Criterion of Instability in Top Heavy Layers of Fluids. - S.L. Malurkar OP
(Abstract)

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The problem of thermal instability in a fluid layer leading to vigorous convection is dependent on the temperature gradient with height, its thermal conductivity, its viscosity the specific heat at constant pressure and the density of the fluid. In general these quantities are taken to be constants and criteria depends on a Rayleigh number containing these quantities. It is interesting to examine the nature of changes introduced when these ~~xxx xx~~ can not be considered as constants. The simple one when the temperature gradient with height is varying, has applications to the lower levels of the air near a heated ground. Malurkar and Ramdas explained the hyperbolic sine temperature height curve by taking account of radiative transfer of heat with water vapour layers as absorbing and emitting entity. This led later for an examination of the stability problem with the incorporation of this type of temperature height curve (1937c) and methods of solution of such differential equations (Malurkar. ~~1x~~ 1937b). The work has been continued for other structures of the temperature height curves (exponential decrease and sine type decrease). When the curve is concave upwards the criterion is increased and when it is convex upwards decreased from the the one for constant temperature gradient.

A reference is made to the earlier method of solving the stability type differential equations with which the corresponding problems with superposition of a magnetic field and Coriolis forces and non-linear temperature height curves have been investigated.

The question of varying the density which ~~xxxxxxxxx~~ were needed for dynamics of Thunderstorms was solved ~~(1937a,1943)~~ by assuming Fick's equation for diffusion instead of the one for heat transport. A criterion similar to the one by Rayleigh could be had (Malurkar. 1937a, 1943).

The variations in the so called constant values in para. 1. lead to useful results. The work connected with varying conductivity and viscosity may not be as important though the field of investigation could be pushed through.

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CRITERION of INSTABILITY in TOP HEAVY LAYERS of FLUIDS

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The problem of thermal instability or of that leading to vigorous convection in a fluid heated from below is dependent on the temperature gradient with height β, the thermal conductivity k and the viscosity ν of the fluid, T the thickness of the layer, C_p the specific heat at constant pressure and ρ the density of the fluid given by the usual formula

$$R = \beta g \rho C_p T^4 / k \nu \theta_0 \pi^4 < K \qquad \ldots (1)$$

K depending on the particular set of boundary conditions. g is the value of gravity and θ_0 is a fixed temperature. In the case of liquids $1/\theta_0$ is replaced by its coefficient of thermal expansion. In most of the discussions, it is assumed that all these quantities are constants. Here, the objective is to discuss the consequences when the temperature gradient is not constant and also refer briefly to the case of variable density.

The problem of thermal instability arose with the classical experiments of Benard (1900,1901) who heated thin layers of liquids from below. Initially the liquid surface broke up into almost regular polygons each with its own convective system. After the temperature of the lower surface exceeded that of the top one by a definite amount, vigorous upturning of the layers occurred. The theoretical deductions by Aichi (1907) and Reyleigh (1916) were based on the hydrodynamical equation of motion, equation of continuity, equation of state and the equation of heat transport (or conduction based on Boussinesq). Assuming that the departures of quantities from steady conditions values were small enough that their squares and products could be neglected, it was shown that the problem depended on a sixth order linear differential equation with constant

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coefficients.

$$\left(d^2/dz^2 - a^2\right)^3 \theta = -Ra^2\theta \qquad \ldots\ldots (2)$$

Rayleigh assumed that both top and bottom layers were Free, Jeffreys (1928) assumed that both the layers were Rigid and Low considered the case of one being Free and the other Rigid.

When dealing with gas layers, it is not possible to consider the quantities in para.1. as all constant.

In the lower levels of the atmosphere, it has been usual after G.I.Taylor(1915) to employ coefficients of conductivity and/of viscosity derived from the motion of eddies instead of the much smaller molecular ones. The equation of heat transport in one dimension can be written as

$$\rho C_p \, d\theta/dt = k \frac{d^2\theta}{dz^2} \quad \text{or} \quad \frac{d\theta}{dt} = K_E \frac{d^2\theta}{dz^2} \qquad \ldots(3)$$

Brunt and Low (1925) obtained the maximum temperature gradient with height in the lower layers of the atmosphere which was, however, much below the observed values. Simpson(1928) had taken account of stratified layers of water vapour in absorbing and emitting long wave heat radiation in the atmosphere. Brunt(1929), taking account of the radiative transfer by water vapour modified the equation of heat transport by adding to the eddy conductivity term one due to radiation (increasing the value of eddy diffusivity term $K_E + K_R$ instead of K_E). To explain the hyperbolic sine temperature height structure near a heated ground which was producing inferior mirages, Malurkar and Ramdas (1932) showed that the effect of radiative transfer in the stratified layers of water vapour added a term distinct from that of the conductivity one in the equation of heat transport. Under steady conditions, the ordinary type of eddy conductivity or heat transport equation would have led to a constant temperature gradient. If the effect of radiative transfer due to absorbing layers is taken as distinct from that due to ordinary or eddy type of conductivity and this is expanded in terms of φ where $\theta = \theta_0 + \varphi$, the equation of heat transport could be written as

$$\rho C_p \, d\varphi/dt = k\left(\nabla^2\varphi - \alpha^2\varphi\right) \qquad \ldots(4)$$

This linear differential equation could not be solved by the 'ad hoc' method of Jeffreys and could not take advantage of symmetry of form allowing Low's analysis. A method of solving such instability equations was put for[illegible] (Malurkar. 1937b) in substituting an assumed expansion in terms of orthogonal functions only in a part of the differential equation (see later). ~~After solving compl[illegible] the modified equation under necessary boundary conditions, the solution co[illegible] expanded in terms of same set of orthogonal functions as before and as a condition of consistency, the coefficients of the expansion could be had explicitly.~~ A sine fourier expansion for $\chi = \sum_{s=1} P_s \sin s\xi$ was substituted to obtain the modified equation

$$\left(d^2/d\xi^2 - a^2\right)^2 \operatorname{Sech} r\xi \left(d^2/d\xi^2 - b^2\right)\chi = -\frac{Ra^2\pi r}{\sinh \pi r}\sum P_s \sin s\xi \quad \ldots (8)$$

This equation was solved taking $\operatorname{Sech} r\xi \left(d^2/d\xi^2 - b^2\right)\chi$ as a single variable. The ~~[illegible]~~ consistency condition for the coefficients of the expansion led to an infinite determinant, whose elements could be evaluated, being equated to zero, both for Rayleigh's type and for Jeffrey's types of boundary conditions. The infinite determinant could be used to solve for the discrete values of Ra^2 and in any particular case by successive approximations. The infinite determinant broke up into two - odd and even solutions of the problem - when the the tendency was to approach constant temperature gradient or $r = 0$. Then the smallest odd solution would have to be chosen.

To examine the nature of change introduced by the non-linearity in temperature height curve, the Rayleigh boundary problem is the simplest and ~~[illegible]~~ neglecting r^4, we have $Ra^2 = (1+a^2)^3\left\{1 + \frac{(5+a^2)r^2}{4(1+a^2)}\right\}$

or $$\frac{\beta g \rho c_p T^4}{k \nu \theta_0 \pi^4} = \frac{1}{a^2}(1+a^2)^3\left\{1 + \frac{(5+a^2)r^2}{4(1+a^2)}\right\}$$

In Rayleigh's problem, the value of $a^2 = 1/2$, To the approximation of r^4, it is sufficient to substitute $a^2 = 1/2$ because the more accurate $a^2 = 1/2 + \frac{1}{6}r^2$ value does not change R to the same approximation. ~~This showed that~~ The limit of Rayleigh criterion was increased because of the departure from linearity to a hyperbolic sine curve and was an increasing function of such departure, in the temperature height curve

While considerable work has appeared on thermal instability of fluid layers, (see references in Sutton.1950; Backus.1955; Lin.1955; Chandrasekhar.1961) they mostly deal with constant gradient of temperature with height in the fluid layers. Following the previous method, detailed calculations have been made to obtain the criteria of instability when the temperature decreased exponentially with height:

With practically same type of notation and stages of working as before the basic differential equation of sixth order becomes

$$(d^2/d\xi^2 - a^2)^2 \, \varepsilon^{-r\xi} \left(d^2/d\xi^2 - b^2\right)\chi = -\frac{Ra^2\pi r}{(\varepsilon^{\pi r}-1)}\chi \qquad \ldots (10)$$

The method of solution was similar to the one used earlier (Malurkar.1937b, 1937c). In the interval $0 < \xi < \pi$ it is assumed that $\chi = \sum_{s=1} P_s \sin s\xi$ and substituted only on the right hand side of equation (10) resulting in:

$$(d^2/d\xi^2 - a^2)^2 \, \varepsilon^{-r\xi} \left(d^2/d\xi^2 - b^2\right)\chi = -\frac{Ra^2\pi r}{(\varepsilon^{\pi r}-1)} \sum_s P_s \sin s\xi \qquad \ldots (11)$$

By taking $\psi = \varepsilon^{-r\xi}\left(d^2/d\xi^2 - b^2\right)\chi$ as a single variable to start with, the equation can be solved <u>completely</u>, though to determine the criterion of instability some steps could be shortened under usual boundary conditions. The consistency conditions can be taken from equation (12) itself. However if more complicated boundary conditions are involved, the complete solution of χ would have to be obtained

$$\begin{aligned}(d^2/d\xi^2 - b^2)\chi = {} & 2\{A_0 + B_0(\pi/2-\xi)\}\,\varepsilon^{r\xi}\cosh a(\pi/2-\xi) \\ & + 2\{A_1 + B_1(\pi/2-\xi)\}\,\varepsilon^{r\xi}\sinh a(\pi/2-\xi) \\ & - \frac{Ra^2\pi r}{(\varepsilon^{\pi r}-1)} \sum_s \frac{P_s\,\varepsilon^{r\xi}\sin s\xi}{(s^2+a^2)^2} \qquad \ldots (12)\end{aligned}$$

The usual boundary conditions that have been taken are:

a) $\chi;\ \psi;\ d^2\psi/d\xi^2 = 0$ at $\xi = 0$ and at $\xi = \pi$. (Rayleigh Type)

b) $\chi;\ \psi;\ d\psi/d\xi = 0$ at $\xi = 0$ and at $\xi = \pi$. (Jeffreys Type)

c) $\chi;\ \psi;\ d^2\psi/d\xi^2 = 0$ at $\xi = 0$ and
$\chi;\ \psi;\ d\psi/d\xi = 0$ at $\xi = \pi$ (Lower Chandrasekhar Type)

$\ldots (13)$

The consistency condition taking account of these boundary conditions lead in each case to an infinite determinant with ns the element J_{ns} and n the diagonal element $J_{nn} - 1/Ra^2$ being equated to zero. The v[illegible] for the three types are given here:.

Rayleigh type boundary conditions (Both Free)

$$J_{ns} = \frac{4r^2 ns\left(\cos n\pi \cos s\pi\, e^{\pi r} - 1\right)/\left(e^{\pi r} - 1\right)}{(n^2+b^2)\left\{(n^2-s^2+r^2)^2 + 4r^2s^2\right\}(s^2+a^2)^2} \quad \ldots (2[illegible]$$

Low or Chandrasekhar type boundary conditions (One Free and the other Rigid)

$$(n^2+b^2)(s^2+a^2)J_{ns} = \frac{4r^2 ns\left(\cos n\pi \cos s\pi\, e^{\pi r} - 1\right)/\left(e^{\pi r} - 1\right)}{\left\{(n^2-s^2+r^2)^2 + 4r^2s^2\right\}}$$

$$+ \frac{r\, ns \cos s\pi \sinh a^2 \pi/2}{\left(e^{\pi r} - 1\right)\left(a\pi \cosh a\pi - \sinh a\pi\right)}\Bigg[$$

$$\frac{2\pi a r\left(1+\cos n\pi\, e^{\pi r}\right)}{\left\{(n^2+b^2)^2 - 4a^2r^2\right\}} + \left(\cosh a\pi + 1\right)\left(1 - \cos n\pi\, e^{\pi r}\right)\left\{\frac{(r-a)}{(n^2+b^2-2ar)^2} - \frac{(r+a)}{(n^2+b^2+2ar)^2}\right\}$$

$$+ \sinh a\pi\left(1+\cos n\pi\, e^{\pi r}\right)\left\{\frac{(r-a)}{(n^2+b^2-2ar)^2} + \frac{(r+a)}{(n^2+b^2+2ar)^2}\right\}\Bigg]$$

$$- \frac{r\, ns \cos s\pi \cosh a^2\pi/2}{\left(e^{\pi r} - 1\right)\left(a\pi \cosh a\pi - \sinh a\pi\right)}\Bigg[\frac{2\pi a r\left(1-\cos n\pi\, e^{\pi r}\right)}{\left\{(n^2+b^2)^2 - 4a^2r^2\right\}}$$

$$- \left(\cosh a\pi - 1\right)\left(1+\cos n\pi\, e^{\pi r}\right)\left\{\frac{(r-a)}{(n^2+b^2-2ar)^2} - \frac{(r+a)}{(n^2+b^2+2ar)^2}\right\}$$

$$- \sinh a\pi\left(1-\cos n\pi\, e^{\pi r}\right)\left\{\frac{(r-a)}{(n^2+b^2-2ar)^2} + \frac{(r+a)}{(n^2+b^2+2ar)^2}\right\}\Bigg]$$

$$\ldots (25)$$

Jeffreys type (Both Rigid):

$$(n^2+b^2)(s^2+a^2)^2\, J_{rs} =$$

$$\frac{4r^2 rs\,(\cos n\pi \cos s\pi\, e^{\pi r}-1)/(e^{\pi r}-1)}{(n^2-s^2+r^2)^2+4r^2s^2}$$

$$+\frac{rns\,(1-\cos s\pi)}{(e^{\pi r}-1)(\sinh a\pi+a\pi)}\left[\frac{2\pi a r\,(1+\cos n\pi\, e^{\pi r})}{\{(n^2+b^2)^2-4a^2r^2\}}\right.$$

$$+(\cosh a\pi+1)(1-\cos n\pi\, e^{\pi r})\left\{\frac{(r-a)}{(n^2+b^2-2ar)^2}-\frac{(r+a)}{(n^2+b^2+2ar)^2}\right\}$$

$$\left.+\sinh a\pi\,(1+\cos n\pi\, e^{\pi r})\left\{\frac{(r-a)}{(n^2+b^2-2ar)^2}+\frac{(r+a)}{(n^2+b^2+2ar)^2}\right\}\right]$$

$$-\frac{rns\,(1+\cos s\pi)}{(e^{\pi r}-1)(\sinh a\pi-a\pi)}\left[\frac{2\pi a r\,(1-\cos n\pi\, e^{\pi r})}{\{(n^2+b^2)^2-4a^2r^2\}}\right.$$

$$-(\cosh a\pi-1)(1+\cos n\pi\, e^{\pi r})\left\{\frac{(r-a)}{(n^2+b^2-2ar)^2}-\frac{(r+a)}{(n^2+b^2+2ar)^2}\right\}$$

$$\left.-\sinh a\pi\,(1-\cos n\pi\, e^{\pi r})\left\{\frac{(r-a)}{(n^2+b^2-2ar)^2}-\frac{(r+a)}{(n^2+b^2+2ar)^2}\right\}\right] \quad \ldots (16)$$

The consistency condition of the infinite determinants vanishing, is satisfied only for discrete values of Ra^2. In any particular case, the solution for Ra^2 can be done by successive approximations. The elements of the determinant decrease rapidly with increase of n or of s. As in the case of hyperbolic sine temperature height curve, the determinant breaks up when $\gamma = 0$ into two portions corresponding to even and odd solutions. The smallest root of Ra^2 has to be chosen and in the degenerate case when $\gamma = 0$, the odd solution has to be selected.

Among the term values, Rayleigh type boundary conditions given simplest forms. For them, the diagonal elements of the infinite determinant have the same form for the hyperbolic sine and exponential temperature height curves. Of course for a given difference of temperature between top and bottom layers, the value of γ are different. Similarly to a first approximation, neglecting γ^4, the criterion of maximum possible temperature difference between top and bottom layers before onset of instability would be same for the two above curves in the Rayleigh type case:

$$R \doteq \frac{(1+a^2)^3}{a^2}\left\{1+\frac{(5+a^2)\gamma^2}{4(1+a^2)}\right\} \qquad (11)$$

The value to be taken for a^2 is $\frac{1}{2}+\frac{1}{8}\gamma^2+O(\gamma^4)$ leading to $R = \frac{27}{4}\left(1+\frac{11}{12}\gamma^2\right)$ neglecting γ^4, which is the same if the value of $a^2 = \frac{1}{2}$ had been taken from the constant temperature gradient case. The value of R is greater in value than in the constant gradient of temperature case.

The general conclusions about increase in the possible maximum temperature between top and bottom layers are true for other type of boundary conditions also.

Temperature decrease Convex upwards: As a representative case where the temperature height decrease curve is convex upwards, the variational part of the temperature is taken as

$$\beta T \sin \pi r (1 - z/T)/\sin \pi r \quad \text{or} \quad \beta T \sin \pi r^2/\sin \pi r \qquad (12)$$

where γ^2 is a constant but small $\gamma^2 \ll 1$. The corresponding basic equation becomes

$$(d^2/d\xi^2 - a^2)^2 \{ \sec r\xi (d^2/d\xi^2 - a^2 + r^2)\chi \} = - \frac{Ra^2\pi r\chi}{\sec r\xi}$$

The equation can be solved exactly as before for different type boundary conditions. It is sufficient to point out that to a first approximation neglecting γ^4 the value of

$$R = \frac{(1+a^2)^3}{a^2} \{ 1 - (5+a^2)/4(1+a^2) \} = \frac{27}{4}\left(1 - \frac{11}{12}r^2\right) \quad \ldots (1)$$

where a^2 can be put as $\frac{1}{2}$ or $\frac{1}{2} - \frac{1}{6}r^2$ without changing the value. This limit is less than when $\gamma = 0$.

The instances when the curve of temperature height is convex upwards may be very rare, but whenever it is so the criterion of instability is less than in the constant temperature gradient one. It follows that for the criterion of instability of a layer of heated fluid particularly in the gasses, the nature of temperature height curves are important. While the temperature difference might be within the limits of criteria of stability for one temperature height structure, it may not be so for another one.

In general, the criterion of instability is greater when the temperature height curve is concave upwards and less when it is convex upwards than when the curve is a linear one (Malurkar. 1939,

High level turbulence in the troposphere noticed even without superadiabatic lapse rates may be due to changes in the structure of temperature height curve there.

Varying Density : Sometimes top heaviness in a fluid may arise when a lighter stream undercuts a relatively heavier one. The incursion of moist air below a drier one could be amgined to give rise to instability and the argument would be appropriate in the case of dynamics of thunderstorms. The equation of heat transport was replaced by equation of diffusion (Fick's), which of course gives a linear change in density. Under simple boundary conditions, the problem can be solved just as in the case of thermal instability. (Malurkar. 1937a, 1943). The problem of varying density has been studied recently (Chandrasekahr. 1961. p. 456, 478)

Stability type Differential Equations: When a practical method of solving the stability type of differential equation arose, when the inclusion of the temperature height structure was introduced into Rayleigh type investigation, Malurkar (1937b) put forward separating a linear differential equation into two suitable parts. An assumed series in terms of fourier or other orthogonal set of functions was substituted in only one part (the one with smaller order of differential symbol). The modified equation was then completely solved taking account of boundary conditions. The solution now obtained was compared with the original assumption in series to explicitly evaluate its successive coefficients

The Rayleigh type equation

$$\left(d^2/d\xi^2 - a^2\right)^3 \chi = -Ra^2\chi$$

exhibits a good degree of symmetry. The introduction of several parameters, representing external forces or non-uniformity in them should lead one to less degenerate equations. As a less degenerate form it could be generalised as: $-Ra^2\chi =$

$$\frac{1}{\phi_1}\left(d/d\xi - a_1\right)\frac{1}{\phi_2}\left(d/d\xi - a_2\right)\frac{1}{\phi_3}\left(d/d\xi - a_3\right)\frac{1}{\phi_4}\left(d/d\xi - a_4\right)\frac{1}{\phi_5}\left(d/d\xi - a_5\right)\frac{1}{\phi_6}\left(d/d\xi - a_6\right) \quad \ldots (20)$$

where $a_1, a_2 \ldots a_6$ could be considered as functions of a^2, and $\phi_1, \phi_2 \ldots \phi_6$ are functions of ξ only. The simple case or generalisation with constant coefficients:

$$\left(d^2/dx^2 - a^2\right)\left(d^2/dx^2 - b^2\right)\left(d^2/dx^2 - c^2\right)y = -\lambda a^2 y$$

was the first application of the method (Malurkar and Srivastava. 1937). When the temperature height structure has to be taken account of ϕ_5 is a function of ξ Still there is a fair degree of symmetry in the first four factors.

Recently the author has tried to extend Chandrasekhar's equations for thermal instability with superposed magnetic fields taking account of non-linear temperature distribution. The basic differential equation at the stage of ordinary instability is found to be with a hyperbolic sine temperature height curve:

$$\left(d^2/d\xi^2 - a^2\right)\left[\left\{\left(d^2/d\xi^2 - a^2\right)^2 - 4M^2 d^2/d\xi^2\right\}\left\{\operatorname{Sech}\pi\xi\left(d^2/d\xi^2 - b^2\right)\chi + Ra^2\pi r \operatorname{Cosech}\pi r\,\chi\right]\right. = 0 \quad \ldots (21)$$

$$\phi_1 = \phi_2 = \phi_3 = \phi_4 = \phi_6 = 1; \quad a_1 = a_3 = a = -a_4 = -a_2$$

$$a_5 = -a_6; \quad a_3^2 = a^2 + r^2 \quad \text{(in particular)}$$

which could be factorised as

$$\left(d^2/d\xi^2 - a^2\right)\left[(D-M-N)(D-M+N)(D+M+N)(D+M-N)\operatorname{sech} r\xi\,(D^2+b^2)\chi + Ra^2\pi r\,\operatorname{cosech}\pi r\,\chi\right] = 0 \quad \cdots (22)$$

The corresponding application to the one with fluid rotation leads to the [illegible] differential equation (for the hyperbolic sine temperature height curve)

$$\left\{\left(d^2/d\xi^2 - a^2\right)^3 + A^2 d^2/d\xi^2\right\}\left\{\operatorname{sech} r\xi\left(d^2/d\xi^2 - a^2 - r^2\right)\chi\right\} + \frac{Ra^2\pi r}{\sinh \pi r}\left(d^2/d\xi^2 - a^2\right)\chi = 0 \quad \cdots (23)^*$$

The method outlined (Malurkar.1937b) is a ~~[illegible]~~ useful workin approach ~~[illegible]~~ to the more rigorous, ~~[illegible]~~ methods of solving the equations ~~[illegible]~~.(Chandrasekhar.1961 ;Backus,1955,Lin.1955) and do not contradict one ~~[illegible]~~ another.

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* these investigations for non-linear temperature height curves have been carried out and are expected to be published early.

It has been found that in all these equations the criterion of ordinary stability increased when the temperature height curve was concave upwards and decreased when it was convex upwards from the [illegible] Chandrasekhar values obtained for constant temperature gradient.

In general, the criterion of ordinary instability, if it exists (not confined to planar strata) increases when the equation of heat transport is taken as $\rho C_p \, d\varphi/dt = k(\nabla^2\varphi - \mathcal{F}(\varphi))$ and decreases when it is taken as $\rho C_p \, d\varphi'/dt = k(\nabla^2\varphi + \mathcal{F}(\varphi)); \mathcal{F}(\varphi) > 0$ from the value derived from the usual thermal conductivity type of equation

$$\rho C_p \, d\varphi'/dt = k(\nabla^2\varphi)$$

If it is possible to change the temperature height structure, after a certain difference of temperature has been kept fixed, stable for one structure, it could be made unstable for another or vice versa.

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