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Transient Conductivity of Some Semiconductors and Plasmas

by

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ABSTRACT

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The decay function of transient conductivity for some cases of semiconductors and plasmas is numerically computed. The results will be useful in analyzing experimental data on conductivity dispersion and to determine the nature of the dispersion mechanism.

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TRANSIENT CONDUCTIVITY OF SOME SEMICONDUCTORS AND PLASMAS

by

spell out

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I. Introduction

The relaxation property of a conductivity dispersion can be measured either in the frequency domain or in the time domain. We are interested in the lightly doped semiconductors and slightly ionized plasmas where the so-called Maxwellian condition applies. In such a system, physical formulation is well founded and specific mathematical functions can be readily derived[1]. The useful mathematical functions of relaxation in the frequency domain have been quite completely computed by Dingle et al.[2] and applied to specific examples by us [3]. The present paper will treat the mathematical functions related to the conductivity in the time domain. The change of the conductivity in this case is called a decay function or a transient.

We noted that in many analyses of experimental data an exponential decay function is used. The origin of this exponential decay function could be purely conventional, or based on over-simplified physical models. One should emphasize that this function is only a very special case of a physically significant relationship between the relaxation time and the activation energy. As it was emphasized in Ref. 3, from the dispersion data, one can, based on general mathematical theory, determine the energy

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dependence of the relaxation time; in extension, then, one can determine the relaxation mechanism. This principle applies to the dispersion measurement in the frequency domain as well as to the transient measurement in the time domain.

In some respects, one loses some preciseness in the measurement of the time domain. Basically, in the frequency domain, one can simultaneously determine two quantities, the so-called in-phase, and the out-of-phase component from a single measurement, and the two components are connected by a fundamental physical principle: the Kronig-Kramers' relation [4]. Therefore, it becomes possible to test the consistency of the measurement itself, even if one has no knowledge of the physical principle involved, which one has to utilize to make the analysis. On the other hand, the experimental arrangement in the time domain is considerably simplified. In fact, under some circumstances, the measurement in the time domain could be a sole source, when the sine-wave source of stimulation is not available. A physical problem which illustrates the situation is the measurement of the relaxation time of recombination of electrons and holes ionized in the semiconductors by the radiation: with one exception known to us [5], all measurements were carried in the time domain.

In the following, first, the transient conductivity integral will be presented, then numerical results from the computer will be given. Some connections between the conductivity integral with special functions are also discussed.

2. Transient Conductivity Integral

We repeat some essential steps leading to the mathematical function of interest given by Ref. 3. The frequency (ω) dependence of the complex conductivity, $\sigma^*(\omega)$, is given by

$$\sigma^*(\omega) = \frac{4\, e^2\, n}{3\sqrt{\pi\, m}} \int_0^\infty \frac{\tau\, \varepsilon^{\frac{2}{3}}\, e^{-\varepsilon}}{1 + i\,\omega\,\tau}\, d\,\varepsilon, \qquad (1)$$

where e, n, and m are effective charge, density and effective mass respectively of the electron. The relaxation time, or the reciprocal collision frequency, τ, is assumed to depend on the reduced energy parameter ε (= $mv^2/2\,kT$, with $mv^2/2$ as the kinetic energy of the electron) as,

$$\tau = a_p\, \varepsilon^{-p} \qquad (2)$$

References to the physical correspondence of the various numerical values of p were given in Ref. 3, they range from p = -.5/2 to + 3/2 in each half integer step.

The decay function A (t) can be obtained from the Fourier transform of $\sigma^*(\omega)$ [3], with substituting Eq. (2) into Eq. (1). But mathematically, in the present problem a great simplification is gained by calculating the Laplace transform of $g(\tau)$ [6], the distribution function of relaxation times. From Eqs. (1) and (2) [7].

$$g(\tau) = \frac{1}{|p|} \left(\frac{a_p}{\tau}\right)^{5/2p} \exp\left[-\left(\frac{a_p}{\tau}\right)^{1/p}\right] \tag{3}$$

In order to normalize A (t), from (3),

$$\int_0^\infty g(\tau)\, d\tau = a_p\, \Gamma(5/2 - p). \tag{4}$$

Set $t/a_p = \theta$, we have the normalized decay function,

$$A(\theta) = \frac{1}{\Gamma(5/2-p)} \int_0^\infty \varepsilon^{3/2-p} \exp\left[-\varepsilon - \theta\, \varepsilon^p\right] d\varepsilon. \tag{5}$$

Some series expansion can be made: for small θ, expanding $\exp(-\theta\varepsilon^p)$ into the power series and we have,

$$A(\theta) = \sum_{n=0}^{\infty} (-1)^n \frac{\Gamma\left[5/2 + (n-1)p\right]}{\Gamma(5/2-p)} \frac{\theta^n}{\Gamma(n+1)} \tag{6}$$

For large θ, we integrate with a variable $\exp(-\theta\varepsilon^p) = \xi$ and we obtain,

$$A(\theta) = \frac{1}{|p|} \sum_{n=0}^{\infty} (-1)^n \frac{\Gamma\left(\frac{2n+5}{2p} - 1\right)}{\Gamma(n+1)} \theta^{-\left[\frac{5+2n}{2p} + 1\right]}. \tag{7}$$

Series (6) can be applied only when $o<p<1$ or p is a negative integer such that the value of the coefficients of θ would not fall into the Riemann holes. In fact, when p is a negative integer, Γ - function approaches zero quite rapidly [8] and this has been utilized well in the numerical computation.

Similarly, series (7) can be applied only when $p > 1$, or p is a negative half integer. The argument is similar to that of series (6).

For some special value of p, closed form of $A(\theta)$ can be obtained. Since this has some analytical interest, we will list them in Section 3.

The numerical results for all $A_p(\theta)$ are given in Table 1, except for the case $p = o$, which is just an ordinary exponential function. For qualitative comparison, some results are also plotted in Figure 1.

3. Special Cases

We list below some special solutions of the transient conductivity integral.

(a) $p = o$. This is the simplest decay function,

$$A = e^{-\theta} \tag{8}$$

(b) $p = 1$.

$$A = (1 + \theta)^{-3/2} \tag{9}$$

In this special case, the inverse Fourier transform of $A(\theta)$ can be obtained immediately, recall $\theta = t/a_1$, and writing a_1 simply as a,

$$\sigma^* (\omega) = \int_0^\infty (1 + t/a)^{-3/2} \quad e^{-i\omega t} \quad dt$$

$$= 2/a \left[1 - 2\pi \sqrt{\frac{i\omega}{a}} \quad e^{i\omega a} \quad \mathrm{Erfc} \sqrt{i\omega a} \right] \qquad (10)$$

The numerical value of Erfc with complex argument can be found in Table [9]. Therefore, Eq. (10) provides a link to the special function of $G_{1.5}$ and $G_{2.5}$ of Dingle <u>et al.</u>[3].

(c) $p = 2$.

$$A = \sqrt{2}\,(8\,\theta)^{-1/4} \quad e^{1/8\,\theta} \quad D_{-1/2} \left[2\,(8\,\theta)^{-1/2} \right] \qquad (11)$$

where D is the parabolic cylindrical function [10].

(d) $p = 1/2$,

$$A = 3\, e^{\theta^2/8} \quad D_{-4}\,(\theta/\sqrt{2}). \qquad (12)$$

4. Discussion

In the plot of log A (θ) versus log (θ), as shown in Fig. 1, we observe that appreciable differences of log A (θ) for different values of p emerge only after $\theta > 10^{-1}$, and large differences are observed after $\theta > 1$. Beyond this region, it is especially interesting to observe that the decay curve with p = o (i.e., exponential decay) has the steepest decay. For any other value of p, irrespective of positive or negative, the decay rate is slower.

Recalling θ is a "normalized" time parameter, it would take about three logarithmic decades of the time parameter in order to determine the value of p from the experimental data. But it seems sufficient to measure just two decades in order to differentiate the exponential decay from other types of decays.

The author would like to express his appreciation to Mr. E. Monasterski for his help in the numerical computation.

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4. Krohig, R. J. J. Opt. Soc. Am. 12 (1926) 547, Kramers, H. A. Atti. Cong. Intern. Fisici, Como 2 (1927) 545.

5. Schultz, B. H., Philips Res. Rep. 12 (1957) 82, van der Pauw, L. J. ibid. 12 (1957) 364. In these experiments, the frequency range covered is restricted by the means of producing sine-wave light sources to 4×10^4 c/s. Recent progress in producing Pockel cells made it possible to extend the frequency to 10^{10} c/s and we are carrying out, at present, experiments in these extended frequencies.

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7. We would like to take this opportunity to correct some errors in Ref. 3.

   (a) the exponent of $(\frac{a_p}{\tau})$ was misprinted as $\frac{5}{2}$ p, it should be 5/2p.

(b) the argument to establish Eq. (12) was wrong; this effects, and only effects Eq. (17). Eq. (17) should be

$$< \tau_g > = \int \tau_g (\tau) \quad d\tau$$

$$= \frac{4}{3\sqrt{\pi}} \quad a_p \; \Gamma \left(\frac{5}{2} - 2p\right)$$

Therefore, the pathological case of p will be $p \geq 5/4$.

(c) in Table 2, $\omega^2 \tau^2$ should be $\omega_1 \tau_1$.

8. See formula for (-n - 0.5) ! in Jahnke, E. and F. Ende, Tables of Functions (Dover Publications, N.Y. 1945) p. 11; also, Davis, H. T., Table for High Math. Func. (Princiton Press, Indiana 1933).

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Fig. 1. Transient decay spectra for different values of p (p = o corresponds to exponential decay).

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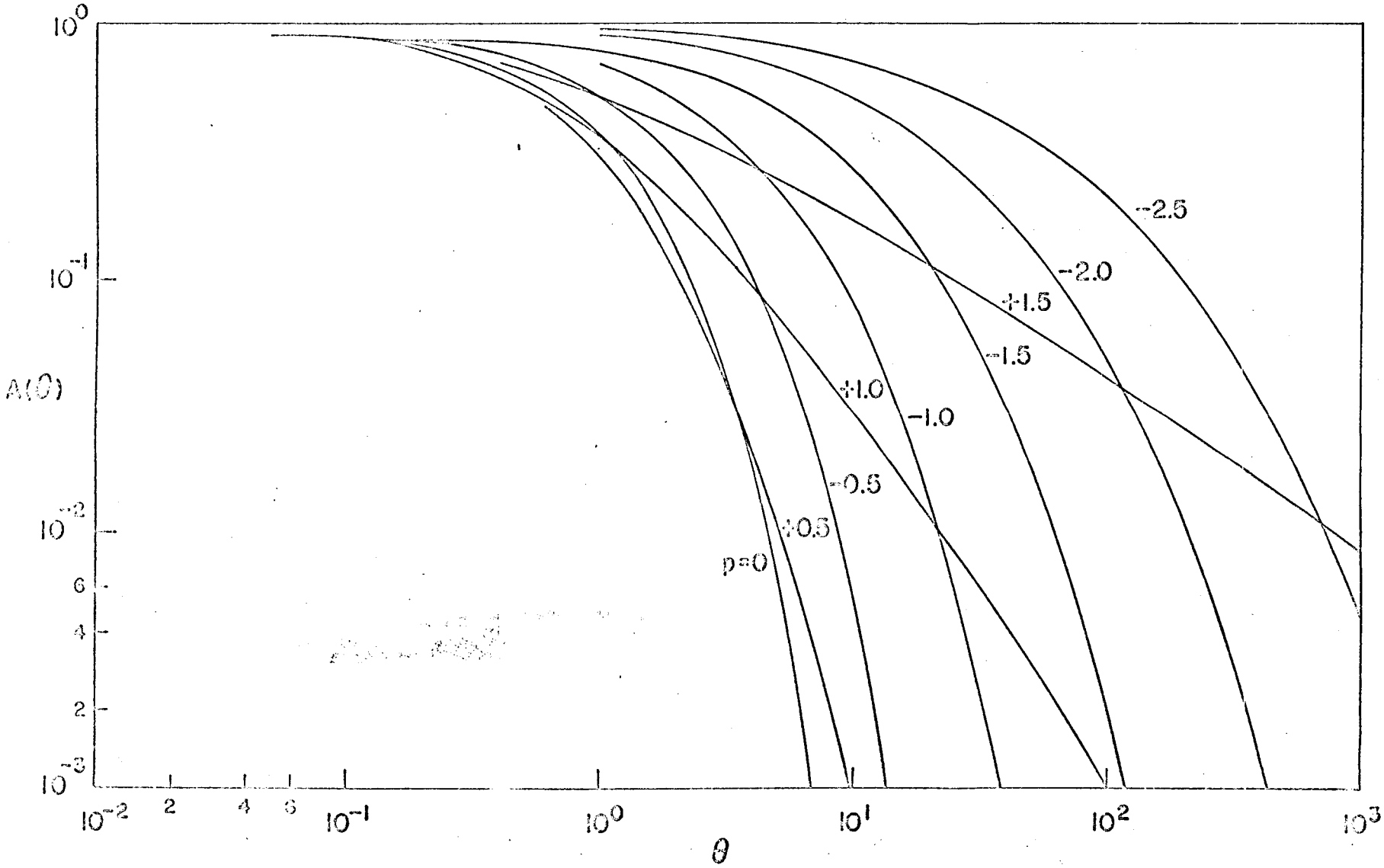
Table 1. Numerical value of A_p (θ) for different p.

Table 2. Continuation of Table 1.


10^0
10^{-1}
10^{-2}
6
4
2
10^{-3}
A(0)
10^{-2}
2
4
6
10^{-1}
10^0
10^1
10^2
10^3
θ
p=0
+0.5
+0.5
+1.0
-1.0
-1.5
+1.5
-2.0
-2.5

λ_p

	p=			
	-0.250E 01	-0.200E 01	-0.150E 01	-0.100E 01
1.00E-05	1.000E-01	1.000E-01	9.999E-01	9.999E-01
1.00E-04	9.999E-01	9.999E-01	9.997E-01	9.996E-01
1.00E-03	9.994E-01	9.988E-01	9.978E-01	9.960E-01
3.00E-02	9.983E-01	9.966E-01	9.934E-01	9.881E-01
5.00E-02	9.973E-01	9.944E-01	9.891E-01	9.803E-01
7.00E-02	9.962E-01	9.922E-01	9.848E-01	9.726E-01
1.00E-01	9.946E-01	9.889E-01	9.785E-01	9.612E-01
2.00E-01	9.896E-01	9.785E-01	9.582E-01	9.248E-01
3.00E-01	9.847E-01	9.684E-01	9.389E-01	8.905E-01
4.00E-01	9.800E-01	9.583E-01	9.204E-01	8.580E-01
5.00E-01	9.754E-01	9.494E-01	9.027E-01	8.272E-01
6.00E-01	9.709E-01	9.404E-01	8.857E-01	7.980E-01
7.00E-01	9.666E-01	9.316E-01	8.694E-01	7.703E-01
8.00E-01	9.623E-01	9.231E-01	8.536E-01	7.439E-01
9.00E-01	9.581E-01	9.147E-01	8.383E-01	7.187E-01
1.00E 00	9.540E-01	9.066E-01	8.236E-01	6.947E-01
1.20E 00	9.460E-01	8.910E-01	7.955E-01	6.499E-01
1.40E 00	9.383E-01	8.760E-01	7.691E-01	6.088E-01
1.60E 00	9.309E-01	8.617E-01	7.442E-01	5.712E-01
1.80E 00	9.237E-01	8.480E-01	7.207E-01	5.365E-01
2.00E 00	9.166E-01	8.347E-01	6.984E-01	5.046E-01
2.50E 00	8.999E-01	8.036E-01	6.474E-01	4.347E-01
3.00E 00	8.841E-01	7.750E-01	6.022E-01	3.767E-01
4.00E 00	8.550E-01	7.238E-01	5.253E-01	2.869E-01
5.00E 00	8.285E-01	6.790E-01	4.624E-01	2.220E-01
6.00E 00	8.042E-01	6.392E-01	4.099E-01	1.740E-01
7.00E 00	7.817E-01	6.036E-01	3.655E-01	1.378E-01
8.00E 00	7.608E-01	5.715E-01	3.276E-01	1.101E-01
9.00E 00	7.411E-01	5.422E-01	2.950E-01	8.874E-02
1.00E 01	7.226E-01	5.155E-01	2.666E-01	7.201E-02
1.20E 01	6.887E-01	4.682E-01	2.200E-01	4.830E-02
1.40E 01	6.581E-01	4.277E-01	1.836E-01	3.308E-02
1.60E 01	6.303E-01	3.926E-01	1.546E-01	2.306E-02
1.80E 01	6.049E-01	3.618E-01	1.313E-01	1.631E-02
2.00E 01	5.814E-01	3.346E-01	1.123E-01	1.170E-02
3.00E 01	4.866E-01	2.357E-01	5.559E-02	2.580E-03
4.00E 01	4.170E-01	1.742E-01	3.026E-02	6.824E-04
5.00E 01	3.632E-01	1.331E-01	1.756E-02	2.046E-04
6.00E 01	3.201E-01	1.041E-01	1.069E-02	6.740E-05
7.00E 01	2.849E-01	8.305E-02	6.757E-03	2.390E-05
8.00E 01	2.555E-01	6.725E-02	4.400E-03	8.998E-06
9.00E 01	2.307E-01	5.515E-02	2.938E-03	3.563E-06
1.00E 02	2.093E-01	4.572E-02	2.003E-03	1.472E-06
1.50E 02	1.371E-01	2.017E-02	3.736E-04	2.866E-08
2.00E 02	9.616E-02	1.018E-02	9.082E-05	9.667E-10
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3.00E 02	5.353E-02	3.289E-03	8.584E-06	2.968E-12
3.50E 02	4.161E-02	2.023E-03	3.086E-06	2.301E-13
4.00E 02	3.299E-02	1.292E-03	1.196E-06	2.102E-14
5.00E 02	2.170E-02	5.747E-04	2.140E-07	2.575E-16
7.50E 02	9.111E-03	1.068E-04	5.759E-09	1.993E-20
1.00E 03	4.488E-03	2.694E-05	2.876E-10	0.

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10.00E-05	9.999E-01	9.998E-01	9.998E-01	9.990E-01
10.00E-04	9.993E-01	9.983E-01	9.903E-01	9.970E-01
10.00E-03	9.933E-01	9.868E-01	9.851E-01	9.861E-01
3.00E-02	9.802E-01	9.610E-01	9.566E-01	9.617E-01
5.00E-02	9.673E-01	9.359E-01	9.294E-01	9.392E-01
7.00E-02	9.546E-01	9.116E-01	9.035E-01	9.184E-01
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4.00E-01	7.699E-01	5.983E-01	6.037E-01	7.014E-01
5.00E-01	7.220E-01	5.291E-01	5.443E-01	6.608E-01
6.00E-01	6.775E-01	4.689E-01	4.941E-01	6.263E-01
7.00E-01	6.360E-01	4.164E-01	4.511E-01	5.964E-01
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1.60E 00	3.663E-01	1.558E-01	2.385E-01	4.356E-01
1.80E 00	3.253E-01	1.277E-01	2.134E-01	4.138E-01
2.00E 00	2.892E-01	1.053E-01	1.924E-01	3.946E-01
2.50E 00	2.165E-01	6.676E-02	1.527E-01	3.556E-01
3.00E 00	1.632E-01	4.378E-02	1.250E-01	3.253E-01
4.00E 00	9.425E-02	2.055E-02	8.943E-02	2.810E-01
5.00E 00	5.548E-02	1.064E-02	6.803E-02	2.496E-01
6.00E 00	3.319E-02	5.955E-03	5.398E-02	2.260E-01
7.00E 00	2.015E-02	3.553E-03	4.418E-02	2.074E-01
8.00E 00	1.238E-02	2.235E-03	3.703E-02	1.923E-01
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1.60E 01	3.508E-04	1.698E-04	1.426E-02	1.278E-01
1.80E 01	1.542E-04	1.076E-04	1.206E-02	1.189E-01
2.00E 01	6.920E-05	7.136E-05	1.038E-02	1.115E-01
3.00E 01	1.622E-06	1.440E-05	5.782E-03	8.663E-02
4.00E 01	5.200E-08	4.529E-06	3.797E-03	7.220E-02
5.00E 01	2.088E-09	1.806E-06	2.734E-03	6.258E-02
6.00E 01	9.975E-11	8.287E-07	2.087E-03	5.562E-02
7.00E 01	5.483E-12	4.154E-07	1.660E-03	5.032E-02
8.00E 01	3.387E-13	2.206E-07	1.360E-03	4.611E-02
9.00E 01	2.311E-14	1.217E-07	1.140E-03	4.268E-02
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1.00E 03	0.	1.603E-27	1.648E-05	8.130E-03