partial \lambda_S} \frac{\partial \lambda_S}{\partial \theta_E} + \frac{\partial \theta_S}{\partial \phi_S} \frac{\partial \phi_S}{\partial \theta_E} \tag{128}$$

$$\frac{\partial \theta_S}{\partial \gamma_{E_1}} = \frac{\partial \theta_S}{\partial \lambda_S} \frac{\partial \lambda_S}{\partial \gamma_{E_1}} \tag{129}$$

$$\frac{\partial \theta_S}{\partial \gamma_{E_2}} = \frac{\partial \theta_S}{\partial \lambda_S} \frac{\partial \lambda_S}{\partial \gamma_{E_2}} \tag{130}$$

The term [illegible]

$$[illegible] \tag{131}$$

[illegible]

[illegible]

$$[illegible] \tag{132}$$

where

$$[illegible] \tag{133}$$

$$[illegible] \tag{134}$$

For the navigation error effects,

$$[illegible] \tag{135}$$

where

$$[illegible] \tag{136}$$

Also

$$[illegible] \tag{137}$$

and

$$[illegible]$$

[illegible]

$$[illegible] \tag{138}$$

$$[illegible] \tag{139}$$

$$[illegible] \tag{140}$$

$$\frac{\partial \theta_c}{\partial \lambda} = \frac{\partial \theta_c}{\partial \lambda_c} \frac{\partial \lambda_c}{\partial \lambda} + \frac{\partial \theta_c}{\partial \lambda_c} \frac{\partial \lambda_c}{\partial \lambda} \tag{141}$$

$$\tag{142}$$

$$\tag{143}$$

The resultant of down-range and cross-range errors is

$$\tag{144}$$

Since cross is always positive, and probability density function is one-sided, and it has a non-zero mean value. Thus

$$\tag{145}$$

However, note that the r.m.s. value is given simply by

$$\tag{146}$$

For adequately large ratios of the semi-major and to semi-minor axis of the dispersion ellipse, the 99% probability value of $\theta_c$ is approximately 5 times the r.m.s. value.

[illegible] Variation of cross effects

The down-range error due to operation effects is given by

$$\tag{147}$$

$$\tag{148}$$

where

$$\tag{149}$$

$$\tag{150}$$

In similar fashion,

$$\overline{\delta s_c}\Big]_c = \left(\frac{\partial \theta_c}{\partial \alpha}\right)^2 \delta(\alpha\alpha) + \frac{\partial \theta_c}{\partial \beta}\,\delta(\beta) \tag{150}$$

where

$$\frac{\partial \theta_c}{\partial \alpha} = \frac{\partial \theta_c}{\partial \alpha}\left[\frac{\partial \theta_c}{\partial \alpha}\,\frac{\partial \theta_c}{\partial \alpha} + \frac{\partial \theta_c}{\partial \beta}\,\frac{\partial \theta_c}{\partial \alpha}\right] \tag{151}$$

$$\frac{\partial \theta_c}{\partial \beta} = \frac{\partial \theta_c}{\partial \beta}\left[\frac{\partial \theta_c}{\partial \alpha}\,\frac{\partial \theta_c}{\partial \beta} + \frac{\partial \theta_c}{\partial \beta}\,\frac{\partial \theta_c}{\partial \beta}\right] \tag{152}$$

and $\frac{\partial \theta_c}{\partial \beta}$ is given by eq. 65. Thus

$$\overline{\delta s_c}\Big]_c = \left(\frac{\partial \theta_c}{\partial \alpha}\right)^2 \delta(\alpha\alpha) + \frac{\partial \theta_c}{\partial \beta}\,\delta(\beta) \tag{153}$$

The [illegible] value of the [illegible]

$$\sqrt{\frac{\partial \theta_c}{\partial \alpha}} = \sqrt{\frac{\partial \theta_c}{\partial \alpha}} \tag{154}$$

It is to be [illegible] the [illegible] factor in eq. 149 is the velocity magnitude [illegible] the [illegible] in eq. 150 is the out-of-plane thrust direction [illegible].

6) [illegible]

[illegible] in [illegible] of the two [illegible]

$$\tag{155}$$

[illegible]

By definition, the [illegible] of the [illegible] components in

[illegible] (158)

whence

[illegible] (159)

Reference to Fig. [illegible] [illegible] [illegible] [illegible] [illegible].

## APPENDIX I

### Nominal Conditions - Equations & Numerical Values

A. Heliocentric Data

For May 19, 1971 launch date, $T_1$ = 200 days, the Julian day of encounter is 2441290.5, from the Planetary Ephemerides, at encounter,

$$\lambda_{M_1} = 29.49[illegible]^\circ, \quad \beta_{M_1} = [illegible]$$

Also for 1972 encounter (see Fig. I-1)

$$\lambda_{M_2} = 1.93^\circ, \quad \lambda_{M_2} = [illegible]^\circ, \quad \lambda_{M_2} = 25.2^\circ, \quad \lambda_{M_2} = 41.[illegible]^\circ$$

Then

$$[illegible] \qquad \text{(I-1)}$$

$$[illegible] \qquad \text{(I-2)}$$

$$\lambda_{M_2} = 54[illegible]^\circ$$

B. Pre Aerocentric Arc

From [illegible] data [illegible] compact to Mars orbit plane is [illegible]

$$[illegible]$$

Then, from Fig. [illegible]

$$[illegible] \qquad \text{(I-3)}$$

$$[illegible]$$

Also [illegible]

It can be shown [illegible] by

$$\tan \ldots \qquad (1\text{-}5)$$

[illegible]

[illegible]

Given

[illegible]

Then

[illegible]

Also,

[illegible]

and

[illegible]

Given

[illegible]

D. Approach Plane Orientation

From Fig. [illegible], or from reference 4, for sun [illegible] landing, and for $\delta_T = [illegible]$ (Landing of [illegible] Fraturn)

$$[illegible] \qquad (I\text{-}28)$$

$$[illegible] = 180^\circ$$

$$[illegible] = 84.01^\circ$$

where $[illegible] = 127.6^\circ$

Also, from Fig. [illegible]

$$[illegible] \qquad (I\text{-}29)$$

$$[illegible] = 180^\circ$$

$$[illegible] = [illegible]$$

Also,

$$[illegible] \qquad (I\text{-}30)$$

$$[illegible]$$

$$[illegible] = [illegible]$$

# [illegible]

## [illegible]

[illegible]

[illegible]

[

$C_{11} = \frac{\partial Z_s}{\partial Z_s} = 0.015$ lbs/lbm $\qquad C_{1\delta} = \frac{\partial Z_s}{\partial \delta} = -.993$ lbs/lbm

$C_{21} = \frac{\partial \theta_s}{\partial Z_s} = 1.00 \times 10^{-5}$ degrees/lbm $\qquad C_{2\delta} = \frac{\partial \theta_s}{\partial \delta} = .75 \times 10^{-5}$ degrees/lbm

$C_{31} = \frac{\partial \dot{\theta}_s}{\partial \dot{Z}_s} = 5.1 \times 10^{-6} \frac{\text{lbs/sec}}{\text{lbs/sec}}$ $\qquad C_{3\delta} = \frac{\partial \dot{\theta}_s}{\partial \dot{\delta}} = 1.0 \frac{\text{lbs/sec}}{\text{lbs/sec}}$

$C_{41} = \frac{\partial \theta_{ss}}{\partial Z_s} = 0.5 \times 10^{-6}$ degrees/lbm $\qquad C_{4\delta} = \frac{\partial \theta_{ss}}{\partial \delta} = .75 \times 10^{-6} \frac{\text{degrees}}{\text{lbm}}$

$C_{45} = \frac{\partial \theta_{ss}}{\partial \dot{Z}_s} = 9.2$ degrees/(lbs/sec) $\qquad C_{4\dot{\delta}} = \frac{\partial \theta_{ss}}{\partial \dot{\delta}} = .104 \frac{\text{degrees}}{(\text{lbs/sec.})}$

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$\frac{\partial \theta_t}{\partial Z_s} = -0.5$ degrees/lbm $\qquad \frac{\partial \theta_t}{\partial \delta} = 5 \times 10^{-6}$ degrees/lbm

$\frac{\partial \theta_t}{\partial \dot{Z}_s} = 1.5$ degrees/(lbs/sec.) $\qquad \frac{\partial \theta_t}{\partial \dot{\delta}} = .09$ degrees/(lbs/sec.)

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$\frac{\partial \dot{\theta}_t}{\partial Z_s} = 0.0 \times 10^{-3} \frac{\text{lbs/sec}}{\text{lbs/lbm}}$ $\qquad \frac{\partial \dot{\theta}_t}{\partial \delta} = 0.93 \times 10^{-6} \frac{(\text{lbs/sec.})}{\text{lbs.}}$

$\frac{\partial \theta_{ss}}{\partial Z_s} = 10.2$ lbs/(lbs/sec) $\qquad \frac{\partial \theta_{ss}}{\partial \dot{\delta}} = 0.7 \times 10^{-3}$ deg s/lbs

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$\frac{\partial \dot{\theta}_t}{\partial Z_s} = 0.0$ deg/(lbs/sec) $\qquad \frac{\partial \dot{\theta}_t}{\partial \dot{\delta}} = 0.05$ deg s/deg

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$\frac{\partial \theta_t}{\partial \dot{\theta}_t} = 0.0 \times 10^{-3}$ s/lb $\qquad \frac{\partial \theta_t}{\partial \dot{\theta}_t} = 0.9 \times 10^{-9}$ (lbs/sec.)

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$\frac{\partial \theta_t}{\partial \dot{\theta}_t} = 0.0 \times 10^{-3}$ $\qquad \frac{\partial \theta_t}{\partial \dot{\theta}_t} = 0.0 \times 10^{-3}$

$\frac{\partial \theta_c}{\partial \Delta V_s} = 163.7$ deg./(km/sec.) ; $\frac{\partial \theta_c}{\partial \beta} = .010$ deg. s/deg.

$\frac{\partial \theta_c}{\partial \delta} = .439$ deg's/deg.

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|---|---|
| $\frac{\partial \theta_c}{\partial X'} = 2.67 \times 10^{-4}$ deg's/km. | $\frac{\partial \theta_c}{\partial Y'} = 1.53 \times 10^{-4}$ deg's/km. |
| $\frac{\partial \theta_c}{\partial \dot{X}'} = -162.6$ deg's/(km/sec.) | $\frac{\partial \theta_c}{\partial \dot{Y}'} = 18.0$ deg's/(km/sec.) |
| $\frac{\partial \theta_c}{\partial V_s} = -5.46 \times 10^{-4}$ deg's/km. | $\frac{\partial \theta_c}{\partial \dot{V}_s} = 164.6$ deg's/(km/sec.) |
| $\frac{\partial \theta_c}{\partial \beta} = 1$ deg/deg. | $\frac{\partial \theta_c}{\partial \delta} = 57.6$ deg's/deg. |
| $\frac{\partial \theta_c}{\partial \Delta V_s} = -1910$ deg's/(km/sec.) | $\frac{\partial \theta_c}{\partial \beta} = .067$ deg's/deg. |
| $\frac{\partial \theta_c}{\partial X'} = .0249$ deg's/km. | $\frac{\partial \theta_c}{\partial Y'} = .00129$ deg's/km. |
| $\frac{\partial \theta_c}{\partial \dot{X}'} = 1670$ deg's/(km/sec.) | $\frac{\partial \theta_c}{\partial \dot{Y}'} = 162.3$ deg's/(km/sec.) |