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ABSTRACT

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Equilibrium solutions for the continuity equation for atomic oxygen ions in the upper F region in terms of exponential series are developed in which account is taken of diffusion, of the effects of a non-divergent vertical flux of ionization, attenuation of the solar radiation, and lack of temperature equilibrium between the electrons and ions. Above the maximum these series converge rapidly and they are well approximated by the first few terms. In this way simple solutions of the continuity equation are developed which allow experimentally observed ion or electron density profiles to be related to the production, loss and transport processes. Modified solutions which include the effect of helium and hydrogen ions on the electron density profile are developed.

The solutions are used to investigate the effects of assuming different neutral atmospheric models on daytime equilibrium profiles. The implications of the solutions to the analysis of experimental profiles is discussed.

1. Introduction

The upper F region is becoming increasingly available to study by means of rockets, satellites and incoherent scatter sounders. In particular, the ionospheric topside sounder satellites provide consecutive measurements of electron density profiles of the top of the layer.

It is of interest to relate these measurements to the basic physical processes of ion production, recombination and transport, to the temperatures, and to the relative densities of the ionic constituents.

Yonezawa (1956, 1958) developed solutions of the continuity equation in the F2 region under daytime equilibrium conditions. The solutions were in terms of somewhat complex integrals which were evaluated numerically.

Rishbeth and Barron (1960) and Gliddon and Kendall (1960) developed techniques for solving the continuity equation using digital computer techniques and have extended the method to non-equilibrium conditions.

Bowhill (1962) simplified the analysis by neglecting attenuation of the ionizing radiation by the atmosphere and obtained analytic solutions for the continuity equation for more general cases than those treated by Yonezawa and which gave results which agreed well with equilibrium solutions obtained by the computer methods.

This analysis technique was later extended, Bowhill (1962), to the cases of layers in which the inflow or outflow of ions from the layer controlled the ion distributions.

Analytical expressions have distinct advantages over computer solutions when it is desired to fit experimental profiles and when it is desired to investigate the range over which changes introduced in the theoretical assumption are effective. For these reasons the analytic approach has been chosen for the present study.

In this work exponential series solutions of the F-region continuity equation are derived in which account is taken of attenuation of the ionizing radiation by the atmosphere, and a scale height for the ionic species different from that for the neutral constituent controlling the diffusion is considered. Different scale heights for the molecular constituent responsible for the recombination, lack of temperature equilibrium between the electrons and ions, and a non-divergent vertical ion flux are also taken into account.

Above the maximum these series are well approximated by the first few terms and quite simple relations for the ion density as a function of height are obtained.

The effects of various assumptions about the atmospheric parameters on the height of the maximum ion density, the ion density at the maximum and the quarter thickness of a parabolic fit to the nose of the ion density profile are discussed.

At higher altitudes helium and hydrogen ions must be considered and modified equations are derived to include their effects on the electron density profiles.

2. Solution of the Continuity Equation

2.1 Region where Atomic Oxygen ions Predominate

The continuity equation for atomic oxygen in the F region may

be written

$$\frac{\partial n}{\partial t_z} = Q_z - L_z - \mathrm{div}(n_i v)_z \tag{1}$$

where

Q_z is the rate of ion production at height z

L_z is the rate of ion recombination at height z

v is the vertical velocity

z is the height measured from the ion density maximum

n_i is the ion density

2.2 Production Term

Recent experimental measurements of solar extreme ultraviolet photon fluxes described by Hinteregger and Watanabe (1962) have indicated that the radiation responsible for the ionization of atomic oxygen in the F region is not monochromatic and that the photon absorption rates are not independent of wavelength.

Under these conditions the production at height z can be represented by

$$Q_z = N(O)_z \, p_z \tag{2}$$

where N(O) is the atomic oxygen density and p depends on the solar flux components and on the appropriate cross sections of atomic oxygen.

Due to absorption, p will decrease with decreasing altitude. Above about 150 km it appears that the experimental data can be fitted adequately for the present purpose by the approximate relation

$$p_z = p_\infty \left\{ 1 - k \sec\psi \int_z^\infty N(O)_m \left[\exp(1-a)\frac{z}{H}\right] dz \right\} \tag{3}$$

where $N(O)_m$ is the atomic oxygen density at the maximum of the ionization density, H is the scale height of atomic oxygen, a and k are empirically determined constants, and ψ is the solar zenith angle.

If the neutral atomic oxygen is assumed to follow a diffusion equilibrium distribution, equation (3) may be combined with equation (2) to give

$$Q_z = p_\infty N(O)_m \exp\left(-\frac{z}{H}\right) + \frac{kp_\infty N(O)_m^2}{(1-a)} H \sec\psi \exp\left(-\frac{az}{H}\right) \quad (4)$$

providing a is greater than one.

Equation (4) may be written

$$Q_z = q_m \exp\left(-\frac{z}{H}\right) - \rho_m \exp\left(-\frac{az}{H}\right) \quad (5)$$

where $q_m \equiv p_\infty N(O)_m$

$$\rho_m = \frac{kp_\infty N(O)_m^2 H \sec \psi}{(a-1)}$$

2.3 Loss Term

It will be assumed that the recombination process for atomic oxygen ions is by the atom ion interchange process

$$O^+ + XY \rightarrow OX^+ + Y \quad (6)$$

followed by dissociative recombination,

$$OX^+ + e \rightarrow O + X \quad (7)$$

and that in the upper F region the recombination rate will be dependent on the rate of the reaction in equation (6). Under these conditions the loss rate is given by

$$L_z = \gamma N(XY)_m \, n_i \, \exp\left(-\frac{bz}{H}\right)$$

or

$$L_z = \beta_m \, n_i \, \exp\left(-\frac{bz}{H}\right) \tag{8}$$

where

$$\beta_m \cong \gamma N(XY)_m$$

or

$$b = \frac{m(XY)}{m(O)} = \frac{H}{H(XY)} \tag{9}$$

For example if XY is presumed to be molecular oxygen then b would be 2.0 and if molecular nitrogen b would be 1.75. At lower altitudes the dissociative recombination process will predominate causing the loss rate L to be lower than would be predicted from equation (8). At these altitudes, however, the effect of diffusion transport is small and photo-equilibrium may be assumed, so that the effect will be confined to the lower F region.

2.4 Transport

The equation of motion for the ions is,

$$\frac{m_i \, \nu_{in} \, v_i}{\sin^2 I} = -m_i g + q E - \frac{kT_i}{n_i} \frac{\partial n_i}{\partial z} \tag{10}$$

and for the electrons is

$$\frac{m_e \, \nu_{en} \, v_e}{\sin^2 I} = -m_e g - q E - \frac{kT_e}{n_e} \frac{\partial n_e}{\partial z} \tag{11}$$

where I is the geomagnetic dip angle,

m_e and m_i are the electron and ion masses,

n_e and n_i are the electron and ion number densities which will be assumed equal,

v_e and v_i are the vertical diffusion velocities of the electrons and ions with respect to the neutral atmosphere,

ν_{en} and ν_{in} are the collision frequencies for the electrons and ions with neutral particles,

q is the charge on an electron,

k is Boltzman's constant,

g is the acceleration of gravity,

T_e and T_i are the equivalent electron and ion temperatures,

and E is the electric field.

Now $$m_e << m_i \,, \tag{12}$$

the ratio $\frac{\nu_{en}}{\nu_{in}}$ is only of the order of 40,

and $$v_e \simeq v_i \,,$$

therefore $$m_e \nu_{en} v_e << m_i \nu_{in} v_i \tag{13}$$

Substituting equations (11), (12), and (13) in equation (10) gives the ion diffusion velocity

$$v_i \approx - g \frac{\sin^2 I}{\nu_{in}} \left[1 + \frac{k(T_i + T_e)}{n_i m_i g} \frac{\partial n_i}{\partial z} \right] \tag{14}$$

It may be assumed that ν_{in} decreases exponentially with height within the region of interest so that, in general,

$$\nu_{in} = \nu_{in_m} \exp\left(- \frac{cz}{H} \right) . \tag{15}$$

In the general case the electron and ion temperatures may be different so that,

$$d \equiv \frac{T_i}{T_e + T_i} \tag{16}$$

may have a value lower than 0.5.

By defining a diffusion coefficient at the ion density maximum

$$D_m = \frac{g\,H\,\sin^2 I}{\nu_{in_m}\,d} \tag{17}$$

equation (14) may be written

$$n_i v_i = -D_m \exp\left(\frac{cz}{H}\right)\left[\frac{n_i d}{H} + \frac{\partial n_i}{\partial z}\right] \tag{18}$$

The divergence of the upward flux of ions assuming horizontal stratification is given by

$$\frac{\partial (n_i v_i)}{\partial z} = -D_m \exp\left(\frac{cz}{H}\right)\left[\frac{\partial^2 n_i}{\partial z^2} + \left(\frac{c+d}{H}\right)\frac{\partial n_i}{\partial z} + \frac{cdn_i}{H^2}\right] \tag{19}$$

An additional upward ion flux will be assumed

$$G = n_i v_t \tag{20}$$

such that

$$\frac{\partial G}{\partial z} = 0 \tag{21}$$

2.5 Exponential Series Solution

Substituting equations (5), (8) and (19) in equation (1) gives,

$$\frac{\partial n_i}{\partial t} = q_m \exp\left(-\frac{z}{H}\right) - \rho_m \exp\left(-\frac{az}{H}\right) - \beta_m n_i \exp\left(-\frac{bz}{H}\right)$$

$$+ D_m \exp\left(\frac{cz}{H}\right)\left[\frac{\partial^2 n_i}{\partial z^2} + \left(\frac{c+d}{H}\right)\frac{\partial n_i}{\partial z} + \frac{cd}{H^2}\, n_i\right] \tag{22}$$

Under equilibrium conditions when

$$\frac{\partial n_i}{\partial t} = 0$$

at all altitudes, equation (22) may be written

$$0 = \frac{\partial^2 n_i}{\partial z^2} + \left(\frac{c+d}{H}\right) \frac{\partial n_i}{\partial z} + \frac{cd}{H^2} n_i + \frac{q_m}{D_m} \exp\left(\frac{-(1+c)z}{H}\right)$$

$$- \frac{\beta_m}{D_m} n_i \exp\left(\frac{-(b+c)z}{H}\right) - \frac{\rho_m}{D_m} \exp\left(- \frac{(a+c)z}{H}\right) \qquad (23)$$

The solution to equation (23) may be obtained in terms of exponential series

$$n_i = \sum_m \sum_{n=o}^{\infty} B_{m+n(b+c)} \exp - \left(\frac{m+n(b+c)z}{H}\right) \qquad (24)$$

By substituting equation (24) in equation (23) it may be shown that equation (23) is satisfied with the following values of m,

$m = 1 + c$

$m = a + c$

and values of m which satisfy the equation

$$m^2 - (c+d)\, m + cd = 0 \qquad (25)$$

namely $m = c$, and $m = d$.

With these values of m substituted in equation (24),

$$n_i' = \sum_{n=o}^{\infty} \left\{ B_{d+n(b+c)} \exp - \left[\frac{d+n(b+c)}{H} \right] z \right.$$

$$+ B_{c+n(b+c)} \exp - \left[\frac{c+n(b+c)}{H} \right] z$$

$$+ B_{1+c+n(b+c)} \exp - \left[\frac{1+c+n(b+c)}{H} \right] z$$

$$\left. + B_{a+c+n(b+c)} \exp - \left[\frac{a+c+n(b+c)}{H} \quad z \right] \right\}. \qquad (26)$$

Equation (26) may be written

$$n_i = B_d S_1 + B_c S_2 + B_{1+c} S_3 + B_{a+c} S_4 \tag{27}$$

where S_1, S_2, S_3 and S_4 are series of exponential terms which are functions of z. By substituting equation (26) into equation (23) and comparing coefficients it may be shown that

$$B_{1+c} = - \frac{q_m H^2}{D_m} \frac{1}{1 + c - d} \tag{28}$$

$$B_{a+c} = \frac{\rho_m H^2}{D_m} \frac{1}{a(a+c-d)} \tag{29}$$

and the recursion relationship for the general coefficient B_k is given by

$$\frac{B_k}{B_{k-(b+c)}} = \frac{H^2 \beta_m}{D_m} \frac{1}{(k-c)(k-d)} \tag{30}$$

Three parameters of equation (27) remain to be determined, B_d, B_c, and $\frac{H^2 \beta_m}{D_m}$.

2.5.1 Location of the Ion Density Maximum

Equation (26) may be differentiated with respect to z to give the rate of change of ion density. The maximum is located where $\frac{\partial n_i}{\partial z} = 0$ so that, at the maximum

$$0 = - H \frac{\partial n_i}{\partial z} = \sum_{n=o}^{\infty} \Big\{ \Big[d + n(b+c) \Big] B_{d+n(b+c)} + \Big[c+n(b+c) \Big] B_{c+n(b+c)}$$

$$+ \Big[1+c+n(b+c) \Big] B_{1+c+n(b+c)} + \Big[a+c+n(b+c) \Big] B_{a+c+n(b+c)} \Big\} \tag{31}$$

and this may be written

$$0 = B_d S_5 + B_c S_6 + B_{1+c} S_7 + B_{a+c} S_8 \,. \tag{32}$$

2.5.2 Upper boundary condition

A convenient upper boundary condition is obtained by imposing the restraint that the downward transport of ionization through the level at which $z = \frac{H}{b+c}$ is equal to the total ion production minus the total ion recombination above that level.

In equations (1), (18) and (20) this gives

$$n_i v_t = - \int_{\frac{H}{b+c}}^{\infty} Q \, dz + \int_{\frac{H}{b+c}}^{\infty} L \, dz + D_m \exp\left(\frac{c}{b+c}\right) \left[\frac{n_i d}{H} + \frac{\partial n_i}{\partial z} \right] \Bigg|_{z = \frac{H}{b+c}} \tag{33}$$

Substituting equations (5) and (8) in equation (33) and integrating gives,

$$n_i v_t = -Hq_m \exp\left(\frac{-1}{b+c}\right) + \frac{H\rho_m}{a} \exp\left(\frac{-a}{b+c}\right) + \beta_m \int_{\frac{H}{b+c}}^{\infty} n_i \exp\left(\frac{-bz}{H}\right) dz$$

$$- D_m \exp\left(\frac{c}{b+c}\right) \left[\frac{n_i d}{H} + \frac{\partial n_i}{\partial z} \right] \Bigg|_{z = \frac{H}{b+c}} \,. \tag{34}$$

The individual terms may be expanded to give

$$n_i v_t = - Hq_m \exp\left(\frac{-1}{b+c}\right) \frac{H\rho_m}{a} \exp\left(\frac{-a}{b+c}\right)$$

$$+ \beta_m H \sum_{n=o}^{\infty} \frac{B_{d+n(b+c)}}{b+d+n(b+c)} \exp - \left(n + \frac{b+d}{b+c}\right)$$

$$- \frac{D_m}{H} \sum_{n=o}^{\infty} \left[n(b+c)\right] B_{d+n(b+c)} \exp - \left(n + \frac{d-c}{b+c}\right)$$

$$+ \beta_m H \sum_{n=o}^{\infty} \frac{B_{c+n(b+c)}}{(n+1)(b+c)} \exp - (n+1)$$

$$- \frac{D_m}{H} \sum_{n=o}^{\infty} \left[c-d+n(b+c)\right] B_{c+n(b+c)} \exp(-n)$$

$$+ \beta_m H \sum_{n=o}^{\infty} \frac{B_{1+c+n(b+c)}}{1+(n+1)(b+c)} \exp - (n+1+\frac{1}{b+c})$$

$$- \frac{D_m}{H} \sum_{n=o}^{\infty} \left[1+c-d+n(b+c)\right] B_{1+c+n(b+c)} \exp - (n+\frac{1}{b+c})$$

$$+ \beta_m H \sum_{n=o}^{\infty} \frac{B_{a+c+n(b+c)}}{a+(n+1)(b+c)} \exp - (n+1+\frac{a}{b+c})$$

$$- \frac{D_m}{H} \sum_{n=o}^{\infty} \left[a+c-d+n(b+c)\right] B_{a+c+n(b+c)} \exp - (n+\frac{a}{b+c}). \tag{35}$$

Using the recursion relationship given in equation (30) it may be shown that equation (35) reduces to

$$n_i v_t = - Hq_m \exp(\frac{-1}{b+c}) + \frac{H\rho_m}{a} \exp(\frac{-a}{b+c})$$

$$- \frac{D_m}{H}(c-d) B_c - \frac{D_m}{H}(1+c-d) B_{1+c} \exp(-\frac{1}{b+c})$$

$$- \frac{D_m}{H}(a+c-d) B_{a+c} \exp(\frac{-a}{b+c}) \tag{36}$$

Using the values for B_{1+c} and B_{a+c} derived in equations (24) and (25) equation (36) reduces to

$$B_c = - \frac{H(n_i v_t)}{D_m(c-d)} \tag{37}$$

2.5.3 Total Production and Loss in the Layer

The total ion production and loss in the layer must be equal under equilibrium conditions.

In equation (1) this gives,

$$\int_{-\infty}^{+\infty} Q\, dz = \int_{-\infty}^{+\infty} L\, dz \tag{38}$$

This may be written

$$\int_{-\infty}^{+\infty} Q\, dz = \Sigma\, \Phi\, \sigma\, \eta \tag{39}$$

where $\Sigma\, \Phi_{\infty}\, \sigma$ is the sum of the products of the solar EUV flux components and the relevant ionization cross sections and η is an efficiency factor related to the absorption of ionization by other constituents.

From equations (26) and (39)

$$\begin{aligned}\int_{-\infty}^{+\infty} L\, dz = \beta_m H \sum_{n=o}^{\infty} \Bigg\{ & \frac{B_{d+n(b+c)}}{b+d+n(b+c)} \exp - \left[\frac{b+d+n(b+c)}{H} \right] z \\ & + \frac{B_{c+n(b+c)}}{(n+1)(b+c)} \exp - \left[\frac{(n+1)(b+c)}{H} \right] z \\ & + \frac{B_{1+c+n(b+c)}}{1+(n+1)(b+c)} \exp - \left[\frac{1+(n+1)(b+c)}{H} \right] z \\ & + \frac{B_{a+c+n(b+c)}}{a+(n+1)(b+c)} \exp - \left[\frac{a+(n+1)(b+c)}{H} \right] z \Bigg\}\end{aligned} \tag{40}$$

which may be written

$$\int_{z}^{\infty} L dz = \beta_m H \left[B_d S_9(z) + B_c S_{10}(z) + B_{1+c} S_{11}(z) + B_{a+c} S_{12}(z) \right] \tag{41}$$

From equations (38), (39) and (41)

$$\frac{\Sigma\ \Phi_\infty \sigma\ \eta}{H\beta_m S_9(-\infty)} = B_d + B_c \frac{S_{10}(-\infty)}{S_9(-\infty)} + B_{1+c} \frac{S_{11}(-\infty)}{S_9(-\infty)} + B_{a+c} \frac{S_{12}(-\infty)}{S_9(-\infty)} \tag{42}$$

The limits of S_9, S_{10}, S_{11} and S_{12} are all infinite as z goes to minus infinity, however, their ratios remain finite and may be evaluated to give,

$$B_d = -B_c \frac{\Gamma(\frac{c-d}{b+c}) + 1}{\Gamma(\frac{d-c}{b+c}) + 1} \left[\frac{H^2\beta_m}{D_m}\right]^{\frac{d-c}{b+c}} \left[b+c\right]^{\frac{2(c-d)}{b+c}}$$

$$-B_{1+c} \frac{\Gamma(\frac{1-d+c}{b+c} + 1)\ \Gamma(\frac{1}{b+c} + 1)}{\Gamma(\frac{d-c}{b+c} + 1)} \left[\frac{H^2\beta_m}{D_m}\right]^{\frac{d-c-1}{b+c}} \left[b+c\right]^{\frac{2(1+c-d)}{b+c}}$$

$$-B_{a+c} \frac{\Gamma(\frac{a-d+c}{b+c} + 1)\ \Gamma(\frac{1}{b+c} + 1)}{\Gamma(\frac{d-c}{b+c} + 1)} \left[\frac{H^2\beta_m}{D_m}\right]^{\frac{d-c-a}{b+c}} \left[b+c\right]^{\frac{2(a+c-d)}{b+c}} \tag{43}$$

Equations (32) and (43) are both functions of B_d and $\frac{H^2\beta_m}{D_m}$. It has been found convenient to solve these equations by using a digital computer to search for the value of $\frac{H^2\beta_m}{D_m}$ that gives the same values of B_d for the two equations.

3. <u>The Effect of Helium and Hydrogen Ions</u>

At levels more than two scale heights above the maximum the atomic oxygen ion density given by equation (27) is, for typical values of a, b, c and d, approximated within one percent using the first term in the series S_1, and,

$$n_i \simeq B_d \exp\left(-\frac{dz}{H}\right) . \tag{44}$$

This is because at these levels the production and loss terms in the continuity equation are small, and the low values of ion-neutral particle collision frequencies allow large diffusion velocities to result for very small departures from a diffusion equilibrium profile.

For a diffusion equilibrium profile equation (10) may be written for each ionic constituent

$$kT_i \frac{\partial n_i}{\partial z} = - m_i n_i g + qEn_i . \tag{45}$$

When more than one type of ion is present equation (45) may be summed over the various ionic species to give,

$$kT_i \sum_i \frac{\partial n_i}{\partial z} = -g \sum_i m_i n_i + q E \sum_i n_i \tag{46}$$

For the electrons, using the same approximation, equation (11) may be written,

$$kT_e \frac{\partial n_e}{\partial z} = -g m_e n_e - q E n_e . \tag{47}$$

It may be assumed in this region that

$$\frac{\partial n_e}{\partial z} = \sum_i \frac{\partial n_i}{\partial z} \tag{48}$$

and

$$n_e = \sum_i n_i \tag{49}$$

From equations (46), (47), (48) and (49)

$$\frac{1}{n_e} \frac{\partial n_e}{\partial z} = - \frac{g \Sigma m_i n_i}{n_e k(T_e + T_i)} = - \frac{g m_+ d}{kT_i} \tag{50}$$

where $m_+ = \dfrac{\Sigma m_i n_i}{\Sigma n_i}$.

Equation (50) may be integrated to give,

$$n_e = n_{eo} \exp - \int_{z_o}^{z} \frac{g m_+ d}{k T_i} dz \tag{51}$$

Substituting equation (45) in (47) gives,

$$\frac{1}{n_i} \frac{\partial n_i}{\partial z} = - \frac{m_i g}{k T_i} - \frac{T_e}{T_i} \frac{1}{n_e} \frac{\partial n_e}{\partial z} . \tag{52}$$

Substituting equation (50) in equation (52) gives,

$$\frac{1}{n_i} \frac{\partial n_i}{\partial z} = - \frac{m_i g}{k T_i} + \frac{g m_+ (1-d)}{k T_i} \tag{53}$$

which may be integrated to give,

$$n_i = n_{io} \exp \left[- \int_{z_o}^{z} \frac{m_i g}{k T_i} dz + \int_{z_o}^{z} \frac{g m_+ (1-d)}{k T_i} dz \right] . \tag{54}$$

Now

$$H_i = \frac{kT}{m_i g} ,$$

so that

$$\sum_i m_i n_i = \exp \left[- \int_{z_o}^{z} \frac{g m_+ (1-d)}{k T_i} dz \right] \sum_i m_i n o_i \exp \left(- \frac{z - z_o}{H_i} \right) \tag{55}$$

Similarly

$$\sum_i n_i = \left[\exp \int_{z_o}^{z} \frac{g m_+ (1-d)}{k T_i} dz \right] \sum_i n_{io} \exp \left(- \frac{z - z_o}{H} \right) . \tag{56}$$

The mean molecular mass is thus given by

$$m_+ = \frac{\sum_i m_i n_i}{\sum_i n_i} = \frac{\sum_i m_i n_{io} \exp \left(- \frac{z - z_o}{H_i} \right)}{\sum_i n_{io} \exp \left(- \frac{z_o}{H_i} \right)} . \tag{57}$$

Let $A = \sum_i n_{io} \exp\left(-\frac{z-z_o}{H_i}\right)$

then,

$$\frac{dA}{dz} = -\frac{kT_i}{g} \sum_i m_i n_i \exp - \left(\frac{z-z_o}{Hi}\right) \tag{59}$$

and

$$-\frac{gm_+}{kT_i} = \frac{\frac{dA}{dz}}{A}. \tag{60}$$

Equation (60) may be integrated with respect to z to give

$$-\int_{z_o}^{z} \frac{gm_+}{kT_i} dz = \log_e \sum_i n_{io} \exp - \left(\frac{z-z_o}{H_i}\right) - n_{io} . \tag{61}$$

Substituting equation (61) in equation (51) gives,

$$n_e = n_{eo} \left[\sum_i \frac{n_{io}}{n_{eo}} \exp - \left(\frac{z-z_o}{H_i}\right) \right]^{\frac{T_i}{T_e + T_i}} \tag{62}$$

In the regions where helium and hydrogen ions are of importance the first term in the series S_1 given in equation (44) may be modified to give the electron density,

$$n_e \simeq B_d \exp\left(\frac{-z_o d}{H}\right) \left[\exp - \left(\frac{z-z_o}{H}\right) + \frac{N(H_e^+)_o}{N(0^+)_o} \exp - \left(\frac{z-z_o}{4H}\right) + \frac{N(H^+)_o}{N(0^+)_o} \exp - \left(\frac{z-z_o}{16H}\right) \right]^d \tag{63}$$

where $N(0^+)_o$, $N(H_e^+)_o$, and $N(H^+)_o$ are the atomic oxygen, helium and hydrogen ion densities at the datum level z_o,

providing $R_1 \equiv \dfrac{N(H^+)_o}{N(0^+)_o} << 1,$

and $R_2 \equiv \dfrac{N(H_e{}^+)_o}{N(0^+)_o} << 1,$

and z_o, the height of the datum level above the maximum, is sufficiently large that the first term in the S_1 series predominates.

The complete solution for the equilibrium electron density may thus be obtained by combining equations (27) and (63) to give,

$$n_e = B_d \left\{ \exp\left(-\frac{zd}{H}\right)\left[1 + R_1 \exp\frac{3(z-z_o)}{4H} + R_2 \exp\frac{15(z-z_o)}{16H}\right]^d + S_1(z) - \exp\left(-\frac{zd}{H}\right)\right\}$$

$$+ B_c S_2(z) + B_{1+c} S_3(z) + B_{a+c} S_4(z) \quad . \tag{64}$$

4. Results

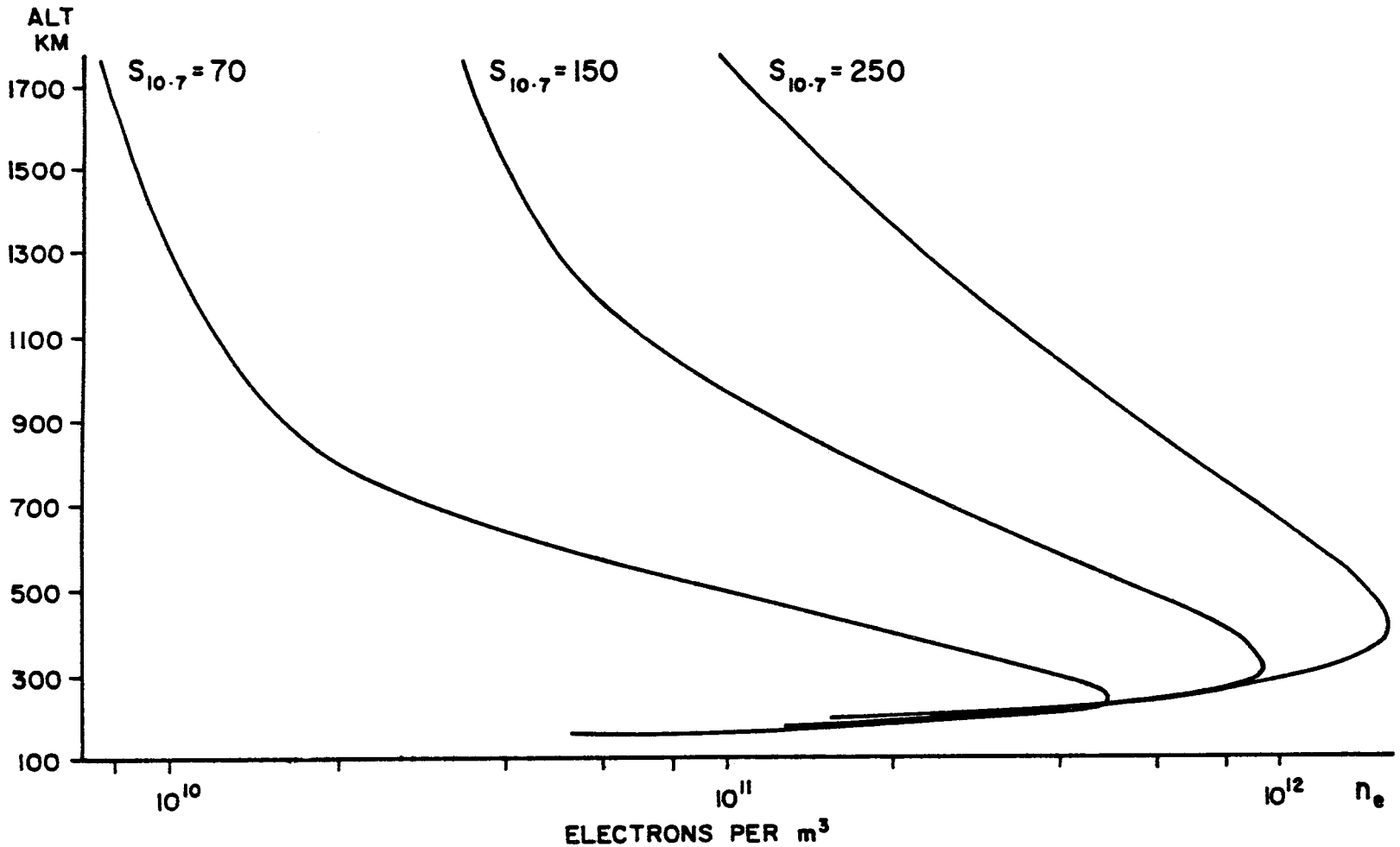
4.1 Profiles

A digital computer program has been used to solve equation (64) using the relations for B_c, B_{1+c}, B_{a+c}, and $\dfrac{H^2\beta_m}{D_m}$ derived in the preceding sections. The program allows the electron density to be calculated and plotted in terms of both the geo-potential altitude z and the true height. Figure 1 shows examples of profiles calculated for three different values of solar activity to illustrate the form of the results.

For this Figure the following values were assumed

$a = 2.3$ $\quad b = 1.75$ $\quad c = 1.3$ $\quad d = .385$

$q = 2 \times 10^{-9} S_{10.7} N(0) \sec^{-1}$ $\quad$ Nisbet (1963)



EXAMPLES OF PROFILES COMPUTED FOR THREE LEVELS OF SOLAR ACTIVITY

FIGURE 1

$$T = 7.05\, S_{10.7} + 372^{\circ}K \qquad \text{Jacchia (1962)}$$

$$\beta = 4 \times 10^{-19} N(N_2)\ \sec^{-1} \qquad \text{Nisbet and Quinn (1963)}$$

$$D = \frac{4.5 \times 10^{19} \sin^2 I\, T^{\frac{1}{2}}}{n(M)}\ m^2 \sec^{-1} \qquad \text{Shimizaki (1957)}$$

$$\frac{\rho_m}{q_m} = 0.1$$

Atmospheric models due to Nicolet (1962) were used for the neutral atmospheric densities. Helium and hydrogen ion density ratios were calculated from models due to Bauer (1963).

4.2 Approximate Profiles

Above the maximum the series S_1, S_2, S_3, and S_4 in equations (27) and (64) are well approximated by the first few terms.

In general, in this region a total of seven terms is required to approximate the electron density within 1% when a nondivergent ion flux is assumed and only five terms is required when it is absent.

With this approximation, equation (64) may be written,

$$n_e \simeq \frac{q_m}{\beta_m} \left\{ K_1 \exp - \frac{L_1 z}{H} \left[1 + R_1 \exp \frac{3(z-z_o)}{4H} + R_2 \exp \frac{15(z-z_o)}{16H} \right]^{L_1} \right.$$

$$+ K_2 \left[\exp - \frac{L_2 z}{H} \right] + K_3 \left[\exp - \frac{L_3 z}{H} \right]$$

$$+ K_4 \left[\exp - \frac{L_4 z}{H} \right] + K_5 \left[\exp - \frac{L_5 z}{H} \right]$$

$$\left. + K_6 \left[\exp - \frac{L_6 z}{H} \right] + K_7 \left[\exp - \frac{L_7 z}{H} \right] \right\} \qquad (65)$$

Where

$$K_1 = B_d \frac{\beta_m}{q_m} , \qquad L_1 = d$$

$$K_2 = B_c \frac{\beta_m}{q_m} , \qquad L_2 = c$$

$$K_3 = B_{1+c} \frac{\beta_m}{q_m} , \qquad L_3 = 1 + c$$

$$K_4 = B_{a+c} \frac{\beta_m}{q_m} , \qquad L_4 = a + c$$

$$K_5 = B_d \frac{\beta_m}{q_m} \frac{H^2\beta_m}{D_m} \frac{1}{(d+b)(b+c)} , \qquad L_5 = d + b + c$$

$$K_6 = B_c \frac{\beta_m}{q_m} \frac{H^2\beta_m}{D_m} \frac{1}{(b+c)(b+2c-d)} , \qquad L_6 = b + 2c$$

$$K_7 = B_{1+c} \frac{\beta_m}{q_m} \frac{H^2\beta_m}{D_m} \frac{1}{(1+b+c)(1+b+2c-d)} , \qquad L_7 = 1 + b + 2c .$$

A computer program has been written to calculate these parameters from equations (28), (29), (30), (32), (37) and (43) and to print them out in the form of tables.

For this program it is necessary to read in the parameters of the atmospheric model a, b, c, d, and $\frac{\rho_m}{q_m}$ and a parameter related to the nondivergent vertical flux $n_i v_t$. The computer also prints out three parameters of the electron density maximum

$$\frac{H^2\beta_m}{D_m} , \quad \frac{n_m\beta_m}{q_m} , \text{ and } SCAT/H = \frac{1}{H} \left[\frac{-n_m}{2 \frac{\partial^2 n}{\partial z^2}_m} \right]^{\frac{1}{2}}$$

SCAT/H is the quarter thickness of a parabola fitted to the peak of the electron density distribution.

Tables 1 to 35 in the Appendix give values of these coefficients for eight different atmospheric models. For each model ranges of parameters for the attenuation of the solar radiation and the vertical nondivergent ion flux have been calculated.

5. Conclusions

The values given in Tables 1 to 35 allow the effects of various assumptions about the neutral atmospheric models and ionizing radiation on the electron density profiles to be studied.

The parameter SCAT/H is of considerable practical interest because of its effect on the calculations of neutral atmospheric temperatures from ionospheric measurements in the region of the maximum, and because its relation to the profile does not depend on assumptions about the diffusion, recombination or production coefficients.

Increasing geographic latitude for a given time of year and neutral atmospheric temperature corresponds to increasing both the value of $\frac{\rho_m}{q_m}$ and a. It is apparent that both these effects produce a decrease in layer thickness as indicated by the SCAT/H values.

Hinteregger and Watanabe (1962) have given experimental measurements of photon fluxes as a function of altitude. These measurements were used to investigate the effect of the non-monochromatic nature of the ionizing radiation on the electron density profile. Tables 33-35 give the parameters of electron

density distribution calculated for models with values of a and $\frac{\rho_m}{q_m}$ chosen to fit the photon flux measurements above 150 km for the wavelengths of importance in F region production. It is apparent that the effect on the thickness of the F_2 region peak of the differing absorption rates is less than 1% at the latitude of White Sands in August. For larger zenith angles the effect would, of course, be larger but it does not appear that measurements of electron density profiles in the region of the F_2 maximum are likely to provide useful information on the relative ionization cross sections of atomic oxygen for the different groups of solar fluxes.

Sets of tables are given for

$$T_e/T_i = 1 \ (d = .5)$$

and

$$T_e/T_i = 1.6 \ (d = .385).$$

It is of interest to compare the effect of the electron ion temperature ratio on the shape of the layer in the region of the maximum and considerably above the maximum. As would be expected in the region far above the maximum the slope is proportional to $\frac{T_e + T_i}{2}$ and there is a 30% difference between the values of L_1 for the two models. The values of SCAT/H which describe the quarter thickness in the region of the maximum show only an 11% difference.

The coefficients for recombination with molecular oxygen and molecular nitrogen are undoubtedly different so that little useful purpose is served by comparing values of $\frac{n_m \beta_m}{q_m}$ for models with different values of b.

The variation of the layer with latitude may be studied by examining the effects of variations in $\frac{\rho_m}{q_m}$ and of $\sin^2 I$ on D_m. As the solar zenith angle increases $\frac{\rho_m}{q_m}$ and a increase. In both cases the effect on the electron density profile is to reduce the value of $n_m \frac{\beta_m}{q_m}$. Because of the effect of the earth's magnetic field on the diffusion coefficient, the height of the layer in middle latitudes decreases with increasing latitude causing $\frac{\beta_m}{q_m}$ to increase. These effects combine to reduce the electron density as the latitude is increased.

The nondivergent vertical ion flux $\frac{G}{q_m H}$ is chiefly of interest in matching boundary conditions when different models are used for successive altitude regions. This approach can be used, for example, to study the equatorial anomaly. It is apparent that this parameter has little effect on the value of $\frac{n_m \beta_m}{q_m}$. The value of $\frac{H^2 \beta_m}{D_m}$ is, however, greatly affected and it appears that the major effect on the peak electron density is the change in $\frac{\beta_m}{q_m}$ produced by the variation in the height of the maximum electron density quite accurately even if there is some uncertainty in the coefficients D and β as a function of height. An error of 50% in the ratio β_m / D_m would result in an error of about 64 km at sunspot maxima and only 24 km at sunspot minima.

The effect of increasing solar zenith angle is to lower the value of $\frac{H^2 \beta_m}{D_m}$. This corresponds to an increase in the height of the layer maximum which might contribute to the winter anomaly at middle latitudes.

The method of analysis provides simple analytic solutions

of the F_2 region equilibrium profile and is sufficiently flexible to allow it to be fitted to various neutral atmosphere models. When the neutral atmosphere parameters are not independent of height a lamination technique may be employed using different models for successive layers.

The application of the method to the prediction of F_2 region parameters will be discussed in a subsequent report.

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Appendix-Tables of Coefficients

Summary of Symbols

PRODUCTION TERM

$$Q_z = q_m \exp\left(-\frac{z}{H}\right) - \rho_m \exp\left(-\frac{az}{H}\right) \tag{5}$$

where

z is the rate of atomic oxygen ion production at height z

H is the scale height of atomic oxygen

and q_m ρ_m and a are chosen to fit assumed profiles of atomic oxygen ion production.

LOSS TERM

$$L_z = \beta_m n_i \exp\left(-\frac{bz}{H}\right) \tag{8}$$

where L_z is the rate of recombination of atomic oxygen ions at height z and β_m is the linear recombination coefficient effective at the ion density maximum

$$\nu_{in} = \nu_{in_m} \exp\left(-\frac{cz}{H}\right) \tag{15}$$

ν_{in} is the ion-neutral particle collision frequency

ν_{in_m} is ν_{in} at the electron density maximum

c is a factor chosen to fit the assumed neutral atmospheric model being used

$$d \equiv \frac{T_i}{T_e + T_i} \tag{16}$$

where T_e is the electron temperature

and T_i is the ion temperature.

DIFFUSION COEFFICIENT

$$D_m = \frac{g\,H \sin^2 I}{\nu_{in_m}\, d} \tag{17}$$

NON-DIVERGENT ION FLUX

$$G = n_i \, v_t \tag{20}$$

with

$$\frac{\partial G}{\partial z} = 0 \tag{21}$$

$$SCAT/H = \frac{1}{H} \left[\frac{-n_m}{2\frac{\partial^2 n}{\partial z^2}_n} \right]^{\frac{1}{2}}$$

$$n_e \simeq \frac{q_m}{\beta_m} \left\{ K_1 \exp - \frac{L_1 z}{H} \left[1 + R_1 \exp \frac{3(z-z_o)}{4H} + R_2 \exp \frac{15(z-z_o)}{16H} \right]^{L_1} \right.$$

$$+ K_2 \left[\exp - \frac{L_2 z}{H} \right] + K_3 \left[\exp - \frac{L_3 z}{H} \right]$$

$$+ K_4 \left[\exp - \frac{L_4 z}{H} \right] + K_5 \left[\exp - \frac{L_5 z}{H} \right]$$

$$\left. + K_6 \left[\exp - \frac{L_6 z}{H} \right] + K_7 \left[\exp - \frac{L_7 z}{H} \right] \right\} \tag{65}$$

where $R_1 \quad \frac{N(H^+)}{N(O^+)}$ at level $z = z_o$

and $R_2 \quad \frac{N(H_e{}^+)}{N(O^+)}$ at level $z = z_o$

TABLE 1

a	2.00000	2.00000	2.00000	2.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.00000	1.00000	1.00000	1.00000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.10000	0.20000	0.30000
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.74660	0.59840	0.44670	0.27990
$n_m\beta_m/q_m$	0.84526	0.70471	0.55186	0.37012
SCAT/H	0.88619	0.86657	0.84464	0.81679
K1	1.21595	1.00703	0.78249	0.51931
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.49773	-0.39893	-0.29780	-0.18660
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.01197	0.01787	0.01679
L4	3.00000	3.00000	3.00000	3.00000
K5	0.14672	0.09739	0.05649	0.02349
L5	3.25000	3.25000	3.25000	3.25000
K6	0.00000	0.00000	0.00000	0.00000
L6	3.75000	3.75000	3.75000	3.75000
K7	-0.02332	-0.01498	-0.00835	-0.00328
L7	4.75000	4.75000	4.75000	4.75000

TABLE 2

a	3.00000	3.00000	3.00000	3.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.00000	1.00000	1.00000	1.00000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.02500	0.05000	0.07500
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.74660	0.65115	0.55355	0.44965
$n_m\beta_m/q_m$	0.84526	0.77456	0.69509	0.60100
SCAT/H	0.88619	0.86483	0.84334	0.82066
K1	1.21595	1.10591	0.98455	0.84373
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.49773	-0.43410	-0.36903	-0.29977
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00155	0.00264	0.00321
L4	4.00000	4.00000	4.00000	4.00000
K5	0.14672	0.11638	0.08608	0.06131
L5	3.25000	3.25000	3.25000	3.25000
K6	0.00000	0.00000	0.00000	0.00000
L6	3.75000	3.75000	3.75000	3.75000
K7	-0.02332	-0.01774	-0.01282	-0.00846
L7	4.75000	4.75000	4.75000	4.75000

TABLE 3

a	4.00000	4.00000	4.00000	4.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.00000	1.00000	1.00000	1.00000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.00600	0.01200	0.01800
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.74660	0.68695	0.62410	0.55615
$n_m\beta_m/q_m$	0.84526	0.80490	0.75929	0.70596
SCAT/H	0.88619	0.86948	0.85250	0.83480
K1	1.21595	1.15160	1.08002	0.99775
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.49773	-0.45797	-0.41607	-0.37077
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00023	0.00042	0.00056
L4	5.00000	5.00000	5.00000	5.00000
K5	0.14672	0.12785	0.10894	0.08968
L5	3.25000	3.25000	3.25000	3.25000
K6	0.00000	0.00000	0.00000	0.00000
L6	3.75000	3.75000	3.75000	3.75000
K7	-0.02332	-0.01974	-0.01629	-0.01294
L7	4.75000	4.75000	4.75000	4.75000

TABLE 4

a	2.00000	2.00000	2.00000	2.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.00000	1.00000	1.00000	1.00000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.00000	0.00000	0.00000
$G/q_m H$	-0.50000	-0.25000	0.25000	0.50000
$H^2\beta_m/D_m$	3.77260	1.25080	0.54220	0.43210
$n_m\beta_m/q_m$	0.84209	0.82992	0.86832	0.89458
SCAT/H	0.64351	0.81308	0.92670	0.95266
K1	-0.91431	0.83759	1.40453	1.53491
L1	0.50000	0.50000	0.50000	0.50000
K2	3.77260	0.62540	-0.27110	-0.43210
L2	1.00000	1.00000	1.00000	1.00000
K3	-2.51507	-0.83387	-0.36147	-0.28807
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00000	0.00000	0.00000
L4	3.00000	3.00000	3.00000	3.00000
K5	-0.55747	0.16932	0.12308	0.10719
L5	3.25000	3.25000	3.25000	3.25000
K6	1.59245	0.08752	-0.01645	-0.02089
L6	3.75000	3.75000	3.75000	3.75000
K7	-0.59535	-0.06544	-0.01230	-0.00781
L7	4.75000	4.75000	4.75000	4.75000

TABLE 5

a	2.00000	2.00000	2.00000	2.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.00000	1.00000	1.00000	1.00000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.10000	0.20000	0.30000
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.56615	0.46355	0.35500	0.23315
$n_m\beta_m/q_m$	0.74664	0.61912	0.48289	0.32702
SCAI/H	0.84996	0.83900	0.82584	0.80797
K1	1.05506	0.87209	0.67745	0.45609
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.37743	-0.30903	-0.23667	-0.15543
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00927	0.01420	0.01399
L4	3.00000	3.00000	3.00000	3.00000
K5	0.07964	0.05390	0.03207	0.01418
L5	3.50000	3.50000	3.50000	3.50000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.00000	4.00000	4.00000	4.00000
K7	-0.01187	-0.00796	-0.00467	-0.00201
L7	5.00000	5.00000	5.00000	5.00000

TABLE 6

a	3.00000	3.00000	3.00000	3.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.00000	1.00000	1.00000	1.00000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.02500	0.05000	0.07500
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.56615	0.50085	0.43330	0.36115
$n_m\beta_m/q_m$	0.74664	0.68204	0.61112	0.53019
SCAT/H	0.84996	0.83616	0.82184	0.80638
K1	1.05506	0.95909	0.85487	0.73732
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.37743	-0.33390	-0.28887	-0.24077
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00119	0.00206	0.00258
L4	4.00000	4.00000	4.00000	4.00000
K5	0.07964	0.06405	0.04939	0.03550
L5	3.50000	3.50000	3.50000	3.50000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.00000	4.00000	4.00000	4.00000
K7	-0.01187	-0.00929	-0.00695	-0.00483
L7	5.00000	5.00000	5.00000	5.00000

TABLE 7

a	4.00000	4.00000	4.00000	4.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.00000	1.00000	1.00000	1.00000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.00600	0.01200	0.01800
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.56615	0.52590	0.48355	0.43810
$n_m\beta_m/q_m$	0.74664	0.70990	0.66940	0.62366
SCAT/H	0.84996	0.83891	0.82752	0.81556
K1	1.05506	0.99946	0.93877	0.87091
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.37743	-0.35060	-0.32237	-0.29207
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00018	0.00032	0.00044
L4	5.00000	5.00000	5.00000	5.00000
K5	0.07964	0.07008	0.06053	0.05087
L5	3.50000	3.50000	3.50000	3.50000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.00000	4.00000	4.00000	4.00000
K7	-0.01187	-0.01024	-0.00866	-0.00711
L7	5.00000	5.00000	5.00000	5.00000

TABLE 8

a	2.00000	2.00000	2.00000	2.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.00000	1.00000	1.00000	1.00000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.00000	0.00000	0.00000
$G/q_m H$	-0.50000	-0.25000	0.25000	0.50000
$H^2\beta_m/D_m$	2.17105	0.88875	0.42230	0.34110
$n_m\beta_m/q_m$	0.76037	0.74082	0.76071	0.77843
SCAT/H	0.64979	0.78523	0.88887	0.91511
K1	-0.19549	0.78470	1.20019	1.30263
L1	0.50000	0.50000	0.50000	0.50000
K2	2.17105	0.44438	-0.21115	-0.34110
L2	1.00000	1.00000	1.00000	1.00000
K3	-1.44737	-0.59250	-0.28153	-0.22740
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00000	0.00000	0.00000
L4	3.00000	3.00000	3.00000	3.00000
K5	-0.05659	0.09299	0.06758	0.05924
L5	3.50000	3.50000	3.50000	3.50000
K6	0.44890	0.03761	-0.00849	-0.01108
L6	4.00000	4.00000	4.00000	4.00000
K7	-0.17457	-0.02925	-0.00661	-0.00431
L7	5.00000	5.00000	5.00000	5.00000

TABLE 9

a	2.00000	2.00000	2.00000	2.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.10000	0.20000	0.30000
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.84475	0.69315	0.53975	0.37855
$n_m\beta_m/q_m$	0.90845	0.77268	0.62776	0.46522
SCAT/H	0.82471	0.80886	0.79151	0.77104
K1	1.24140	1.05045	0.84846	0.62422
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.46931	-0.38508	-0.29986	-0.21031
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.01238	0.01928	0.02028
L4	3.30000	3.30000	3.30000	3.30000
K5	0.15281	0.10610	0.06673	0.03443
L5	3.55000	3.55000	3.55000	3.55000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.35000	4.35000	4.35000	4.35000
K7	-0.02018	-0.01359	-0.00824	-0.00405
L7	5.35000	5.35000	5.35000	5.35000

TABLE 10

a	3.00000	3.00000	3.00000	3.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.02500	0.05000	0.07500
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.84475	0.75385	0.66270	0.56970
$n_m\beta_m/q_m$	0.90845	0.84657	0.77929	0.70439
SCAT/H	0.82471	0.80879	0.79296	0.77689
K1	1.24140	1.15061	1.05325	0.94640
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.46931	-0.41881	-0.36817	-0.31650
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00165	0.00291	0.00375
L4	4.30000	4.30000	4.30000	4.30000
K5	0.15281	0.12640	0.10171	0.07857
L5	3.55000	3.55000	3.55000	3.55000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.35000	4.35000	4.35000	4.35000
K7	-0.02018	-0.01607	-0.01242	-0.00918
L7	5.35000	5.35000	5.35000	5.35000

TABLE 11

a	4.00000	4.00000	4.00000	4.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.00600	0.01200	0.01800
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.84475	0.79335	0.74075	0.68640
$n_m\beta_m/q_m$	0.90845	0.87695	0.84289	0.80560
SCAT/H	0.82471	0.81331	0.80191	0.79042
K1	1.24140	1.19404	1.14340	1.08856
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.46931	-0.44075	-0.41153	-0.38133
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00025	0.00046	0.00064
L4	5.30000	5.30000	5.30000	5.30000
K5	0.15281	0.13804	0.12342	0.10888
L5	3.55000	3.55000	3.55000	3.55000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.35000	4.35000	4.35000	4.35000
K7	-0.02018	-0.01780	-0.01552	-0.01333
L7	5.35000	5.35000	5.35000	5.35000

TABLE 12

a	2.00000	2.00000	2.00000	2.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.00000	0.00000	0.00000
$G/q_m H$	-0.50000	-0.25000	0.25000	0.50000
$H^2\beta_m/D_m$	4.36135	1.40275	0.62025	0.49945
$n_m\beta_m/q_m$	0.86612	0.87467	0.94819	0.99004
SCAT/H	0.61449	0.76652	0.85505	0.87369
K1	0.01289	1.00421	1.38078	1.49009
L1	0.50000	0.50000	0.50000	0.50000
K2	2.72584	0.43836	-0.19383	-0.31216
L2	1.30000	1.30000	1.30000	1.30000
K3	-2.42297	-0.77931	-0.34458	-0.27747
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00000	0.00000	0.00000
L4	3.30000	3.30000	3.30000	3.30000
K5	0.00819	0.20527	0.12480	0.10845
L5	3.55000	3.55000	3.55000	3.55000
K6	1.01242	0.05237	-0.01024	-0.01328
L6	4.35000	4.35000	4.35000	4.35000
K7	-0.53799	-0.05565	-0.01088	-0.00706
L7	5.35000	5.35000	5.35000	5.35000

TABLE 13

a	2.00000	2.00000	2.00000	2.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.10000	0.20000	0.30000
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.64945	0.54375	0.43325	0.31375
$n_m\beta_m/q_m$	0.81790	0.69243	0.56036	0.41562
SCAT/H	0.79325	0.78422	0.77369	0.76043
K1	1.10132	0.93002	0.75031	0.55421
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.36081	-0.30208	-0.24069	-0.17431
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00971	0.01547	0.01681
L4	3.30000	3.30000	3.30000	3.30000
K5	0.08670	0.06130	0.03940	0.02108
L5	3.80000	3.80000	3.80000	3.80000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.01069	-0.00749	-0.00476	-0.00249
L7	5.60000	5.60000	5.60000	5.60000

TABLE 14

a	3.00000	3.00000	3.00000	3.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.02500	0.05000	0.07500
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.64945	0.58665	0.52285	0.45710
$n_m\beta_m/q_m$	0.81790	0.76010	0.69828	0.63094
SCAT/H	0.79325	0.78285	0.77222	0.76115
K1	1.10132	1.01987	0.93345	0.84008
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.36081	-0.32592	-0.29047	-0.25394
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00129	0.00229	0.00301
L4	4.30000	4.30000	4.30000	4.30000
K5	0.08670	0.07252	0.05916	0.04655
L5	3.80000	3.80000	3.80000	3.80000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.01069	-0.00872	-0.00693	-0.00529
L7	5.60000	5.60000	5.60000	5.60000

TABLE 15

a	4.00000	4.00000	4.00000	4.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.00600	0.01200	0.01800
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.64945	0.61425	0.57810	0.54075
$n_m\beta_m/q_m$	0.81790	0.78841	0.75698	0.72322
SCAT/H	0.79325	0.78559	0.77783	0.76992
K1	1.10132	1.05900	1.01418	0.96635
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.36081	-0.34125	-0.32117	-0.30042
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00019	0.00036	0.00051
L4	5.30000	5.30000	5.30000	5.30000
K5	0.08670	0.07885	0.07107	0.06334
L5	3.80000	3.80000	3.80000	3.80000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.01069	-0.00956	-0.00847	-0.00741
L7	5.60000	5.60000	5.60000	5.60000

TABLE 16

a	2.00000	2.00000	2.00000	2.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00000	0.00000	0.00000	0.00000
$G/q_m H$	-0.50000	-0.25000	0.25000	0.50000
$H^2\beta_m/D_m$	2.47035	1.00730	0.49050	0.40085
$n_m\beta_m/q_m$	0.79386	0.79471	0.84765	0.88022
SCAT/H	0.62222	0.74117	0.82302	0.84242
K1	0.36876	0.92570	1.21219	1.30114
L1	0.50000	0.50000	0.50000	0.50000
K2	1.54397	0.31478	-0.15328	-0.25053
L2	1.30000	1.30000	1.30000	1.30000
K3	-1.37242	-0.55961	-0.27250	-0.22269
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00000	0.00000	0.00000
L4	3.30000	3.30000	3.30000	3.30000
K5	0.11042	0.11303	0.07207	0.06322
L5	3.80000	3.80000	3.80000	3.80000
K6	0.28190	0.02344	-0.00556	-0.00742
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.15460	-0.02570	-0.00609	-0.00407
L7	5.60000	5.60000	5.60000	5.60000

TABLE 17

a	2.00000	2.00000	2.00000	2.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.00000	1.00000	1.00000	1.00000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.10000	0.20000	0.30000
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.52615	0.40985	0.28955	0.14480
$n_m\beta_m/q_m$	0.79667	0.64973	0.48608	0.26735
SCAT/H	0.98126	0.95970	0.93495	0.89851
K1	1.03818	0.84226	0.62614	0.34099
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.32579	-0.25378	-0.17929	-0.08966
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00784	0.01107	0.00831
L4	3.00000	3.00000	3.00000	3.00000
K5	0.09304	0.05879	0.03088	0.00841
L5	3.13500	3.13500	3.13500	3.13500
K6	0.00000	0.00000	0.00000	0.00000
L6	3.75000	3.75000	3.75000	3.75000
K7	-0.01047	-0.00635	-0.00317	-0.00079
L7	4.75000	4.75000	4.75000	4.75000

TABLE 18

a	3.00000	3.00000	3.00000	3.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.00000	1.00000	1.00000	1.00000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.02500	0.05000	0.07500
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.52615	0.44450	0.35870	0.00005
$n_m\beta_m/q_m$	0.79667	0.71406	0.61651	0.00013
SCAT/H	0.98126	0.95569	0.92925	0.81637
K1	1.03818	0.92463	0.79276	0.00016
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.32579	-0.27523	-0.22211	-0.00003
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00102	0.00165	0.00000
L4	4.00000	4.00000	4.00000	4.00000
K5	0.09304	0.07000	0.04843	0.00000
L5	3.13500	3.13500	3.13500	3.13500
K6	0.00000	0.00000	0.00000	0.00000
L6	3.75000	3.75000	3.75000	3.75000
K7	-0.01047	-0.00747	-0.00487	-0.00000
L7	4.75000	4.75000	4.75000	4.75000

TABLE 19

a	4.00000	4.00000	4.00000	4.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.00000	1.00000	1.00000	1.00000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.00600	0.01200	0.01800
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.52615	0.46910	0.00005	0.00005
$n_m\beta_m/q_m$	0.79667	0.74320	0.00013	0.00013
SCAT/H	0.98126	0.95932	0.80766	0.80858
K1	1.03818	0.96358	0.00016	0.00016
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.32579	-0.29046	-0.00003	-0.00003
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00015	0.00000	0.00000
L4	5.00000	5.00000	5.00000	5.00000
K5	0.09304	0.07699	0.00000	0.00000
L5	3.13500	3.13500	3.13500	3.13500
K6	0.00000	0.00000	0.00000	0.00000
L6	3.75000	3.75000	3.75000	3.75000
K7	-0.01047	-0.00832	-0.00000	-0.00000
L7	4.75000	4.75000	4.75000	4.75000

TABLE 20

a	2.00000	2.00000	2.00000	2.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.00000	1.00000	1.00000	1.00000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.00000	0.00000	0.00000
G/q_mH	-0.50000	-0.25000	0.25000	0.50000
$H^2\beta_m/D_m$	2.88660	0.88970	0.37980	0.30100
$n_m\beta_m/q_m$	0.81725	0.75074	0.81142	0.82976
SCAF/H	0.69656	0.89770	1.02785	1.05791
K1	-0.13195	0.84279	1.13821	1.20962
L1	0.38500	0.38500	0.38500	0.38500
K2	2.34681	0.36166	-0.15439	-0.24471
L2	1.00000	1.00000	1.00000	1.00000
K3	-1.78737	-0.55000	-0.23517	-0.18638
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00000	0.00000	0.00000
L4	3.00000	3.00000	3.00000	3.00000
K5	-0.06487	0.12771	0.07363	0.06201
L5	3.13500	3.13500	3.13500	3.13500
K6	0.73206	0.03477	-0.00634	-0.00796
L6	3.75000	3.75000	3.75000	3.75000
K7	-0.31520	-0.02994	-0.00546	-0.00343
L7	4.75000	4.75000	4.75000	4.75000

TABLE 21

a	2.00000	2.00000	2.00000	2.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.00000	1.00000	1.00000	1.00000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.10000	0.20000	0.30000
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.39025	0.31130	0.22670	0.12515
$n_m\beta_m/q_m$	0.68938	0.55866	0.41635	0.24018
SCAT/H	0.94420	0.93234	0.91755	0.89472
K1	0.88719	0.71717	0.53272	0.30562
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.24164	-0.19276	-0.14037	-0.07749
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00595	0.00867	0.00718
L4	3.00000	3.00000	3.00000	3.00000
K5	0.04839	0.03120	0.01688	0.00535
L5	3.38500	3.38500	3.38500	3.38500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.00000	4.00000	4.00000	4.00000
K7	-0.00511	-0.00325	-0.00172	-0.00053
L7	5.00000	5.00000	5.00000	5.00000

TABLE 22

a	3.00000	3.00000	3.00000	3.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.00000	1.00000	1.00000	1.00000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.02500	0.05000	0.07500
G/q_mH	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.39025	0.33555	0.27750	0.21125
$n_m\beta_m/q_m$	0.68938	0.61558	0.53140	0.42671
SCAT/H	0.94420	0.92774	0.91015	0.88968
K1	0.88719	0.78898	0.67801	0.54145
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.24164	-0.20777	-0.17183	-0.13080
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00077	0.00128	0.00146
L4	4.00000	4.00000	4.00000	4.00000
K5	0.04839	0.03700	0.02630	0.01599
L5	3.38500	3.38500	3.38500	3.38500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.00000	4.00000	4.00000	4.00000
K7	-0.00511	-0.00378	-0.00258	-0.00150
L7	5.00000	5.00000	5.00000	5.00000

TABLE 23

a	4.00000	4.00000	4.00000	4.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.00000	1.00000	1.00000	1.00000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.00600	0.01200	0.01800
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.39025	0.35265	0.31155	0.00005
$n_m\beta_m/q_m$	0.68938	0.64196	0.58682	0.00013
SCAT/H	0.94420	0.92977	0.91440	0.80858
K1	0.88719	0.82338	0.74984	0.00016
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.24164	-0.21836	-0.19291	-0.00003
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00011	0.00020	0.00000
L4	5.00000	5.00000	5.00000	5.00000
K5	0.04839	0.04058	0.03265	0.00000
L5	3.38500	3.38500	3.38500	3.38500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.00000	4.00000	4.00000	4.00000
K7	-0.00511	-0.00417	-0.00326	-0.00000
L7	5.00000	5.00000	5.00000	5.00000

TABLE 24

a	2.00000	2.00000	2.00000	2.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.00000	1.00000	1.00000	1.00000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.00000	0.00000	0.00000
$G/q_m H$	-0.50000	-0.25000	0.25000	0.50000
$H^2\beta_m/D_m$	1.53490	0.61625	0.28940	0.23250
$n_m\beta_m/q_m$	0.72317	0.69213	0.69566	0.70624
SCAT/H	0.71701	0.87097	0.98859	1.01881
K1	0.26407	0.75532	0.95933	1.01200
L1	0.38500	0.38500	0.38500	0.38500
K2	1.24788	0.25051	-0.11764	-0.18902
L2	1.00000	1.00000	1.00000	1.00000
K3	-0.95040	-0.38158	-0.17920	-0.14396
L3	2.00000	2.00000	2.00000	2.00000
K4	0.00000	0.00000	0.00000	0.00000
L4	3.00000	3.00000	3.00000	3.00000
K5	0.05665	0.06505	0.03880	0.03288
L5	3.38500	3.38500	3.38500	3.38500
K6	0.17661	0.01423	-0.00314	-0.00405
L6	4.00000	4.00000	4.00000	4.00000
K7	-0.07902	-0.01274	-0.00281	-0.00181
L7	5.00000	5.00000	5.00000	5.00000

TABLE 25

a	2.00000	2.00000	2.00000	2.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.30000	1.30000	1.30000	1.30000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.10000	0.20000	0.30000
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.59890	0.48025	0.35970	0.22930
$n_m\beta_m/q_m$	0.86004	0.71918	0.56643	0.38695
SCAT/H	0.91442	0.89713	0.87785	0.85373
K1	1.08088	0.90023	0.70573	0.47918
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.31274	-0.25078	-0.18783	-0.11974
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00824	0.01234	0.01180
L4	3.30000	3.30000	3.30000	3.30000
K5	0.09941	0.06639	0.03898	0.01687
L5	3.43500	3.43500	3.43500	3.43500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.35000	4.35000	4.35000	4.35000
K7	-0.00931	-0.00599	-0.00336	-0.00137
L7	5.35000	5.35000	5.35000	5.35000

TABLE 26

a	3.00000	3.00000	3.00000	3.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.30000	1.30000	1.30000	1.30000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.02500	0.05000	0.07500
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.59890	0.52240	0.44475	0.36330
$n_m\beta_m/q_m$	0.86004	0.78942	0.71042	0.61809
SCAT/H	0.91442	0.89564	0.87673	0.85693
K1	1.08088	0.98768	0.88465	0.76570
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.31274	-0.27279	-0.23225	-0.18971
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00111	0.00189	0.00232
L4	4.30000	4.30000	4.30000	4.30000
K5	0.09941	0.07924	0.06042	0.04272
L5	3.43500	3.43500	3.43500	3.43500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.35000	4.35000	4.35000	4.35000
K7	-0.00931	-0.00709	-0.00514	-0.00343
L7	5.35000	5.35000	5.35000	5.35000

TABLE 27

a	4.00000	4.00000	4.00000	4.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.30000	1.30000	1.30000	1.30000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.00600	0.01200	0.01800
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.59890	0.55145	0.50195	0.44920
$n_m\beta_m/q_m$	0.86004	0.82009	0.77535	0.72380
SCAT/H	0.91442	0.89988	0.88515	0.86990
K1	1.08088	1.02733	0.96795	0.90025
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.31274	-0.28796	-0.26211	-0.23457
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00017	0.00031	0.00041
L4	5.30000	5.30000	5.30000	5.30000
K5	0.09941	0.08700	0.07461	0.06210
L5	3.43500	3.43500	3.43500	3.43500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.35000	4.35000	4.35000	4.35000
K7	-0.00931	-0.00790	-0.00654	-0.00524
L7	5.35000	5.35000	5.35000	5.35000

TABLE 28

a	2.00000	2.00000	2.00000	2.00000
b	1.75000	1.75000	1.75000	1.75000
c	1.30000	1.30000	1.30000	1.30000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.00000	0.00000	0.00000
$G/q_m H$	-0.50000	-0.25000	0.25000	0.50000
$H^2\beta_m/D_m$	3.38390	1.00510	0.43680	0.34975
$n_m\beta_m/q_m$	0.84279	0.83664	0.89029	0.92312
SCAT/H	0.66469	0.84712	0.94973	0.97163
K1	0.32731	0.94039	1.16766	1.23839
L1	0.38500	0.38500	0.38500	0.38500
K2	1.84913	0.27462	-0.11934	-0.19112
L2	1.30000	1.30000	1.30000	1.30000
K3	-1.76705	-0.52486	-0.22809	-0.18264
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00000	0.00000	0.00000
L4	3.30000	3.30000	3.30000	3.30000
K5	0.17009	0.14515	0.07833	0.06651
L5	3.43500	3.43500	3.43500	3.43500
K6	0.51742	0.02282	-0.00431	-0.00553
L6	4.35000	4.35000	4.35000	4.35000
K7	-0.29737	-0.02623	-0.00495	-0.00318
L7	5.35000	5.35000	5.35000	5.35000

TABLE 29

a	2.00000	2.00000	2.00000	2.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.10000	0.20000	0.30000
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.45090	0.36975	0.28435	0.18955
$n_m\beta_m/q_m$	0.76037	0.63237	0.49596	0.34118
SCAT/H	0.88209	0.87238	0.86078	0.84524
K1	0.94575	0.78500	0.61414	0.42100
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.23546	-0.19308	-0.14849	-0.09898
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00634	0.00975	0.00975
L4	3.30000	3.30000	3.30000	3.30000
K5	0.05418	0.03688	0.02219	0.01014
L5	3.68500	3.68500	3.68500	3.68500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.00473	-0.00318	-0.00188	-0.00084
L7	5.60000	5.60000	5.60000	5.60000

TABLE 30

a	3.00000	3.00000	3.00000	3.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.02500	0.05000	0.07500
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.45090	0.39910	0.34580	0.28945
$n_m\beta_m/q_m$	0.76037	0.69561	0.62477	0.54462
SCAT/H	0.88209	0.86986	0.85717	0.84355
K1	0.94575	0.86268	0.77241	0.67100
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.23546	-0.20841	-0.18057	-0.15115
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00085	0.00147	0.00185
L4	4.30000	4.30000	4.30000	4.30000
K5	0.05418	0.04374	0.03394	0.02468
L5	3.68500	3.68500	3.68500	3.68500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.00473	-0.00371	-0.00278	-0.00195
L7	5.60000	5.60000	5.60000	5.60000

TABLE 31

a	4.00000	4.00000	4.00000	4.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.00600	0.01200	0.01800
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.45090	0.41910	0.38590	0.35065
$n_m\beta_m/q_m$	0.76037	0.72373	0.68363	0.63881
SCAT/H	0.88209	0.87237	0.86238	0.85195
K1	0.94575	0.89820	0.84646	0.78899
L1	0.38500	0.38500	0.38500	0.38500
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.23546	-0.21885	-0.20151	-0.18311
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00013	0.00024	0.00032
L4	5.30000	5.30000	5.30000	5.30000
K5	0.05418	0.04783	0.04150	0.03515
L5	3.68500	3.68500	3.68500	3.68500
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.00473	-0.00409	-0.00347	-0.00286
L7	5.60000	5.60000	5.60000	5.60000

TABLE 32

a	2.00000	2.00000	2.00000	2.00000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.38500	0.38500	0.38500	0.38500
ρ_m/q_m	0.00000	0.00000	0.00000	0.00000
$G/q_m H$	-0.50000	-0.25000	0.25000	0.50000
$H^2\beta_m/D_m$	1.76400	0.70395	0.33855	0.27520
$n_m\beta_m/q_m$	0.75886	0.74714	0.78096	0.80490
SCAT/H	0.68647	0.82270	0.91635	0.93889
K1	0.53760	0.84701	1.01131	1.06620
L1	0.38500	0.38500	0.38500	0.38500
K2	0.96394	0.19233	-0.09250	-0.15038
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.92115	-0.36760	-0.17679	-0.14371
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00000	0.00000	0.00000	0.00000
L4	3.30000	3.30000	3.30000	3.30000
K5	0.12049	0.07576	0.04350	0.03728
L5	3.68500	3.68500	3.68500	3.68500
K6	0.12225	0.00973	-0.00225	-0.00298
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.07246	-0.01154	-0.00267	-0.00176
L7	5.60000	5.60000	5.60000	5.60000

TABLE 33

	283A	303.8A	335A	368.1A
a	3.73000	3.17000	2.66000	2.59000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.00400	0.01240	0.03500	0.03400
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.63100	0.61370	0.58320	0.58880
$n_m\beta_m/q_m$	0.80235	0.78603	0.75347	0.75819
SCAT/H	0.78944	0.78695	0.78387	0.78500
K1	1.07904	1.05616	1.01133	1.01806
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.35056	-0.34094	-0.32400	-0.32711
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00015	0.00060	0.00222	0.00228
L4	5.03000	4.47000	3.96000	3.89000
K5	0.08253	0.07857	0.07149	0.07266
L5	3.80000	3.80000	3.80000	3.80000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.01009	-0.00954	-0.00862	-0.00878
L7	5.60000	5.60000	5.60000	5.60000

TABLE 34

	554A	584.3A	610A	630A
a	2.76000	3.24000	2.34000	2.45000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.03800	0.01200	0.06300	0.05900
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.57120	0.61275	0.55860	0.55585
$n_m\beta_m/q_m$	0.74284	0.78541	0.72216	0.72193
SCAT/H	0.78157	0.78662	0.78270	0.78150
K1	0.99626	1.05523	0.96903	0.96826
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.31733	-0.34042	-0.31033	-0.30881
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00221	0.00056	0.00479	0.00412
L4	4.06000	4.54000	3.64000	3.75000
K5	0.06898	0.07837	0.06561	0.06524
L5	3.80000	3.80000	3.80000	3.80000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.00827	-0.00951	-0.00790	-0.00783
L7	5.60000	5.60000	5.60000	5.60000

TABLE 35

	770A	790A	833-835A	865-885A
a	2.94000	2.61000	2.61000	2.94000
b	2.00000	2.00000	2.00000	2.00000
c	1.30000	1.30000	1.30000	1.30000
d	0.50000	0.50000	0.50000	0.50000
ρ_m/q_m	0.01600	0.04100	0.01900	0.00900
$G/q_m H$	0.00000	0.00000	0.00000	0.00000
$H^2\beta_m/D_m$	0.61130	0.57500	0.61500	0.62800
$n_m\beta_m/q_m$	0.78284	0.74459	0.78440	0.79831
SCAT/H	0.78709	0.78298	0.78853	0.78979
K1	1.05191	0.99911	1.05452	1.07369
L1	0.50000	0.50000	0.50000	0.50000
K2	0.00000	0.00000	0.00000	0.00000
L2	1.30000	1.30000	1.30000	1.30000
K3	-0.33961	-0.31944	-0.34167	-0.34889
L3	2.30000	2.30000	2.30000	2.30000
K4	0.00089	0.00265	0.00131	0.00051
L4	4.24000	3.91000	3.91000	4.24000
K5	0.07794	0.06964	0.07861	0.08173
L5	3.80000	3.80000	3.80000	3.80000
K6	0.00000	0.00000	0.00000	0.00000
L6	4.60000	4.60000	4.60000	4.60000
K7	-0.00947	-0.00838	-0.00958	-0.00999
L7	5.60000	5.60000	5.60000	5.60000